

**THE TORSION OF  
STELLAR STREAMS  
AND THE SHAPE OF GALACTIC  
GRAVITY'S SOURCE**

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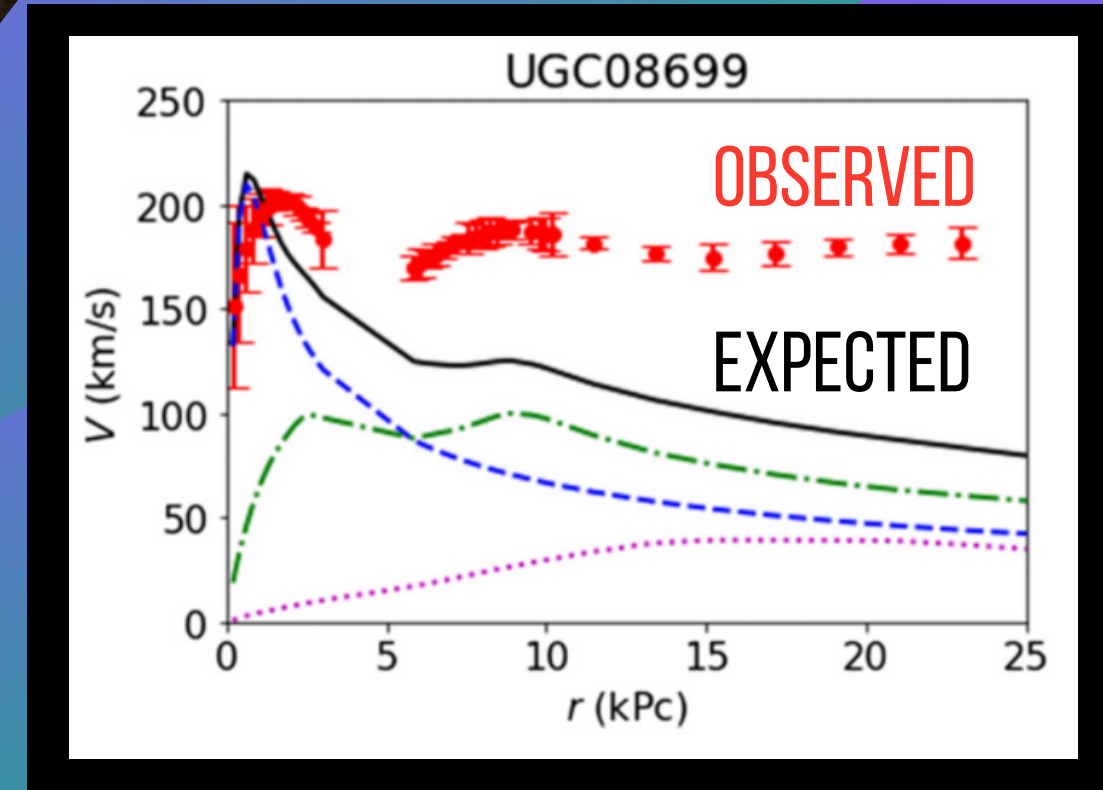
# 1. ROTATION CURVE PROBLEM

The problem of galactic rotation is the empirical statement that rotational velocity around the galactic center seems to flatten out for a large fraction of the galaxy population [1].

This is at odds with orbital equilibrium outside a spherical source.

$$\frac{v^2}{r} = \frac{GM}{r^2} \rightarrow v = \sqrt{\frac{GM}{r}}$$

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Modifications of mechanics, such as MOND, solve the issue but run into problems at larger, cosmological scales.

