Skewness, kurtosis, 5th and 6th order cumulants of conserved charge fluctuations and the comparison to STAR data

UNIVERSITÄT BIELEFELD
Faculty of Physics

Christian Schmidt
HotQCD Collaboration

CRC-TR 211
Strong-interaction matter under extreme conditions

HotQCD Collaboration

A. Bazavov, D. Bollweg, H.-T. Ding, P. Enns, J. Goswami, P. Hegde, O. Kaczmarek, F. Karsch, A. Lahiri, S.-T. Li, Swagato Mukherjee, H. Ohno, P. Petreczky, C. Schmidt, S. Sharma, P. Steinbrecher,

➡See also QM'19 talk by D. Bollweg

Workshop on QCD in the Nonperturbative Regime,
18-20 Nov 2019, TIFR, Mumbai, India

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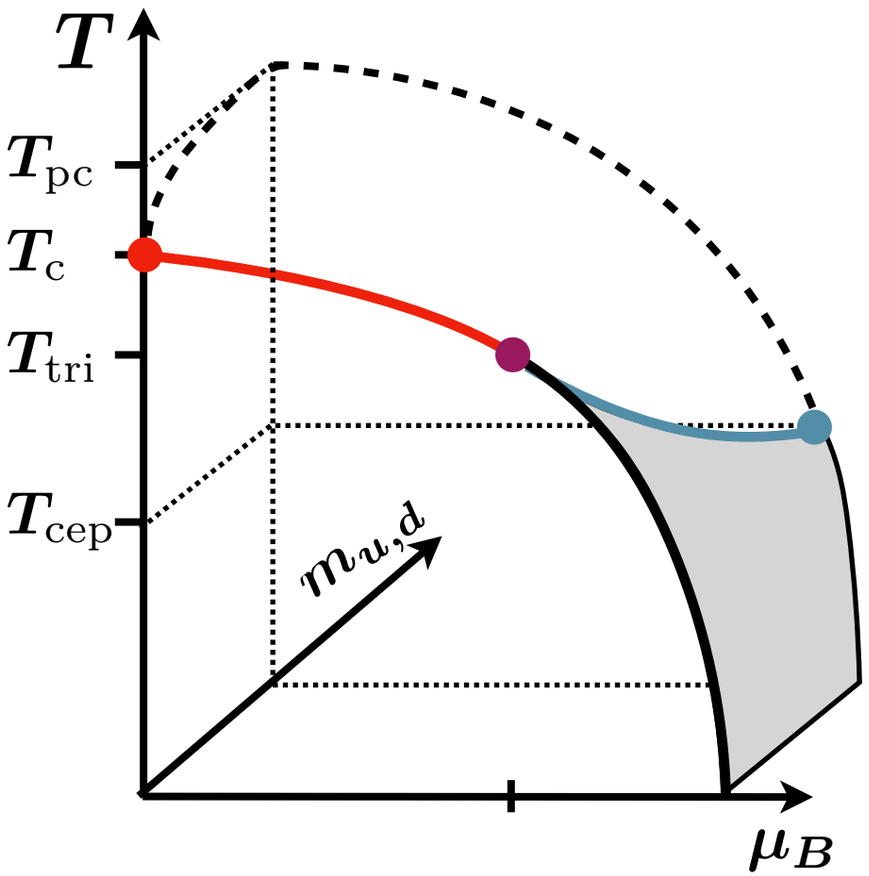
Motivation: The QCD Phase diagram



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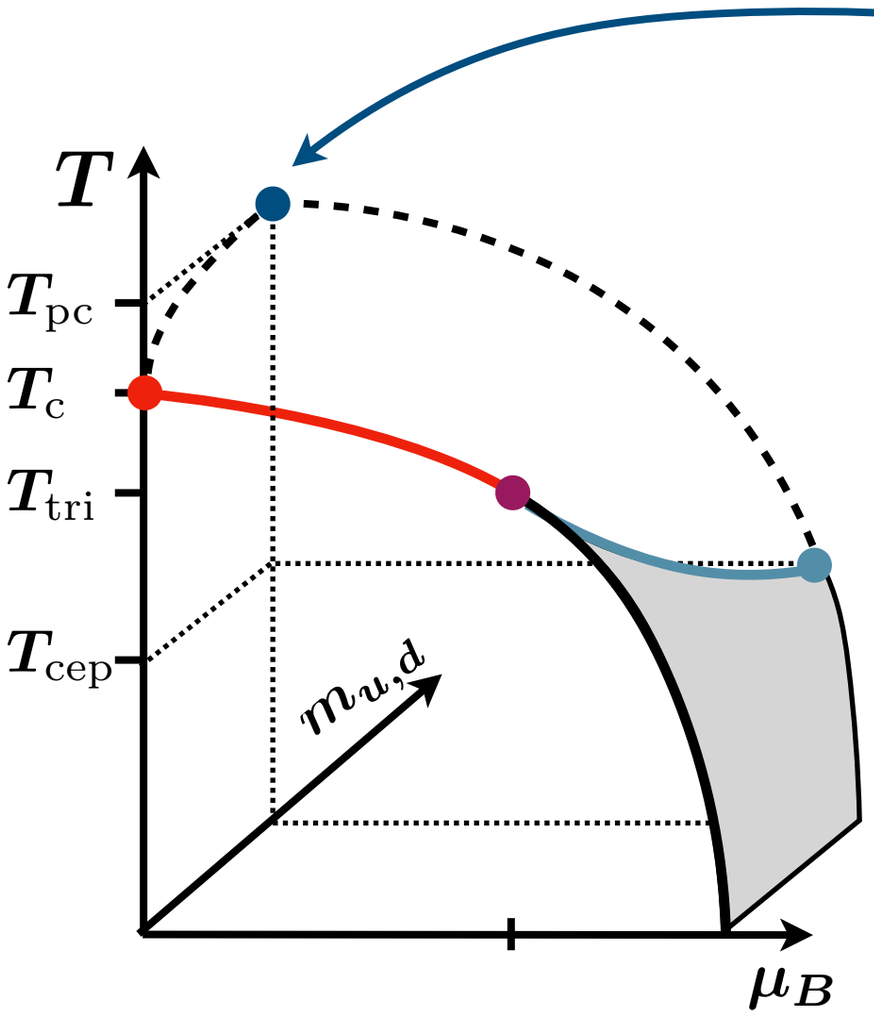

- New precise determination of the QCD transition temperature $T_{\rm pc} = \mathbf{156.5 \pm 1.5}\ \mathrm{MeV}$

HotQCD: PLB 795 (2019) 15

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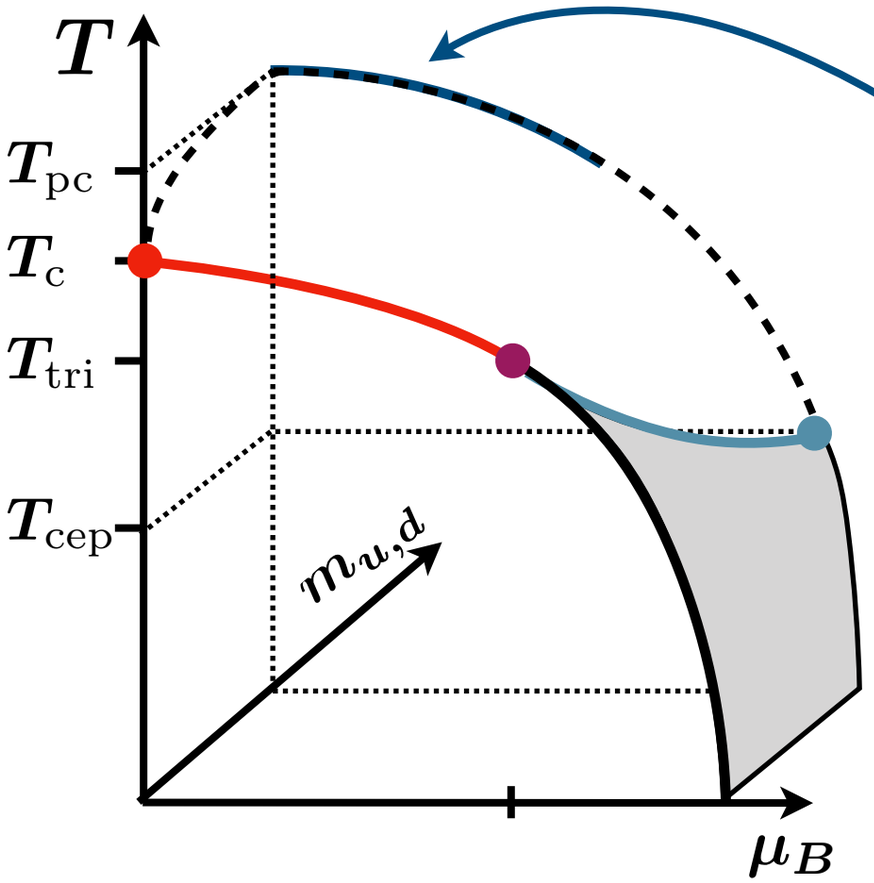

