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Stefano Profumo
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz

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Neutron Star Quantum Death by Small Black Holes

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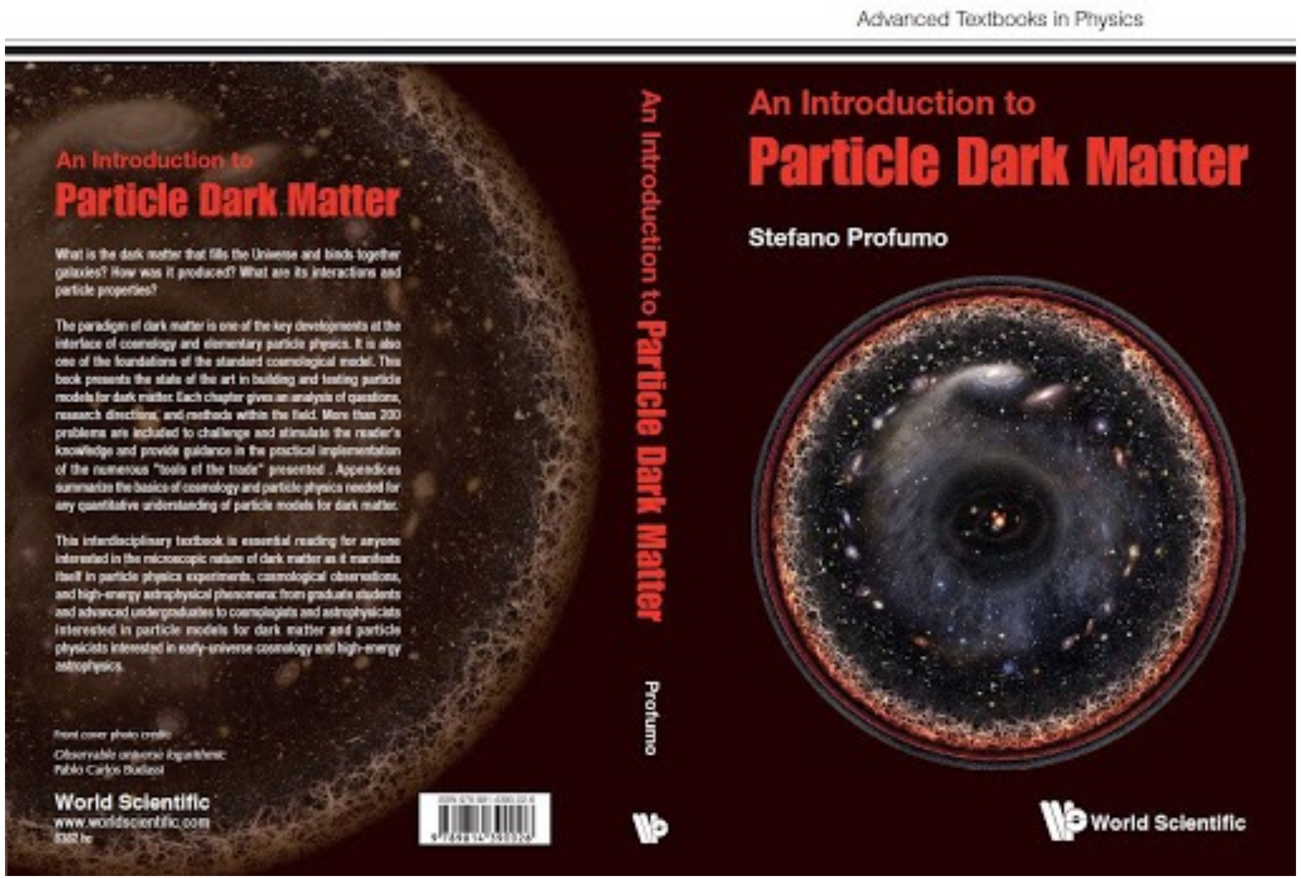
Most particle models seek to explain the **genesis** of **dark matter** as a result of **freeze-out**

If the dark matter pair annihilates via ~ **weak-interaction** cross sections,

its thermal **relic abundance is ~ right**, and the particle is sufficiently **cold**

Late universe **annihilation** offers a way to **search** for dark matter today



Late universe **annihilation** offers a way to **search** for dark matter today

Ordinary (**baryonic matter**) was not born out as a result of a “freeze-out” process

but, rather, from a **matter-antimatter** asymmetry

Similarly, the dark matter could originate from a (**dark sector**) **asymmetry**

