Verification of pairwise non-locality trade-off in pure symmetric 3-qubit states using the IBM open access quantum computer

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Majorana construction of 3-qubit pure symmetric state with 2 distinct spinors

$$|\Psi_{3,2}^{ABC}\rangle = \mathcal{N}_{3,2}\sum_{P}\hat{P}\{|0\rangle\otimes|0\rangle\otimes|\beta\rangle\}, \qquad |\beta\rangle = \cos\frac{\beta}{2}|0\rangle + \sin\frac{\beta}{2}|1\rangle,\ 0<\beta<\pi$$

$$= \frac{1}{\sqrt{2+\cos\beta}}\left(\sqrt{3}\cos\frac{\beta}{2}\,|0_A\,0_B\,0_C\rangle + \sin\frac{\beta}{2}\,|\mathrm{W}\rangle\right).$$

$$|\mathrm{W}\rangle = \mathcal{N}\sum_{P}\hat{P}\{|0\rangle\otimes|0\rangle\otimes|1\rangle\}$$

$$= \frac{1}{\sqrt{3}}\left(|0\,0\,1\rangle + |0\,1\,0\rangle + |1\,0\,0\rangle\right)$$

2-qubit reduced state of $|\Psi_{3,2}^{ABC}\rangle$

$$\rho_{\mathrm{R}} = \mathrm{Tr}_A\left(|\Psi\rangle_{\mathrm{sym}}\langle\Psi|\right) = \mathrm{Tr}_B\left(|\Psi\rangle_{\mathrm{sym}}\langle\Psi|\right) = \mathrm{Tr}_C\left(|\Psi\rangle_{\mathrm{sym}}\langle\Psi|\right)$$

$$\rho_{\mathrm{R}} = \frac{1}{4}\left[I\otimes I + \sum_{i=1}^{3} s_i\,(\sigma_i\otimes I + I\otimes\sigma_i) + \sum_{i,j=1}^{3} t_{ij}\,(\sigma_i\otimes\sigma_j)\right]$$

$$s_i = \mathrm{Tr}\left[\rho_R\,(\sigma_i\otimes I)\right] = \mathrm{Tr}\left[\rho_R\,(I\otimes\sigma_i)\right]$$

$$t_{ij} = \mathrm{Tr}\left[\rho_R\,(\sigma_i\otimes\sigma_j)\right] = t_{ji}.$$

$\sigma_i,\ i = 1,\ 2,\ 3$ are the Pauli matrices; I denotes 2×2 identity matrix

Bell-CHSH Inequality

$$\langle CHSH\rangle_{AB} = \langle A_1 \otimes B_1\rangle + \langle A_1 \otimes B_2\rangle + \langle A_2 \otimes B_1\rangle - \langle A_2 \otimes B_2\rangle$$

$$\langle A_i \otimes B_j\rangle = Tr[\rho_{AB} A_i \otimes B_j],\ A_i = \vec{\sigma}.\vec{a}_i, B_j = \vec{\sigma}.\vec{b}_j,\quad i,j = 1,2$$

Pauli observables with orientation directions $\vec{a}_i, \vec{b}_j$ of qubits A, B.

$$\langle A_i \otimes B_j\rangle = \sum_{a_i, b_j = \pm 1} a_i\, b_j\ p(a_i, b_j | A_i, B_j),\quad i,\ j = 1,\ 2.$$

Correlation probabilities evaluated based on the measurement outcomes a_i, b_j of the observables A_i, B_j of Alice and Bob