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HotQCD: PLB 795 (2019) 15

- The chiral crossover line with respect to μ_B

$$T_{\rm pc}(\mu_B) = T^0_{\rm pc} \left(1 - \kappa_2^{B,f} \left(\frac{\mu_B}{T^0_{\rm pc}} \right)^2 - \kappa_4^{B,f} \left(\frac{\mu_B}{T^0_{\rm pc}} \right)^4 \right)$$

$$\kappa_2^{B,f} = 0.012(4), \quad \kappa_4^{B,f} = 0.00(4)$$

HotQCD: PLB 795 (2019) 15

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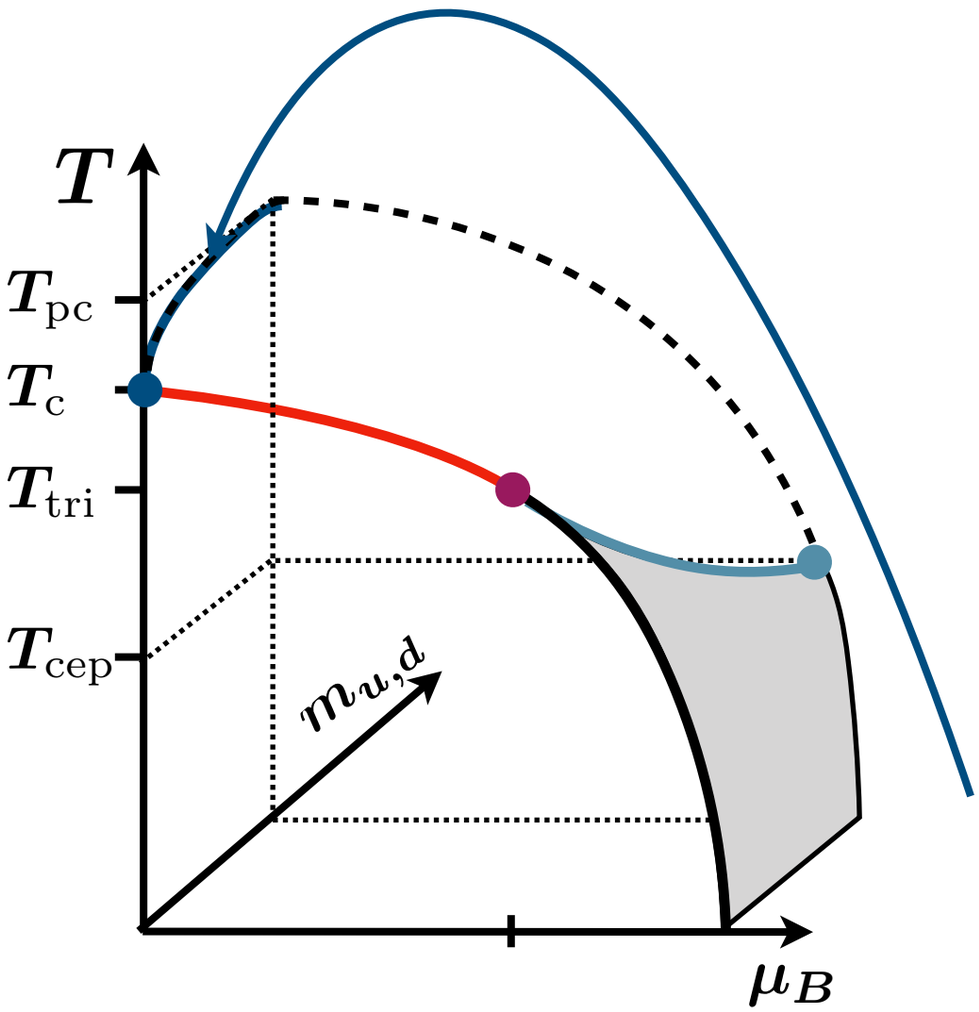

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HotQCD: PLB 795 (2019) 15

- The chiral phase transition temperature and pseudo-critical line $T_{\rm c} = 132^{+3}_{-6}$ MeV

HotQCD: PRL 123 (2019) 062002

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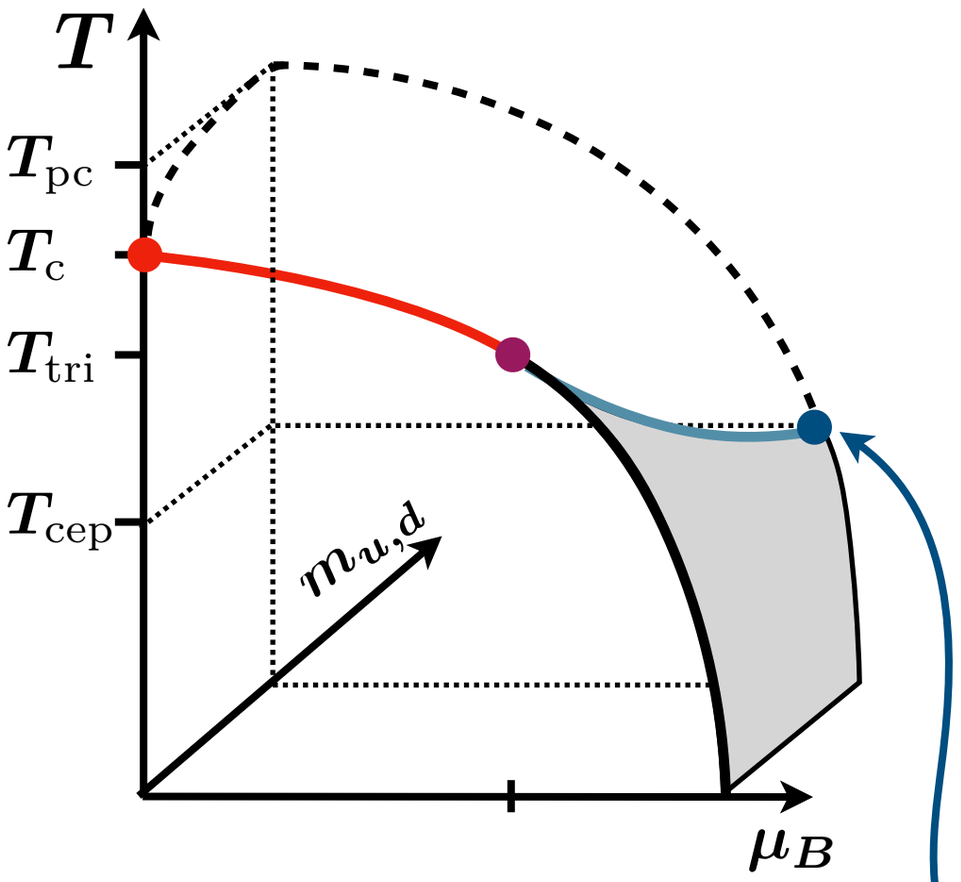

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HotQCD: PLB 795 (2019) 15

- The chiral phase transition temperature and pseudo-critical line $T_{\rm c} = 132^{+3}_{-6}$ MeV

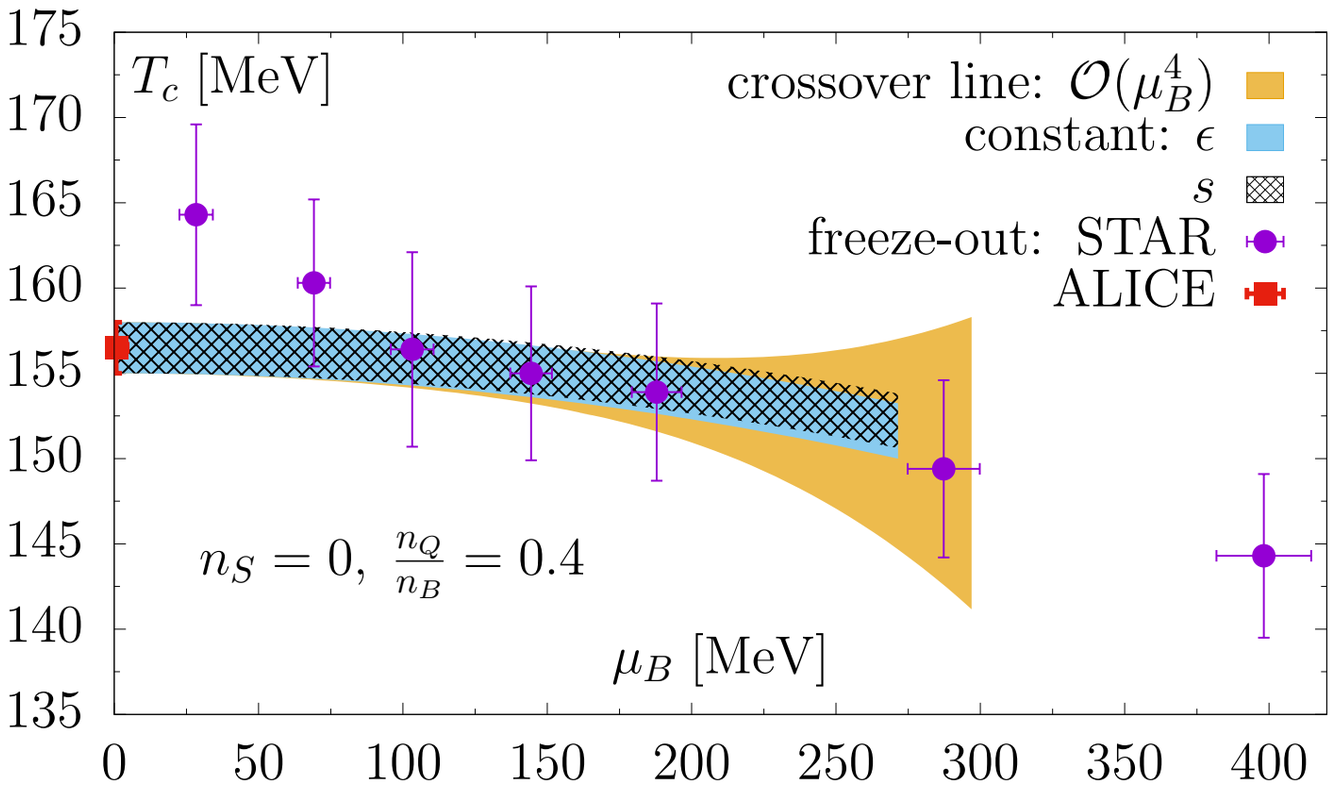
HotQCD: PRL 123 (2019) 062002

- Expected bounds on the QCD critical end-point

$$T_{\rm cep} < T_{\rm c} = 132^{+3}_{-6} \text{ MeV}$$

$$\mu_B^{\rm cep} > (300 - 400) \text{ MeV}$$

Motivation: Freeze-out data vs. lattice QCD



- Established Method to extract freeze-out parameter from HIC experiments uses hadronic yields and fits to a statistical (HRG) model.
  - ➡ Find good agreement with the chiral crossover line from QCD
  - ➡ Model dependent analysis
  - ➡ No phase transition in the HRG