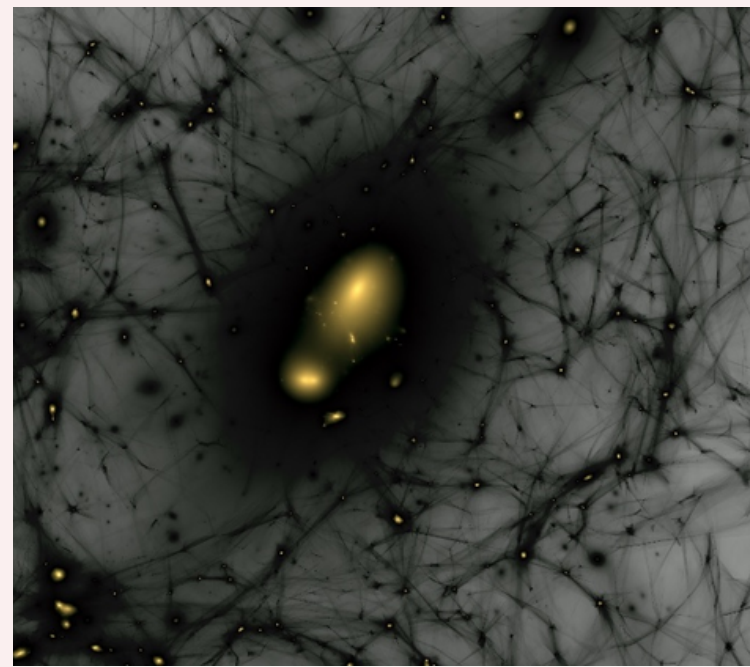
We typically turn to a modification of the gravity source, typically in the form of a spherical Dark Matter halo [2] envelope.

In a 2 dimensional cosmos  
We expect  $\rightarrow v^2 \propto \text{constant}$   
 $F \propto 1/r$

In our 3 dimensional cosmos  
We expect  $\rightarrow v^2 \propto 1/r$   
 $F \propto 1/r^2$

We observe  $\rightarrow v^2 \propto \text{constant}$   
 $F \propto 1/r$

We do not live in a 2D cosmos!  
But a cylindrical matter source achieves the same dimensional reduction.





## 2. CYLINDRIC VS SPHERICAL HALO

A spherical DM distribution has to be fine-tuned to have an isothermal  $\rho(r) \propto 1/r^2$  profile to explain the flatness of the rotation curve.

A cylindrical source of linear density  $\lambda$  naturally explains constant rotation curves [3].

