

# Anomaly Cancellation With an Extra Gauge Boson

by

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- Anomaly cancellation in the SM
- Lie algebra  $su(3) \oplus su(2) \oplus u(1) \oplus u(1)$
- General solution (BCA, Gripaios, Tooby-Smith, [arXiv:2006.03588](https://arxiv.org/abs/2006.03588))



Cambridge Pheno Working Group

Where data and theory collide



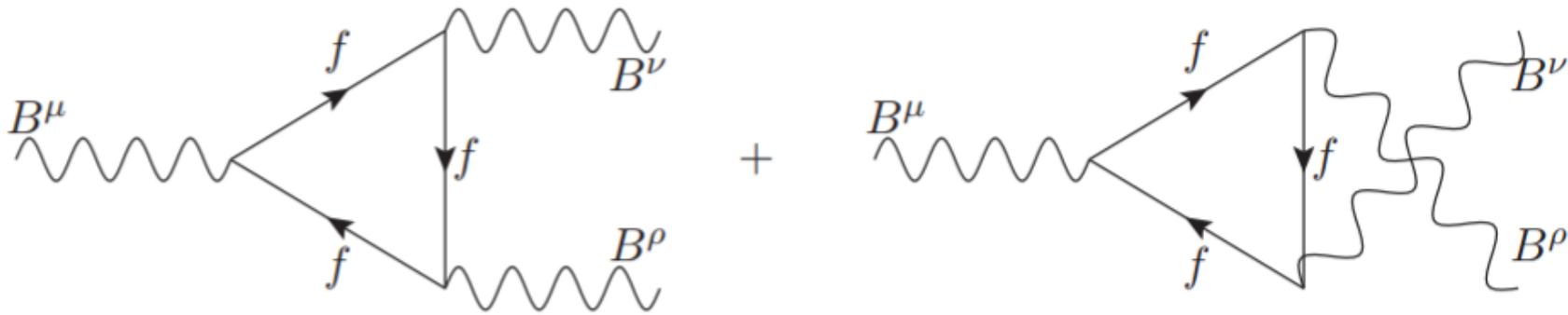
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# Gauge Rank++

Extend SM gauge Lie algebra to  $su(3) \oplus su(2) \oplus u(1) \oplus u(1)$

- $Z'$  phenomenology: coupling to  $\psi$
- axions
- $(g - 2)_\mu$
- anomalies in  $B$ -meson decays
- fermion mass hierarchies
- unification: our analysis will cover extensions for which this is the algebra of a subgroup eg non-abelian extensions

# Local QFT Anomalies in the 4d SM



$$A \equiv \sum_{LH f_i} Y_i^3 - \sum_{RH f_i} Y_i^3$$

Also, replace two  $B$  fields by gravitons, gluons or  $SU(2)$   $W$  bosons. From now on, write all fields as left-handed.

$$Y^3 : \quad 0 = \sum_{j=1}^3 \left( 6Y_{Q_j}^3 + 3Y_{U_j}^3 + 3Y_{D_j}^3 + 2Y_{L_j}^3 + Y_{E_j}^3 \right),$$

$$3^2 Y : \quad 0 = \sum_{j=1}^3 \left( 2Y_{Q_j} + Y_{U_j} + Y_{D_j} \right),$$

$$2^2 Y : \quad 0 = \sum_{j=1}^3 \left( 3Y_{Q_j} + Y_{L_j} \right),$$

$$\text{grav}^2 Y : \quad 0 = \sum_{j=1}^3 \left( 6Y_{Q_j} + 3Y_{U_j} + 3Y_{D_j} + 2Y_{L_j} + Y_{E_j} \right).$$

*Three family* **A**nomaly **C**ancellation **C**onditions

# *Family Universal SM* **anomaly cancellation**

If the hypercharges are quantised, FU but otherwise free, the gauge ACC implies the gravitational ACC<sup>1</sup>.

Deforming the FU SM to  $SU(3) \times SU(2) \times \mathbb{R}_Y$ , and allowing the hypercharges  $Y$  of the chiral fermionic fields to float, the combination of gauge ACC and gravitational ACC implies that the hypercharges must be commensurate<sup>2</sup>.

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<sup>1</sup>Lohitsiri and Tong, [arXiv:1907.00514](https://arxiv.org/abs/1907.00514)

<sup>2</sup>Weinberg, *The Quantum Theory of Fields (1995)*, Cambridge University Press

# Extra $u(1)$ plus SM-singlets

- RH neutrino  $N :=$  SM singlet. Default number is 3
- Can explain neutrino oscillation data
- 0 RH neutrinos equivalent to  $N_i = 0$  subset
- Now, field labels denote the **extra**  $u(1)$  charge
- ACCs become

$$3^2 X : 0 = \sum_{j=1}^3 (2Q_j + U_j + D_j),$$

$$2^2 X : 0 = \sum_{j=1}^3 (3Q_j + L_j),$$

$$Y^2 X : 0 = \sum_{j=1}^3 (Q_j + 8U_j + 2D_j + 3L_j + 6E_j),$$

$$\text{grav}^2 X : 0 = \sum_{j=1}^3 (6Q_j + 3U_j + 3D_j + 2L_j + E_j + N_j),$$

$$YX^2 : 0 = \sum_{j=1}^3 (Q_j^2 - 2U_j^2 + D_j^2 - L_j^2 + E_j^2),$$

$$X^3 : 0 = \sum_{j=1}^3 (6Q_j^3 + 3U_j^3 + 3D_j^3 + 2L_j^3 + E_j^3 + N_j^3).$$

We had a *partial solution* to these in [BCA, Gripaos, Tooby-Smith, arXiv:1912.10022](#): could tell you what the SM fermions' charges had to be, but not what  $N_i$  were, in general.

For 5  $N$  fields, there were always some full solutions (but we didn't capture all of them). For less than 5  $N$ s, one has to do further work to tell whether there are solutions or not, and we didn't say what they are.

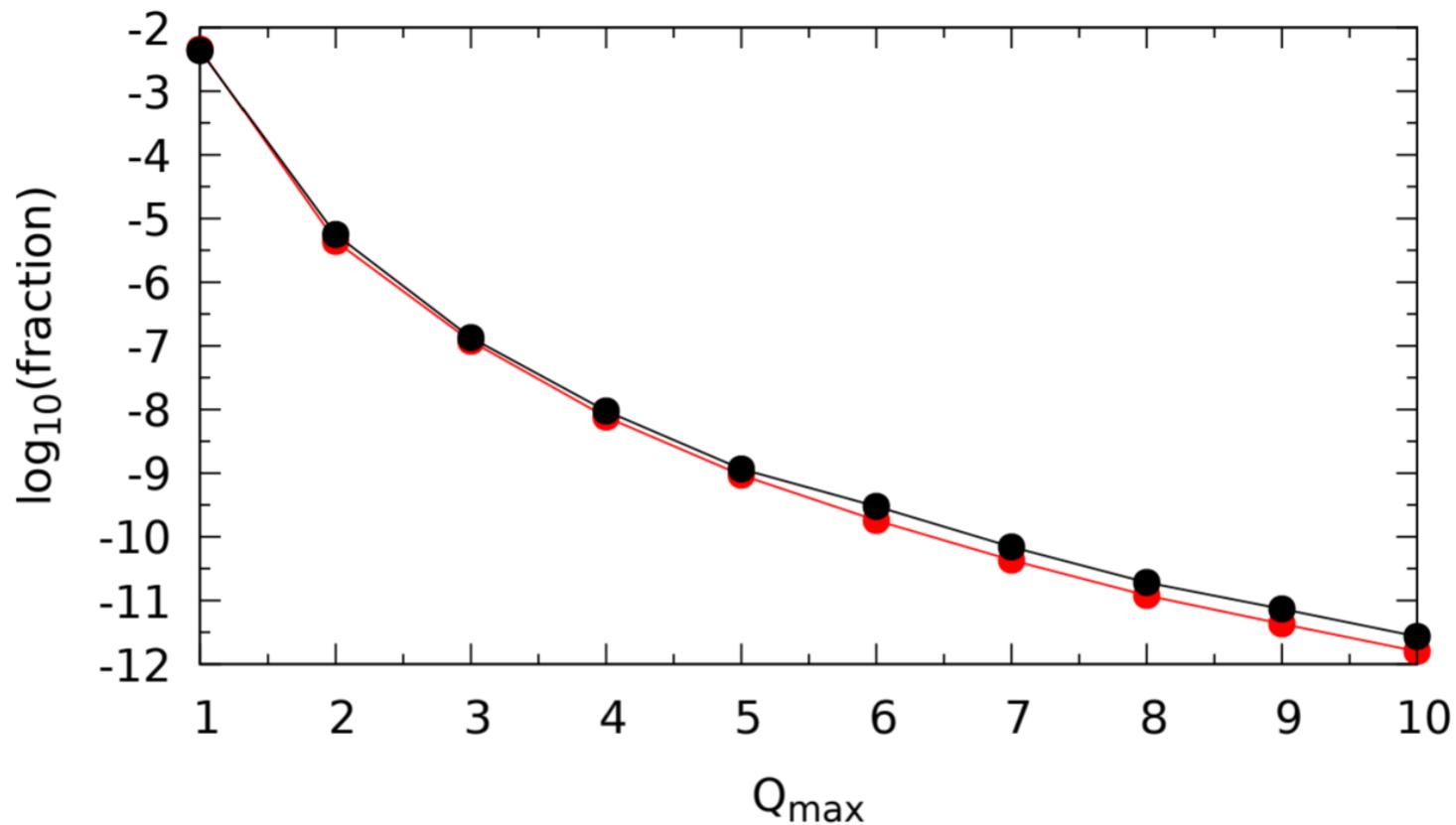
# Diophantine Equations

- Since this is  $u(1)$ , charges are **commensurate**: looking for **compact** extensions like the SM
- Thus we are looking for solutions over  $\mathbb{Z}^{18}$ .
- Any overall real factor in charge can be absorbed in  $u(1)$  gauge coupling:  $\mathcal{L} \supset -g \sum_{\psi} X_{\psi} \bar{\psi} X \psi$
- General diophantine equations are difficult to solve analytically over the integers
- Number theory state-of-the art for general analytic solution of generic diophantine equations is roughly **one cubic in three unknowns**

# Anomaly-free Atlas

To find solutions for fixed  $n \leq 3$  and charges between -10 and 10, we did a numerical scan ( ~~$21^{18} \sim 10^{24}$~~ ): [BCA, Davighi, Melville, arXiv:1812.04602](#).

An [Anomaly-Free Atlas](#) is available for public use:  
<http://doi.org/10.5281/zenodo.1478085>



We begin with 18 charges and 6 anomaly equations reduce these to a 12-dimensional surface of solutions, extending out to infinity, but sparser away from  $\mathbf{0}$ .

$Q_{\max}$	<b>Solutions</b>	Symmetry	Quadratics	Cubics	Time/sec
1	<b>38</b>	16	144	38	0.0
2	<b>358</b>	48	31439	2829	0.0
3	<b>4116</b>	154	1571716	69421	0.1
4	<b>24552</b>	338	34761022	932736	0.6
5	<b>111152</b>	796	442549238	7993169	6.8
6	<b>435305</b>	1218	3813718154	49541883	56
7	<b>1358388</b>	2332	24616693253	241368652	312
8	<b>3612734</b>	3514	127878976089	978792750	1559
9	<b>9587085</b>	5648	558403872034	3432486128	6584
10	<b>21546920</b>	7540	2117256832910	10687426240	24748

Inequivalent solutions with 3 RH  $\nu$

$Q$	$Q$	$Q$	$\nu$	$\nu$	$\nu$	$e$	$e$	$e$	$u$	$u$	$u$	$L$	$L$	$L$	$d$	$d$	$d$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	-1	0	1
0	0	0	0	0	0	-1	0	1	0	0	0	-1	0	1	0	0	0
0	0	0	0	0	0	-1	0	1	-1	0	1	0	0	0	-1	0	1
-1	0	1	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0
-1	0	1	0	0	0	0	0	0	-1	0	1	0	0	0	-1	0	1
-1	0	1	0	0	0	-1	0	1	-1	0	1	0	0	0	0	0	0
-1	0	1	0	0	0	-1	0	1	-1	0	1	-1	0	1	-1	0	1

eg:  $Q_{max} = 1$ ,  $N_i = 0$ . Charges within a species are listed in *increasing order*.

# Known Solutions

	$Q_1$	$Q_2$	$Q_3$	$U_1$	$U_2$	$U_3$	$D_1$	$D_2$	$D_3$	$L_1$	$L_2$	$L_3$	$E_1$	$E_2$	$E_3$	$N_1$	$N_2$	$N_3$
$A$	0	0	1	0	0	-4	0	0	-2	0	0	-3	0	0	6	0	0	0
$B$	1	1	1	-1	-1	-1	-1	-1	-1	-3	-3	-3	3	3	3	3	3	3
$C$	-1	0	1	-1	0	1	-1	0	1	-1	0	1	-1	0	1	0	0	0

- $A$  is TFHM (BCA, Davighi, arXiv:1809.01158)
- $B$  is  $B - L$ , *vector-like*
- $C$  has inter-family cancellation

# Analytic Solution

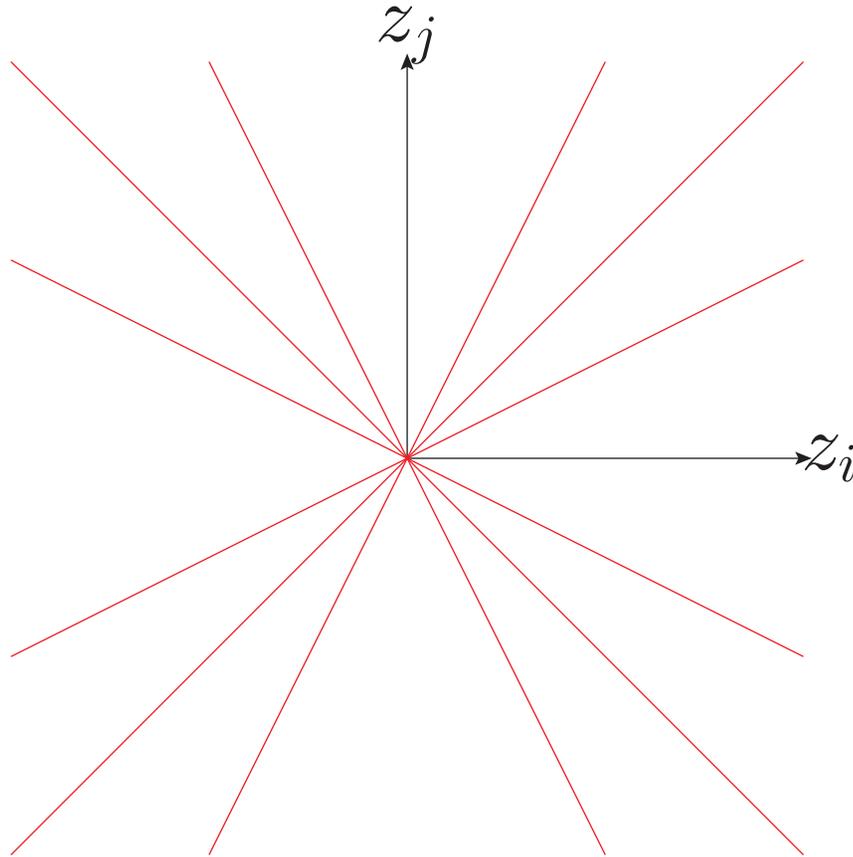
Want a full, general analytic solution for any  $Q_{\max}$ .

First step is to convert it into a problem in geometry by noting that solutions over  $\mathbb{Q}$  are equivalent to those over  $\mathbb{Z}$  by clearing all denominators. Since  $\mathbb{Q}$  is a field, you can define geometry on it.

We start with  $\mathbb{Q}^{18}$  solution space.

All solutions where charges  $z_i$  differ by a common multiple are physically equivalent so we define an equivalence class to obtain  $P\mathbb{Q}^{17}$ .

# Projective space $P\mathbb{Q}^{17}$



2d surface through origin becomes a line in projective space  
and a line through origin becomes a point

# Preliminaries

4 linear equations restrict  $P\mathbb{Q}^{17}$  to a projective subspace isomorphic to  $P\mathbb{Q}^{13}$ . Within this, we look for the intersection of a quadratic surface

$$0 = \sum_{j=1}^3 (Q_j^2 - 2U_j^2 + D_j^2 - L_j^2 + E_j^2)$$

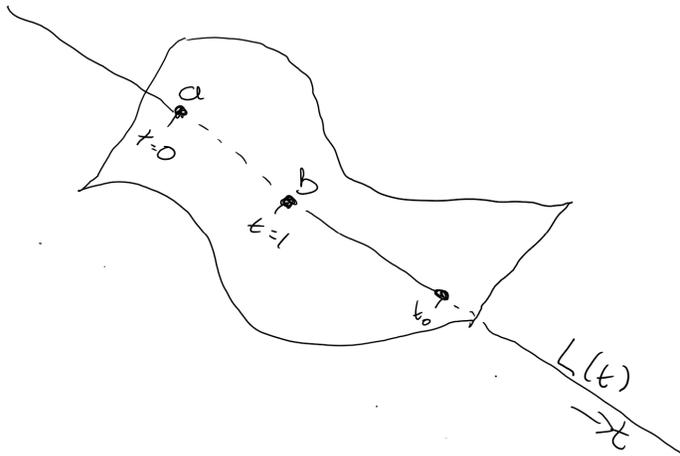
and a cubic surface

$$0 = \sum_{j=1}^3 (6Q_j^3 + 3U_j^3 + 3D_j^3 + 2L_j^3 + E_j^3 + N_j^3)$$

# The Method of Chords<sup>3</sup>

*“A chord intersecting a rational cubic surface at two known rational points intersects it at 1 other  $\mathbb{Q}$  point”*

eg Rational cubic  $c(z_i) = 0$ . Put a line through 2 known intersections  $a, b$ :  $L(t) = a + t(b - a)$ . Along line,  $c(L(t)) = kt(t - 1)(t - t_0)$ , where  $k, t_0 \in \mathbb{Q}$ .



*Caveat:* It is possible that the line lies *entirely within the cubic surface*, i.e.  $c(L(t)) = 0$  irrespective of  $t$ .

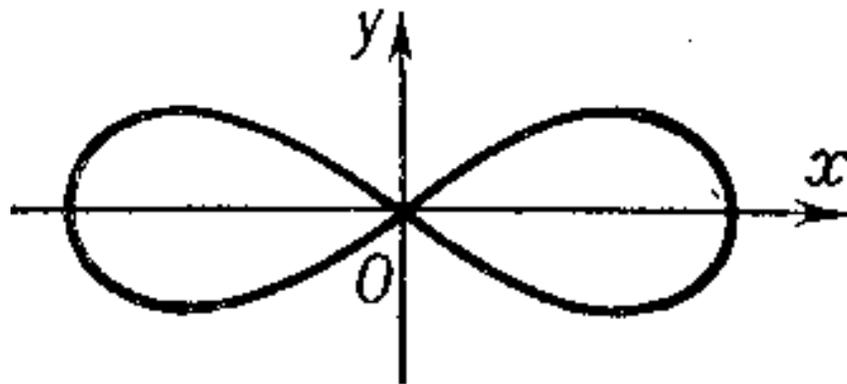
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<sup>3</sup>Newton, Fermat, C17<sup>th</sup>

# Double Points

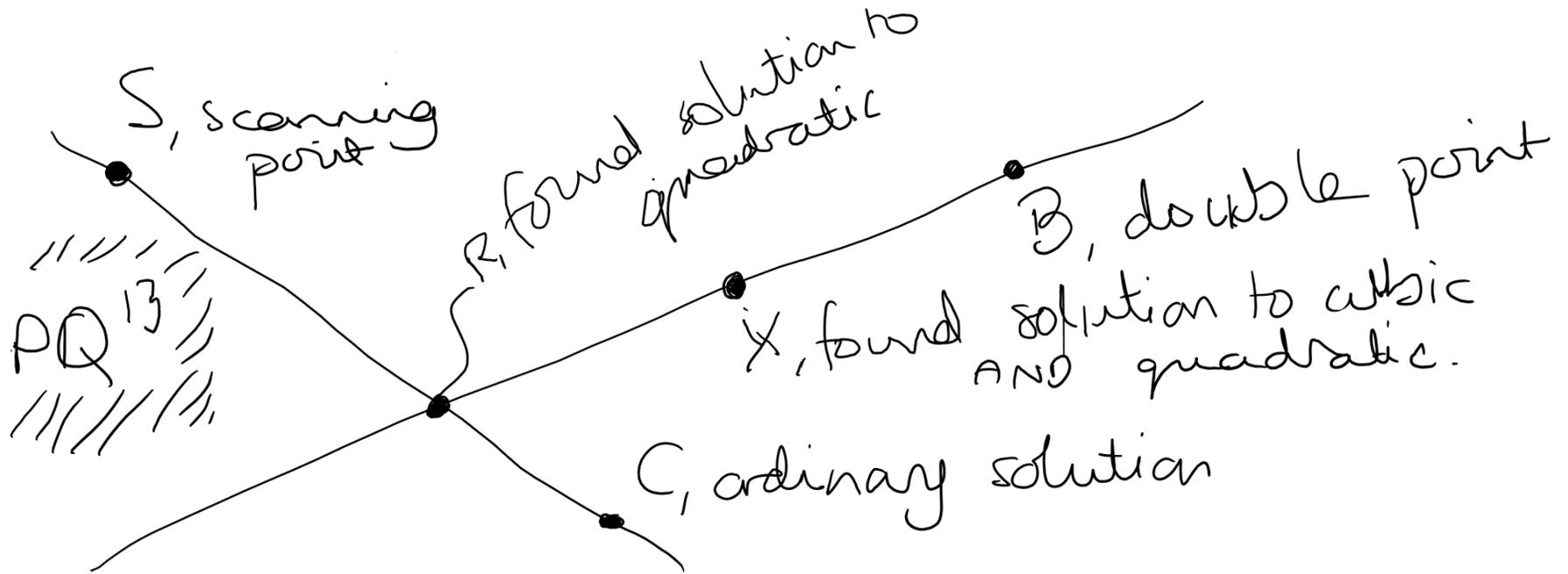
Points which are solutions of multiplicity two. All partial derivatives of the surface vanish there, eg  $(x, y) = (0, 0)$  of the curve

$$(x^2 + y^2 + a^2)^2 - 4a^2x^2 - a^4 = 0$$



Point  $B$  is a *double point* of the quadratic and the cubic

# Method



- Every solution to quadratic  $R$  lies on *some* line  $SC$
- $B$  is double point of quadratic  $\Rightarrow RB$  in quadratic
- Scan  $S$  (parameters) to find *all*  $R$ , extend line to find *all*  $X$ .