- Higher oder cumulants of conserved charge fluctuations are the ideal probes to study the freeze-out and chiral crossover line.
  - ➡ Can be calculated in (lattice) QCD
  - ➡ Show well defined divergencies close to the critical end-point and extrema at the chiral crossover

Goal: Provide QCD predictions for cumulant ratios ($M/\sigma^2,\ S\sigma,\ \kappa\sigma^2,\ \ldots$) of conserved charge fluctuations as net baryon-number (B), net electric charge (Q), net strangeness (S).

Plan

- **Methodology**
  - Definition of cumulant ratios
  - Formulation of observables on the lattice
  - Recent improvements
- **Results: A comparison to HRG**
  - Using HRG to estimate difference of net-baryon and net-proton number fluctuations (only M/σ^2)
- **Results: A comparison to STAR**
  - Thermodynamic consistency of the new $\sqrt{s_{NN}} = 54.4$ GeV proton-number fluctuations
- **Summary and Outlook**

Methodology: The Taylor expansion method

- Compute expansion coefficients of the pressure

$$\frac{p}{T^4} = \frac{\ln Z}{T^3 V} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi^{BQS}_{ijk,0} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Gavai, Gupta (2001)
Bielefeld-Swansea (2002)

Cumulants of conserved charge fluctuations, can also be measured as event-by-event fluctuations in heavy ion collisions

$$\chi^{BQS}_{ijk,0} = \frac{\partial^i}{\partial(\mu_B/T)} \frac{\partial^j}{\partial(\mu_Q/T)} \frac{\partial^k}{\partial(\mu_S/T)} \frac{\ln Z}{T^3 V}$$

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- **Notation:** relevant operators are derivatives of the determinant

$$D^f_i = \frac{\partial^i}{\partial \mu_f^i} \det\left[M_f(\mu_f)\right]^{1/4} = \frac{\partial^i}{\partial \mu_f^i} e^{\frac{1}{4}\mathrm{Tr}\ln M_f(\mu_f)}, \quad f \in \{u,d,s\}$$

up to 4th-order in μ:

exponential dependence:

$$D^f_1 = \mathrm{Tr}\left[M_f^{-1} M_f^{(1)}\right]$$

$$D^f_2 = -\mathrm{Tr}\left[M_f^{-1} M_f^{(1)} M_f^{-1} M_f^{(1)}\right] + \mathrm{Tr}\left[M_f^{-1} M_f^{(2)}\right]$$

$$\vdots$$

from 6th-order in μ onwards:

linear dependence:

$$D^f_1 = \mathrm{Tr}\left[M_f^{-1} M_f^{(1)}\right]$$

$$D^f_2 = -\mathrm{Tr}\left[M_f^{-1} M_f^{(1)} M_f^{-1} M_f^{(1)}\right]$$

all $M_f^{(k)} = 0$, for k>1

⇒ much less operators to measure!

Gavai, Sharma 2015

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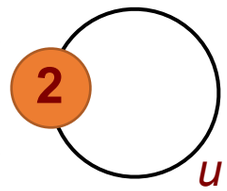
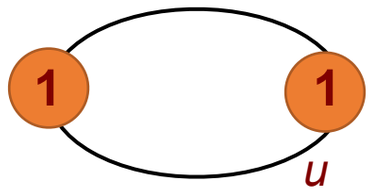
Gavai, Gupta (2001)
Bielefeld-Swansea (2002)

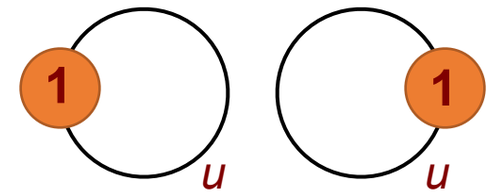
- **Interpretation:**

$$\chi^{uds}_{200} = \frac{\partial^2}{\partial(\mu_u/T)^2}\left(\frac{\ln Z}{T^3 V}\right) = \left\langle D^u_2 \right\rangle + \left\langle \left(D^u_1\right)^2 \right\rangle$$

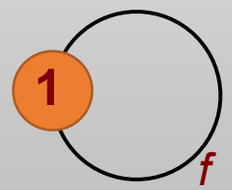
$$= \left\langle \mathrm{Tr}\left[M_u^{-1} M_u^{(2)}\right] \right\rangle - \left\langle \mathrm{Tr}\left[M_u^{-1} M_u^{(1)} M_u^{-1} M_u^{(1)}\right] \right\rangle + \left\langle \mathrm{Tr}\left[M_u^{-1} M_u^{(1)}\right] \mathrm{Tr}\left[M_u^{-1} M_u^{(1)}\right] \right\rangle$$

$$= \left\langle \right.$$ 

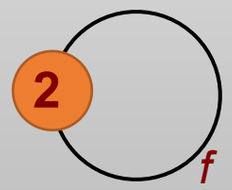
$$\left. \right\rangle - \left\langle \right.$$ 

$$\left. \right\rangle + \left\langle \right.$$ 

$$\left. \right\rangle$$



$= \left[M_f^{-1} M_f^{(1)}\right]_{xx}$
f ≙ local f-quark density



$= \left[M_f^{-1} M_f^{(2)}\right]_{xx}$
f ≙ un-physical contact term

Methodology: The Taylor expansion method

- **Random noise method:**
  - Choose a number of random vectors $\eta^{(k)}$ with
  - The trace of a matrix A is approximated as

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k}^{N} \eta_i^{(k)} \cdot \eta_j^{(k)} = \delta_{ij}$$

$$\mathrm{Tr}A \approx \frac{1}{N} \sum_{k}^{N} \eta^{(k)\dagger} A\, \eta^{(k)}$$

$$\mathrm{std}\left[\mathrm{Tr}A\right] \sim \frac{1}{\sqrt{N}}$$

⇒ A:=M $^{-1}$: matrix inversion can be reduced to a linear problem Mx= η.

- **Unbiased estimators:**
  - Need unbiased estimators for powers of traces: (TrA)^{m}

$$(\mathrm{Tr}A)^m \approx \frac{1}{\mathcal{N}} \sum_{k_1\neq k_2,\cdots\neq k_m} \left(\eta^{(k_1)\dagger} A\, \eta^{(k_1)}\right) \cdot \left(\eta^{(k_2)\dagger} A\, \eta^{(k_2)}\right) \cdots \left(\eta^{(k_m)\dagger} A\, \eta^{(k_m)}\right)$$

⇒ need at least m random vectors, more might be necessary to improve precision

(signal to noise ratio can be large)

⇒ have developed efficient recursive method to calculate unbiased estimators

Methodology: Cumulant ratios of conserved charge fluctuations

- Combine quark number fluctuations (χ^{uds}_{ijk}) to obtain hadronic fluctuations (χ^{BQS}_{ijk}).
- Determine strangeness (μ_S/T) and electric charge chemical (μ_Q/T) potentials by imposing strangeness neutrality $n_S = 0$ and $n_Q/n_B = 0.4$ (order by order in the expansion).
- From the pressure expansion we readily obtain the expansions for the nth-order cumulants:

$$\chi^B_n(T,\mu_B) = \sum_{k=0}^{k_{\rm max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B, \quad \text{with} \quad \hat{\mu}_B = \mu_B/T$$

- Define ratios to eliminate the leading order volume dependence

$$R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,mu_B)} = \frac{\sum_{k=0}^{k_{\rm max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=0}^{l_{\rm max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

- In terms of the shape parameters of the distribution we find

$$R_{12} = M/\sigma^2,\ R_{31} = S\sigma^3/M,\ R_{32} = S\sigma,\ R_{42} = \kappa\sigma^2,\ \ldots$$

- Eventually we want calculate observables along the crossover (and freeze-out) line, we thus need spline interpolations of our data at discrete temperature values.