If so, (asymmetric) dark matter can be completely **elusive**,

bar an (**unnecessary**) scattering cross section off of ordinary matter

Much more promising than direct detection (i.e. operative for much

weaker scattering cross sections) is the process of **destruction of**

neutron stars that asymmetric dark matter could trigger

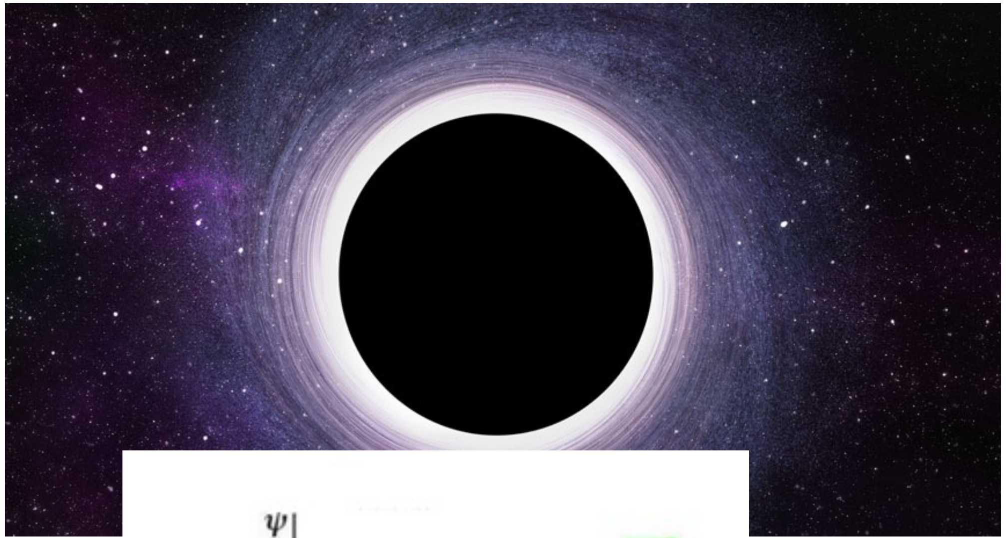
Dark matter can be **captured** in celestial bodies via repeated scattering off of ordinary matter

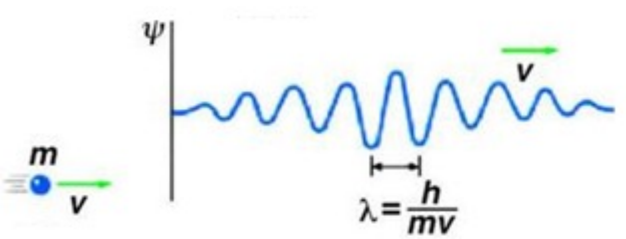
If dark matter **cannot pair-annihilate**, it will **accrete** over time, possibly thermalize, possibly form a Bose-Einstein condensate, and possibly eventually **collapsing** into a **black hole**

the **dark matter-triggered black hole** can **destroy** the neutron star

the very **existence** of (old, ~ 10^{10} yr) **neutron stars** in **dark matter-rich regions** therefore constrains (asymmetric, or non-annihilating) dark matter

similarly, accretion of **macroscopic dark matter** with large scattering cross sections off of matter can also trigger the formation of black holes before discussing the black hole formation process in detail, let me note the following: what if the **black hole size** (its Schwarzschild radius) is **smaller** than the size of the **neutron** star constituents?





the description of matter accretion must necessarily use **quantum mechanics**

and **not** (classical) **fluid dynamics** (massless point particles)!

Central question: how **big** is the **black hole** that forms inside a NS?

1. number of **accreted** dark matter particles
2. gravitational vs degeneracy pressure – **collapse**

1. number of accreted dark matter particles

in theory, complicated process where **orbits shrink** passage after passage till they are confined inside the NS and thermalize

in practice, particles are (**eventually**) **accreted** as long as they scatter once

there is a **critical cross section** such that the mean free path is the same size as the NS radius…

$$\lambda = \frac{1}{\sigma \cdot n_n} = \frac{m_n}{\sigma \rho} \qquad \rho = \frac{M}{\frac{4\pi}{3} R^3} \qquad \sigma_{\rm crit} = \frac{3 m_n R^2}{4\pi M} \simeq 10^{-45}\ {\rm cm}^2$$

1. number of accreted dark matter particles

a subtlety has to do with the fact that baryons in a NS are in a

~**degenerate Fermi gas** momentum distribution

$$p_F = \hbar \left(3\pi^2 n_n\right)^{1/3} \simeq 0.2 \text{ GeV}$$

if the momentum transfer to the neutron is larger than p_F ,

the scattered neutron can be **excited** above the Fermi surface.