# The Nitty-Gritty

$$\begin{aligned}
 Q_1 &= \Gamma - \Sigma + \Lambda S_{Q_1}, \\
 Q_2 &= \Gamma + \Lambda S_{Q_2}, \\
 Q_3 &= \Gamma + \Sigma + \Lambda S_{Q_3}, \\
 U_1 &= -\Gamma - \Sigma + \Lambda S_{U_1}, \\
 U_2 &= -\Gamma + \Lambda S_{U_2}, \\
 U_3 &= -\Gamma + \Sigma + \Lambda S_{U_3}, \\
 D_1 &= -\Gamma - \Sigma + \Lambda S_{D_1}, \\
 D_2 &= -\Gamma + \Lambda S_{D_2}, \\
 D_3 &= -\Gamma + \Sigma + \Lambda S_{D_3}, \\
 L_1 &= -3\Gamma - \Sigma + \Lambda S_{L_1}, \\
 L_2 &= -3\Gamma + \Lambda S_{L_2}, \\
 L_3 &= -3\Gamma + \Sigma + \Lambda S_{L_3}, \\
 E_1 &= 3\Gamma - \Sigma + \Lambda S_{E_1}, \\
 E_2 &= 3\Gamma + \Lambda S_{E_2}, \\
 E_3 &= 3\Gamma + \Sigma + \Lambda S_{E_3}, \\
 N_1 &= 3\Gamma + \Lambda S_{N_1}, \\
 N_2 &= 3\Gamma + \Lambda S_{N_2}, \\
 N_3 &= 3\Gamma + \Lambda S_{N_3},
 \end{aligned}$$

$$\begin{aligned}
 \Gamma &= c(R, R, R) + r\delta_{c(B,R,R),0}\delta_{c(R,R,R),0}, \\
 \Sigma &= (-3c(B, R, R) + t\delta_{c(B,R,R),0}\delta_{c(R,R,R),0}) \\
 &\quad (q(S, S) + a\delta_{q(S,S),0}\delta_{q(C,S),0}), \\
 \Lambda &= (-3c(B, R, R) + t\delta_{c(B,R,R),0}\delta_{c(R,R,R),0}) \\
 &\quad (-2q(C, S) + b\delta_{q(S,S),0}\delta_{q(C,S),0}).
 \end{aligned}$$

$$\begin{aligned}
 q(P, P') &:= \sum_{i=1}^3 (Q_i Q_i' - 2U_i U_i' + D_i D_i' \\
 &\quad - L_i L_i' + E_i E_i'), \\
 c(P, P', P'') &:= \sum_{i=1}^3 (6Q_i Q_i' Q_i'' + 3U_i U_i' U_i'' + 3D_i D_i' D_i'' \\
 &\quad + 2L_i L_i' L_i'' + E_i E_i' E_i'' + N_i N_i' N_i''). \quad (3)
 \end{aligned}$$

$$R = q(S, S)C - 2q(C, S)S + \delta_{q(S,S),0}\delta_{q(C,S),0}(aC + bS),$$

$$S_{Q_3} = \frac{1}{2} \left[ -2S_{Q_1} - 2S_{Q_2} + \sum_{i=1}^3 (S_{D_i} + S_{N_i}) \right],$$

$$S_{U_3} = - \left[ S_{U_1} + S_{U_2} + \sum_{i=1}^3 (2S_{D_i} + S_{N_i}) \right],$$

$$S_{L_3} = -\frac{1}{2} \left[ 2S_{L_1} + 2S_{L_2} + 3 \sum_{i=1}^3 (S_{D_i} + S_{N_i}) \right],$$

$$S_{E_3} = -S_{E_1} - S_{E_2} + \sum_{i=1}^3 (3S_{D_i} + 2S_{N_i}).$$

# Solution Space

Is called a projective *variety*, i.e. *not* a manifold (in  $\mathbb{Q}$  anyway, but also there are singular cases of lines within planes where the dimensionality decreases).

**Over-**parameterisation in terms of 18 integers

$$S_{Q_1}, S_{Q_2}, S_{U_1}, S_{U_2}, S_{D_1}, S_{D_2}, S_{D_3}, S_{L_1}, S_{L_2}, S_{E_1}, S_{E_2},$$

$$S_{N_1}, S_{N_2}, S_{N_3}, a, b, r, t \in \mathbb{Q}$$

It is at most **11-dimensional**.  $S \cdot C = S \cdot B = 0$ . An inverse ( $S = T, a = 0, b = 1, r = 0, t = 1$ ), was checked against 21 549 920 all Anomaly-free Atlas solns.

# Caveat?

Anomalies can be cancelled by a Wess-Zumino term, a higher dimension  $\mathcal{L}$  operator of topological origin. These can eg be obtained by integrating out heavy states.

Generic ones are hard to generate whilst making the relevant heavy states heavy from  $u(1)$  spontaneous breakdown.

# Other Constraints

Consider perturbativity:

$$\frac{d \ln g}{d \ln \mu} = \frac{g^2 \sum_{i \in \chi_{UV}} z_i^2}{24\pi^2} < 1$$

$$\Leftrightarrow g < \frac{2\pi\sqrt{6}}{\sqrt{\sum_{i \in \chi_{UV}} z_i^2}}.$$

# Summary

Using techniques from number theory and algebraic geometry

We have a general solution to the full set of anomaly equations for SM rank extensions with 3 RH neutrinos.

The couplings and phenomenology of a resulting  $Z'$  depend upon these. Model extensions also depend upon them.

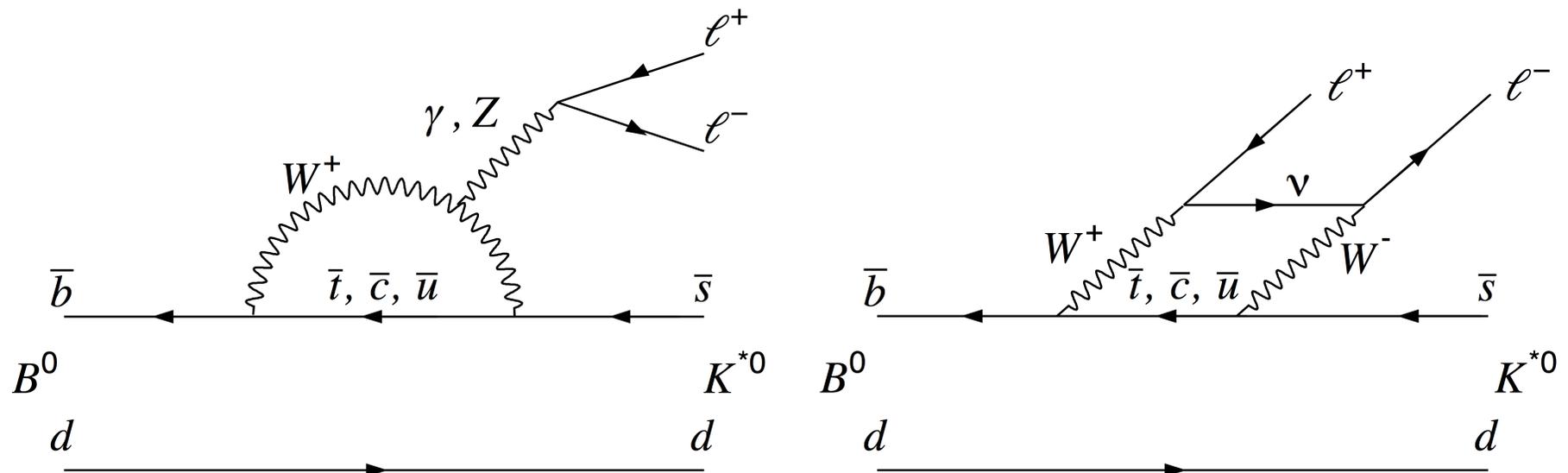
# Strange *b* Activity



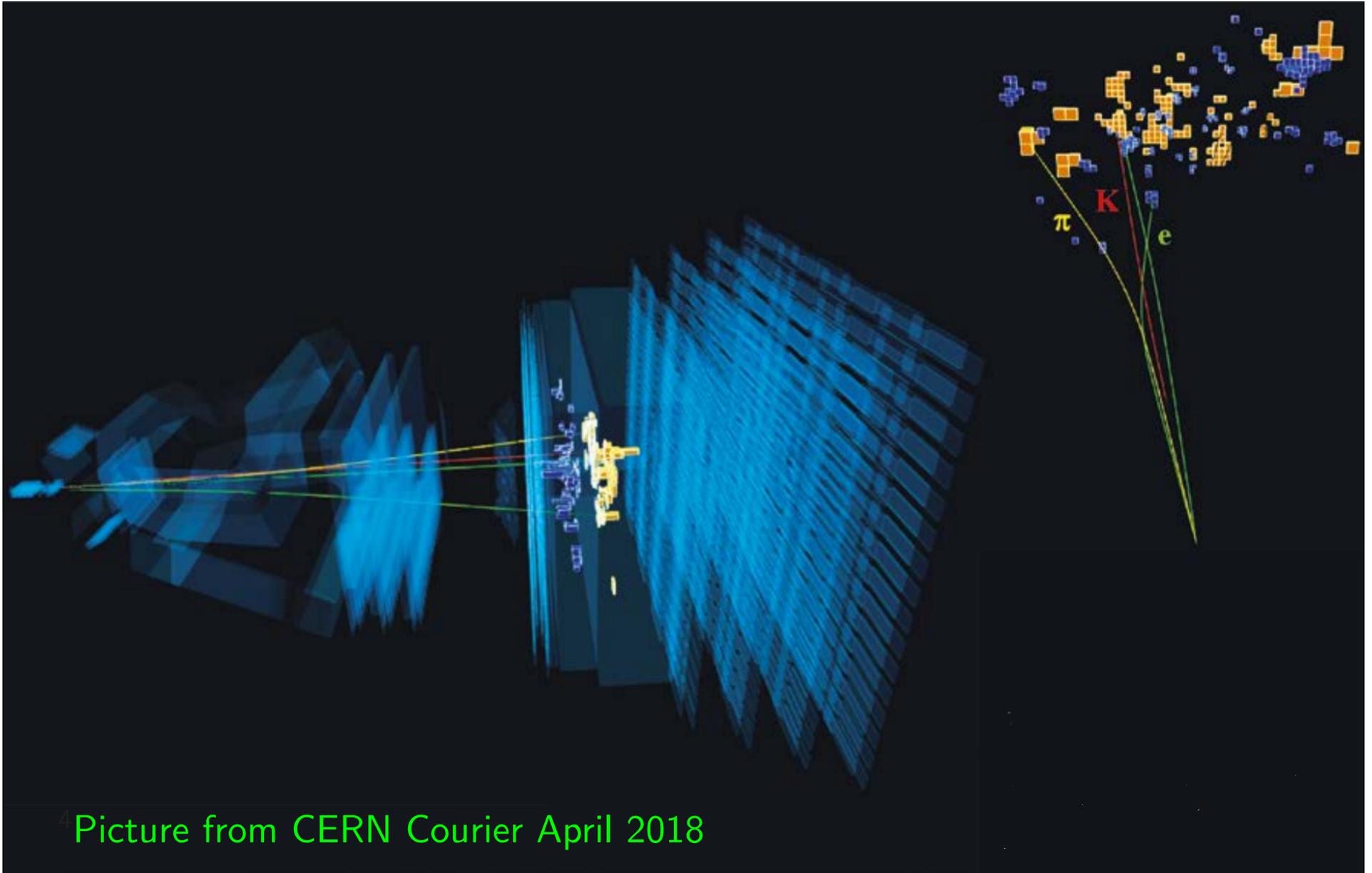
# $R_K^{(*)}$ in Standard Model

$$R_K = \frac{BR(B \rightarrow K \mu^+ \mu^-)}{BR(B \rightarrow K e^+ e^-)}, \quad R_{K^*} = \frac{BR(B \rightarrow K^* \mu^+ \mu^-)}{BR(B \rightarrow K^* e^+ e^-)}.$$

These are **rare decays** (each  $BR \sim \mathcal{O}(10^{-7})$ ) because they are absent at tree level in SM.



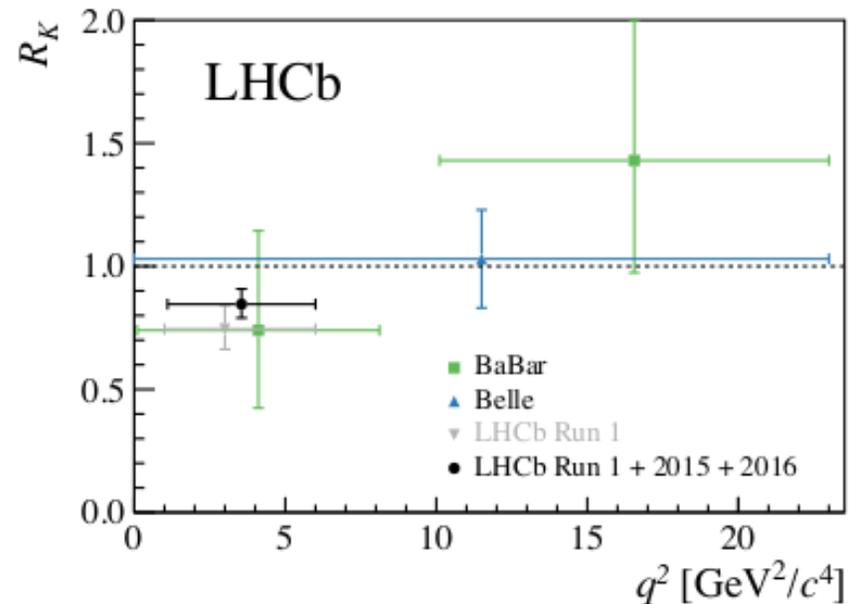
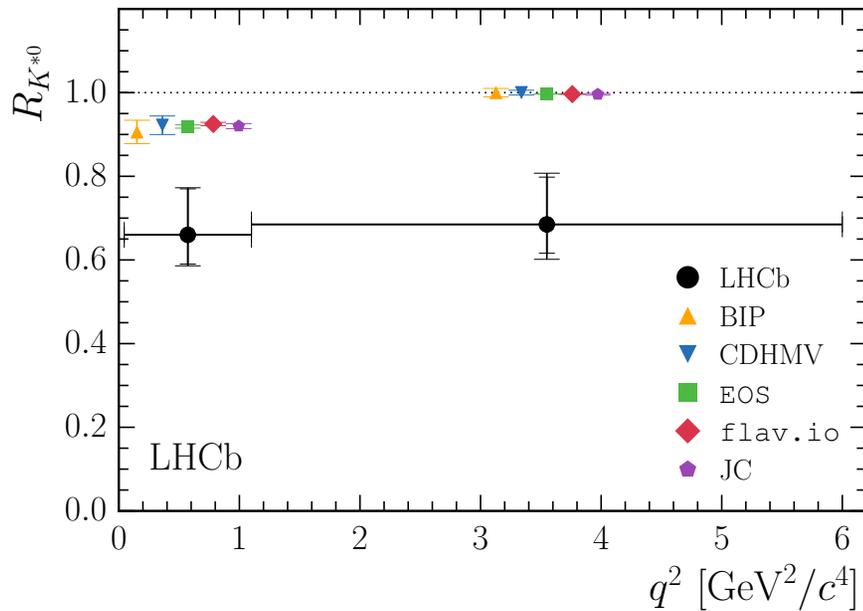
# LHCb $B^0 \rightarrow K^{0*} e^+ e^-$ Event<sup>4</sup>



$$R_{K^{(*)}}$$

LHCb results:  $q^2 = m_{ll}^2$ .

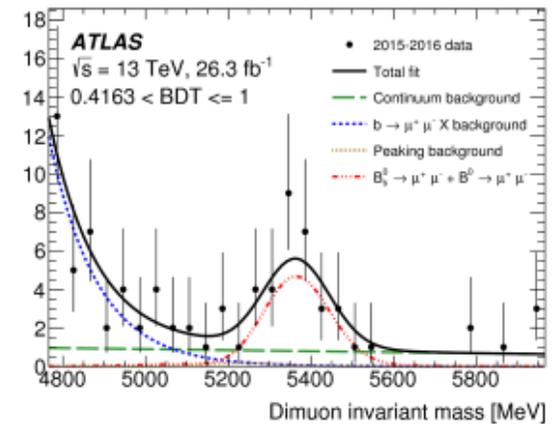
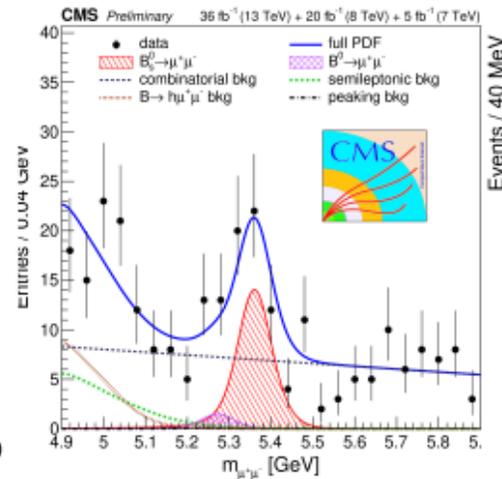
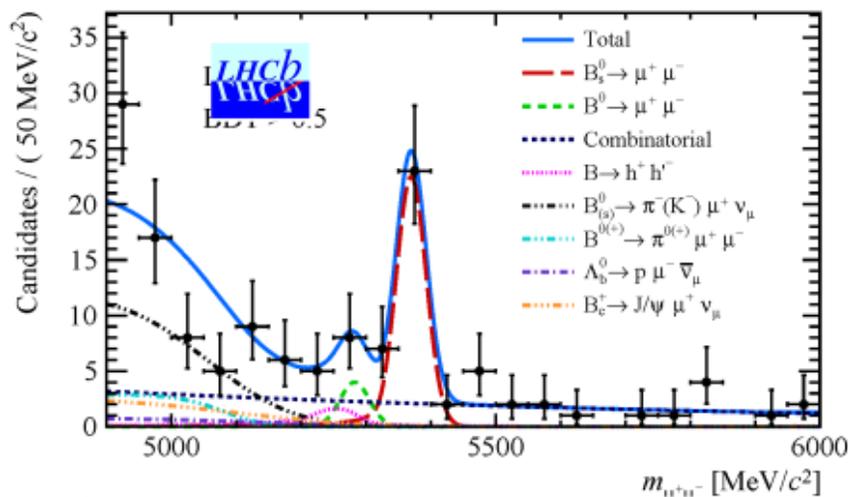
	$q^2/\text{GeV}^2$	SM	LHCb $3 \text{ fb}^{-1}$	$\sigma$
$R_K$	[1, 6]	$1.00 \pm 0.01$	$0.846 \pm 0.06$	2.5
$R_{K^*}$	[0.045, 1.1]	$0.91 \pm 0.03$	$0.66^{+0.11}_{-0.07}$	2.2
$R_{K^*}$	[1.1, 6]	$1.00 \pm 0.01$	$0.69^{+0.11}_{-0.07}$	2.5