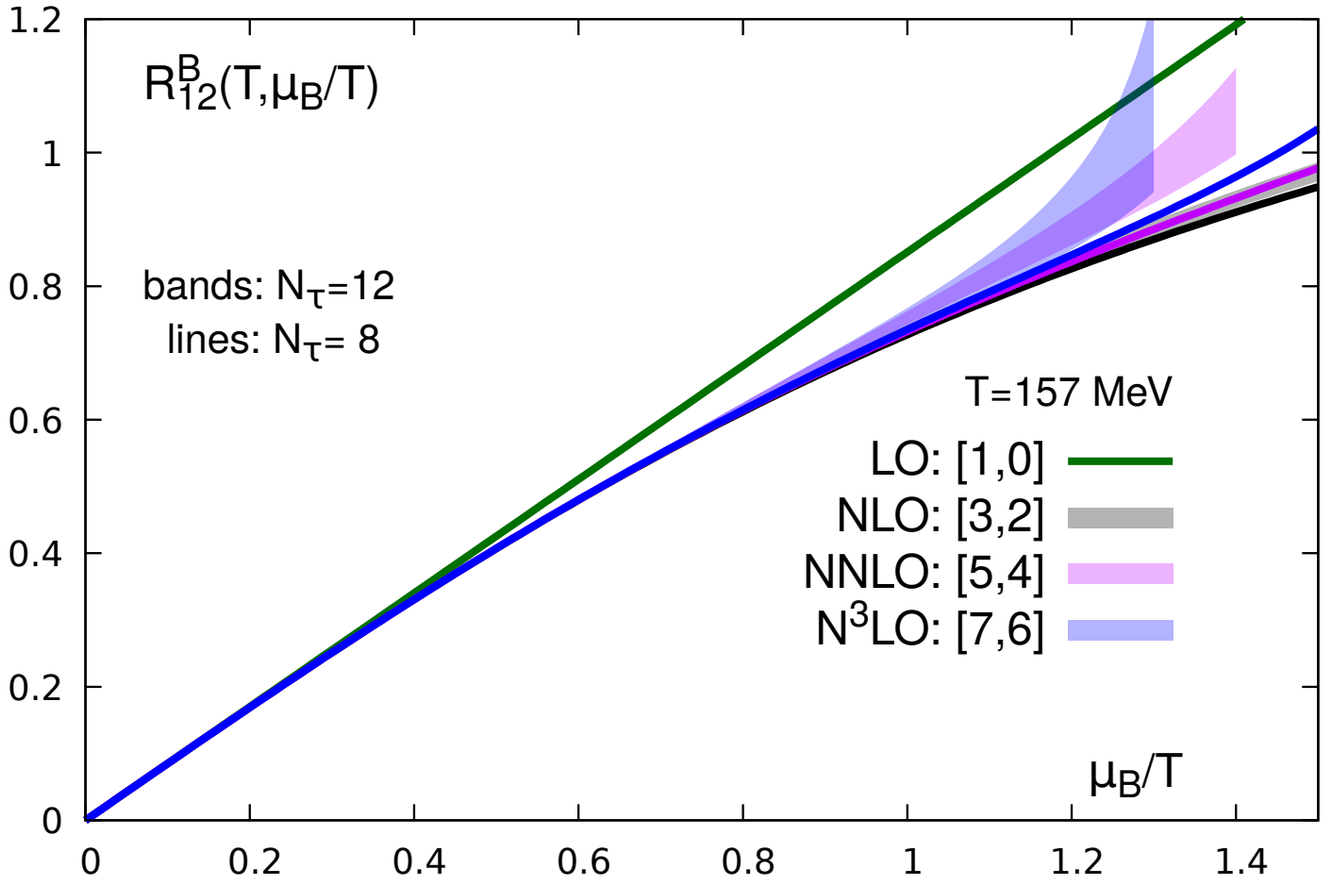
Methodology: Lattice setup and statistics

- Use (2+1)-flavor of HISQ-fermions, with physical strange and light quark masses.
- **Lattices sizes** are $32^3 \times 8,\ 48^3 \times 12,\ 64^3 \times 16$, at 9 different temperature values.
- **Statistics:** Compared to our previous analysis of skewness and kurtosis [HotQCD, PRD 96 (2017) 074510] we increased the statistics on ($N_\tau = 8$)-lattices by a factor 3-4 and on ($N_\tau = 12$)-lattices by a factor 6-8. I.e. we have now

N_τ	8	12	16
#conf.	$1.2 \cdot 10^6$	$(2-3) \cdot 10^5$	10^4

- **Order of the expansion:** We can now go to N^{3}LO, compared to NLO in our previous study. I.e., we include 8-th order expansion coefficients of the pressure.
- This years calculations were performed on Summit, using Nvidia's V100 GPU's.

Results: The net baryon-number density R_{12}^B



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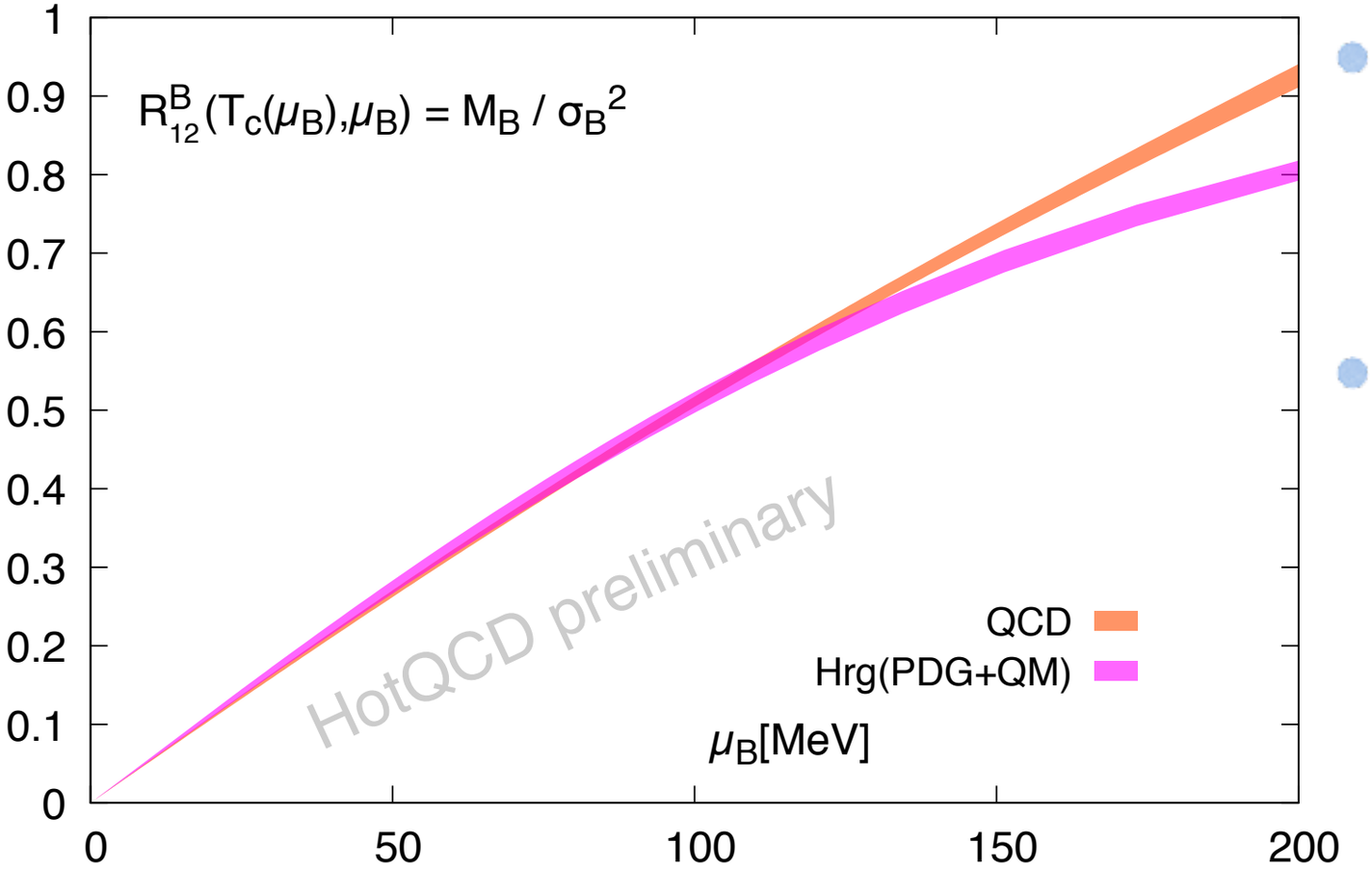