On the other hand, if the momentum transfer δp is less than p_F , only those neutrons with momentum larger than $\sim p_F - \delta p$ can **participate** in the capture process. The fraction of these neutrons is $\sim \delta p/p_F$, so we can approximate ξ as $\xi \simeq \delta p/p_F$

1. number of accreted dark matter particles

$\xi \simeq 1$ for all $m_X > 1$ GeV. In contrast, the capture rate is suppressed by a factor $\sim m_X v_{esc}/p_F$ if the **DM mass smaller than the neutron mass**.

On the other hand, if the momentum transfer δp is less than p_F , only those neutrons with momentum larger than $\sim p_F - \delta p$ can **participate** in the capture process. The fraction of these neutrons is $\sim \delta p/p_F$, so we can approximate ξ as $\xi \simeq \delta p/p_F$

1. number of accreted dark matter particles

$$m_X \gtrsim 1 \text{ GeV}$$

$$N_X \simeq 2.3 \times 10^{44} \left(\frac{100 \text{ GeV}}{m_X}\right) \left(\frac{\rho_X}{10^3 \text{ GeV/cm}^3}\right) \left(\frac{\sigma_{XB}}{2.1 \times 10^{-45} \text{ cm}^2}\right) \left(\frac{t}{10^{10} \text{ years}}\right)$$

$$\sigma_{XB} = \text{Min}\left[\sigma_n, \sigma_{max}\right]$$

$$m_X \lesssim 1 \text{ GeV}$$

$$N_X \simeq 3.4 \times 10^{46} \left(\frac{\rho_X}{10^3 \text{ GeV/cm}^3}\right) \left(\frac{\sigma_{XB}}{2.1 \times 10^{-45} \text{ cm}^2}\right) \left(\frac{t}{10^{10} \text{ years}}\right)$$

2. gravitational vs degeneracy pressure – collapse

The critical particle number *N* that triggers **gravitational collapse** depends on the **spin** of the dark matter. In the case of fermions, the onset of the gravitational collapse occurs when the potential energy of the dark matter exceeds the Fermi energy, and therefore **Pauli blocking** cannot prevent the collapse anymore:

$$\frac{GNm^2}{r} > E_F = \left(\frac{3\pi^2 N}{V}\right) = \left(\frac{9\pi}{4}\right)^{1/3} \frac{N^{1/3}}{r},$$

$$N^f = \left(\frac{9\pi}{4}\right)^{1/2} \left(\frac{M_{\rm Pl}}{m}\right)^3 \qquad M^f = N^f m \frac{M_{\rm Pl}^3}{m^2} = \simeq 9 \times 10^{30} \left(\frac{\rm GeV}{m}\right)^2 \ {\rm kg}.$$

…collapse happens at this mass, as long as enough particles are **accreted**

2. gravitational vs degeneracy pressure – collapse

In the case of **bosons**, the energy for a particle is

$$E \sim -\frac{GNm^2}{R} + \frac{1}{R},$$

the second term from the **zero-point energy** due to the uncertainty principle. As a result, the critical number of particles is here