Maximum value $\langle \mathrm{CHSH}\rangle_{\mathrm{opt}}$

$$\langle \mathrm{CHSH}\rangle_{\mathrm{opt}} = 2\sqrt{t_1^2 + t_2^2}$$

$t_1^2, t_2^2 \Longrightarrow$ **two largest eigenvalues of** $T^\dagger T$

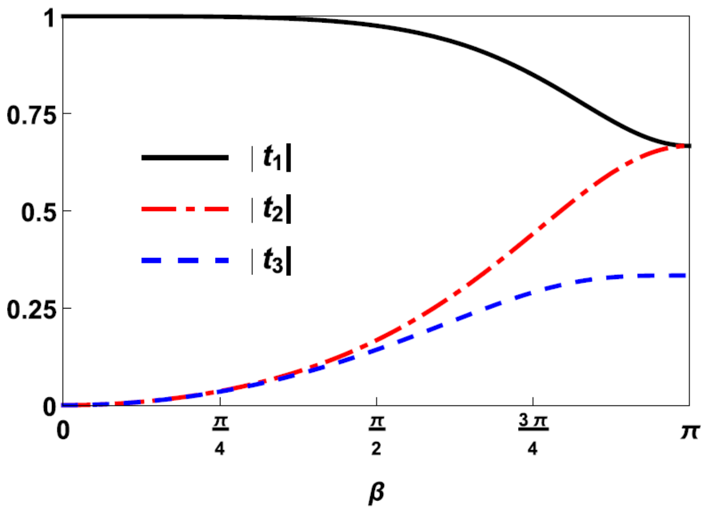
Correlation matrix of ρ_R

$$T = \frac{1}{3(2+\cos\beta)} \begin{pmatrix} 1-\cos\beta & 0 & 3\sin\beta \\ 0 & 1-\cos\beta & 0 \\ 3\sin\beta & 0 & 4+5\cos\beta \end{pmatrix}$$

$$t_1 = \frac{5 + 4\cos\beta + 3\sqrt{5 + 4\cos\beta}}{6(2+\cos\beta)}$$

$$t_2 = \frac{1 - \cos\beta}{3(2+\cos\beta)}$$

$$t_3 = \frac{5 + 4\cos\beta - 3\sqrt{5 + 4\cos\beta}}{6(2+\cos\beta)}.$$



Absolute values

$$|t_1| \geq |t_2| \geq |t_3|$$

Monogamy trade-off relation in the case of 3-qubit states

Monogamy constraint imposes the trade-off relation

$$\mathfrak{M}_{ABC} \equiv \langle CHSH \rangle_{AB}^2 + \langle CHSH \rangle_{BC}^2 + \langle CHSH \rangle_{AC}^2 \leq 12$$

- ❑ **3-qubit permutation symmetric states**

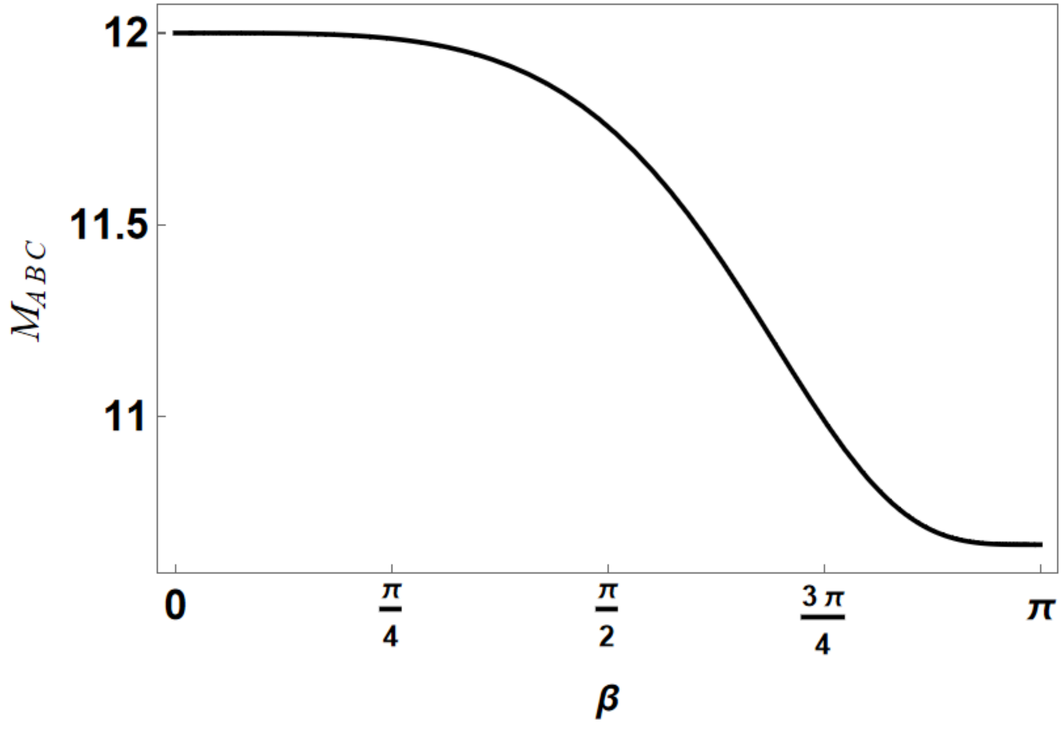
$$\langle CHSH \rangle_{AB} = \langle CHSH \rangle_{BC} = \langle CHSH \rangle_{AC}$$

