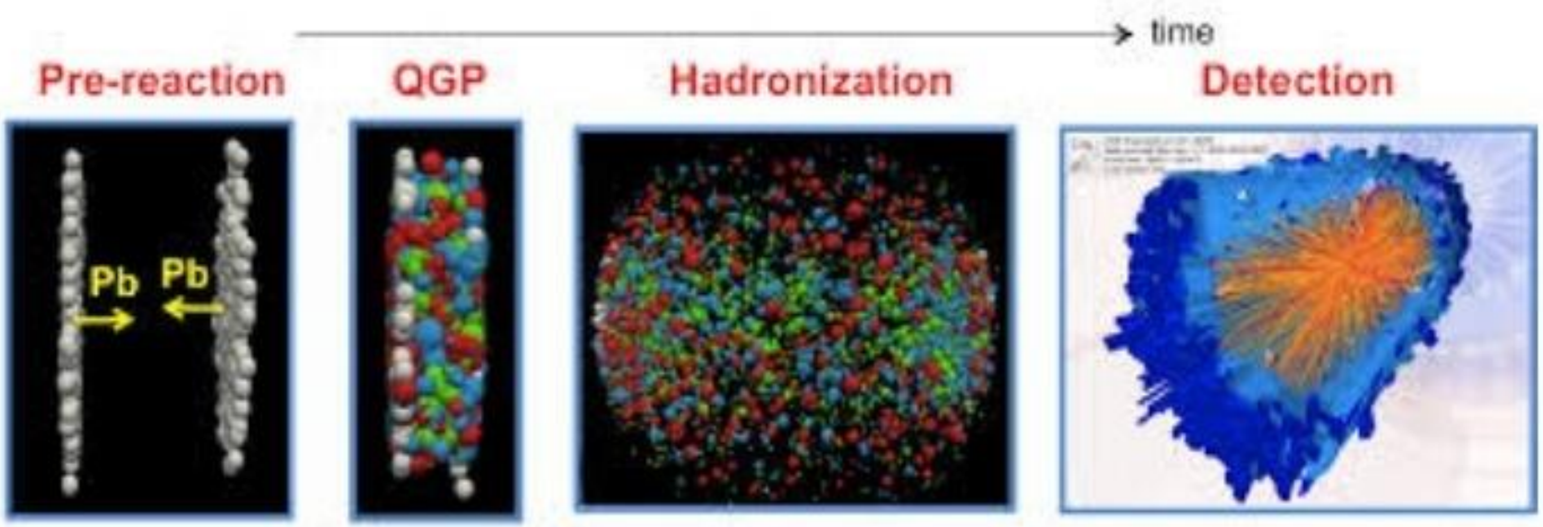
Heavy quark dynamics in QCD matter



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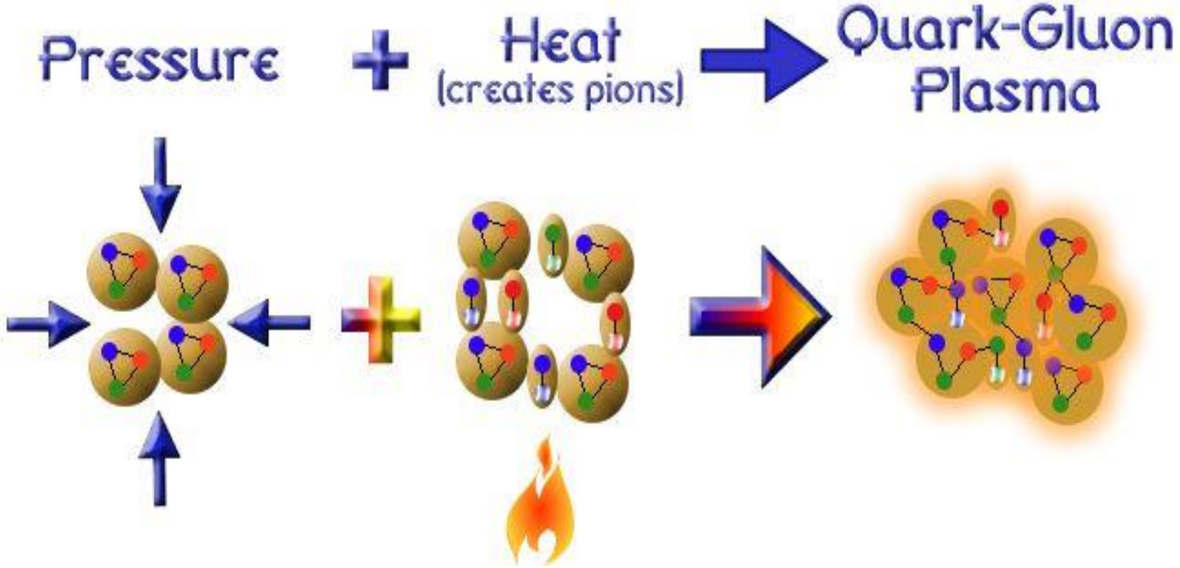
OUTLINE ……………

- ☐ Introduction
- ☐ Quark Gluon Plasma - the primordial fluid
- ☐ Heavy quark dynamics in QGP
- ☐ Probing of initial electromagnetic field by heavy quarks
- ☐ Heavy quark dynamics in small system
- ☐ Summary and outlook

Quark Gluon Plasma (QGP)

On the basis of asymptotic freedom, first Collins and Perry suggest that super dense matter consist of quarks rather than of hadrons.



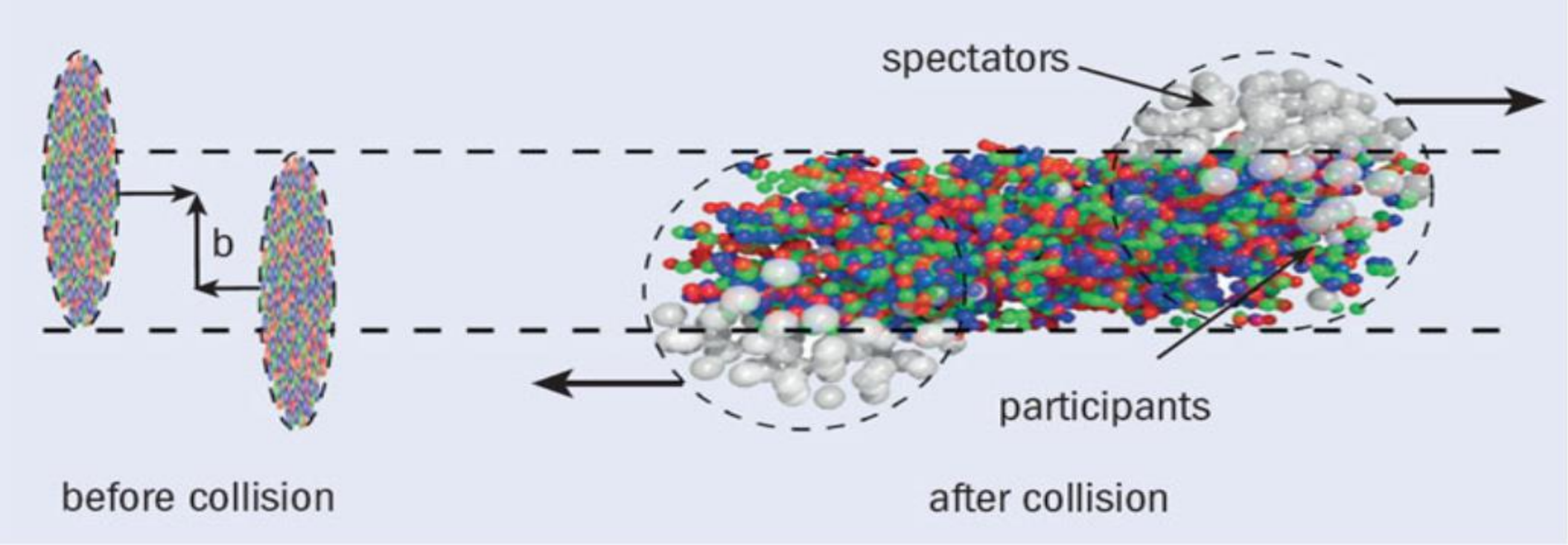
At very high temperature and density hadrons (proton, neutron,) melt to a new phase of matter called Quark Gluon Plasma (QGP).

Why QGP ???

Relevant for:
- Early Universe
- Compact Astrophysical Objects (Neutron Star)

Offers Opportunity to Study:
- Non-abelian Field Theory (QCD) in Thermal Bath
- High Temperature & Density – Phase Transition (Deconfinement)

How to create QGP



Nuclear Collisions at Relativistic Energies – Tool to Create High Temperature (Early Universe) & Density (Neutron Star)

Quark-Hadron transition occurred at T~170 MeV.

Density ~10^{18} kg/m^3

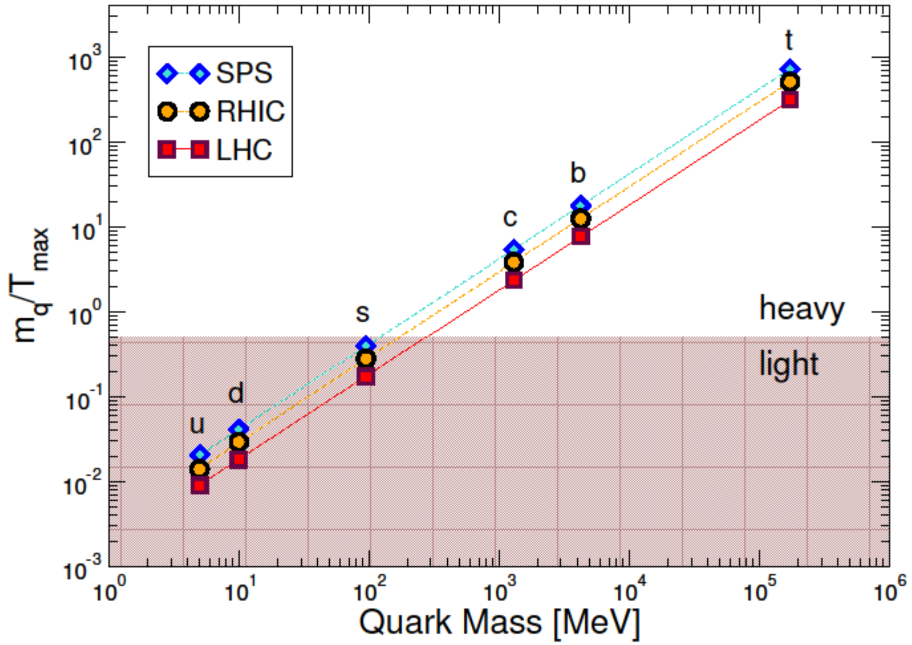
Temperature of sun~10 KeV.

More than 10000 times of the temperature of sun.

Experimental facility:

Relativistic Heavy Ion Collider (RHIC), BNL, USA
Large Hadron Collider (LHC), CERN, Switzerland
Facility for Antiproton and Ion Research (FAIR), Darmstadt, Germany
Nuclotron-based Ion Collider fAcility (NICA), Dubna, Russia
Future Circular Collider (FCC), CERN, Switzerland

Heavy Quark & QGP



SPS to LHC

$\sqrt{s} = 17.3 GeV \ to \ 2.76 TeV$ **~100 times**

$T_i = 200 \ MeV \ to \ 600 \ MeV$ **~3 times**

τ **relaxation time**

$$M_{c,b} >> \Lambda_{QCD}$$

Produced by pQCD process (before equilibrium)
(Early production)

$$\tau_{c,b} >> \tau_{QGP}$$

They go through all the QGP life time

$$M_{c,b} >> T_0$$

No thermal production

Boltzmann Kinetic equation

- The plasma is uniform ,i.e., the distribution function is independent of x.
- In the absence of any external force, F=0

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,\, p,\, t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

$$\boxed{R(p,t)=\left(\frac{\partial f}{\partial t}\right)_{col}=\int d^3k\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]}$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}\longrightarrow$$

is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

$$\boxed{\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p}_\mathbf{i}}\left[\mathbf{A}_\mathbf{i}(\mathbf{p})\mathbf{f}+\frac{\partial}{\partial \mathbf{p}_\mathbf{j}}\left[\mathbf{B}_\mathbf{ij}(\mathbf{p})\mathbf{f}\right]\right]}$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

,

$$\mathbf{A}_\mathbf{i}=\int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_\mathbf{i}\quad\rightarrow \textbf{Drag Coefficient}$$

$$\mathbf{B}_\mathbf{ij}=\int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_\mathbf{i}\mathbf{k}_\mathbf{j}\quad\rightarrow \textbf{Diffusion Coefficient}$$

Langevin Equation

The Fokker-Planck equation can be recast to Langevin equation:

$$\mathbf{dr} = \frac{p}{E}\mathbf{d}t$$

$$\frac{dp}{dt} = -\gamma(p)p + \zeta$$

with $< \zeta_i(t)\zeta_k(t') >= D\delta(t-t')\delta_{jk}$

where γ **is the deterministic friction (drag) force**

ζ **is stochastic force**

Das, Scardina, Plumari, Greco
Phys. Lett. B 747 (2015)260-264

For the numerical implementation of Langevin dynamics we use pre-point Ito discretization.

Transport coefficients are connected by Fluctuation dissipation theorem : $D = M\gamma T$

Heavy quark initialization

✧ r-space: N_coll (Glauber mode)

✧ p-space: NLO (pQCD)

Nuclear Modification Factor (R_{AA}) and Elliptic Flow (v2)

$$R_{AA} = \frac{\left(\frac{dN}{d^2 p_T dy}\right)^{Au+Au}}{N_{coll}\left(\frac{dN}{d^2 p_T dy}\right)^{p+p}} = \frac{f^{FP}_{final}}{f^{FP}_{initial}}$$

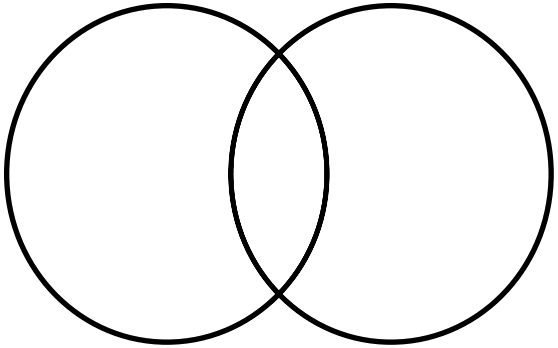
If $R_{AA} = 1$ → **No medium/ No interaction**

If $R_{AA} < 1$ → **Medium/Interaction**

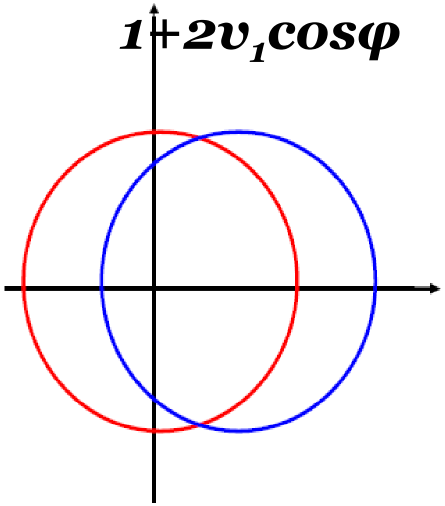
A direct measure of the energy loss.

