

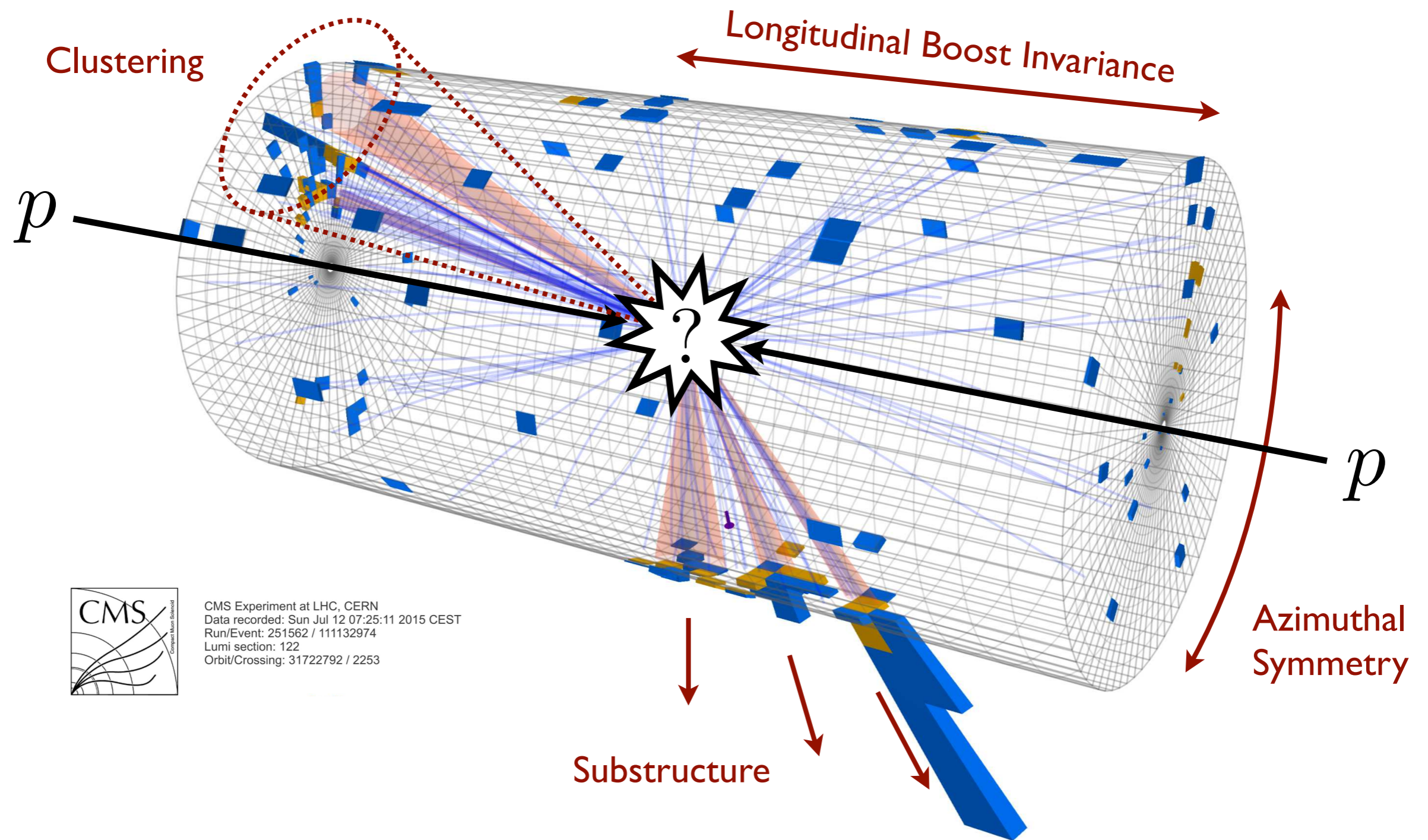
The Hidden Geometry of Particle Collisions

Jesse Thaler

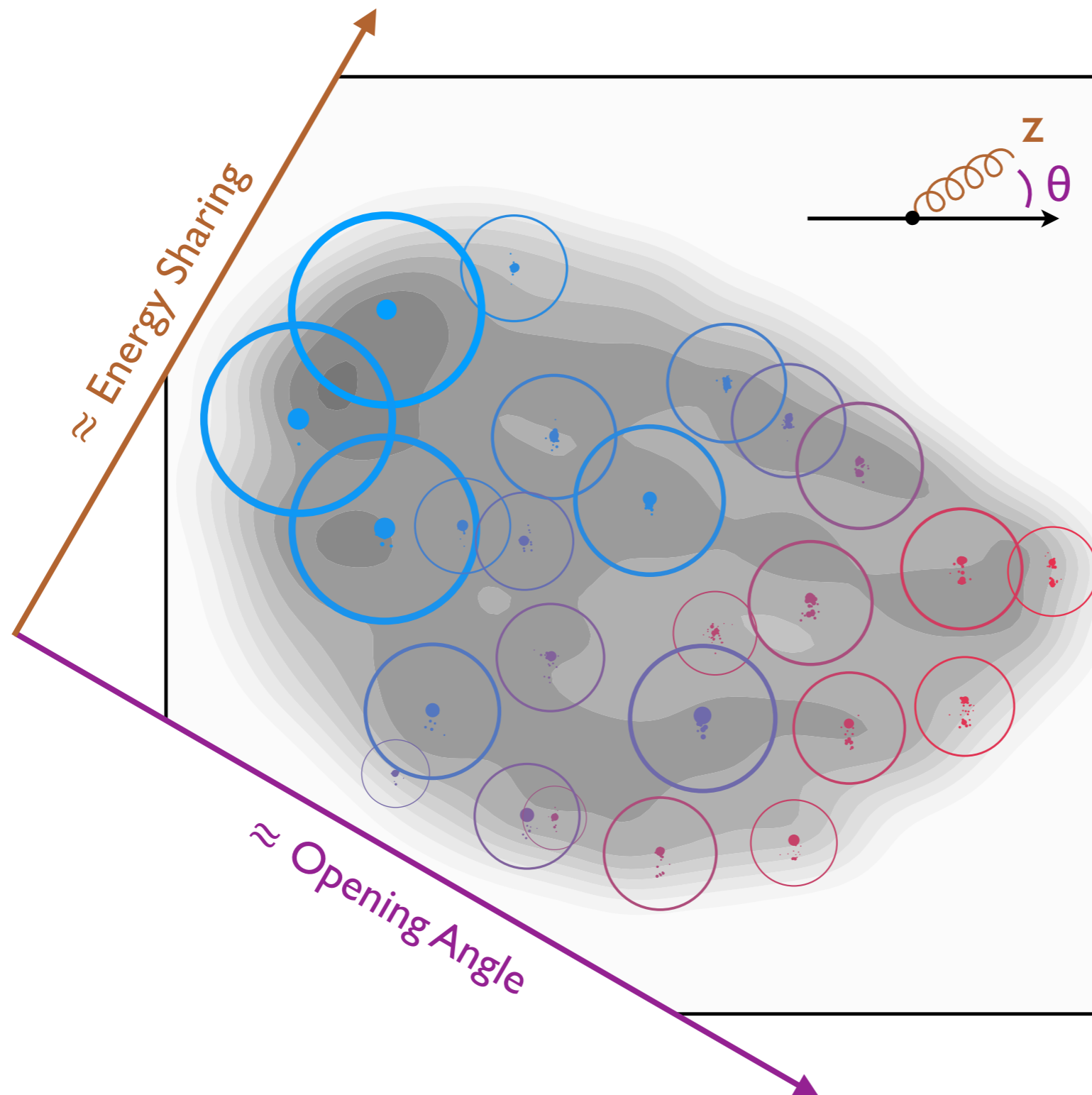


Free Meson Seminar, Tata Institute of Fundamental Research — June 18, 2020

The Manifest Geometry of One Collision



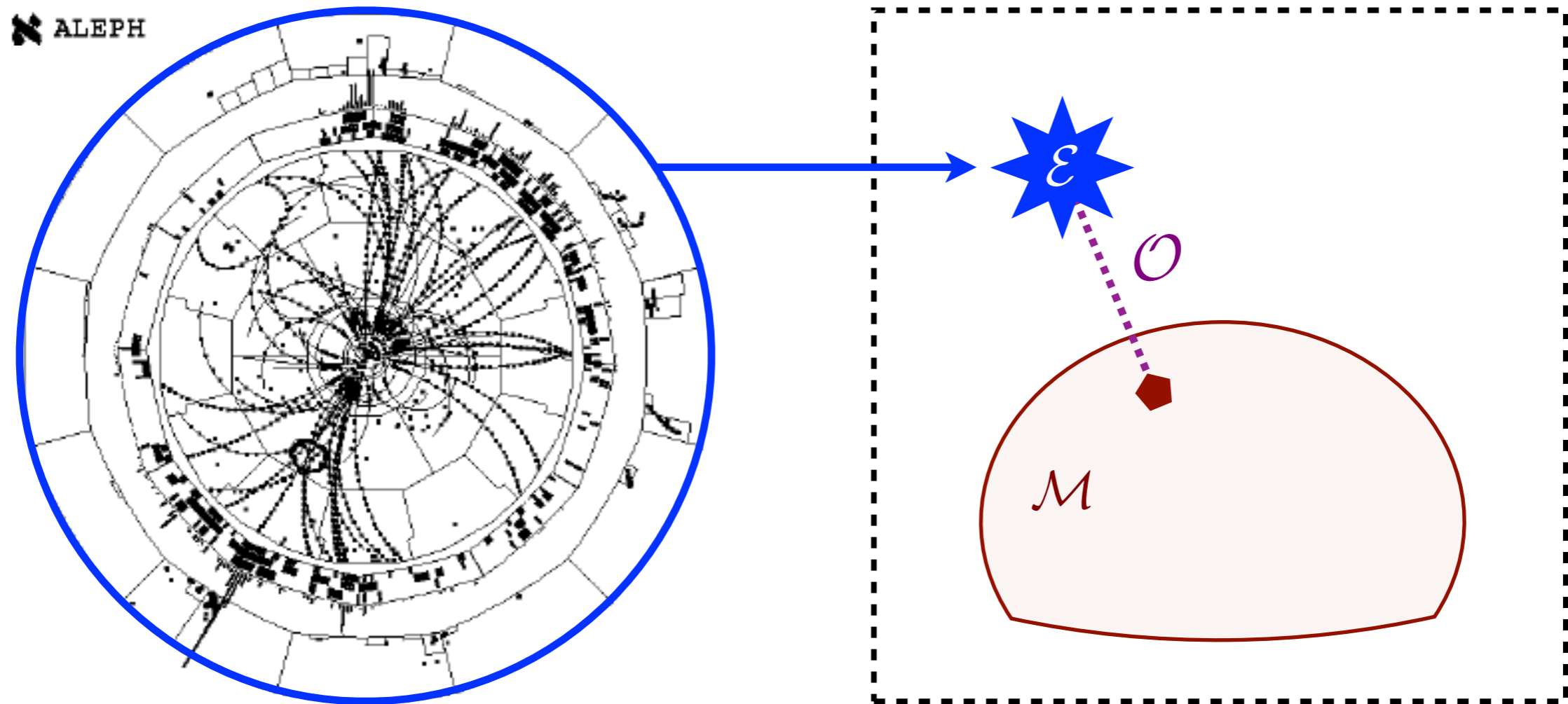
The Emergent Geometry of Many Collisions



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
based on Komiske, Metodiev, JDT, PRL 2019; using EnergyFlow and CMS Open Data]

The Hidden Geometry of Particle Collisions

E.g. Classic QCD Event Shapes



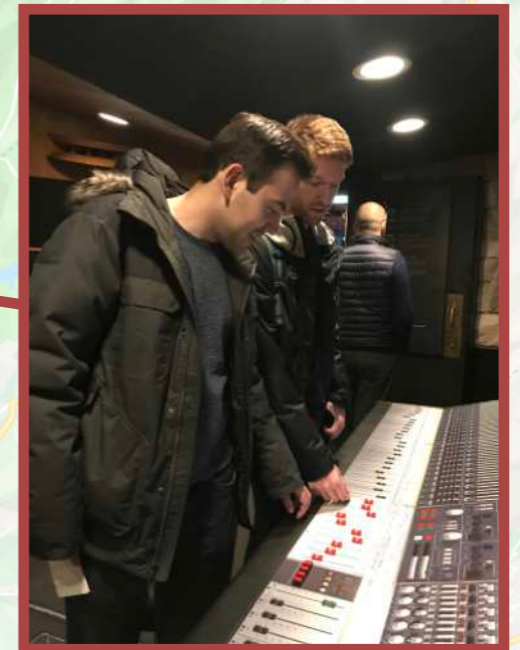
One Electron-Positron Event

Distance to a Manifold in Event Space

[Komiske, Metodiev, JDT, arXiv 2020]

[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977]

Scenes from My Sabbatical



Eric Methodiev

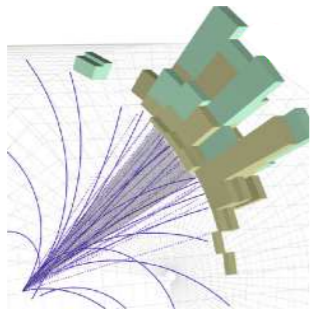
Patrick Komiske

[February 2019;
Simons Sabbatical Fellowship]

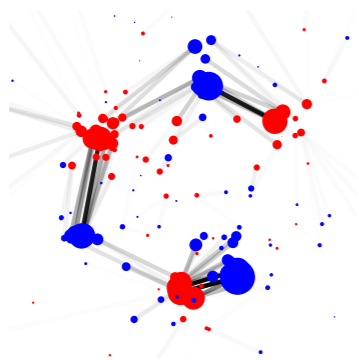


Grand Combin

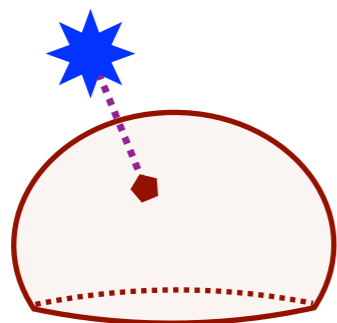
Outline



What is a Collider Event?



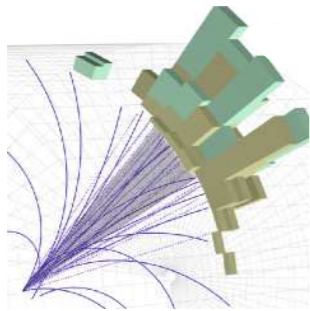
When are Events Similar?



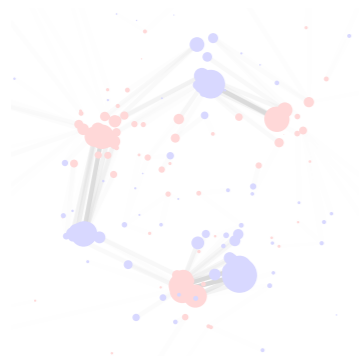
What can be Geometrized?

Pause

Interrupt me or drop questions in the chat, and I'll try to answer them as I go



What is a Collider Event?



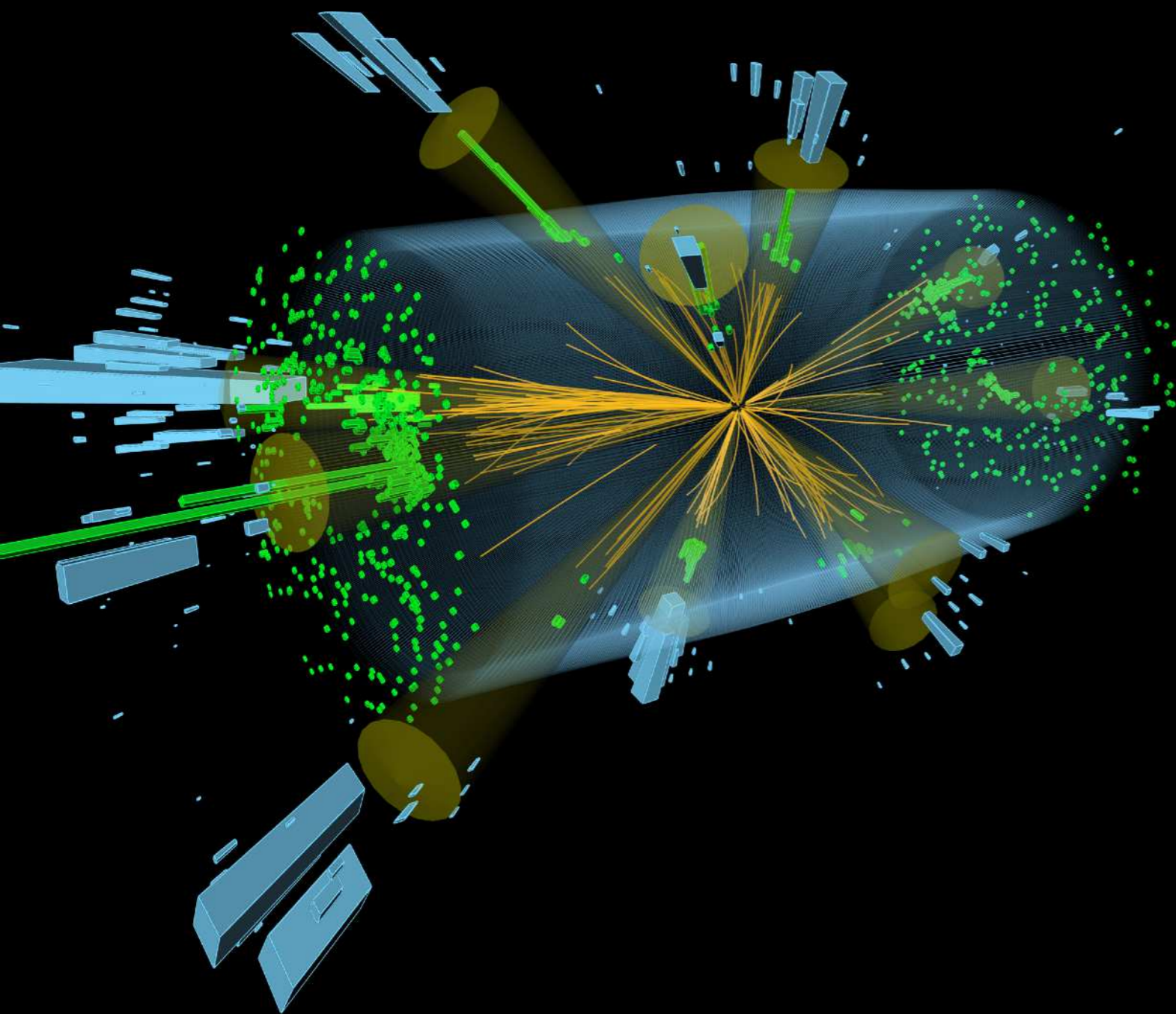
When are Events Similar?



What can be Geometrized?