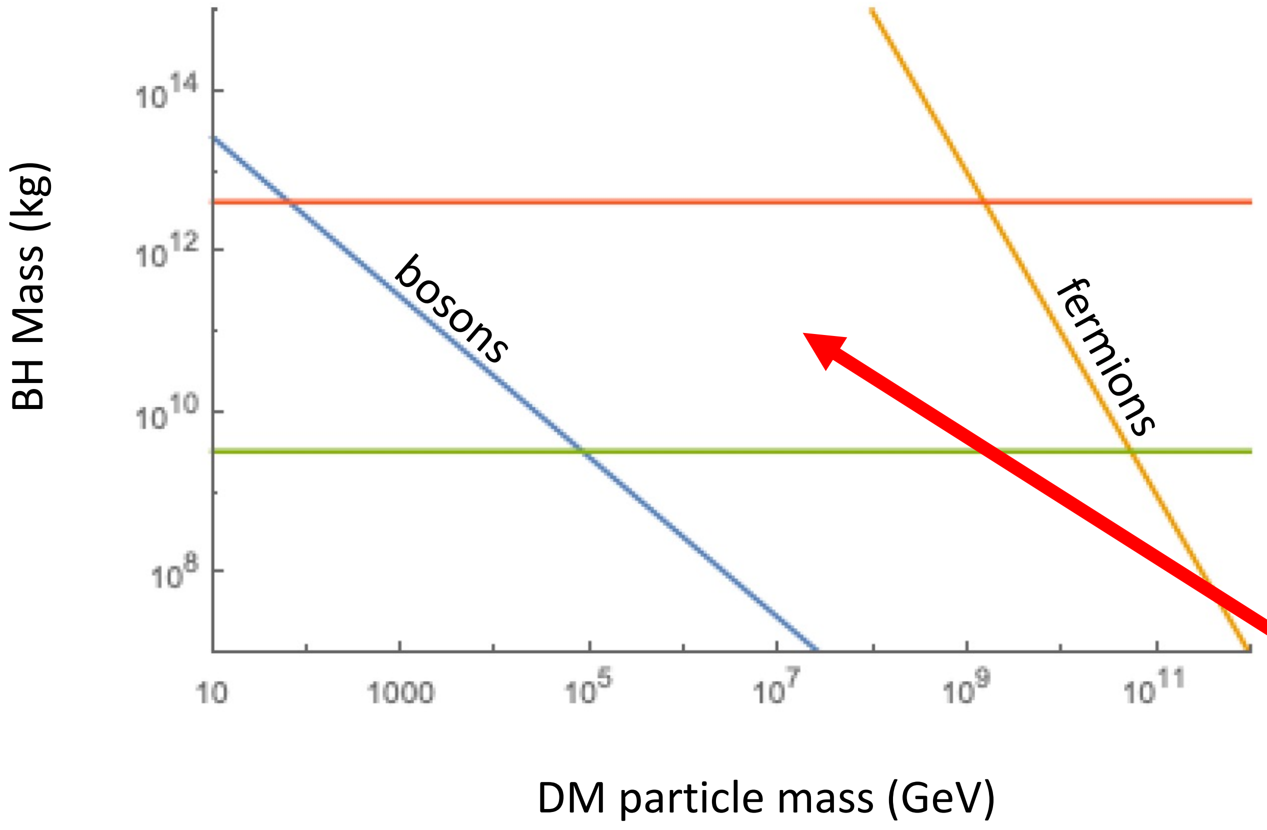
$$N^b \sim \left(\frac{M_{\rm Pl}}{m}\right)^2$$

and the **black hole** mass

$$M^b = N^b m = \frac{M_{\rm Pl}^2}{m} =\simeq 3 \times 10^{14} \frac{\rm GeV}{m}\ {\rm kg}.$$

…again collapse happens at this mass, as long as enough particles are **accreted**

2. gravitational vs degeneracy pressure – collapse



$$R_{\text{Schw}} = 2GM_{\text{BH}} = \lambda_{DB} = \frac{2\pi}{p_F}$$

$$M_{\text{Unruh}} \equiv \frac{\pi M_P^2}{p_F} = 4.1 \times 10^{12}\ \text{kg}.$$

neutrons are

"bigger" than the

black hole!

How does **accretion** onto a black hole work inside a neutron star?

The **Bondi-Hoyle** absorption cross section generalizes the classical **Hoyle-Lyttleton** result for the accretion of massless point particles of density ρ by a star of mass M moving at a steady asymptotic speed v

$$\left(\frac{dM}{dt}\right)_{\mathrm{HL}} = \pi\zeta_{\mathrm{HL}}^2 v\rho = \frac{4\pi G^2 M^2 \rho}{v^3},$$

where ζ_{HL} is the Hoyle-Lyttleton radius, corresponding to the maximal impact parameter yielding capture. Augmenting the Hoyle-Lyttleton treatment with fluid effects, but maintaining the assumption that the accreted particles be **massless** and **point-like**, gives

$$\left(\frac{dM}{dt}\right)_{\mathrm{BH}} = \frac{4\pi\lambda_s(\gamma) G^2 M^2 \rho}{(c_s^2 + v^2)^{3/2}}.$$