$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{2\pi p_T dp_T dy}\left(1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + ...\right)$$

$$v_2^{HF}(p_T) = \langle\cos(2\phi)\rangle = \frac{\int d\phi \frac{dN}{dydp_T d\phi}\cos(2\phi)}{\int d\phi \frac{dN}{dydp_T d\phi}}$$

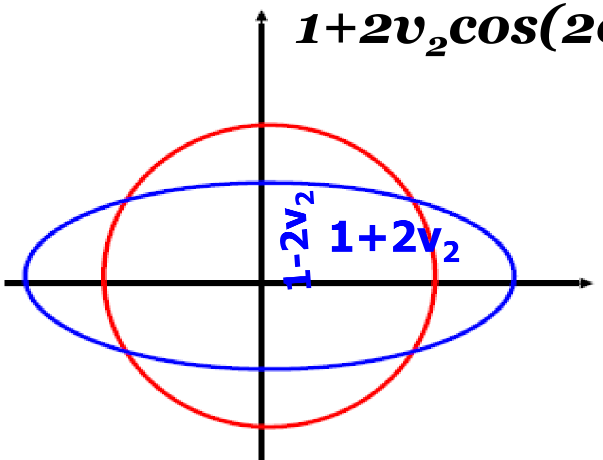


$1+2v_1\cos\varphi$

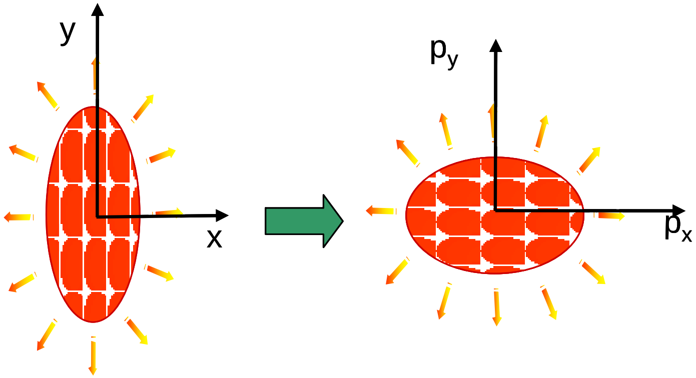


Overall shift

$1+2v_2\cos(2\varphi)$

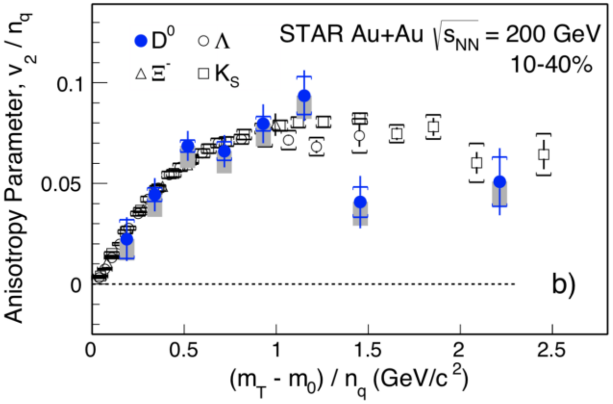


Major axis = $1+2v_2$
Minor axis = $1-2v_2$



Heavy quark physics at different scales

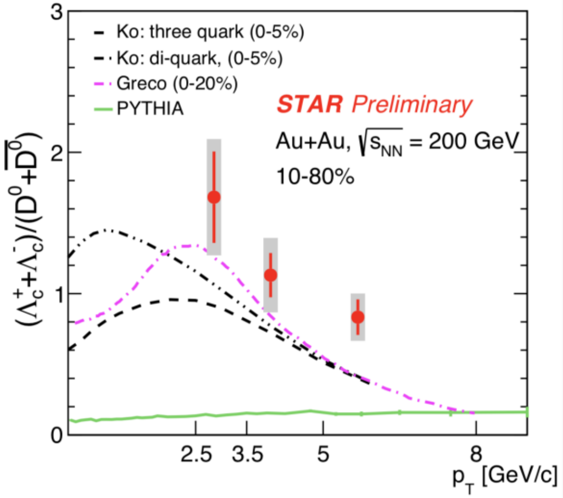
low p_T



Study thermalization process of HQ

Constrain diffusion coefficient D_s

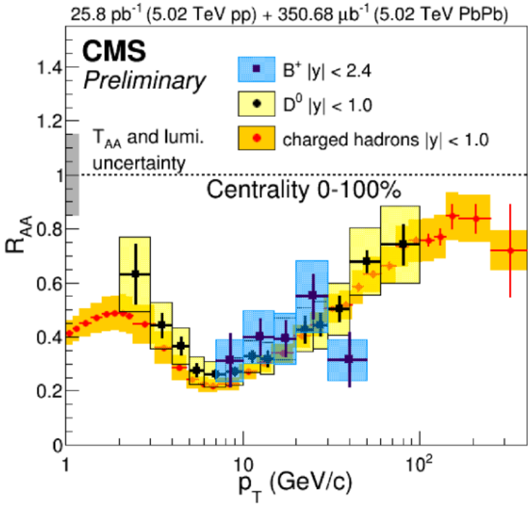
medium p_T



Study hadronization process of HQ

Constrain hadron wave-function

high p_T

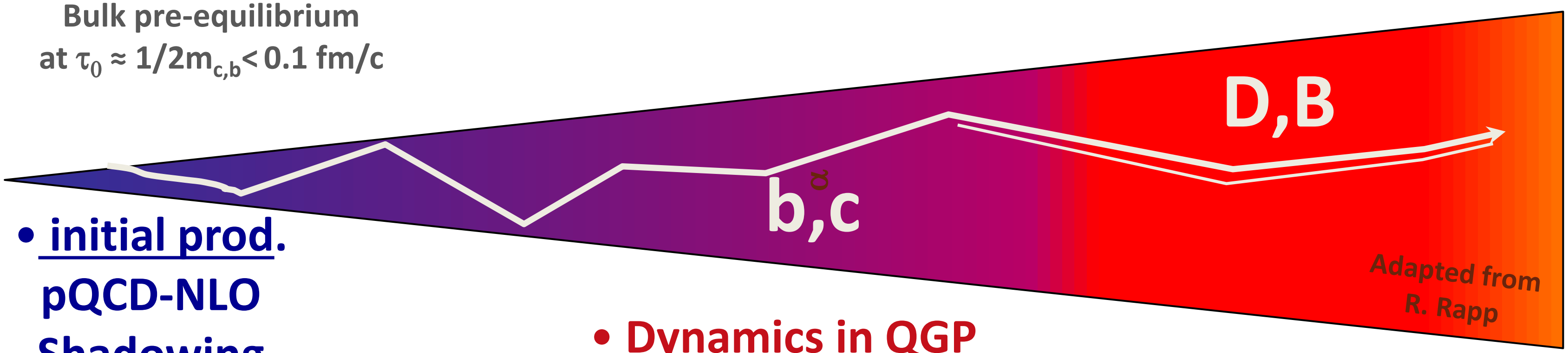


Study parton energy loss and mass effect

Constrain jet transport parameter $\hat{q}$

Studying the HF at RHIC and LHC

Bulk pre-equilibrium
at $\tau_0 \approx 1/2m_{c,b} < 0.1$ fm/c



- **initial prod.**

pQCD-NLO
Shadowing
Pre-equilibrium
Effect/Glasma
Electromagnetic
filed

- **Dynamics in QGP**

Heavy quark QGP interaction

Transp. coeff. of QCD matter

-> thermalization ?!

Mass & color in Jet quenching

Heavy quark momentum evol.

(Langevin/Boltzmann/E. loss model)

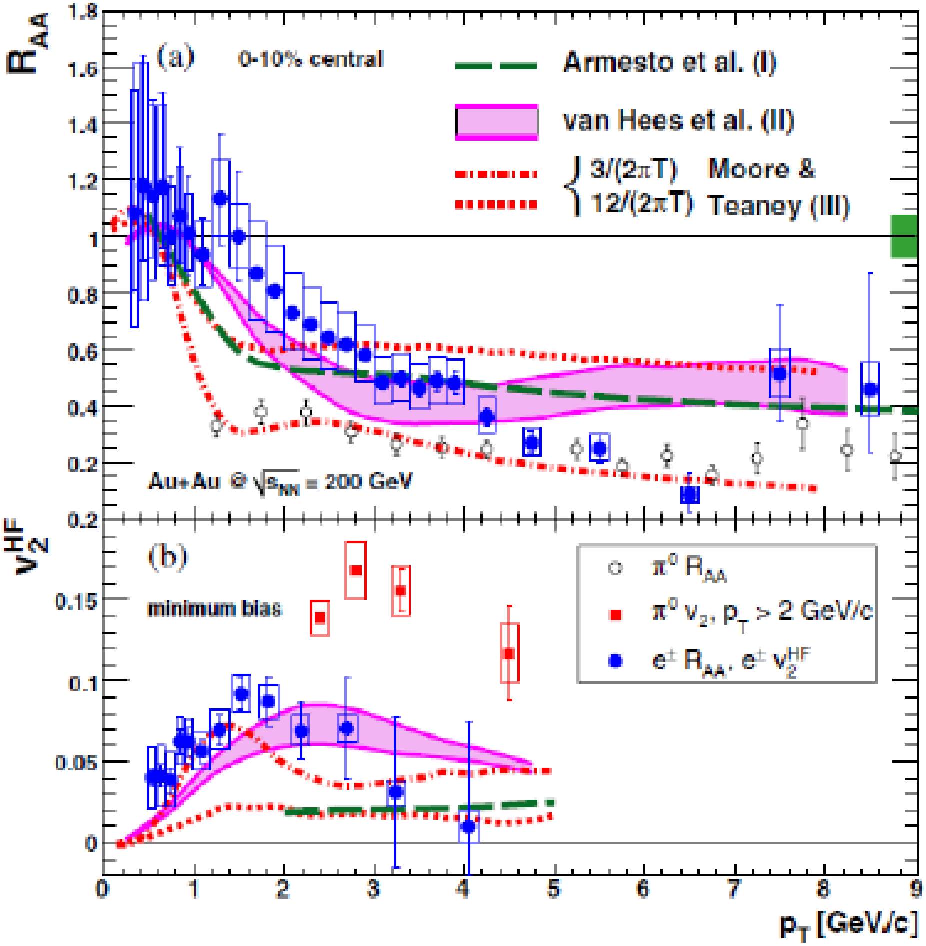
- **hadronization:**

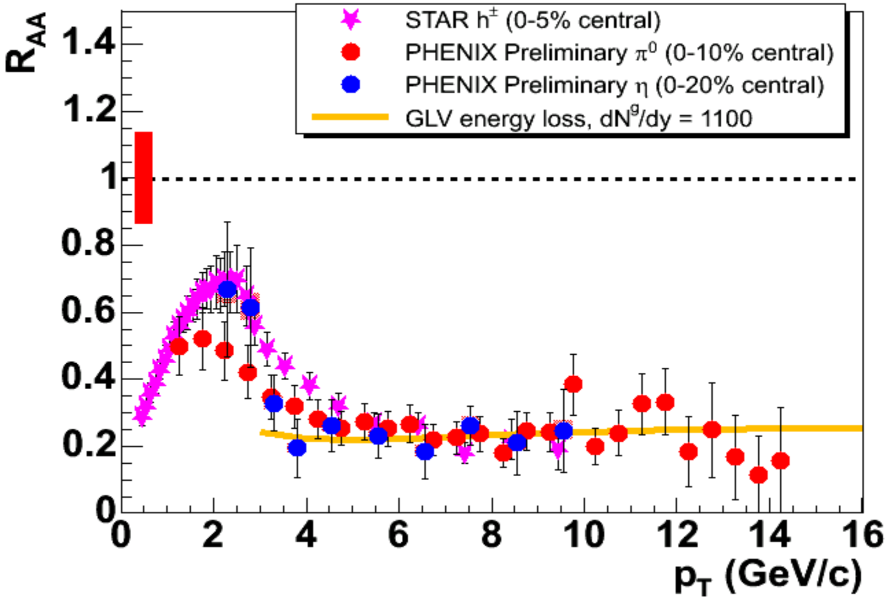
coalescence and/or

fragmentation.

Hadronic rescattering

Heavy flavor at RHIC (2007)

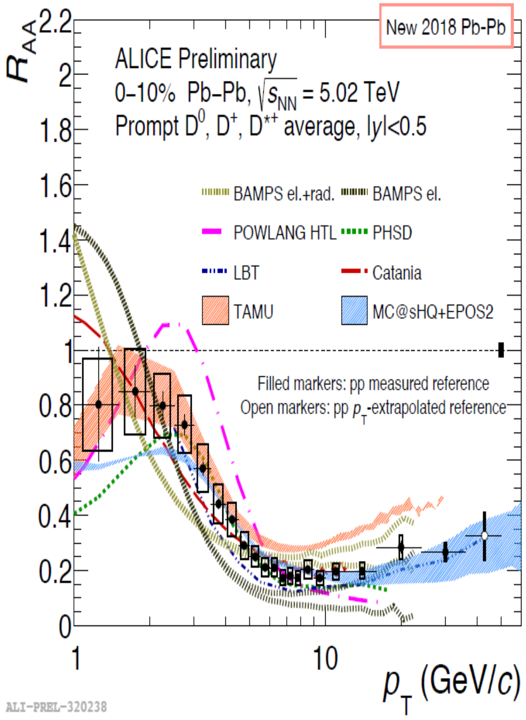




At RHIC energy heavy flavor suppression is similar to light flavor

Simultaneous description of RAA and v2 is a tough challenge for all the models.

R_{AA} and v_2
Comparison with models



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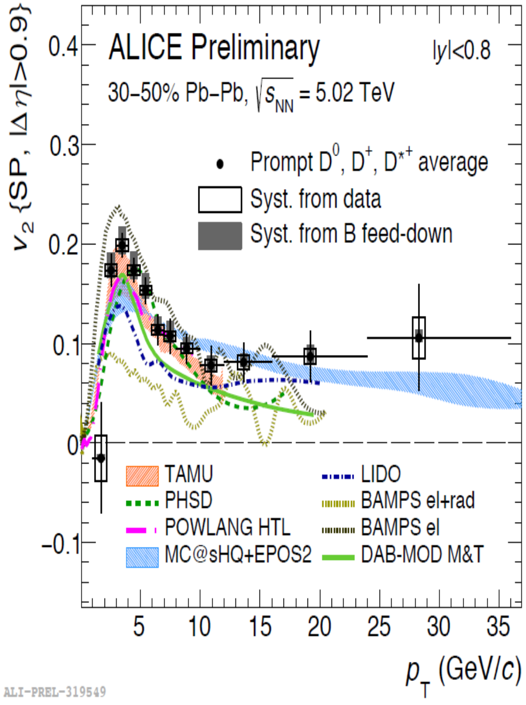

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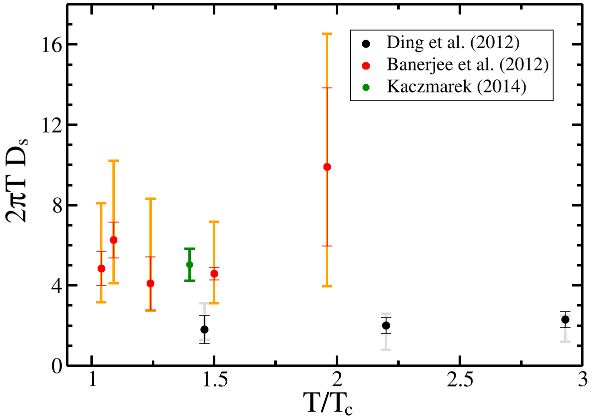
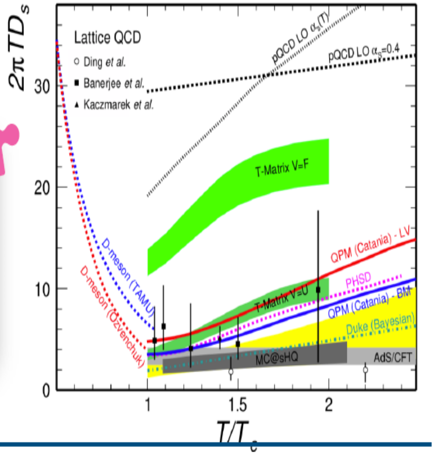


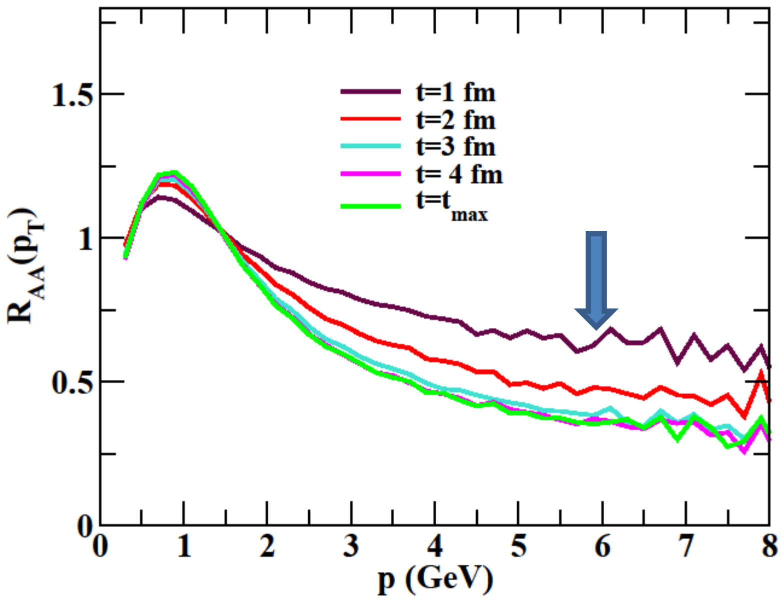
TAMU: PLB 735,445-450(2014), arXiv:1905.09216
PHSD: PRC 92, 014910 (2015)
POWLANG: EPJC 75,121(2015)
MC@sHQ+EPOS2: PRC 89 014905 (2014)
LBT: PLB 777 (2018) 255-259
LIDO: arXiv:1810.08177
BAMPS: JPG 42, 115106 (2016)
Djordevic: PRC 92, 024918 (2015)
CUJETS3.0: JHEP 02 (2016) 169
SCET: JHEP 03 (2017) 146
DAB-MOD: PRC 96 (2017) 064903
Catania: Eur. Phys. J. C (2018) 78: 348

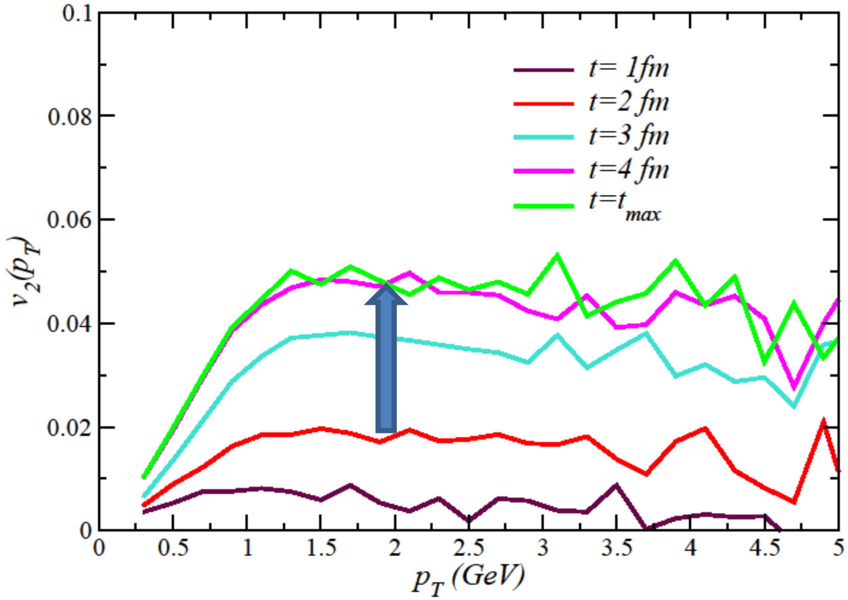
- ➡ Simultaneous description of R_{AA} and v_2 **is challenging** in the whole measured p_T range!
- ➡ Experimental measurements start to **provide constraint to the models** for the characterization of the charm and beauty interaction with the medium
  - ➡**constraints on plasma transport parameters**, such as the heavy-quark diffusion coefficient



