Quantum Dissipation of Quarkonium in Quark Gluon Plasma

Free meson seminar ＠online

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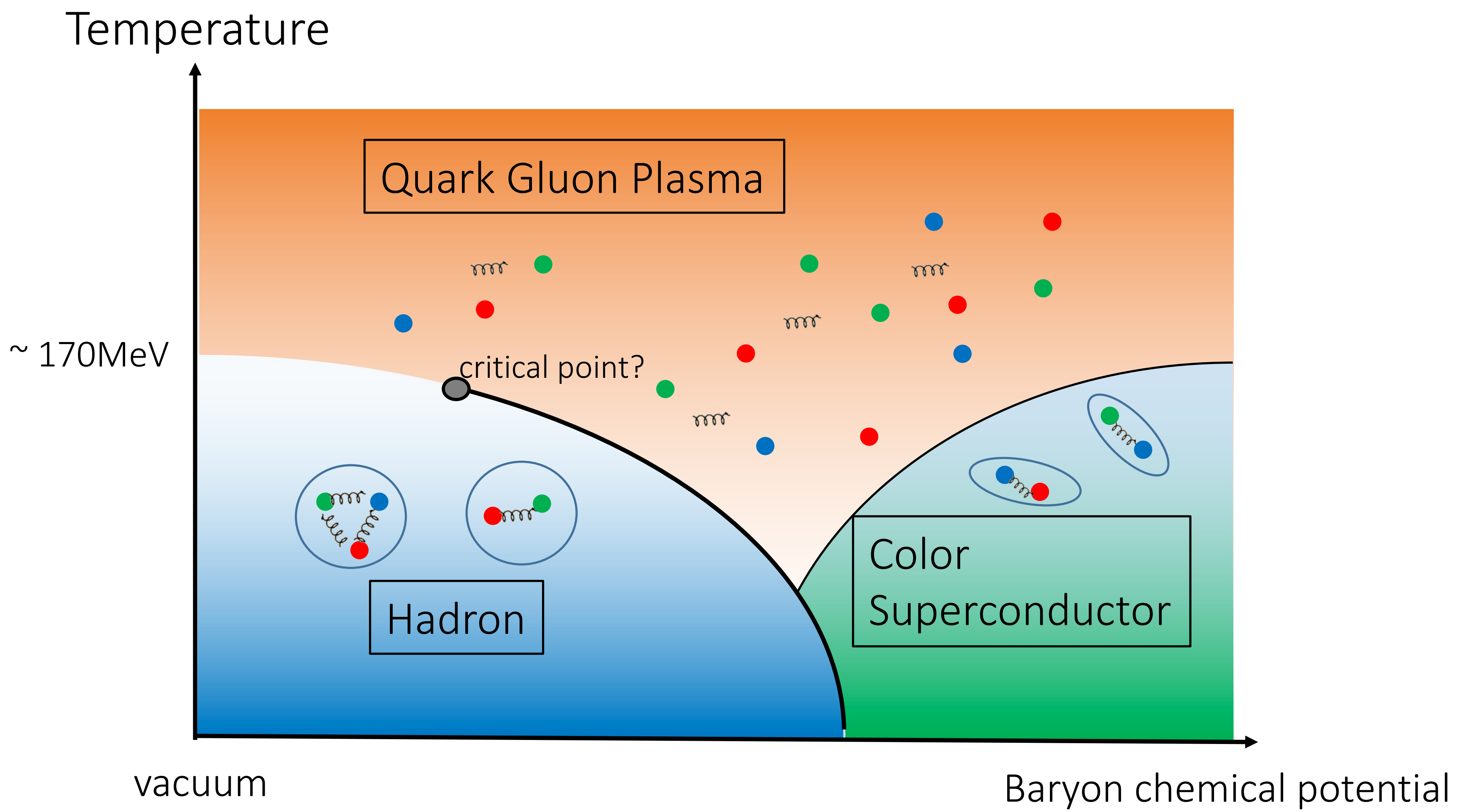
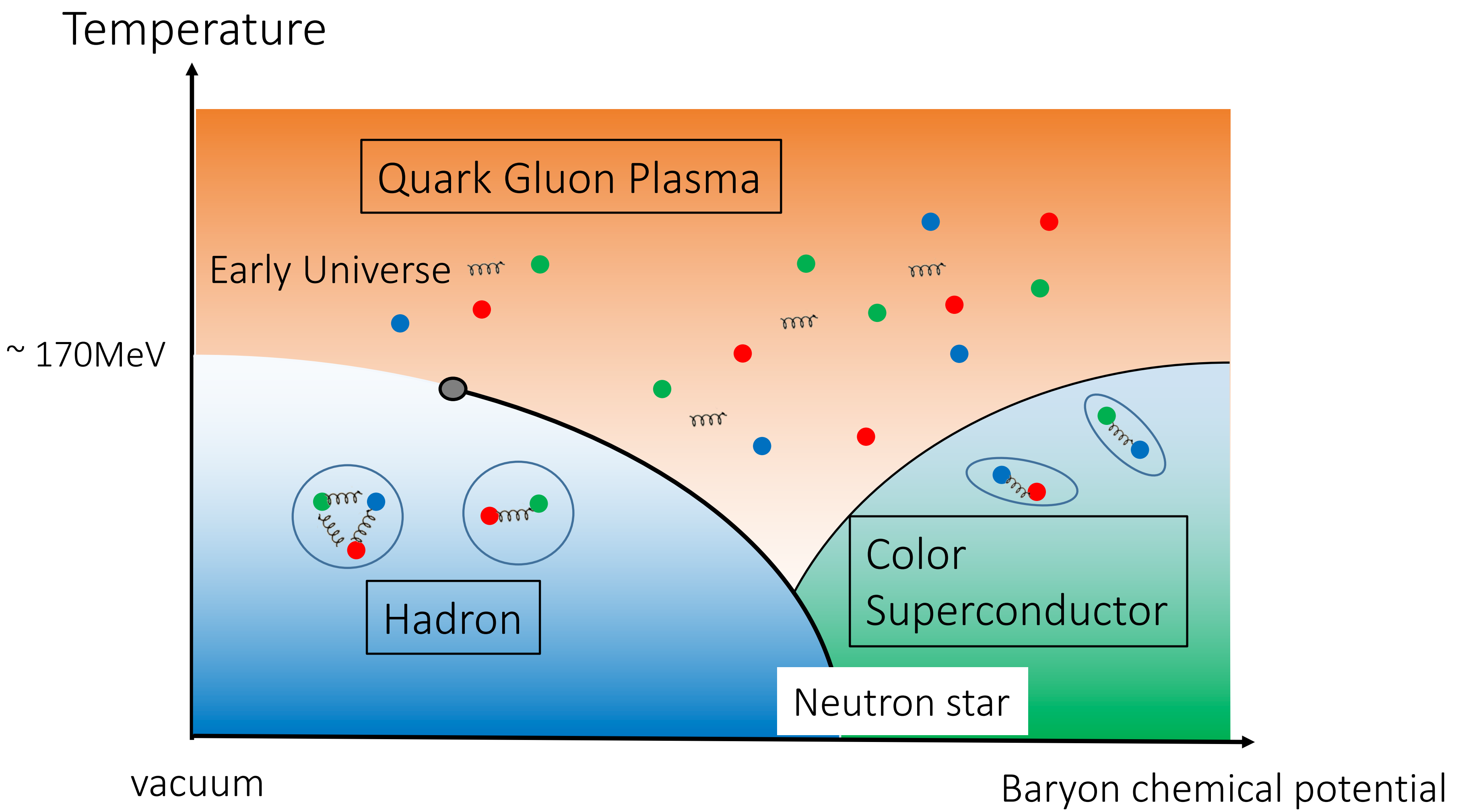
Based on PRD.101.034011(2019)