Collider Event

Collection of points in (momentum) space



T E H M

	●	γ	photon		
●	●	e^{\pm}	electron		
●	●	●	●	μ^{\pm}	muon
●	●	●		π^{\pm}	pion
●	●	●		K^{\pm}	kaon
	●	●		K_L^0	K-long
●	●	●		p/\bar{p}	proton
	●	●		n/\bar{n}	neutron

elementary

composite

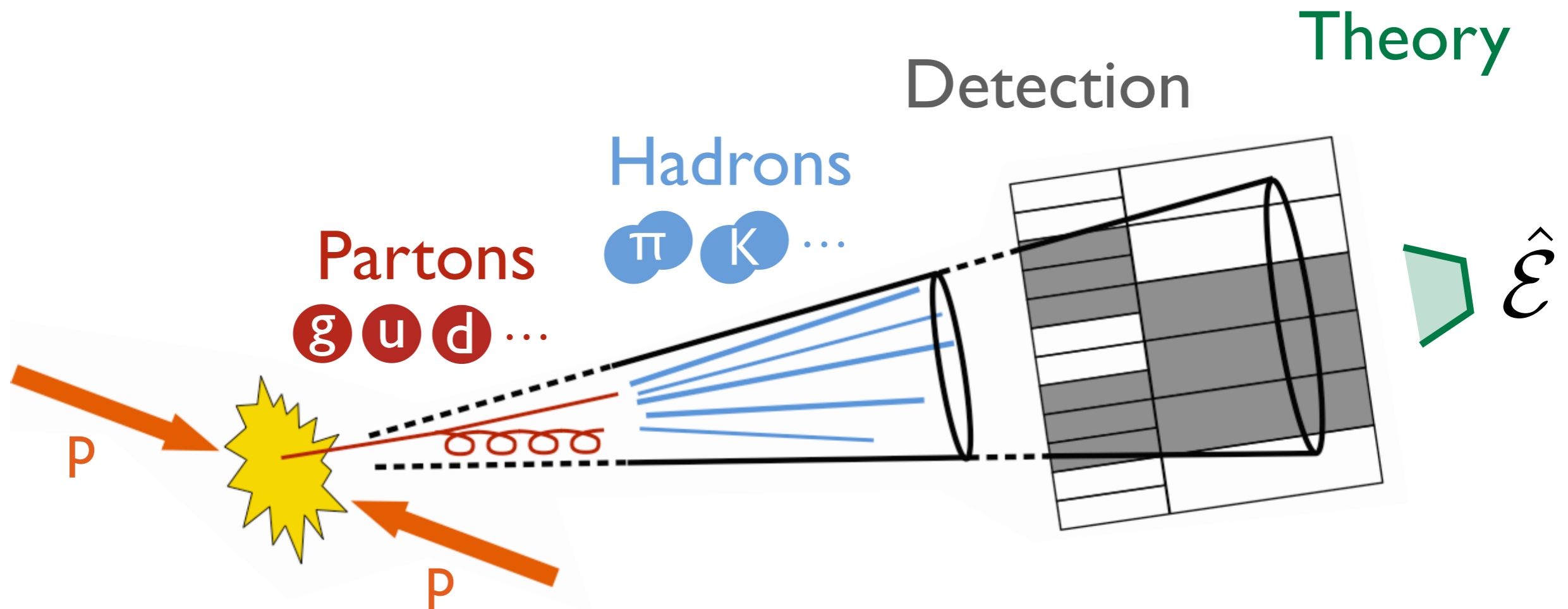
Point Cloud

Collection of points in (position) space



[Popular Science, 2013]

Jet Formation Process



Stress-energy flow:

Robust to non-perturbative and detector effects
Well-defined for massless gauge theories

$$\hat{\mathcal{E}} \simeq \lim_{t \rightarrow \infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

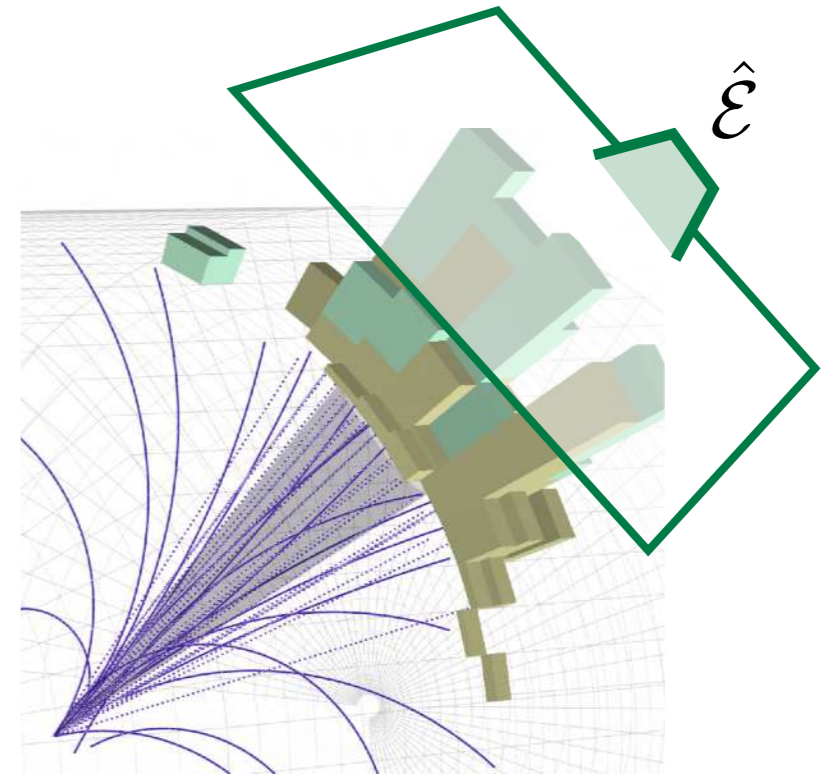
[see e.g. Sveshnikov, Tkachov, [PLB 1996](#); Hofman, Maldacena, [JHEP 2008](#); Mateu, Stewart, [JDT, PRD 2013](#); Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, [PRL 2014](#); Chen, Mout, Zhang, Zhu, [arXiv 2020](#)]

Jets as **Weighted Point Clouds**

- **Energy-Weighted Directions**

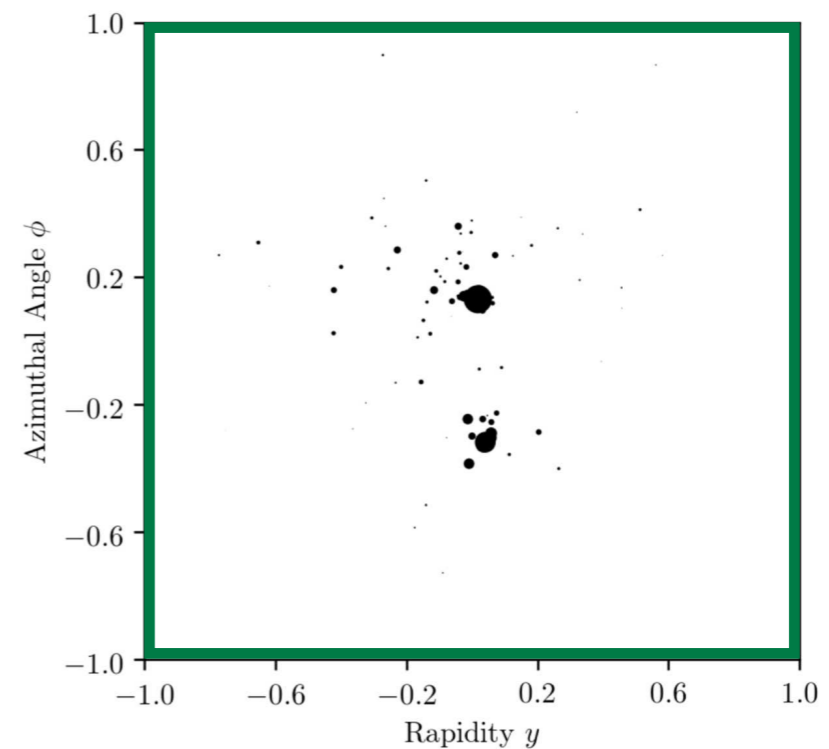
$$\vec{p} = \left\{ \underset{\substack{\uparrow \\ \text{Energy}}}{E}, \underbrace{\hat{n}_x, \hat{n}_y, \hat{n}_z}_{\text{Direction}} \right\}$$

(suppressing “unsafe” charge/flavor information)



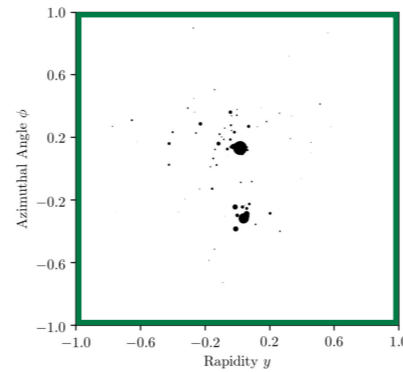
- Equivalently: **Energy Density**

$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} \underset{\substack{\uparrow \\ \text{Energy}}}{E_i} \delta^{(2)}(\hat{n} - \underset{\substack{\uparrow \\ \text{Direction}}}{\hat{n}_i})$$



Pause

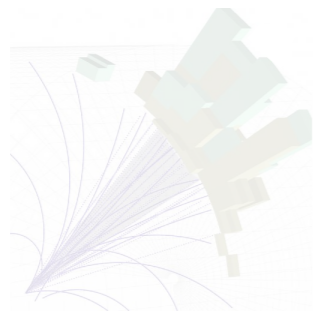
What is a Collider Event?



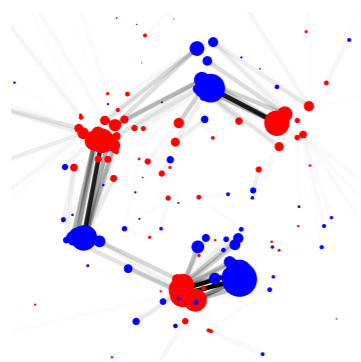
“Calo” Energy Density

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

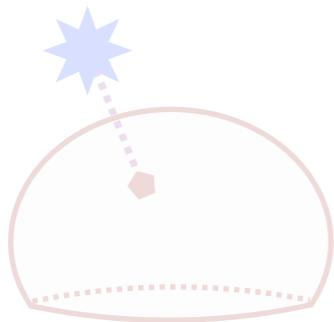
(see backup for relevance to ML)



What is a Collider Event?



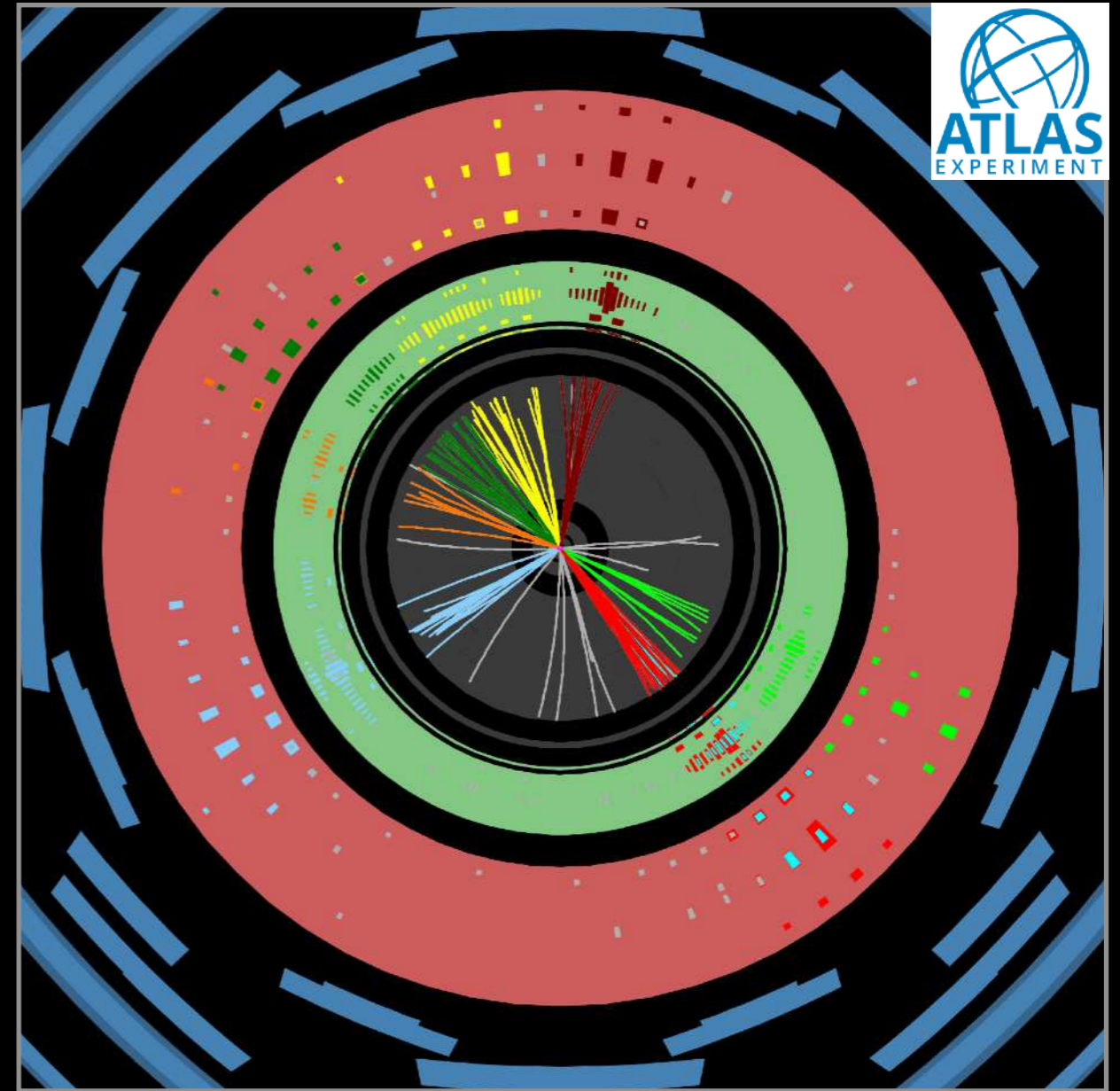
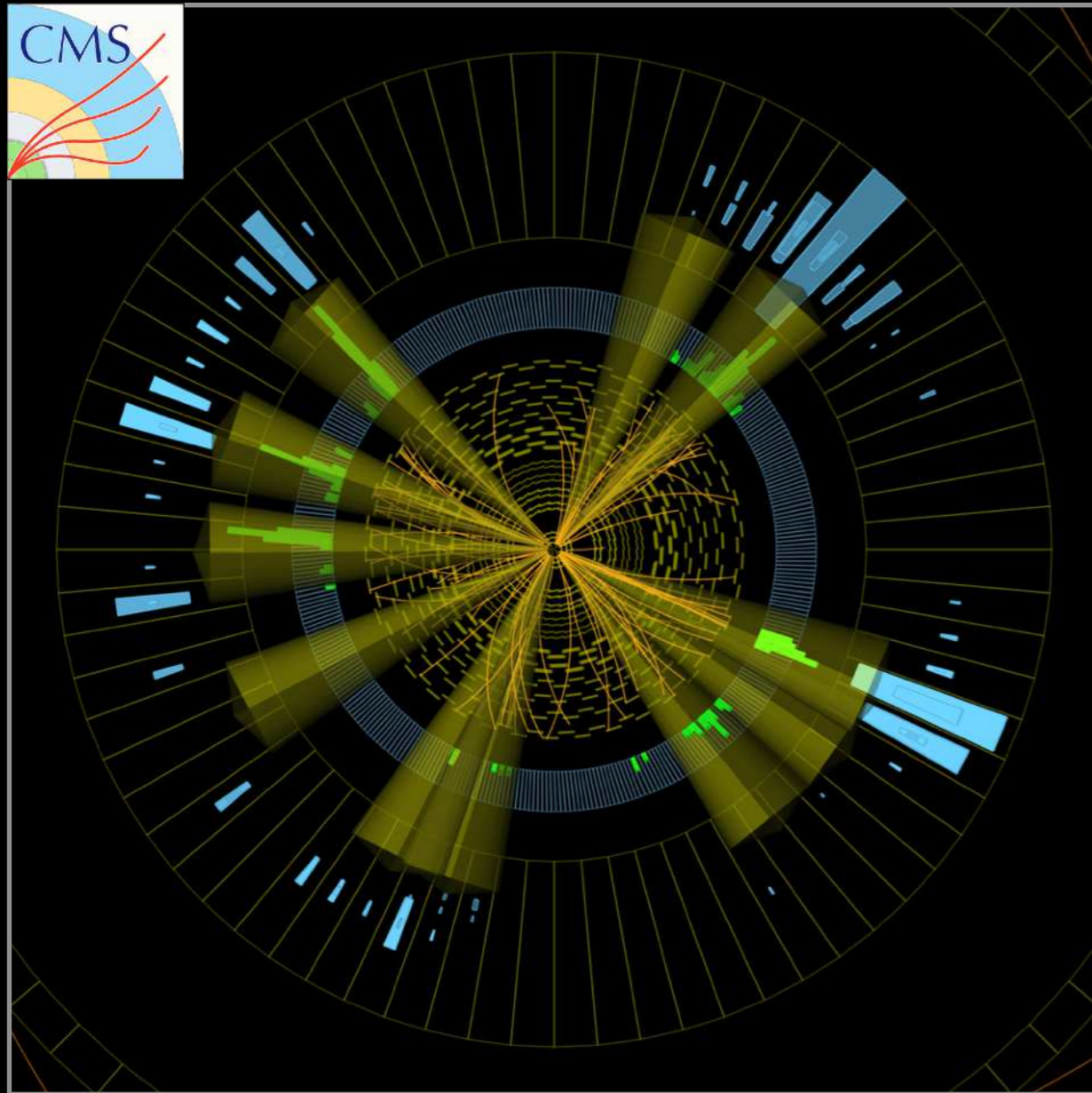
When are Events Similar?



What can be Geometrized?

Two Collider Events

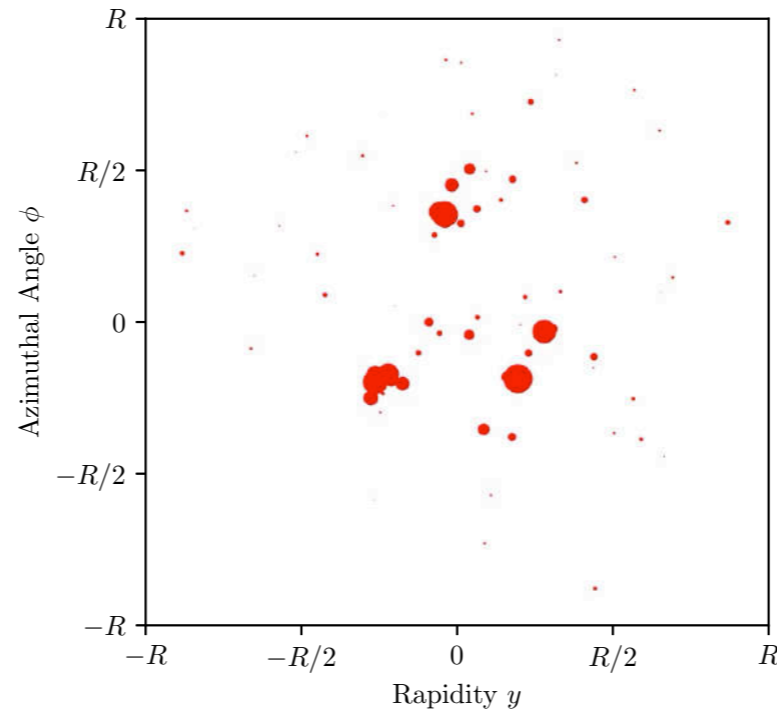
Two collections of points in (momentum) space



How “close” are these? (8.5 km?)

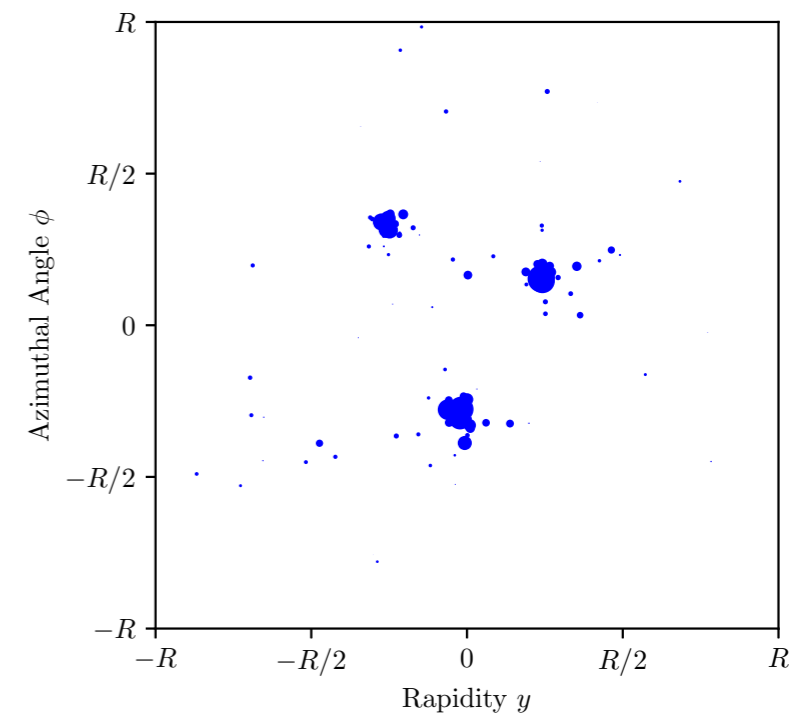
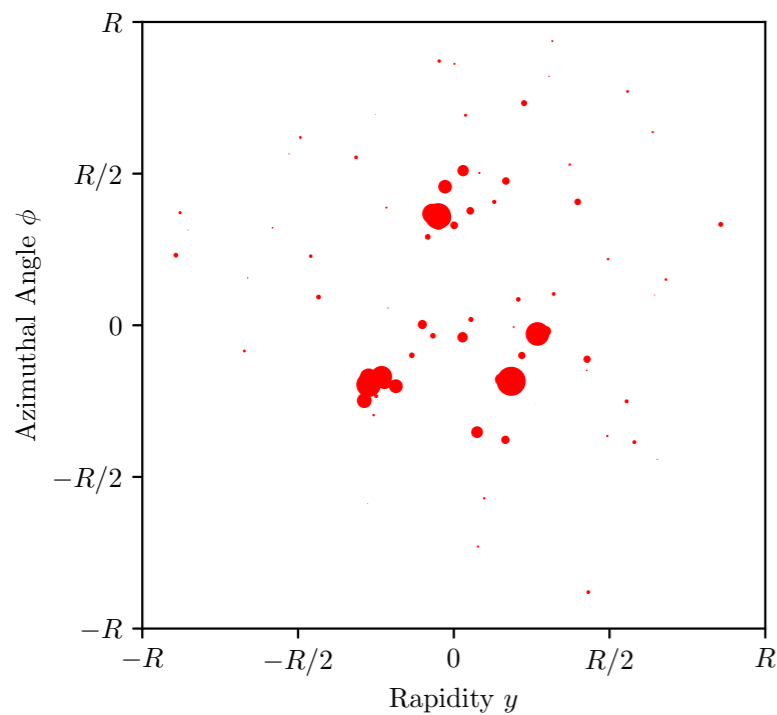
Similarity of Two Energy Flows?