$$\frac{v^2}{r} = \frac{2G\lambda}{r} \rightarrow v = \sqrt{2G\lambda}$$

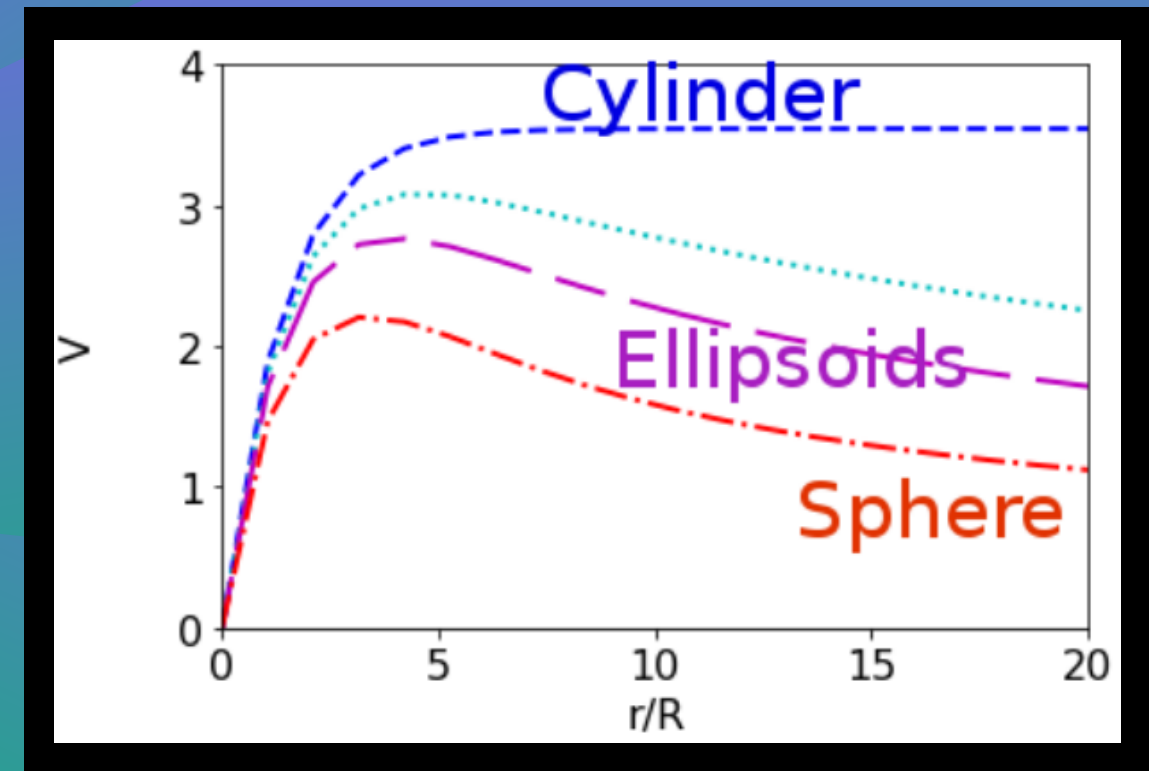
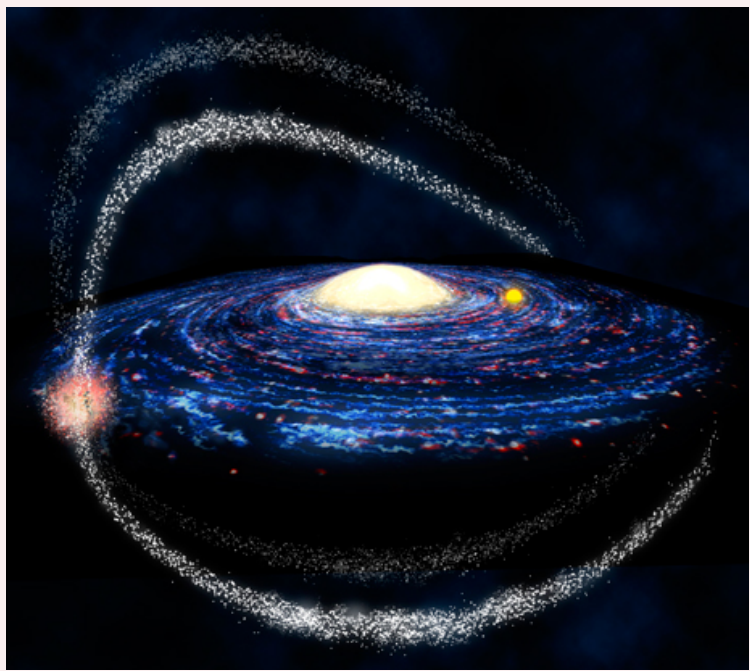
Since the rotation curve is measured to a finite  $r$  the source does not need to be infinitely cylindrical: it is sufficient that it be prolate (elongated) DM halo [4].

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Rotation curves cannot distinguish spherical haloes with isothermal profiles from elongated haloes with arbitrary profile.