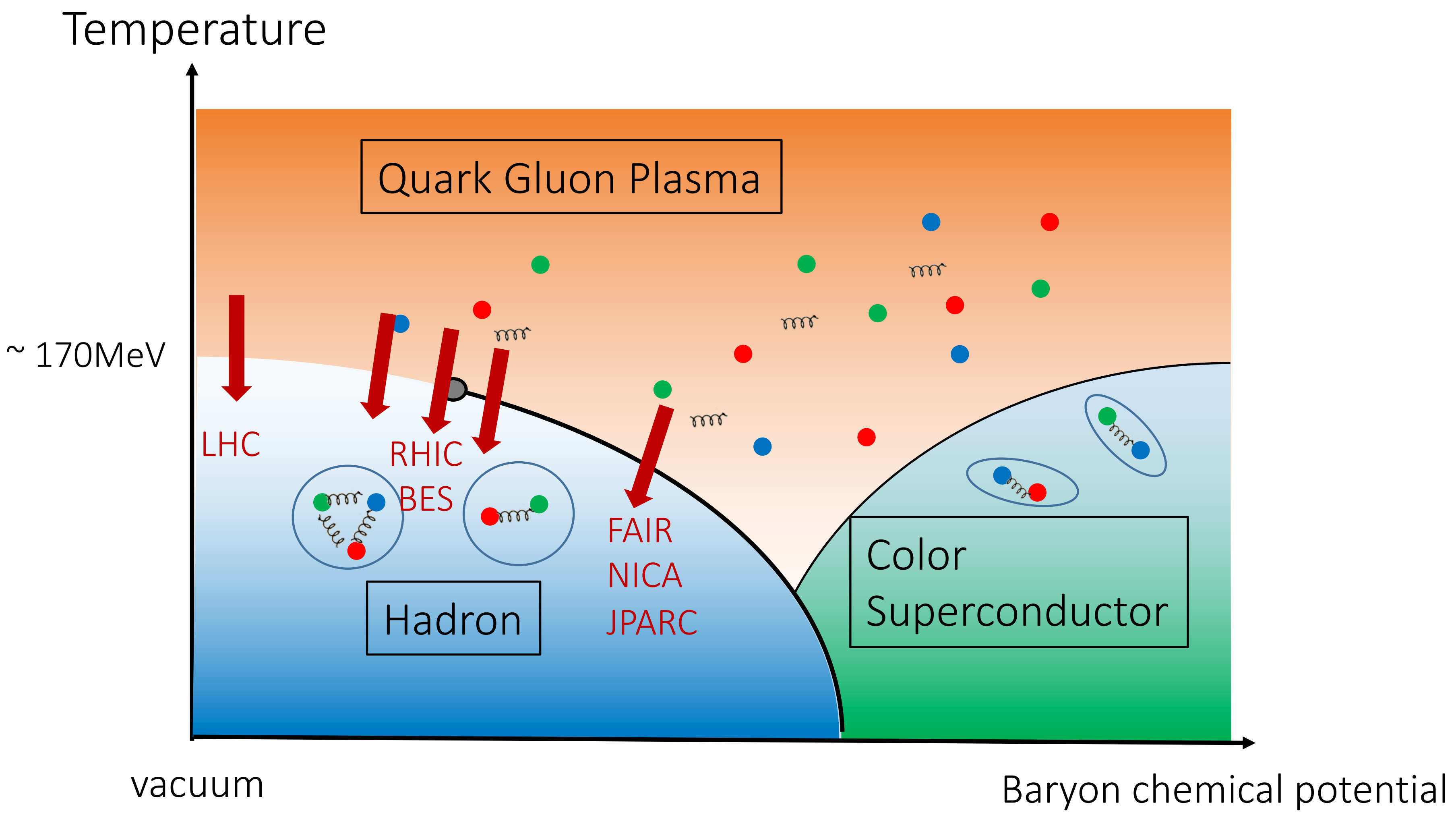
QCD Phase Diagram



QCD Phase Diagram

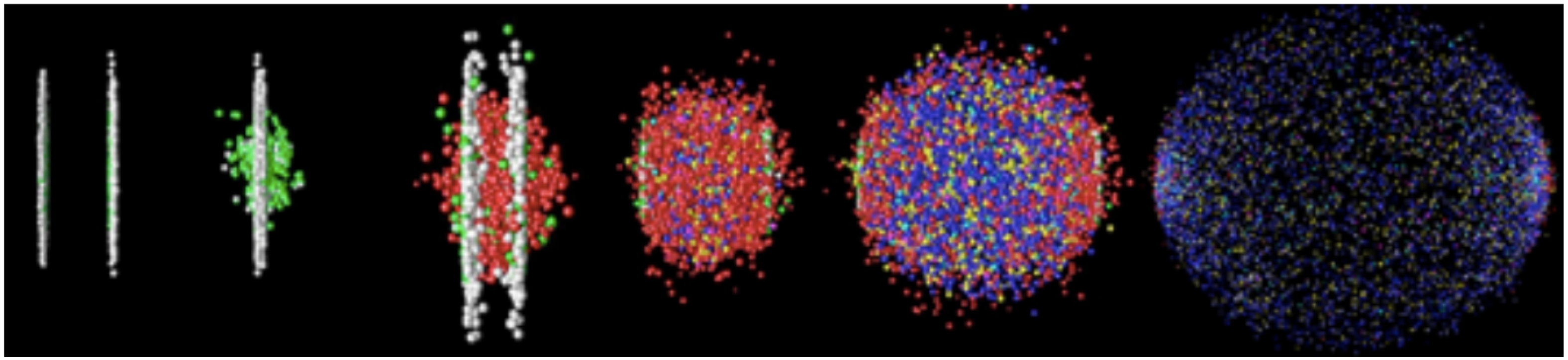


QCD Phase Diagram



Heavy Ion Collisions

Dynamics in HICs（LHC,RHIC） figure taken from http://alice-j.org/



→ → → → time

nuclei collision　equilibration　QGP　hadronization

↓

observed

Observables reflect the properties of QGP

*JET QUENCHING → stopping power

*ELIPTIC FLOW → shear viscosity

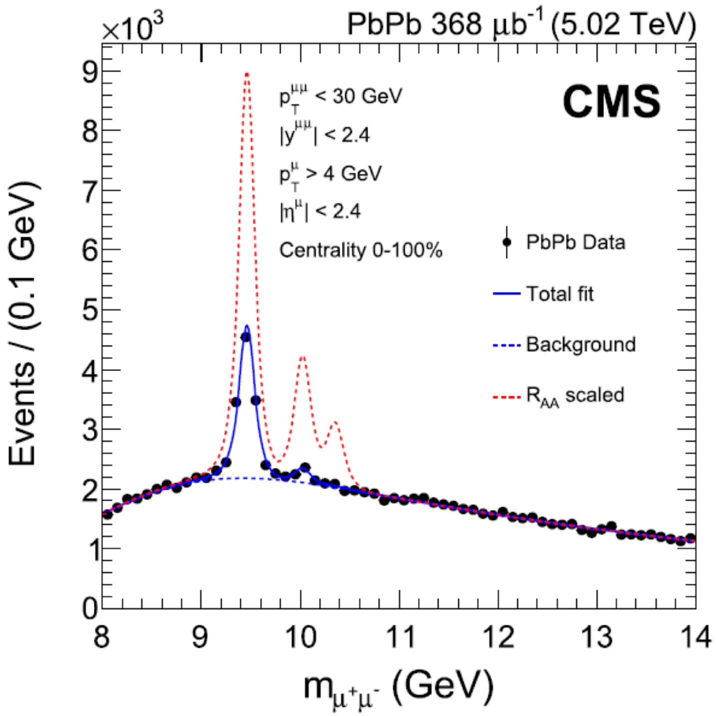
*QUARKONIUM SUPPRESSION → screening length
(focused in this talk)

