Why is chemical freezeout at the chiral crossover temperature?

Sushant Kumar Singh
(sushantsk@vecc.gov.in)
Variable Energy Cyclotron Centre, Kolkata

Talk based on
Sourendu Gupta, Jajati K Nayak & SKS, PRD 103, 054023 (2021)

QCD chiral crossover

- A deconfined state of quarks and gluons (popularly called as QGP) is formed when normal nuclear/hadronic matter ($\rho_0 \approx 0.17$ fm^{-3}) is compressed to high densities ($\gtrsim 5\ \rho_0$) or heated to high temperatures ($\sim 10^{12}$ K).
- The phase change from QGP to hadrons is a smooth crossover at $\mu_B = 0$ characterized by breaking of approx chiral symmetry.
- Chiral crossover temperature $T_{CO} \approx 155$ MeV at $\mu_B = 0$.

Quark-Gluon Plasma in Laboratory

- When heavy nuclei like Au, Pb are collided at relativistic energies, a locally equilibrated fireball of quarks and gluons is formed. Hadrons emitted from the fireball are measured experimentally.
- Large internal pressure makes the fireball expand and cool. Typical timescale $\tau_{exp} \sim 10 - 20$ fm.
- The hadrons may further interact changing the yields and momentum distribution of individual species. Let τ_R denotes the typical timescale over which yields change.
- If $\tau_R < \tau_{exp}$, then chemical equilibrium is maintained.
- If $\tau_R \gtrsim \tau_{exp}$, then system falls out-of-chemical equilibrium. Yields do not change significantly. Chemical Freeze Out occurs at temperature T_{CFO} and chemical potential $\mu_{B,CFO}$.

Hadron Resonance Gas model

- Number and identity of hadrons (yields) are described by HRG.
- Non-interacting gas of hadrons and resonances. The grand canonical partition function is

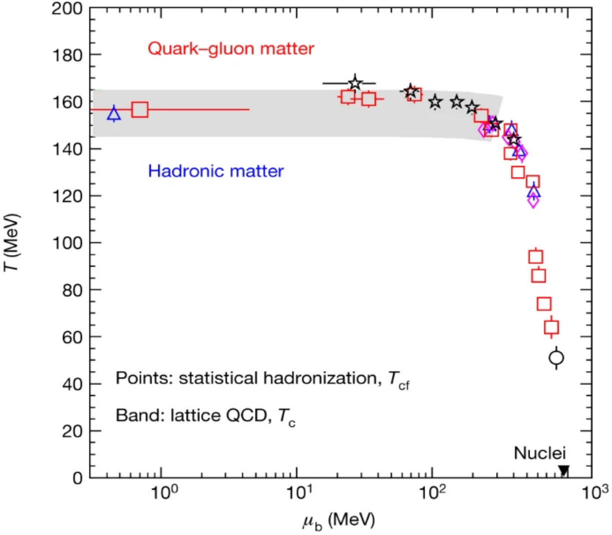
$$\ln Z = \sum_{i=1}^{N} \ln Z_i \quad , \quad \ln Z_i = \pm \frac{V g_i}{2\pi^2} \int dp \; p^2 \ln\left[1 \pm \exp[-(E_i - \mu_i)/T]\right]$$

and

$$N_i = -T \frac{\partial}{\partial \mu_i} \ln Z = \frac{g_i V}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} \quad , \quad \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$

- The yields have been analyzed using the HRG model (NPA 772 (2006) 167-199 and Nature 561, 321-330 (2018)). Mean yields have been fitted fairly successfully. Fitting provides freezeout parameters T_{CFO} and $\mu_{B,CFO}$.

$T_{CO} \approx T_{CFO}$: a coincidence ?