$$B_s \rightarrow \mu^+ \mu^-$$

Lattice QCD provides important input to<sup>5</sup>

$$BR(B_s \rightarrow \mu\mu)_{SM} = (3.65 \pm 0.23) \times 10^{-9},$$



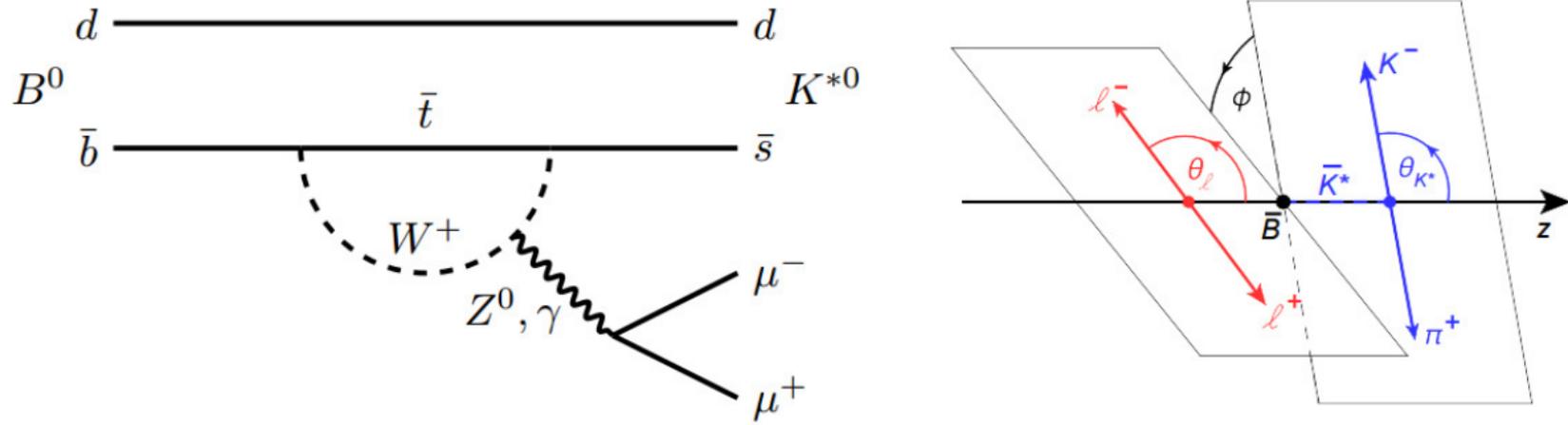
$$B(B_s \rightarrow \mu^+ \mu^-) (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$$

$$(2.9^{+0.7}_{-0.6} \pm 0.2) \times 10^{-9}$$

$$(2.8^{+0.8}_{-0.7}) \times 10^{-9}$$

<sup>5</sup>Bobeth et al, 1311.0903

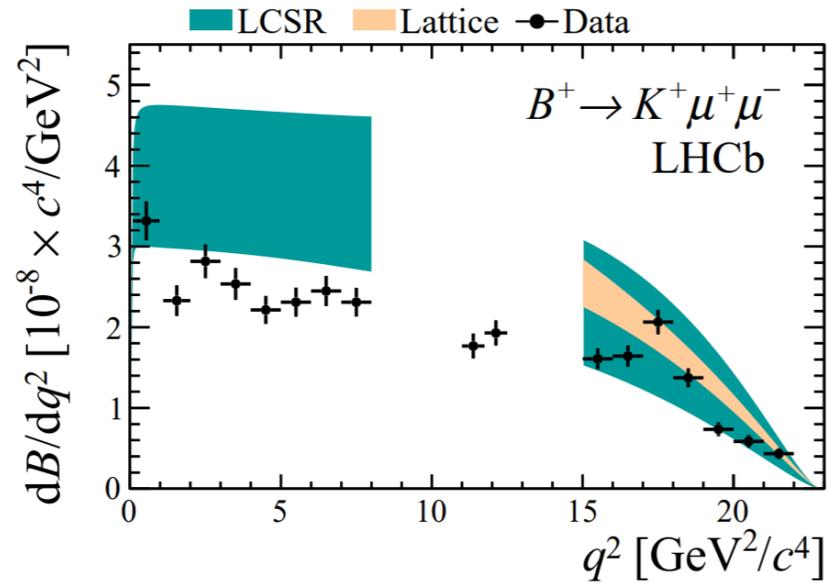
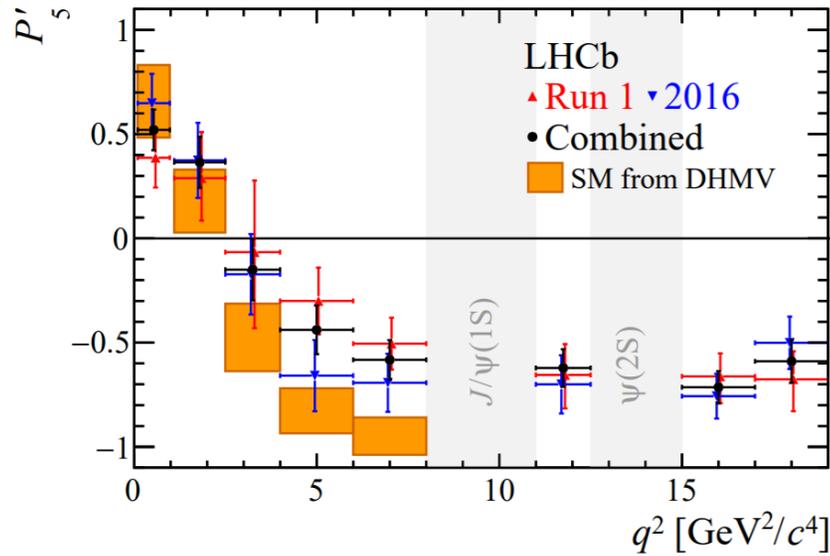
$$B^0 \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \mu^+ \mu^-$$



Decay fully described by three helicity angles  $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$  and  $q^2 = m_{\mu\mu}^2$

$$\begin{aligned} \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\vec{\Omega}} &= \frac{9}{32\pi} \left[ \frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell \right. \\ &\quad - F_L \cos^2 \theta_K \cos 2\theta_\ell + S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi \\ &\quad + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi \\ &\quad + \frac{4}{3} A_{\text{FB}} \sin^2 \theta_K \cos \theta_\ell + S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi \\ &\quad \left. + S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right] \end{aligned}$$

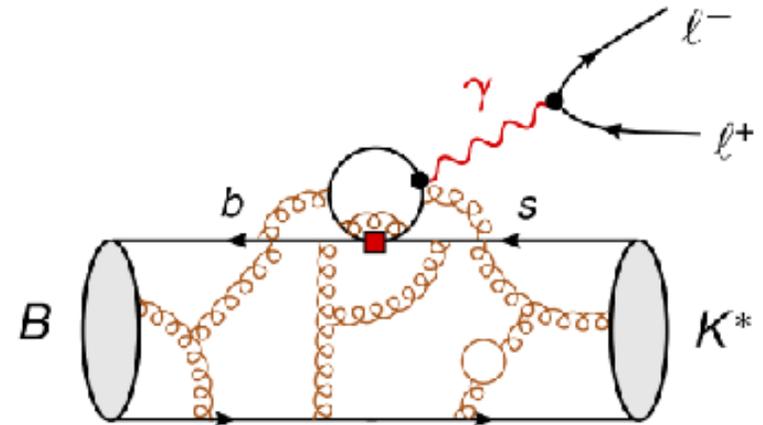
# $P'_5$



$P'_5 = S_5 / \sqrt{F_L(1 - F_L)}$ , leading form factor uncertainties cancel [2003.04831](#)

# Hadronic Uncertainties

- ▶ Hadronic effects like charm loop are photon-mediated  $\Rightarrow$  vector-like coupling to leptons just like  $C_9$



- ▶ How to disentangle NP  $\leftrightarrow$  QCD?
  - ▶ Hadronic effect can have different  $q^2$  dependence
  - ▶ Hadronic effect is lepton flavour universal ( $\rightarrow R_K!$ )

# Wilson Coefficients $\bar{c}_{ij}^l$

In SM, can form an **EFT** since  $m_B \ll M_W$ :

$$\mathcal{O}_{ij}^l = (\bar{s}\gamma^\mu P_i b)(\bar{l}\gamma_\mu P_j l).$$

$$\begin{aligned}\mathcal{L}_{\text{eff}} &\supset \sum_{l=e,\mu,\tau} \sum_{i=L,R} \sum_{j=L,R} \frac{c_{ij}^l}{\Lambda^{2,l,ij}} \mathcal{O}_{ij}^l, \\ &= \sum_{l=e,\mu,\tau} V_{tb} V_{ts}^* \frac{\alpha}{4\pi v^2} (\bar{c}_{LL}^l \mathcal{O}_{LL}^l + \bar{c}_{LR}^l \mathcal{O}_{LR}^l \\ &\quad + \bar{c}_{RL}^l \mathcal{O}_{RL}^l + \bar{c}_{RR}^l \mathcal{O}_{RR}^l) \\ \Rightarrow \bar{c}_{ij}^l &= (36 \text{ TeV}/\Lambda)^2 c_{ij}^l.\end{aligned}$$

$c_{ij}^l \sim \pm \mathcal{O}(1)$  all predicted by weak interactions in SM.

# Which Ones Work?

Options for a single *BSM* operator:

- $\bar{c}_{ij}^e$  operators fine for  $R_{K^{(*)}}$  but are disfavoured by global fits including other observables.
- $\bar{c}_{LR}^\mu$  disfavoured: predicts *enhancement* in both  $R_K$  and  $R_{K^*}$
- $\bar{c}_{RR}^\mu, \bar{c}_{RL}^\mu$  disfavoured: they pull  $R_K$  and  $R_{K^*}$  in *opposite directions*.
- $\bar{c}_{LL}^\mu = -1.06$  fits well globally<sup>6</sup>.

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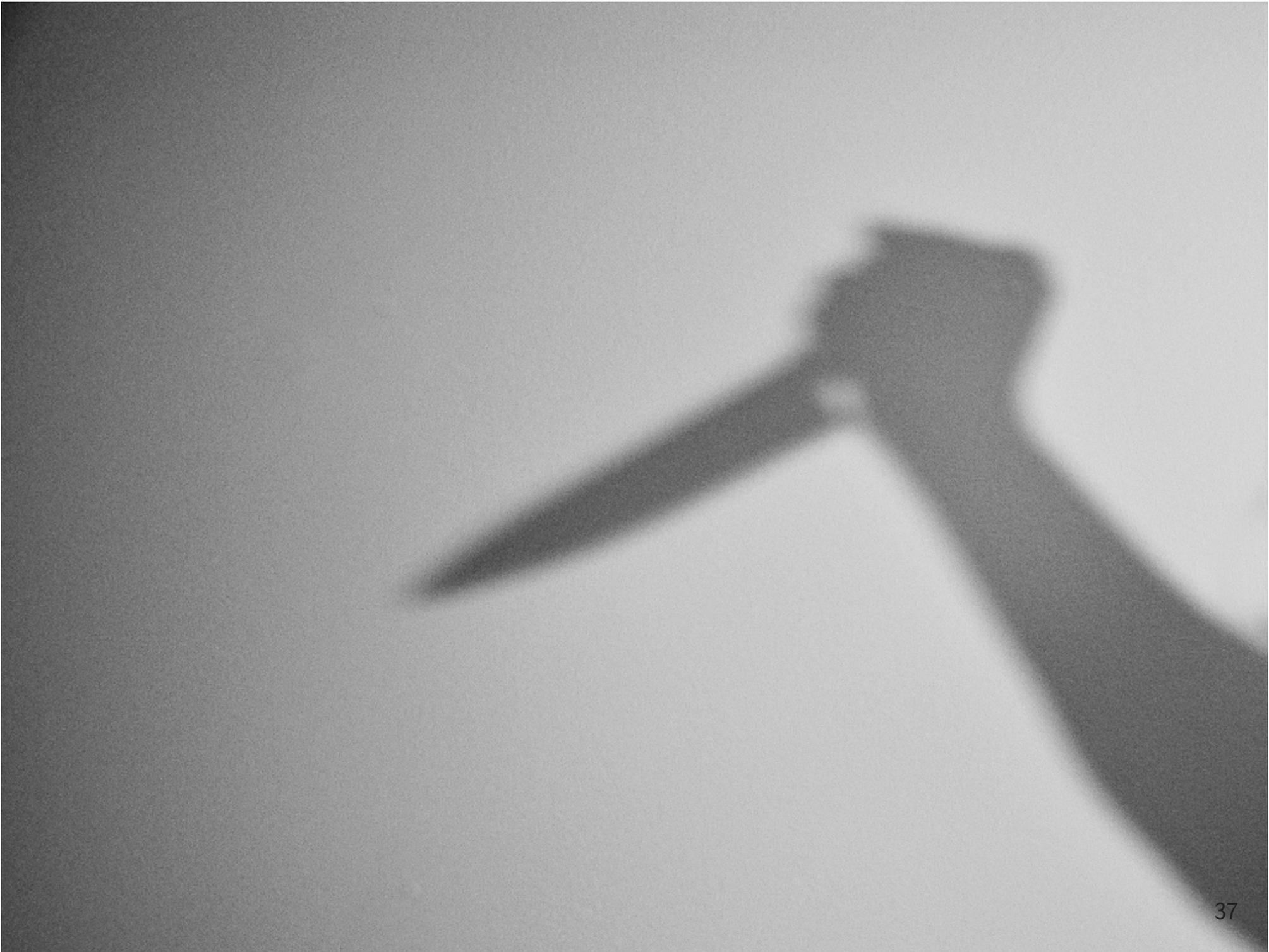
<sup>6</sup>D'Amico et al, 1704.05438; Aebischer et al 1903.10434.

# Statistics<sup>7</sup>

	$\bar{c}_{LL}^\mu$	$\sqrt{\chi_{SM}^2 - \chi_{best}^2}$
clean	$-1.33 \pm 0.34$	4.1
dirty	$-1.33 \pm 0.32$	4.6
all	$-1.06 \pm 0.16$	6.5
	$C_9^\mu = (\bar{c}_{LL}^\mu + \bar{c}_{LR}^\mu)/2$	$\sqrt{\chi_{SM}^2 - \chi_{best}^2}$
clean	$-1.51 \pm 0.46$	3.9
dirty	$-1.15 \pm 0.17$	5.5
all	$-0.95 \pm 0.15$	5.8

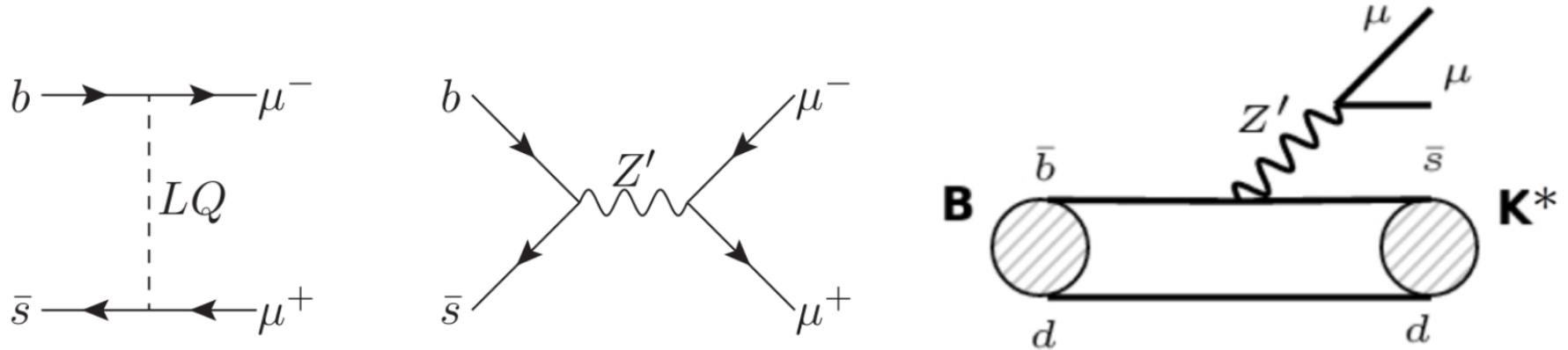
<sup>7</sup>'clean' ( $R_K, R_{K^*}, B_s \rightarrow \mu\mu$ ) and 'dirty' ( $P_5', B \rightarrow \phi\mu\mu$ +100 others).  
D'Amico, Nardecchia, Panci, Sannino, Strumia, Torre, Urbano 1704.05438;  
Aebischer, Altmanshoffer, Guadagnoli, Reboud, Stangl, Straub, 1903.10434. SM  

$p$ -value around  $3\sigma$  for NCBA's.

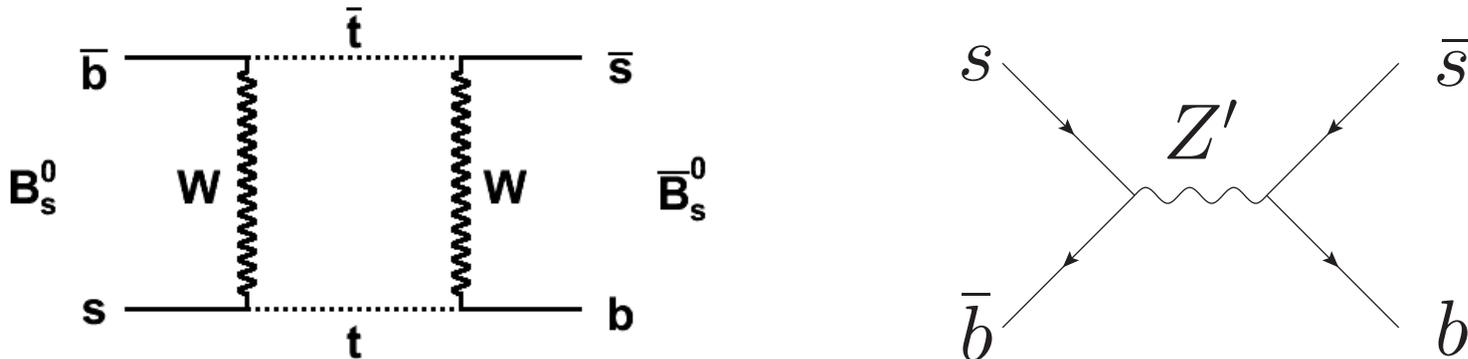


# Simplified Models for $C_{LL}^{\mu}$

At tree-level, we have:



# $B_s - \bar{B}_s$ Mixing



$$\bar{g}_L^{sb} \lesssim \frac{M_{Z'}}{194 \text{ TeV}}$$

from QCD sum rules and lattice<sup>8</sup>

<sup>8</sup>King, Lenz, Rauh, arXiv:1904.00940

# $Z' \rightarrow \mu\mu$ ATLAS 13 TeV 139 $\text{fb}^{-1}$

ATLAS analysis: look for two track-based isolated  $\mu$ ,  
 $p_T > 30$  GeV. One reconstructed primary vertex. Keep  
only highest scalar sum  $p_T$  pair<sup>9</sup>

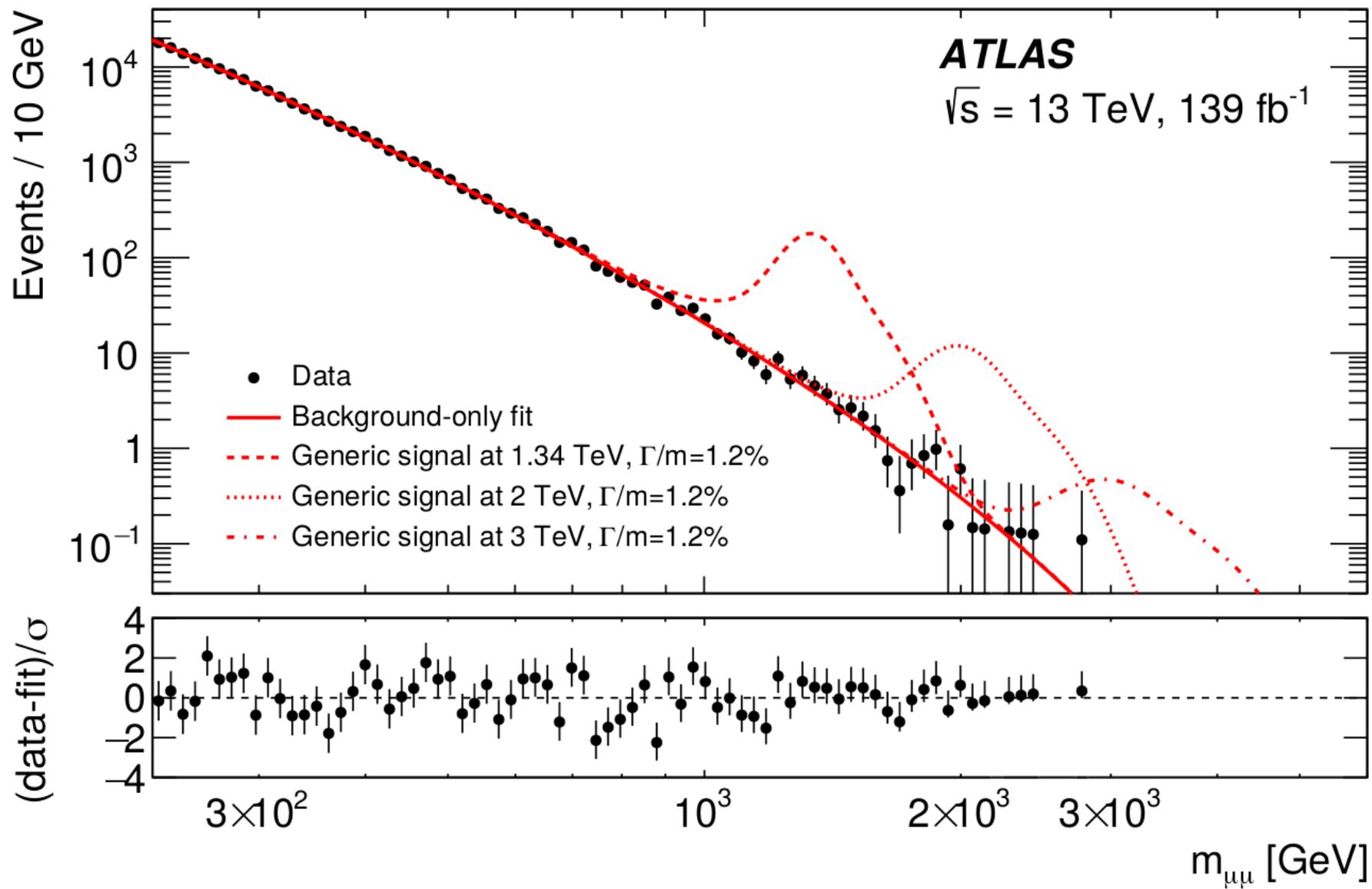
$$m_{\mu_1\mu_2}^2 = (p_1^\mu + p_2^\mu) (p_{1\mu} + p_{2\mu})$$

CMS also have released<sup>10</sup> a similar 36  $\text{fb}^{-1}$  analysis.

