Flow, nonflow, and flow fluctuations

Sergei A. Voloshin
WAYNE STATE UNIVERSITY

Flow (as any other analysis)

Start with formulating the goals, including needed "precision"

Make clear, unambiguous definitions

Determine limitations and uncertainties

Do not "invent" new terminology, unless really needed.

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Do not "invent" new terminology, unless really needed.

Flow:

- Definitions, and first measurements
- Centrality/energy dependence, and ideal fluid
- Constituent Quark Scaling, and deconfinement
- Nonflow and flow fluctuations, and "ridge", initial geometry
- Participants/flow planes.
- Flow correlations and decorrelations
- Linear and nonlinear flow modes

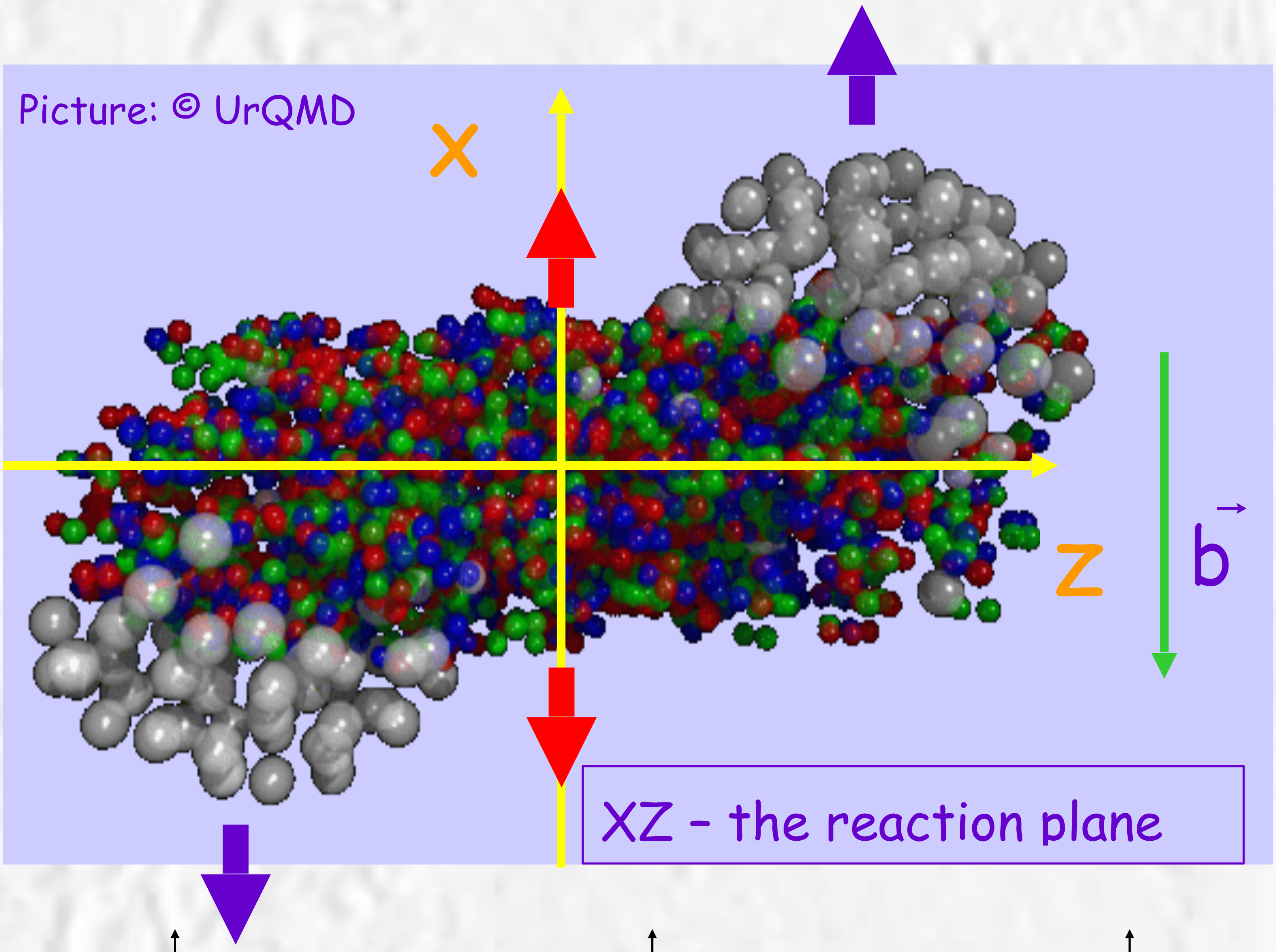
Anisotropic flow

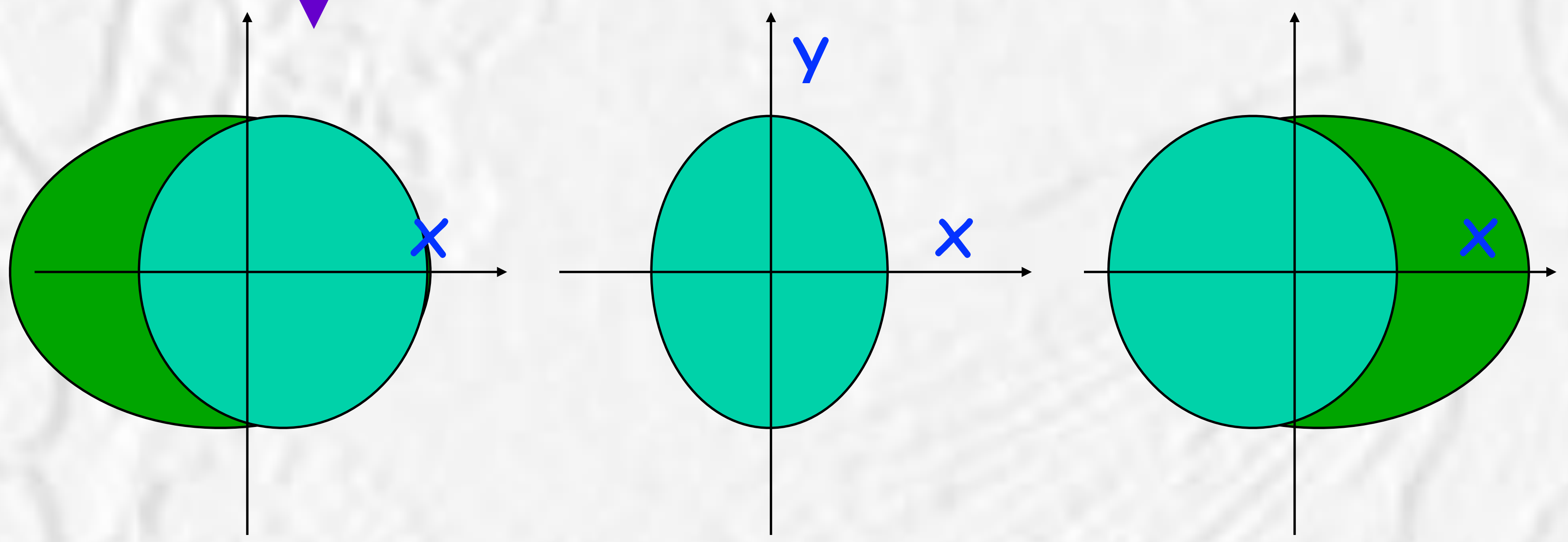
Anisotropic flow ≡ correlations with respect to the reaction plane, system response to azimuthally asymmetric initial conditions

Term "flow" does not mean necessarily "hydro" flow – used only to emphasize the collective behavior
↔ multiparticle azimuthal correlation.