In the limit where the particles being accreted are **neither massless** (rather, they have mass m) **nor point-like** and possess a **quantum wavelength** (de Broglie wave- length) **larger than the Schwarzschild radius** of the accreting mass *M*, the absorption cross section was computed by **Unruh** in 1976

$$\left(\frac{dM}{dt}\right)_{\mathrm{U}} = \sigma_U(M,m,v)\rho v,$$

$$\sigma_U(M,m,v) = \frac{2\pi G^2 M^2}{v}\frac{\xi}{1-e^{-\xi}}$$

$$\xi = 2\pi GMm\frac{1+v^2}{v\sqrt{1-v^2}} = \pi\frac{1+v^2}{v^2\sqrt{1-v^2}}\frac{R_{\mathrm{Schw}}}{\lambda_{\mathrm{DB}}}$$

$$R_{\mathrm{S}}/\lambda_{\mathrm{DB}} \ll 1$$

$$\left(\frac{dM}{dt}\right)_{\mathrm{U}}(M) = m_n n_n \int_0^1 dv f_F(v) \sigma_U(M, m_n, v) c(\hbar c)^2$$

The key assumption is that the particles being absorbed are falling into the black hole **undisturbed** as plane waves

The black hole mass at which we expect the breakdown of the assumption that there exists an "**infinite reservoir**" of neutrons inflowing and scattering off the black hole corresponds to the mass for which accretion rates are comparable with the neutron-neutron scattering rate (that fuels the accreting neutrons)

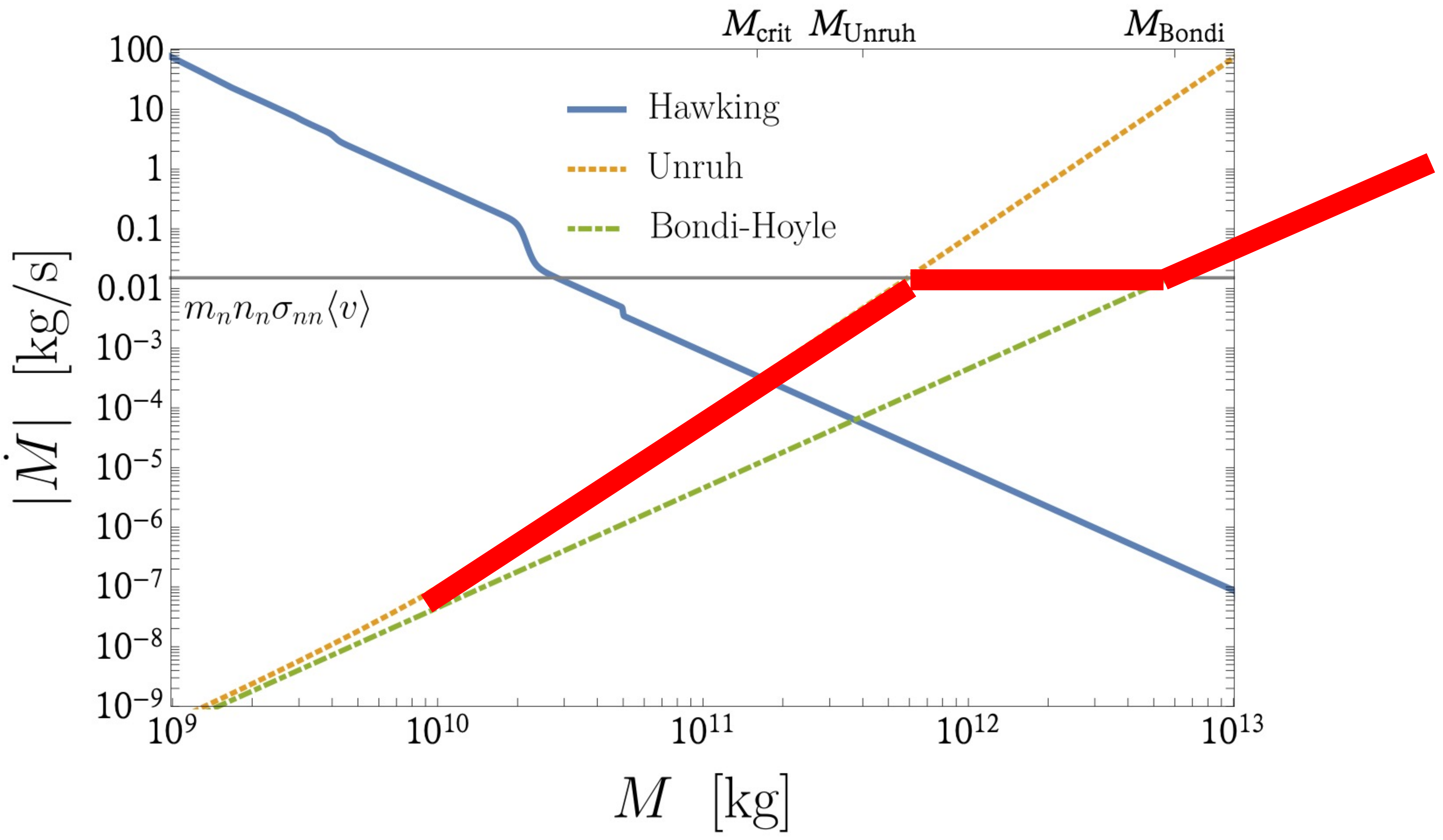
$$\dot{M}_{\mathrm{U}} \simeq m_n n_n \langle \sigma_{nn} v \rangle$$