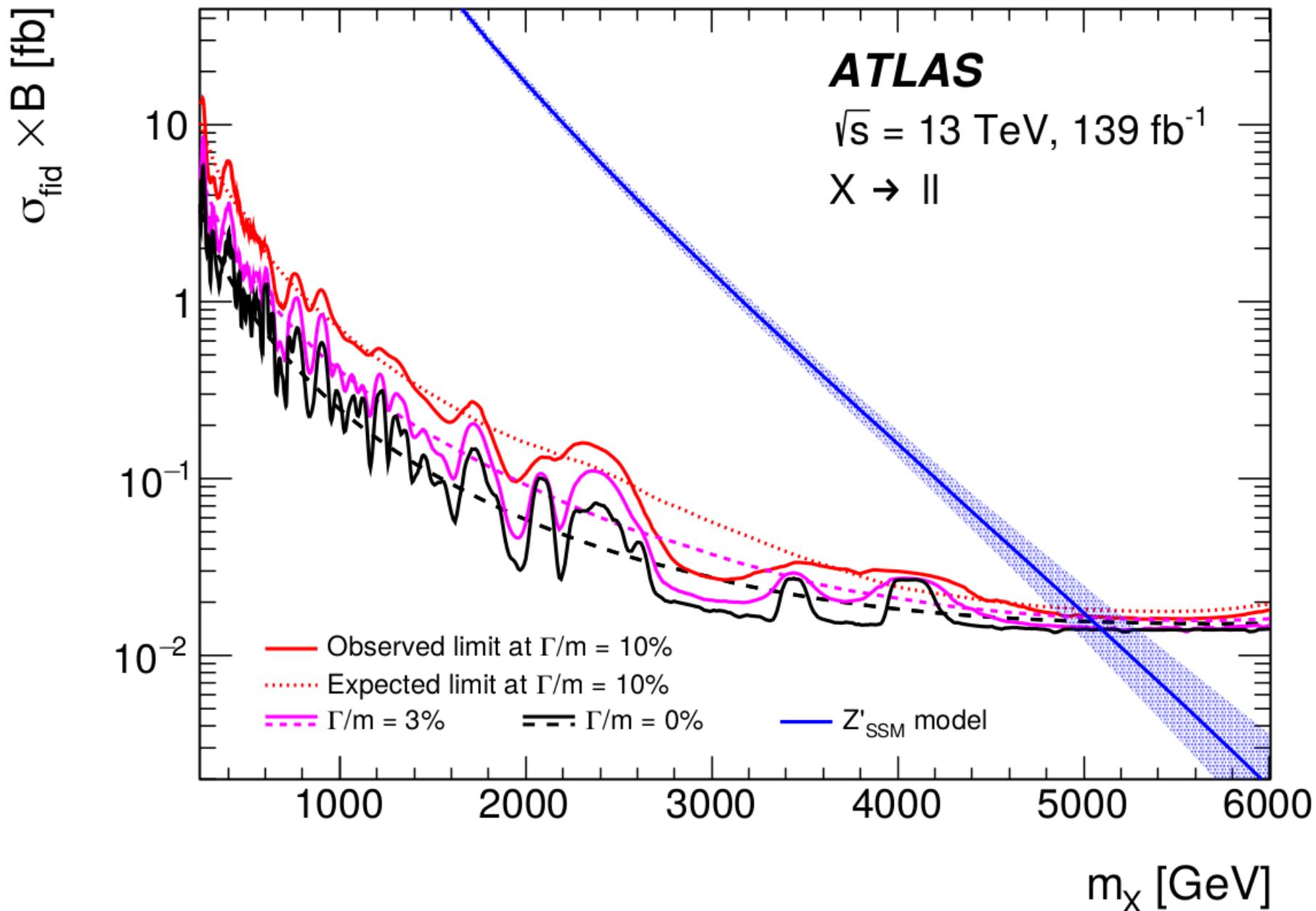
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<sup>9</sup>1903.06248

<sup>10</sup>1803.06292



# ATLAS $l^+l^-$ limits



# During the **1990s**

We wanted to be the Grand Architects, searching for **the** string model to rule them all



# During the 2010s

We are happy with **any** beyond the Standard Model  
roof



# A Model

BCA, Davighi, [arXiv:1809.01158](https://arxiv.org/abs/1809.01158): Add complex SM singlet scalar  $\theta$  and gauged  $U(1)_F$ :

$$\begin{array}{c} SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_F \\ \downarrow \langle \theta \rangle \sim \text{Several TeV} \\ SU(3) \times SU(2)_L \times U(1)_Y \\ \downarrow \langle H \rangle \sim 246 \text{ GeV} \\ SU(3) \times U(1)_{em} \end{array}$$

- SM fermion content
- anomaly cancellation
- 0  $F$  charges for first two generations

# The Flavour Problem

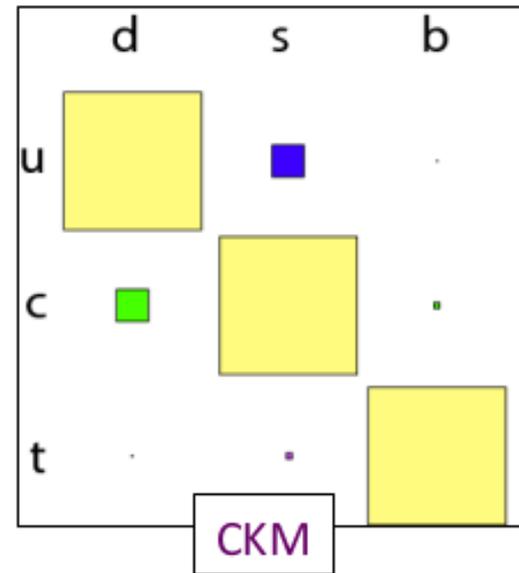
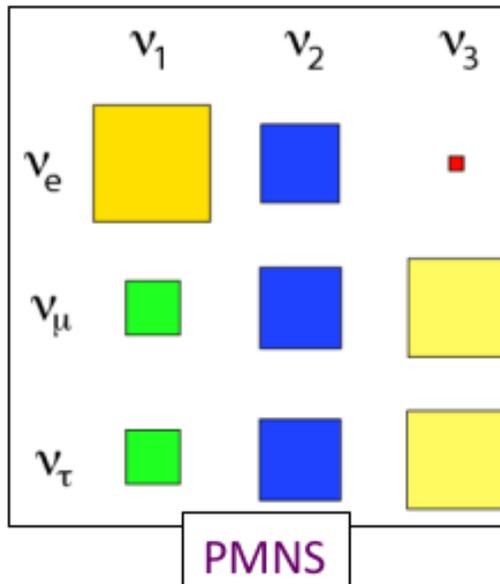
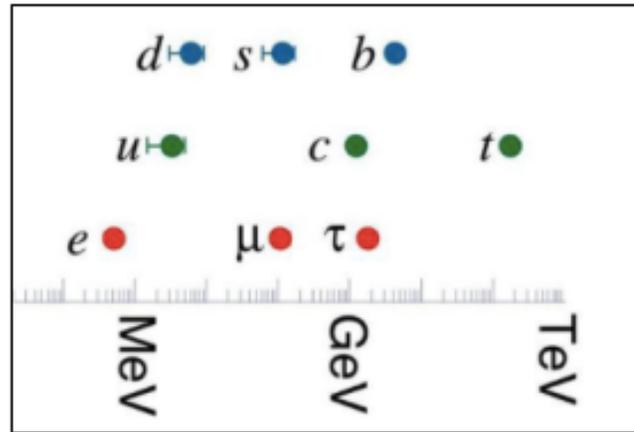
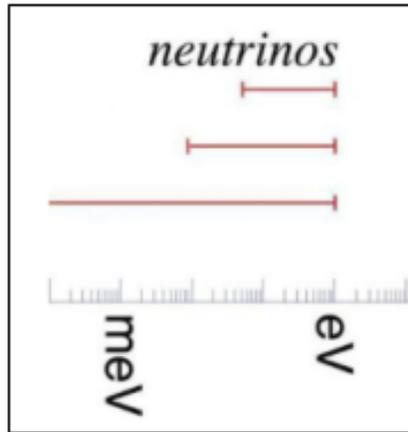


up

charm

top

# The Flavour Problem



# Unique Solution

$F_{Q'_i} = 0$	$F_{u_{R'_i}} = 0$	$F_{d_{R'_i}} = 0$	$F_{L'_i} = 0$
$F_{e_{R'_i}} = 0$	$F_H = -1/2$	$F_{Q'_3} = 1/6$	$F_{u'_{R3}} = 2/3$
$F_{d'_{R3}} = -1/3$	$F_{L'_3} = -1/2$	$F_{e'_{R3}} = -1$	$F_\theta \neq 0$

$$\mathcal{L} = Y_t \overline{Q'_{3L}} H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + Y_\tau \overline{L'_{3L}} H^c \tau'_R + H.c.,$$

$$\left( \begin{array}{c|c} & \\ \hline & \\ \hline & \blacksquare \end{array} \right) \approx \left( \begin{array}{c|c} \cdot & \cdot \\ \hline \cdot & \blacksquare \\ \hline & \blacksquare \end{array} \right)$$

# Yukawa Advantages

- First two families massless at renormalisable level
- Their masses and fermion mixings generated by small non-renormalisable operators

This explains the hierarchical heaviness of the third family and small CKM angles

# $Z - X$ mixing

Because  $F_H = -1/2$ ,  $Z - X$  mix:

$$\mathcal{M}_N^2 = \frac{v^2}{4} \begin{pmatrix} g'^2 & -gg' & g'g_F \\ -gg' & g^2 & -gg_F \\ g'g_F & -gg_F & g_F^2(1 + 4F_\theta^2 r^2) \end{pmatrix} \begin{matrix} -B_\mu \\ -W_\mu^3 \\ -X_\mu \end{matrix}$$

- $v \approx 246$  GeV is SM Higgs VEV
- $g_F = U(1)_F$  gauge coupling
- $r \equiv v_F/v \gg 1$ , where  $v_F = \langle \theta \rangle$
- $F_\theta$  is  $F$  charge of  $\theta$  field

# $Z - X$ mixing angle

$$\sin \alpha_z \approx \frac{g_F}{\sqrt{g^2 + g'^2}} \left( \frac{M_Z}{M'_Z} \right)^2 \ll 1.$$

This gives small non-flavour universal couplings to the  $Z$  boson proportional to  $g_F$  and:

$$Z_\mu = \cos \alpha_z \left( -\sin \theta_w B_\mu + \cos \theta_w W_\mu^3 \right) + \sin \alpha_z X_\mu,$$

$$\mathcal{L}_{X\psi} = g_F \left( \frac{1}{6} \overline{\mathbf{u}_L} \Lambda^{(u_L)} \gamma^\rho \mathbf{u}_L + \frac{1}{6} \overline{\mathbf{d}_L} \Lambda^{(d_L)} \gamma^\rho \mathbf{d}_L - \frac{1}{2} \overline{\mathbf{n}_L} \Lambda^{(n_L)} \gamma^\rho \mathbf{n}_L - \frac{1}{2} \overline{\mathbf{e}_L} \Lambda^{(e_L)} \gamma^\rho \mathbf{e}_L + \frac{2}{3} \overline{\mathbf{u}_R} \Lambda^{(u_R)} \gamma^\rho \mathbf{u}_R - \frac{1}{3} \overline{\mathbf{d}_R} \Lambda^{(d_R)} \gamma^\rho \mathbf{d}_R - \overline{\mathbf{e}_R} \Lambda^{(e_R)} \gamma^\rho \mathbf{e}_R \right) Z'_\rho,$$

$$\Lambda^{(I)} \equiv V_I^\dagger \xi V_I, \quad \xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**$Z'$  couplings**,  $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$

# A simple limiting case

$$V_{u_R} = V_{d_R} = V_{e_R} = 1$$

for simplicity and the ease of passing bounds.

$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{sb} & -\sin \theta_{sb} \\ 0 & \sin \theta_{sb} & \cos \theta_{sb} \end{pmatrix}, \quad V_{e_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\Rightarrow V_{u_L} = V_{d_L} V_{CKM}^\dagger \text{ and } V_{\nu_L} = V_{e_L} U_{PMNS}^\dagger.$$

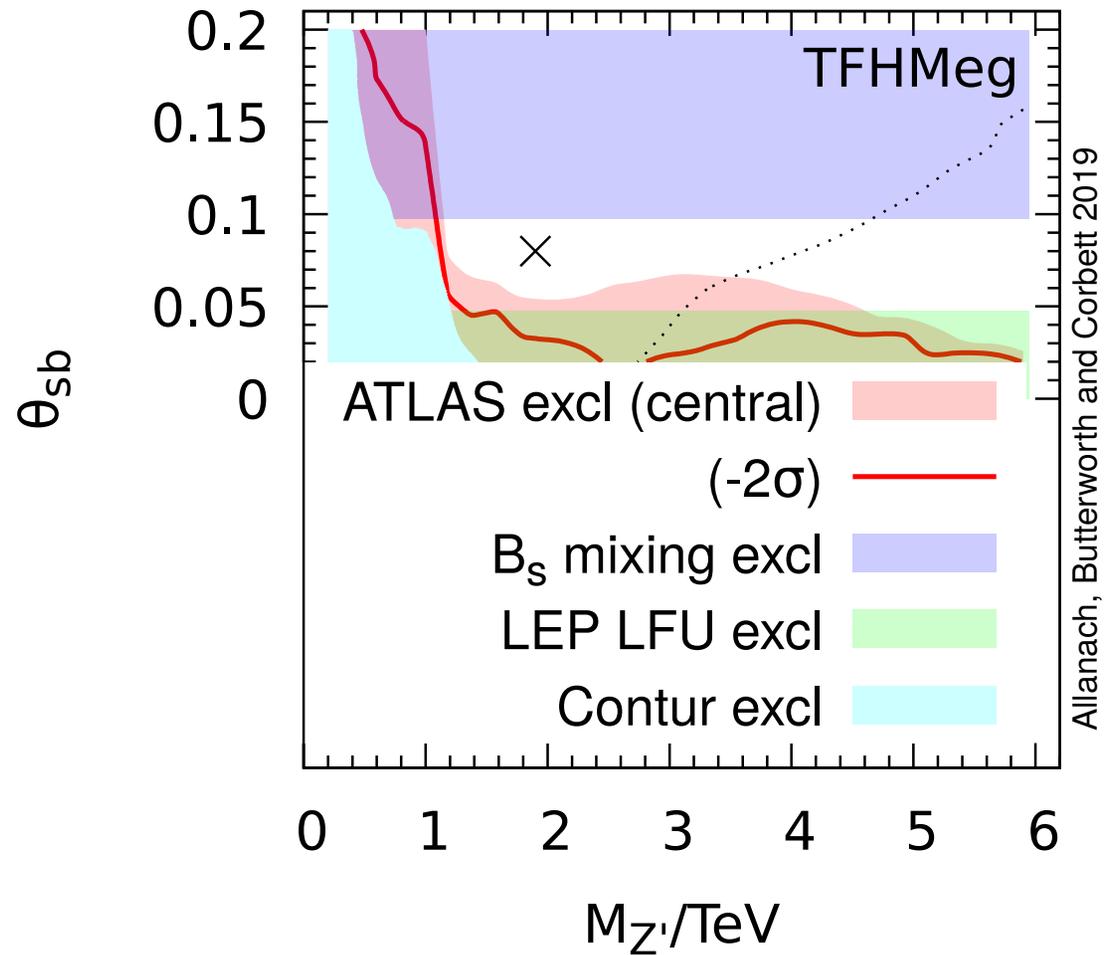
# Important $Z'$ Couplings

$$g_F \left[ \frac{1}{6} \overline{\mathbf{d}_L} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2 \theta_{sb} & \frac{1}{2} \sin 2\theta_{sb} \\ 0 & \frac{1}{2} \sin 2\theta_{sb} & \cos^2 \theta_{sb} \end{pmatrix} \not{Z}' \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \right. \\ \left. - \frac{1}{2} \overline{\mathbf{e}_L} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \not{Z}' \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \right]$$

Put  $|\theta_{sb}| \sim \mathcal{O}(|V_{ts}|) = 0.04$ , so  $|g_{\mu\mu}| \gg |g_{bs}|$ , which helps us survive  $B_s - \overline{B}_s$  constraint.

$$c_{LL} = g_F^2 \sin 2\theta_{sb} / (24M_{Z'}^2).$$

$$g_F \propto M_{Z'} / \sqrt{\sin 2\theta_{bs}}$$



# Example Case Predictions

Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.42	$b\bar{b}$	0.12	$\nu\bar{\nu}'$	0.08
$\mu^+\mu^-$	0.08	$\tau^+\tau^-$	0.30	other $f_i f_j$	$\sim \mathcal{O}(10^{-4})$

LEP LFU

$$g_F^2 \left( \frac{M_Z}{M_{Z'}} \right)^2 \leq 0.004 \Rightarrow g_F \leq \frac{M_{Z'}}{1.3 \text{ TeV}}.$$

It's worth chasing  $BR(B \rightarrow K^{(*)}\tau^\pm\tau^\mp)$ .

# Backup



# A Warm Up: $U(1)$

Pioneering solution to ACCs: [Costa, Dobrescu, Fox, arxiv:1905.13729](#).  $n$  chiral fermions with charges  $z_i$ :

$$\begin{aligned}z_1^3 + \dots + z_n^3 &= 0, \\z_1 + \dots + z_n &= 0.\end{aligned}\tag{1}$$

Given 2 solutions  $\underline{x}$ ,  $\underline{y}$ , construct a third by “merger”

$$\{\underline{x}\} \oplus \{\underline{y}\} := \left( \sum_{i=1}^n x_i y_i^2 \right) \{\underline{x}\} - \left( \sum_{i=1}^n x_i^2 y_i \right) \{\underline{y}\}.$$

Want to find suitably general solutions  $\underline{x}$ ,  $\underline{y}$ .