Quarkonium in Heavy Ion Collisions

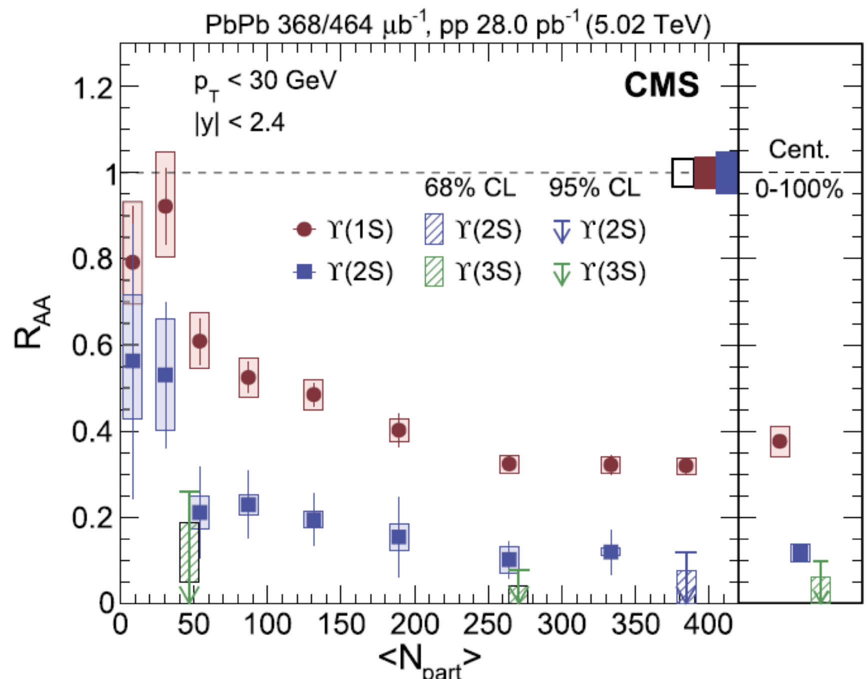
Quarkonium yield suppression

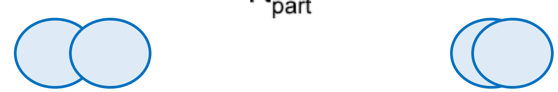
[CMS collaboration]

Spectra in pp and PbPb



R_{AA} ~ degree of suppression

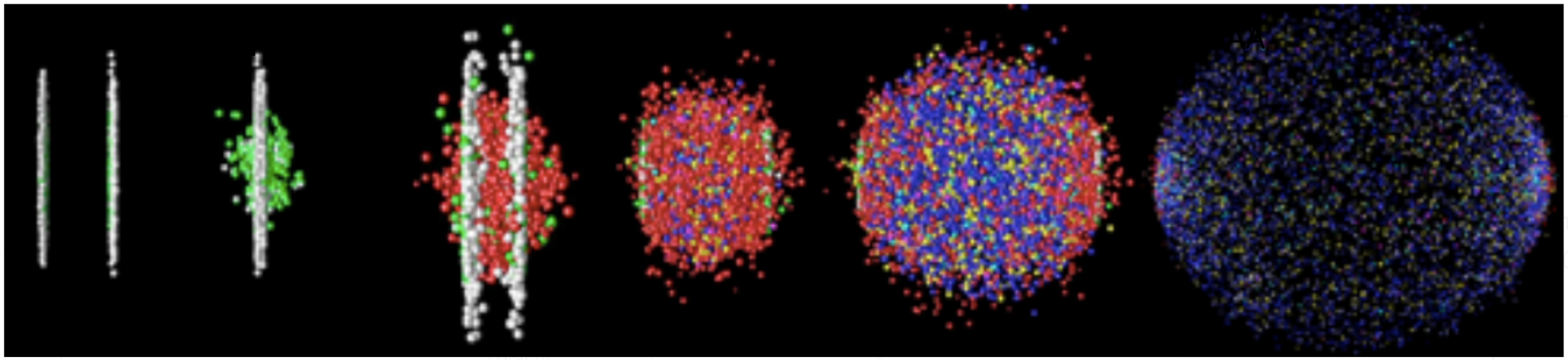




What determines these experimental data?

Quarkonium in Heavy Ion Collisions

Dynamics in HICs



time

nuclei collision → equilibration → QGP → hadronization

Quarkonium dynamics

interaction

production → observed

How does quarkonium evolve in QGP?

Quarkonium:Theory1

Matsui & Satz Scenario

[Matsui, Satz(86)]

J/ψ as a probe of color charge screening

confined phase $T < T_c$

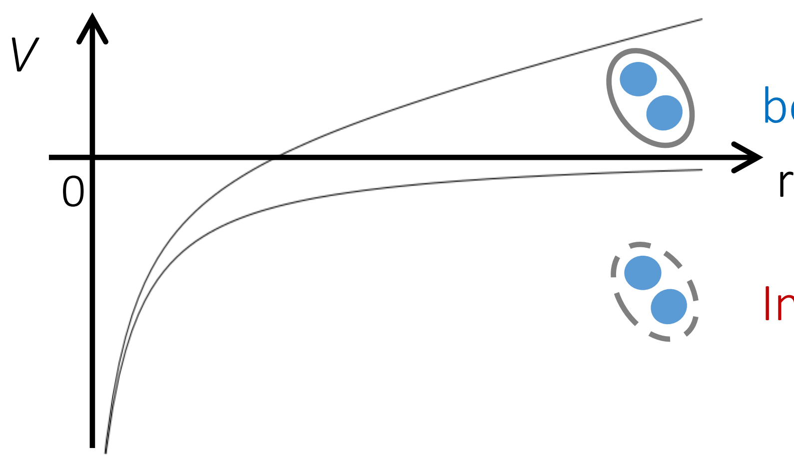
$$V = -\frac{\alpha}{r} + \sigma r$$

Confinement potential

QGP phase $T > T_c$

$$V = -\frac{\alpha}{r} \mathrm{e}^{-m_D r}$$

Debye screening potential
by liberated colors



bound at low T

In QGP, no bound state at high T

Quarkonium dissociation based on potential change

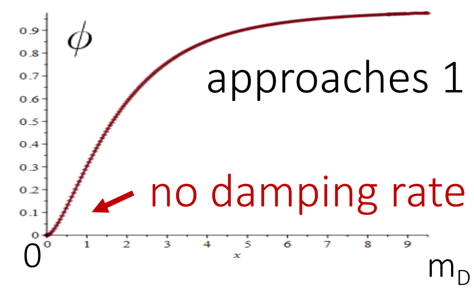
Quarkonium:Theory2

Real-time potential $V(r) \equiv \left.\frac{i\partial_t W_{\text{loop}}}{W_{\text{loop}}}\right|_{t\to\infty}$

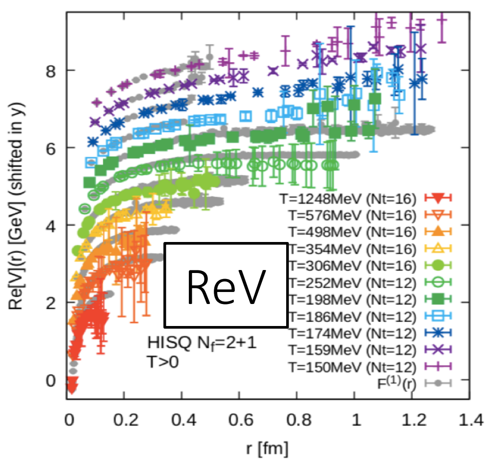
- From perturbation theory [Laine+(08),Beraudo+(08),etc.]

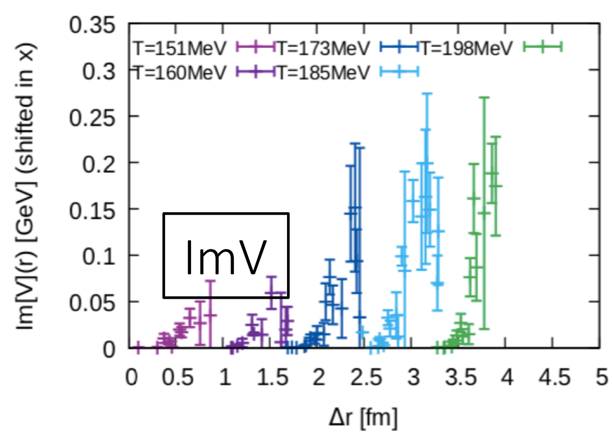
$$V(r) = -\alpha C_F\left[m_D + \frac{e^{-m_D r}}{r}\right] - i\alpha C_F T\phi(r) \in \mathbb{C}$$

imaginary part



- From lattice QCD results [Rothkopf+(12-), Petreczky+(18),etc.]