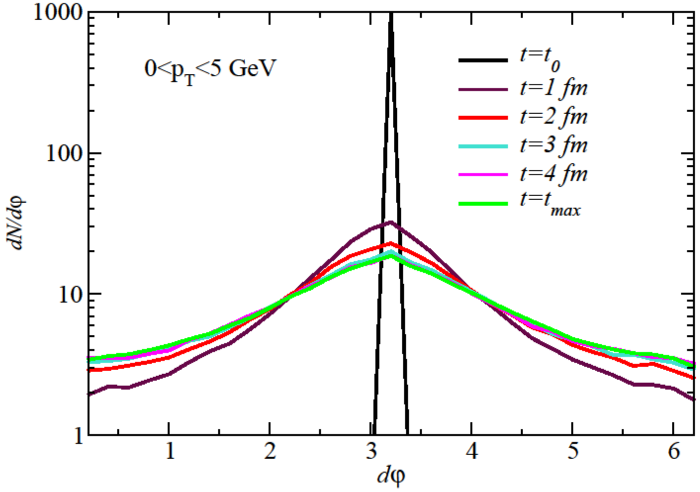
Terrevoli (SQM 2019)

Time evolution of Heavy quarks observables





Das, Scardina, Plumari, Greco
arXiv:1509:06307

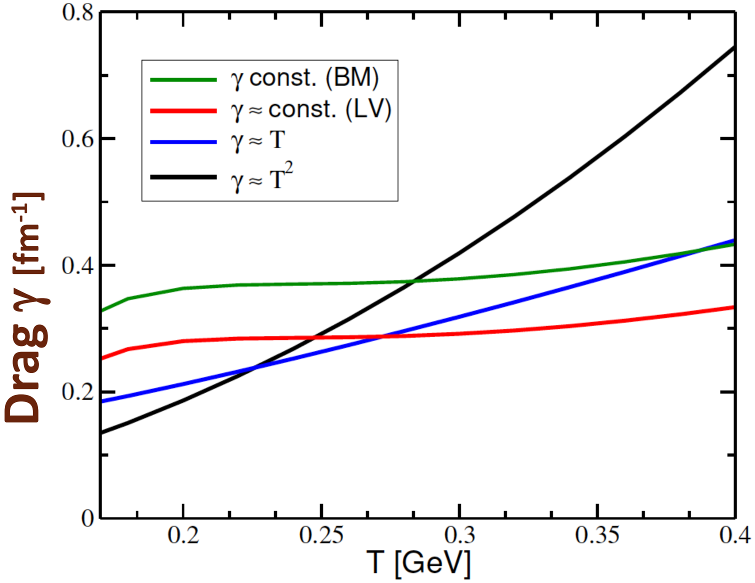


RAA and dN/dphi_ccbar developed during the early stage of the evolution → T_i

V2 developed duringing the later stage of the evolution → T_c

T dependence of the interaction i.e the transport Coefficients are the essential ingradiant for the symultanious description of HQ observabbles

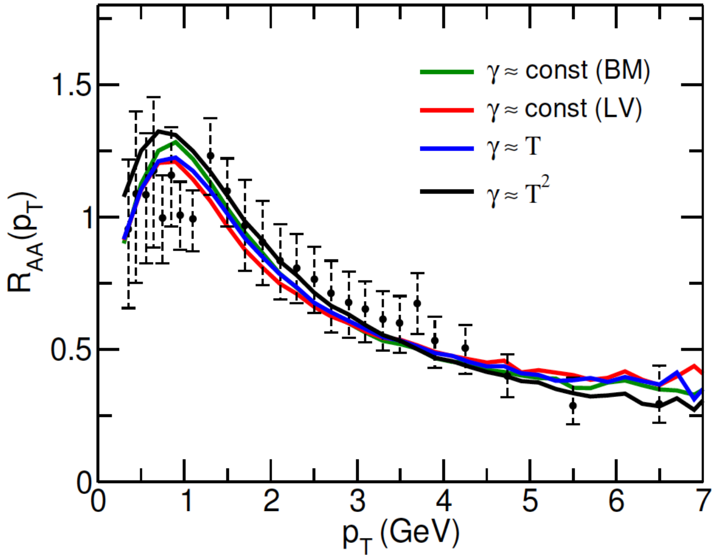
Impact of T dep. interaction on $R_{AA} - v_2$

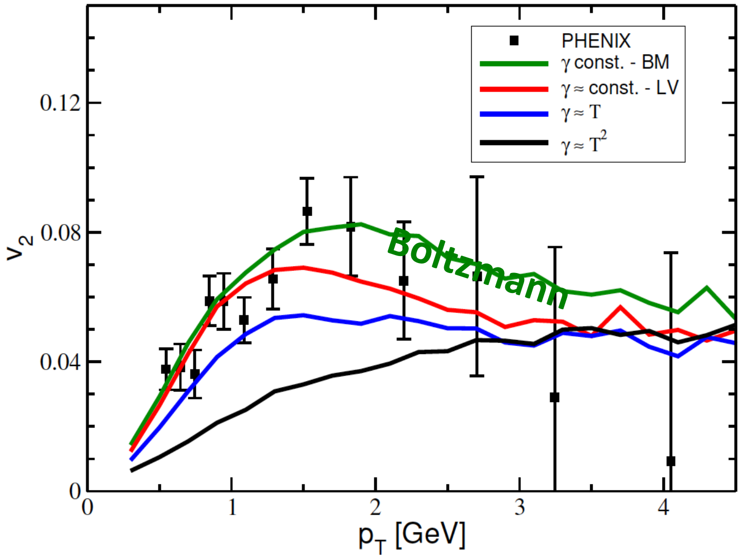


Looking at it beyond the specific modelings

- ➢ $\gamma \approx T^2$ [Ads/CFT, pQCD α_s=const]
- ➢ $\gamma \approx T$ [pQCD strong α_s running]
- ➢ $\gamma \approx$ const. [QPM, PHSD, T-matrix]

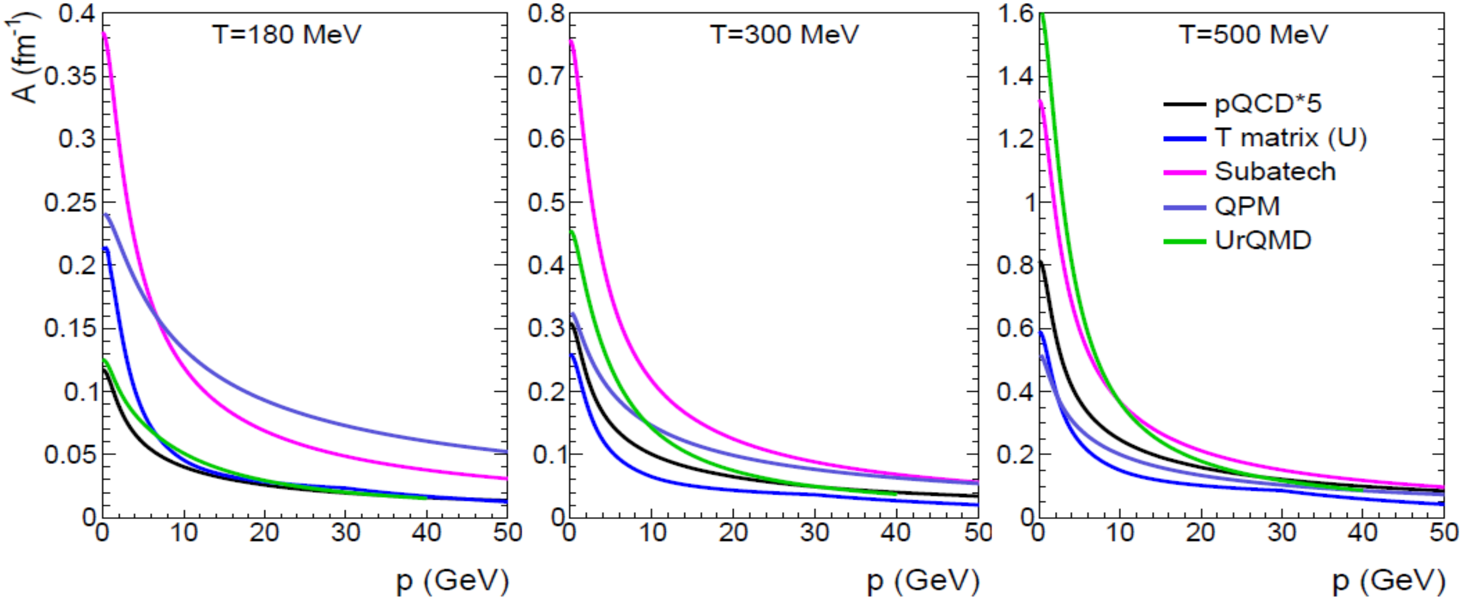
γ rescaled to fit $R_{AA}(p_T)$, D from FDT



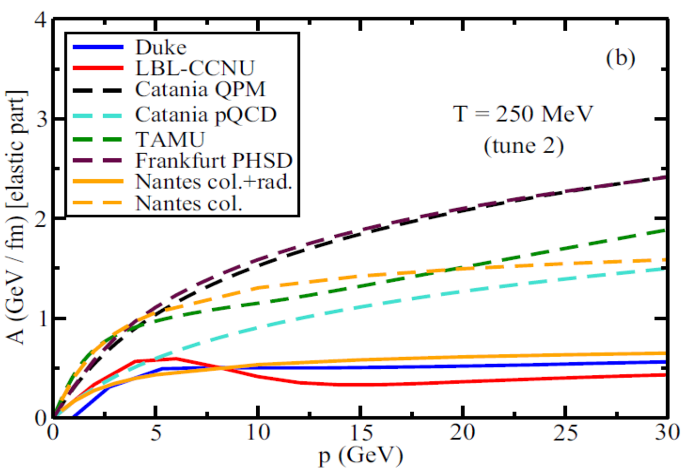


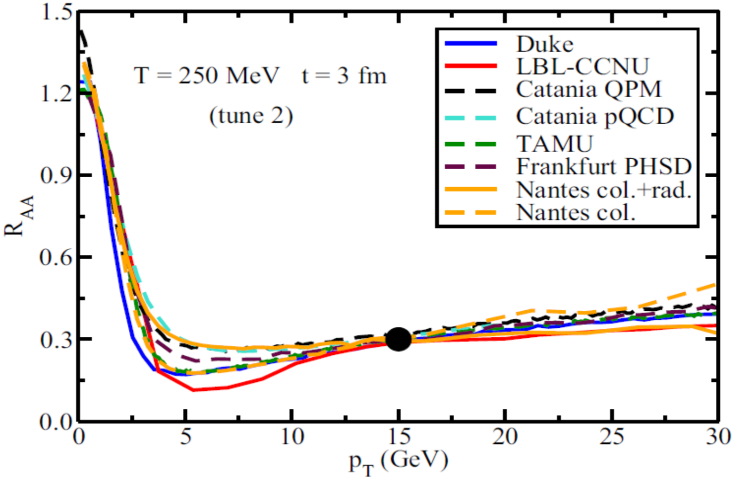
Das, Scardina, Plumari, Greco
Phys. Lett. B 747 (2016)260-264

A systematic attempts are going on within the EMMI-RRTF and “JET-HQ” working groups to find a common agreement between different groups:



R. Rapp et.al NPA 979 (2018)   (EMMI-RRTF)





S. Cao et. al PRC 99, 054907 (2019) (JET-HQ)

Heavy quark momentum evolution: Langevin vs Boltzmann

$$\omega(p+k,k)\,f(p+k) \approx \omega(p,k)\,f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j\,\frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It will be interesting to study both the equation in a identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin dynamics:

$$dx_j = \frac{p_j}{E}\,dt$$

$$dp_j = -\Gamma p_j\,dt + \sqrt{dt}\,C_{jk}(t, p + \xi dp)\,\rho_k$$

Das, Scardina, Plumari and Greco
Phys. Rev. C,90,044901(2014)

Γ **is the deterministic friction (drag) force**

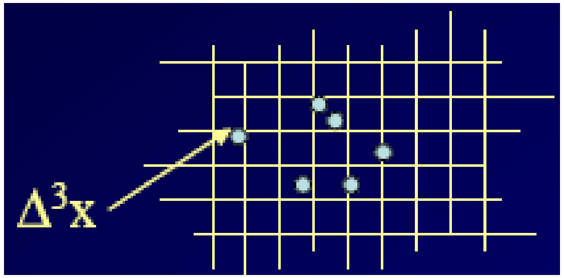
C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable.**

Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22}$$

We consider two body collisions

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$
$$- \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2)$$



$$\Delta t \to 0$$

$$\Delta^3 x \to 0$$

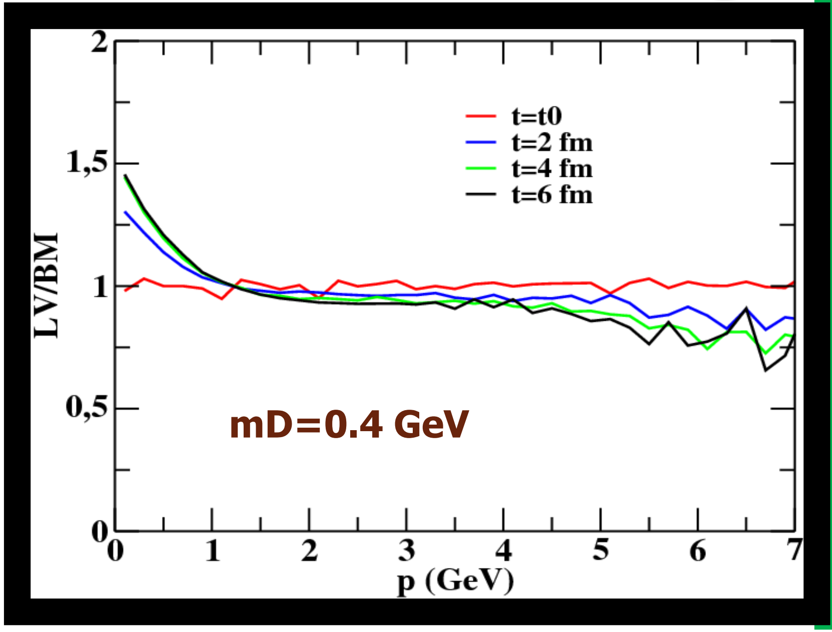
Exact solution

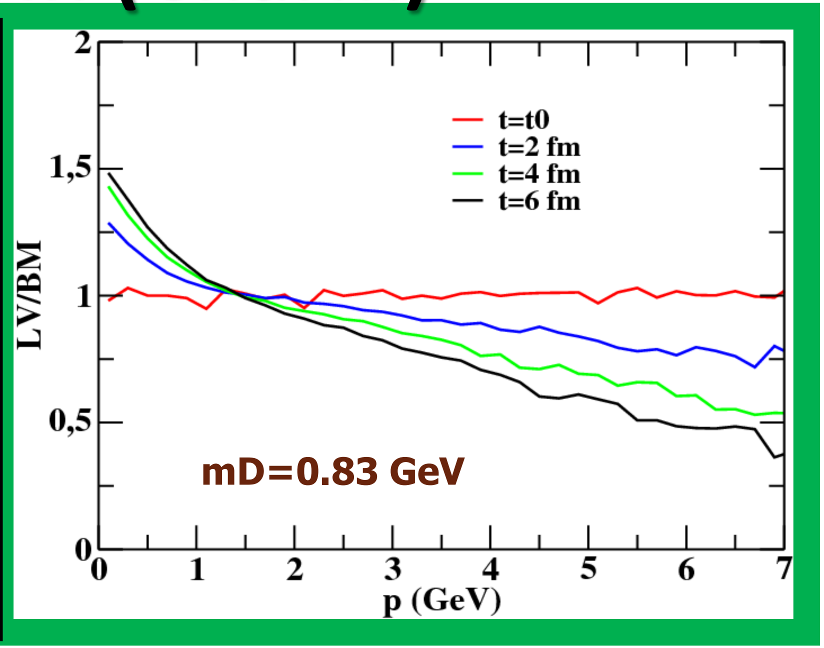
Collision integral is solved with a local stochastic sampling

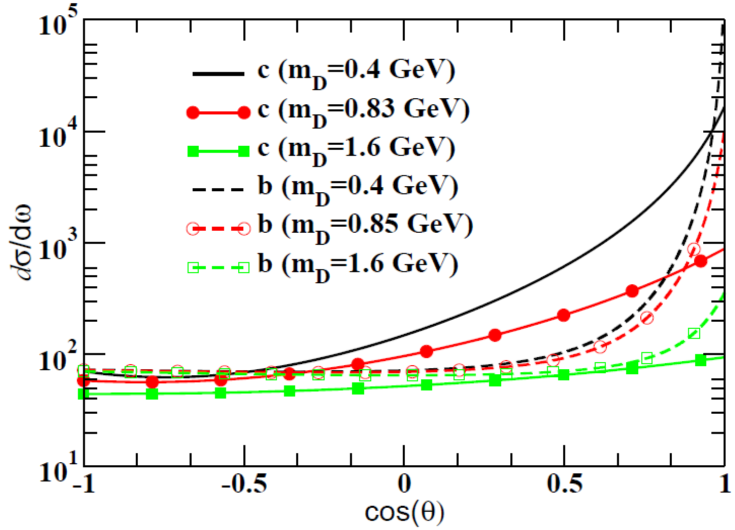
Das, Scardina, Plumari and Greco
Phys. Rev. C,90,044901(2014)