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$



Optimal Transport:

Earth Mover's Distance
a.k.a. *1-Wasserstein metric*



[Komiske, Metodiev, JDT, [PRL 2019](#); code at Komiske, Metodiev, JDT, [energyflow.network](#)]

The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff x distance) to make one distribution look like another distribution



Déblai

Remblai

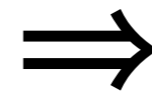
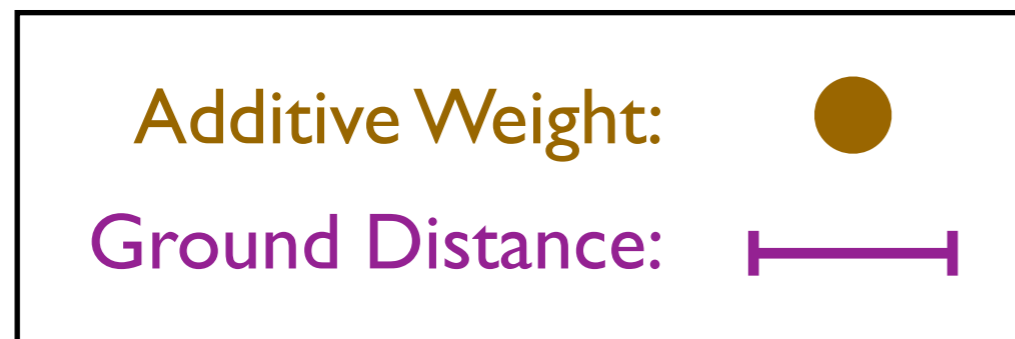
[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Vaserštejn, 1969; [Wikipedia](#)]

The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (**stuff** x **distance**) to make
one distribution look like **another distribution**

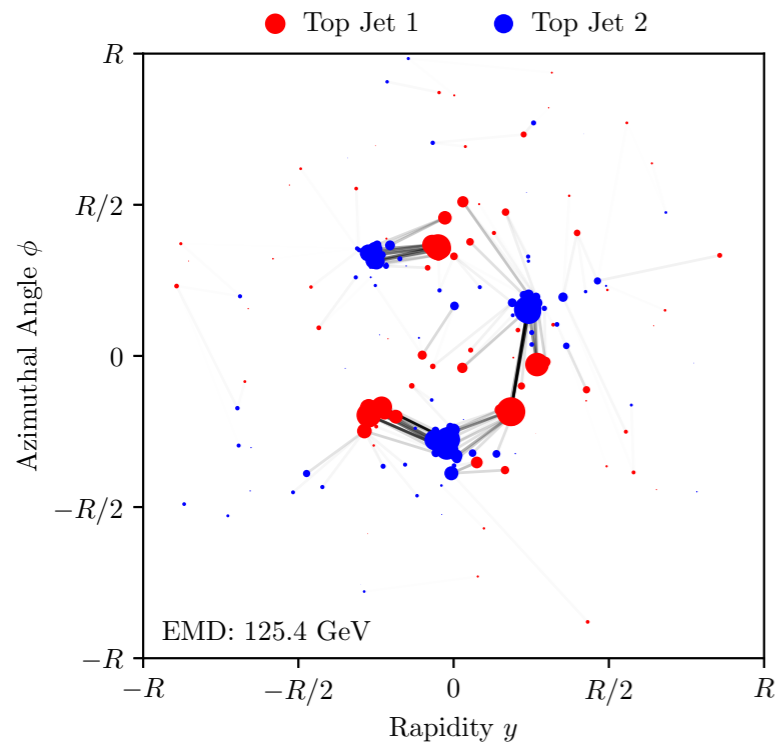
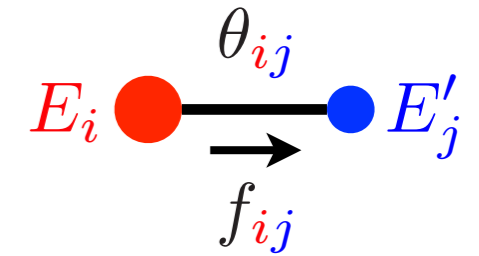


Distance Between
Distributions



[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Vaserštejn, 1969; [Wikipedia](#)]

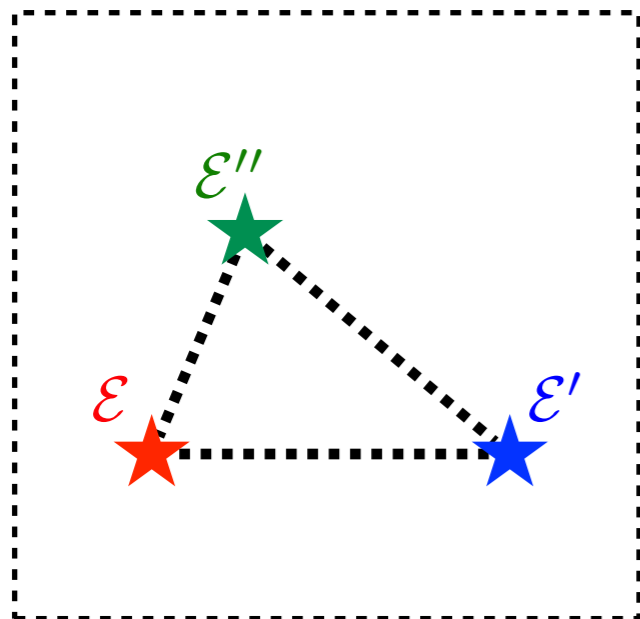
The Energy Mover's Distance



Optimal transport between energy flows...

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \underbrace{\sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R}}_{\text{Cost to move energy}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost to create energy}}$$

in GeV



...defines a metric on the space of events

$$0 \leq \text{EMD}(\mathcal{E}, \mathcal{E}') \leq \text{EMD}(\mathcal{E}, \mathcal{E}'') + \text{EMD}(\mathcal{E}', \mathcal{E}'')$$

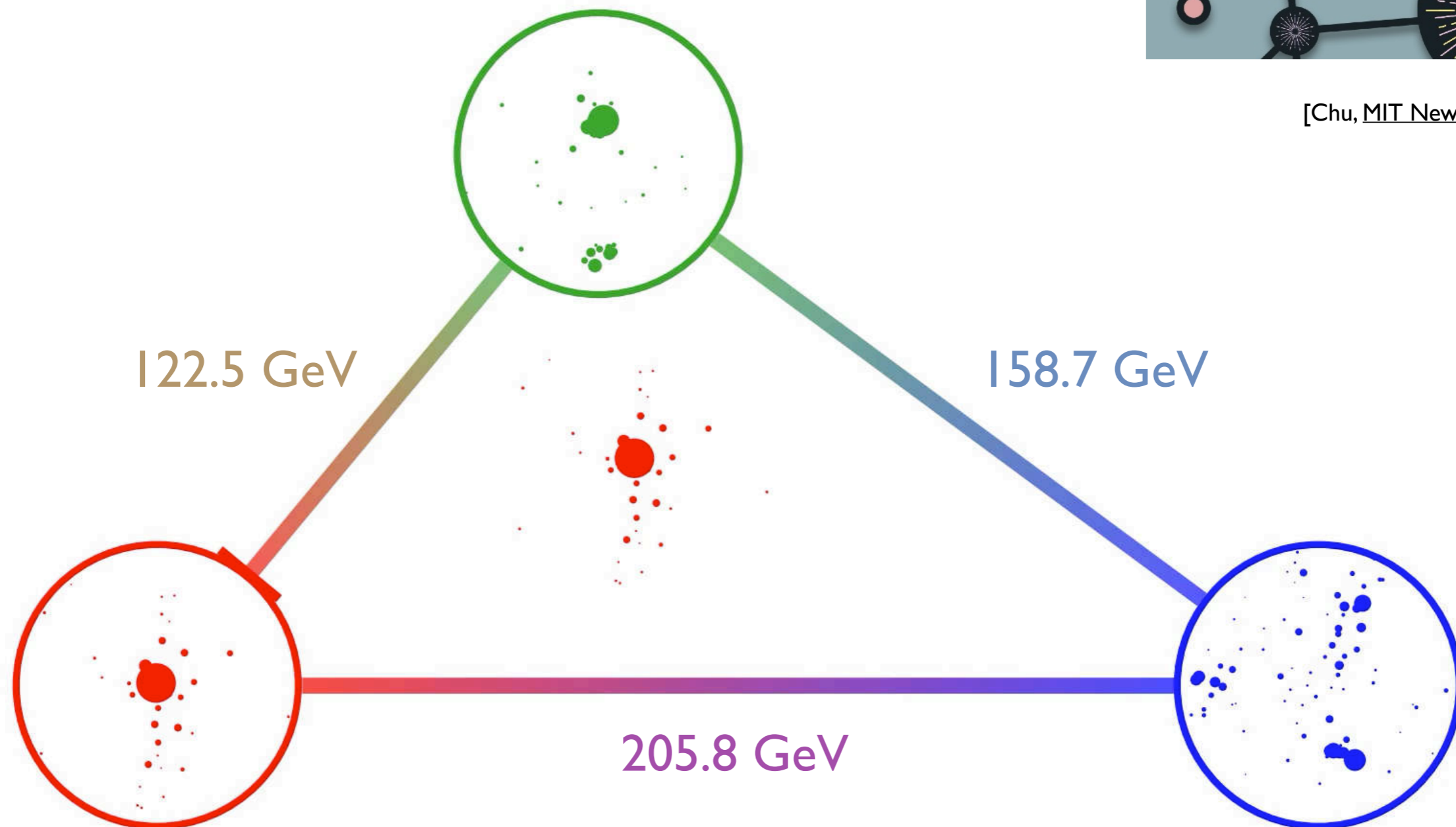
(assuming $R \geq \theta_{\max}/2$, i.e. $R \geq$ jet radius for conical jets)

[Komiske, Metodiev, JDT, [PRL 2019](#);
see also Pele, Werman, [ECCV 2008](#); Pele, Taskar, [GSI 2013](#);
see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, [arXiv 2020](#)]

Similarity of Three Energy Flows?



[Chu, [MIT News July 2019](#)]



[Komiske, Metodiev, JDT, [PRL 2019](#); code at Komiske, Metodiev, JDT, [energyflow.network](#)]

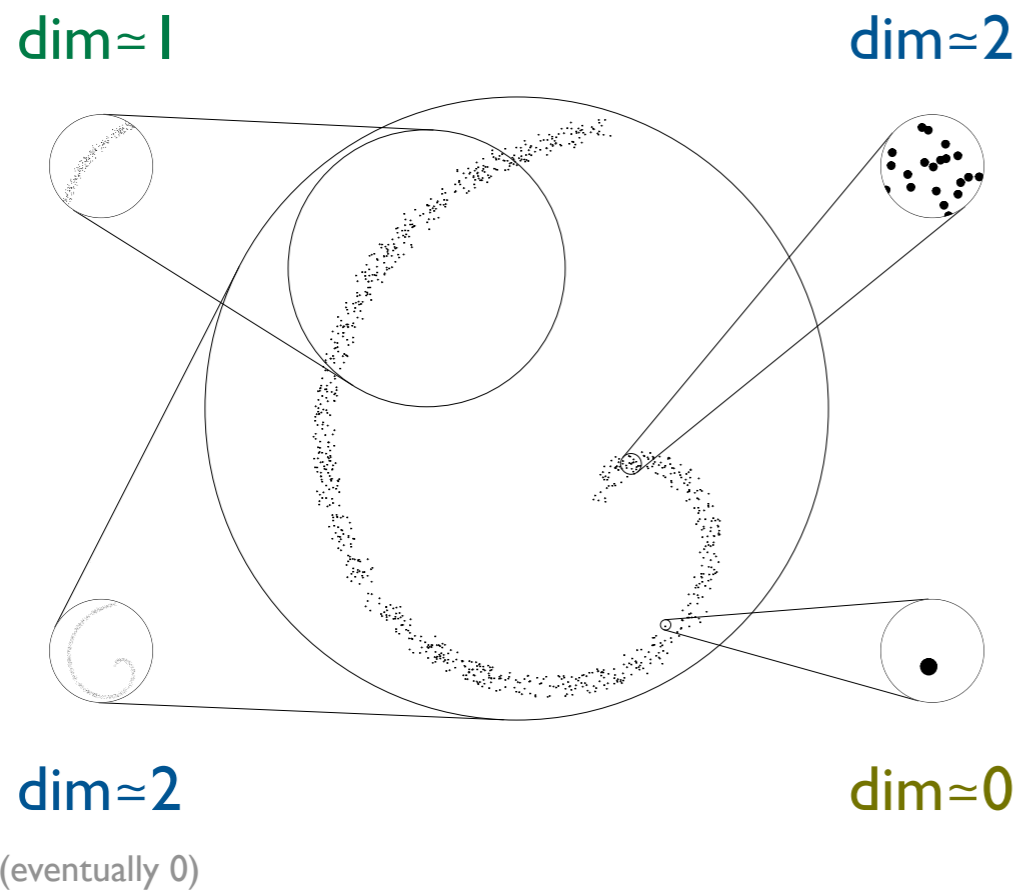
What can you do with a metric?

Dimensionality of Space of Jets

$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

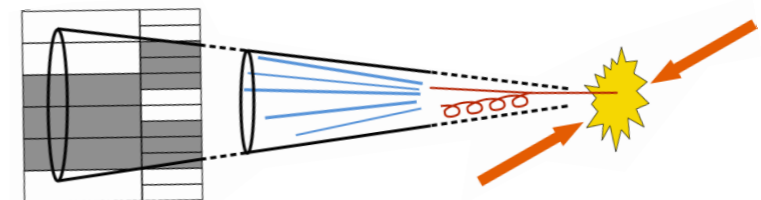
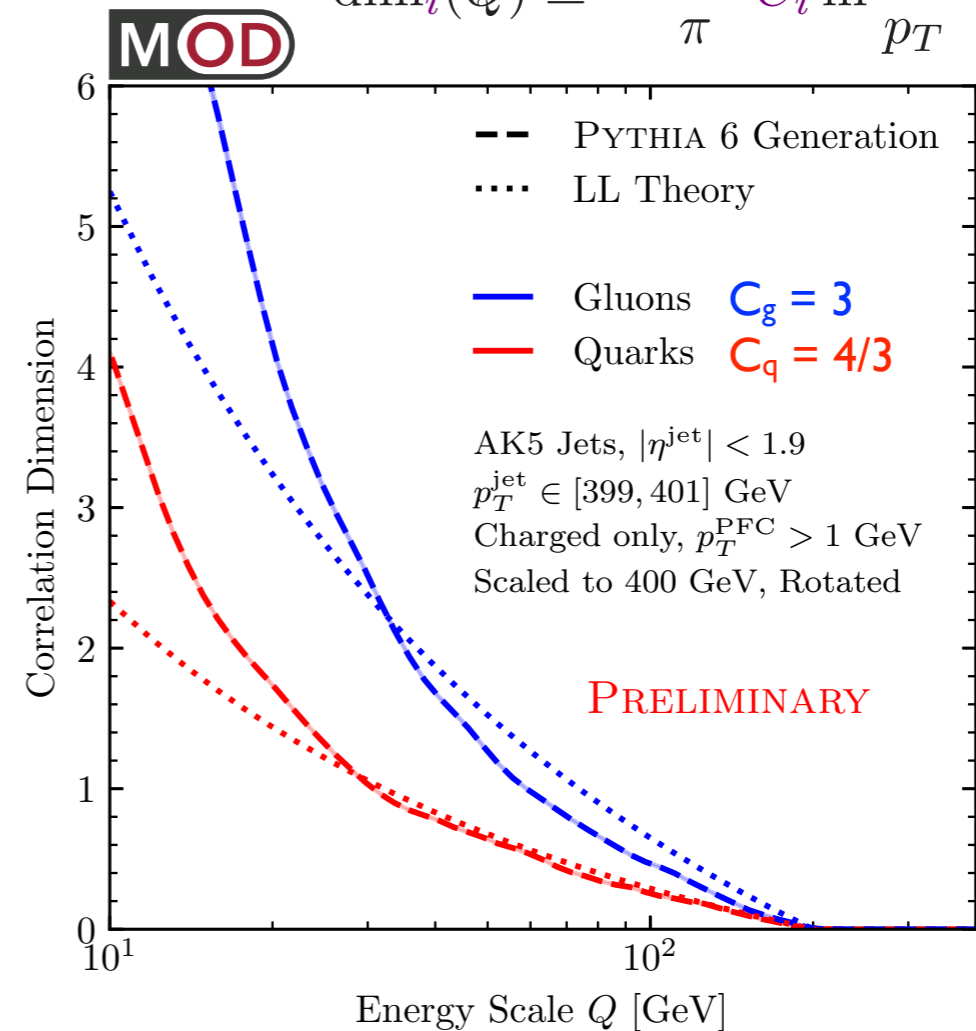
$$\Rightarrow \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]



QCD Calculation

$$\text{dim}_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T}$$



Dimensionality of Space of Jets

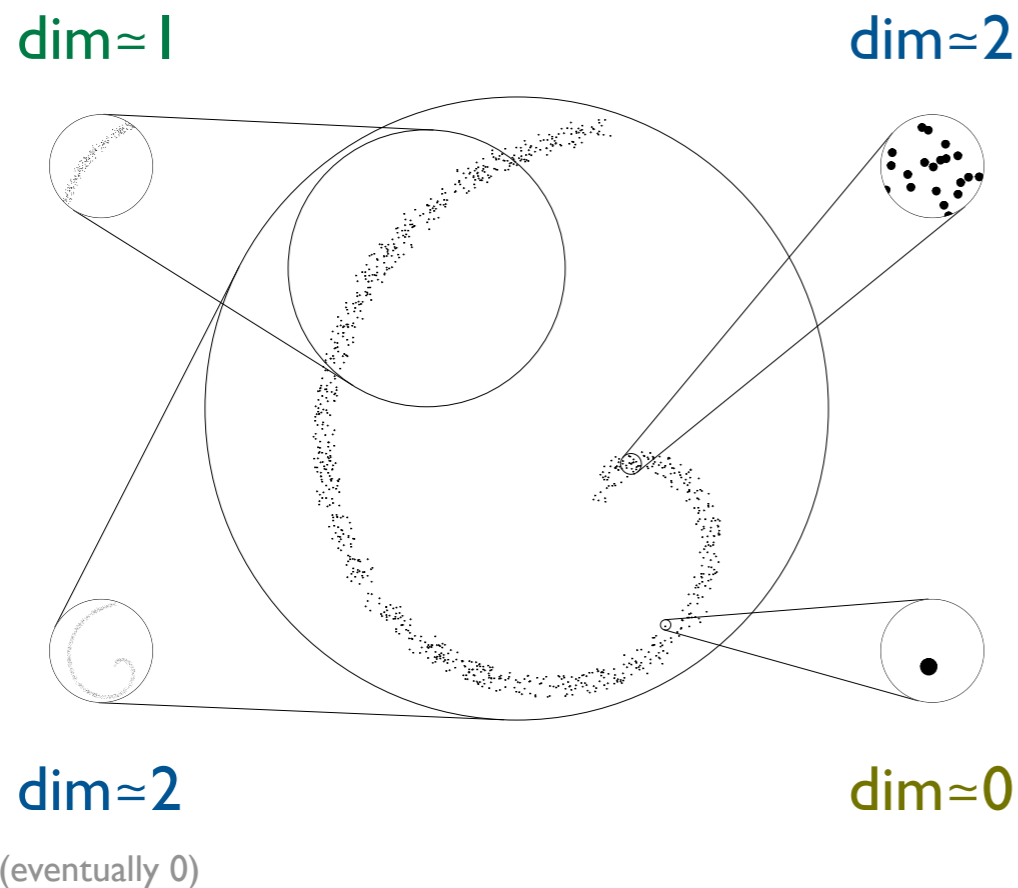


[<http://opendata.cern.ch/>]