For larger black hole masses, effectively one must impose momentum and mass conservation, as in the **Bondi-Hoyle** picture;
correspondingly, the hole's radius is large enough that **wave effects** can be neglected.

For $M > M_{Unruh}$ we therefore conclude that the **Bondi picture** is warranted and qualitatively correct.

$$\dot{M}_{\rm acc} = \max\left[\dot{M}_{\rm BH}, \min(\dot{M}_{\rm U}, m_n n_n \langle \sigma_{nn} v \rangle)\right]$$

note that the **classical limit** of the Unruh rate is **not** the Bondi-Hoyle rate,
as the two pictures make qualitatively **different assumptions**!



$M_{\rm crit}$
$M_{\rm Unruh}$
$M_{\rm Bondi}$
Hawking
Unruh
Bondi-Hoyle
$m_n n_n \sigma_{nn} \langle v \rangle$
$|\dot{M}|$ [kg/s]
M [kg]
100
10
1
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}

In addition to accretion, the black hole mass changes because of **Hawking evaporation**, at a rate

$$\left(\frac{dM}{dt}\right)_{\rm H}(M) \simeq -5\times 10^{16} f(M)\left(\frac{\rm kg}{M}\right)^2 \frac{\rm kg}{\rm s},$$

where f(M) is a function of the degrees of freedom **kinematically available** for evaporation: only those particles for which the Hawking temperature T_H>m, where m is the particle that the black hole evaporates into, can be produced by the black hole.

For M ~ 10^9 kg, T_H ~ 10 GeV and f(M)≃15, while for M ~10^{13} kg, T_H ~1 MeV and f(M) ≃ 2.

$$M(t) = \int_{t_0}^{t} dt \left[\left(\frac{dM}{dt} \right)_{\text{acc}} + \left(\frac{dM}{dt} \right)_{\text{H}} \right]$$

$$M_{\text{crit}} \simeq 1.6 \times 10^{11} \text{ kg}$$

$$\tau_{\text{evap}}(M) \simeq 8 \times 10^{9} \text{ sec } \left(\frac{M}{10^{10} \text{ kg}} \right)^{3}, \quad (M < M_{\text{crit}})$$

Note that unlike the case of evaporation of a black hole inside the Earth or the Sun, evaporation inside a NS is **not expected to yield any observable signature**: comparing the rest-mass energy of the largest hole that would evaporate quicker than accrete, $M \sim M_{crit} \simeq 8 \times 10^{34}$ ergs, with the lower limit to the specific heat of a NS, $c_{NS} \sim 2 \times 10^{36}$ ergs/K: the deposited heat would **never yield a detectable temperature change** to the NS.

Nevertheless, it is possible that this sudden deposition of energy in the NS core will have a transient effect such as a **glitch**.