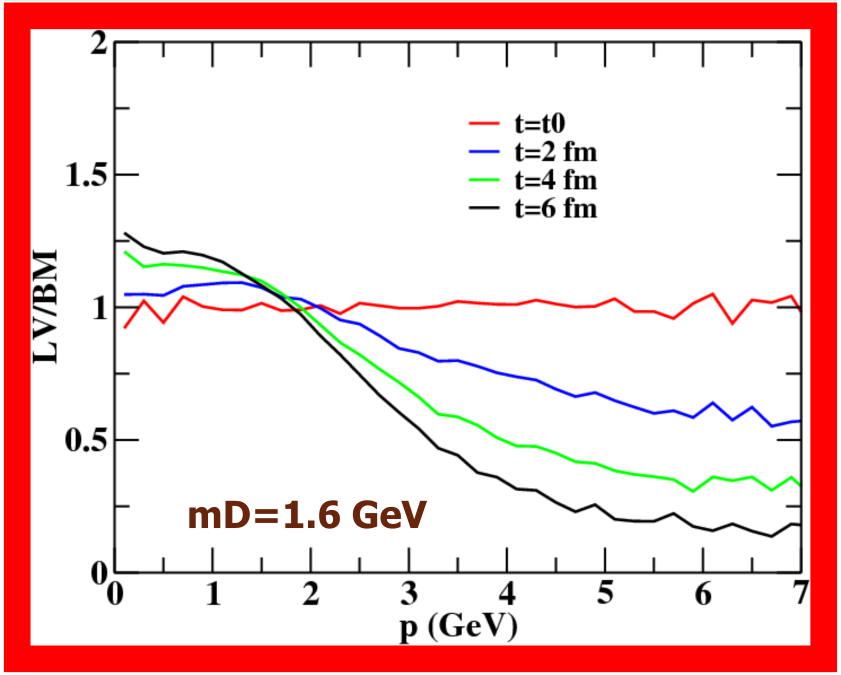
$$P_{22} = \frac{\Delta N^{2\to 2}_{\text{coll}}}{\Delta N_1 \Delta N_2} = v_{\text{rel}}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Boltzmann vs Langevin (Charm)



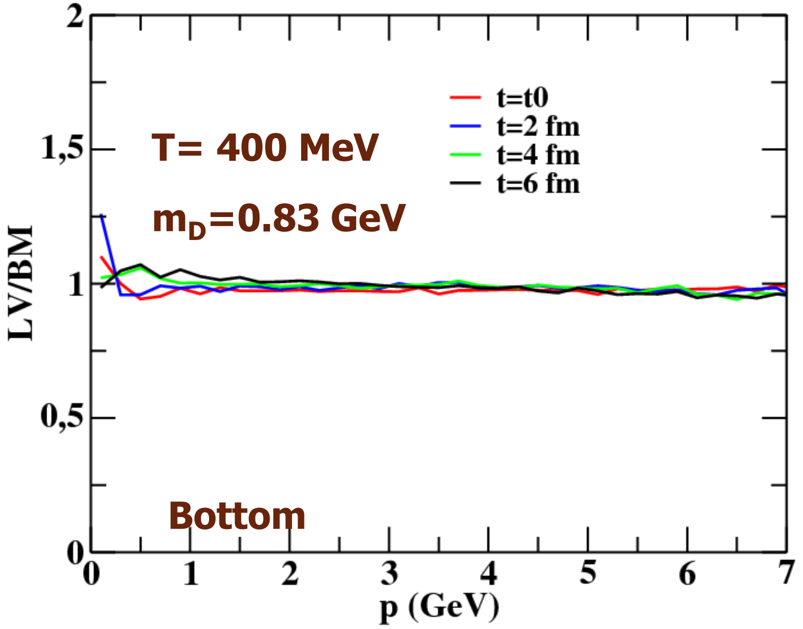


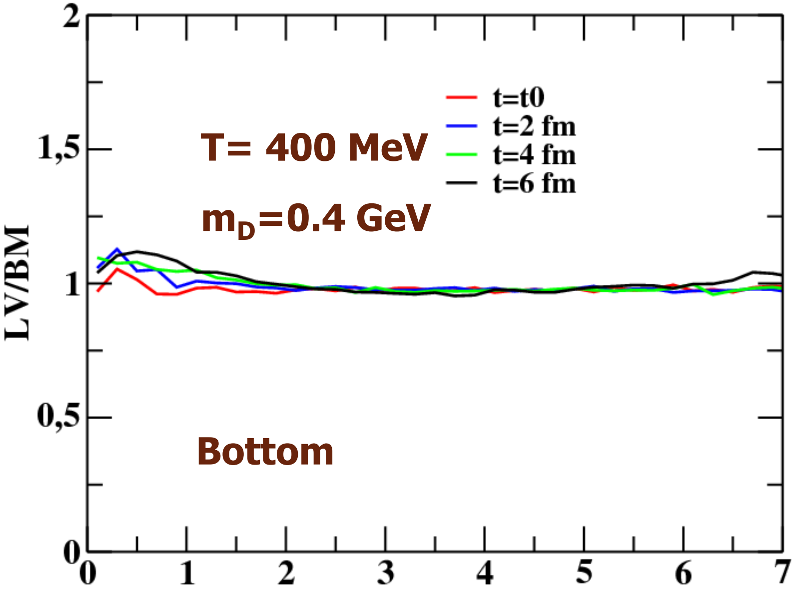


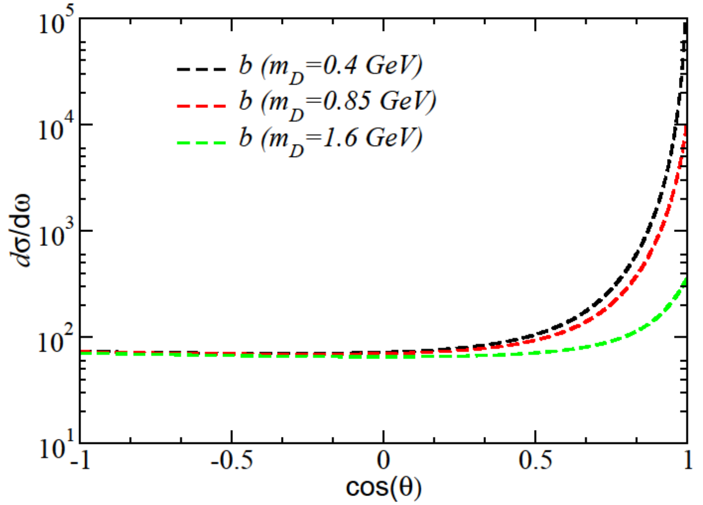


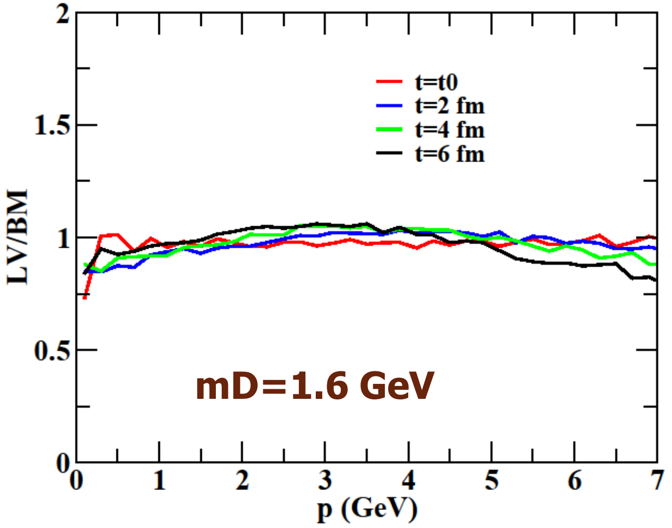
Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

Bottom: Boltzmann = Langevin









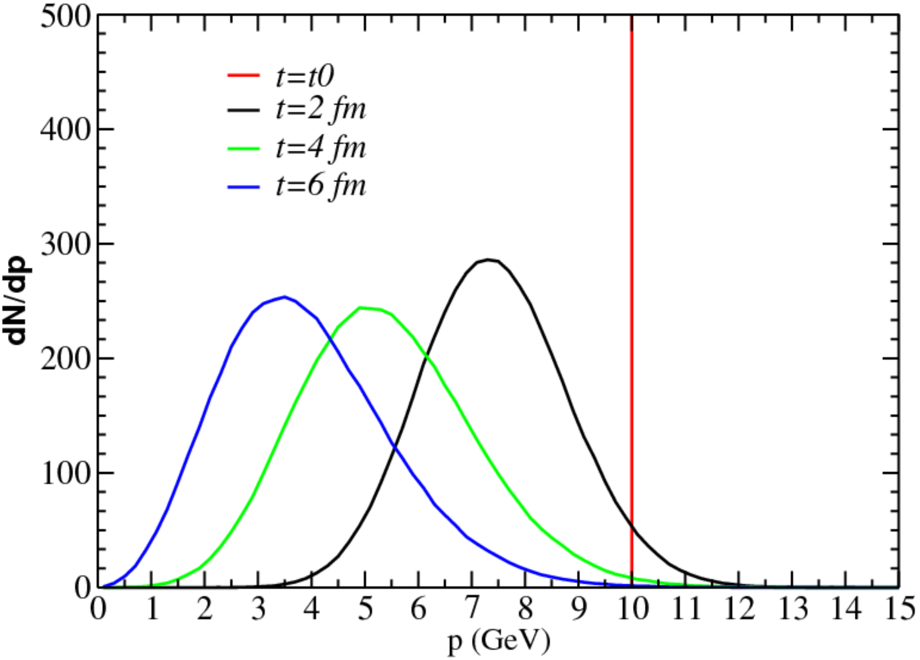
But Larger M_b/T (≈ 10) the better Langevin approximation works

Evolution: Boltzmann vs Langevin (Charm)

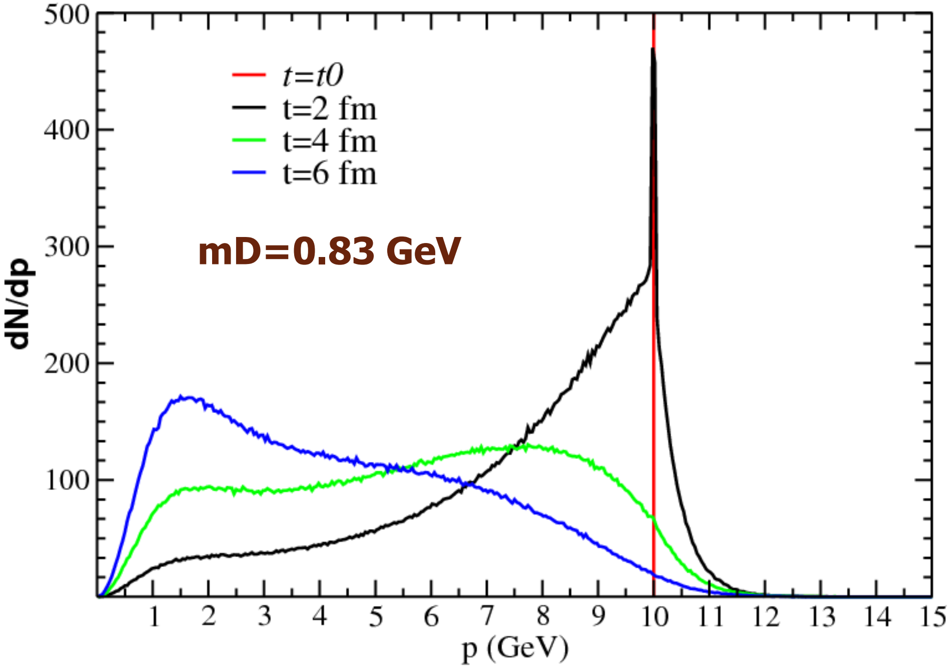
Momentum evolution starting from a δ (Charm) in a Box

$$\left.\frac{dN}{d^3p}\right|_{initial} = \delta(p - 10GeV)$$

Langevin



Boltzmann



In case of Langevin the distributions are Gaussian as expected by construction

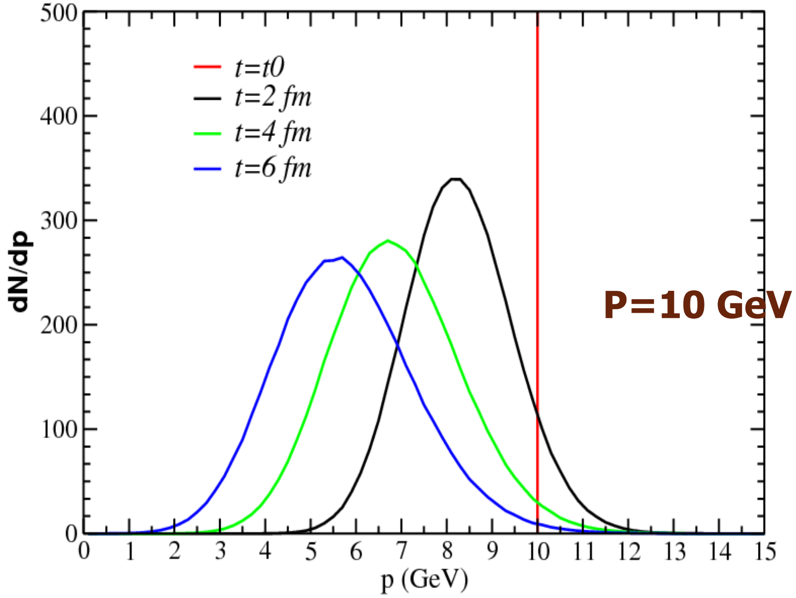
In case of Boltzmann the charm quarks does not follow the Brownian motion

Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

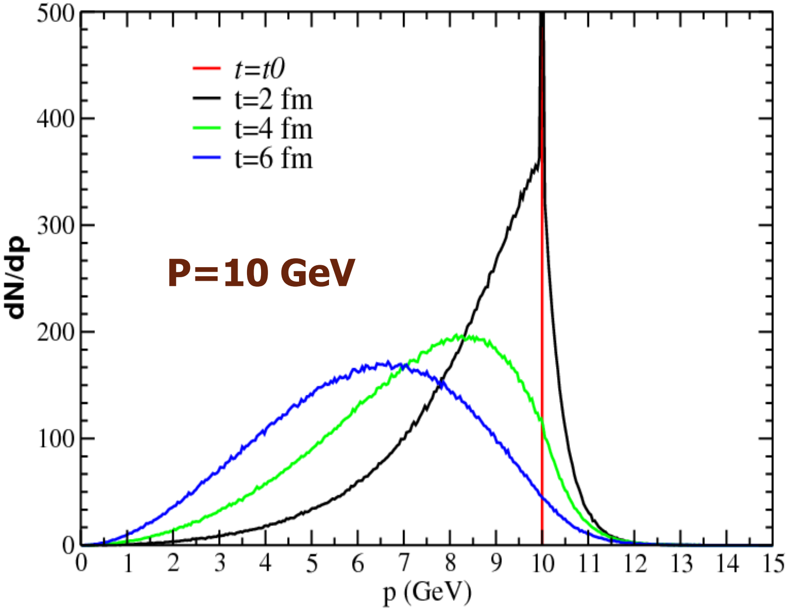
Momentum evolution starting from a δ (Bottom)

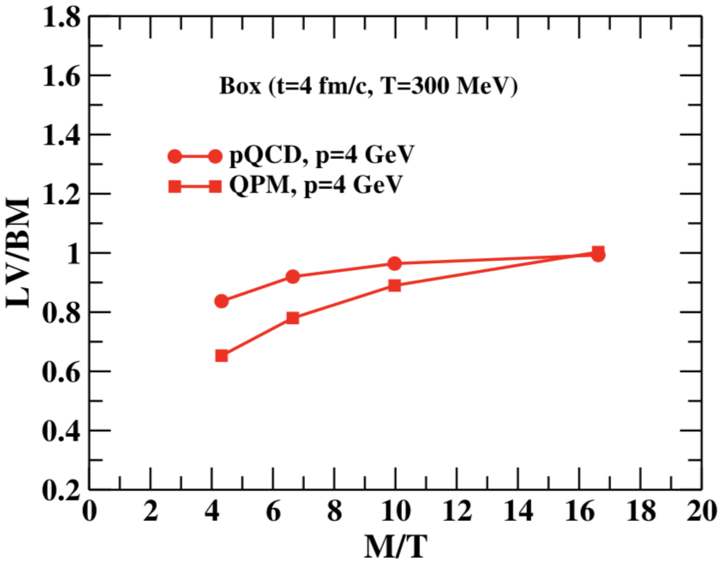
In a Box

Langevin



Boltzmann





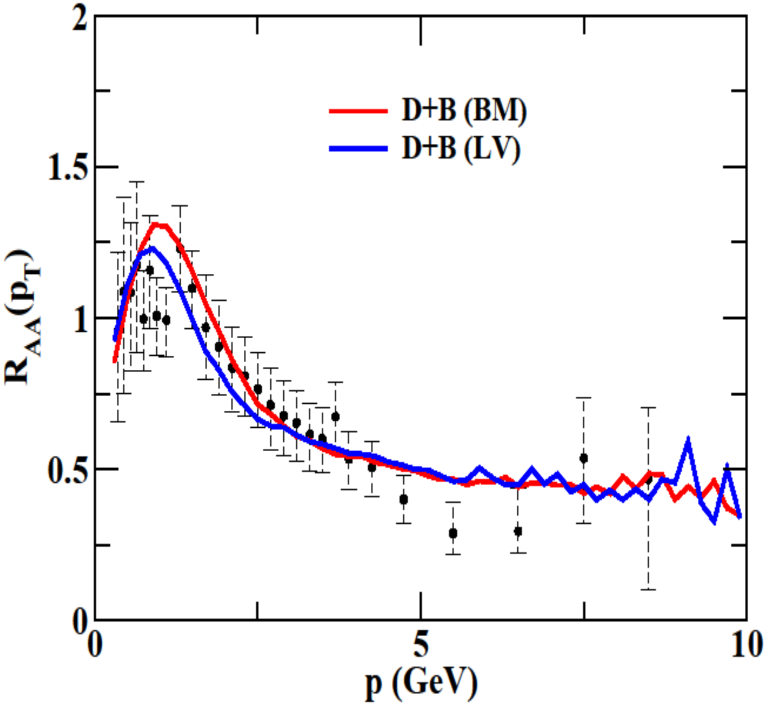
Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

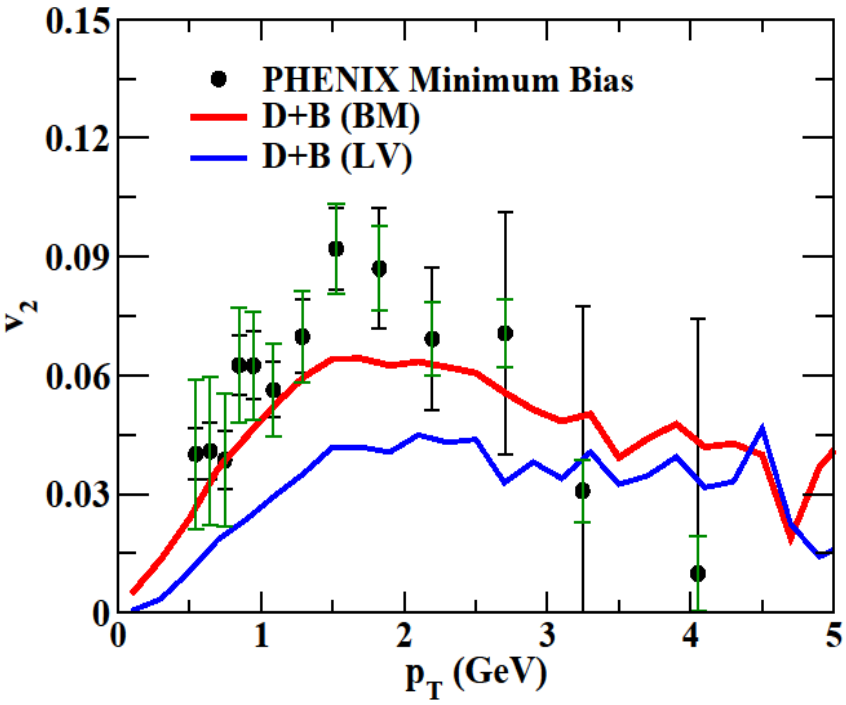
Langevin dynamics overestimate the interaction
Boltzmann generate more v2 for the same RAA.