⬇

$$\mathfrak{M}_{ABC} = 3\langle CHSH \rangle_{AB}^2 \leq 12$$

⬇

$$|\langle CHSH \rangle_{AB}| < 2$$



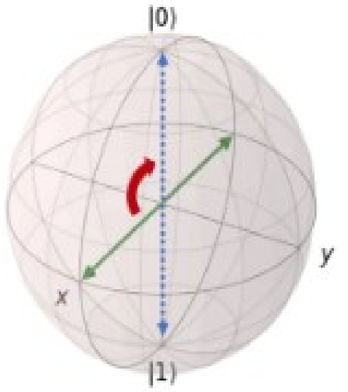
CHSH Monogamy relation

Any arbitrary 2-qubit state extracted from 3-qubit permutation symmetric system cannot violate CHSH inequality, even though the constituent qubits are entangled.

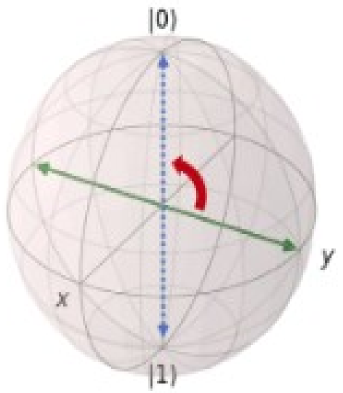
Verification of monogamy relations using open access IBM quantum computer *ibmq_lima*

- Using IBM open-source software kit Qiskit we initialize the 3-qubit state $|\Psi_{3,2}\rangle$ for $\beta = \pi/6, \pi/4, 3\pi/8, 9\pi/16, \pi$.
- Recording measurement data on the 2-qubit reduced density matrices and construction of correlation matrix T
- Experimental verification of monogamy relation $\mathfrak{M}_{ABC} \leq 12$

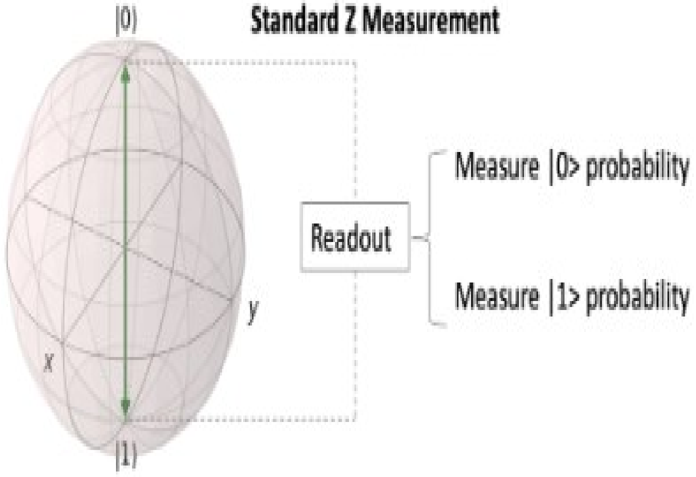
Measurements of Pauli gates X, Y and Z



Rotation for X Measurement

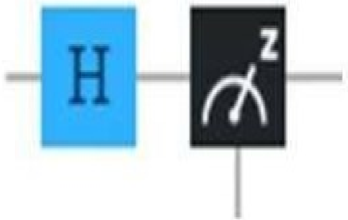


Rotation for Y Measurement

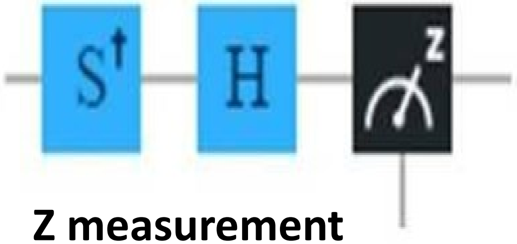


Measurement of the X or Y Pauli matrices requires application of unitary rotation operation so as to rotate the X - or Y –axis to be the Z-axis.