# Example: even $n$

$$\{\underline{x}\} = \{l_1, k_1, \dots, k_m, -l_1, -k_1, \dots, -k_m\}$$

$$\{\underline{y}\} = \{0, 0, l_1, \dots, l_m, -l_1, \dots, -l_m\},$$

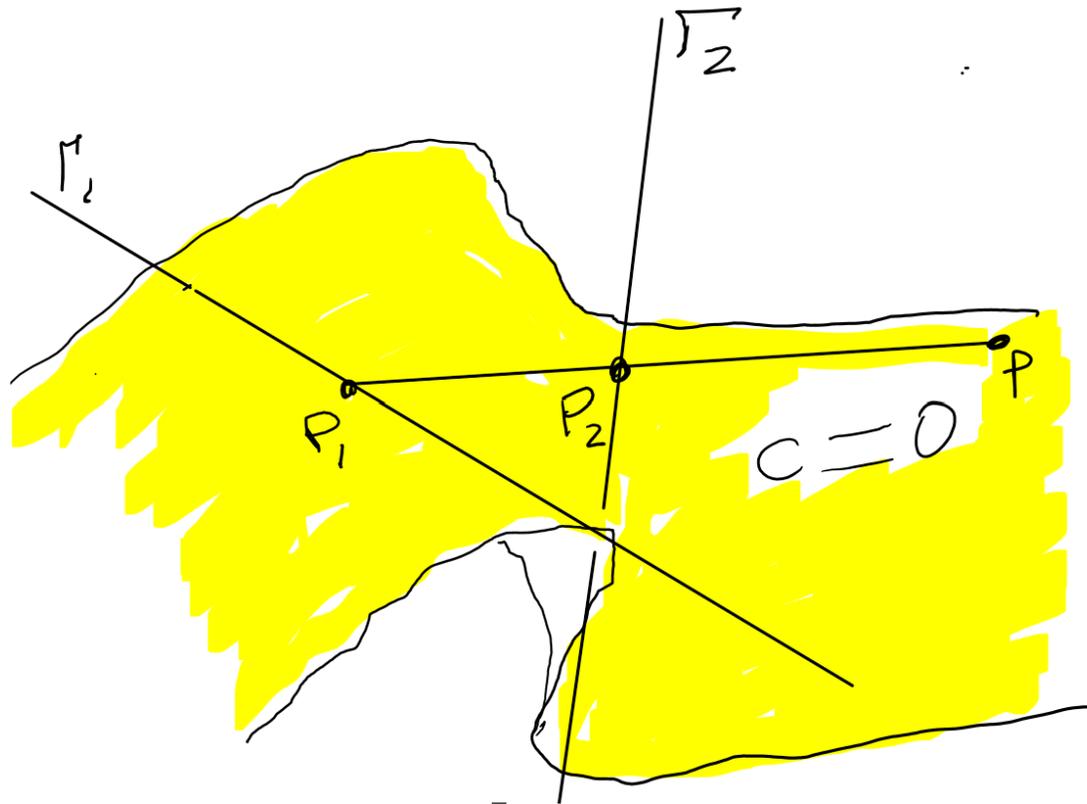
$$m = n/2 - 1 \geq 2, \quad 1 \leq i \leq m$$

$\{\underline{x}\}$  and  $\{\underline{y}\}$  are each vector-like solutions but it turns out that  $\{\underline{x}\} \oplus \{\underline{y}\}$  is a new **chiral** solution.

$\{\underline{x}\} \oplus \{\underline{y}\}$  parameterises all solutions up to permutations.  
There is a *similar story* for odd  $n$ .

# Mordell's Theorem<sup>11</sup>

Skew  $\Gamma_1, \Gamma_2$  in  $c = 0 \Rightarrow$  all rational points on  $c$  can be found this way.



<sup>11</sup>Mordell (1969) *Diophantine Equations*

# Geometric Understanding

In [BCA, Gripaos, Tooby-Smith, arXiv:1912.04804](#), we provide a geometric understanding of this. First, note that each solution in  $\mathbb{Q}$  is equivalent to one in  $\mathbb{Z}$  by clearing denominators. Using gravitational anomaly cancellation, eliminate  $z_n$  to obtain the homogeneous cubic

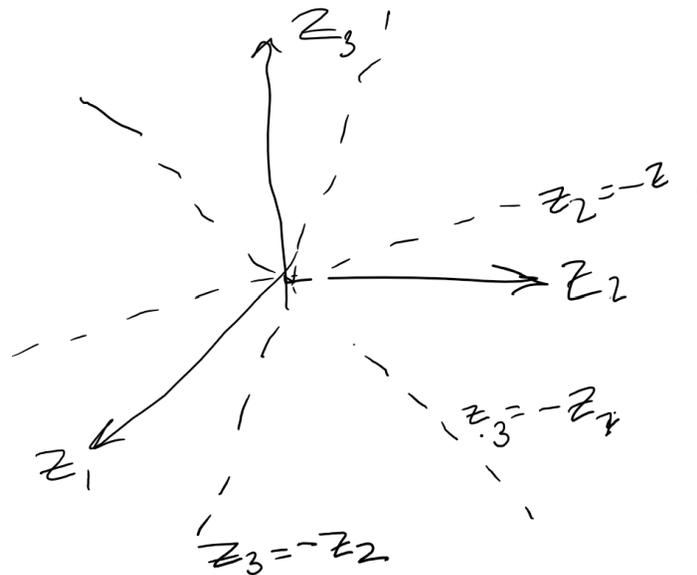
$$\sum_{i=1}^{n-1} z_i^3 - \left( \sum_{i=1}^{n-1} z_i \right)^3 = 0$$

defining a cubic hypersurface in  $\mathbb{Q}^{n-1}$ .

# Special Surface

In fact, our cubic hypersurface is rather special: no purely cubic terms in any one variable: (add perms)

$n = 3$ :  $\underline{z} = [-a : 0 : a]$ , ie three lines  $z_3 = -z_1$ ,  $z_2 = 0$



$n = 4$ :  $\underline{z} = [-x : -y : x : y]$ ,  $x, y \in \mathbb{Q}$  ie three planes

# Strategy

1. Find solutions for SM fermions charges from first 4

2. Apply  $GL(3, \mathbb{Z})$  transformation to species  $F$ :

$$F_+ := F_1 + F_2 + F_3, F_\alpha := F_1 - F_2, F_\beta := F_2 + F_3.$$

3. Linear equations become

$$D_+ = -2Q_+ - U_+, L_+ = -3Q_+, E_+ = 2Q_+ - U_+.$$

4. Quadratic is a solveable homogeneous diophantine equation of degree 2 in the 12-tuple

$$X := (Q_+, U_+, Q_\alpha, Q_\beta, U_\alpha, U_\beta, D_\alpha, D_\beta, L_\alpha, L_\beta, E_\alpha, E_\beta).$$

$X^T H X = 0$  defines hypersurface  $\Gamma \in P\mathbb{Q}^{11}$ .

$$H = \begin{pmatrix} 0 & 0 & -2 & -4 & 0 & 0 & 4 & 8 & -6 & 0 & -4 & -8 \\ & 0 & 0 & 0 & 4 & 8 & 2 & 4 & 0 & 0 & 2 & 4 \\ & & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & -4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & -12 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & 2 & 3 & 0 & 0 & 0 & 0 \\ & & & & & & & 6 & 0 & 0 & 0 & 0 \\ & & & & & & & & -2 & 3 & 0 & 0 \\ & & & & & & & & & 6 & 0 & 0 \\ & & & & & & & & & & 2 & 3 \\ & & & & & & & & & & & 6 \end{pmatrix}.$$

# Quadratic

$$X^T H X = 0$$

Consider lines  $L = \alpha \tilde{X} + \beta R$  through a known solution  $\tilde{X} \in P\mathbb{Q}^{11}$ , where  $R \in P\mathbb{Q}^{11}$ , and  $[\alpha : \beta] \in P\mathbb{Q}^1$ : (eg  $\tilde{X}$  has all zero except  $Q_\alpha = L_\alpha = 1$ )

$$\beta(2R^T H \tilde{X} \alpha + R^T H R \beta) = 0.$$

Using same trick as before

$$X = (R^T H R) \tilde{X} - 2(R^T H \tilde{X}) R.$$

# Solution In Detail

$$Q_\alpha = 2R_{Q_\alpha}\Lambda + \Sigma, \quad L_\alpha = 2R_{L_\alpha}\Lambda + \Sigma,$$

where  $\Sigma = R^T H R$  and

$$\Lambda = (8R_{Q_+} + 2R_{L_\alpha} + 3R_{L_\beta} - 2R_{Q_\alpha} - 3R_{Q_\beta}).$$

All other charges  $X$  are  $2R_X\Lambda$ , where  $R_X \in \mathbb{Z}$ .

$$R := \{R_{Q_+}, R_{U_+}, R_{Q_\alpha}, R_{Q_\beta}, R_{U_\alpha}, R_{U_\beta}, R_{D_\alpha}, R_{D_\beta}, \\ R_{L_\alpha}, R_{L_\beta}, R_{E_\alpha}, R_{E_\beta}\}.$$

Then, invert the  $GL(3, \mathbb{Z})$ .

# SM Singlets

Adding  $n$  SM singlets with  $U(1)$  charges decouples the last two equations. Results:

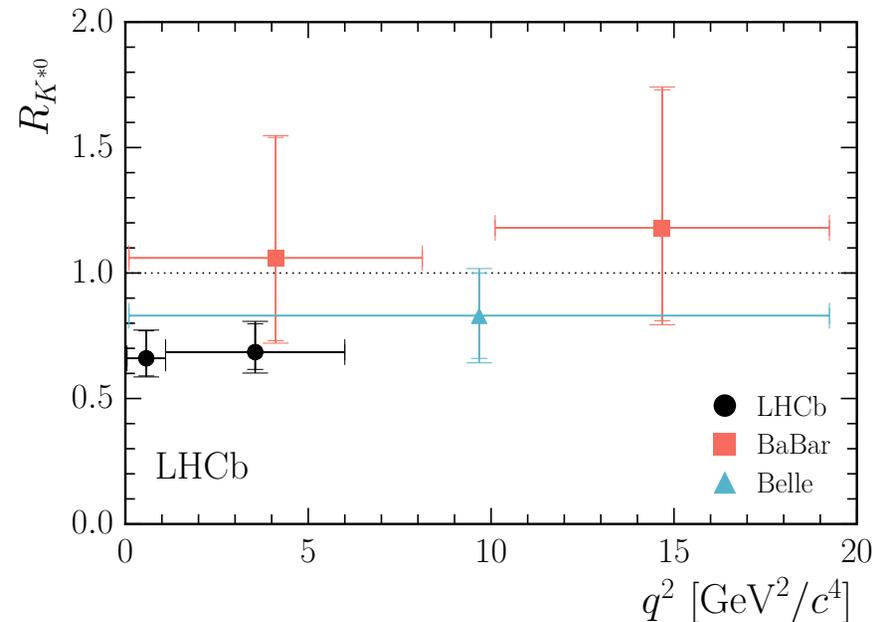
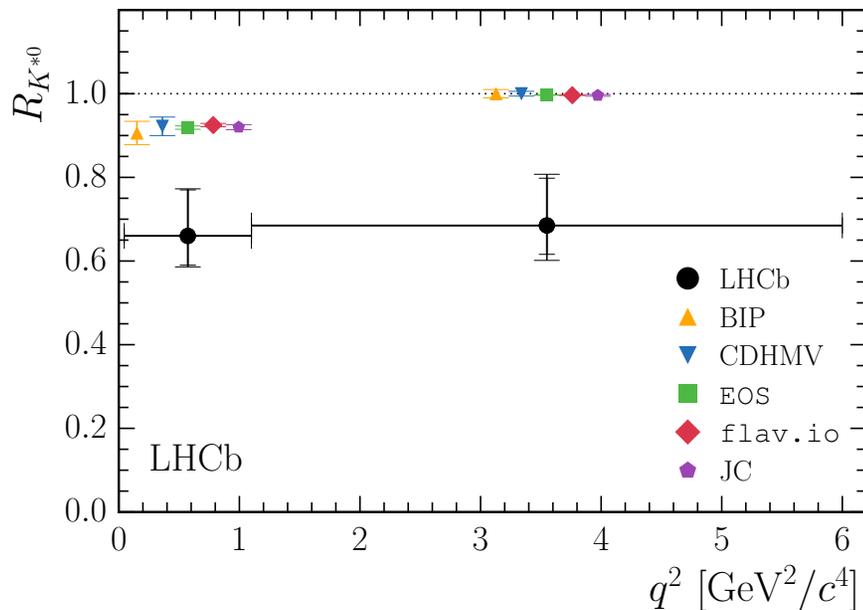
- We can always find a full solution for  $n \geq 5$ , eg:  
 $(M/6 \in \mathbb{Z}) \{M/6 + 1, M/6 - 1, -M/6, -M/6, J\}$
- For lower  $n$ , we give restrictions on  $M, J$  for when a solution exists.

However, annoyingly, we only have a **partial** solution for the full 6 equations together.

# $R_{K^{(*)}}$ pre Moriond 2019

LHCb results from 7 and 8 TeV:  $q^2 = m_U^2$ .

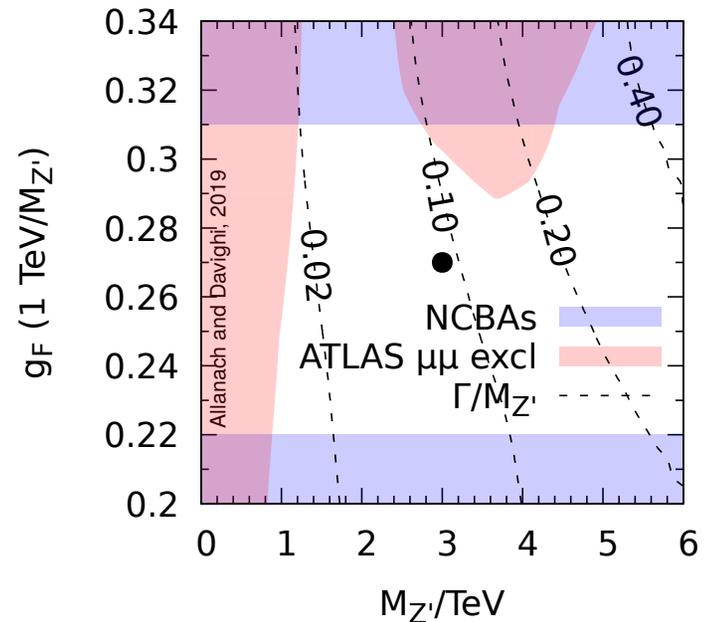
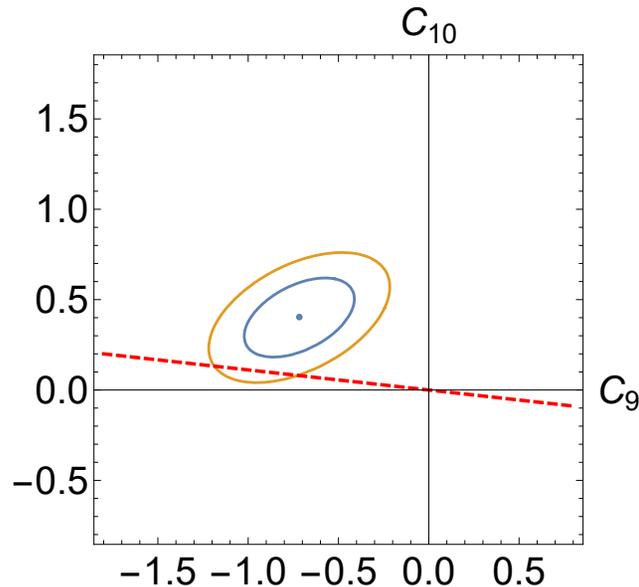
	$q^2/\text{GeV}^2$	SM	LHCb $3 \text{ fb}^{-1}$	$\sigma$
$R_K$	[1, 6]	$1.00 \pm 0.01$	$0.745^{+0.090}_{-0.074}$	2.6
$R_{K^*}$	[0.045, 1.1]	$0.91 \pm 0.03$	$0.66^{+0.11}_{-0.07}$	2.2
$R_{K^*}$	[1.1, 6]	$1.00 \pm 0.01$	$0.69^{+0.11}_{-0.07}$	2.5



# Deformed TFHM

$$\begin{array}{cccc}
 F_{Q'_i} = 0 & F_{u_{R'_i}} = 0 & F_{d_{R'_i}} = 0 & F_H = -1/2 \\
 F_{e_{R'_1}} = 0 & F_{e_{R'_2}} = 2/3 & F_{e_{R'_3}} = -5/3 & \\
 F_{L'_1} = 0 & F_{L'_2} = 5/6 & F_{L'_3} = -4/3 & \\
 F_{Q'_3} = 1/6 & F_{u'_{R3}} = 2/3 & F_{d'_{R3}} = -1/3 & F_\theta \neq 0
 \end{array}$$

$$\mathcal{L} = Y_t \overline{Q'_{3L}} H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + H.c.,$$



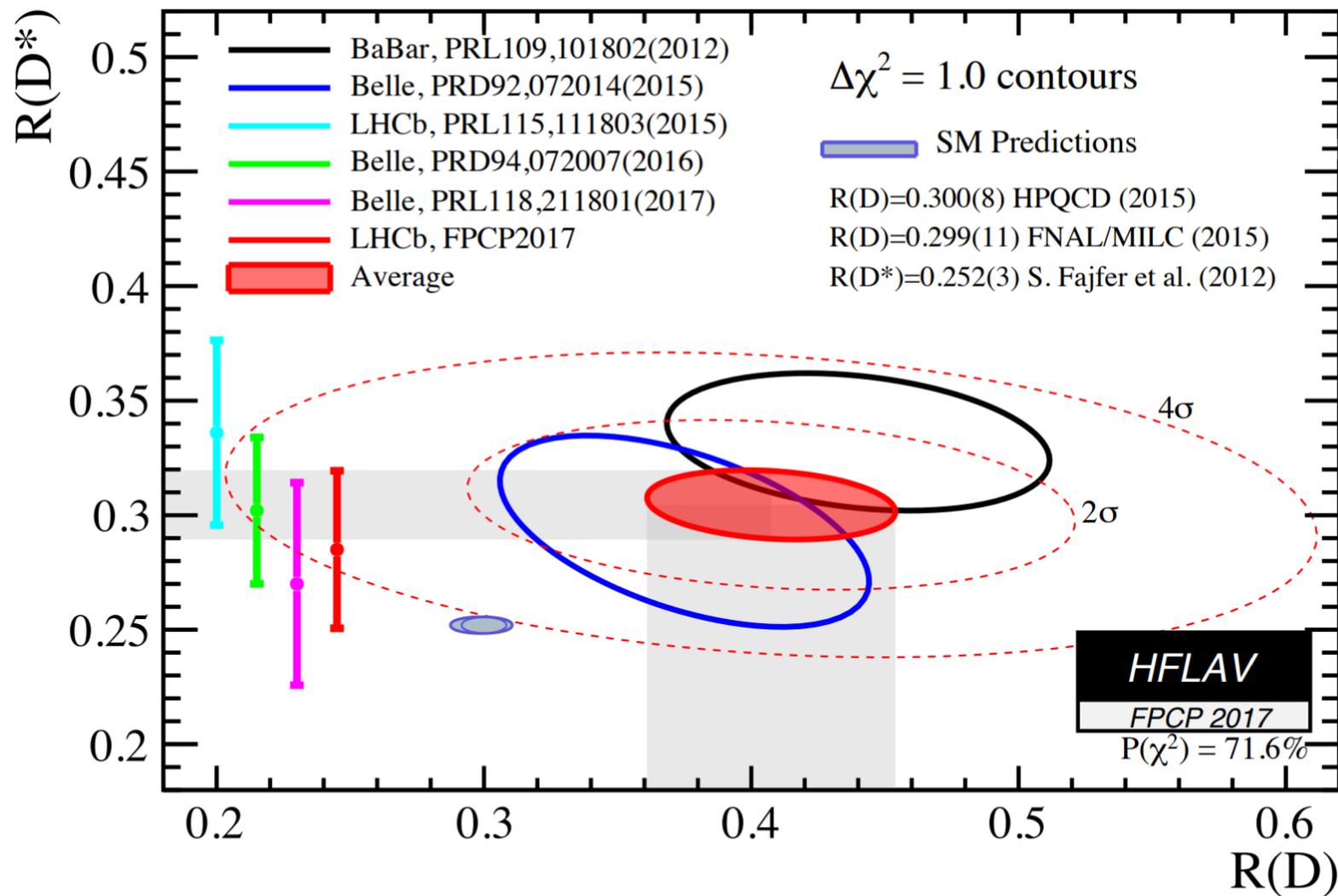
# Invisible Width of $Z$ Boson

$$\Gamma_{\text{inv}}^{(\text{exp})} = 499.0 \pm 1.5 \text{ MeV}, \text{ whereas } \Gamma_{\text{inv}}^{(\text{SM})} = 501.44 \text{ MeV}.$$

$$\Rightarrow \Delta\Gamma^{(\text{exp})} = \Gamma_{\text{inv}}^{(\text{exp})} - \Gamma_{\text{inv}}^{(\text{SM})} = -2.5 \pm 1.5 \text{ MeV}.$$

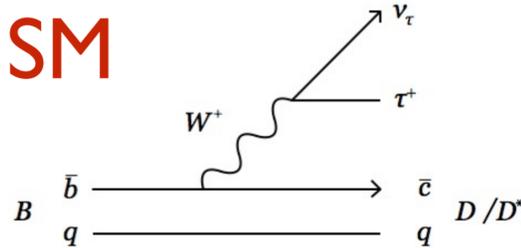
$$\begin{aligned} \mathcal{L}_{\bar{\nu}\nu Z} = & -\frac{g}{2 \cos \theta_w} \bar{\nu}'_{Le} \not{Z} P_L \nu'_{Le} \\ & -\bar{\nu}'_{L\mu} \left( \frac{g}{2 \cos \theta_w} + \frac{5}{6} g_F \sin \alpha_z \right) \not{Z} \nu'_{L\mu} \\ & -\bar{\nu}'_{L\tau} \left( \frac{g}{2 \cos \theta_w} - \frac{8}{6} g_F \sin \alpha_z \right) \not{Z} \nu'_{L\tau}. \end{aligned}$$

$$R_{D^{(*)}} = BR(B^- \rightarrow D^{(*)}\tau\nu) / BR(B^- \rightarrow D^{(*)}\mu\nu)$$



# $R_{D^{(*)}}$ : BSM Explanation

SM



... has to compete with

$$\mathcal{L}_{eff} = -\frac{2}{\Lambda^2} (\bar{c}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \nu_{\tau L}) + H.c.$$

$$\Lambda = 3.4 \text{ TeV}$$

*A factor 10 lower than required for  $R_{K^{(*)}} \Rightarrow$  different explanation?*

PMP  $\Rightarrow$  we ignore  $R_{D^{(*)}}$ .