EMMI-RRTF, NPA 979 (2018)

R_{AA} and v2 at RHIC

(With near isotropic cross-section)





Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

With isotropic cross section one can describe both RAA and V2 simultaneously within the Boltzmann approach !

Hadronization: Coalescence plus Fragmentation

Fragmentation function gives the probability to get a hadron from a parton:

$$f_H(p_T) = \sum_p f_p(p_T / z) \otimes D_{p\to H}(z)$$

<z>~0.9 for charm quark and <z>~0.5 for light quark

Das, Torres-Rincon, Tolos, Minissale, Scardina, Greco, PRD,94,114039,2016

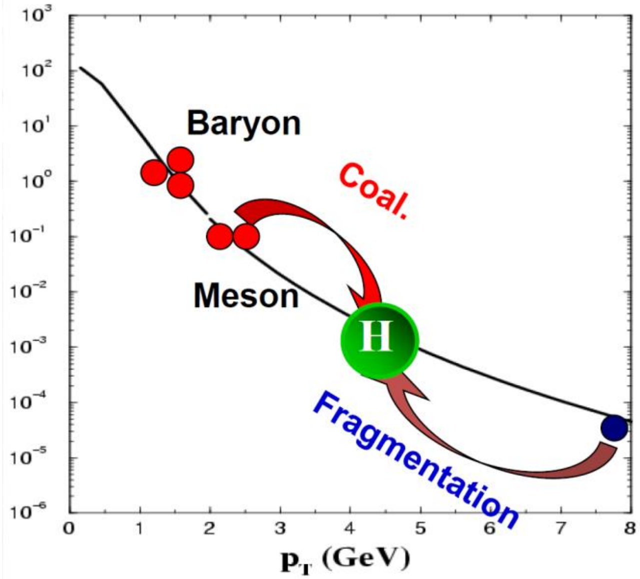
Coalescence is the convolution of two /three parton distribution folded by a wave function:

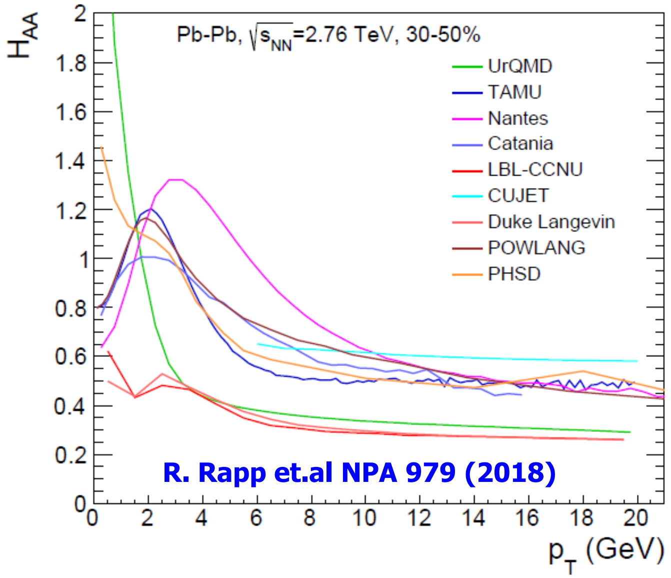
$$\frac{dN_{Meson}}{d^2 p_T} = g_M \sum_{i,j} P_q(i) P_q(j) \delta^{(2)}(p_T - p_{iT} - p_{jT}) f_M(x_i, x_j; p_i, p_j)$$

V. Greco, C.M. Ko, and P. L´evai PRL 90 , 202302 (2003)

Hadron wave function

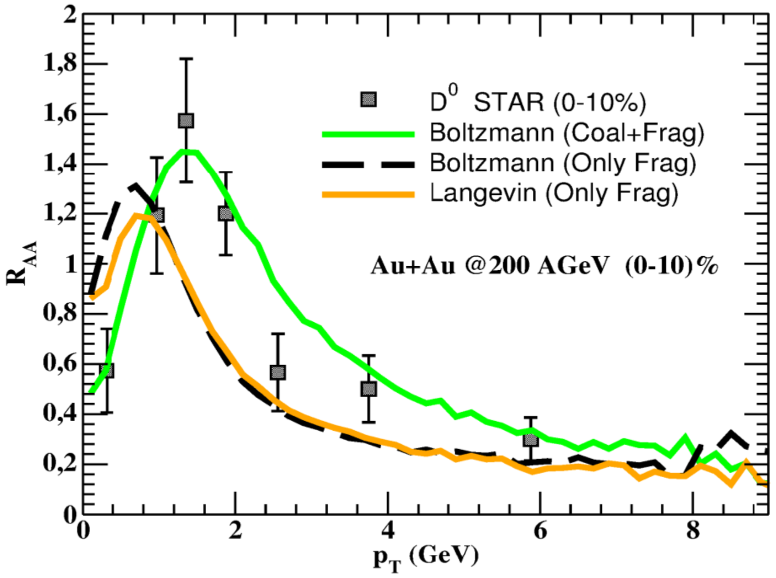
$$\frac{dN_{Baryon}}{d^2 p_T} = g_B \sum_{i,j,k} P_q(i) P_q(j) P_q(k) \delta^{(2)}(p_T - p_{iT} - p_{jT} - p_{kT}) f_B(x_i, x_j, x_k; p_i, p_j, p_k)$$





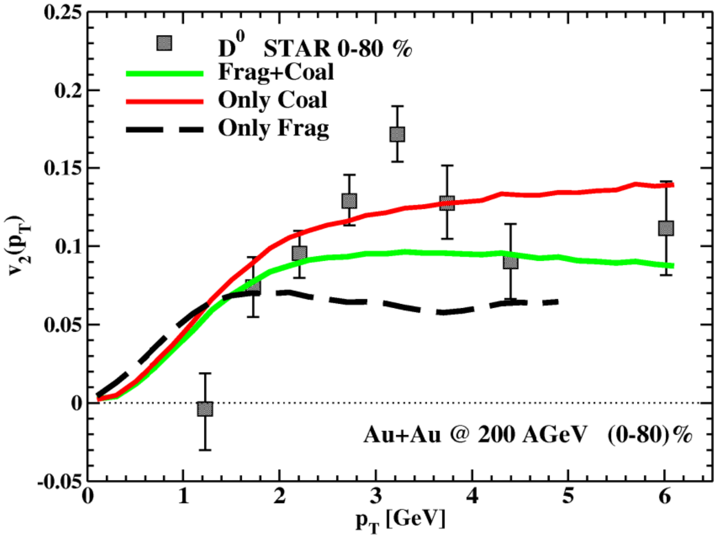
R. Rapp et.al NPA 979 (2018)

RHIC results: RAA vs v2



The impact of coalescence decreases with p_T.

This indicates charm quark get about 35% of v2 due to coalescence and about 65% due to in medium interaction (diffusion).



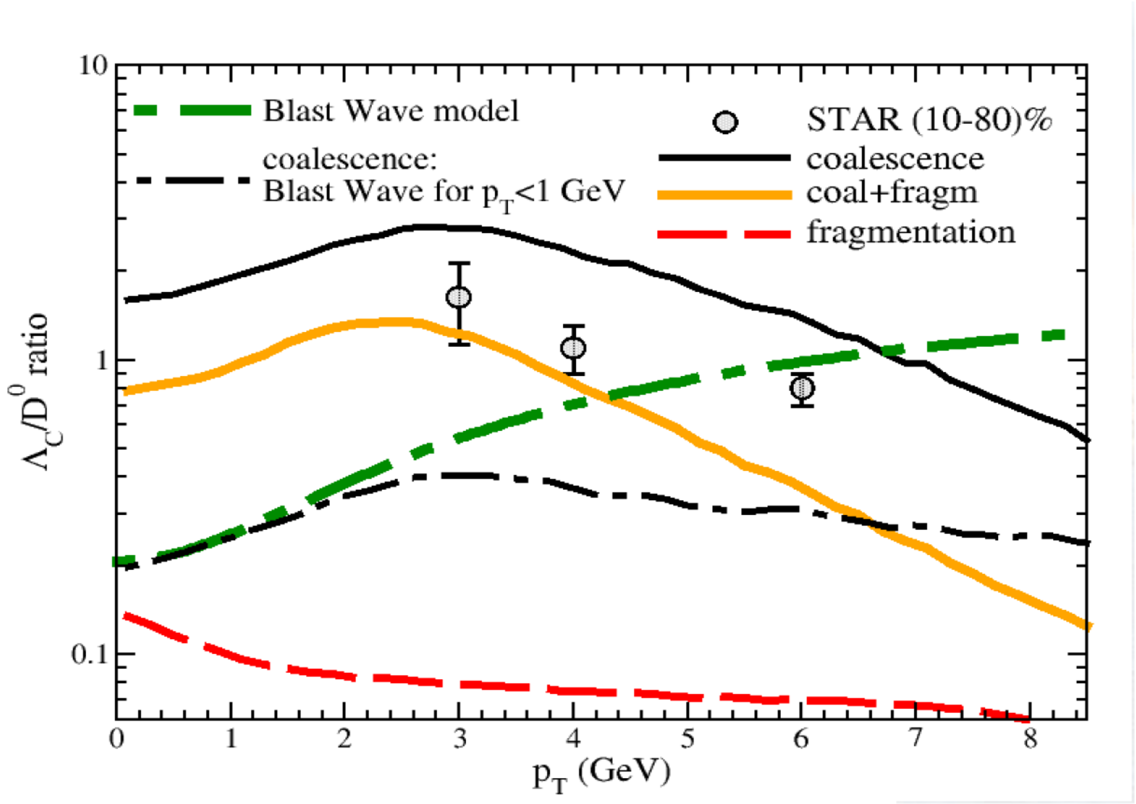
In (0-80)% the $v_2(p_T)$ due to only coalescence increase a factor 2 compared to the $v_2(p_T)$ charm

Scardina, Das, Minissale, Plumari, Greco
PRC, 96,044905 (2017)

BEFORE THE STUDY OF Λ_c / D^0 (early 2017)

Heavy Baryon to meson ratio

(Serve as a tool to disentangle different hadronization mechanisms)



We set:

$P_{coal}=1$ at p=0

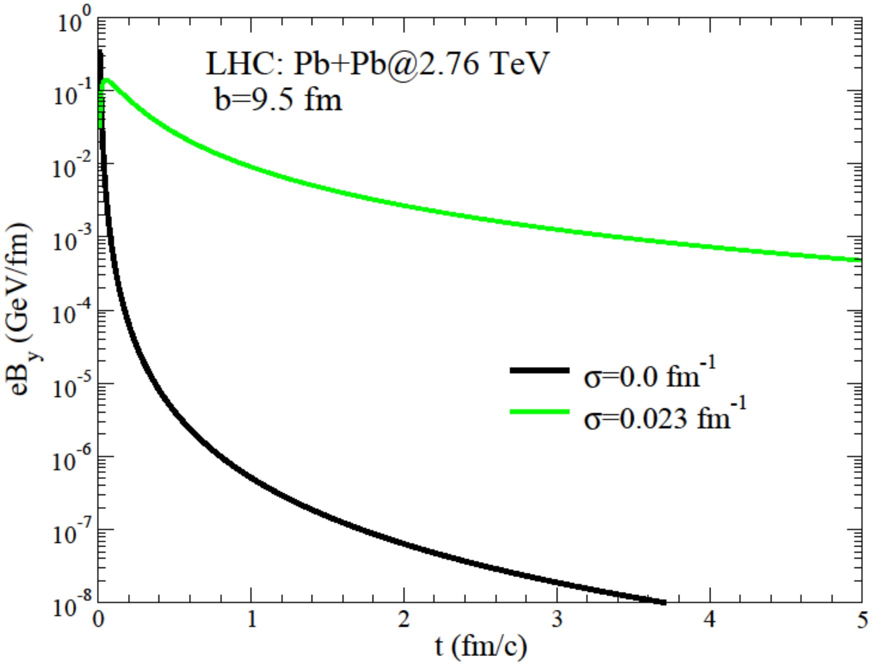
Plumari, Minissale, Das, Coci, Greco
EPJC, 78 (2018) 348

Impact of EM field on heavy quark dynamics at LHC

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\, C_{jk}(t, p + \xi dp)\rho_k + F_{ext} dt$$

$$F_{ext} = q(E' + v \times B')$$

$$E' = \gamma (E + v \times B) - (\gamma - 1)(E \cdot \hat{v})\hat{v}$$
$$B' = \gamma (B - v \times E) - (\gamma - 1)(B \cdot \hat{v})\hat{v}$$



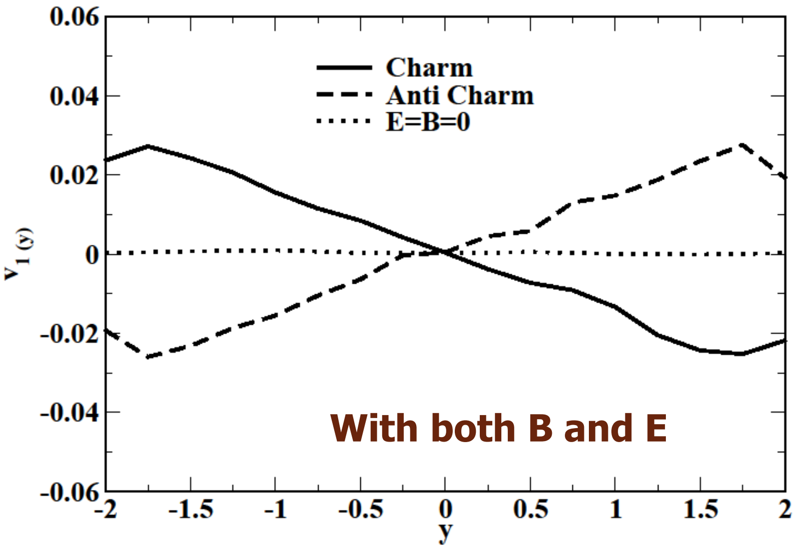
Electromagnetic field has been included in the Langevin equation as a external force.