Ref : A. Andronic et al, Nature 561, 321-330(2018)

Setting-up the Solution

- The hadron yields provide information (*i.e.* T_{CFO} and $\mu_{B,CFO}$) only on the last scattering surface.
- To show that $T_{CO} \approx T_{CFO}$, we need to show
  - Chemical relaxation time in the chiral symmetric phase is small. This has been done in PRL 122, 142301 (2019). Typical timescale $\sim 1 - 2.5$ fm.
  - Chemical relaxation time in the chiral broken phase is large.
- In order to understand FO, we concentrate on broken phase and calculate τ_R of a hadron gas which is not very far from equilibrium.
- Set-up a transport theory for hadrons.
- Choice of hadrons ? Lightest : π, Strangeness: K, Flavor symmetry: η. Gives full octet of pseudoscalar mesons.
- For $\mu_B = 0$, $n_p^{\mathrm{eq}}/n_\pi^{\mathrm{eq}} \sim 0.01$ near T_{CO}. Neglect baryons and higher mass hadrons.

Kinetic Theory of Hadrons

- To calculate τ_R, sufficent to consider a fluid at rest.
- System is dilute (Nucl.Phys.B 321 (1989) 387). Hence, use of Classical Boltzmann equation. Also $2 \to 2$ reactions.
- In order to understand freeze out, sufficent to calculate close to equilibrium. Hence, use of linear approximation *i.e.* for i^{th} species $n_i = n_i^{\mathrm{eq}} + \delta n_i$ and keep terms linear in δn_i.
- Equation determining the approach towards equilibrium

$$\frac{d\delta n_a}{dt} = -\sum_r \langle \sigma_r v_{ab} \rangle \left(n_a^{\mathrm{eq}} \delta n_b + n_b^{\mathrm{eq}} \delta n_a \right) + \sum_{\bar{r}} \langle \sigma_{\bar{r}} v_{cd} \rangle \left(n_c^{\mathrm{eq}} \delta n_d + n_d^{\mathrm{eq}} \delta n_c \right)$$

where $\langle \cdot \rangle \equiv$ averaging over the thermal distribution, σ_r denotes cross-section of reactions where a is in initial state, $\sigma_{\bar{r}}$ denotes cross-section of reactions where a is in final state.

Role of Symmetries

- Isospin symmetry : since mass diff. b/w isospin partners $\Delta m \ll T_{CO}$, a reasonable approximation

$$\pi \ : \ (\pi^+, \pi^0, \pi^-) \quad , \quad K \ : \ (K^+, K^0) \quad , \quad \bar{K} \ : \ (\bar{K}^0, K^-)$$

  Instead of 8, only 4 independent densities *i.e.* δn_π, δn_K, $\delta n_{\bar{K}}$, δn_η.
- Strangeness : $S = n_{\bar{K}} - n_K$ conserved
- Accidental conservation : $N = n_\pi + n_K + n_{\bar{K}} + n_\eta$ conserved.
- S and N conservation can be taken into account through following parametrization of δn_i's

$$\delta n_\pi = h_\pi, \ \delta n_\eta = h_\eta, \ \delta n_K = \delta n_{\bar{K}} = -(h_\pi + h_\eta)/2$$

  so that

$$\frac{d}{dt}\begin{pmatrix} h_\pi \\ h_\eta \end{pmatrix} = \mathbb{C}\begin{pmatrix} h_\pi \\ h_\eta \end{pmatrix} \quad , \quad \text{elements of } \mathbb{C} \text{ have dimension } [t]^{-1}$$

Cross-sections from ChPT

- Octet of pseudoscalar mesons : $(\pi^+,\ \pi^-,\ \pi^0,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ \eta)$ forms the Goldstone bosons of chiral symmetry breaking of QCD $(SU(3)_L \times SU(3)_R \to SU(3)_V)$.

- Chiral symmetry constrains the interactions among the pseudo-scalar mesons. At leading order

$$\mathcal{L} = \frac{f_0^2}{4} \mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U + M_0(U + U^\dagger)\right]$$

  where $U = \exp(i\sqrt{2}\Phi/f_0)$ and Φ is a SU(3) matrix containing the 8 meson fields and $M_0 = \mathrm{diag}(M_{0\pi}^2, M_{0\pi}^2, 2M_{0K}^2 - M_{0\pi}^2)$.