Schrödinger equation with complex potential

→application to phenomenology [Krouppa+(17),Islam+(20)etc.]

Quarkonium:Theory3

Langevin dynamics [Blaizot+(16,18)]

Interference between HQ and anti HQ is included

HQ
$$M\ddot{r} + \frac{\beta g^2}{2}\left(\mathcal{H}(0)\dot{r} - \mathcal{H}(s)\dot{\bar{r}}\right) - g^2\nabla V(s) = \xi(s,t)$$

anti HQ
$$M\ddot{\bar{r}} + \frac{\beta g^2}{2}\left(\mathcal{H}(0)\dot{\bar{r}} - \mathcal{H}(s)\dot{r}\right) + g^2\nabla V(s) = \bar{\xi}(s,t)$$

$$\langle \xi(s,t)\xi\left(s,t'\right)\rangle = g^2\mathcal{H}(0)\delta\left(t-t'\right)$$
$$\left\langle \xi(s,t)\bar{\xi}\left(s,t'\right)\right\rangle = -g^2\mathcal{H}(s)\delta\left(t-t'\right)$$

$$s = r - \bar{r}$$

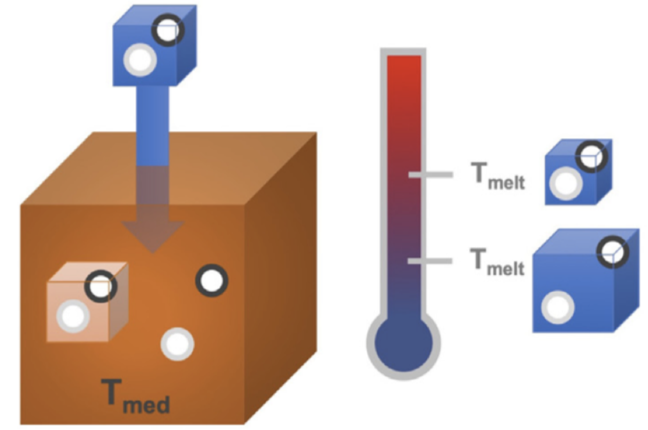
relative distance

Quarkonium as two interacting random walking particles

Quarkonium as Open System

Problems in descriptions

figure taken form Rothkopf(20)



- Debye screening phenomenon

  static picture of quarkonium

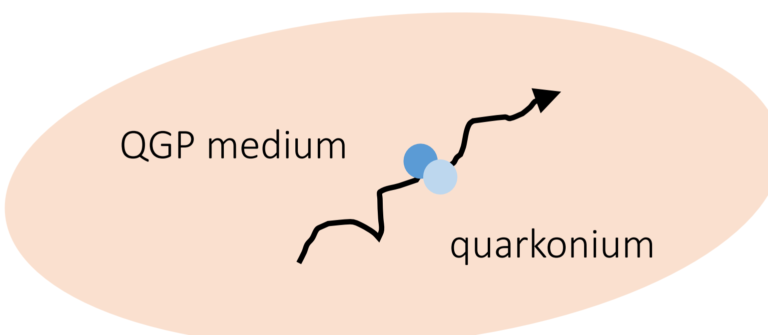
- Complex potential

  not unitary evolution

$$i\partial_t \psi = [-\frac{\nabla^2}{M} + \mathrm{Re}V + i\,\mathrm{Im}V]\psi$$

- Langevin equation

  in classical limit



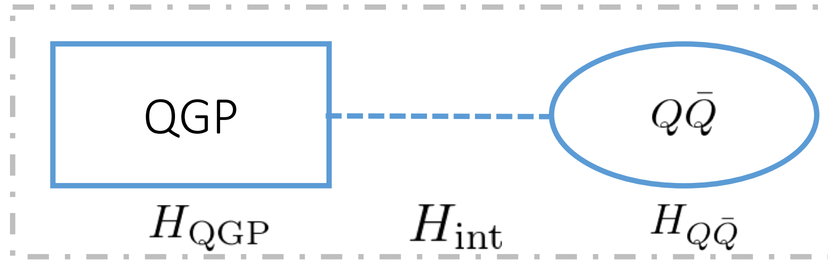
To overcome, apply OPEN QUANTUM SYSTEMS framework

Open Quantum Systems

We describe quarkonium in QGP just by quarkonium variables

von Neumann equation

$$\frac{d}{dt}\rho_{\mathrm{total}} = -i[H_{\mathrm{total}}, \rho_{\mathrm{total}}]$$



$$H_{\mathrm{total}} = H_{\mathrm{QGP}} + H_{\mathrm{Q\bar{Q}}} + H_{\mathrm{int}}$$

$$H_{\mathrm{int}} = \sum_i S_i \otimes E_i$$

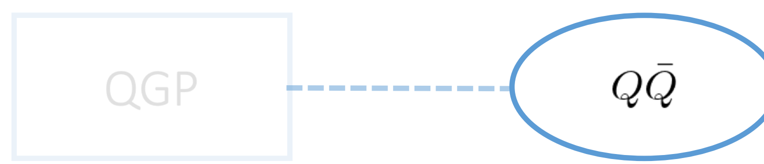
integrate out QGP

"reduced" density matrix

$$\rho_{Q\bar{Q}} = \mathrm{Tr}_{\mathrm{QGP}}\ \rho_{total}$$

master equation

$$\frac{d}{dt}\rho_{Q\bar{Q}} = \quad ?$$



What form of the master equation is derived?

Time Scale Hierarchies in OQSs

Three time scales τ_S , τ_E , τ_R are involved

Here $\tau_E \ll \tau_S$, $\tau_E \ll \tau_R$ are assumed for QBM

system is slow

- System time scale

$$\tau_S \sim \frac{1}{\Delta E_S}$$

- Environmental relaxation time scale

$$\langle A(t)A(0)\rangle \sim \mathrm{e}^{-\frac{t}{\tau_E}}$$

- System relaxation time scale

$$\langle p(t)\rangle \propto \mathrm{e}^{-\frac{t}{\tau_R}}$$

Quarkonium(=S) in QGP(=E)

$$\tau_S \sim {\Delta E_S}^{-1} \sim 2\mathrm{fm}$$

from Coulombic binding energy

$$\tau_E \sim T^{-1} \sim 0.5\mathrm{fm}$$

QGP temperature ~ 400MeV

$$\tau_R \sim M/T^2 \qquad M \gg T$$

kinetic equilibration

Lindblad Master Equation

Following conditions are imposed

- Trace conservation $\mathrm{Tr}\rho_S = 1$
- Hermicity $\rho_S^\dagger = \rho_S$
- Positive $\forall \left|\alpha\right\rangle, \left\langle\alpha\right| \rho_S \left|\alpha\right\rangle \geq 0$
- Markov No memory effect $\tau_E \ll \tau_R$

$$\frac{d}{dt}\rho_S = -i\left[H_S, \rho_S\right] + \sum_k \gamma_k \left[L^k \rho_S L^{k\dagger} - \frac{1}{2}\left\{L^{k\dagger}L^k, \rho_S\right\}\right] \qquad \gamma_k > 0$$

mathematically proven [Lindblad(76)]

Equivalent form

$$\frac{d}{dt}\rho_S = -i\left[H_S, \rho_S\right] + \sum_{i,j} a_{ij} \left[F^i \rho_S F^{j\dagger} - \frac{1}{2}\left\{F^{j\dagger}F^i, \rho_S\right\}\right] \qquad a_{ij} \text{ positive}$$