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https://aeon.co/essays/has-the-quest-for-top-down-...

# Going nowhere fast

After the success of the Standard Model, experiments have stopped answering to grand theories. Is particle physics in crisis?

*Photo by Getty*

Ben Allanach is a professor in the department of applied mathematics and theoretical physics at the University of Cambridge. Along with other members of the Cambridge Supersymmetry Working Group, his research focuses on collider searches for new physics.

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2,900 words

SYNDICATE THIS ESSAY

**I**n recent years, physicists have been watching the data coming in from the Large Hadron Collider (LHC) with a growing sense of unease. We've spent decades devising elaborate accounts for the behaviour of the quantum zoo of subatomic particles, the most basic components of the known universe. The Standard Model is

# Other conclusions

- The answers to the questions raised by  $R_{K^{(*)}}$  may provide a **direct experimental probe into the flavour problem**.
- Focused on *tree-level* explanations of  $R_{K^{(*)}}$  as they are usually harder to discover:  $Z'$  and leptoquarks.
- News on  $R_K^{(*)}$  expected *in 2019*. At the current central value, Belle II can reach  $5\sigma$  by mid 2021. LHCb's  $R_{K^*}$  would be close to<sup>12</sup>  $5\sigma$  by 2020.
- $R_{K^{(*)}} \Rightarrow$  *HL-LHC, HE-LHC and FCC-hh*

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<sup>12</sup>Albrecht *et al*, 1709.10308

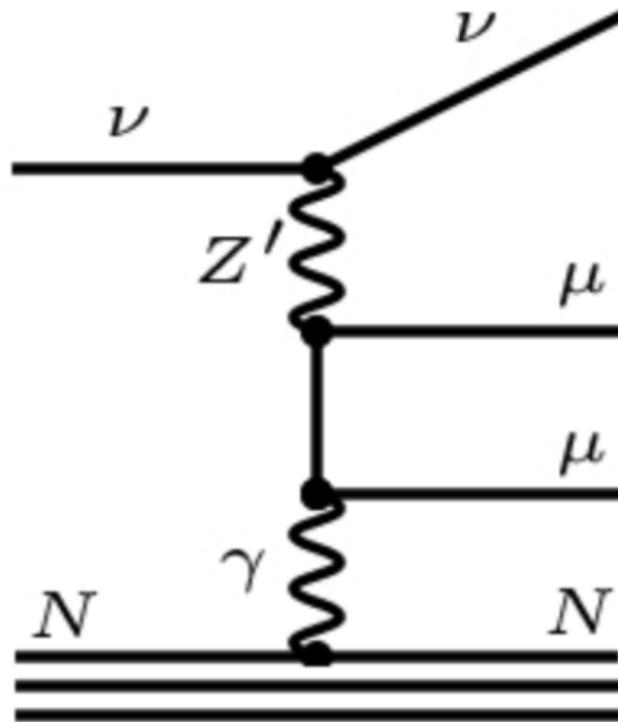
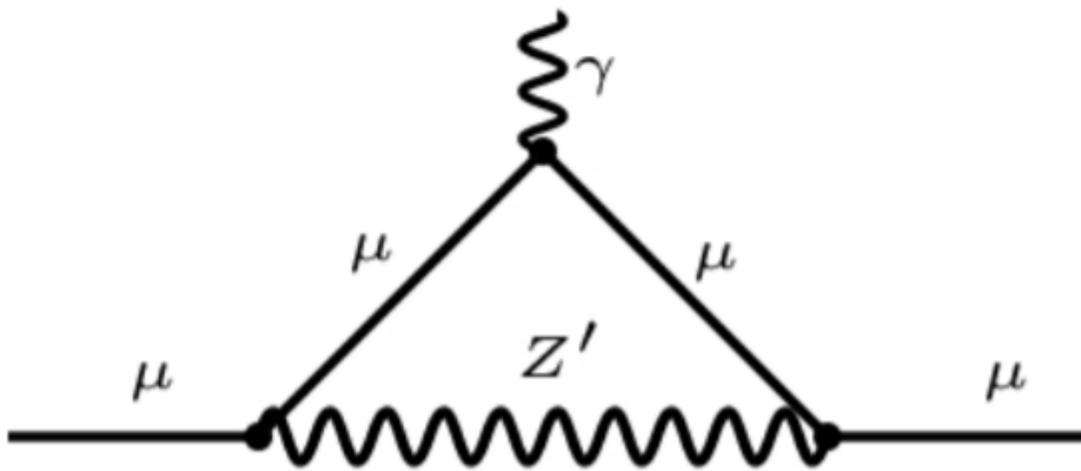
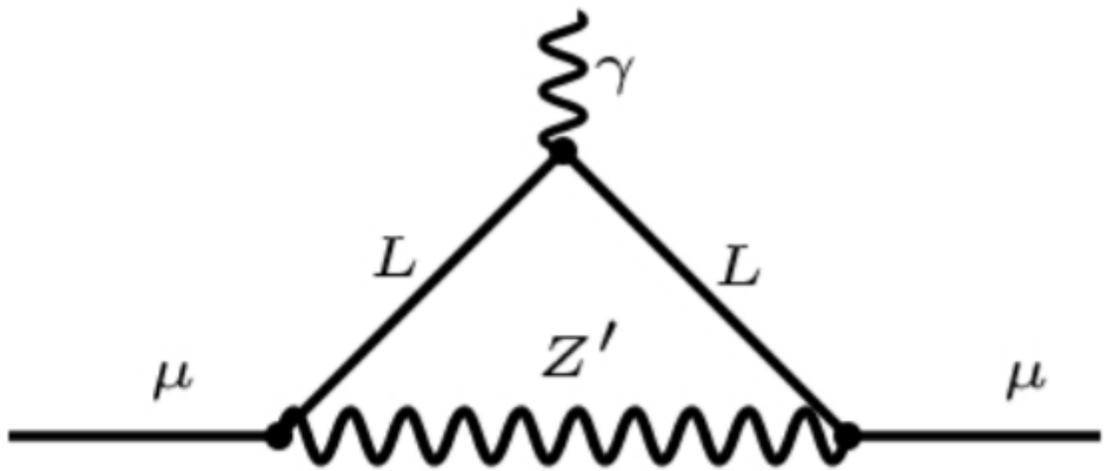


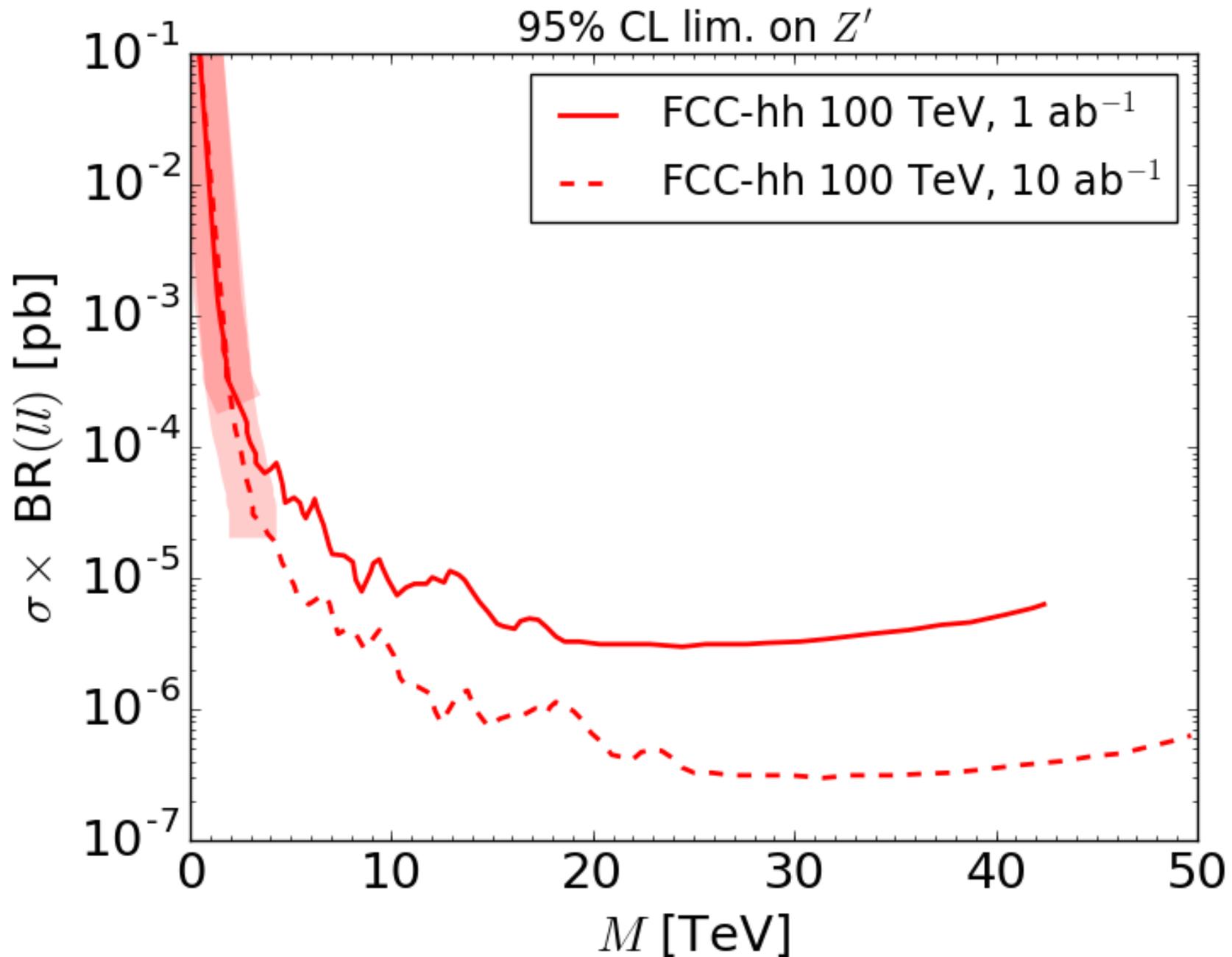
FIG. 10. Neutrino trident process that leads to constraints on the  $Z^\mu$  coupling strength to neutrinos-muons, namely  $M_{Z'}/g_{\nu\mu} \gtrsim 750$  GeV.



$Q_{\max}$	<b>Solutions</b>	Symmetry	Quadratics	Cubics	Time/sec
1	<b>38</b>	16	144	38	0.0
2	<b>358</b>	48	31439	2829	0.0
3	<b>4116</b>	154	1571716	69421	0.1
4	<b>24552</b>	338	34761022	932736	0.6
5	<b>111152</b>	796	442549238	7993169	6.8
6	<b>435305</b>	1218	3813718154	49541883	56

SM + 3  $\nu_R$ : number of solutions etc

# 13 TeV ATLAS 3.2 fb<sup>-1</sup> $\mu\mu$



# Neutrino Masses

At dimension 5:

$$\mathcal{L}_{SS} = \frac{1}{2M} (L_3'^T H^c) (L_3' H^c),$$

but if we add RH neutrinos, then integrate them out

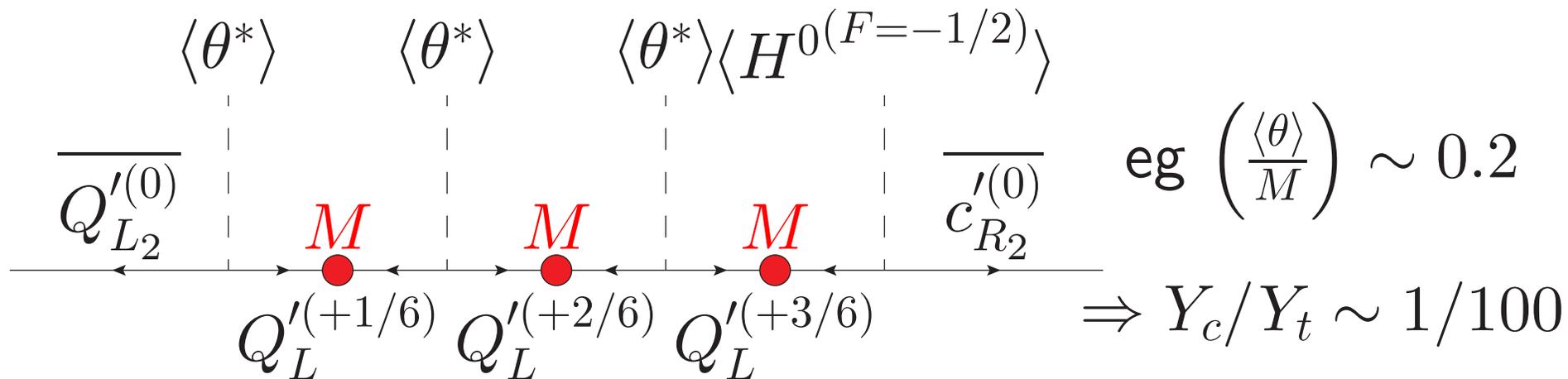
$$\mathcal{L}_{SS} = 1/2 \sum_{ij} (L_i' H^c) (M^{-1})_{ij} (L_j' H^c),$$

where now  $(M^{-1})_{ij}$  may well have a non-trivial structure. If  $(M^{-1})_{ij}$  are of same order, large PMNS mixing results.

# Froggatt Neilsen Mechanism<sup>13</sup>

A means of generating the non-renormalisable Yukawa terms, e.g.  $F_\theta = 1/6$ :

$$Y_c \overline{Q'_{L2}}^{(F=0)} H^{(F=-1/2)} c'_{R2}{}^{(F=0)} \sim \mathcal{O} \left[ \left( \frac{\langle \theta \rangle}{M} \right)^3 \overline{Q'_{L2}} H c'_{R2} \right]$$



<sup>13</sup>C Froggatt and H Neilsen, NPB147 (1979) 277

# LQ Models

**Scalar**<sup>14</sup>  $S_3 = (\bar{3}, 3, 1/3)$  of  $SU(2) \times SU(2)_L \times U(1)_Y$ :

$$\mathcal{L} = \dots + y_{3b\mu} Q_3 L_2 S_3 + y_{3s\mu} Q_2 L_2 S_3 + y_q Q Q S_3^\dagger + \text{h.c.}$$

**Vector**  $V_1 = (\bar{3}, 1, 2/3)$  or  $V_3 = (3, 3, 2/3)$

$$\mathcal{L} = \dots + y'_3 V_3^\mu \bar{Q} \gamma_\mu L + y_1 V_1^\mu \bar{Q} \gamma_\mu L + y'_1 V_1^\mu \bar{d} \gamma_\mu l + \text{h.c.}$$

$$\Rightarrow \bar{c}_{LL}^\mu = \kappa \frac{4\pi v^2}{\alpha_{\text{EM}} V_{tb} V_{ts}^*} \frac{y_{3b\mu}^* y_{3s\mu}}{M^2}.$$

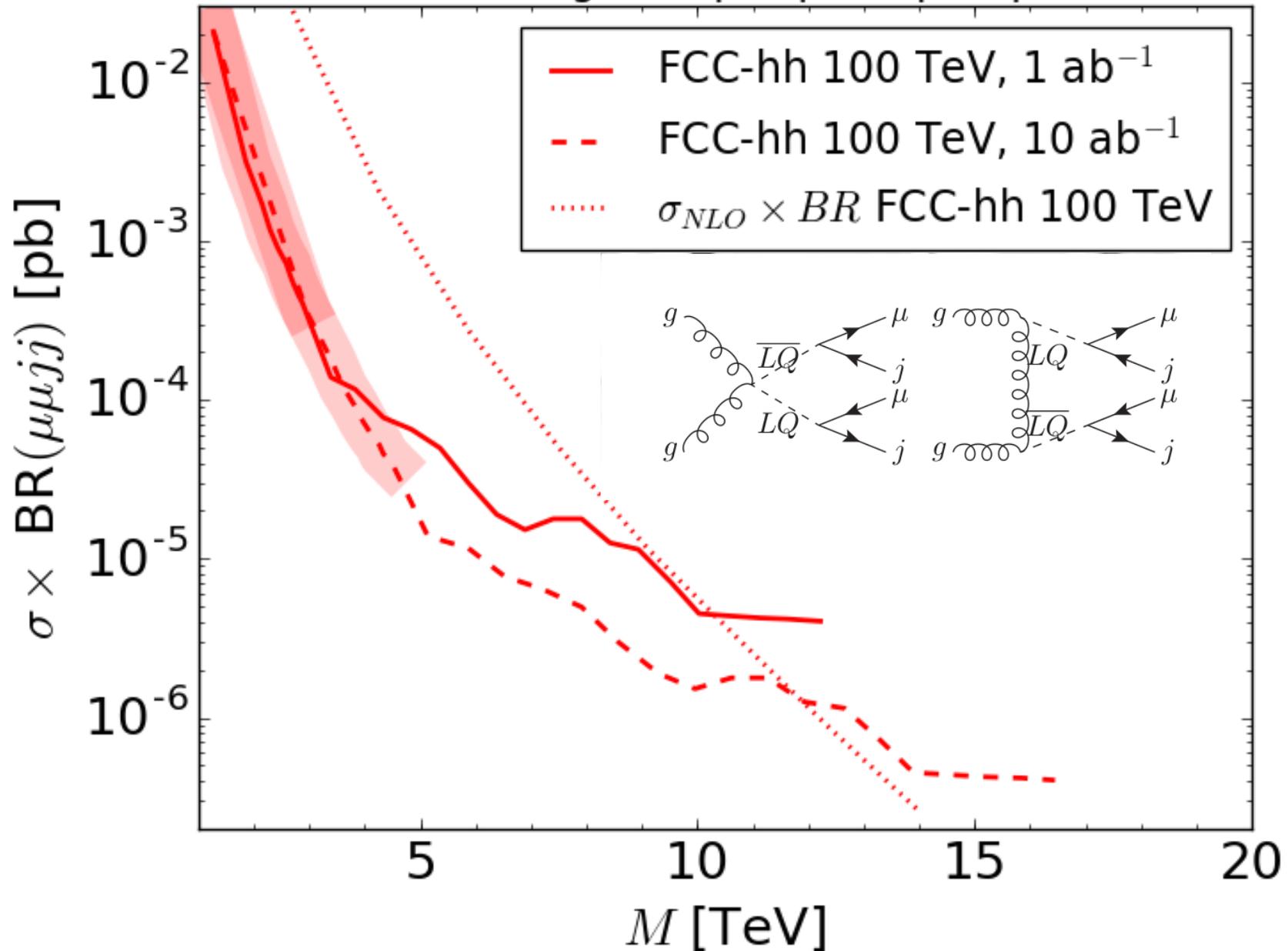
$\kappa = 1, -1, -1$  and  $y = y_3, y_1, y'_3$  for  $S_3, V_1, V_3$ .

<sup>14</sup>Capdevila *et al* 1704.05340, Hiller and Hisandzic 1704.05444, D'Amico *et al* 1704.05438.

# CMS 8 TeV 20fb<sup>-1</sup> 2nd gen

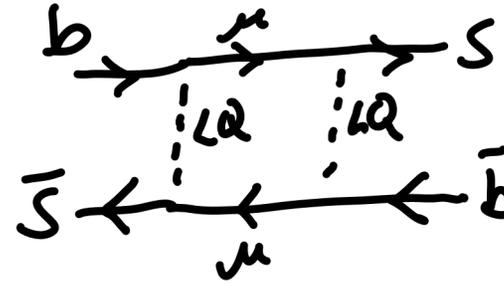
CMS-PAS-EXO-12-042:  $M > 1.07$  TeV.

