Finite volume effects on the QCD chiral phase transition using Nambu–Jona-Lasinio (NJL) model

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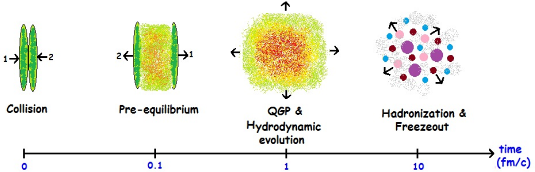
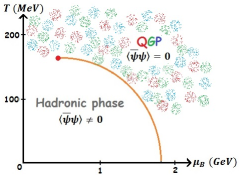
Introduction and Motivation

- Quark-Gluon Plasma (QGP) is a deconfined phase of quarks and gluons as the effective degrees of freedom. It exist at high temperature and/or high baryon density, e.g. in heavy-ion collision experiments at the time scale of $\sim \mathcal{O}(1 \text{ fm/c})$.
- Finite-sized phase of matter of the order of few fermi comparable to the characteristic QCD interaction scale. Geometry of the QGP system and the boundary condition is important for theoretical understanding and realistic analysis.
- We tried to estimate the size of the finite volume effects with different boundary conditions, starting from simple geometries where other theoretical treatments are available, and then proceeding towards those with closer resemblence to the actual fireball geometry.





NJL model with cubic geometry

- Nambu–Jona-Lasinio (NJL) Model with $N_f = 2$ and $N_c = 3$.

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\bar{\tau}\psi)^2\right]$$

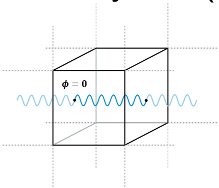
- Iterative solving of the gap equation : $M = m + \sigma$ where $\sigma = -2G\langle\bar{\psi}\psi\rangle$
- Chiral condensate $\langle\bar{\psi}\psi\rangle$ obtained from trace over fermionic propagator with dressed quark mass M.

$$\langle\bar{\psi}\psi\rangle = -\int \frac{d^4p}{(2\pi)^4}\, Tr\left\{\frac{1}{\not{p} - M}\right\} = -2\, N_c\, N_f \int \frac{d^3p}{(2\pi)^3} \frac{M}{E}\left(1 - \frac{2}{1 + e^{E/T}}\right)$$

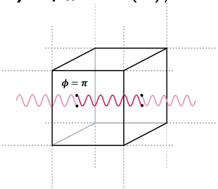
- Cubic geometry with different boundary conditions.

  For earlier studies, see **Wang, Xia and Zong, arXiv : 1802.00258, 1806.05315**

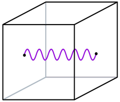
  - **Periodic (PBC)** : $p_n = 2\pi(n)/L$, $n = 0, \pm1, \pm2, \ldots$
  - **Anti-periodic (APBC)** : $p_n = 2\pi(n + 1/2)/L$, $n = 0, \pm1, \pm2, \ldots$
  - **Stationary wave (SWC)** : $p_n = \pi(n)/L$, $n = 1, 2, 3, \ldots$



PBC

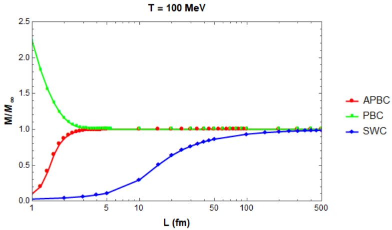


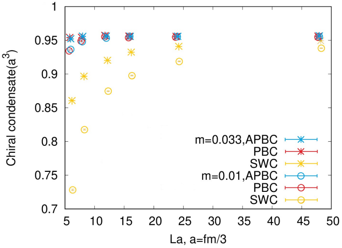
APBC



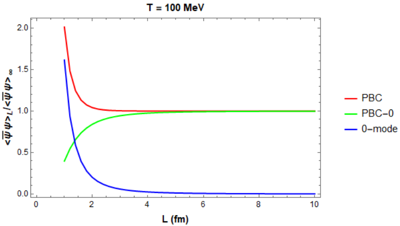
SWC

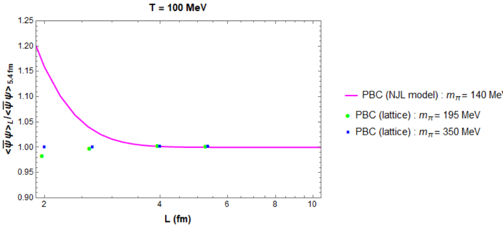
Results for cubic geometry





- SWC approach infinite limit for larger L compared to APBC and PBC.





- PBC has opposite nature near smaller volumes for lattice and NJL model.
- PBC zero-mode treatment in MF is slightly arbitrary and could be the cause of discrepancy.

NJL model with cylindrical geometry

- Cylinder with MIT b.c. along the transverse direction (finite radius R) and infinite length. **Chernodub and Gongyo, JHEP01 (2017) 136**

$$[i\gamma^{\mu} n_{\mu}(\varphi) - 1]\psi(t, z, \rho, \varphi)|_{\rho=R} = 0$$

- Solving the Dirac equation in curved space-time :

$$[i\gamma^{\mu}(\partial_{\mu} + \Gamma_{\mu}) - M]\psi_j = 0$$

- Solution with energy eigenvalue $E_j = \sqrt{k_z^2 + \frac{q_{ml}^2}{R^2} + M^2}$:

$$\psi_j = \frac{1}{2\pi} e^{-i\tilde{E}_j t + ik_z z} u_j(\rho, \varphi)$$

- q_{ml} is l^{th} positive root of the equation ($J_m(x)$ is the Bessel function) :

$$j_m^2 + \frac{2MR}{q} j_m(q) - 1 = 0 \quad \text{where} \quad j_m(x) = \frac{J_m(x)}{J_{m+1}(x)}$$

- Constraining the cylinder along length (L) :

$$\int_{-\infty}^{\infty} \frac{d^3k}{(2\pi)^3} \xrightarrow{\text{finite R}} \frac{1}{\pi R^2} \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \int \frac{dk_z}{(2\pi)} \xrightarrow{\text{finite L}} \frac{1}{\pi R^2 L} \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \sum_{n}$$

Results for cylindrical geometry

- UV cutoff : $\Lambda \approx 1 GeV$