Out-of-plane observables:  
**Stellar streams**





### 3. TORSION IN STELLAR STREAMS

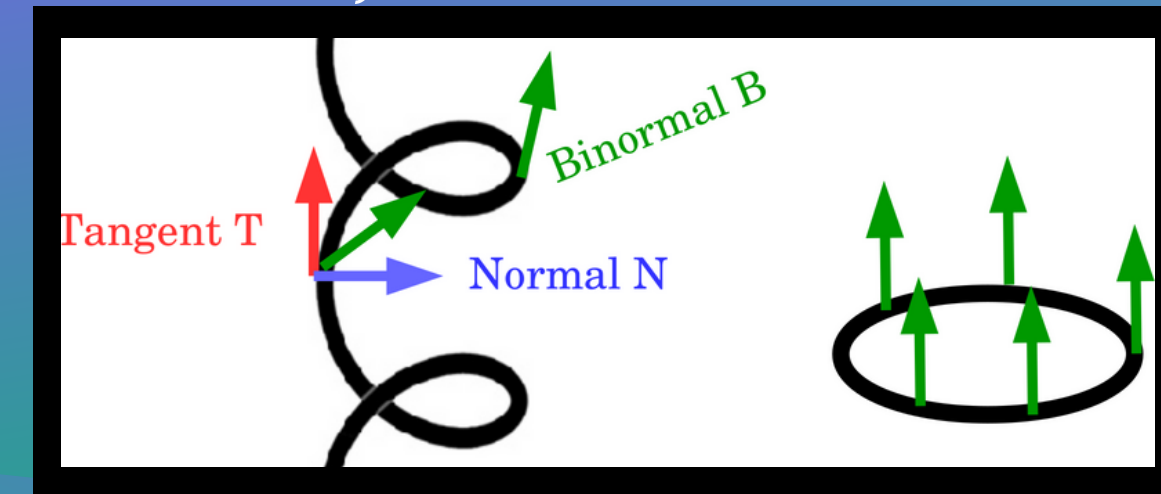
The torsion of a curve measures how sharply it is twisting out of the osculating plane, it is defined by the velocity and normal acceleration.

The planarity of the orbit around a central potential is a consequence of the conservation of the direction of the angular momentum vector  $L$ , it is parallel to the binormal vector.

If the curve is perfectly planar the tangent and normal vectors will always lie in the same plane, and in such case the binormal vector stays parallel to itself along the curve.

$$\tau = 0$$

Illustrated by the Frenet-Serret trihedron:



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$$\left. \begin{aligned} \mathbf{T} &= \frac{d\mathbf{r}}{ds} \\ \mathbf{N} &= \frac{d\mathbf{T}}{ds} \\ \mathbf{B} &= \mathbf{T} \times \mathbf{N} \end{aligned} \right\}$$

TORSION:

$$\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N}$$

$$= \frac{(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r}'''}{|\mathbf{r}' \times \mathbf{r}''|^2}$$

CYLINDER ORBIT

SPHERE ORBIT

If the curve twists out of the plane the binormal vector will acquire a rotation  
In a uniformly advancing helix:

$$\tau = \text{constant}$$

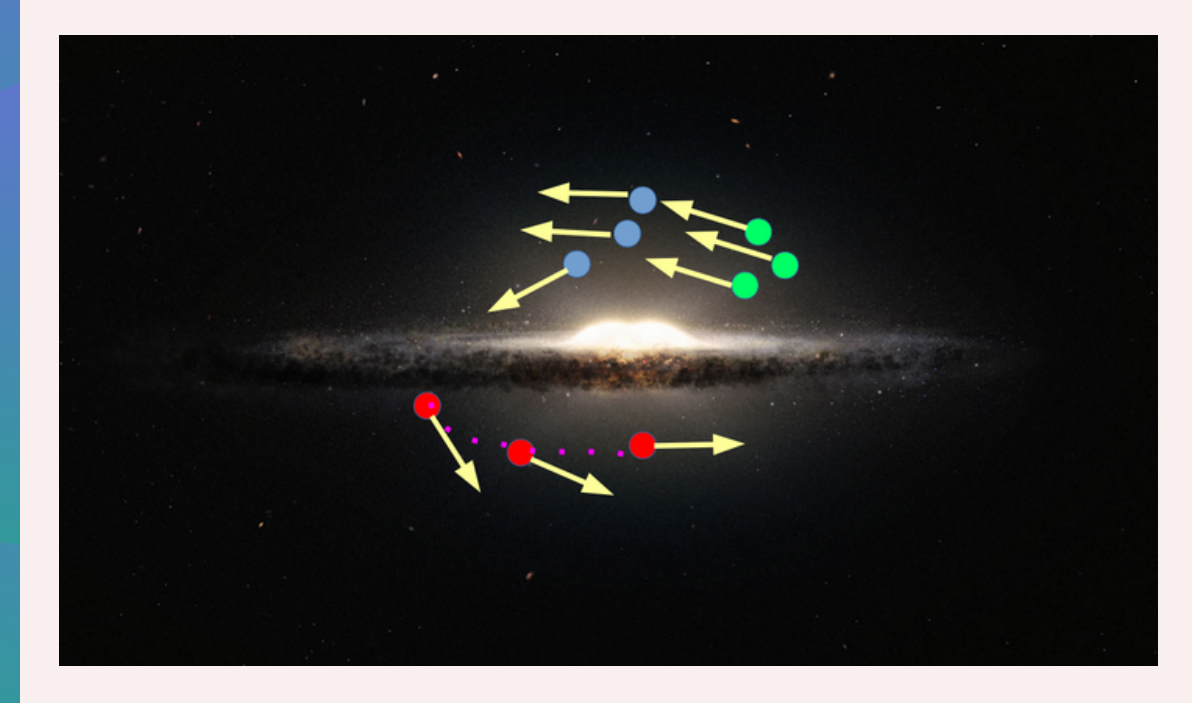


# 4. N-BODY SIMULATIONS OF STELLAR STREAMS

A test body moving in a central field has null torsion. A cluster made of test bodies moving around a galaxy would lose dust grains forming a kind of contrail, its shape through space would be a planar curve.

TIDAL DISTORTION OF A CLUSTER AROUND A GALAXY  
Stellar streams are the result of the tidal stretching of a globular cluster or dwarf galaxy [5].

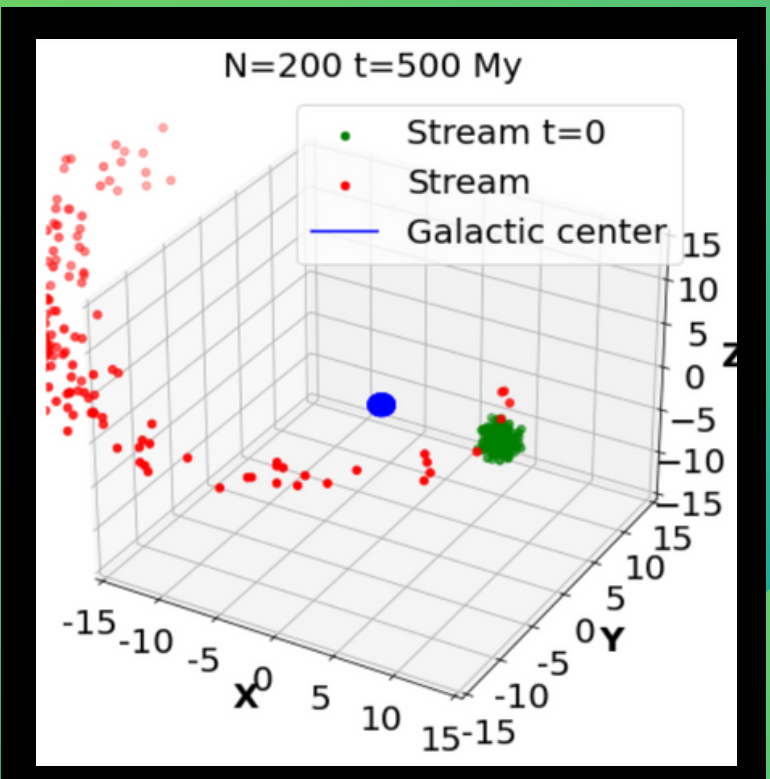
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Stars at different heights can deviate the orbit from a planar curve. We show that the effect is negligible