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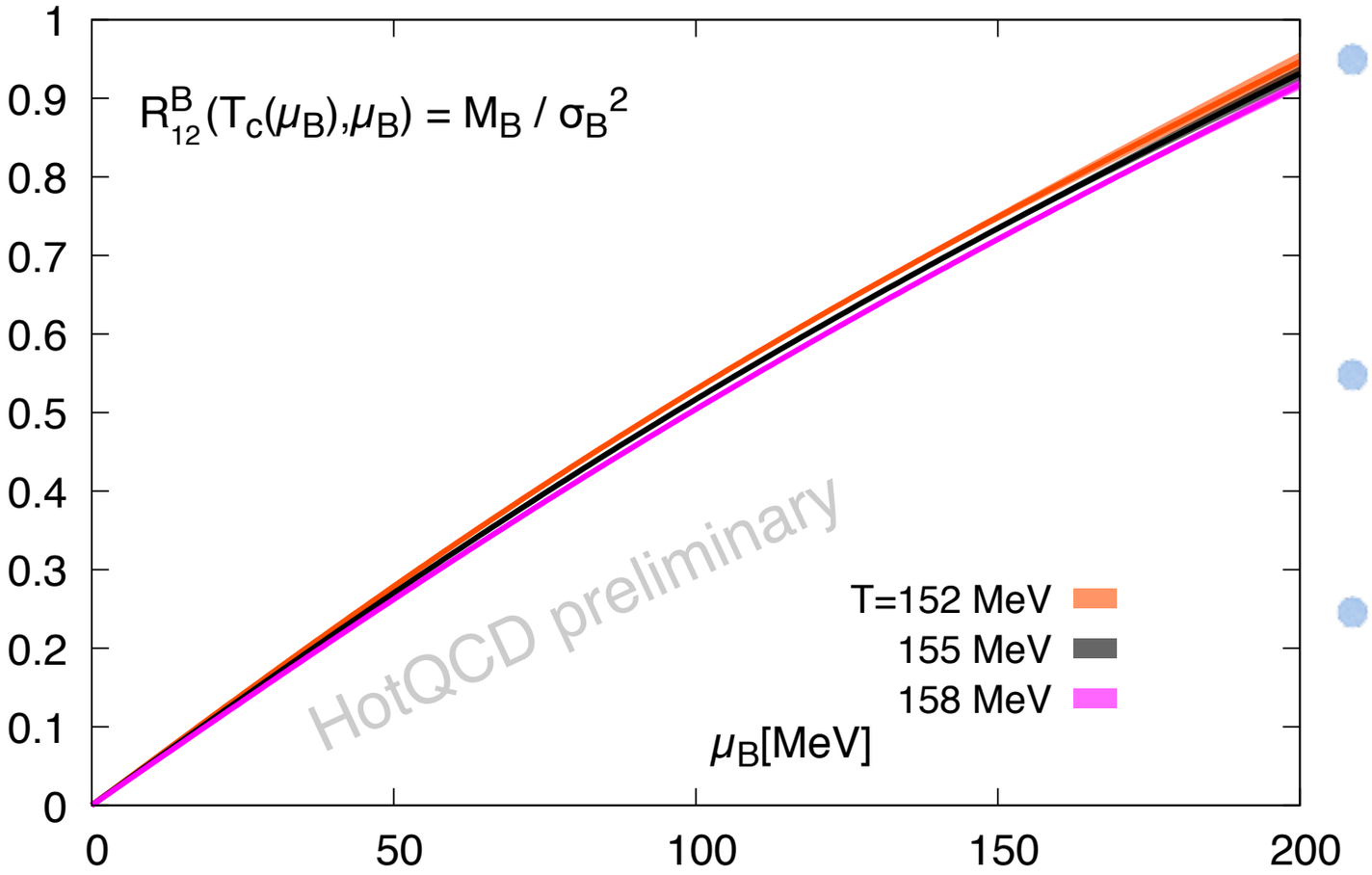

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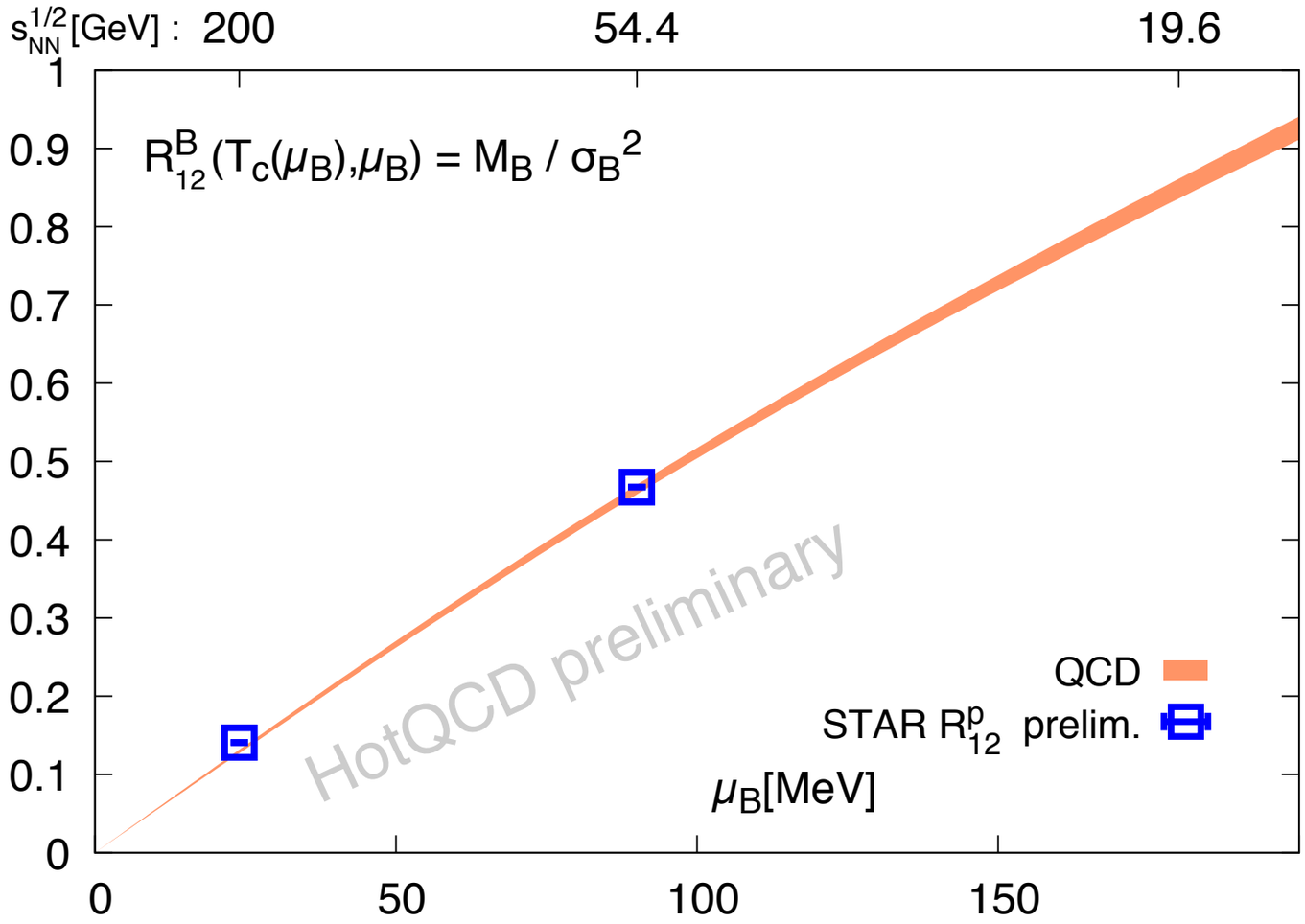

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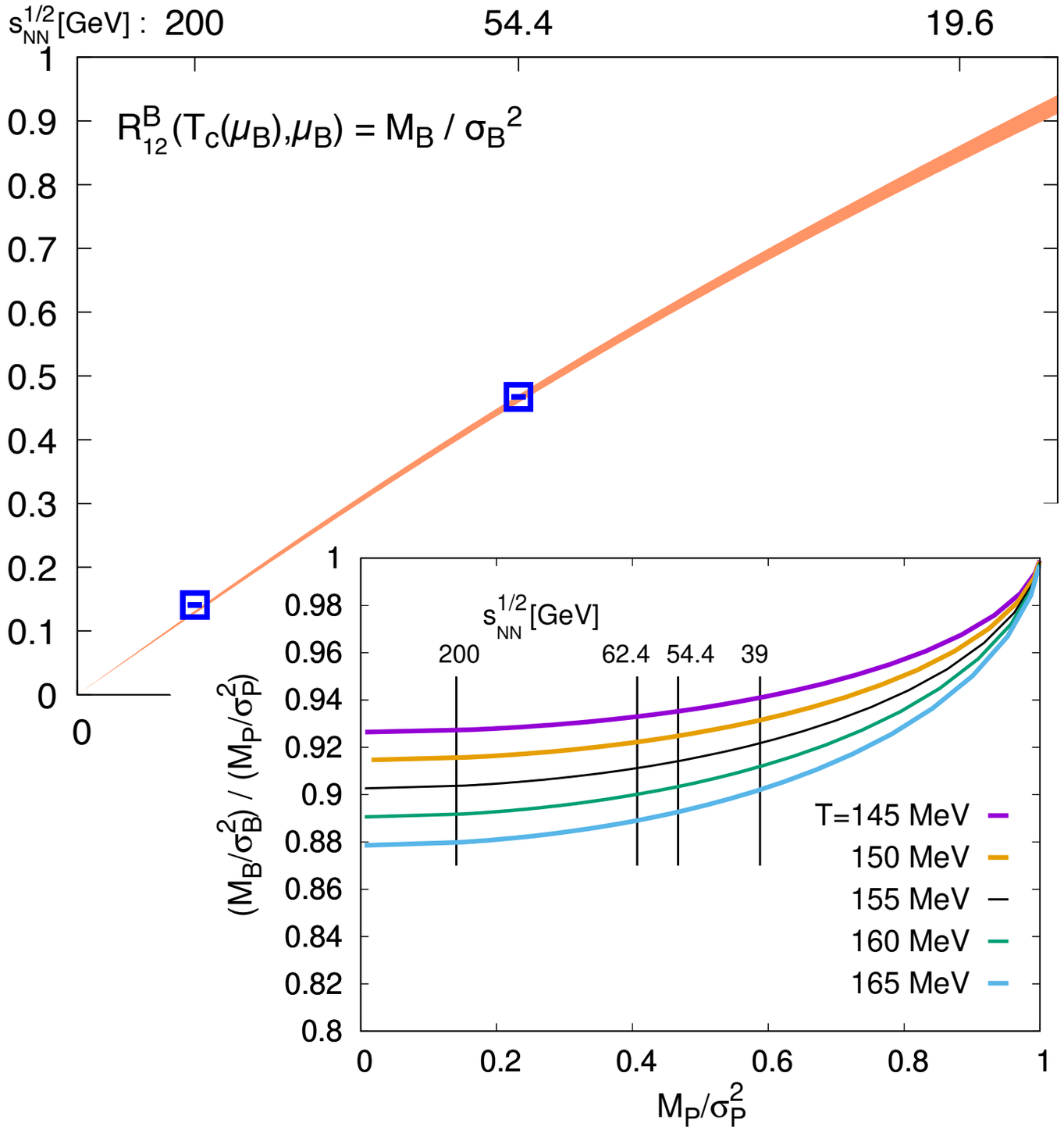