$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

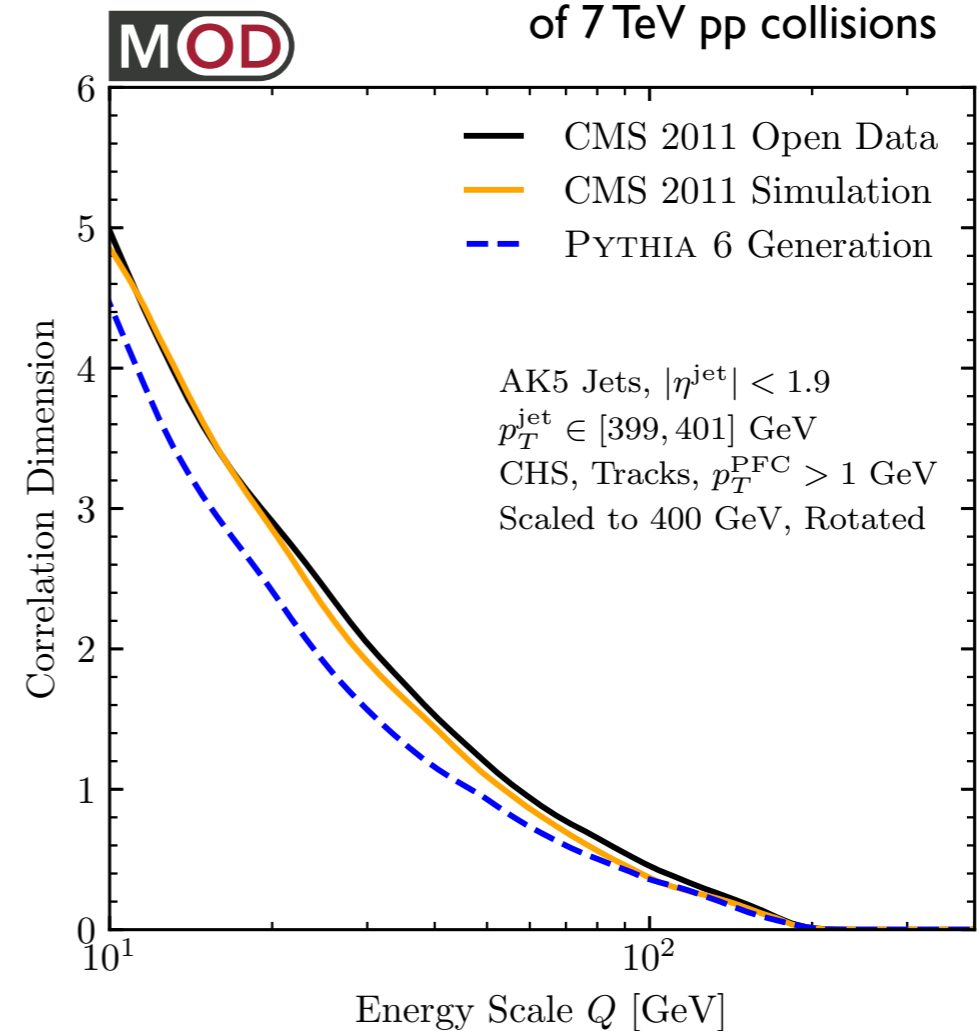
$$\Rightarrow \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

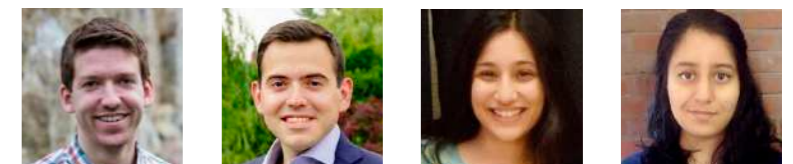


CMS Open Data

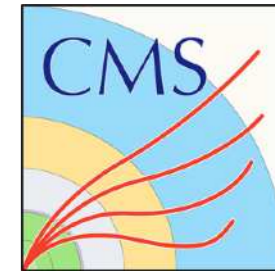
400 GeV jets in 2.3 fb⁻¹
of 7 TeV pp collisions



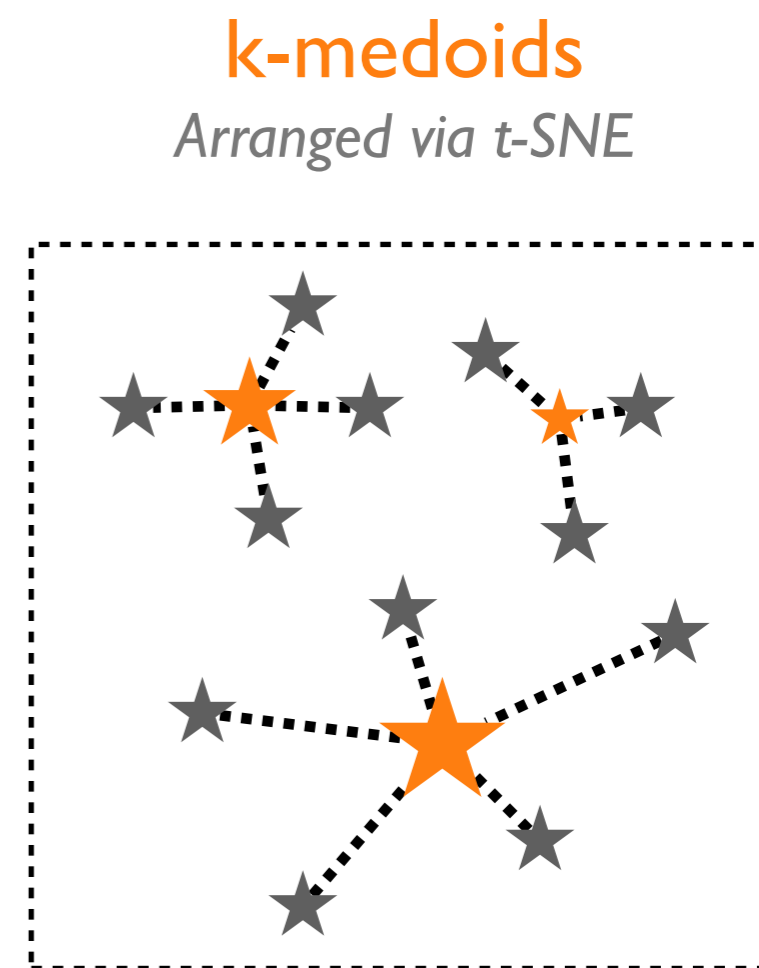
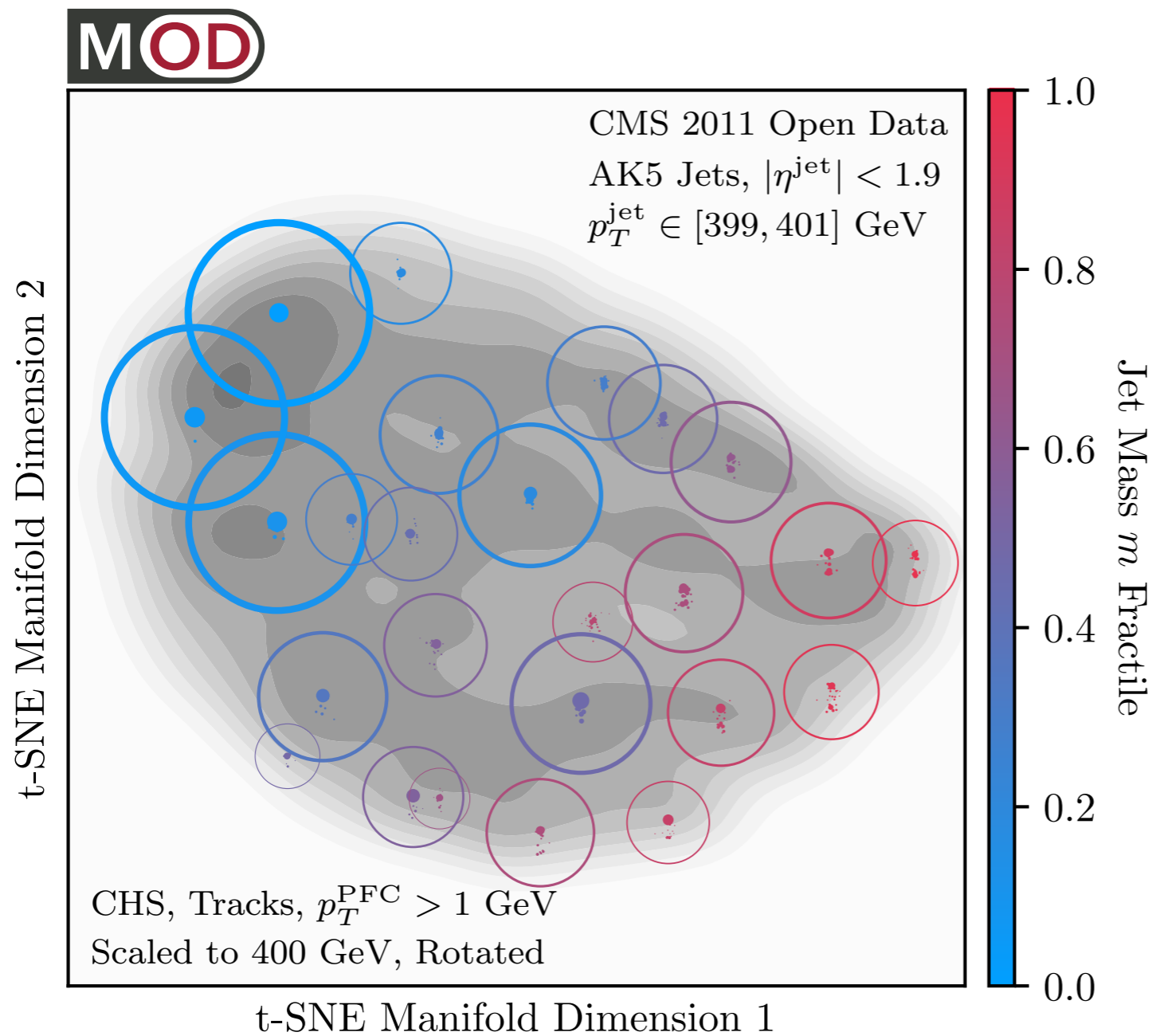
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]



Most Representative Jets

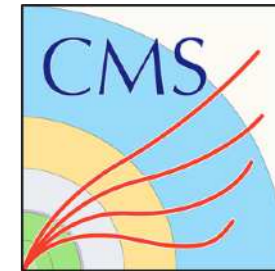


[<http://opendata.cern.ch/>]

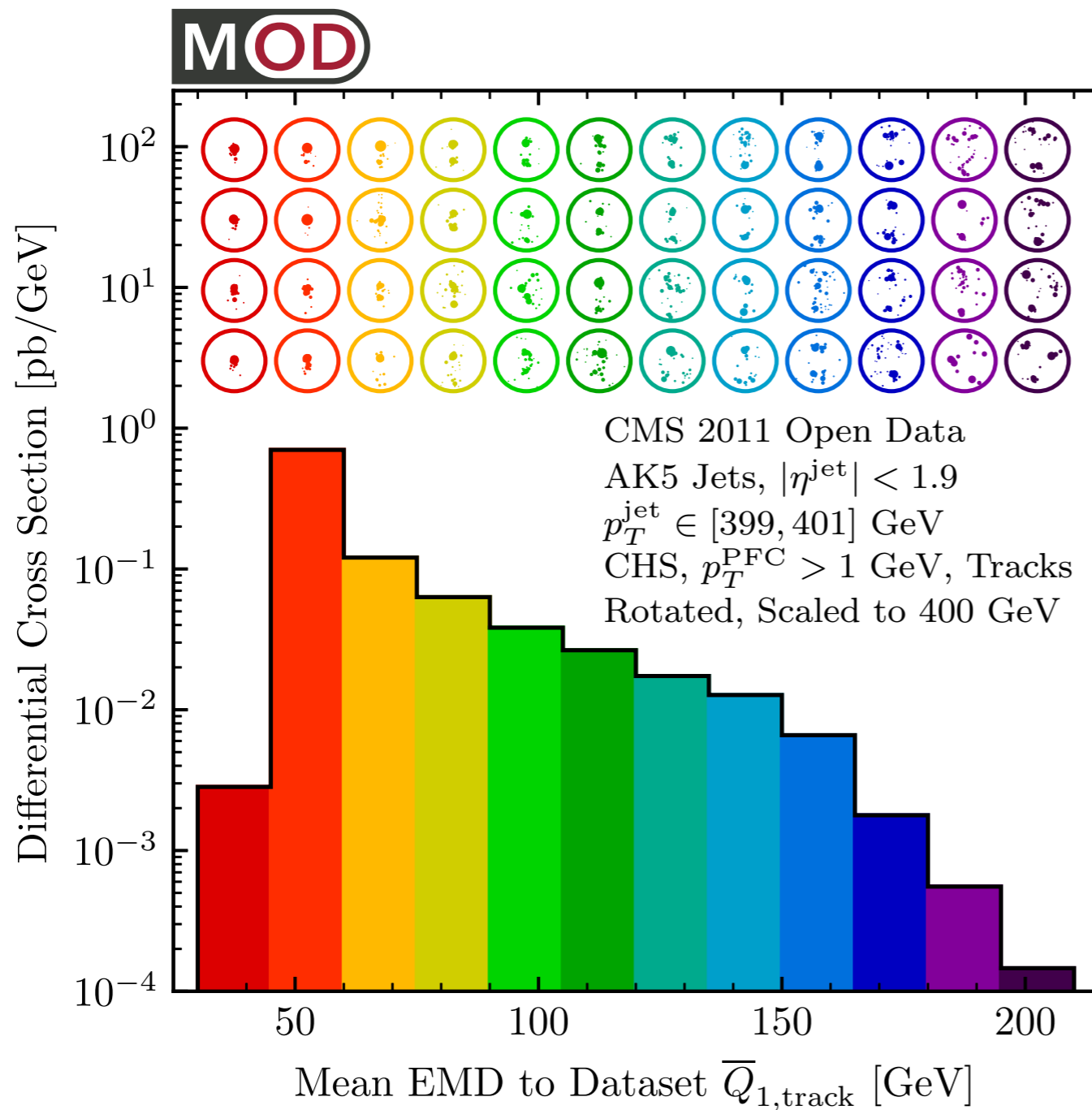


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

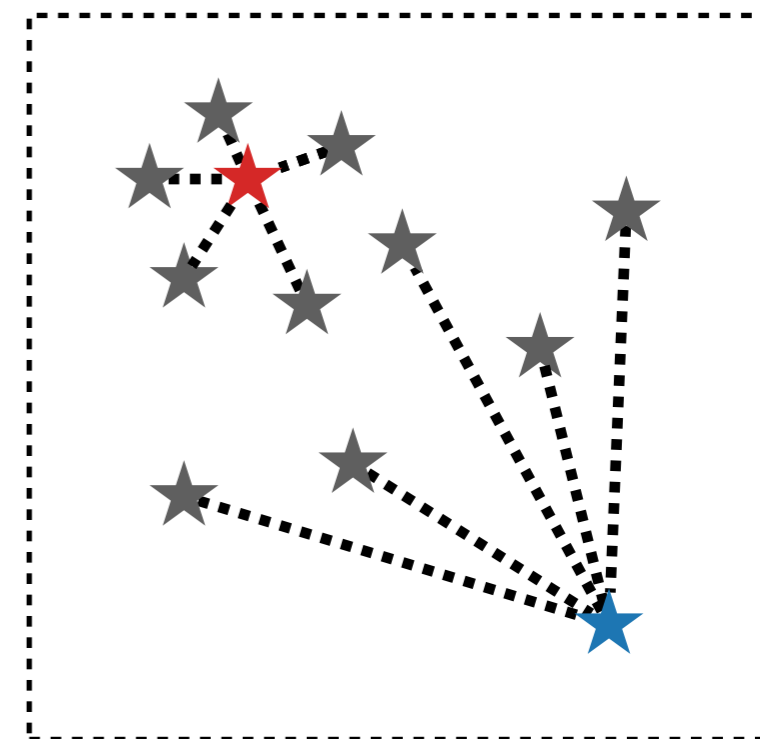
Least Representative Jets



[<http://opendata.cern.ch/>]



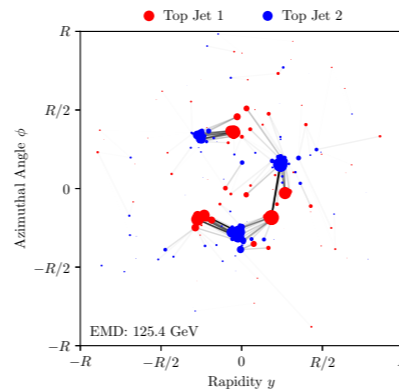
New Physics?
 Or tails of QCD?



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

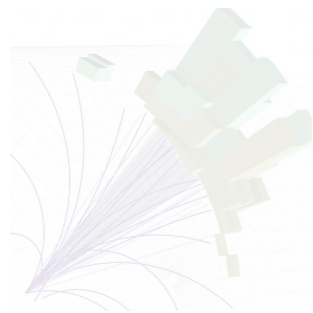
Pause

When are Events Similar?

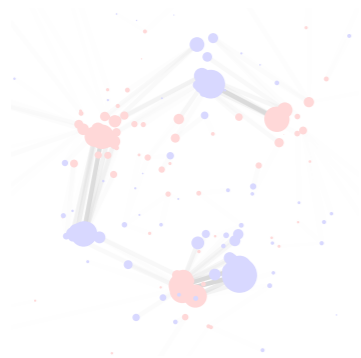


Small Energy Mover's Distance

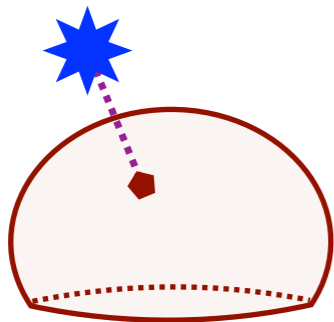
$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_i E_i - \sum_j E'_j \right|$$



What is a Collider Event?



When are Events Similar?

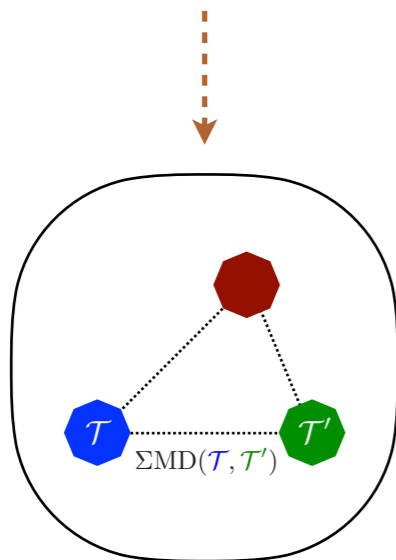


What can be Geometrized?