200 STAR CLUSTER MOVING FOR 500 My AROUND A SPHERICAL DM HALO

The orbit times commensurate with Hubble time →→→→→  
We cannot observe complete orbits





# 5. MILKY WAY STREAMS

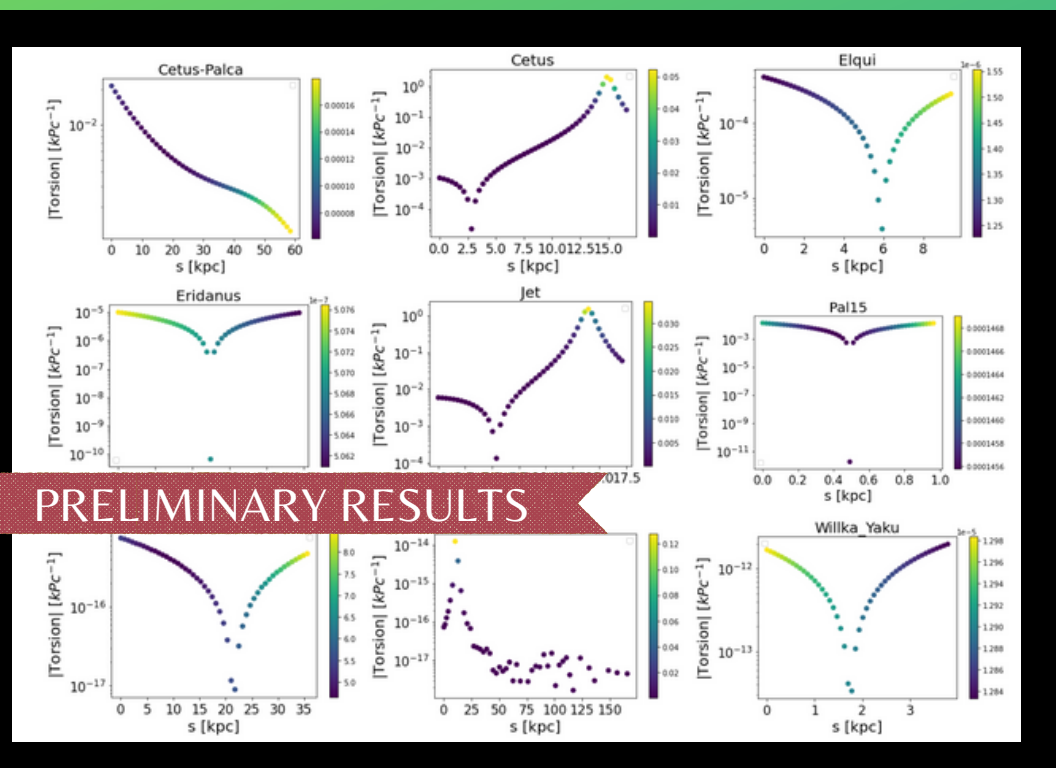
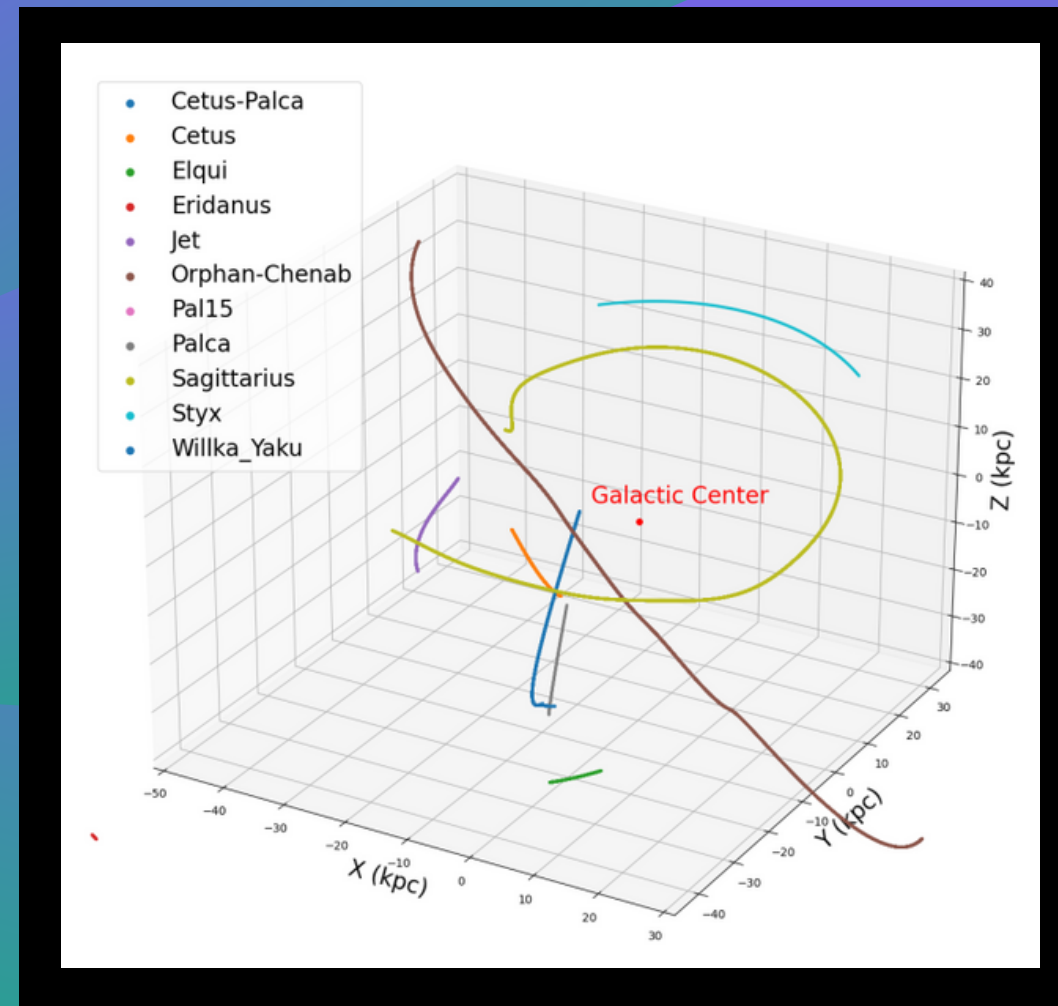
We select as relevant streams found at distances  $d > 30$  kPc from the galactic center, so that the internal structure of the galaxy produces the minimum possible alteration in the stream [6].

Torsion (as curvature) has dimensions of inverse length, we would expect stellar streams to perhaps show an inverse relation with respect to their distance from the galactic center.

$$\tau = \frac{(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r}'''}{|\mathbf{r}' \times \mathbf{r}''|^2}$$

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In a galaxy such as the Milky Way the torsion of galactic streams should have a characteristic scale of  $10 / \text{kPc}$ .



PRELIMINARY RESULTS

Our selection of streams at 30 kpc or more means that we would consider values of the torsion of order 0.03 in units of inverse kPc to be sizeable and very different from zero

REFERENCES:  
 [1] Rubin, Ford & Thonnard, 1978, ApJ  
 [2] Frenk, White, Efstathiou & Davis, 1985, Nature, 317, 595  
 [3] Llanes-Estrada 2021, Universe, 7, 346.  
 [4] Bariego-Quintana, Llanes-Estrada & Manzanilla Carretero, Physical Review D 2022, 107, 083524  
 [5] Noreña, Muñoz-Cuartas, Quiroga, & Libeskind, N. 2019, Rev. Mexicana Astron. Astrofis., 55, 273  
 [6] Mateu. 2023, Monthly Notices of the Royal Astronomical Society, 520, 5225