We also estimate that the neutrino mean free path inside a NS is too short for **neutrinos** to escape

$$\lambda_\nu \simeq \frac{1}{n_n \sigma_{n\nu}} \simeq \frac{1}{n_n G_F^2 E_\nu^2} \simeq 2 \times 10^{-8}\ \mathrm{cm} \left(\frac{\mathrm{GeV}}{E}\right)^2$$

so that the predicted flux would be **too small** to be detectable above the atmospheric neutrino background

For initial black hole masses larger than M_{crit}, we can determine the neutron star **lifetime** via

$$\tau(M_0) = \int_{M_0}^{M_{\rm NS}} \frac{dM}{\left(\frac{dM}{dt}\right)_{\rm acc} + \left(\frac{dM}{dt}\right)_{\rm H}}$$

$$\frac{\tau(M)}{\tau_{\rm NS}} \simeq \begin{cases} 0.2\left(\frac{10^{10}{\rm kg}}{M}\right)^2 & M_{\rm crit} < M < M_{\rm Unruh} \\ 6\times 10^9{\rm kg}/M & M > M_{\rm Unruh} \end{cases}$$

where we have taken the typical NS age to be **$\tau_{NS} \simeq$ 10 Gyr**.

Because $M_{crit} \sim 10^{11}$ kg, the **neutron star destruction time is shorter than τ_{NS}** if the black hole mass is sufficiently large

…back to the black hole masses expected in given **particle physics** models…

barring strong self-interactions, the **fermionic** prediction is rather generic

the bosonic prediction is significantly more model dependent,
with important effects from **self-interactions** and **BEC** formation

$$E \sim -\frac{GNm_b^2}{R} + \frac{1}{2m_b R^2} - \frac{\lambda N}{32\pi m_b^2 R^3}$$

(positive: attractive)

$$N_{\rm max}^b = \left(\frac{M_{\rm Pl}}{m_b}\right)^2 \sqrt{\frac{17}{20}\left(1 - \frac{3\lambda M_{\rm Pl}^2}{34\pi m_b^2}\right)}.$$

$$M_{\rm max}^b \simeq 2.5 \times 10^{14}\ {\rm kg}\,\frac{\rm GeV}{m_b}\sqrt{1 - 4\times 10^{36}\lambda\left(\frac{\rm GeV}{m_b}\right)^2}.$$

Accumulated bosonic dark matter can form a **Bose-Einstein** condensate (BEC).

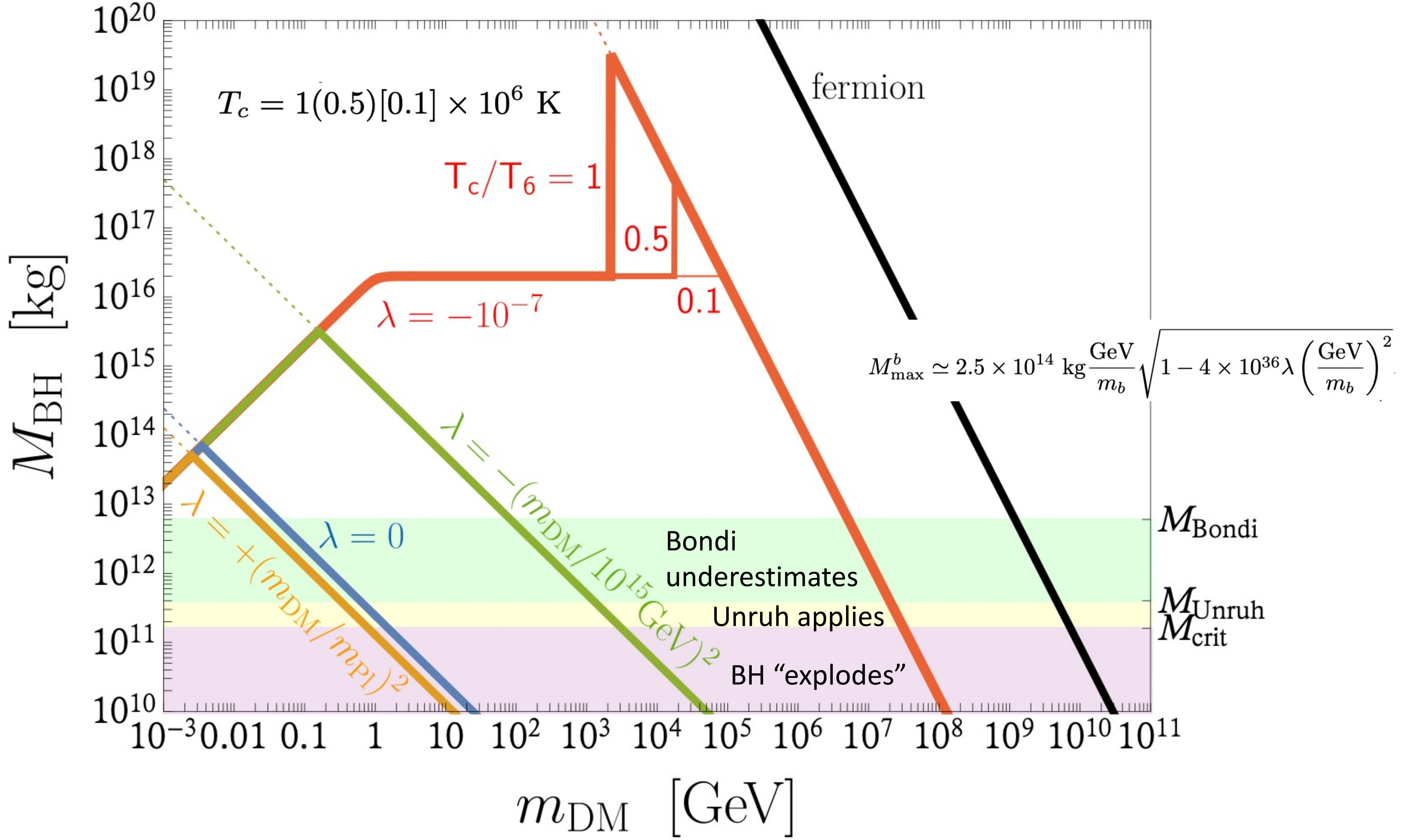
This can trigger black hole formation from the **condensate sub-component** of the dark matter rather than the entire thermal population.