Forces in Lindblad Master Equation

Quantum descriptions of forces in Brownian Motion

$$\frac{d}{dt}\rho_{Q\bar{Q}} =$$

1. Debye screening potential between heavy quarks V(x)

\+ 2.Random force

Previous study, i.e. stochastic potential

$$H = -\frac{\nabla^2}{M} + V(r)(t^a \otimes t^{a*}) + \theta^a(\frac{r}{2})(t^a \otimes 1) - \theta^a(-\frac{r}{2})(1 \otimes t^{a*})$$

[Kajimoto+(18)]

\+ 3. Drag force

→ Essential for equilibration

[Akamatsu+(18), Miura+(19)]

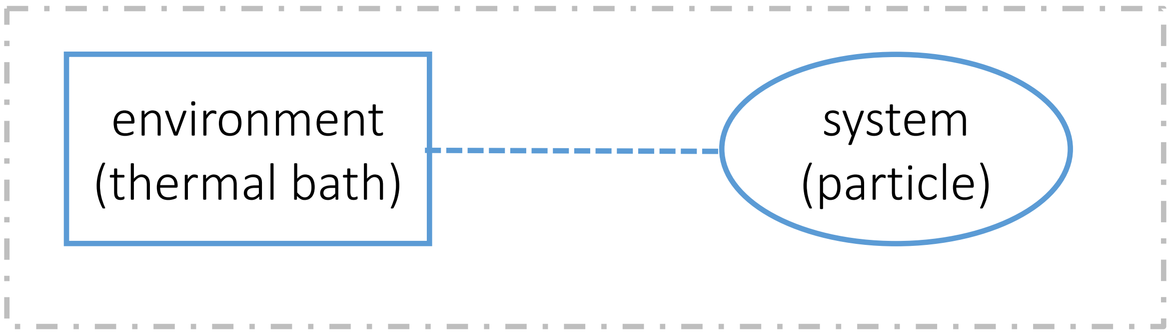
so far U(1) only / SU(3) in progress

How does quantum dissipation affect quarkonium dynamics in QGP?

Remark: What about Caldeira Leggett?

What is Caldeira Legette model? [Caldeira-Leggett(83)]

- Prototype of quantum Brownian motion



- Limited application
  - Particles are localized, i.e. wave packet limit
  - Not in Lindblad from

More general description is required

[Akamatsu(14,15), Blaizot(16,18), Brambilla+(17),etc]

Steps in Derivation

$$\frac{d}{dt}\rho_{\mathrm{total}}(t) = -\,i\left[\sum_i S_i(t)\otimes E_i(t), \rho_{\mathrm{total}}(t)\right]$$ in interaction picture

after iteratively solving $\rho_{\mathrm{total}}(t)$ with Born-Markov approximation

1. Trace QGP part

a_{ij} in Lindblad master equation

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -\int_0^\infty ds \sum_{i,j} \mathrm{Tr}_{QGP}\left(\rho_{QGP}(t)E^i(t)E^j(t-s)\right)$$

correlation decays in small s

$$\times\left[S^j(t-s)\rho_{Q\bar{Q}}(t)S^i(t) - S^i(t)S^j(t-s)\rho_{Q\bar{Q}}(t)\right] + h.c.$$

2. Gradient Expansion

$\tau_{QGP} \ll \tau_{Q\bar{Q}}$

$$S^j(t+\tau_{QGP}) \sim S^j(t) + \partial_t S^j(t)\tau_{QGP} + \cdots$$

leading next to leading

Lindblad master equation with Lindblad operator $L^i \sim S^i + \frac{i}{4T}\dot{S}^i$

Lindblad Operator for Quarkonium

Interaction Hamiltonian

$$H_{\text{int}} = \sum_i S_i \otimes E_i = \int d^3x \left[\underbrace{\delta\left(\vec{x}-\vec{x}_Q\right) t_Q^a - \delta\left(\vec{x}-\vec{x}_{\bar{Q}}\right) t_{\bar{Q}}^{a*}}_{\text{quarkonium}} \right] \otimes \underbrace{g A_0^a(\vec{x})}_{\text{QGP}}$$

i → position

Following the steps, Lindblad master equation for quarkonium is derived

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i\left[H'_{Q\bar{Q}}, \rho_{Q\bar{Q}}\right] + \int dx dy\, a(x-y) \left[F^a(y)\rho_{Q\bar{Q}} F^{a\dagger}(x) - \frac{1}{2}\left\{ F^{a\dagger}(x) F^a(y), \rho_{Q\bar{Q}} \right\} \right]$$

Fourier transform to momentum space

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[H'_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \int dk \{ 2L_k^a \rho_{Q\bar{Q}} L_k^{a\dagger} - L_k^{a\dagger} L_k^a \rho_{Q\bar{Q}} - \rho_{Q\bar{Q}} L_k^{a\dagger} L_k^a \}$$

- potential $V(x_Q - x_{\bar{Q}}) \in H'_{Q\bar{Q}}$
- Lindblad operator $L_{\vec{k}}^a = L_{\vec{k}}(\vec{x}_Q, \vec{p}_Q, t_Q^a; \vec{x}_{\bar{Q}}, \vec{p}_{\bar{Q}}, t_{\bar{Q}}^{a*})$

Lindblad operator is represented by both HQ and anti HQ d.o.f.

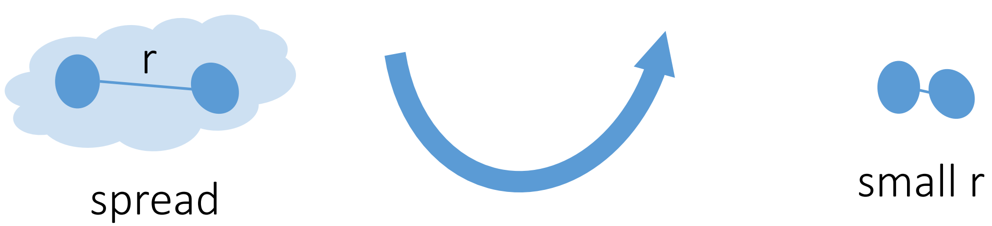
Our study vs related studies

Two approaches to Lindblad equation

dissipation	NRQCD	pNRQCD
No LO grad. exp.	Kajimoto+(18) ※stochastic Schroedinger eq.	Brambilla+(18,19)
Yes NLO grad. exp.	Miura+(in progress)	Akamatsu(20)

※De Boni(17) not in Lindblad form
Blaizot+(18) in classical limit

※Yao+(19) in quantum optical & classical limit



Dipole approximation

Is quarkonium dipole in whole process?

Interpretation of Lindblad Operator

- LO gradient expansion(relative motion)

$$L^{1}_{\vec{k}} = \sqrt{\frac{D(\vec{k})}{2}} \#_1 \left[\mathrm{e}^{i\vec{k}\cdot\hat{\vec{r}}/2} - \mathrm{e}^{-i\vec{k}\cdot\hat{\vec{r}}/2} \right] \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

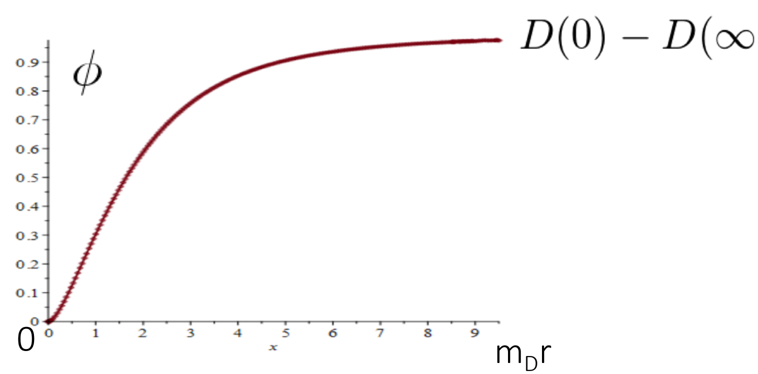
+ other transitions

rate$^{1/2}$ momentum transfer color rotation(octet→singlet)

note: D function is related to imaginary potential [Akamatsu+(12),Rothkopf(13)]

$$H_{\mathrm{complex}} = -\frac{\nabla^2}{M} + V(r) + i[D(r) - D(0)]$$

→imaginary potential



Interpretation of Lindblad Operator

- LO gradient expansion(relative motion)