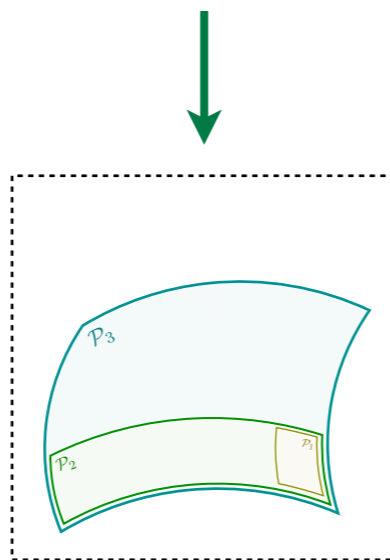
Master Formula for Collider Physics

$$\sigma_{\text{obs}} \simeq \frac{1}{2E_{\text{CM}}^2} \sum_{n=2}^{\infty} \int d\Phi_n |\mathcal{M}_{AB \rightarrow 12\dots n}|^2 f_{\text{obs}}(\Phi_n)$$

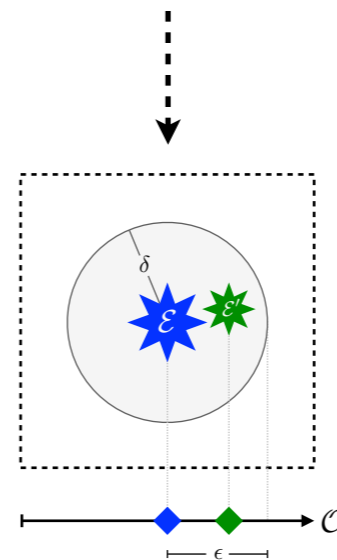
cross section



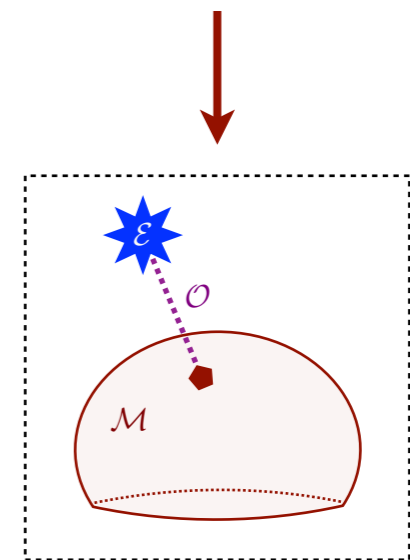
phase space



amplitude



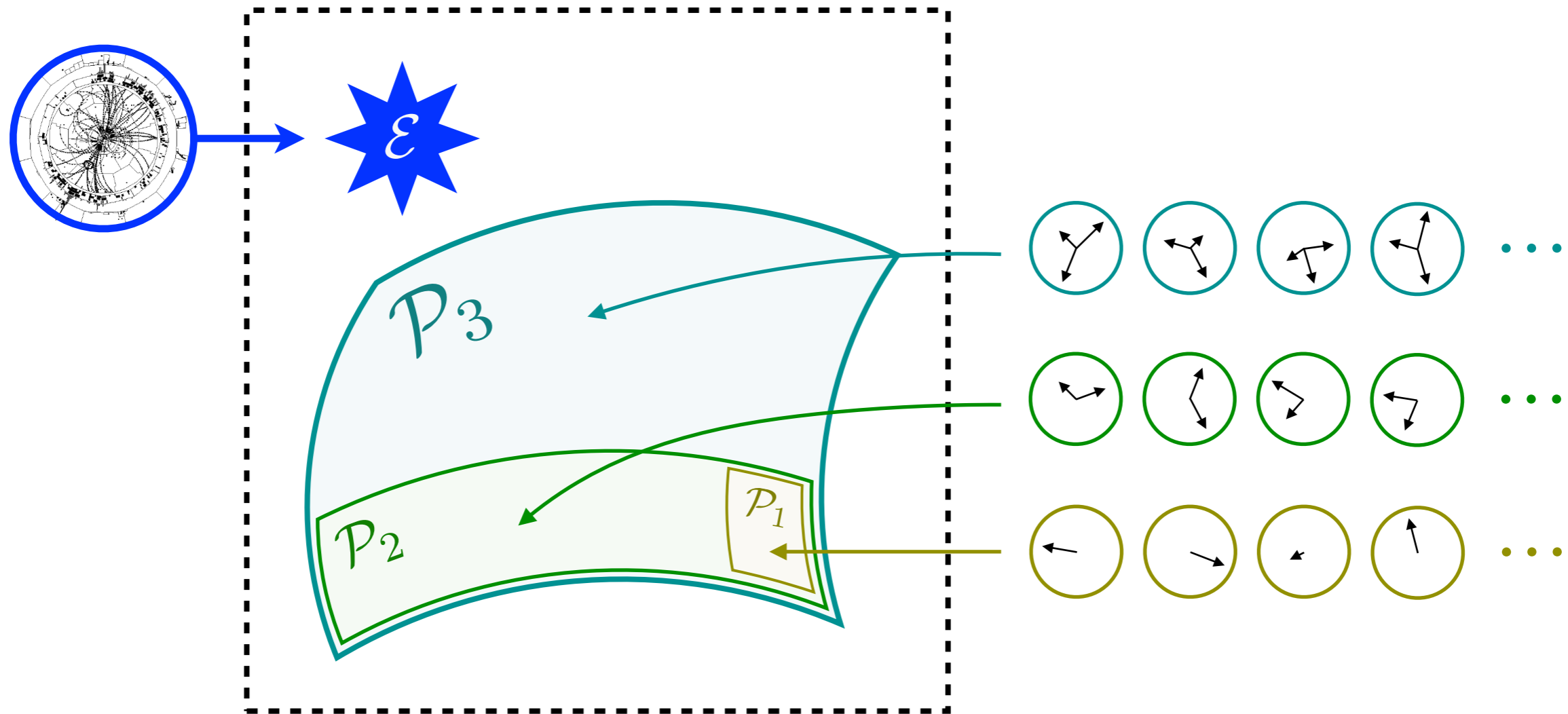
observable



Introducing N-particle Manifolds

$$\sum_{n=2}^{\infty} \int d\Phi_n$$

\mathcal{P}_N = set of all N-particle configurations

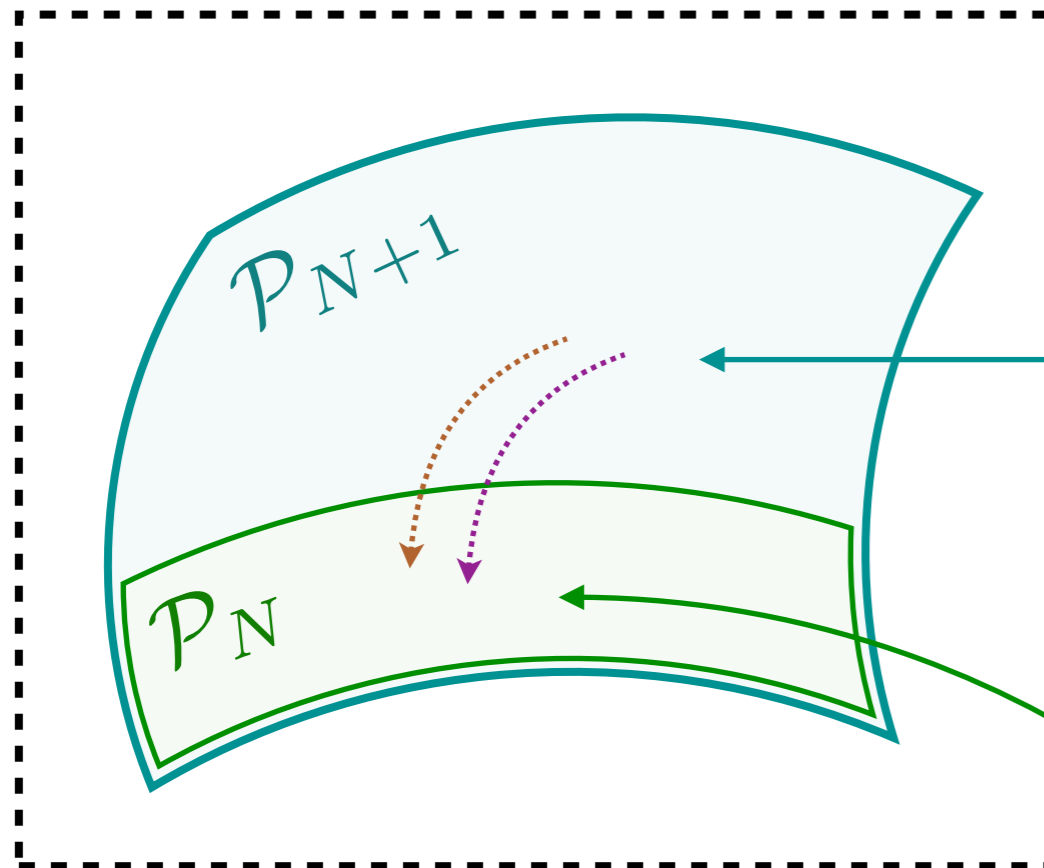
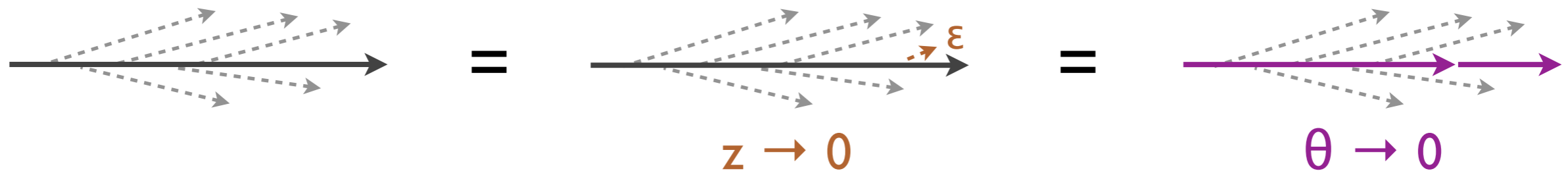


$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \dots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by **soft/collinear** limits

When are Two Events **the Same**?

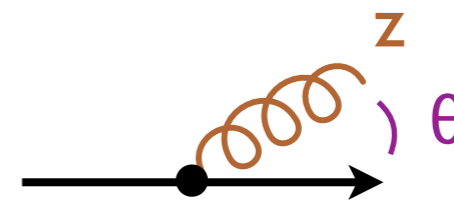
$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

Energy Flow unchanged by infinitesimal **soft/collinear** emissions



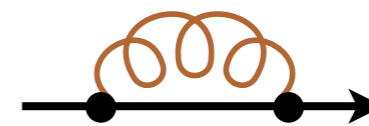
Infrared divergences “live” together!

Real:



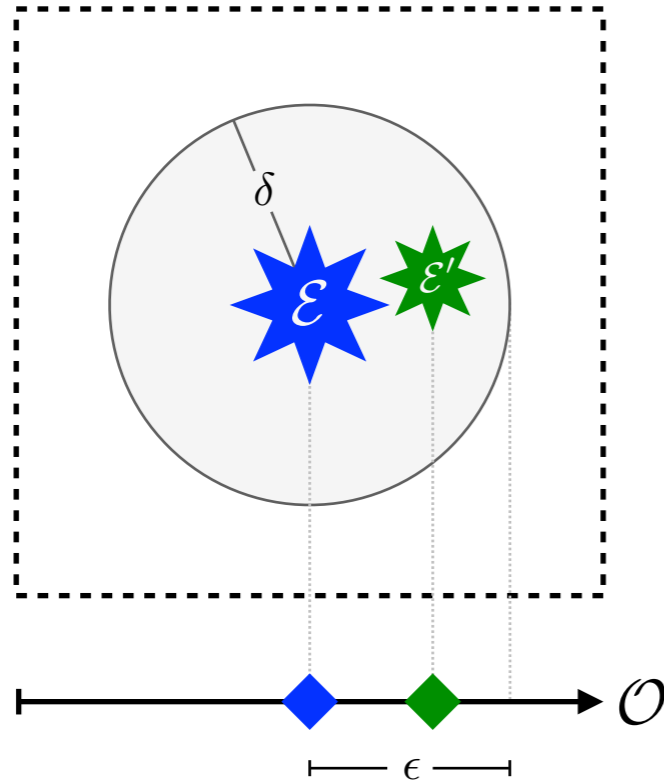
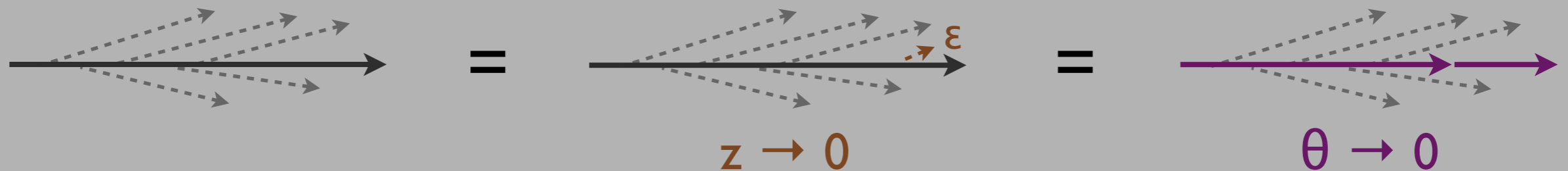
$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{dz}{z} \frac{d\theta}{\theta}$$

Virtual:



When are Two Events the Same?

Energy Flow unchanged by infinitesimal *soft/collinear* emissions



Infrared & Collinear Safety

\approx calculable in perturbative quantum field theory

is^*

Continuity in EMD Space

[Komiske, Metodiev, JDT, arXiv 2020]
 [Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]
 [see also Banfi, Salam, Zanderighi, JHEP 2005]

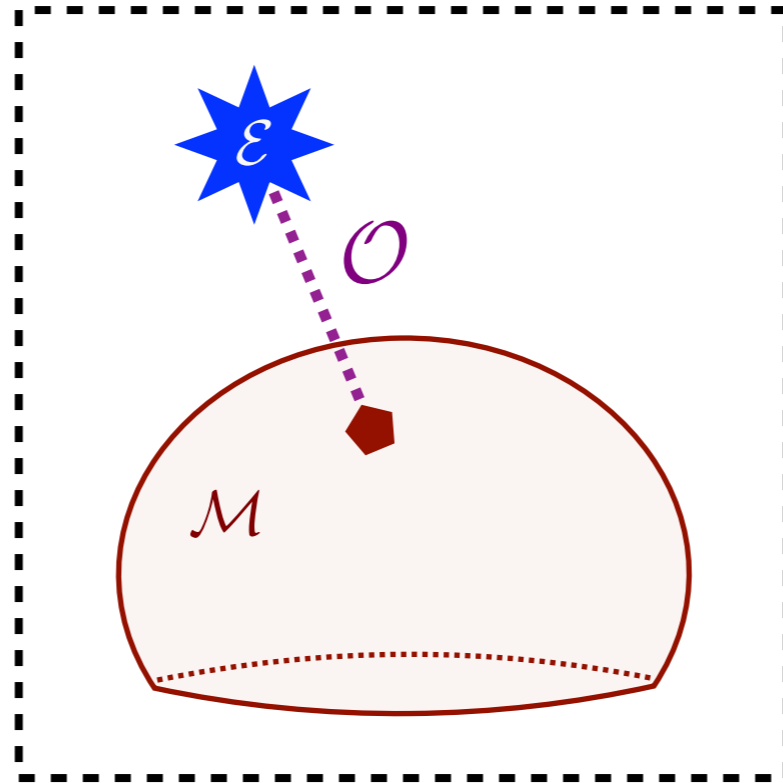
*EMD seems to define the “natural”
geometry for massless gauge theories*

Open question: Can you define $|\mathcal{M}_{AB \rightarrow 12 \dots n}|^2$ directly in this space?

Manifolds for Observables

$f_{\text{obs}}(\Phi_n)$

One Event



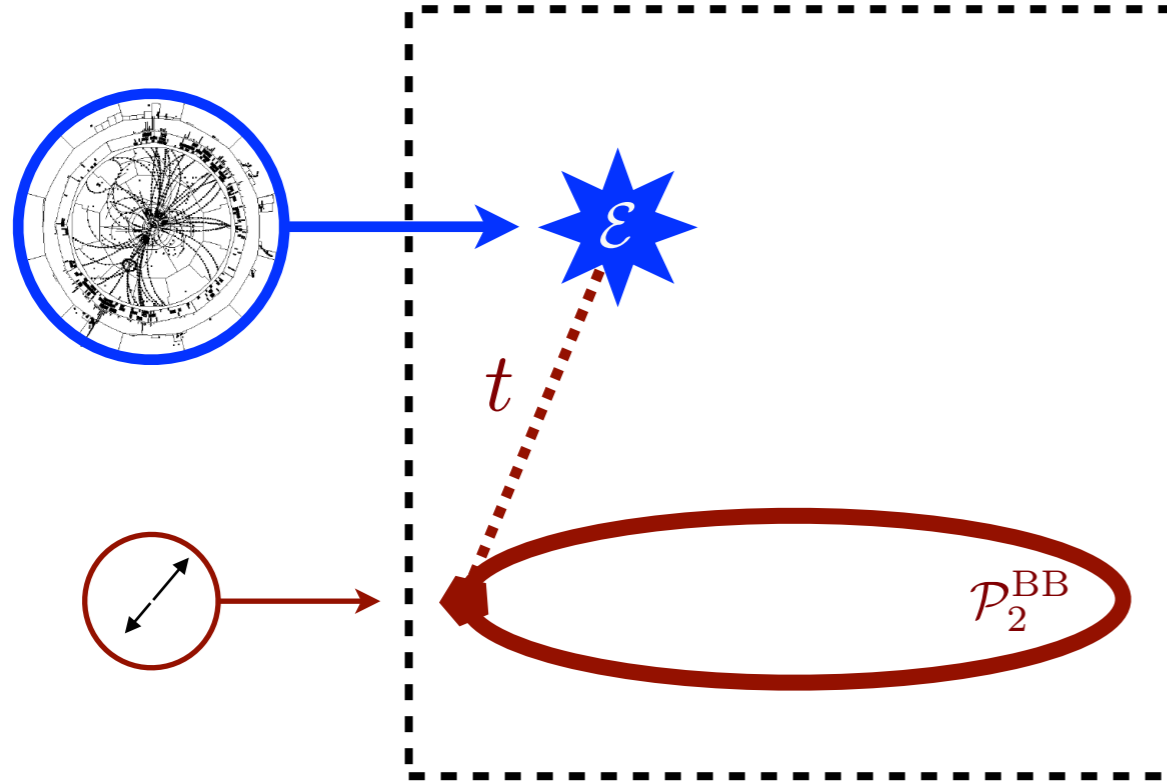
Set of Events

Distance of Closest Approach \Rightarrow Observable

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}(\mathcal{E}, \mathcal{E}')$$

E.g. Thrust

How dijet-like is an event?