The **fraction** of dark matter particles in the BEC if the star is below the critical temperature is

$$N_{\rm BEC}/N^b = \Theta(T_{\rm crit} - T_c)[1 - (T_c/T_{\rm crit})^{3/2}]$$

$$T_{\rm crit} = \frac{2\pi}{m}\left[\frac{3N^b}{4\pi\zeta(3/2)r_{\rm th}^3}\right]^{2/3} \qquad r_{\rm th} \propto \sqrt{T_c/m_X}$$

$$g_{\rm BEC} = \frac{\rho_X}{\rm GeV/cm^3}\frac{\sigma_{XN}}{10^{-45}{\rm cm}^2}\frac{t}{\rm 10Gyr} \qquad M_{\rm BEC}^b \simeq 2\times 10^{16}\min\left(\frac{m}{\rm GeV},1\right) g_{\rm BEC}\,{\rm kg}$$


$T_c = 1(0.5)[0.1] \times 10^6$ K
fermion
$T_c/T_6 = 1$
0.5
0.1
$\lambda = -10^{-7}$
$M^b_{\max} \simeq 2.5 \times 10^{14}\ \mathrm{kg}\frac{\mathrm{GeV}}{m_b}\sqrt{1 - 4\times 10^{36}\lambda\left(\frac{\mathrm{GeV}}{m_b}\right)^2}$
$\lambda = -(m_{\mathrm{DM}}/10^{15}\,\mathrm{GeV})^2$
$\lambda = 0$
$\lambda = +(m_{\mathrm{DM}}/m_{\mathrm{Pl}})^2$
Bondi underestimates
Unruh applies
BH "explodes"
M_{Bondi}
M_{Unruh}
M_{crit}
M_{BH} [kg]
m_{DM} [GeV]

Concluding remarks

When the quantum size of neutrons exceeds the Schwarzschild radius of a black hole at the center of a neutron star, **accretion** cannot be described with the **Bondi-Hoyle** picture; rather, it should be described by an appropriate **cross section** that accounts for both the space-time geometry of the black hole, and the **quantum nature** of the particles being accreted.

We **corrected** the predictions for neutron star destruction by black holes formed by non-annihilating dark matter accumulating at the neutron star interior using the **correct capture cross section** for light black holes. While the key results in the existing literature are not dramatically affected, we find a significant **change in the minimal mass necessary to prevent black hole evaporation**, and in the predicted **neutron star lifetime**.

Open Questions

- Fermion accretion onto Schwarzschild black holes at **finite temperature** and **chemical** potential

- **Intermediate** regime between Unruh and Bondi?

- What happens to the cross section outside the **Unruh range $R_S \ll \lambda_{DB}$**?

- Are accreting neutrons really **plane waves**? [no!]

- Any hope to **detect** a BH "**explosion**" inside a NS? what about other celestial bodies?