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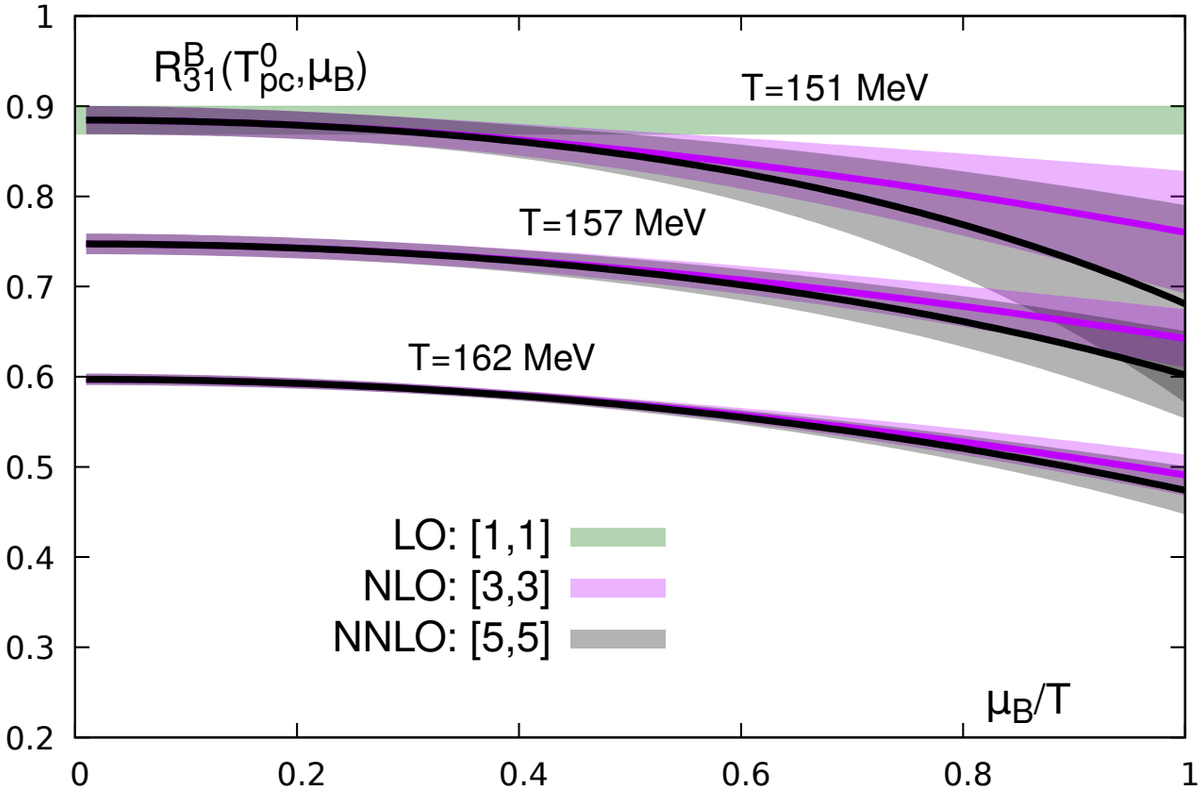
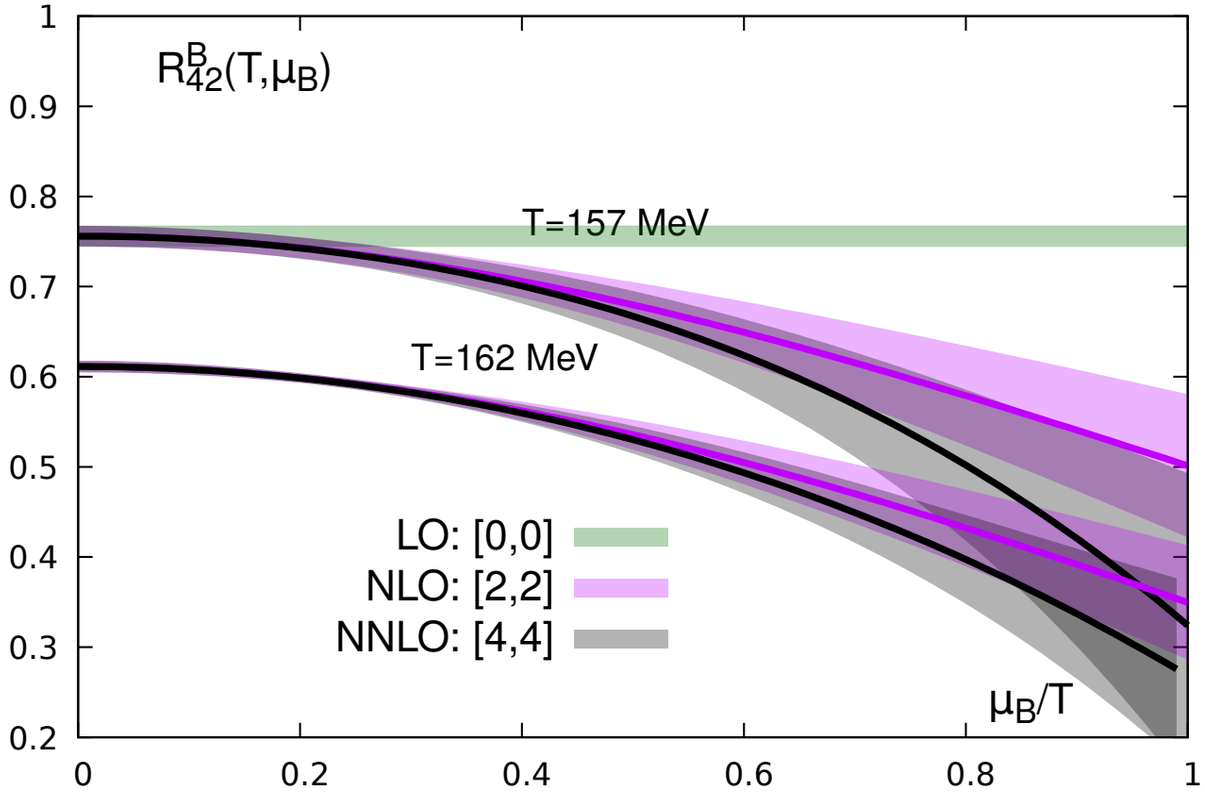
- Temperature dependence is very mild. The curvature of the freeze-out line varies the temperature by less than 3 MeV.
- R_{12}^B may be used to eliminate μ_B in studies of higher order cumulants
- The difference of R_{12}^B and R_{12}^P is less than 10% in the HRG, one may or may not account for this difference in the determination of μ_B .