95% CL lim. 2nd gen. leptoquark pair production



# Other Constraints On LQs

Note that the extrapolation is **very rough** for **pair** production. Fix  $M = 2M_{LQ}$ , assuming they are produced



close to threshold:  $\Delta = 0.1$ .

**mixing** is at one-loop:

$B_s - \bar{B}_s$

$$\mathcal{L}_{\bar{b}s\bar{b}s} = k \frac{|y_{b\mu} y_{s\mu}^*|^2}{32\pi^2 M_{LQ}^2} (\bar{b} \gamma_\mu P_L s) (\bar{s} \gamma^\mu P_L b) + \text{h.c.}$$

$y = y_3, y_1, y'_3$  and  $k = 5, 4, 20$  for  $S_3, V_1, V_3$ .

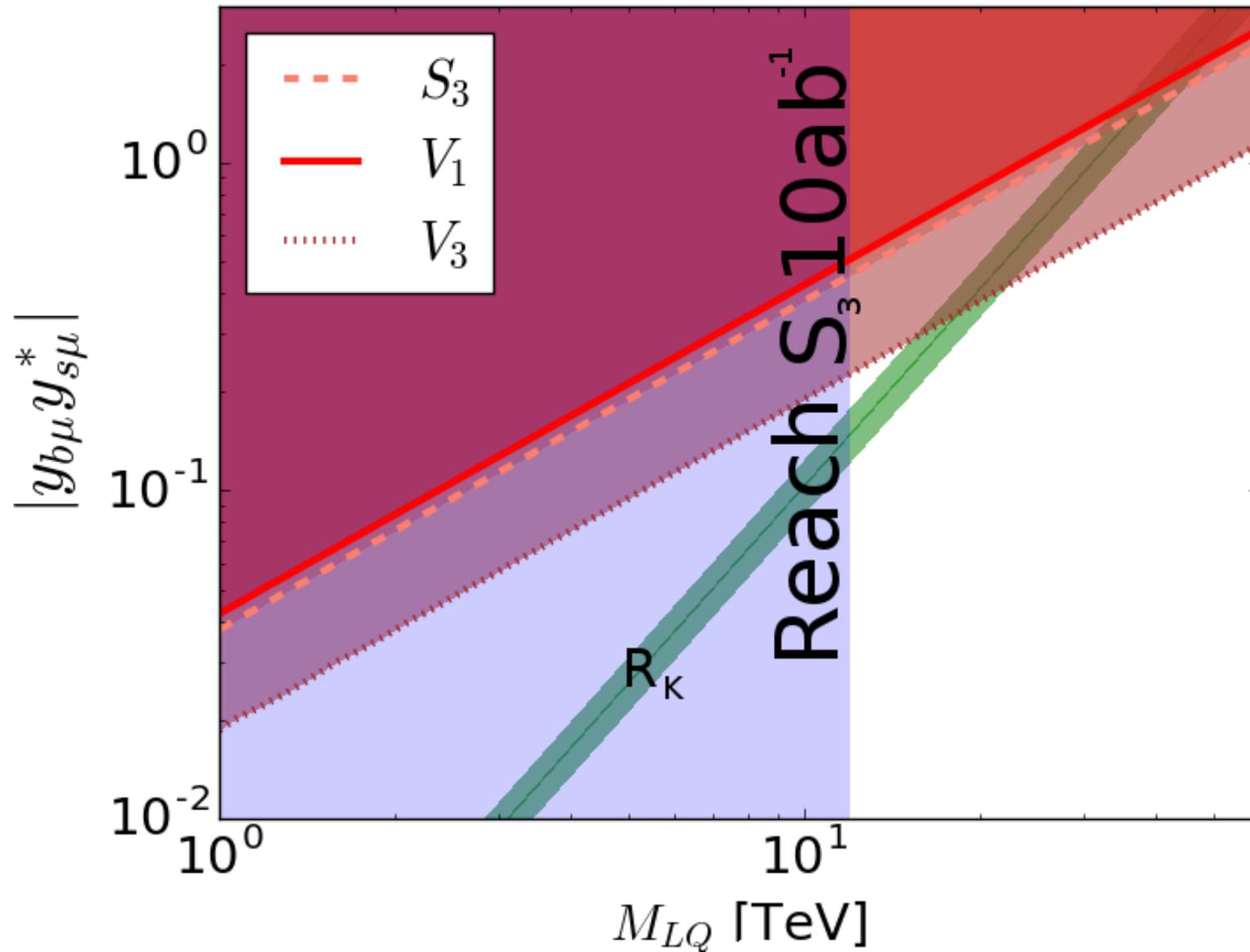
Data  $\Rightarrow c_{LL}^{bb} < 1/(210\text{TeV})^2$ .

# Mass Constraints: Summary

$S_3$	41 TeV
$V_1$	41 TeV
$V_3$	18 TeV

Upper mass limits for leptoquarks that satisfy neutral current  $B$ -anomaly fits and  $B_s$ -mixing constraints.

# 8 TeV CMS $20\text{fb}^{-1}$ 2nd gen



Up to 14 TeV LQs with 100 TeV  $10\text{ab}^{-1}$  FCC-hh.  $M_{LQ} < 41\text{ TeV}$ .

# LQ Mass Limits

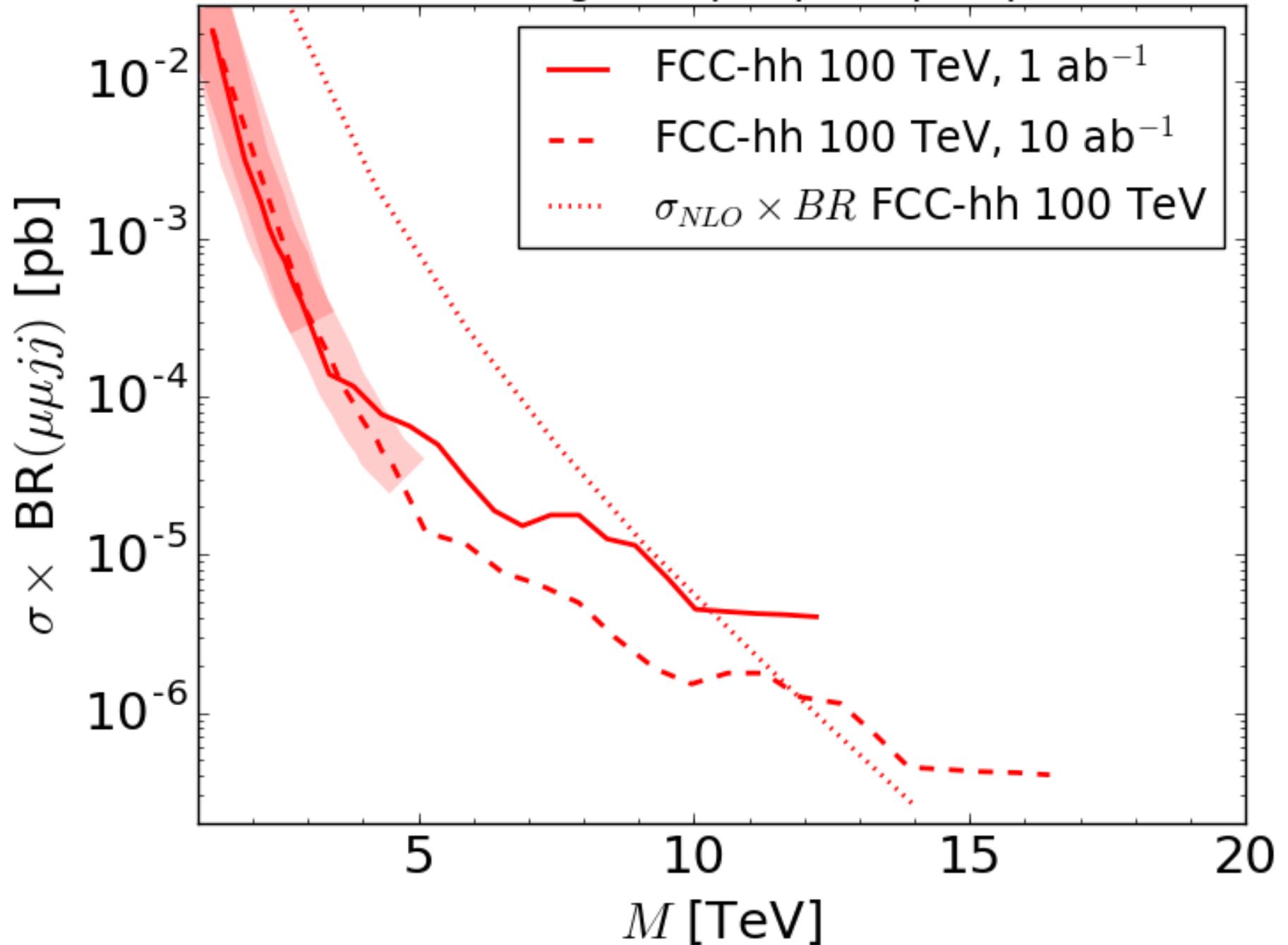
$S_3$	41 TeV
$V_1$	41 TeV
$V_3$	18 TeV

From  $B_s - \bar{B}_s$  mixing and fitting  $b$ -anomalies.

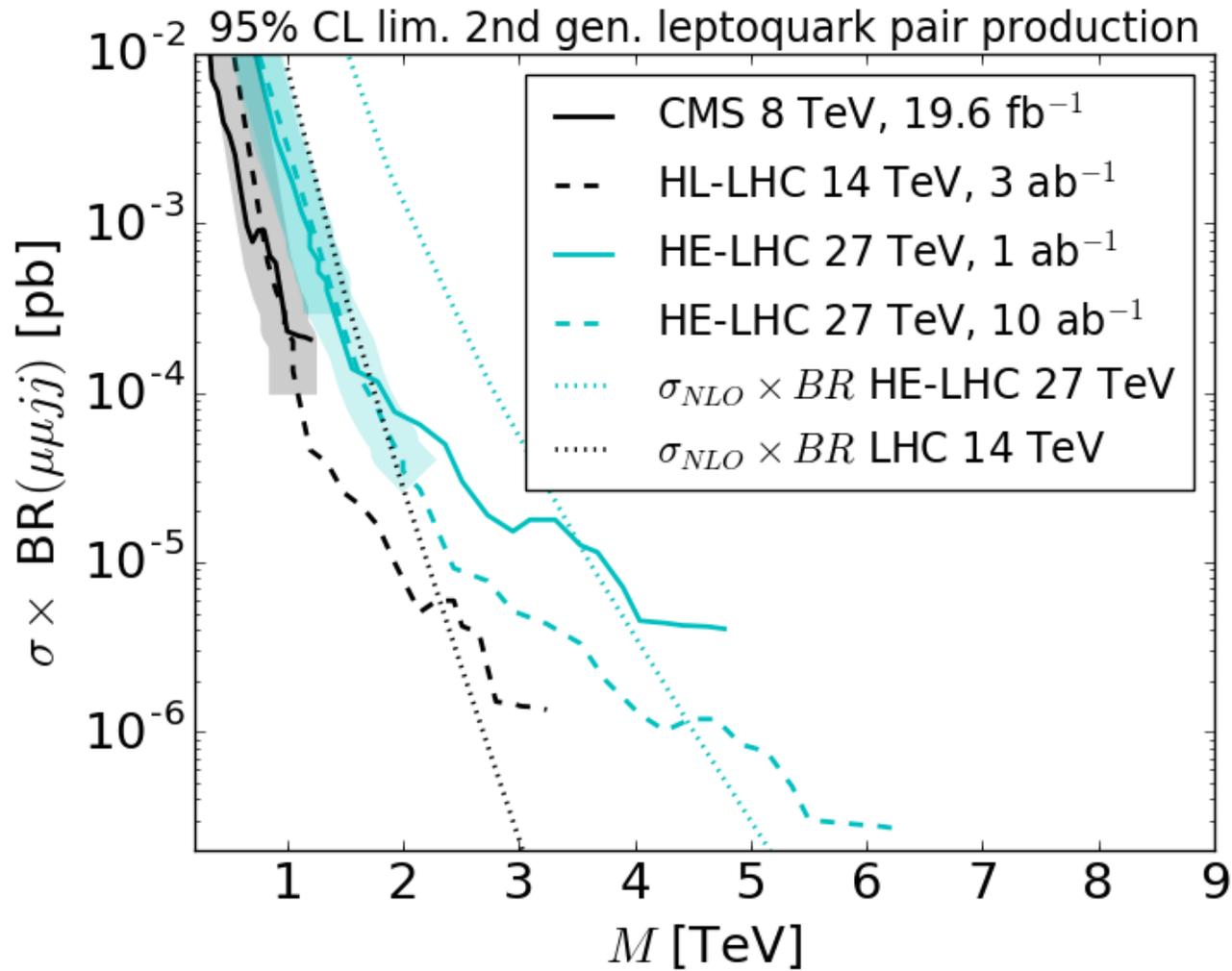
Pair production has a reach up to 12 TeV.

The pair production cross-section is insensitive to the representation of  $SU(2)$  in this case.

95% CL lim. 2nd gen. leptoquark pair production



# HL-LHC/HE-LHC LQs

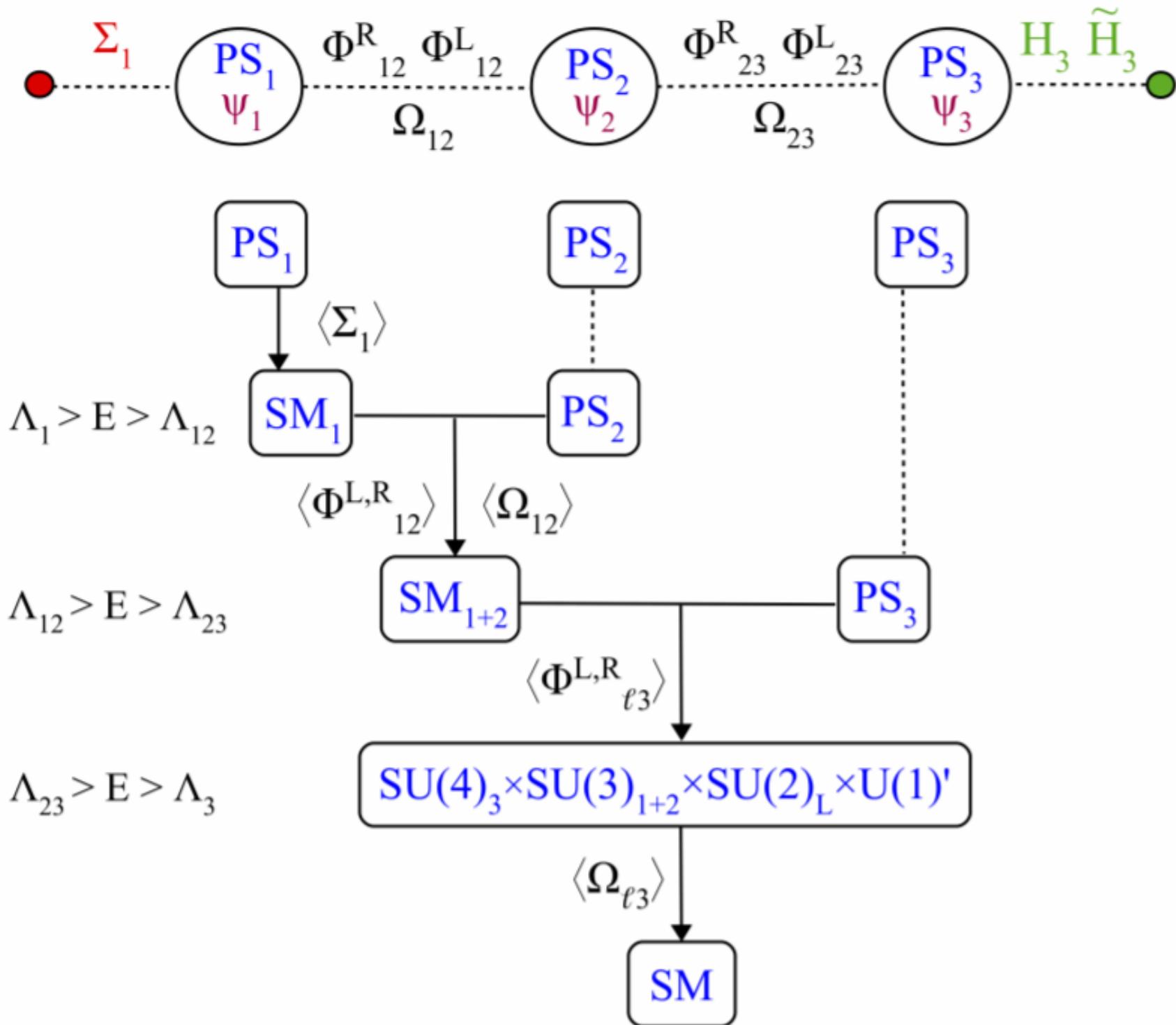


# Other Flavour Models

Realising<sup>15</sup> the vector LQ solution based on  $PS = [SU(4) \times SU(2)_L \times SU(2)_R]^3$ . SM-like Higgs lies in third generation PS group, explaining large Yukawas (others come from VEV hierarchies). Get  $U(2)_Q \times U(2)_L$  approximate global flavour symmetry.

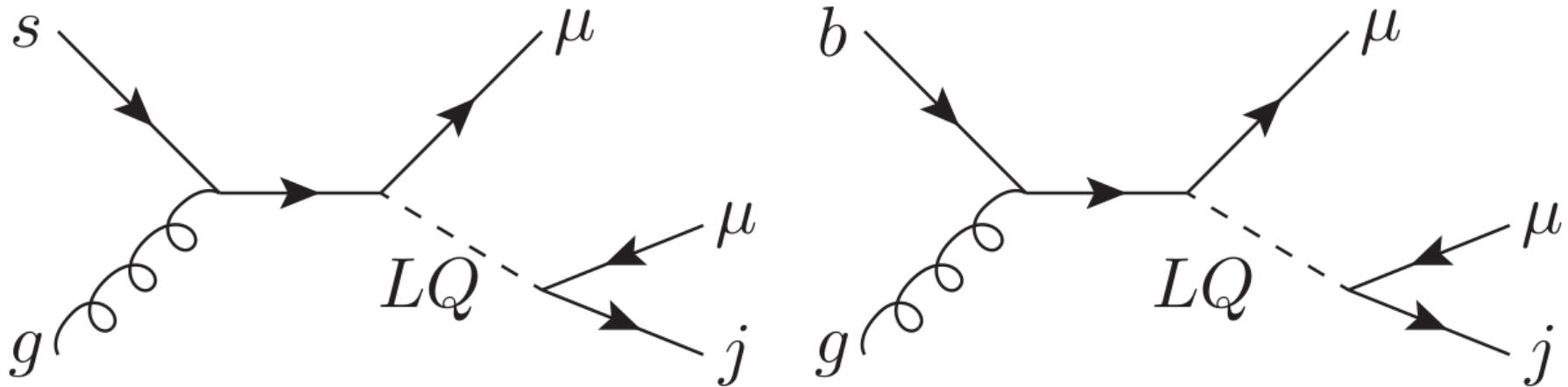
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<sup>15</sup>Di Luzio Greljo, Nardecchia [arXiv:1708.08450](#), Bordone, Cornella, Fuentes-Martin, Isidori, [arXiv:1712.01368](#)

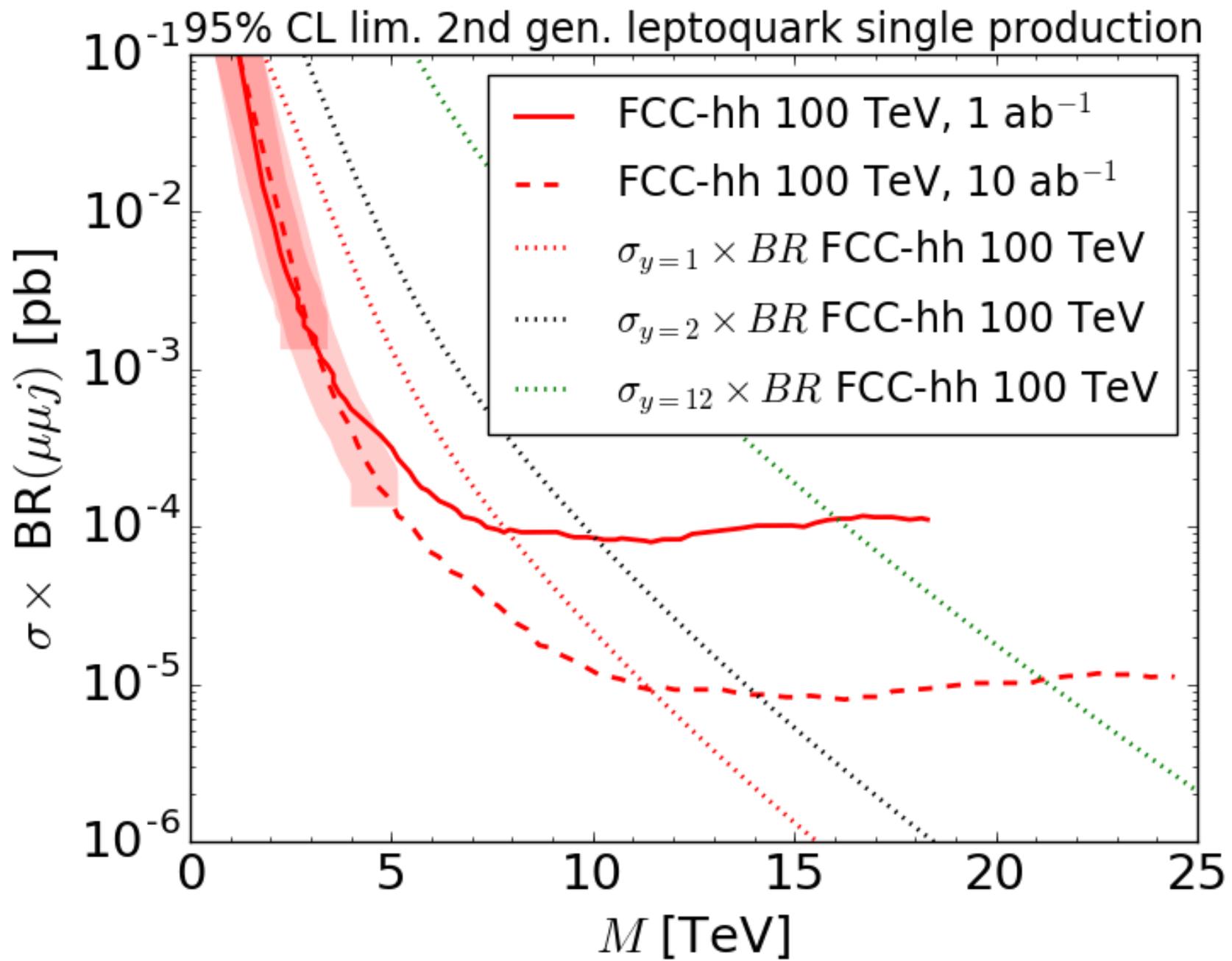


# Single Production of LQ

Depends upon **LQ coupling** as well as LQ mass



Current bound by CMS from 8 TeV  $20 \text{ fb}^{-1}$ :  $M_{LQ} > 660$  GeV for  $s\mu$  coupling of 1. We include  $b$  as well from NNPDF2.3LO ( $\alpha_s(M_Z) = 0.119$ ), re-summing large logs from initial state  $b$ . Integrate  $\hat{\sigma}$  with LHAPDF.