- NLO ChPT scattering-amplitudes have been computed in Ref. PRD65, 054009 (2002). Lagrangian contains 8 LECs.

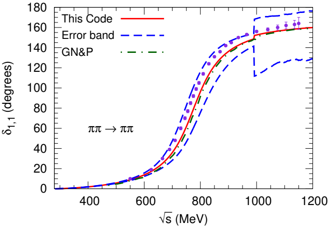
- Unitarity $(SS^\dagger = \mathbb{I})$ enforced in different (I, J) channels through Inverse Amplitude Method. At every mass threshold, number of states increases by one. So dimension of S-matrix increases by one. So discontinuities in amplitudes at thresholds.

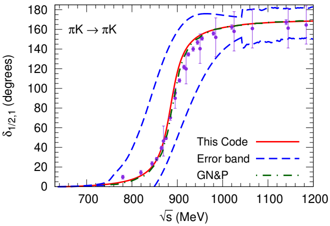
Some features of the Calculation

- Calculation involves 12 input parameters :

$$m_\pi,\; m_K,\; m_\eta,\; f_\pi,\; L_1, \ldots, L_8$$

- Error uncertainties due to L_i's only. Errors in others negligible.
- Dynamical generation of resonances in Unitarized ChPT. Scalar and vector resonances till 1.2 GeV are reproduced. Need not include resonances as explicit degrees of freedom.





- **No UV cutoff needed** as the amplitudes are unitarized.

Results - I

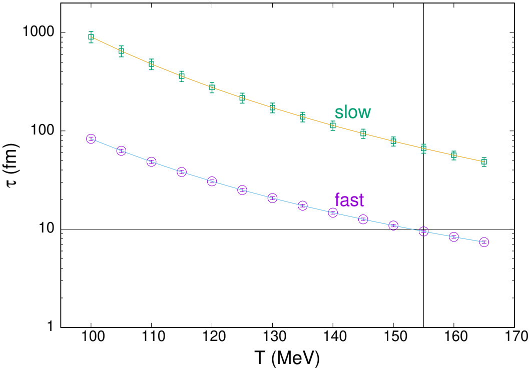


Figure: Relaxation time of normal modes as a function of the temperature. Ref: PRD 103, 054023 (2021)

- The slow mode has a relaxation time of 100 fm at 150 MeV and 1000 fm at 100 MeV.
- The fast mode has a relaxation time of 10 fm at 150 MeV and 100 fm at 100 MeV.

Results - II

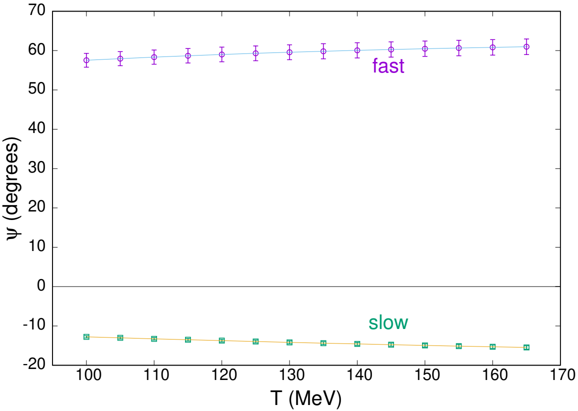


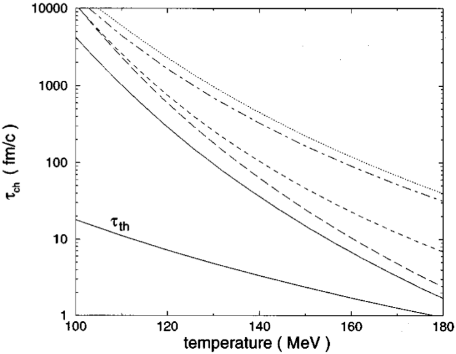
Figure: Angle between normal modes and pion direction shown as a function of the temperature. Ref: PRD 103, 054023 (2021)

- The slow mode is dominantly π's.
- The fast mode is dominantly η's.