+ dissipation

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+ other transitions

rate$^{1/2}$ momentum transfer color rotation(octet→singlet)

- NLO gradient expansion

+ anti HQ part
+ other transitions

$$L^1_{\vec{k}} = \sqrt{\frac{D(\vec{k})}{2}} \#_1 e^{i\vec{k}\cdot\hat{\vec{r}}/2} \left[1 - \frac{\vec{k}\cdot\hat{\vec{p}}}{4MT} - \frac{\vec{k}^2}{8MT} + \frac{N_c V(\vec{r})}{8T} \right] \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

① ②

Meaning①

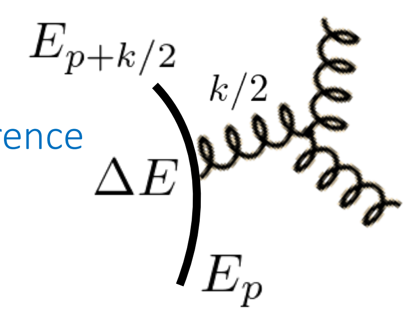
Collision rate depends on HQ momentum

$$\vec{p} \to \vec{p} + \vec{k}/2$$

Including NLO,

$$\frac{\Gamma_{\vec{p}\to\vec{p}+\vec{k}/2}}{\Gamma_{\vec{p}+\vec{k}/2\to\vec{p}}} \sim \exp\left[-\frac{\Delta E}{T} \right]$$

energy difference



Detailed balance approximately holds

Interpretation of Lindblad Operator

- LO gradient expansion(relative motion)

+ dissipation

$$L^1_{\vec{k}} = \sqrt{\frac{D(\vec{k})}{2}} \#_1 \left[\mathrm{e}^{i\vec{k}\cdot\hat{\vec{r}}/2} - \mathrm{e}^{-i\vec{k}\cdot\hat{\vec{r}}/2} \right] \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

+ other transitions

rate$^{1/2}$ momentum transfer color rotation(octet→singlet)

- NLO gradient expansion

+ anti HQ part
+ other transitions

$$L^1_{\vec{k}} = \sqrt{\frac{D(\vec{k})}{2}} \#_1 \mathrm{e}^{i\vec{k}\cdot\hat{\vec{r}}/2} \left[1 - \frac{\vec{k}\cdot\hat{\vec{p}}}{4MT} - \frac{\vec{k}^2}{8MT} + \frac{N_c V(\vec{r})}{8T} \right] \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

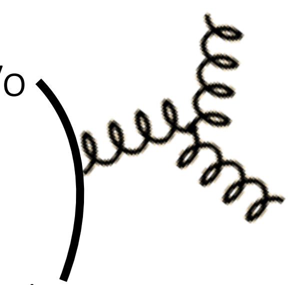
① ②

Meaning②

Different color state transition
singlet ⇆ octet

Including NLO,

$$\frac{\Gamma_{s\to o}}{\Gamma_{o\to s}} \sim \exp\left[-\frac{1}{T}\{V_o - V_s\} \right]$$



Detailed balance approximately holds

NUMERICAL ANALYSIS

SOLVE RELATIVE MOTION WITH LINDBLAD OPERATORS IN U(1)/SU(3)

Quantum State Diffusion(QSD) method

- Stochastic unravelling

[Gisin, Persival (92)]

Lindblad master eq.	via QSD ⇔ equivalent	nonlinear stochastic Schrödinger eq.
density matrix		wave function

$$\rho_{Q\bar{Q}}(x,y,t) = \langle \psi(x,t)\psi^*(y,t)\rangle$$

nonlinear stochastic Schrödinger eq. form

$$|d\psi\rangle = -iH'_{Q\bar{Q}}\,|\psi(t)\rangle\, dt + \int d\vec{k}\Big(2\underline{\langle L^\dagger_{\vec{k}}\rangle_\psi} L_{\vec{k}} - L^\dagger_{\vec{k}} L_{\vec{k}} - \underline{\langle L^\dagger_{\vec{k}}\rangle_\psi \langle L_{\vec{k}}\rangle_\psi}\Big)\,|\psi(t)\rangle\, dt$$
$$+ \int d\vec{k}\Big(L_{\vec{k}} - \underline{\langle L_{\vec{k}}\rangle_\psi}\Big)\,|\psi(t)\rangle\, d\xi_{\vec{k}}$$

→nonlinearity

complex noise property $d\xi_{\vec{k}} d\xi^*_{\vec{k}'} = \delta(\vec{k} - \vec{k}')$

Apply QSD method to Lindblad master equation

QSD Simulation Setups

For simplicity, in one spatial dimension

Parameter setups in heavy quark mass unit

Δx	Δt	N_x	T	γ	$l_{\rm corr}$	α	m_D	$r_{\rm c}$
$1/M$	$0.1M(\Delta x)^2$	254		T/π	$1/T$	0.3	$2T$	$1/M$

correlation function

$$D(r) = \gamma \exp(-r^2/l_{\rm corr}^2)$$

Debye screening potential

$$V(r) = -\frac{\alpha}{\sqrt{r^2 + r_c^2}} \mathrm{e}^{-m_D r}$$

note: in SU(3) $C_F V(r)$

- Fixed temperature case

$$T = 0.1M,\ 0.3M$$

- Bjorken expanding QGP case

$$T(t) = T_0 \left(\frac{t_0}{t + t_0} \right)^{1/3}$$

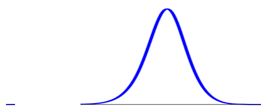
$$T_0 = 470\,\mathrm{MeV}$$
$$t_0 = 0.84\,\mathrm{fm}$$

QSD Simulation Outline

For simplicity, in one spatial dimension

Outline of numerical calculations

initial state → QSD evolution event by event → occupations

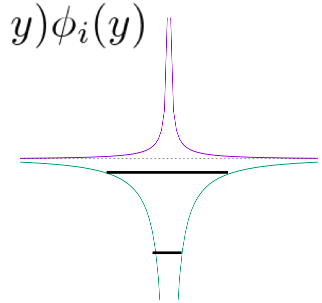


singlet eigenstate $\phi_i(x)$

$$H = \frac{p^2}{M} + V_{\text{Debye}}$$

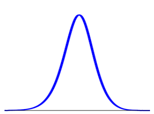
$$N_i = \int dxdy\, \phi_i^*(x)\rho(x,y)\phi_i(y)$$