Results: Skewness and kurtosis

- Skewness and kurtosis ratios R_{31}^B and R_{42}^B on $(N_\tau = 8)$-lattices

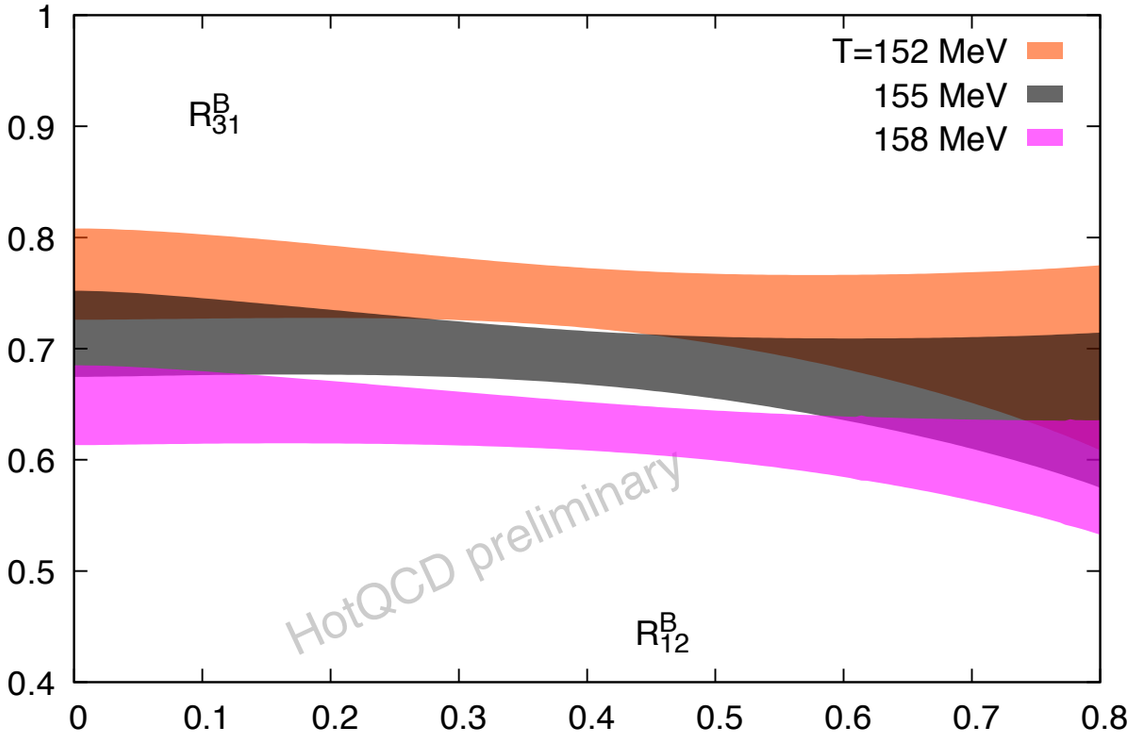


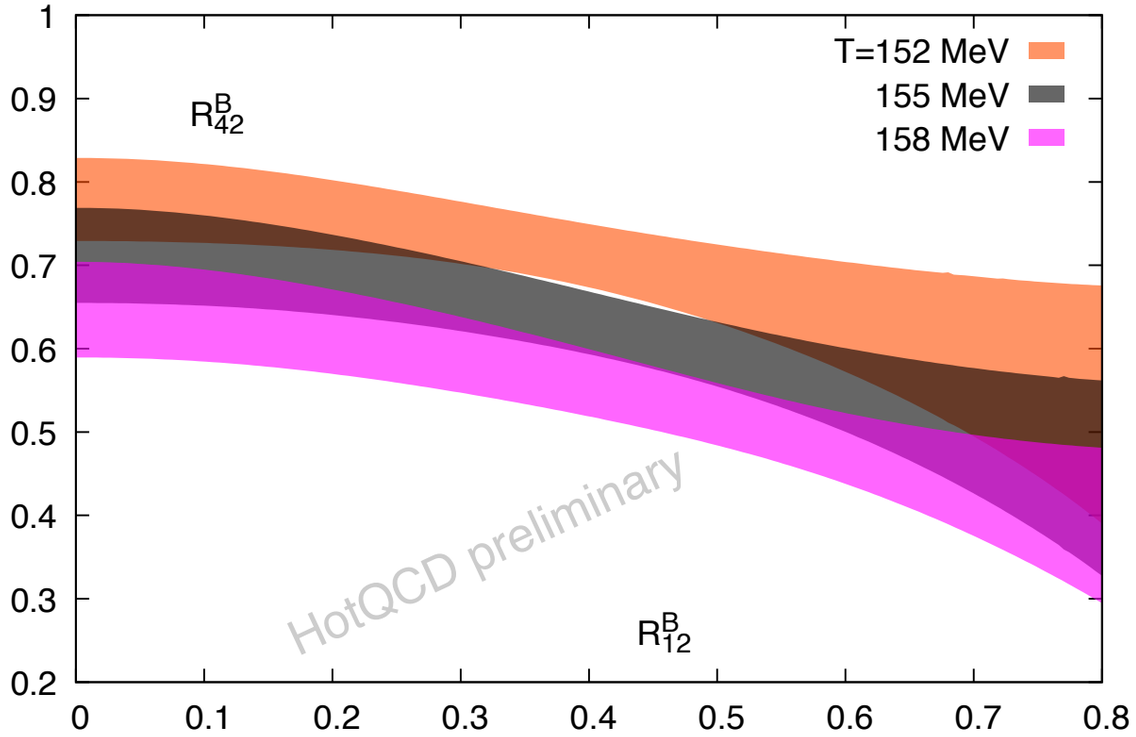


- Convergence gets worth with increasing order of the cumulant and with decreasing temperature.
- NLO and NNLO corrections are negative.

Results: Skewness and kurtosis

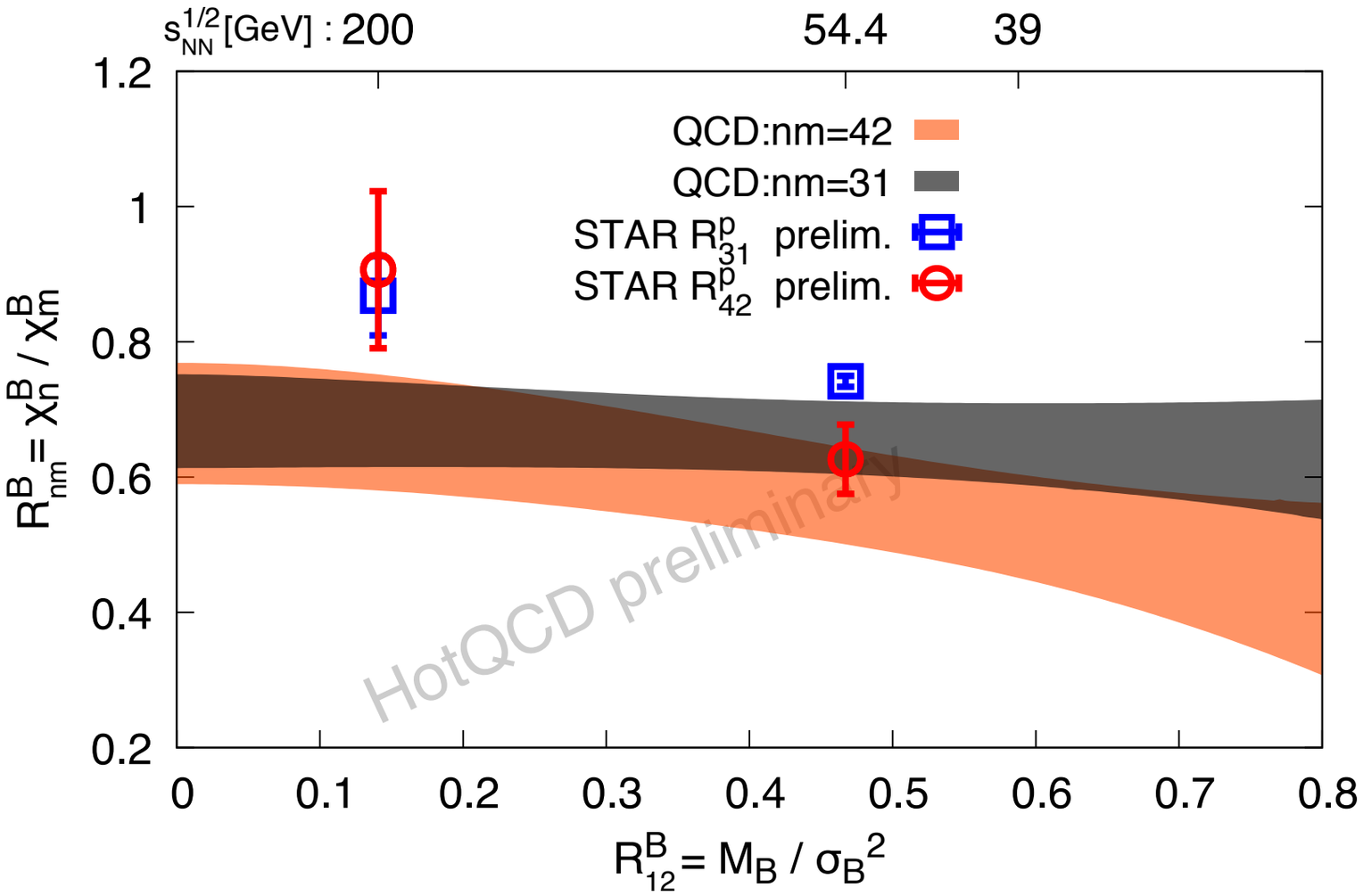
- Continuum estimates of R_{31}^B and R_{42}^B as function of R_{12}^B for various temperatures.





- Ratios drop with increasing R_{12}^B and with increasing temperature.