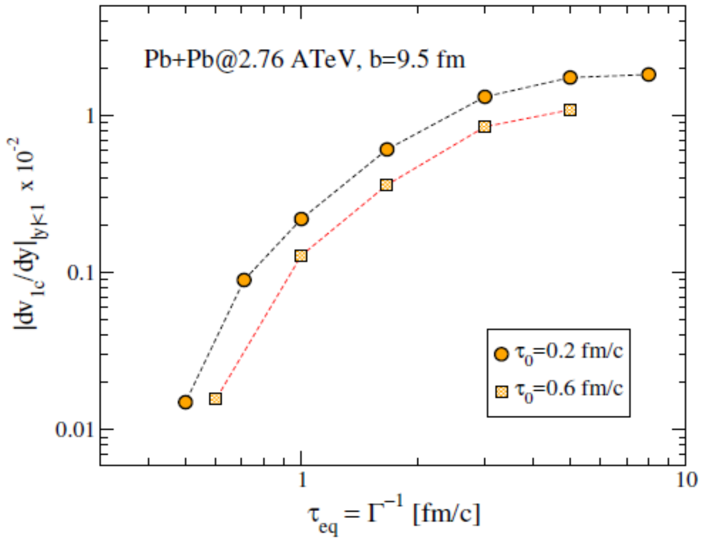
We consider both E and B.
Bx=Bz=0
And Ey=Ez=0

$$v_1 = < \frac{p_x}{p_T} >$$

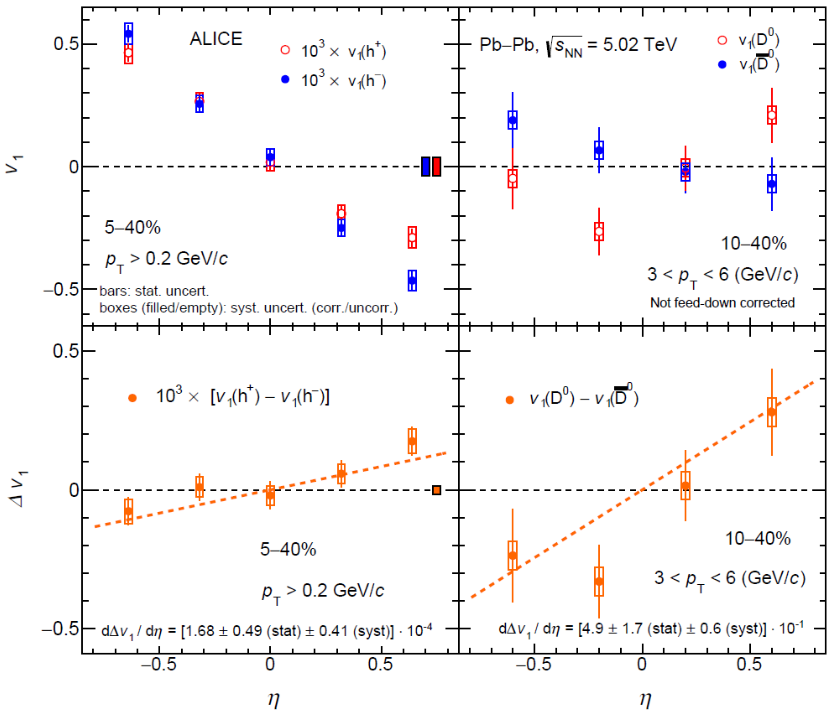
Das, Plumari, Chartarjee, Scardina, Greco, Alam
Phys. Lett. B, 768 (2017) 260

Heavy quark v1@LHC





Das, Plumari, Chartarjee, Scardina, Greco, Alam
Phys. Lett. B, 768 (2017) 260

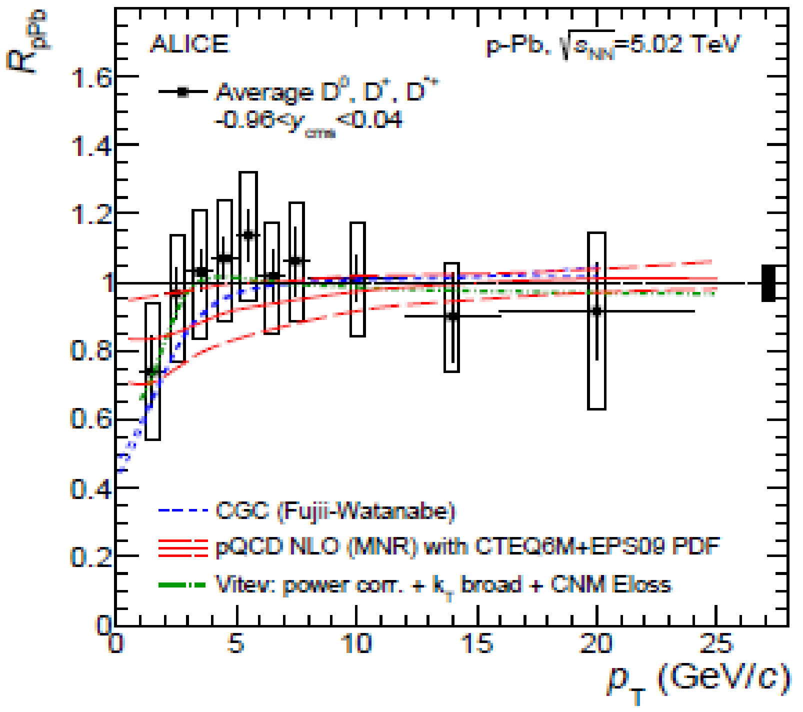


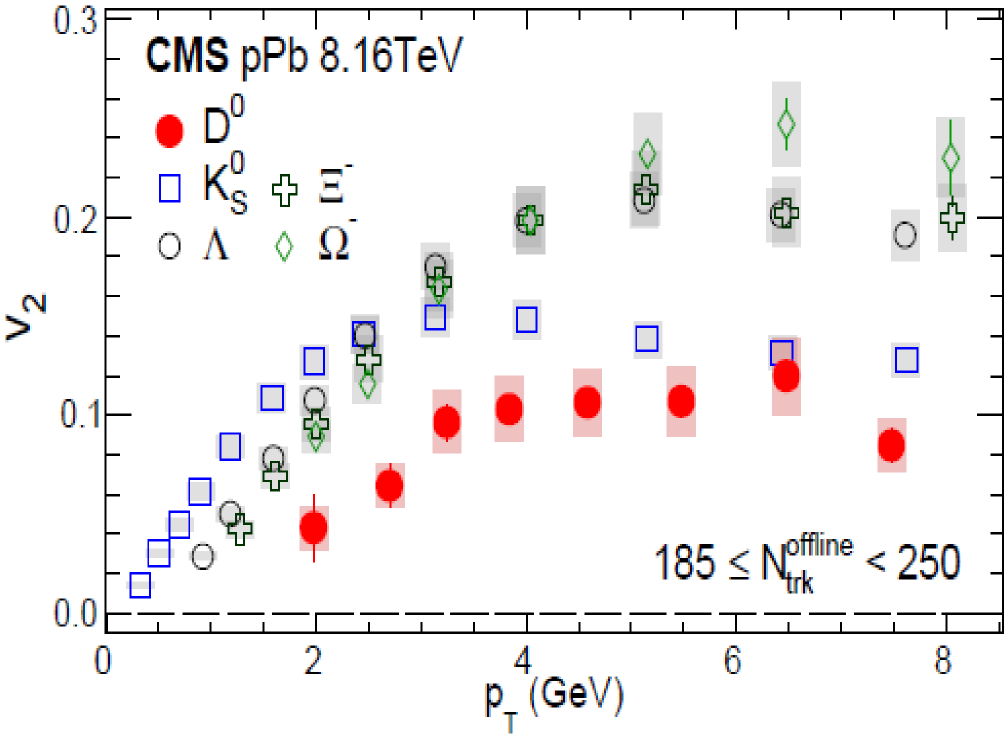
Heavy quark v1 is larger than light quark v1.

Recent data from ALICE indicates splitting in D and Dbar v1.

arxiv:1910.14406 (ALICE Collaboration)

Heavy quark in small system (p-nucleus)



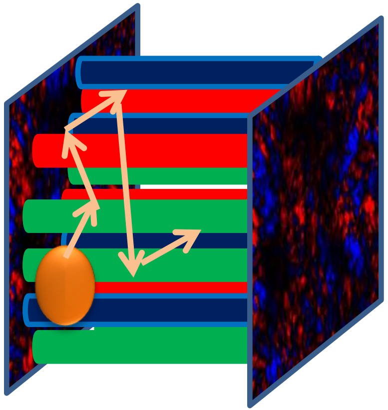


ALICE Collaboration
Phys. Rev. Lett. 113 (2014) 232301

CMS Collaboration
arXiv:1804.09767v2

What mechanism could build up v_2 without energy loss?

Heavy quarks as probes of the evolving Glasma



(Adapted from M. Ruggieri)

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06\ \text{fm/c}$$

HQs can probe the very early evolution of the Glasma fields

Hamilton equations of motion of *c*-quarks:

$$\frac{dx_i}{dt} = \frac{p_i}{E}$$

$$E = \sqrt{\boldsymbol{p}^2 + m^2}$$

$$\boldsymbol{v} \equiv \frac{\boldsymbol{p}}{E}$$

(Relativistic) Velocity

$$E\frac{dp_i}{dt} = gQ_a F^a_{i\nu} p^\nu .$$

$$\frac{d\boldsymbol{p}}{dt} = q\boldsymbol{E} + q\left(\boldsymbol{v} \times \boldsymbol{B}\right)$$

Lorentz force

$$E\frac{dQ_a}{dt} = -gQ_c \varepsilon^{cba} \boldsymbol{A}_b \cdot \boldsymbol{p}$$

Wong (1979)

$$D_\mu J^\mu_a = 0$$

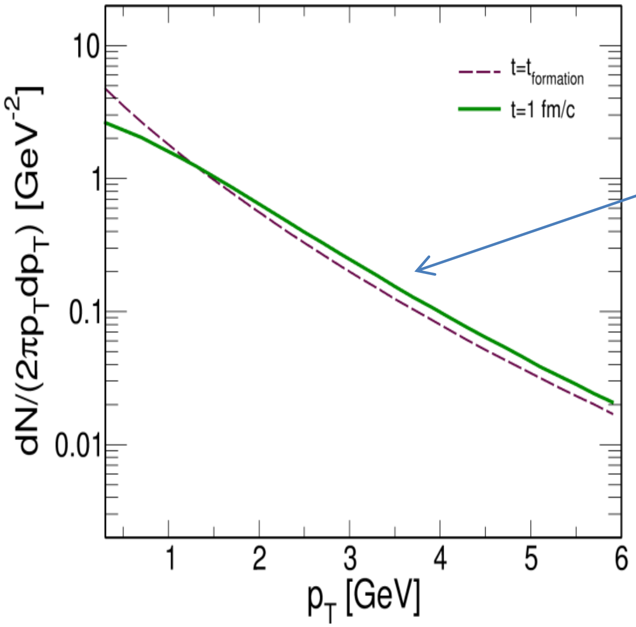
$$J^\mu_a = \bar{c}\gamma^\mu T_a c$$

Gauge-invariant conservation of the color Current carried by charm quaks + gluons

Equations of motion of heavy quarks are solved in the background given by the evolving Glasma fields

p-Pb @ 5.02 TeV

Nuclear modification factor (R_{pPb}) for p-Pb collisions

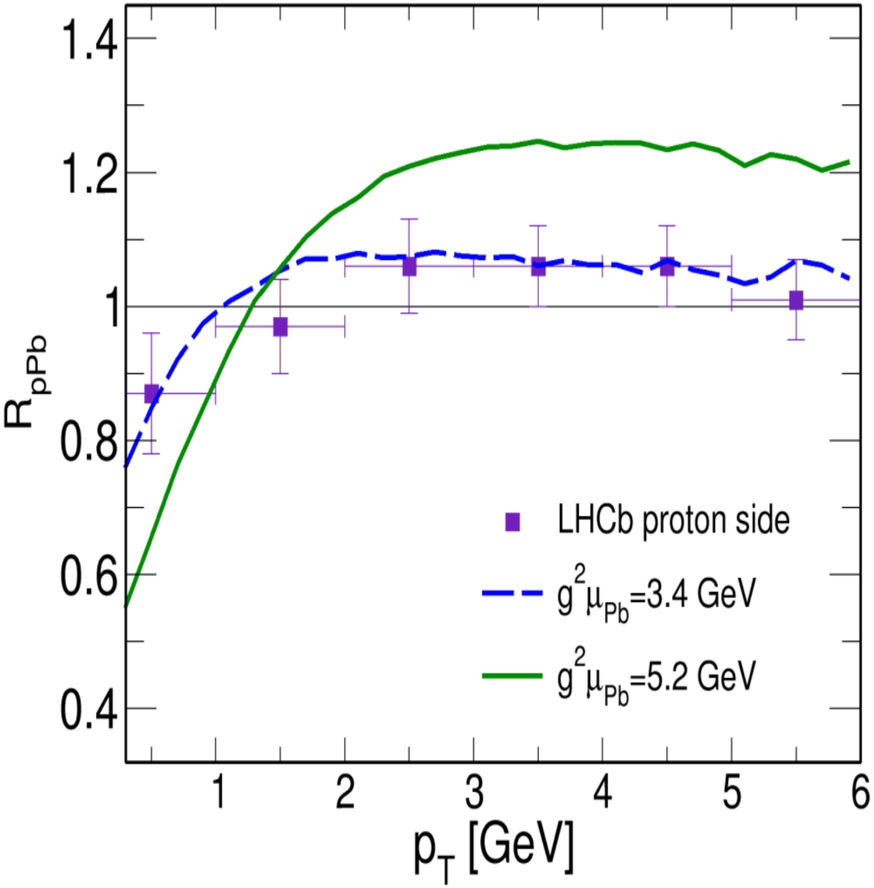


Initial distribution: from perturbative QCD

Evolution: interaction with the Glasma

D-mesons R_{pPb}

Standard fragmentation **[Peterson *et al.*(1983)]**



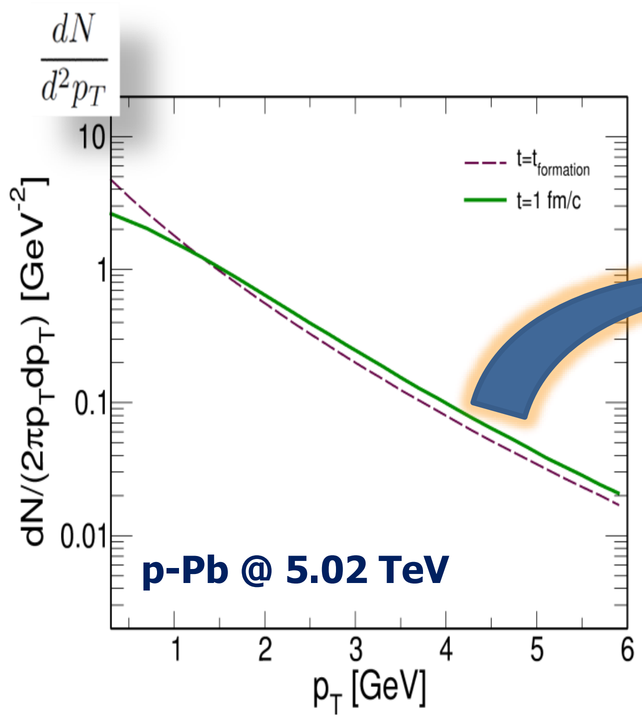
$$R_{\mathrm{pPb}} = \frac{(dN/d^2p_T)_{\mathrm{final}}}{(dN/d^2p_T)_{\mathrm{pQCD}}}$$

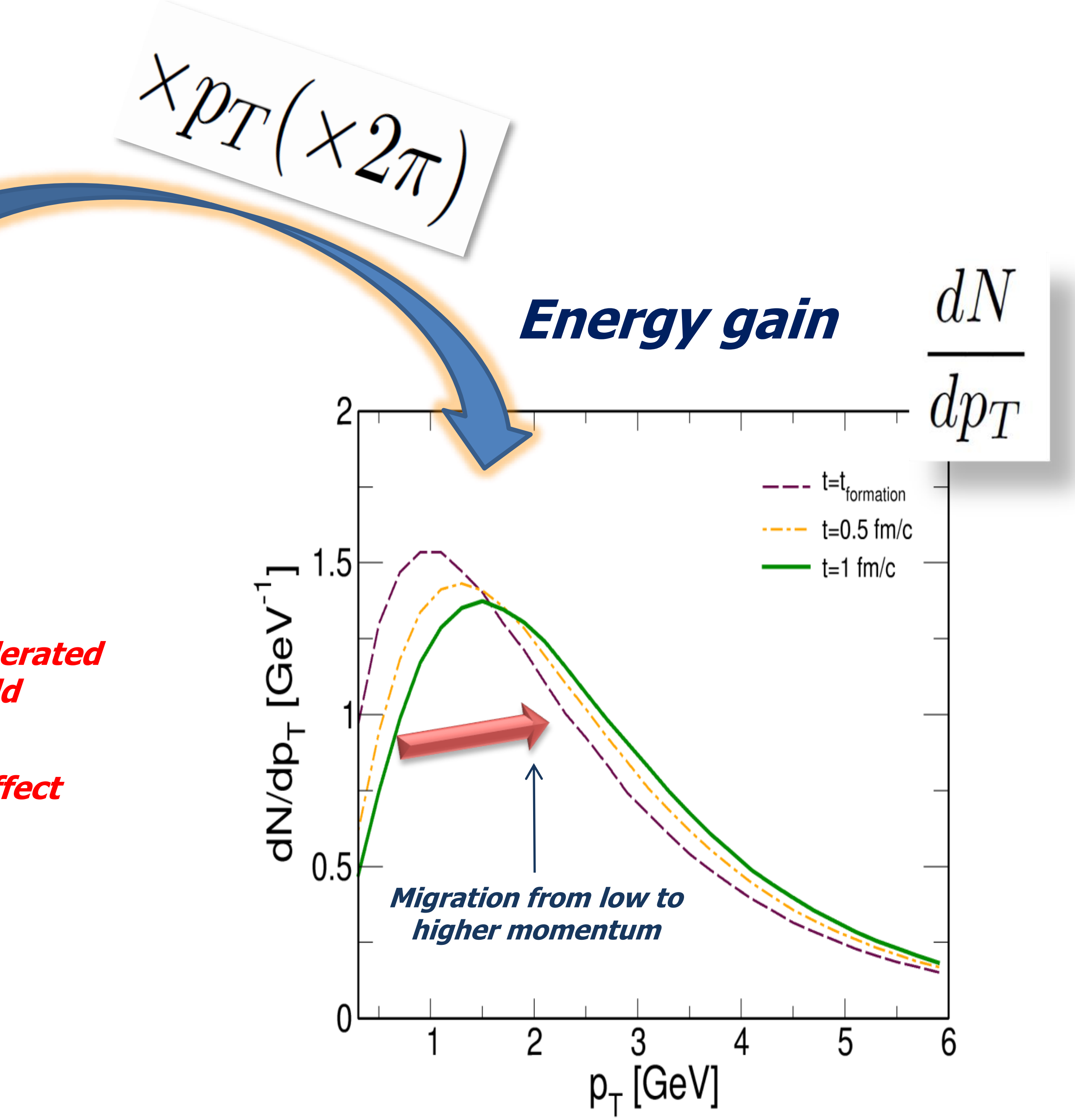
$R_{pPb} \neq 1$

Interaction with the fields created by the collision

Ruggieri and Das
PRD, 98, 094024(2018)