$$H = \frac{p^2}{M} + V_{\text{Debye}}$$



✓ Bjorken expanding QGP case

initial state → QSD evolution event by event → occupations

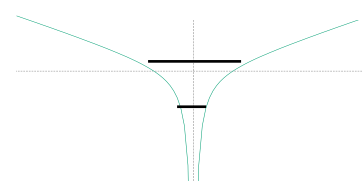


vacuum eigenstate $\phi_i(x)$

$$H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r$$

$$\sigma = 0.01 M_b^2$$

$$H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r$$

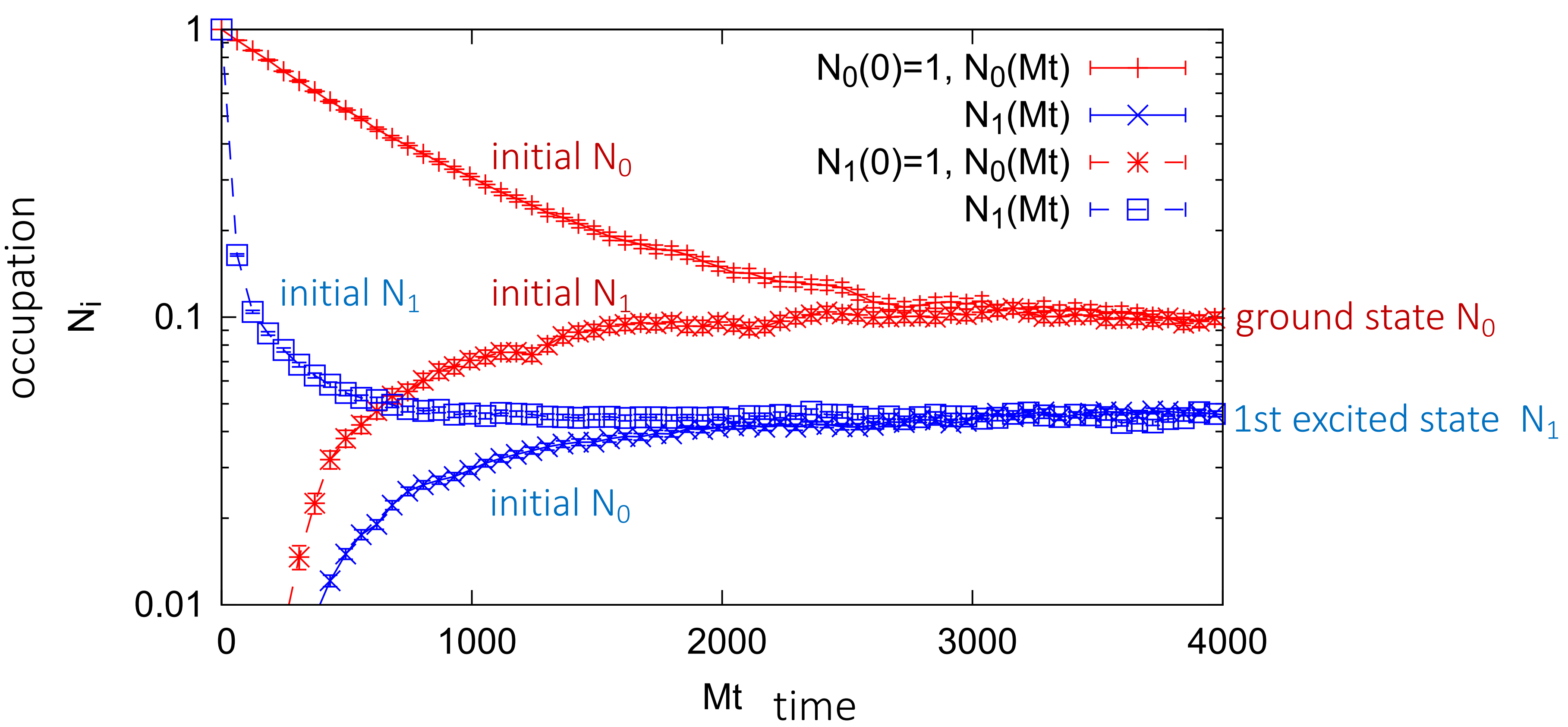


Fixed temperature case

Results - Equilibration(U1)

■ Time evolution of occupation number of eigenstates $H = \frac{p^2}{M} + V_{\text{Debye}}$

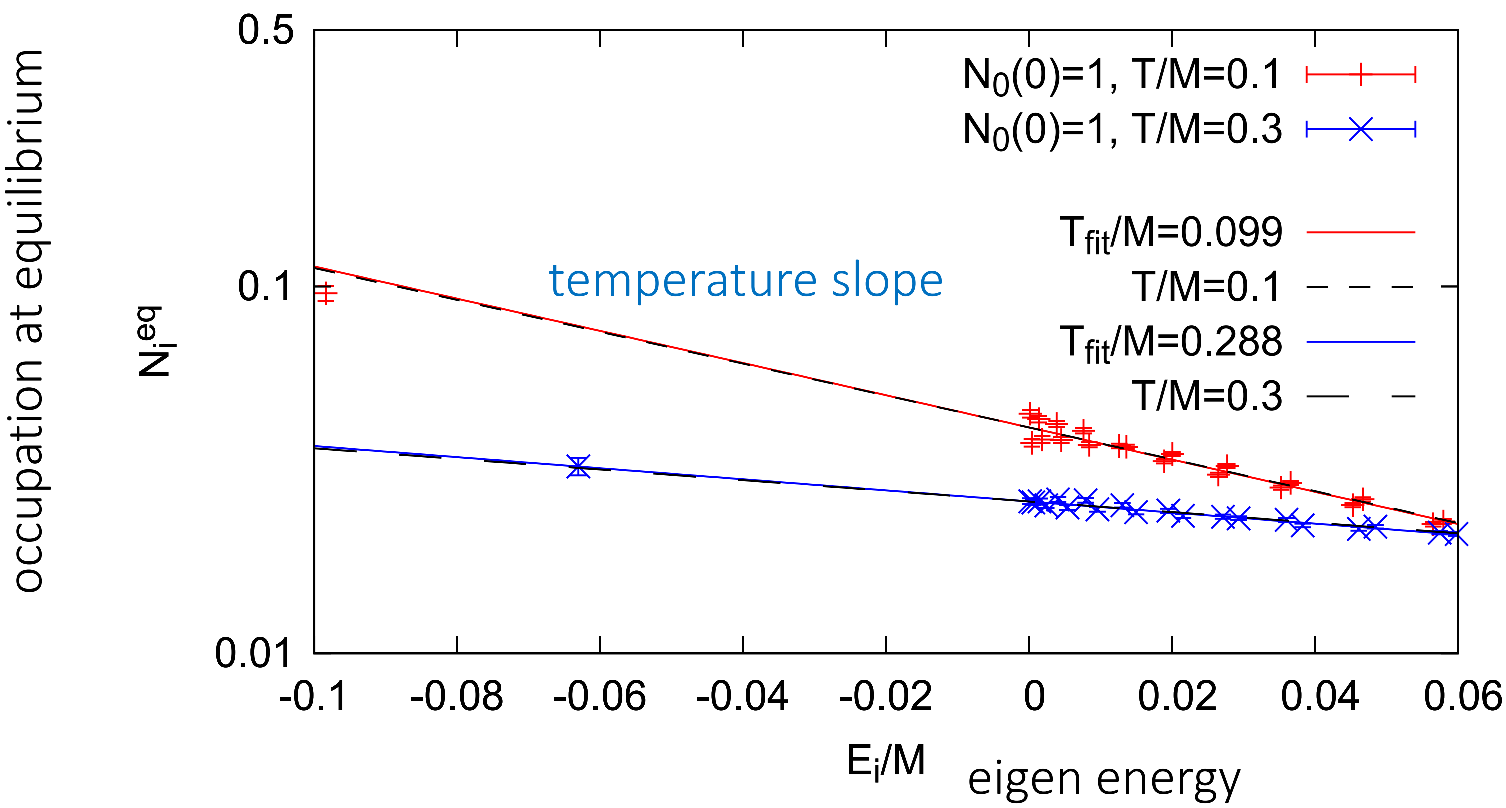
Initial state
ground/1st excited state



Each occupation approaches the value independent of initial state

Results - Distribution at Equilibrium(U1)