X measurement



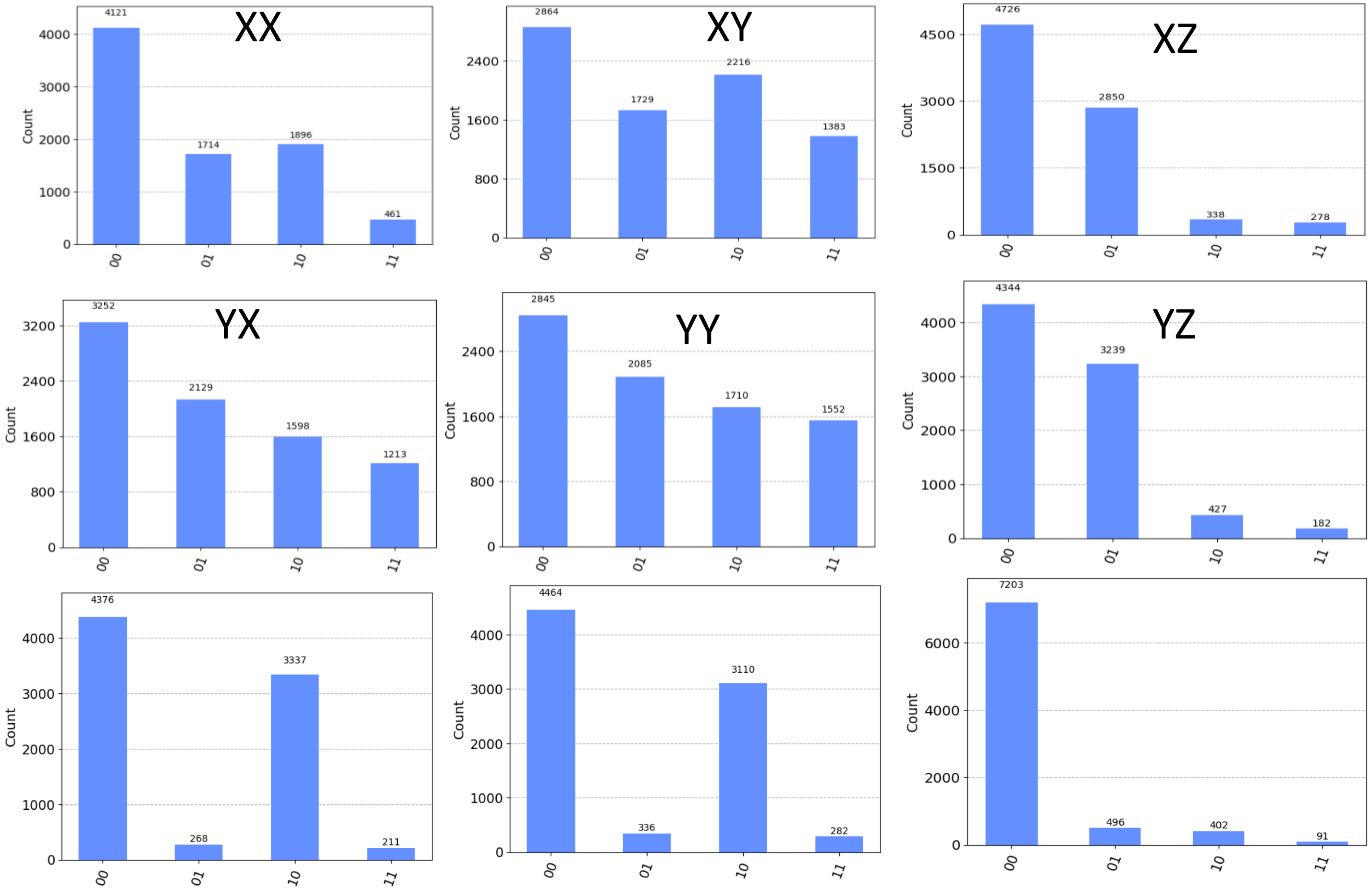
Y measurement

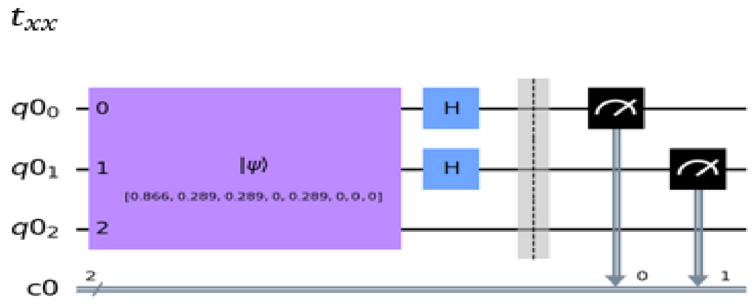


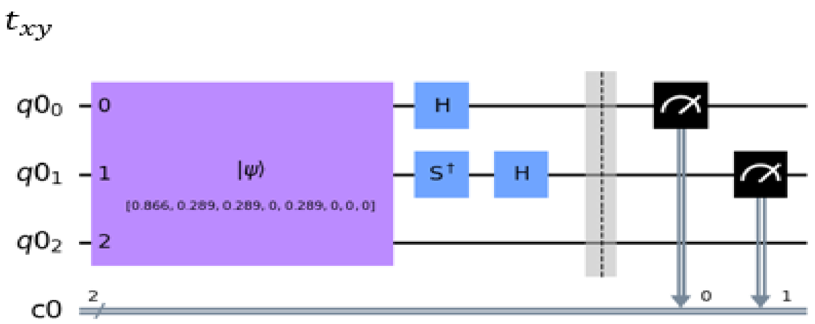
Z measurement

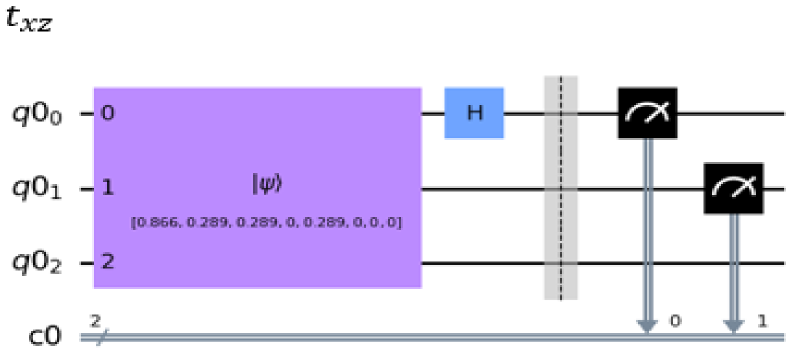
Measurement on qubits 1 & 2: *Counts* for $\beta = {}^{\pi}/_{4}$ (ibmq_lima)
Total number of trials: 8192