Diffusion results in acceleration





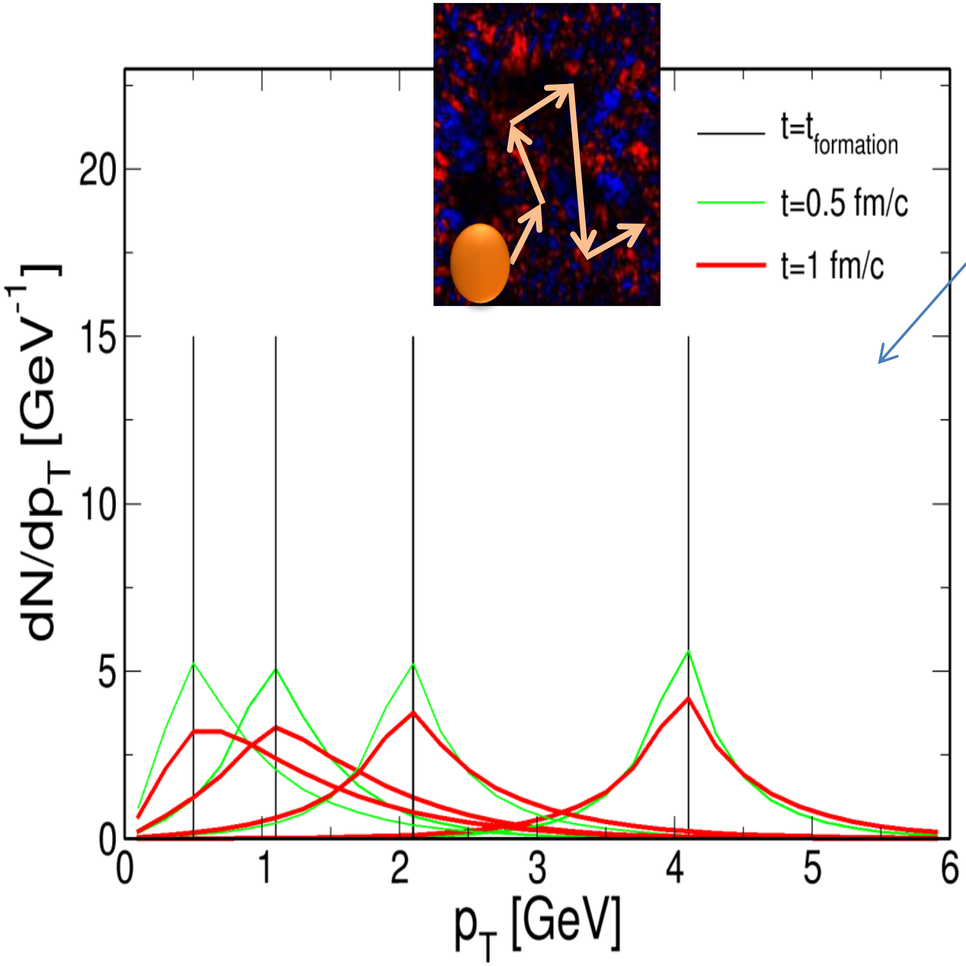
Heavy quarks seem to be accelerated by the (color-)electric field

We called it cathode tube effect

Ruggieri and Das
PRD, 98, 094024(2018)

Heavy quarks diffusion: Plasma vs Glasma

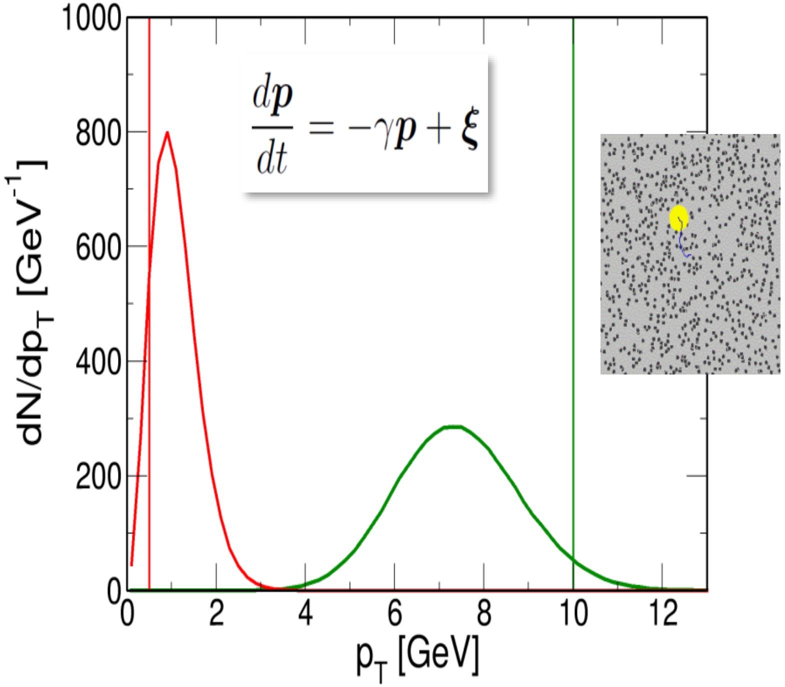
HQs in Glasma



Diffusion in momentum space

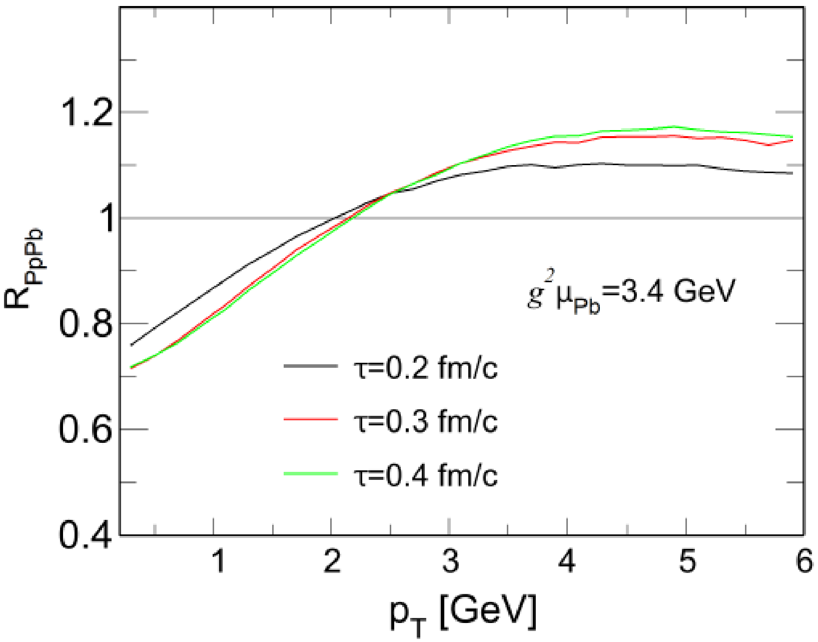
Ruggieri and Das
PRD, 98, 094024(2018)

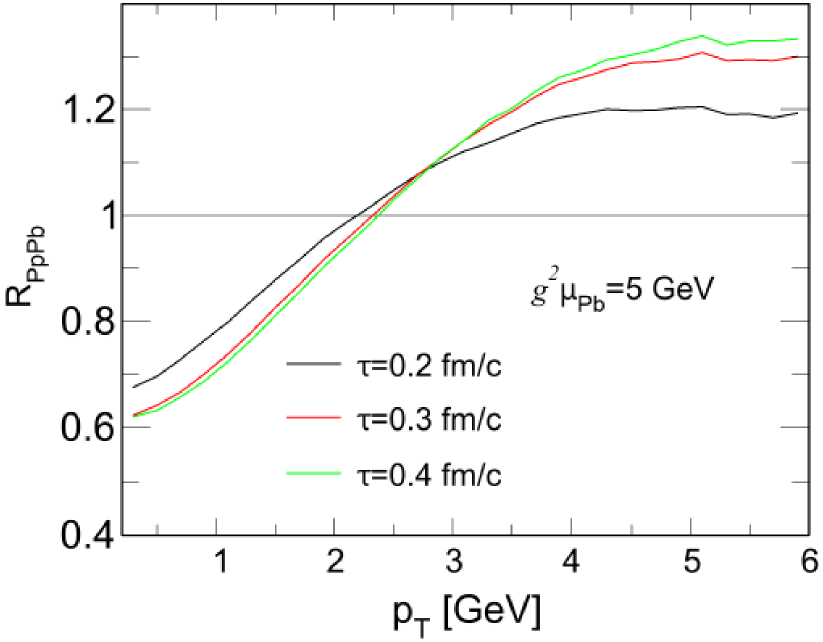
T=500 MeV ***HQs in hot plasma***

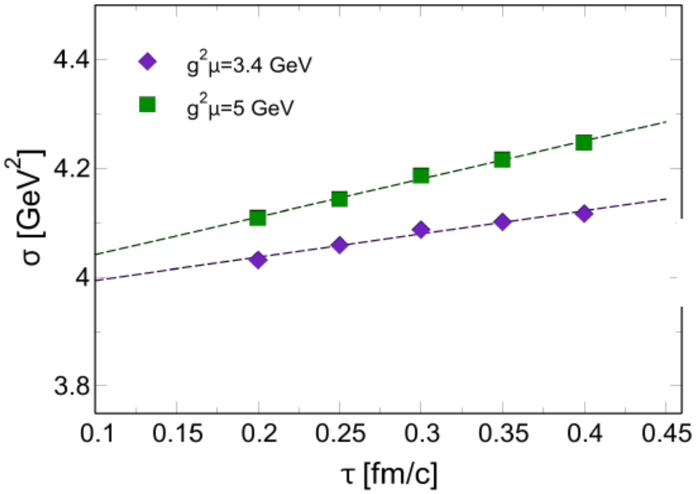


$$\frac{dp}{dt} = -\gamma p + \xi$$

Heavy quark dynamics in Expanding Glasma

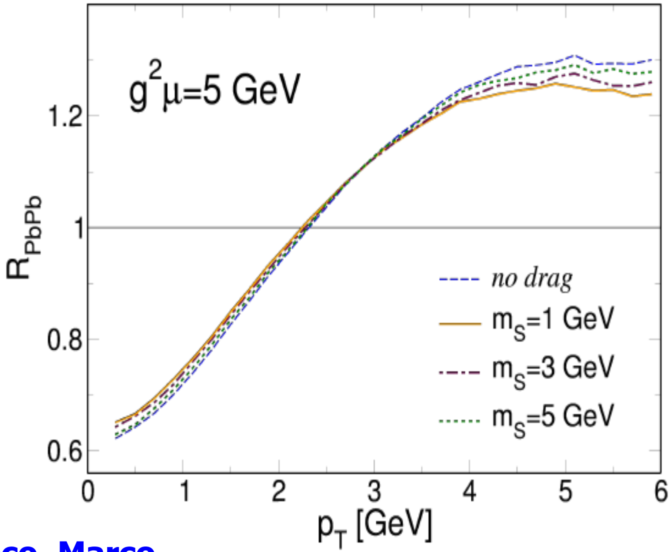






$$E\frac{dp_i}{dt} = Q_a F^a_{i\nu} p^\nu - E\Gamma p_i,$$

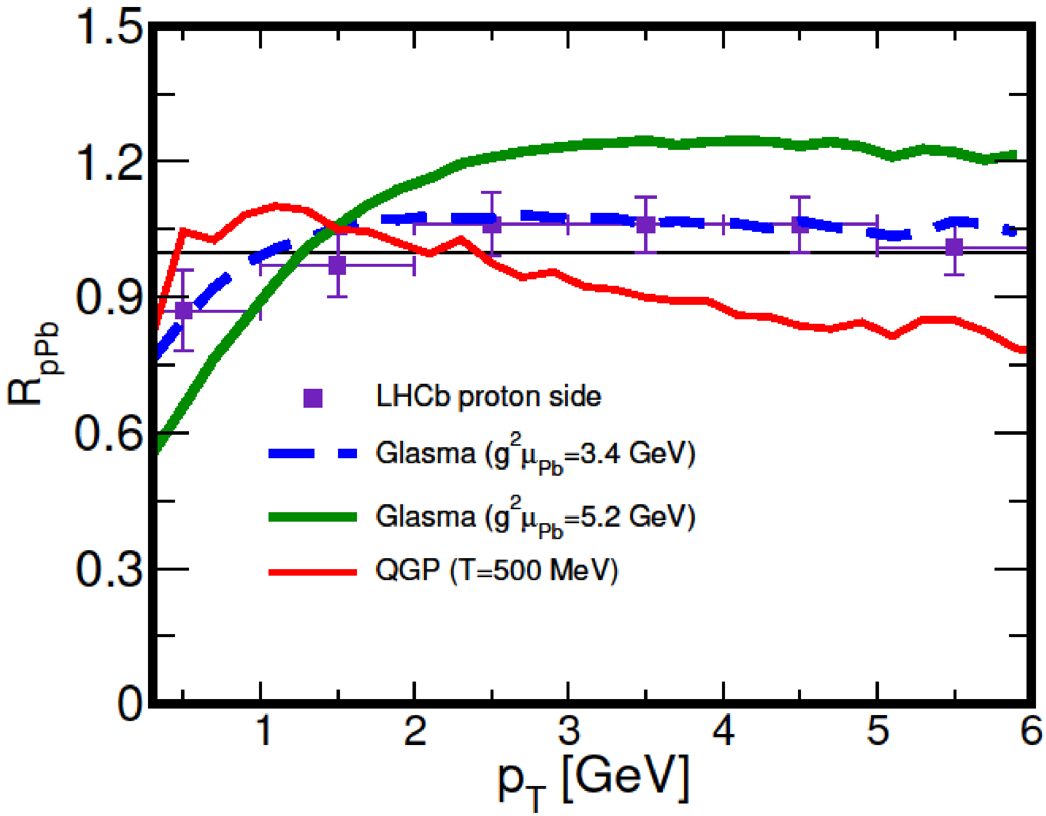
$$D = \Gamma E T,$$



$$\sigma \equiv \langle (p_T - \langle p_T \rangle)^2 \rangle = 2D\tau \ + \ \text{constant}$$

Liu, Plumari, Das, Greco, Marco
arxiv:1911.02480

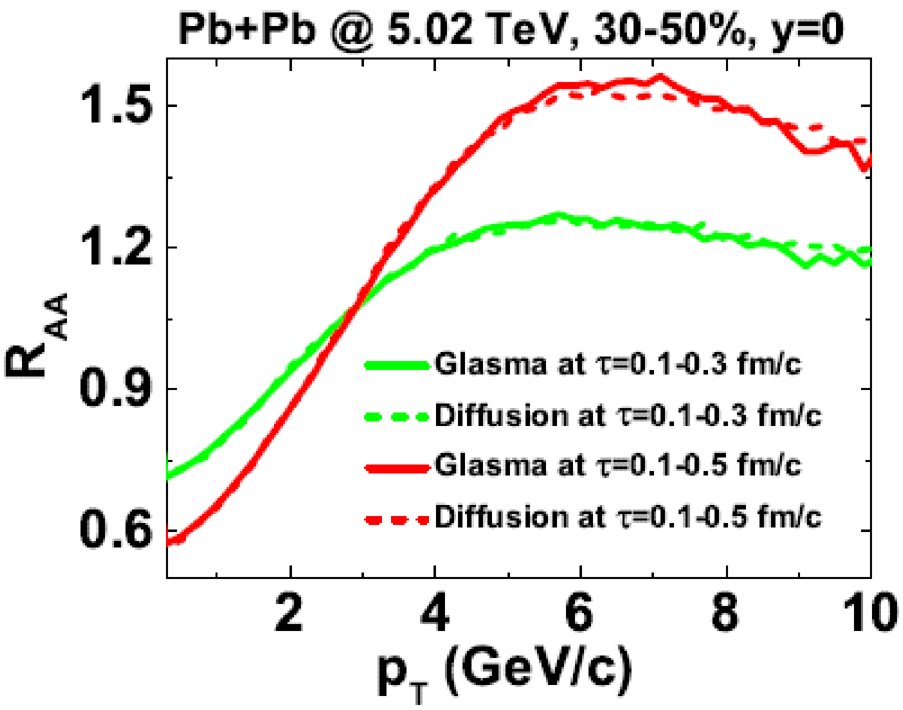
Heavy quark suppression in pPb: Glasma vs Plasma

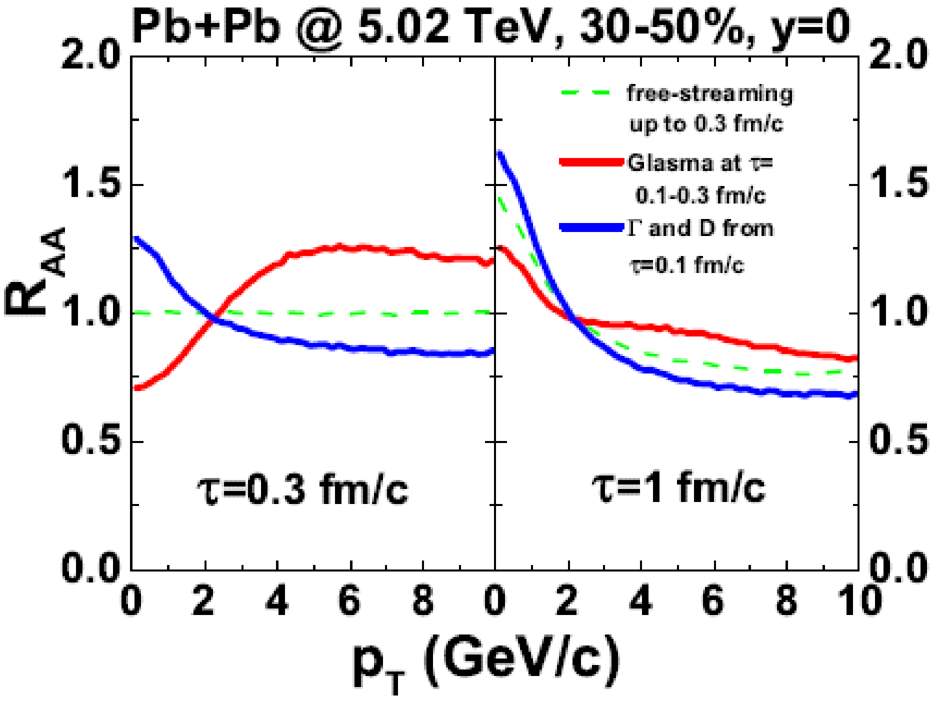


In Plasma: high momentum particle loose energy shifted to low momentum domain.
In Glasma: low momentum particle get accelerated and shifted to high momentum domain