$\sigma$ s for  $S_3$  with  $y_{s\mu} = y_{b\mu} = y$ .

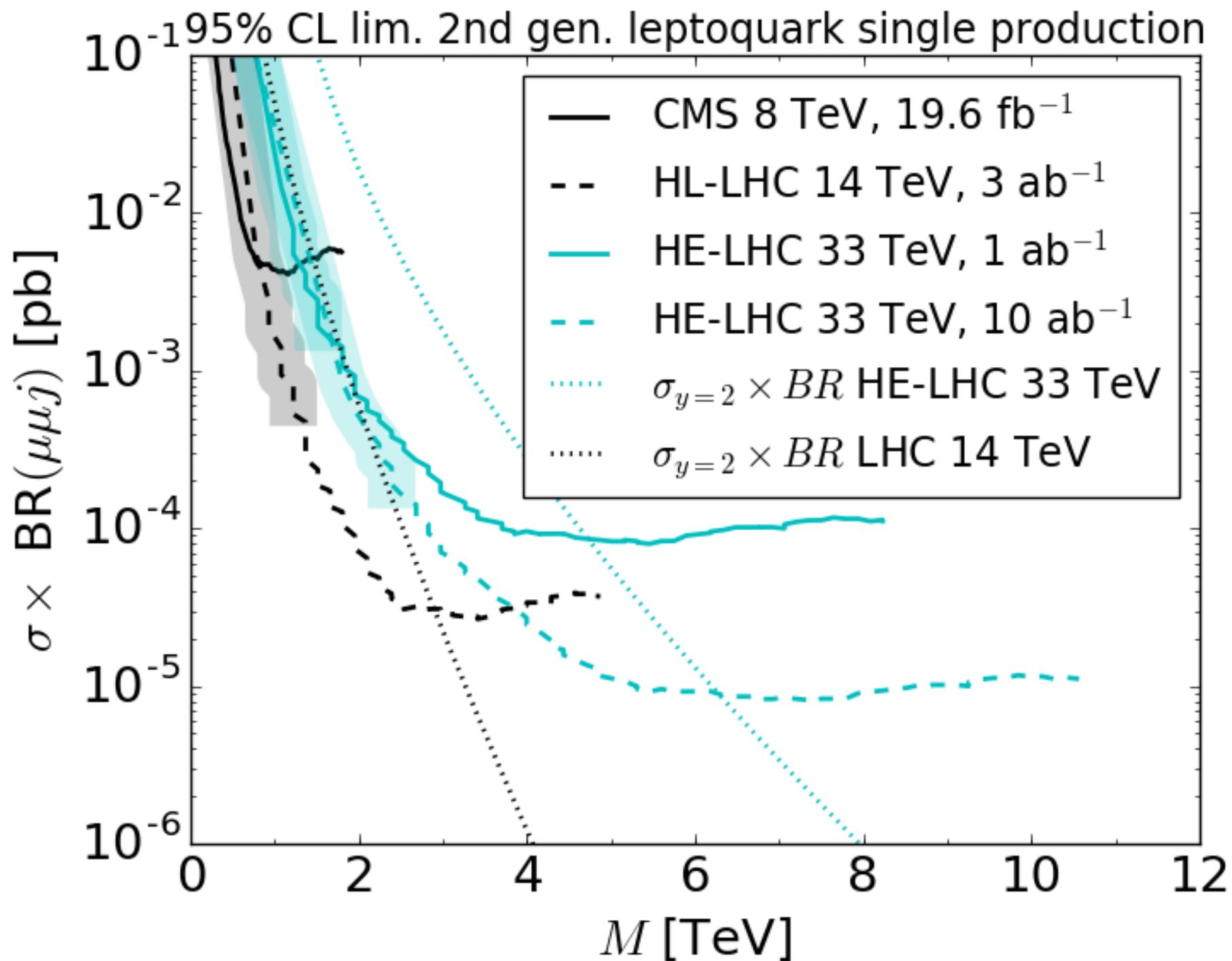
# Single LQ Production $\sigma$

$$\hat{\sigma}(qg \rightarrow \phi l) = \frac{y^2 \alpha_S}{96 \hat{s}} (1 + 6r - 7r^2 + 4r(r+1) \ln r) ,$$

where<sup>16</sup>  $r = M_{LQ}^2 / \hat{s}$  and we set  $y_{s\mu} = y_{b\mu} = y$ .

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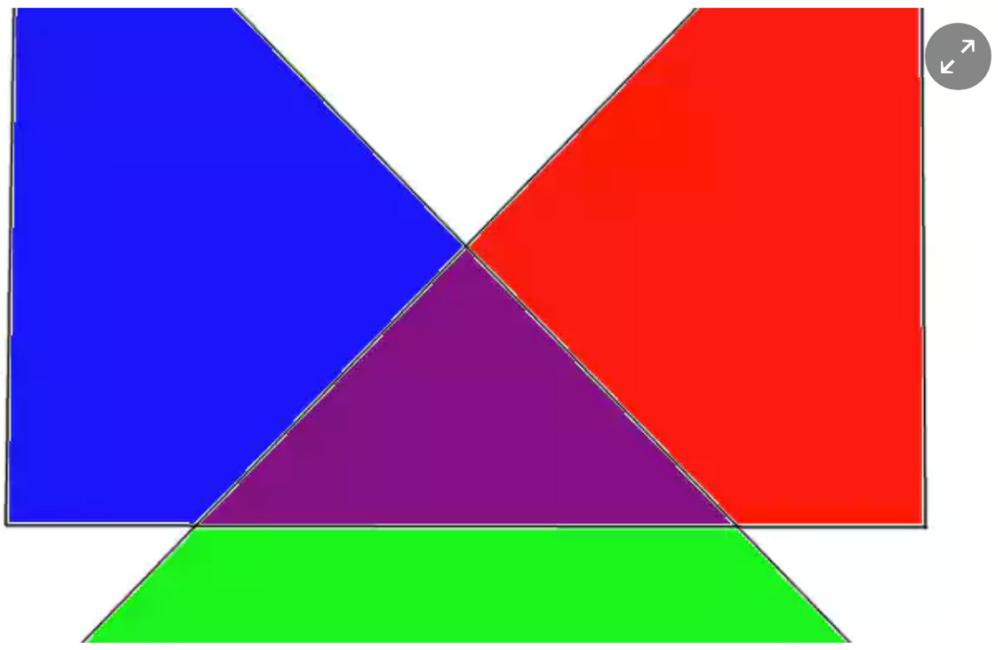
<sup>16</sup>Hewett and Pakvasa, PRD **57** (1988) 3165.



Science Life and Physics

## Modelling the fourth colour: dispatch from de Moriond

At the particle physics conference, it's clear inconclusive LHCb data are stimulating strange new ideas



▲ Four colours (or colors?) Photograph: Ben Allanach

Ben Allanach  
Sat 17 Mar 2018 10:15 GMT

In the middle of the [Rencontres de Moriond](#) particle physics conference in Italy, the scientific talks stopped to allow a standing ovation dedicated to the memory and achievements of my inspirational colleague Stephen Hawking, who we heard had died earlier that day.

The talks quickly resumed, which I think Stephen would have approved of. The most striking thing about the scientific content of the conference this year was that a whole day was dedicated to the weirdness in bottom particles that [Tevong You and I wrote about](#) last November. As Marco Nardecchia reviewed in his talk ([PDF](#)), bottom particles produced in the LHCb detector in proton collisions are decaying too often in certain particular ways, compared to predictions from the Standard Model of particle physics. Their decay products are coming out with the wrong angles too often compared with predictions, too.



Anomalous bottoms at Cern and the case for a new collider  
[Read more](#)

We were hoping for an update on the data at the conference: the amount of data has roughly doubled since they were last released, and we need to see the new data to be convinced that something really new is happening in the collisions. I strongly suspect that if the effect is seen in the new data, the theoretical physics community will “go nuts” and we will quickly see the resulting avalanche of papers. If the new data look ordinary, the effect will be forgotten and everyone will move on. Taking such measurements correctly takes care and time, however, and the LHCb experiment didn't release them.

We shall have to wait until other conferences later this year for the LHCb to present its analyses of the new data.

There were interesting theory talks on how new forces could explain the strange properties of the bottom particle decays. The full mathematical models look quite baroque: they need a lot of “bells and whistles” in order to pass other experimental tests. But these models prove that it can be done, and they are quite different to what has been proposed before.

[One of them](#) even unifies different classes of particle (leptons and quarks), describing the lepton as the “fourth colour” of a quark. We are used to the idea that quarks come in three (otherwise identical) copies: physicists label them red, green and blue to distinguish them. As Javier Fuentes-Martin describe ([PDF](#)), once you design the mathematics to make leptons the fourth colour, the existence of a new force-carrying particle with just the correct properties to break up the

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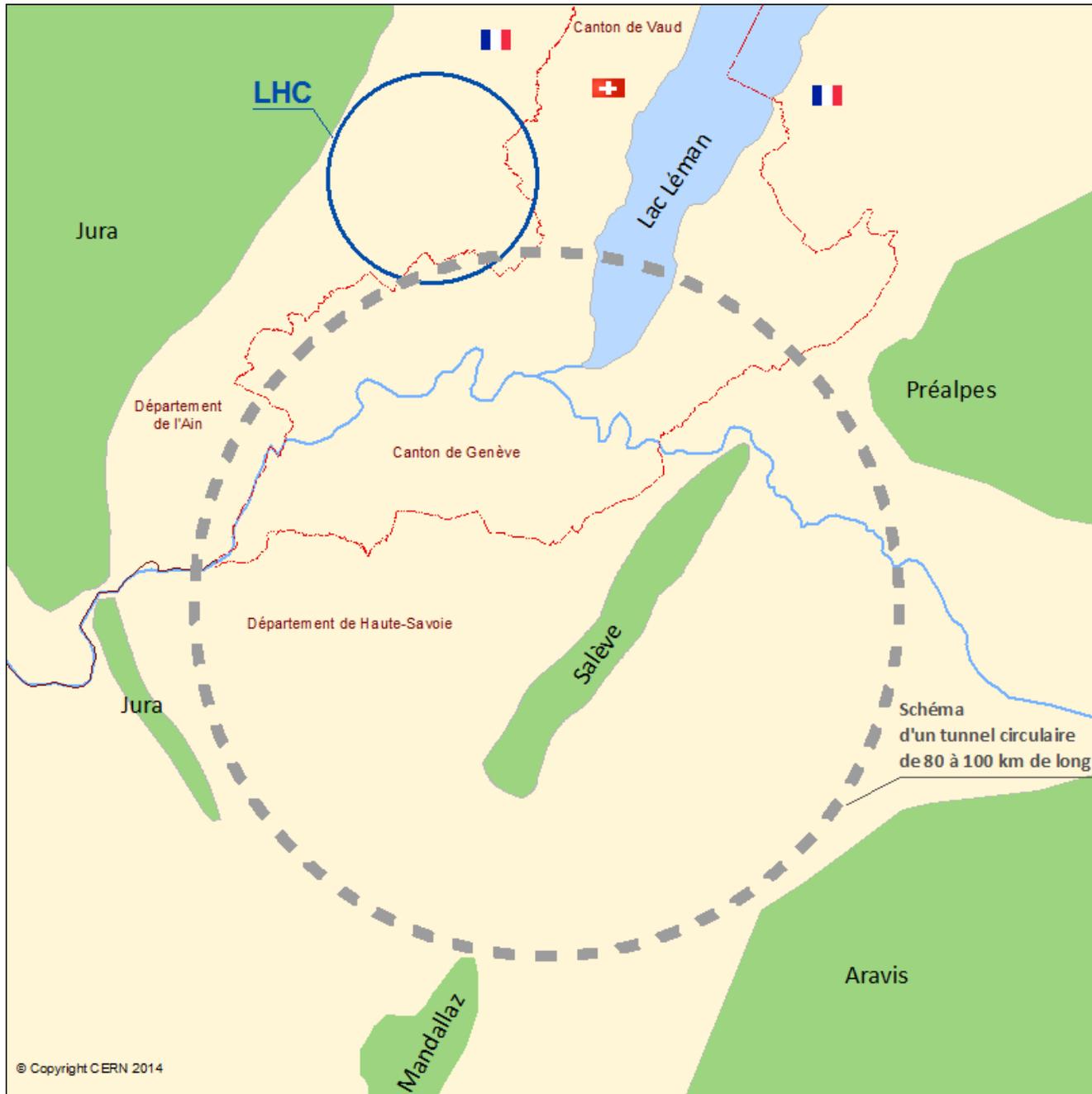
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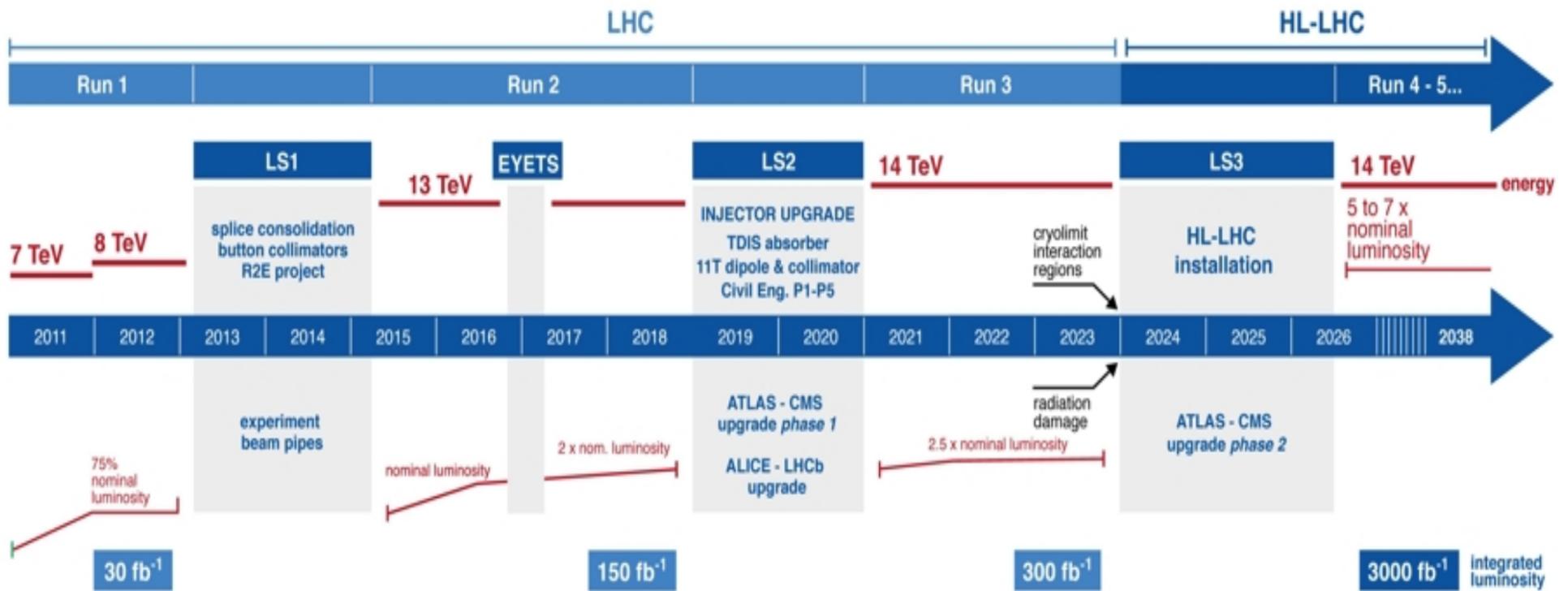
# Cern draws up plans for collider four times the size of Large Hadron

**The Future Circular Collider would smash particles together in a tunnel 100km long**

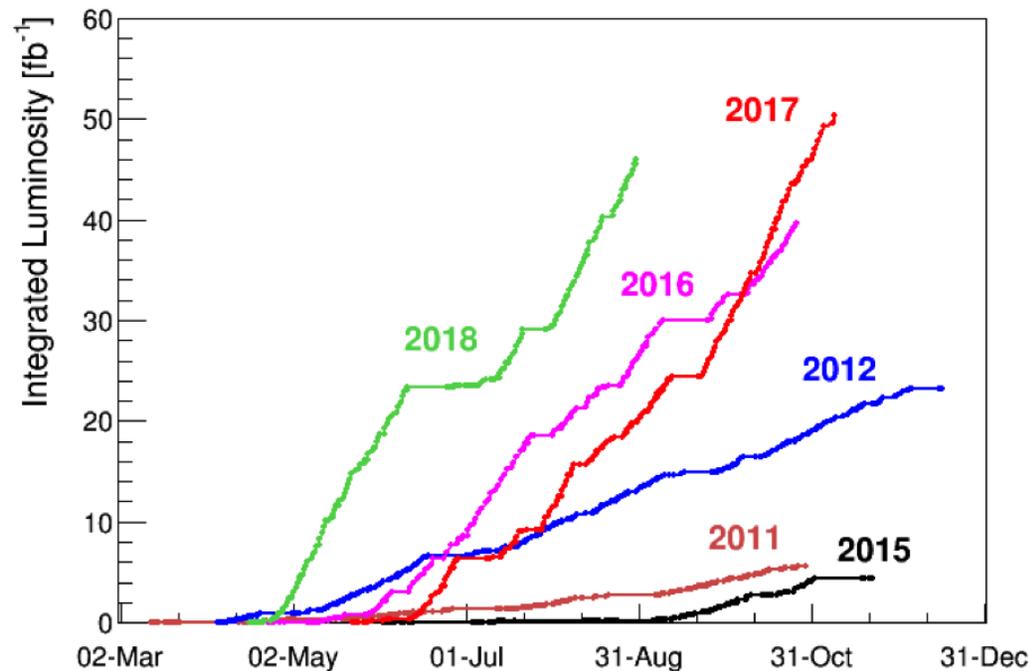




# LHC / HL-LHC Plan



# LHC Upgrades



High Luminosity (HL) LHC: go to  $3000 \text{ fb}^{-1}$  ( $3 \text{ ab}^{-1}$ ).

High Energy (HE) LHC: Put FCC magnets (16 Tesla rather than 8.33 Tesla) into LHC ring: roughly *twice* collision energy: 27 TeV.

# Properties of anomaly-free solutions