Earlier studies in this direction

T (MeV)	$\tau^{(A)}$ (fm)	$\tau^{(B)}$ (fm)
100	2200, 1200	1060, 470, 20, 8
	1300, 700	1030, 280, 20, 8
120	260, 130	165, 65, 15, 8
	200, 60	140, 40, 15, 8
140	65, 25	50, 17, 9, 7
	50, 13	35, 15, 8, 5.5
160	25, 9	22, 10, 8, 4
	17, 4	12, 9, 8, 4
180	10, 4	13, 9, 7, 4
	7, 2	9.5, 7, 5, 4
200	5, 2	11, 9, 6, 4
	4, 1	9, 7, 4, 3

PLB319 (1993) 401



PRC55, 3026 (1997)

- Relaxation time at high temperatures underestimated due to unphysical inputs.

Goldstone Physics

- Goldstone bosons are non-interacting. This means in the limit of exact chiral symmetry $\pi, \ K, \ \eta$ do not interact.
- Explicit breaking of symmetry gives rise to interaction proportional to symmetry breaking parameter. Here the symmetry breaking parameters are masses.
- Fast mode being η and slow mode being π suggests Goldstone behaviour.
- Best check should be to vary the masses and see whether $m \to 0$ gives $\tau_R \to \infty$. Unfortunately this cannot be done as LECs are fixed.
- Another check will be to remove η from the system. The relaxation time must increase. This we have checked and results consistent with Goldstone physics.

Naturalness

- Define the dimensionless ratio $\Pi = \tau(T) n^{\rm eq}(m, T) m^2/(4 f_\pi^4)$.
- For slow mode ($m = m_\pi$ and $\tau = \tau_s$): Π is of order unity (changes from 1 to 2 in the temperature range 100-150 MeV).
- With our values of τ_s, we find that $1/\tau_s \approx \sigma n^{\rm eq}(m_\pi, T)$ with $\sigma = m_\pi^2/(4 f_\pi^4) \approx 25$ mb.
- *The result agrees with kinetic theory arguments !!!*
- This simple result is only obtained when the complexity of NLO ChPT and meson resonances are included.
- For fast mode ($m = m_\eta$ and $\tau = \tau_f$): Π is of order unity (changes from $2/3$ to $4/3$ in the temperature range 100-150 MeV).

Is $T_{CFO} \approx T_{CO}$?

- Slow mode relaxation time is 100 fm near T_{CO} much larger that τ_{exp} typically 10-20 fm. Meson gas cannot remain in chemical equilibrium. This together with "short equilibration time in chiral symmetric phase" suggests $T_{CFO} \approx T_{CO}$.

- Inclusion of baryons will modify the slow mode relaxation time as

$$\frac{1}{\tau_s'} \approx \sigma_{\pi\pi} n_\pi^{\mathrm{eq}} + \sigma_{\pi N} n_N^{\mathrm{eq}}$$

  At low energies, $\sigma_{\pi N}/\sigma_{\pi\pi} \simeq 2$ and whereas $n_N^{\mathrm{eq}}/n_\pi^{\mathrm{eq}}$ varies between 0.001 and 0.01 when T changes from 100 to 150 MeV. Therefore, the effect of adding baryons will be a few percent.

- Future calculation to include baryons are being undertaken but our results and arguments give a strong indication that we understand the basic physics of chemical freezeout.

Conclusions

- Constructed a kinetic theory of yields using pseudoscalar mesons and unitarized cross sections from NLO ChPT. This takes care of the fact that pseudoscalar mesons are pseudo-Goldstone bosons of chiral symmetry breaking in QCD.
- Neglected baryons. Although πN cross sections are similar to $\pi\pi$ cross sections but densities are 2 to 3 orders of magnitude smaller.
- Interactions are completely fixed by chiral symmetry but due to low densities relaxation times are large.
- Clear physical reason why fireball cannot be in equilibrium after chiral symmetry breaking.

Thank You !!!