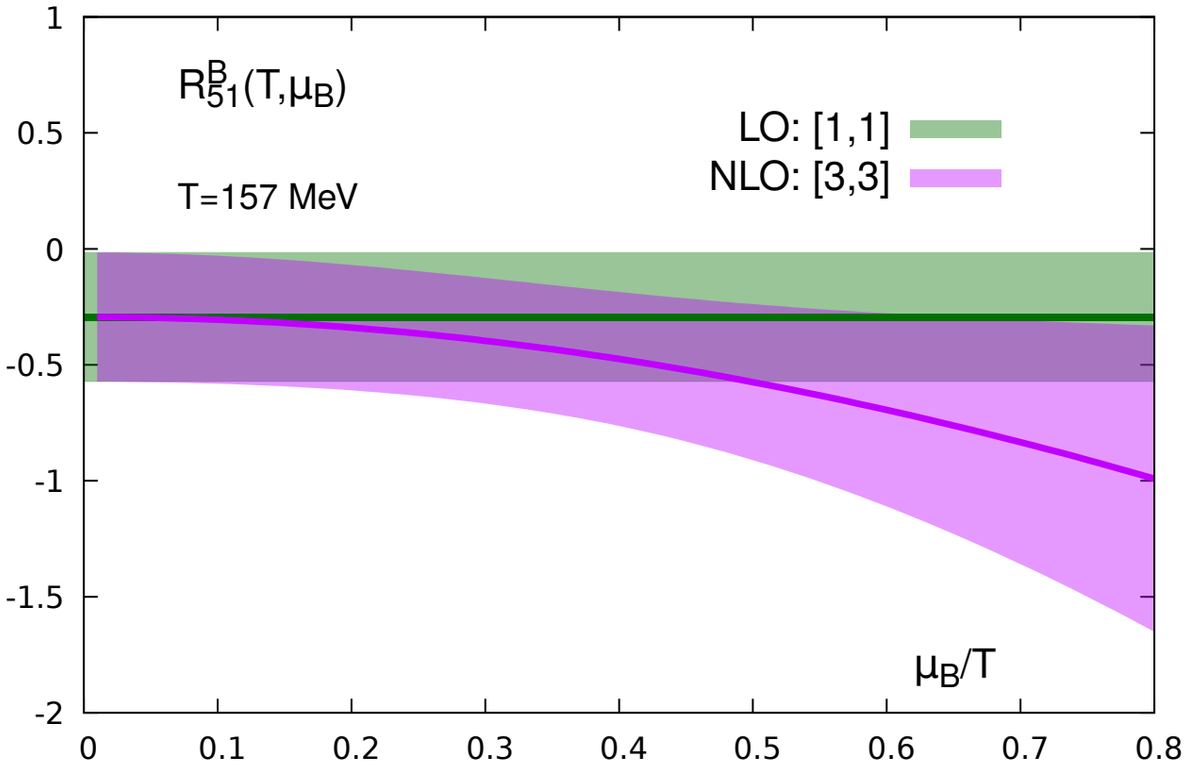
Results: Skewness and kurtosis

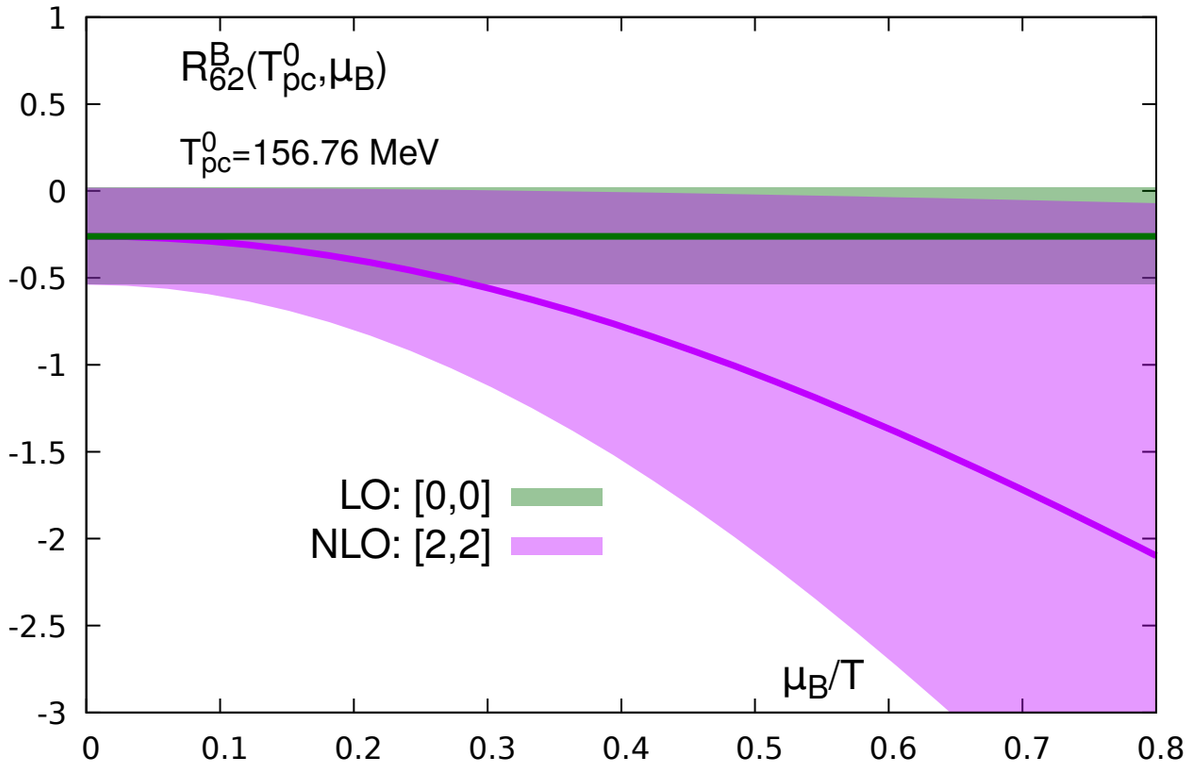


- Continuum estimates of R^B_{31} and R^B_{42} as function of R^B_{12} on the crossover line.
- Star data at $\sqrt{s_{NN}} = 54.4$ GeV favors a freeze-out temperature slightly below the crossover.
- The estimate of the freeze-out temperature $T_f = 165$ MeV for $\sqrt{s_{NN}} = 200$ GeV (from a statistical model analysis) is not consistent with a determination of T_f from the skewness and kurtosis data by STAR.

Results: Fifth and sixth order cumulant ratios R_{51}^B and R_{62}^B

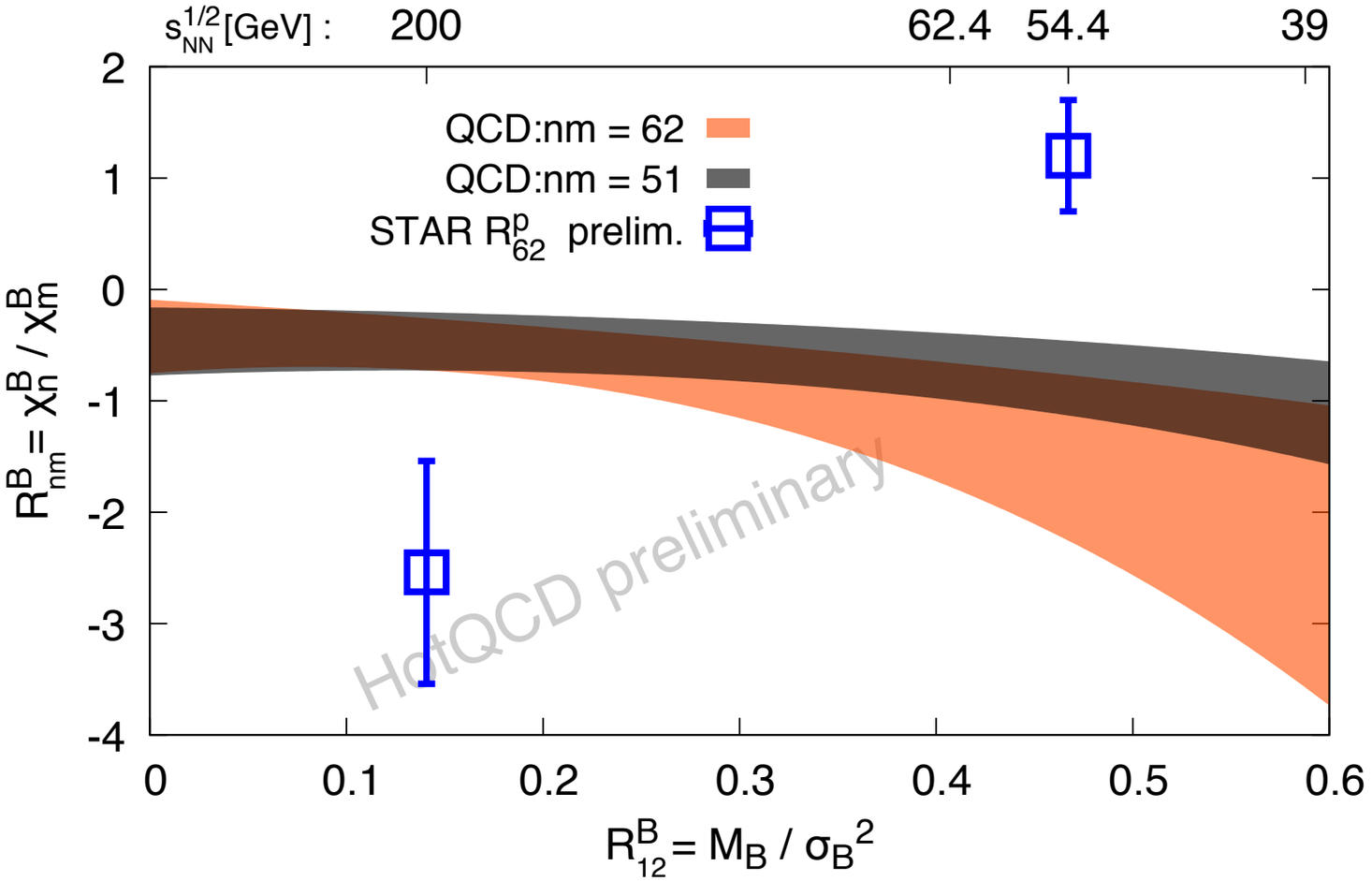
- R_{51}^B and R_{62}^B on $(N_\tau = 8)$-lattices





- Large statistical uncertainties
- NLO corrections are negative

Results: Fifth and sixth order cumulant ratios R^B_{51} and R^B_{62}



- R^B_{51} and R^B_{62} on ($N_\tau = 8$)-lattices
- Not consistent with STAR data:
  - ➡ A. Pandav@SQM19

$$\sqrt{s_{NN}} = 200 \text{ GeV}: R^P_{62} < 0$$

$$\sqrt{s_{NN}} = 54.4 \text{ GeV}: R^P_{62} > 0$$

  - ➡ Lattice QCD predictions

$\sqrt{s_{NN}}$	R^B_{51}	R^B_{62}
200	$-0.5(3)$	$-0.7(3)$
54.4	$-0.7(4)$	$-2(1)$

Summary and outlook

- Lattice QCD calculators show significant increase in precision for cumulant ratios along the crossover (or any other) line in the QCD phase diagram for $\mu_B/T \leq 1.2$ due to increase in the statistics and thus also the order of the expansion.
- Presented first calculations of R^B_{51} and R^B_{62} along the crossover line
- Freeze-out temperature at $\sqrt{s_{NN}} = 200$ GeV seems not consistent with higher order cumulants
- At $\sqrt{s_{NN}} = 54.4$ GeV we find thermodynamic consistency of the STAR data except for the R^P_{62} data.
- **Outlook:**
  - Need to increase statistics for ($N_\tau = 12$)- and ($N_\tau = 16$)-lattices further.
  - Need to perform similar analysis for net-electric charge fluctuations, thus need larger volumes.