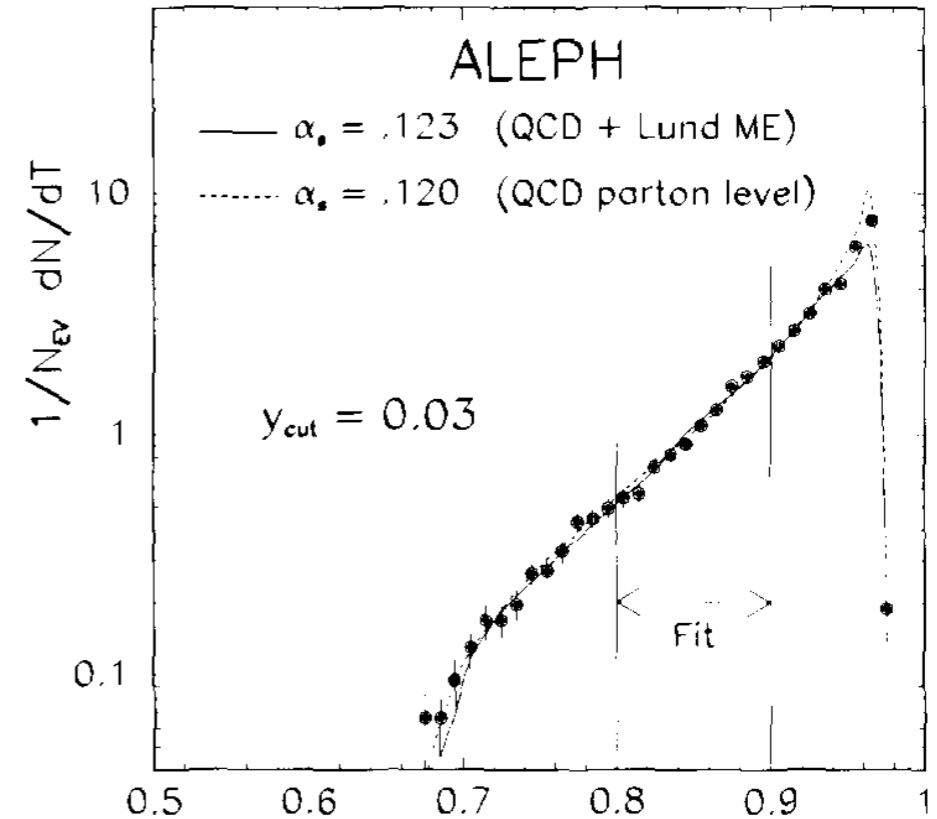


All Back-to-Back Two Particle Configurations

$$\mathcal{P}_2^{\text{BB}} = \left\{ \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \text{---} \end{array} \dots \right\}$$

(using $\beta=2$ EMD variant)

$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_2(\mathcal{E}, \mathcal{E}')$$



$$1 - \frac{t}{2E_{\text{CM}}}$$

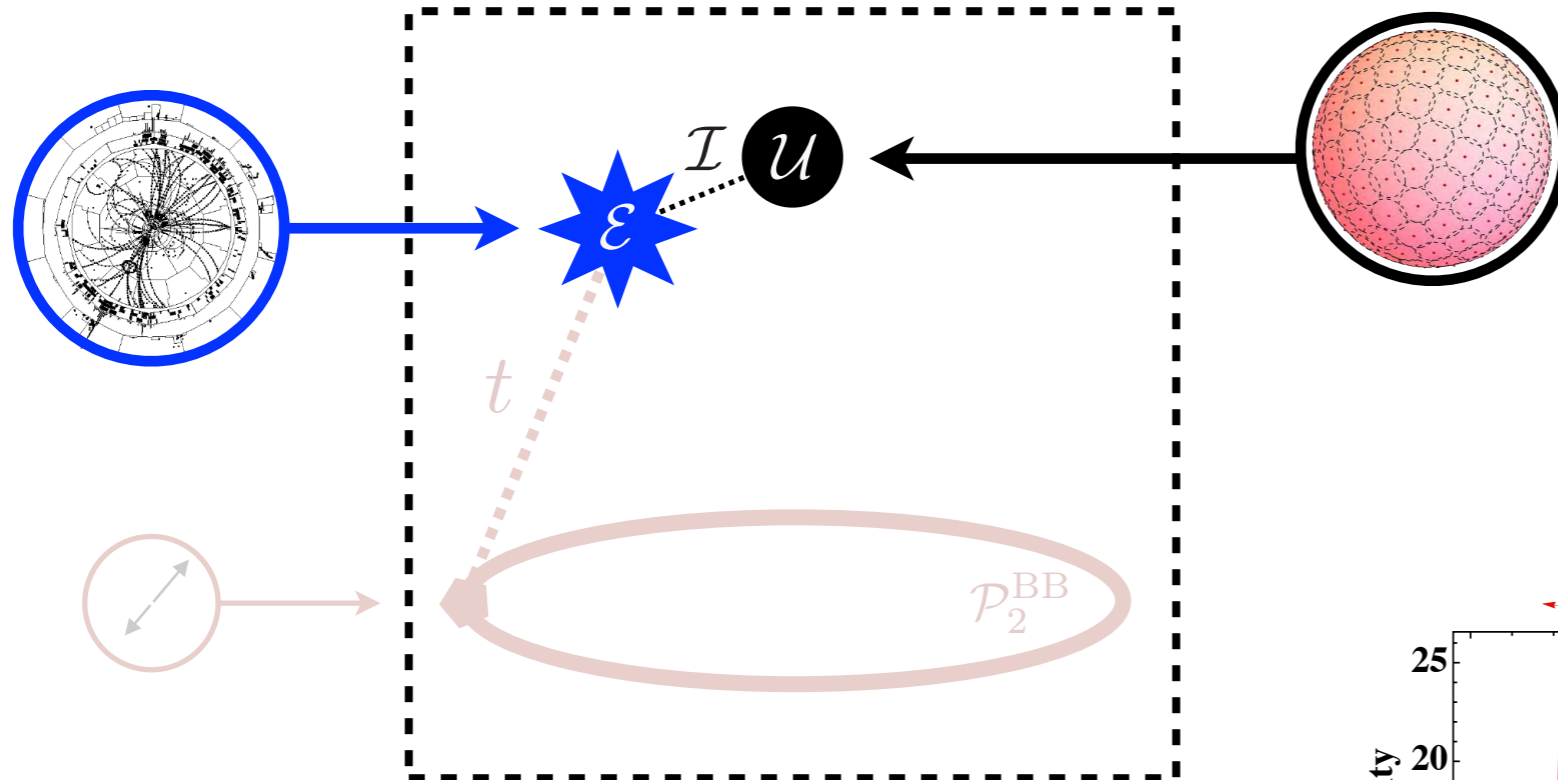
cf. $T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$

[Komiske, Metodiev, JDT, arXiv 2020]

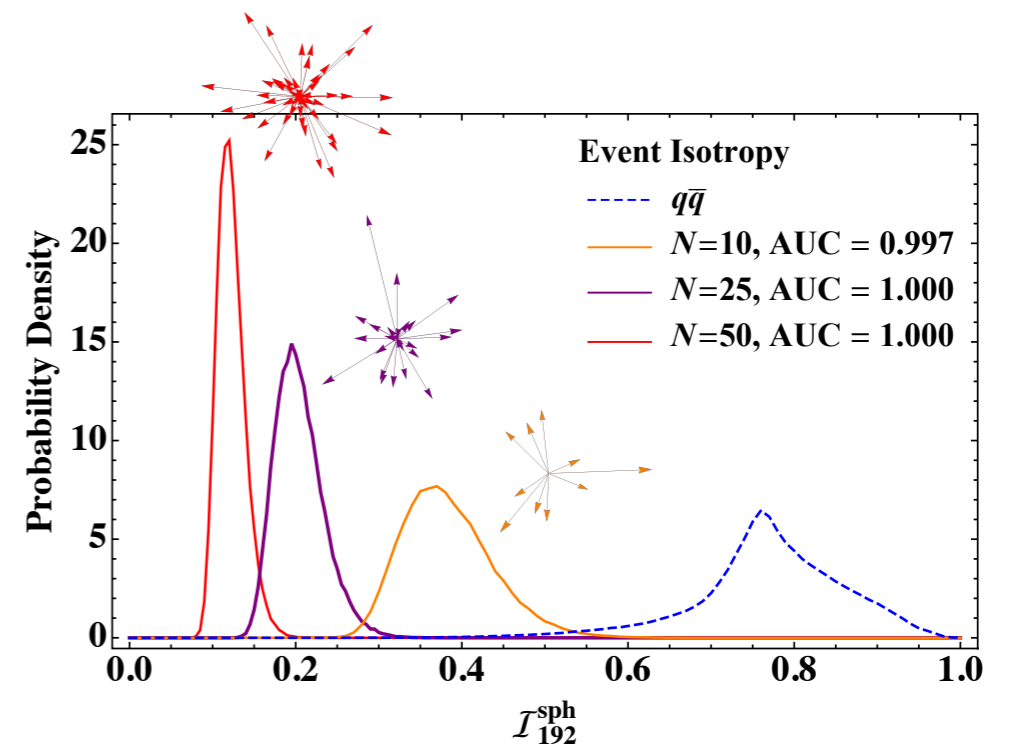
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

How isotropic is an event?



$$\mathcal{I}(\mathcal{E}) = \text{EMD}(\mathcal{E}, \mathcal{U})$$

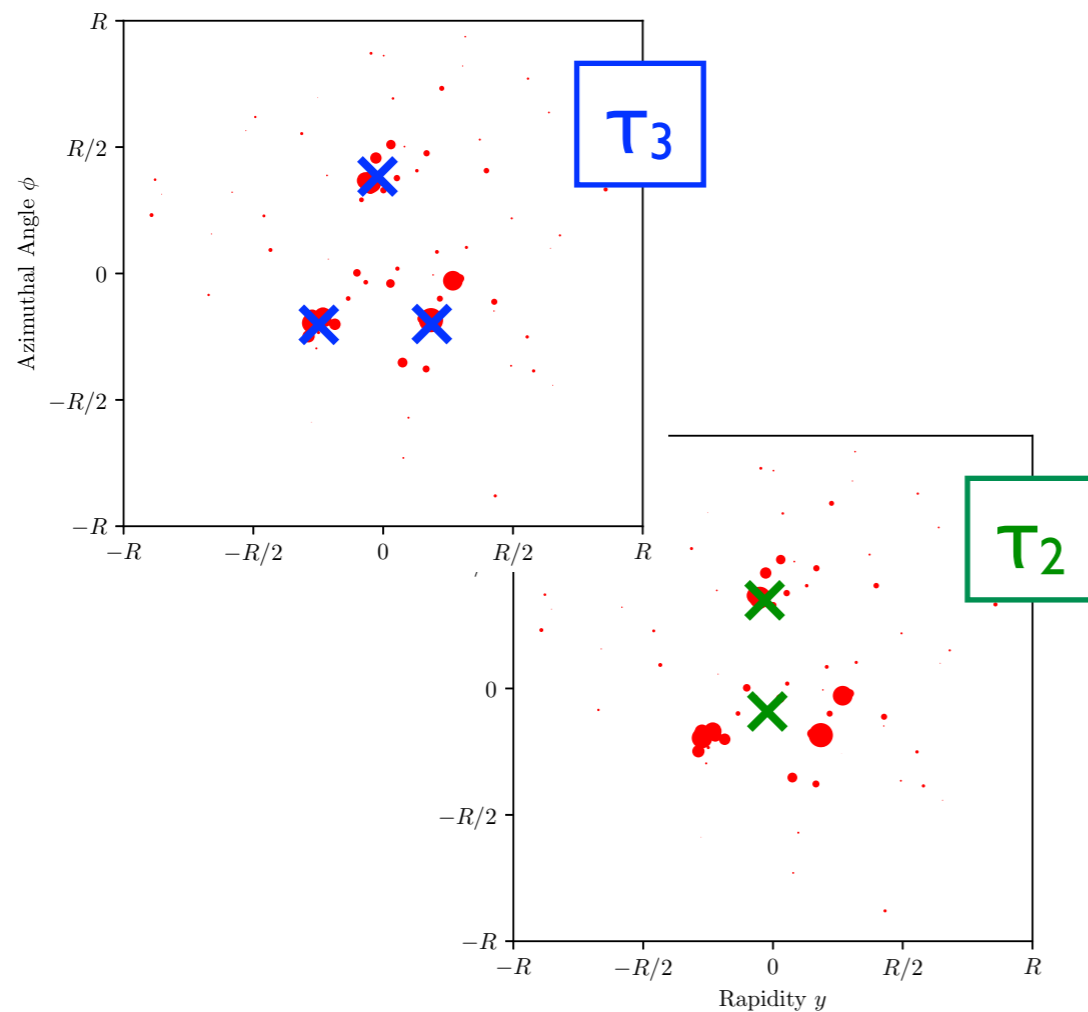


[Cesarotti, JDT, arXiv 2020]

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade...

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_i E_i \min \{ \theta_{1,i}, \theta_{2,i}, \dots, \theta_{N,i} \}$$

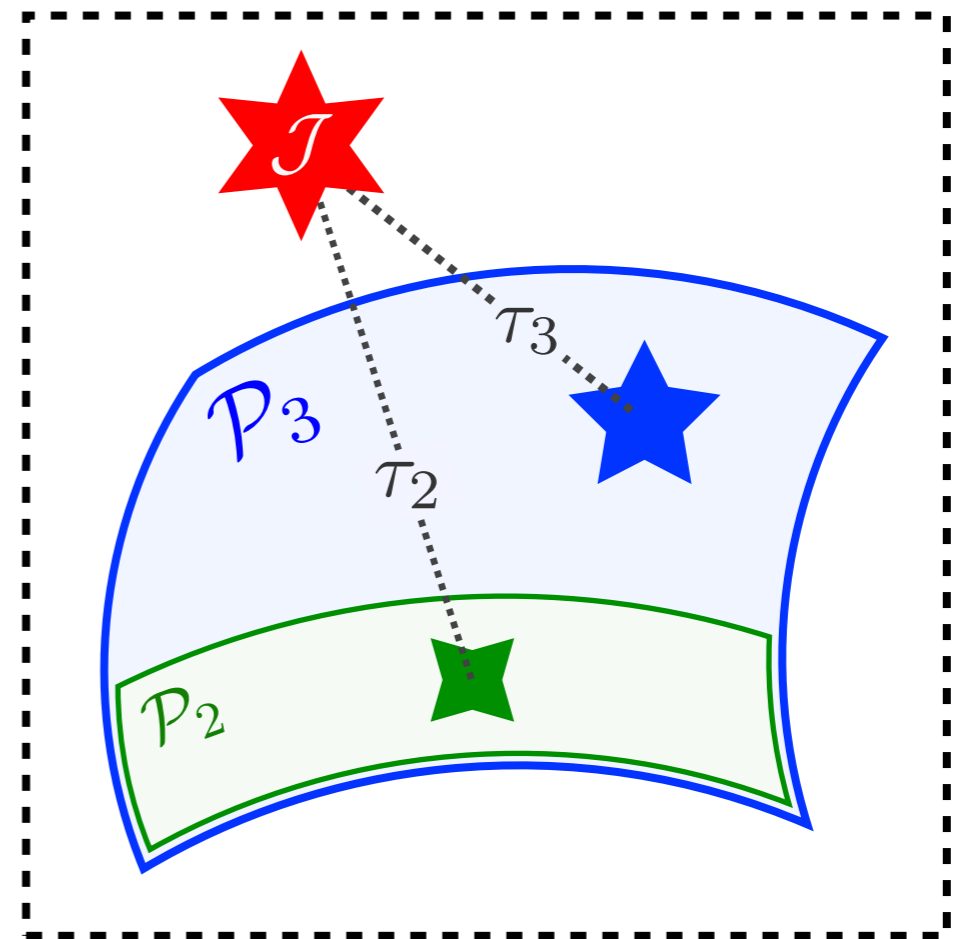
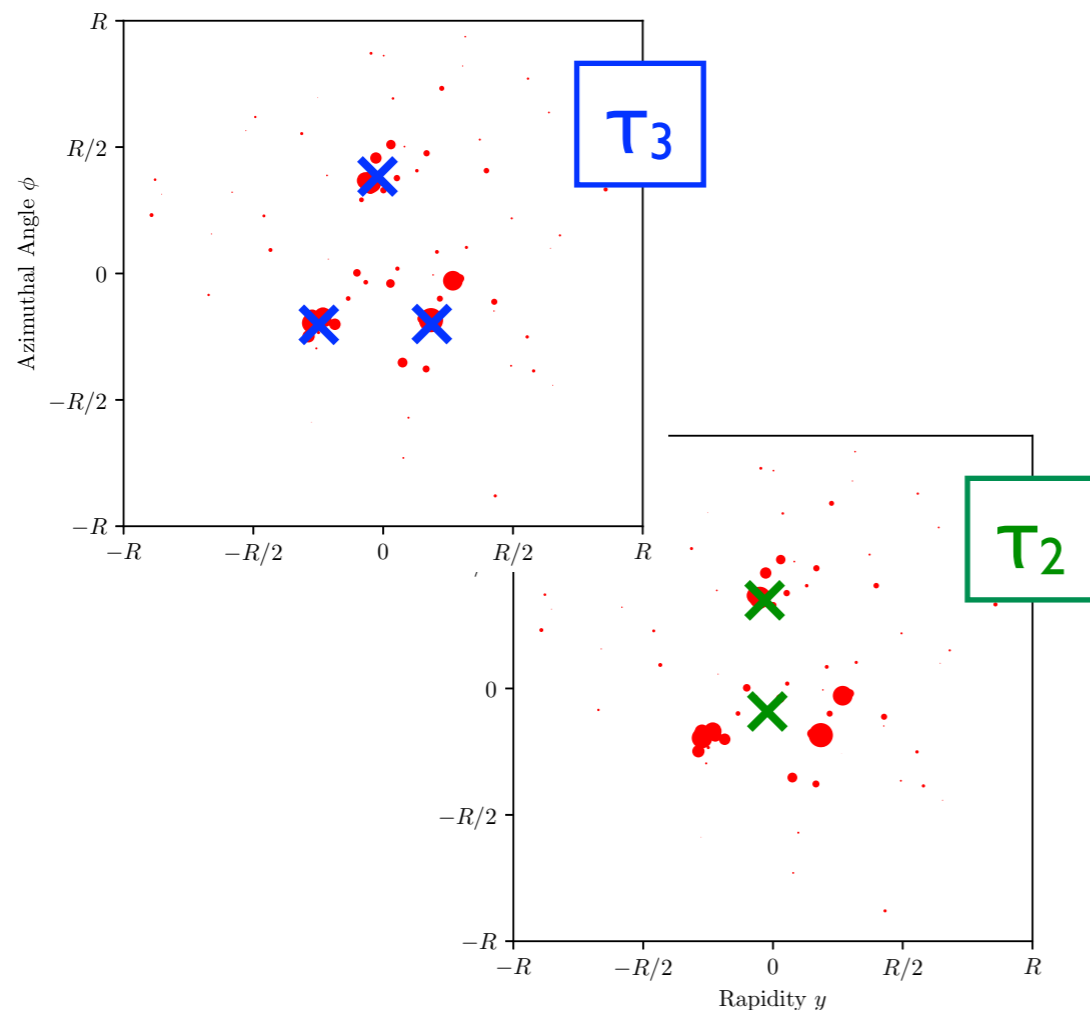


[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
based on Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

N-subjettiness = Point to Manifold EMD

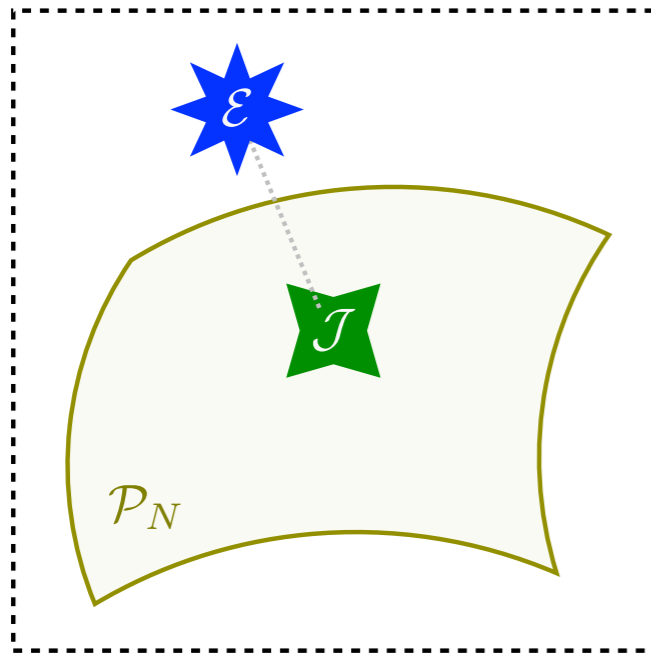
...is secretly an optimal transport problem

$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \text{EMD}(\mathcal{J}, \mathcal{J}')$$



[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]

More Fun with N-particle Manifolds



N-jettiness

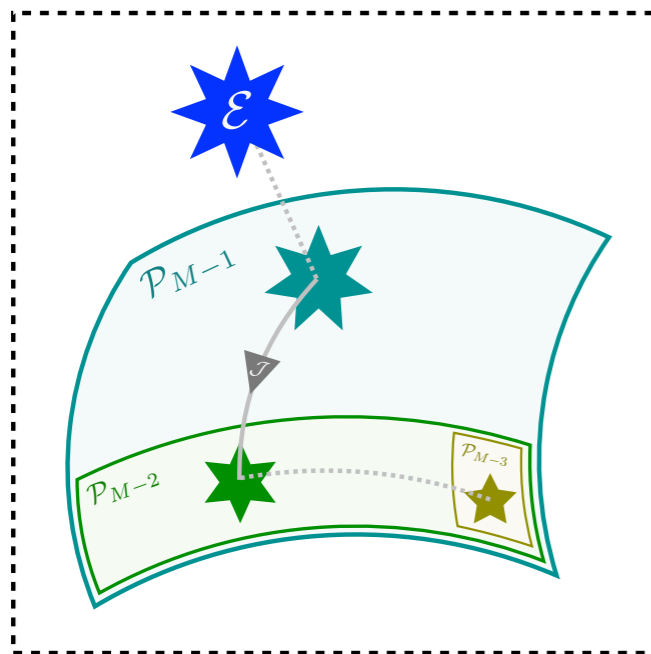
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, [ZPC 1979](#); Stewart, Tackmann, Waalewijn, [PRL 2010](#)]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, [JHEP 2015](#)]



Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

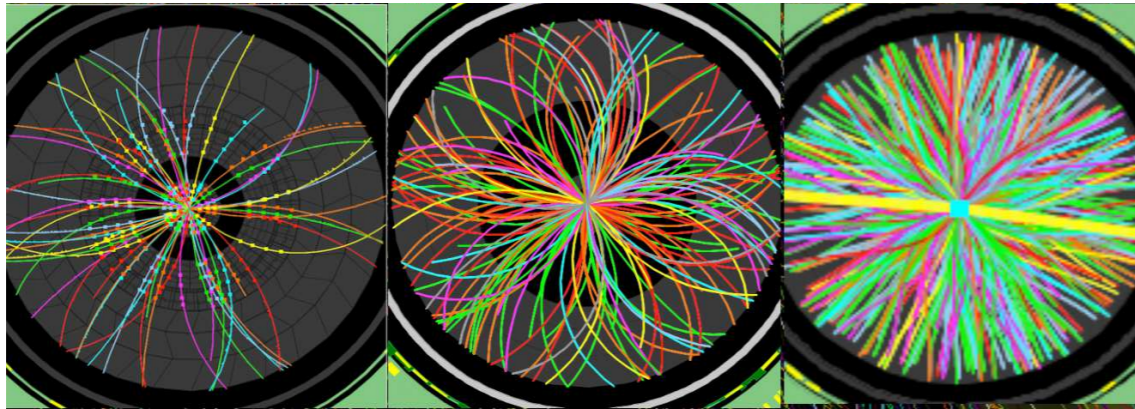
[Catani, Dokshitzer, Seymour, Webber, [NPB 1993](#); Ellis, Soper, [PRD 1993](#)]

[Dokshitzer, Leder, Moretti, Webber, [JHEP 1997](#); Wobisch, Wengler, [arXiv 1999](#)]

[Butterworth, Couchman, Cox, Waugh, [CPC 2003](#); Larkoski, Neill, JDT, [JHEP 2014](#)]

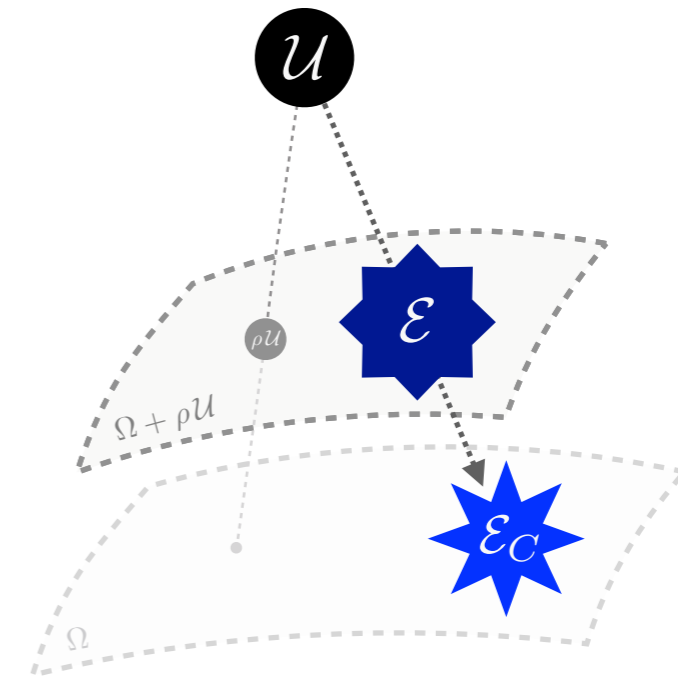
[Komiske, Metodiev, JDT, [arXiv 2020](#)]

Pileup Mitigation



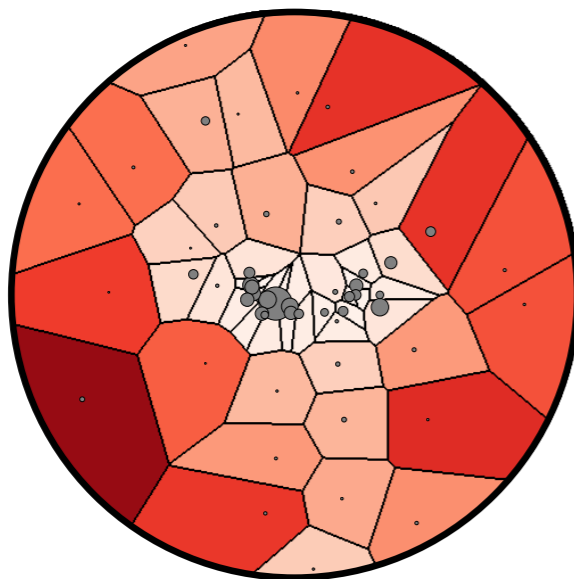
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



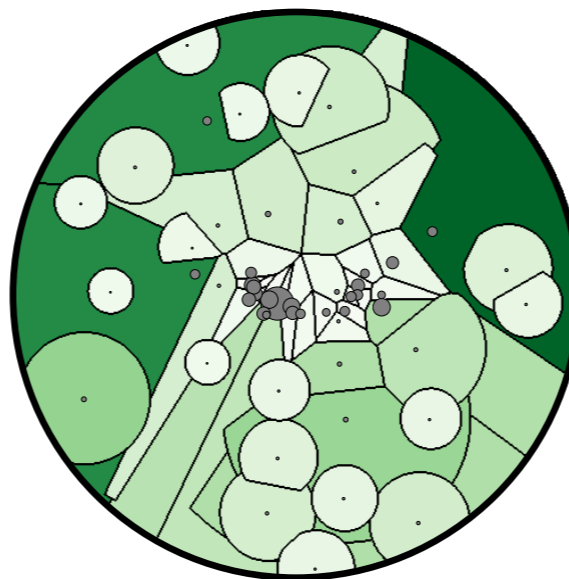
Pileup Mitigation:
“Move away” from uniform event

Voronoi



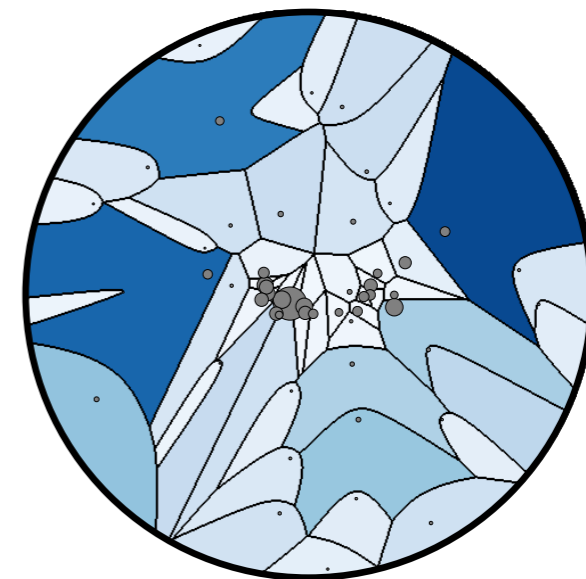
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction



[Berta, Spousta, Miller, Leitner, JHEP 2014]

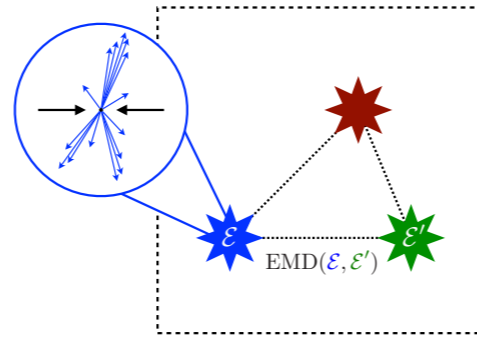
Apollonius



[Komiske, Metodiev, JDT, arXiv 2020]

Pause

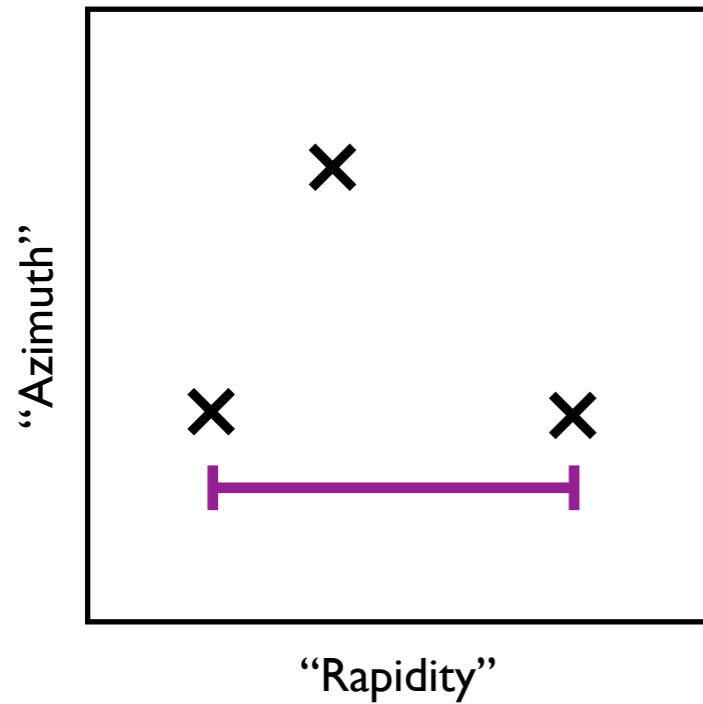
What can be Geometrized?



IRC Safety, Observables,
Jet Algorithms, Pileup Mitigation

How far down does this rabbit hole go?

Direction Space



x = Direction

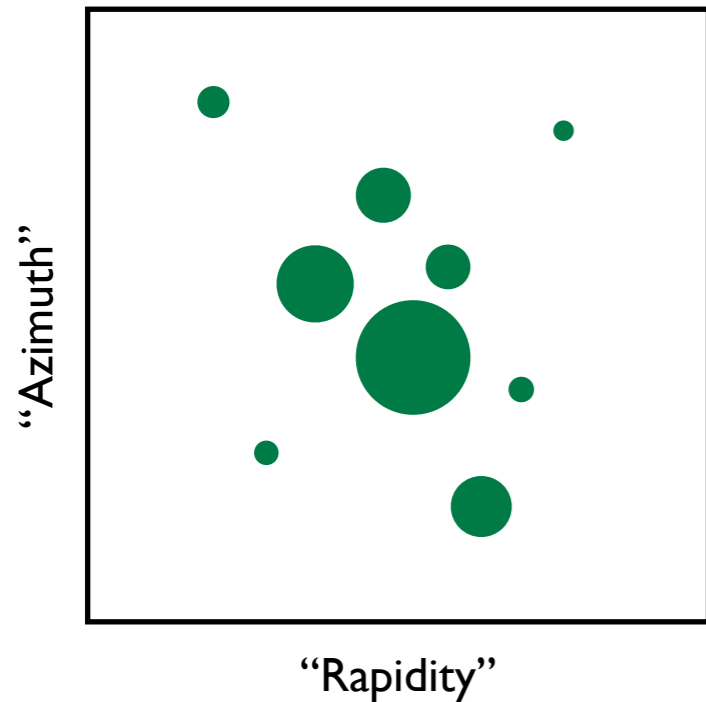
— = Angular Distance

$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$\theta_{ij} = \sqrt{2n_i^\mu n_{j\mu}}$$

(for massless particles)

Direction Space Distribution



● = Weighted Direction

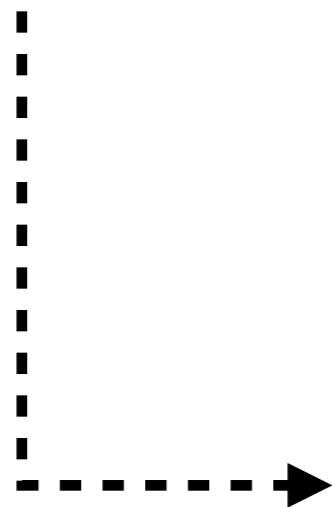
— = Angular Distance

$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$w_i = E_i$$

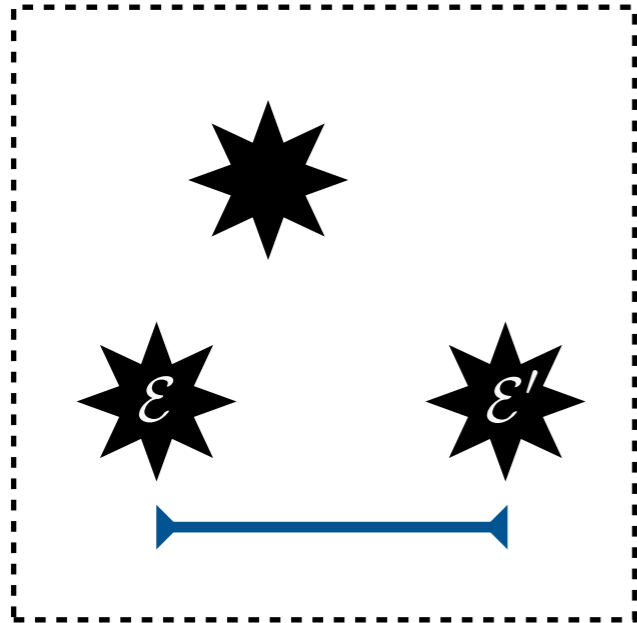
$$\theta_{ij} = \sqrt{2n_i^\mu n_{j\mu}}$$

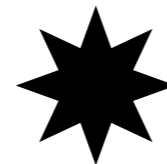
(for massless particles)



★ \mathcal{E} = Event

Event Space



 = Event

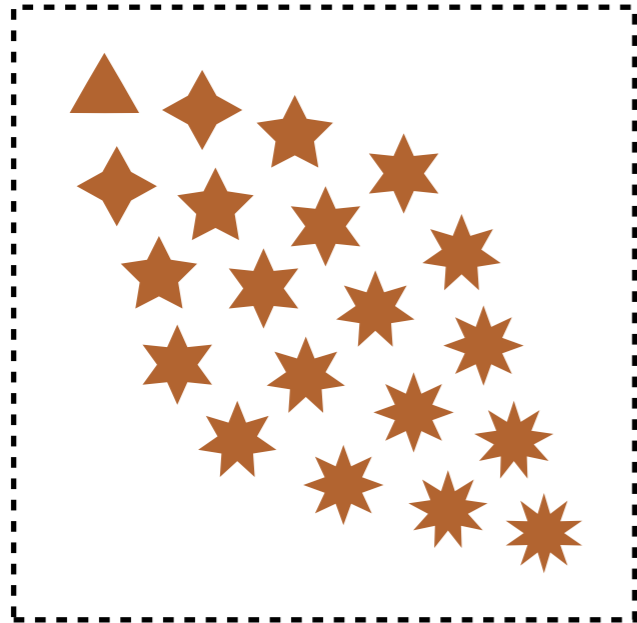
 = EMD
Energy
Mover's Distance


$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

Event Space Distribution



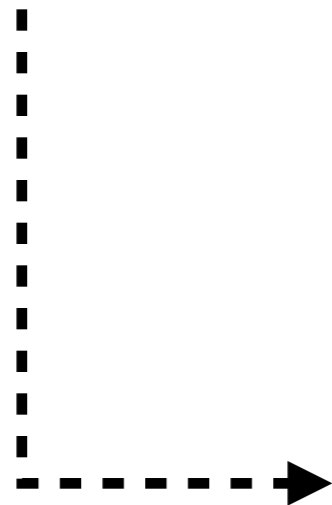
 = **Weighted Event** $\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$


$w_a = \sigma_a$

 = **EMD** $\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$

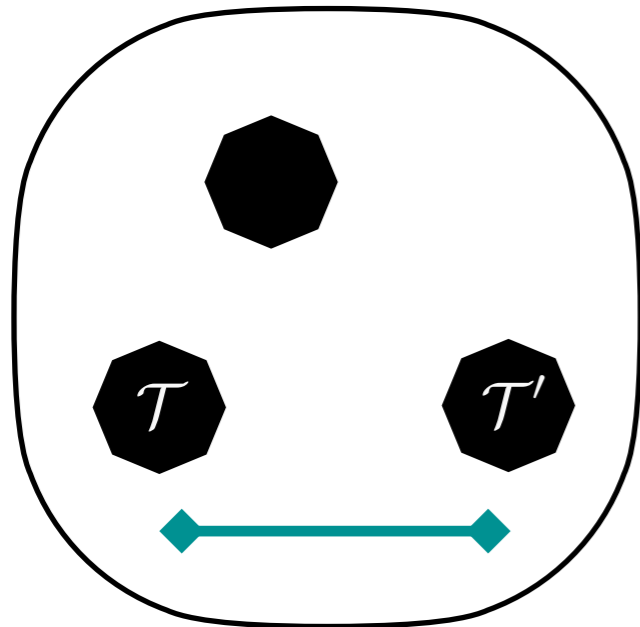
Energy
Mover's Distance

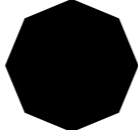
(for equal total energy)



 = **Theory**

Theory Space



 = Theory

 = ΣMD
Cross-Section
Mover's Distance

$$\mathcal{T}(\mathcal{E}) = \sum_a \sigma_a \delta(\mathcal{E} - \mathcal{E}_a)$$

$$\Sigma\text{MD}(\mathcal{T}, \mathcal{T}') = \min_{\{\mathcal{F}\}} \sum_a \sum_b \mathcal{F}_{ab} \text{EMD}(\mathcal{E}_a, \mathcal{E}'_b)$$

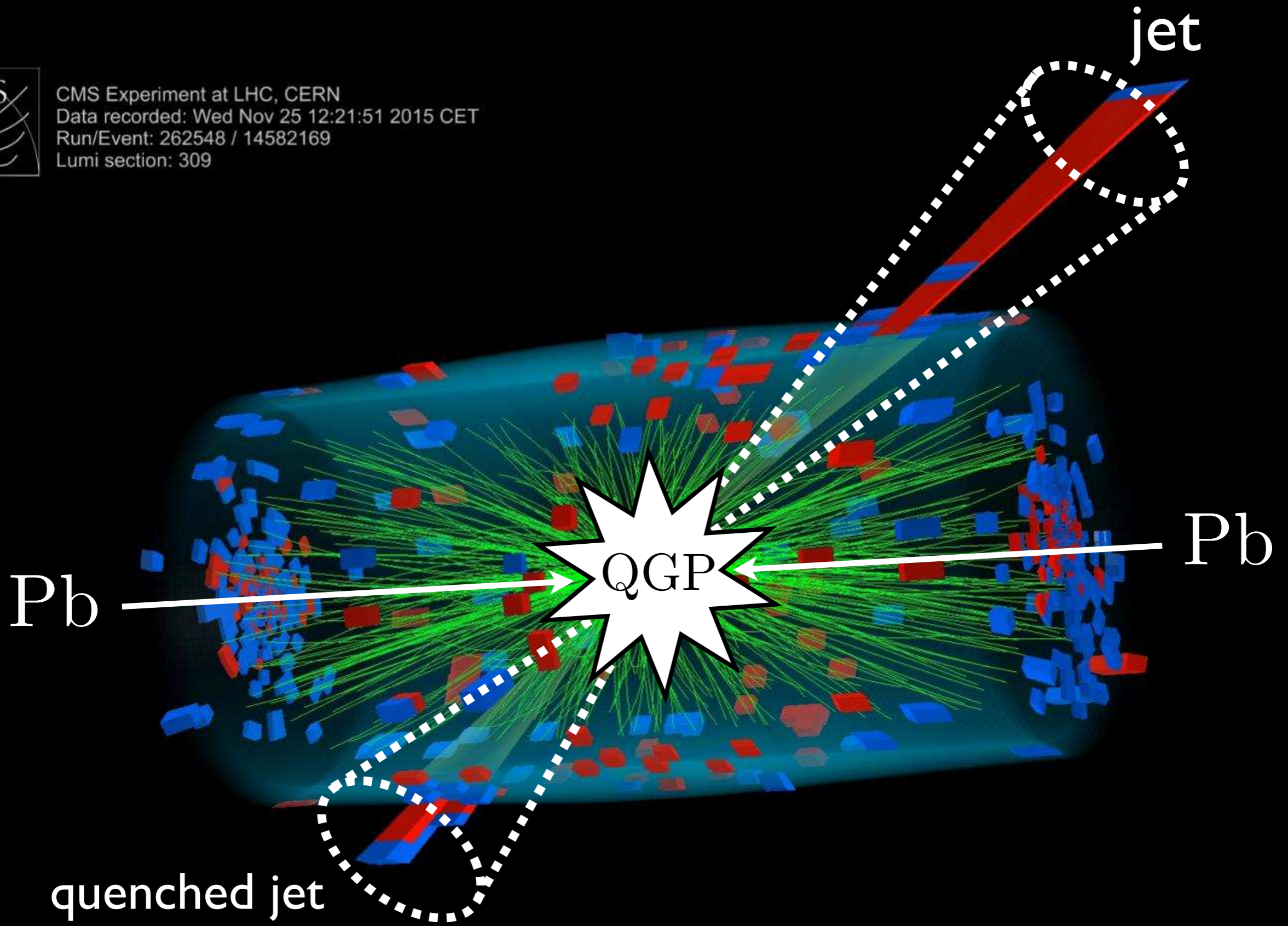
(for equal total xsec)

A distance between theories!