Total number of measurements for each β: 27







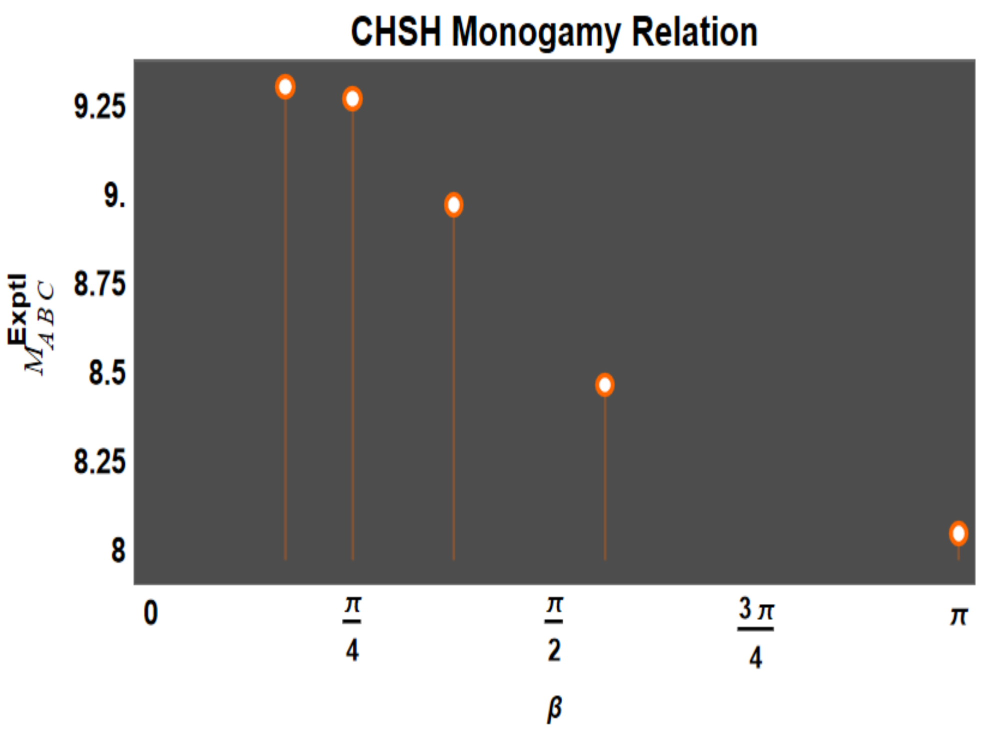


$$T_{12}^{Exptl} = \begin{pmatrix} 0.0315 & 0.03686 & 0.2216 \\ 0.0288 & 0.07348 & 0.105 \\ 0.11987 & 0.1586 & 0.7807 \end{pmatrix}; \; t_1^{Exptl} = 0.7149, \;\; t_2^{Exptl} = 0.0027$$

$$t_1^{Th} = 0.9987, \qquad t_2^{Th} = 0.0360$$

$\langle CHSH\rangle_{AB}^{Exptl}$ =1.6943, $\mathfrak{M}_{ABC}^{Exptl}$ =9.2728 ; $\langle CHSH\rangle_{AB}^{Th}$ =1.9987, $\mathfrak{M}_{ABC}^{Th}$ =11.9855

β	qubit pairs	$t_{1,2}^{Exptl}$	$t_{1,2}^{Th}$	$\mathfrak{M}_{ABC}^{Exptl}$	$\mathfrak{M}_{ABC}^{Th}$
$\frac{\pi}{6}$	12	0.8035, 0.0148	0.9997, 0.0155	9.3071	11.9972
	23	0.6908,0.0252			
	13	0.7752,0.0170			
$\frac{\pi}{4}$	12	0.7149, 0.0027	0.9987, 0.0360	9.2728	11.9855
	23	0.8063,0.01688			
	13	0.7526,0.0246			
$\frac{3\pi}{8}$	12	0.8143, 0.0129	0.9930, 0.0860	8.9751	11.9242
	23	0.6818,0.02534			
	13	0.6612,0.0480			
	23	0.8264, 0.0302			
	13	0.8483,0.0520			
$\frac{9\pi}{16}$	12	0.6342, 0.06255	0.9586, 0.2207	8.4692	11.6100
	23	0.6115,0.0688			
	13	0.6951,0.0455			
	23	0.5677,0.1243			
	13	0.5956,0.1297			
π	12	0.3936, 0.3125	0.4444, 0.4444	8.04924	10.6667
	23	0.3268,0.2607			
	13	0.3950,0.3236			



Conclusion: Shareability places restrictions on CHSH non-locality.

References

1. J. S. Bell, On the Einstein–Podolsky–Rosen paradox, Physics 1, 195 (1964)
2. J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt, Proposed experiment to test local hidden-variable theories, Phys. Rev. Lett. 23, 880 (1969)
3. H. -H. Qin, S. -M. Fei, X. Li-Jost, Trade-off relations of Bell violations among pairwise qubit systems, Phys. Rev. A 92, 062339 (2015)
4. K. Anjali, S. H. Akshata, H. S. Karthik, S. Sahu, Sudha, A. R. Usha Devi, Characterizing nonlocality of pure symmetric three-qubit states, Quantum Information Processing, 20, 18 (2021)
5. IBM quantum computing platform (2019). https://quantum-computing.ibm.com/

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Thank you!