Liu , Das, Ruggieri, Plumari, Greco
Under preparation

Impact of Glasma on a heavy quark observables at LHC
(Glasma vs Plasma)

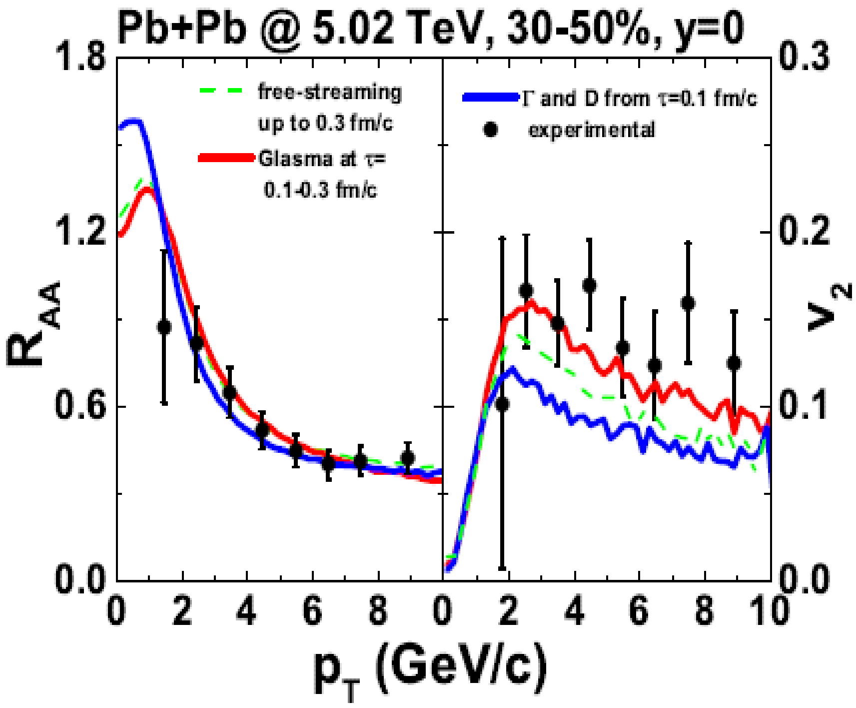


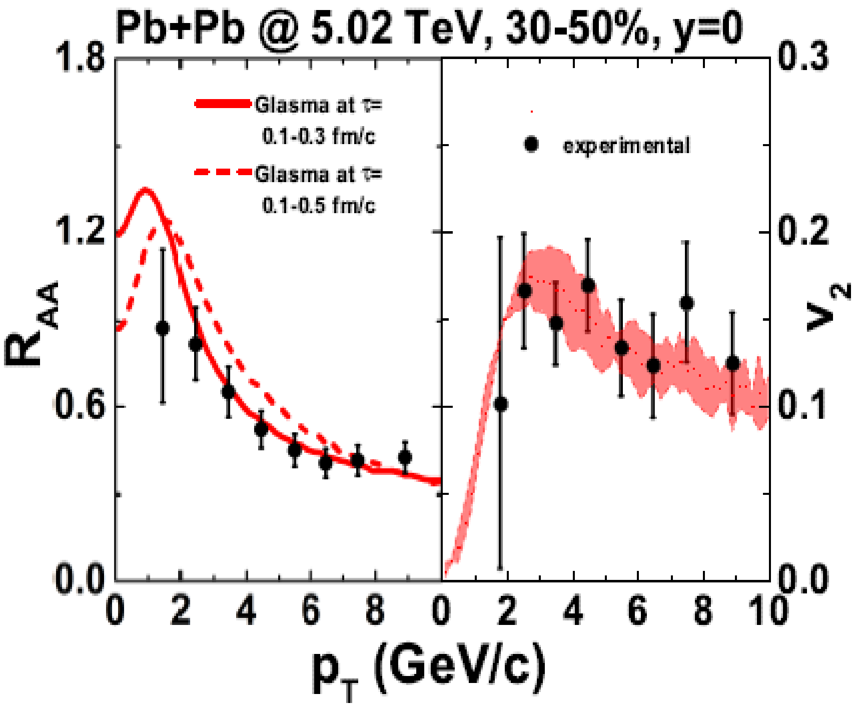


Glasma induce a diffusion of charm quarks in momentum space resulting in a tilt of their spectrum without a significant drag.

Sun, Coci, Das, Plumari, Ruggieri, Greco
PLB, 798 (2019) 134933

Impact of Glasma on a heavy quark observables at LHC
(Heavy quark dynamics in Glasma plus Plasma)





This indicates an initial pre-thermal stage is unlikely to be described in terms of a standard drag and diffusion dynamics, because even if one tune such coefficients to reproduce the same $R_{AA}(p_T)$ this would imply a significantly smaller v_2.

Sun, Coci, Das, Plumari, Ruggieri, Greco
PLB, 798 (2019) 134933

Summary & Outlook ……

- Heavy quarks are the novel probe to characterized QGP and to probe initial state.
- Our study indicates the temperature of the system produced at RHIC (T=340 MeV) and LHC (T=510 MeV) energies are much larger than the temperature needed to create the QGP.
- Several new experiments are coming up (FAIR, FCC) and we are looking for new observables which help us to understand several basic issues ...

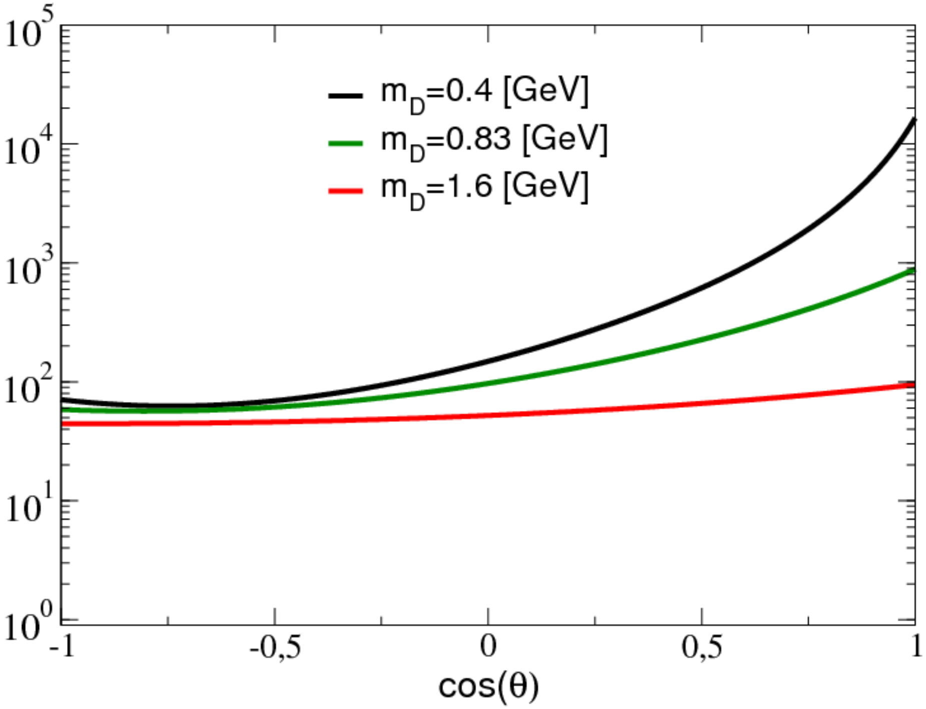
  - Heavy quark diffusion coefficient in QGP and in Glasma.
  - Hadronization.
  - Einstein relation will be studied.
  - Heavy quark thermalization.
  - QGP in small system (p-Au)
  - To probe the effect of initial magnetic field.

Thank You

Boltzmann vs Langevin (Charm)

T=400 MeV

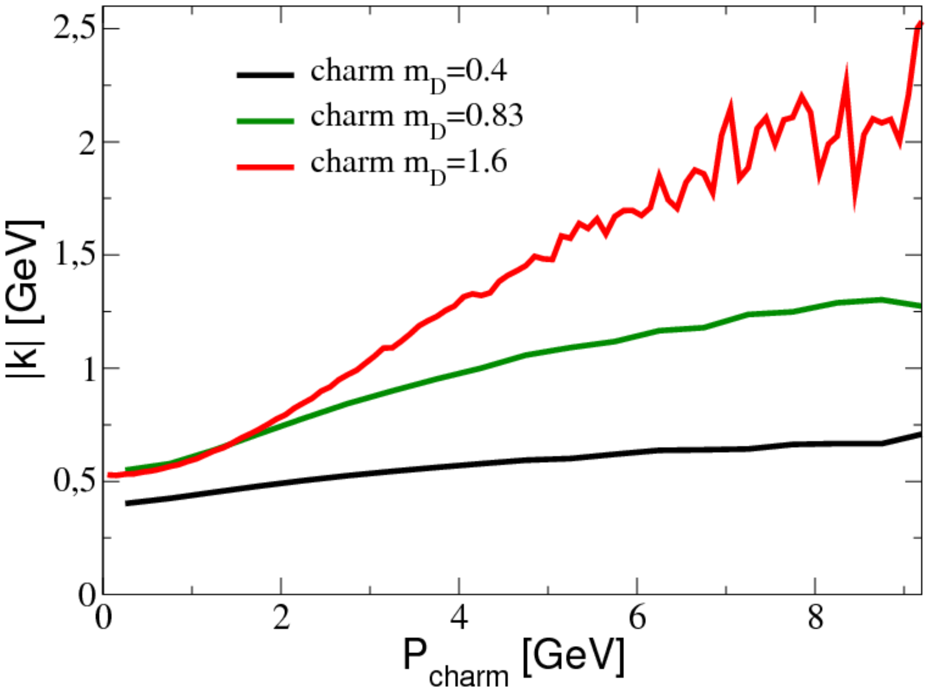
Angular dependence of σ



Decreasing m_D makes the σ more anisotropic

Hees,Greco,Rapp,PRC,73,034913 (2006)

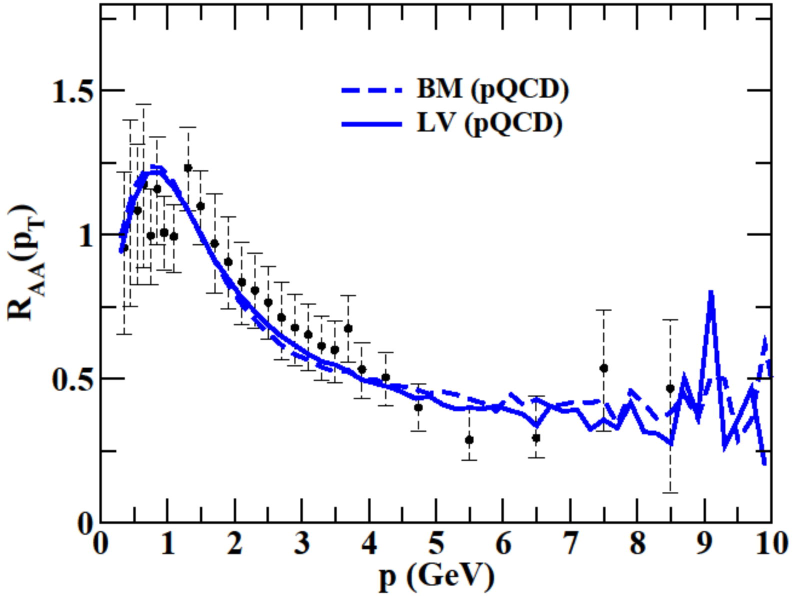
Mometum transfer vs P

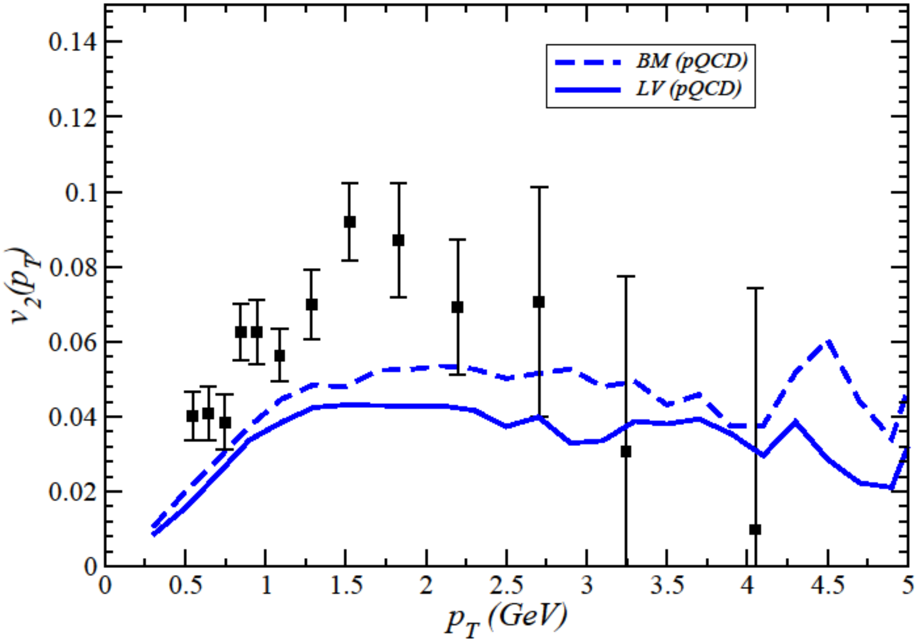


➡ **Smaller average momentum transfer**

Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

R_{AA} and v2 at RHIC at mD=gT





Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

I) LPM effect : Suppression of bremsstrahlung and pair production.

Formation length ($l_f = \frac{\hbar}{q_\perp}$) : The distance over which interaction is spread out

1) It is the distance required for the final state particles to separate enough that they act as separate particles.

2) It is also the distance over which the amplitude from several interactions can add coherently to the total cross section.

As $q_\perp$ increase → ℓ_f reduce → Radiation drops proportional

S. Klein, Rev. Mod. Phys **71** (1999)1501

(II) Dead cone Effect : Suppression of radiation due to mass

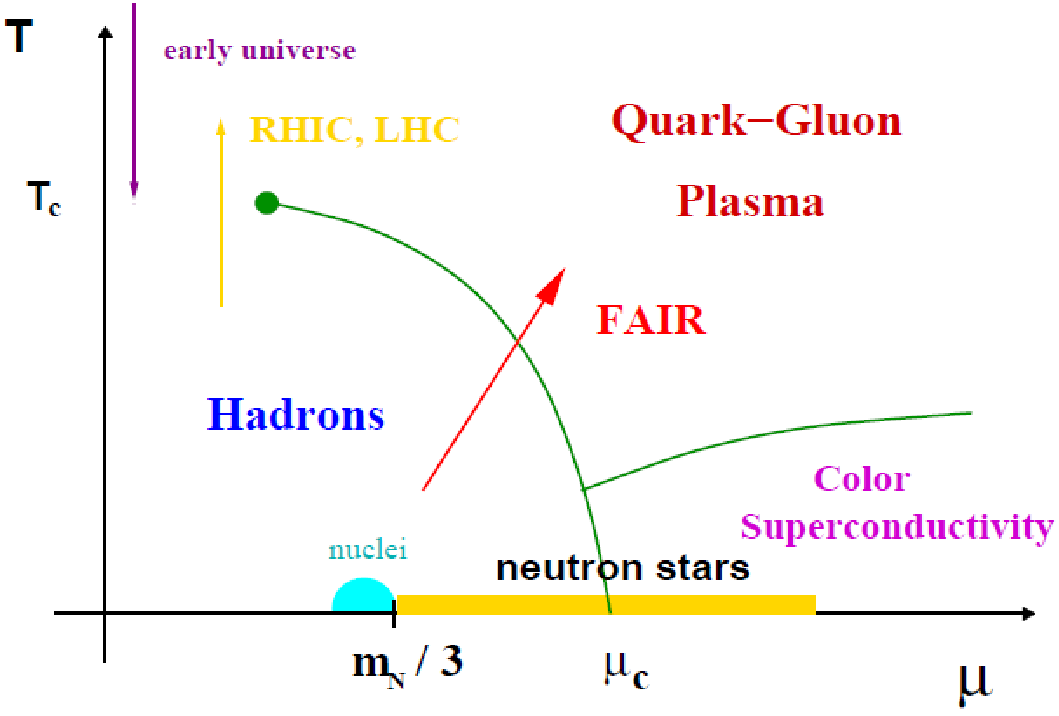
$$\boxed{\frac{1}{\sigma}\frac{d^2\sigma}{dz\,d\theta^2} \sim C_F\frac{\alpha_s}{\pi}\frac{1}{z}\frac{\theta^2}{\left(\theta^2+4\gamma\right)^2}}$$

where $z = 2 - x_1 - x_2$ *and* $\gamma = \frac{m^2}{s}$

Where $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ $\longrightarrow$ the energy fraction of the final state quark and anti-quark.

Radiation from heavy quarks suppress in the cone from $\theta = 0$ (minima) to $\theta = 2\sqrt{\gamma}$ (maxima)

Landscape of different phase of matter



Deconfined phases of QCD matter at two extreme conditions.

Life time of QGP ~ 8-10 fm