Verification of pairwise non-locality trade-off in pure symmetric 3-qubit states using the IBM open access quantum computer ibmq_lima

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Violation of Bell inequality reveals inherent non-locality in quantum entangled systems [1]. In particular, the Clauser-Horne-Shimony-Holt (CHSH) inequality [2] may be used to verify pairwise non-locality of constituent two-qubits of multiqubit systems. Yet another essential feature of entangled multiparty systems is monogamy i.e., restriction placed on the shareability of entanglement [3]. Non-local correlations recorded by the violation of CHSH inequalities obey monogamy trade-off relations. Monogamy trade-off relation in the case of 3-qubit states ρ_{ABC} is given by [4]:

$$\mathfrak{M}_{ABC} \equiv \langle CHSH\rangle_{AB}^2 + \langle CHSH\rangle_{BC}^2 + \langle CHSH\rangle_{AC}^2 \leq 12$$

where $\langle CHSH\rangle_{AB} = \langle A_1 \otimes B_1\rangle + \langle A_1 \otimes B_2\rangle + \langle A_2 \otimes B_1\rangle - \langle A_2 \otimes B_2\rangle$; $\langle A_i \otimes B_j\rangle = Tr\left[\rho_{AB} A_i \otimes B_j\right]$ and $A_i = \vec{\sigma}.\vec{a}_i, B_j = \vec{\sigma}.\vec{b}_j, i,j = 1,2$ are Pauli observables with orientation directions $\vec{a}_i, \vec{b}_j$ of qubits A, B respectively. While violation of the CHSH inequality $|\langle CHSH\rangle_{AB}| < 2$ reveals non-locality, monogamy constraint imposes the trade-off relation $\mathfrak{M}_{ABC} \leq 12$ on 3-qubit states. In the special case of 3-qubit permutation symmetric states for which $\langle CHSH\rangle_{AB} = \langle CHSH\rangle_{BC} = \langle CHSH\rangle_{AC}$, one obtains $\mathfrak{M}_{ABC} = 3\langle CHSH\rangle_{AB}^2 \leq 12$, in turn indicating that $|\langle CHSH\rangle_{AB}| < 2$. Hence one ends up with the monogamy restriction on non-locality: Any arbitrary 2-qubit state extracted from 3-qubit permutation symmetric system cannot violate CHSH inequality, even though the constituent qubits are entangled.

In this work, we verify monogamy relations obeyed by one parameter family of symmetric 3-qubit states[5]: $|\Psi_\beta\rangle = \frac{1}{\sqrt{2+\cos\beta}} \left(|0\rangle \otimes |0\rangle \otimes |\beta\rangle + |\beta\rangle \otimes |0\rangle \otimes |0\rangle + |0\rangle \otimes |\beta\rangle \otimes |0\rangle\right)$, $|\beta\rangle = \cos\frac{\beta}{2}|0\rangle + \sin\frac{\beta}{2}|1\rangle$, $0 < \beta \leq \pi$ (known as W-class states) using open access IBM quantum computer ibmq_belem. A scheme of of the paper is outlined here:

- Building quantum circuit using the IBM open-source software kit Qiskit to prepare the 3-qubit state $|\Psi_\beta\rangle$ for $\beta = {}^{\pi}/_{6}, {}^{\pi}/_{4}, {}^{3\pi}/_{8}, {}^{9\pi}/_{16}, \pi$.
- Preparation the quantum state using ibmq_belem
- Collecting measurement data (based on 8192 statistical trials) and constructing 2-qubit correlation matrices.
- Verification of monogamy relation $\mathfrak{M}_{ABC} \leq 12$

Our results agree with theoretical predictions and establish how shareability places restrictions on CHSH non-locality.

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