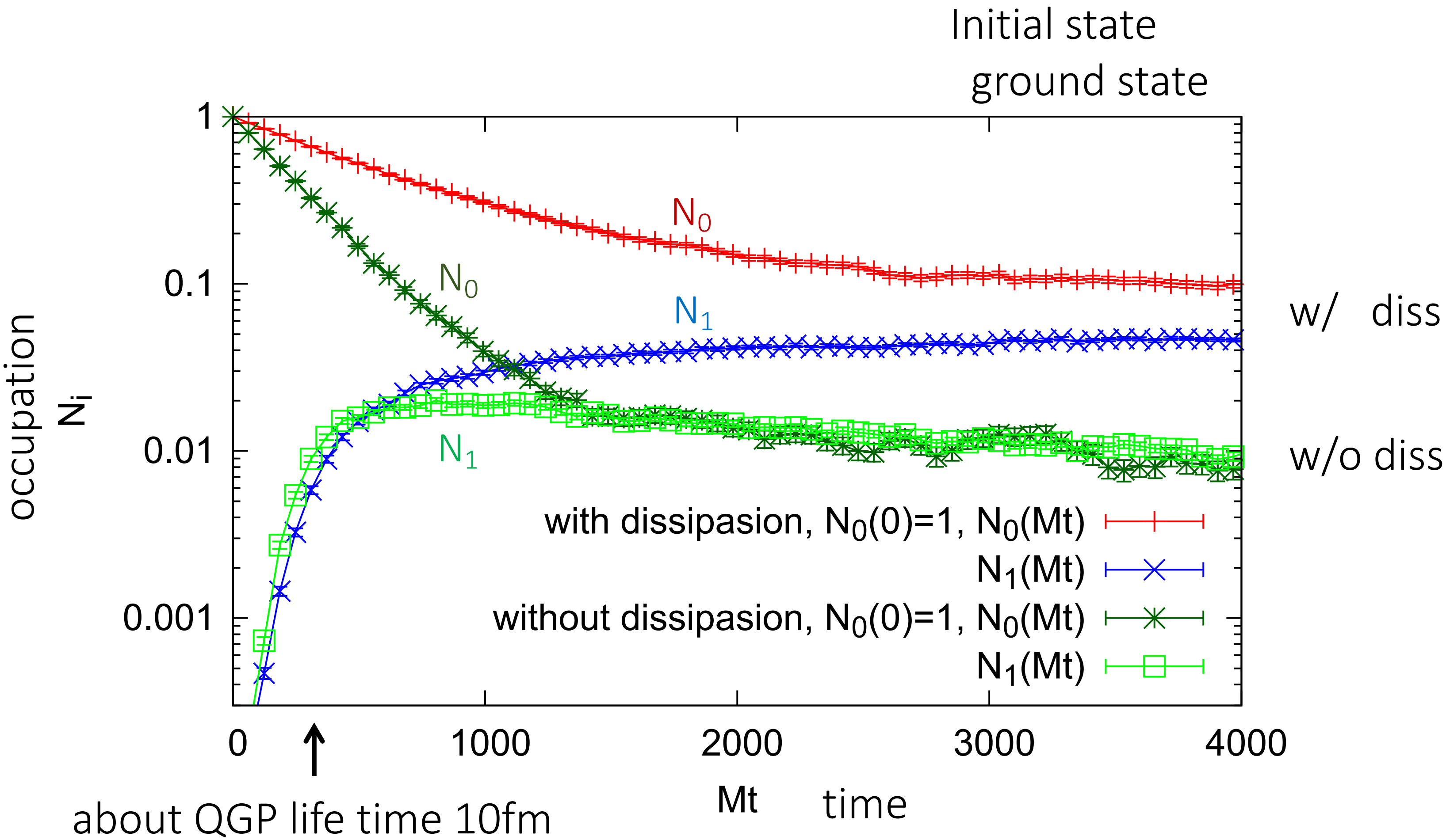
- Eigenstate steady distribution at Mt=4650 (T/M=0.1) $H = \frac{p^2}{M} + V_{\mathrm{Debye}}$
  at Mt=900 (T/M=0.3)



Eigenstate distribution approches the Boltzmann distribution

Results - Dissipative Effect(U1)

■ Time evolution of occupation number of eigenstates $H = \frac{p^2}{M} + V_{\mathrm{Debye}}$



Without dissipation, occupations are underestimated
Dissipation can be effective in QGP life time

Bjorken expanding QGP case

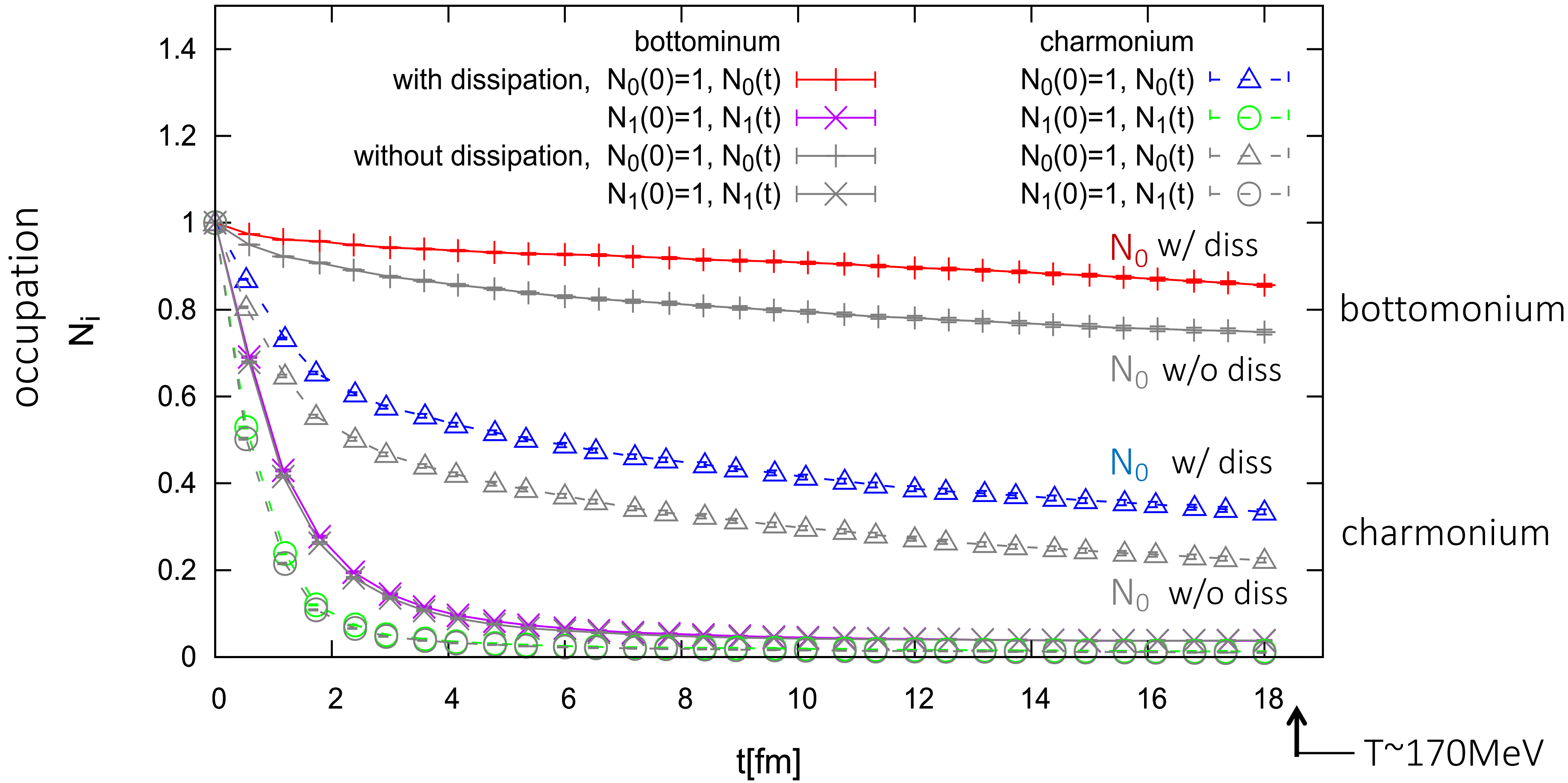
Results - Bjorken Expanding QGP(U1)

- Time evolution of occupations of eigenstates

$$H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r$$

T_{const}/M=0.1 → bottom mass
=0.3 → charm mass

Initial state
ground/1st excited state



Dissipative effect is not negligible in short lived QGP

Summary

- Quarkonium as an open system
  - It is described by Lindblad master equation with positivity
  - Lindblad operator with heavy quark color is derived

- Numerical Simulations via Quantum State Diffusion
  - We confirmed thermalization with dissipation
  - Dissipation affects even in short time scale, in Bjorken expanding QGP

Outlook

3D analysis

Comparison to semi-classical description