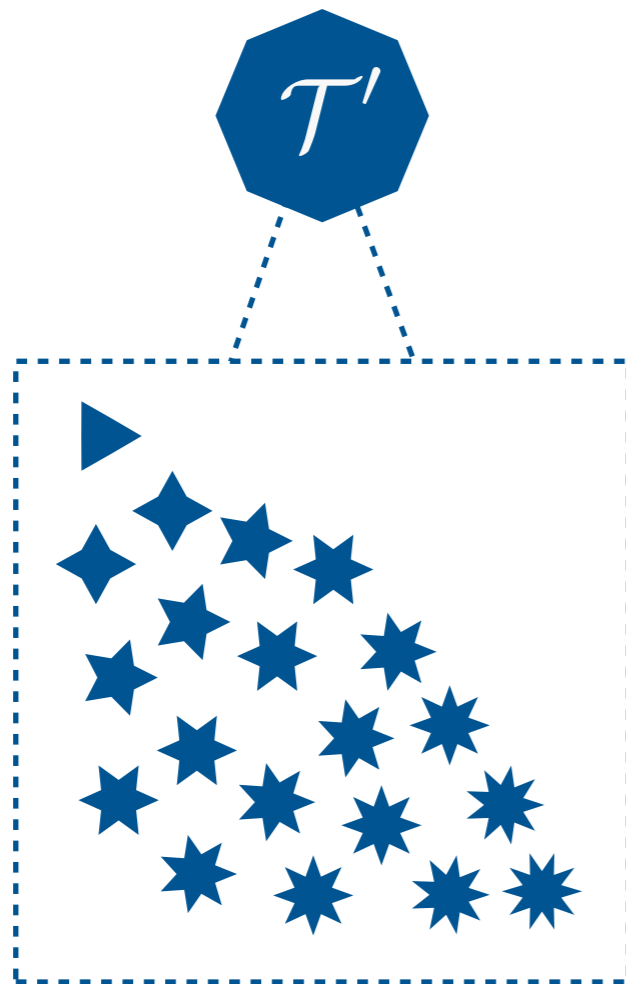
(e.g. EMD : N-jettiness :: ΣMD : k-eventiness)



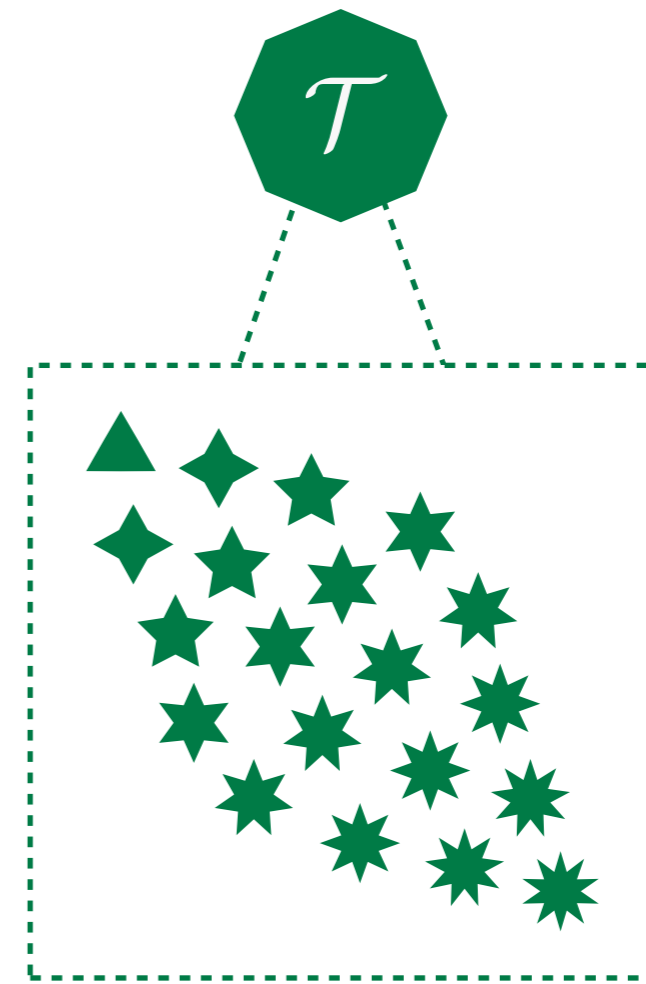
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 25 12:21:51 2015 CET
Run/Event: 262548 / 14582169
Lumi section: 309



Theory Prime: In-Medium QCD



Theory: Vacuum QCD



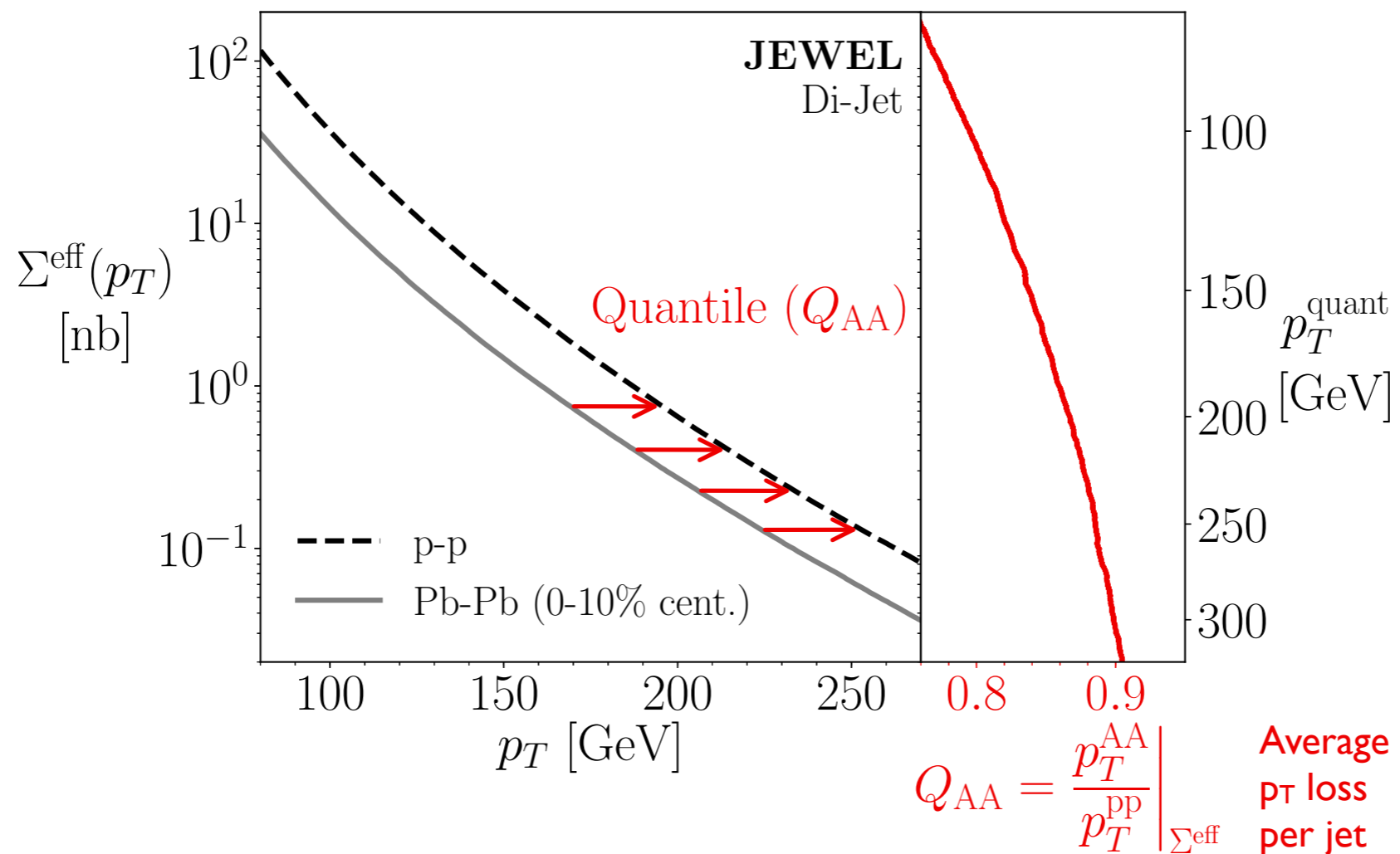
Σ MMD



*Optimal transportation plan defines mapping
between in-medium jets and vacuum jets!*

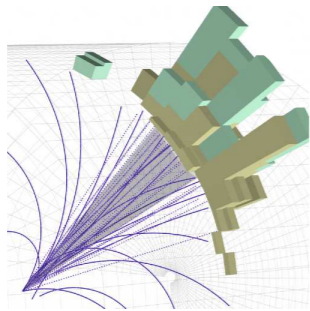
Jet Quenching via Quantile Matching

Equivalent to following a geodesic in theory space (!)



[Brewer, Milhano, JDT, PRL 2019]

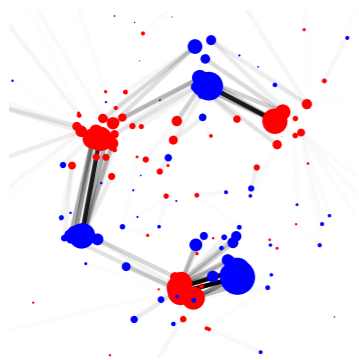
Summary



What is a Collider Event?

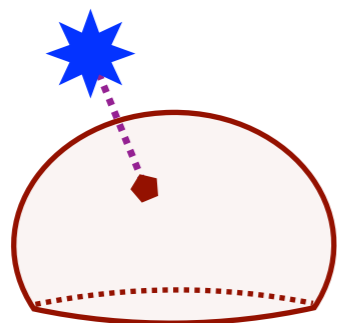
An unordered set of particles that describes the energy flow away from the collision point

(ask me about ML!)



When are Events Similar?

When they are close in the geometric space triangulated by the energy mover's distance



What can be Geometrized?

Many concepts/techniques in quantum field theory and collider physics from the last half century

Fin

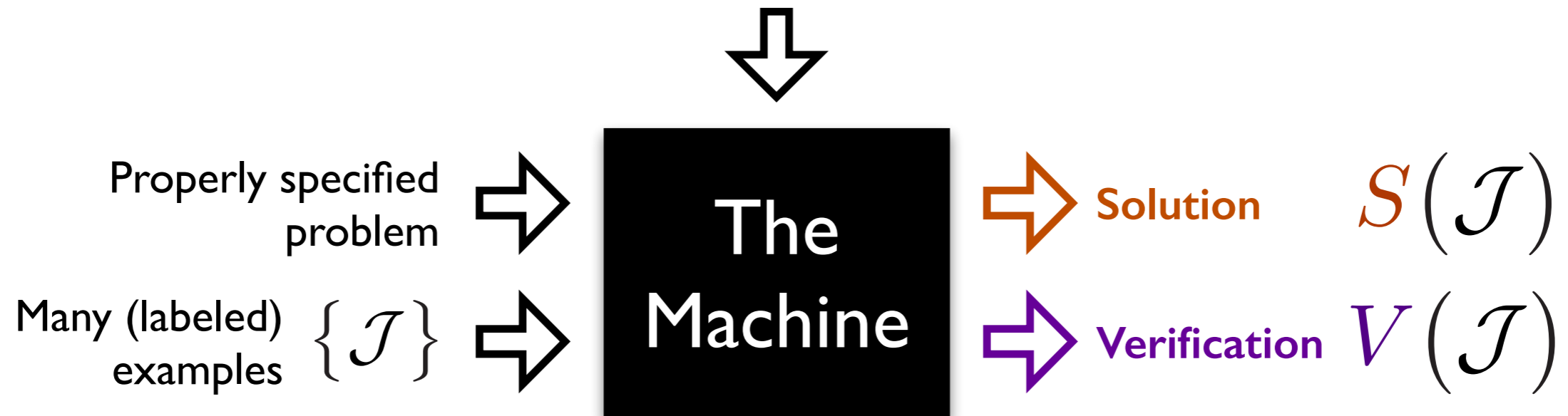
Backup Slides

Aside: Machine Learning for Jets

“ML4Jets”
NYU, January 2020

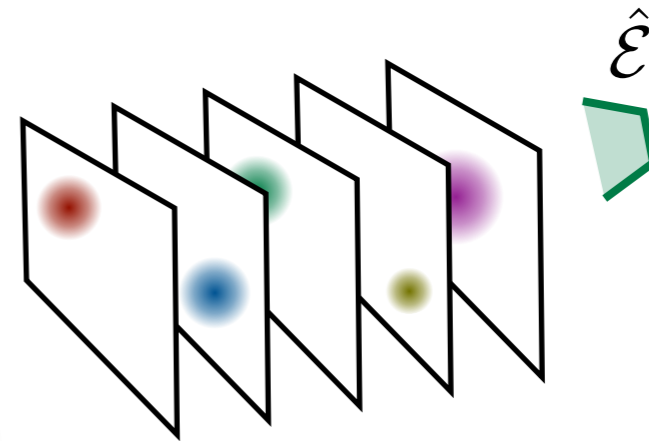
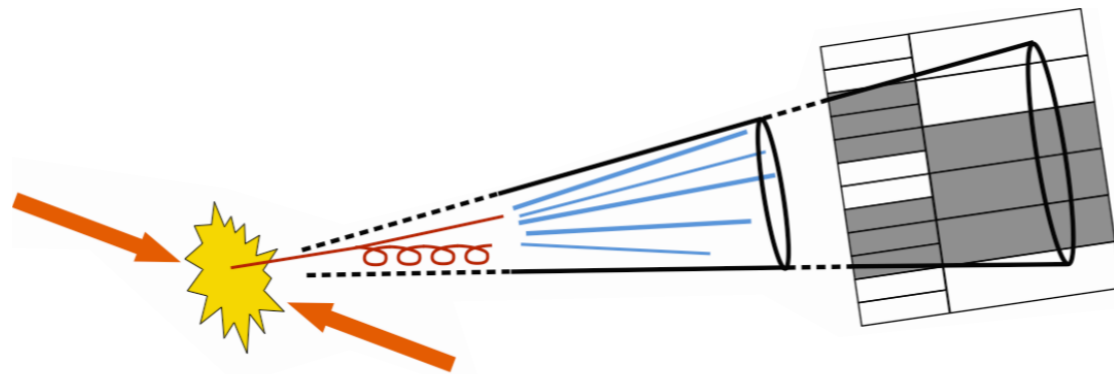
Symmetry: $\mathcal{J} = \{ \vec{p}_1, \vec{p}_2, \vec{p}_3, \dots, \vec{p}_N \}$
Unordered, Variable Length Set (QM!)

Safety: $\vec{p} = \{ E, \hat{n}_x, \hat{n}_y, \hat{n}_z \}$
Energy weighting (QFT!)

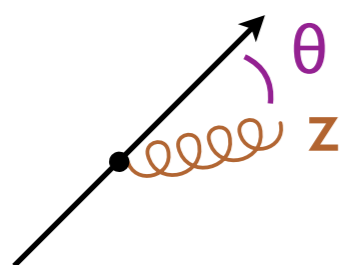
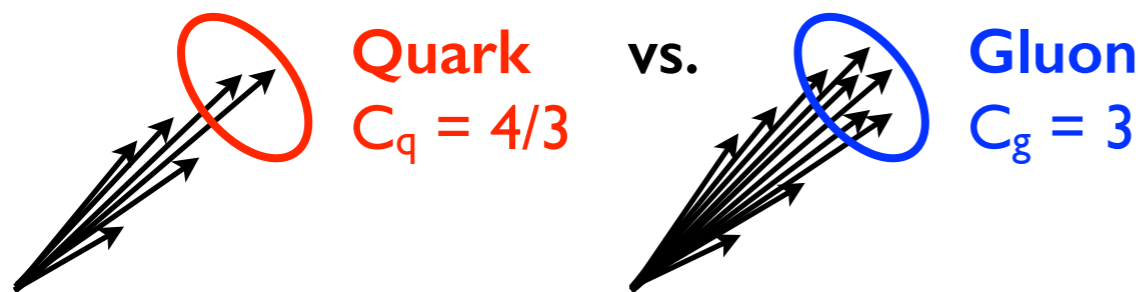


*Check that answer
is physically sensible*

E.g. Energy Flow Networks

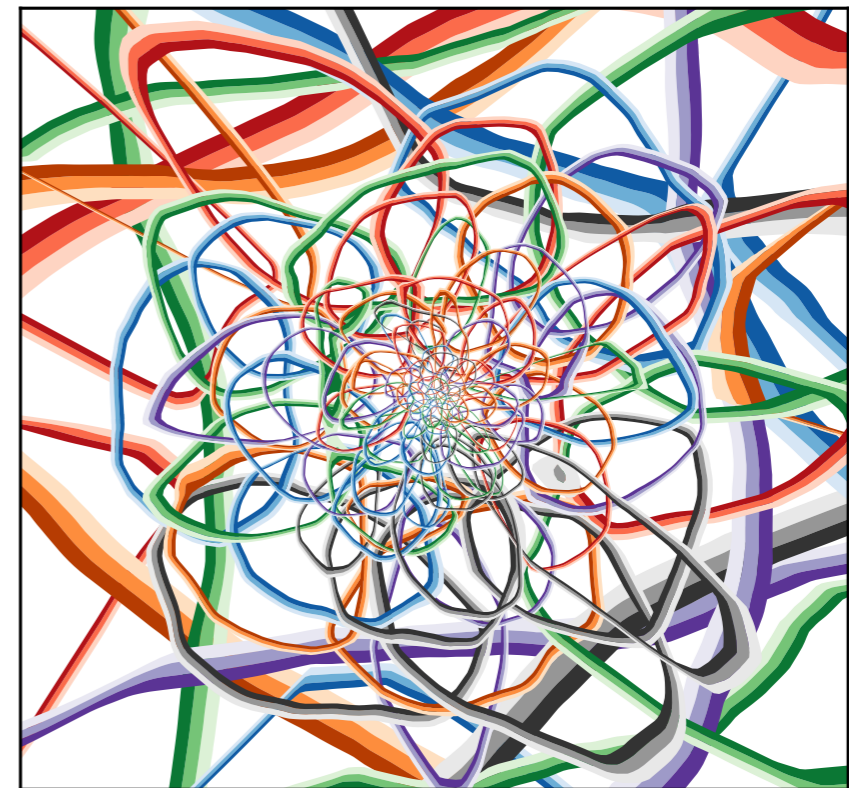


Learning QCD singularities!



AP splitting probability:

$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$



[Komiske, Metodiev, JDT, JHEP 2019; see also Komiske, Metodiev, JDT, JHEP 2018, PRD 2020; special case of Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, NIPS 2017]

Additional Travel Documentation