$$E\frac{d^3n}{d^3p} = \frac{1}{2\pi\, p_T}\frac{d^2n}{dp_t\, dy}\left(1 + \sum_n 2v_n \cos[n(\phi - \Psi_{RP})]\right)$$

$$v_n(p_T, y) = \langle \cos[n(\phi_i - \Psi_{RP})] \rangle$$





Advantages:

- Describes different kind of anisotropies in a common way
- Possibility to "fully" correct the results and compare directly to theory and other experiments

Flow study in relativistic nuclear collisions by Fourier expansion of Azimuthal particle distributions #14

S. Voloshin (Pittsburgh U.), Y. Zhang (SUNY, Stony Brook) (Jun, 1994)

Published in: *Z.Phys.C* 70 (1996) 665-672 • e-Print: hep-ph/9407282 [hep-ph]

pdf DOI cite claim reference search 945 citations

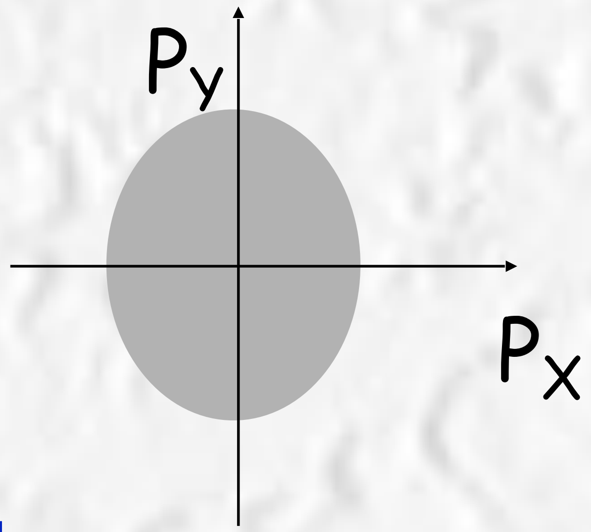
In-plane elliptic flow

Anisotropy as a signature of transverse collective flow #5

Jean-Yves Ollitrault (Saclay) (Mar 20, 1992)

Published in: *Phys.Rev.D* 46 (1992) 229-245

DOI cite claim reference search 1,401 citations



In "v_n" language: $v_2 > 0$

Hydrodynamics: stronger in-plane expansion of the system

$$S_{ij} = \langle p_i p_j \rangle; \quad i,j = x,y$$

2d sphericity

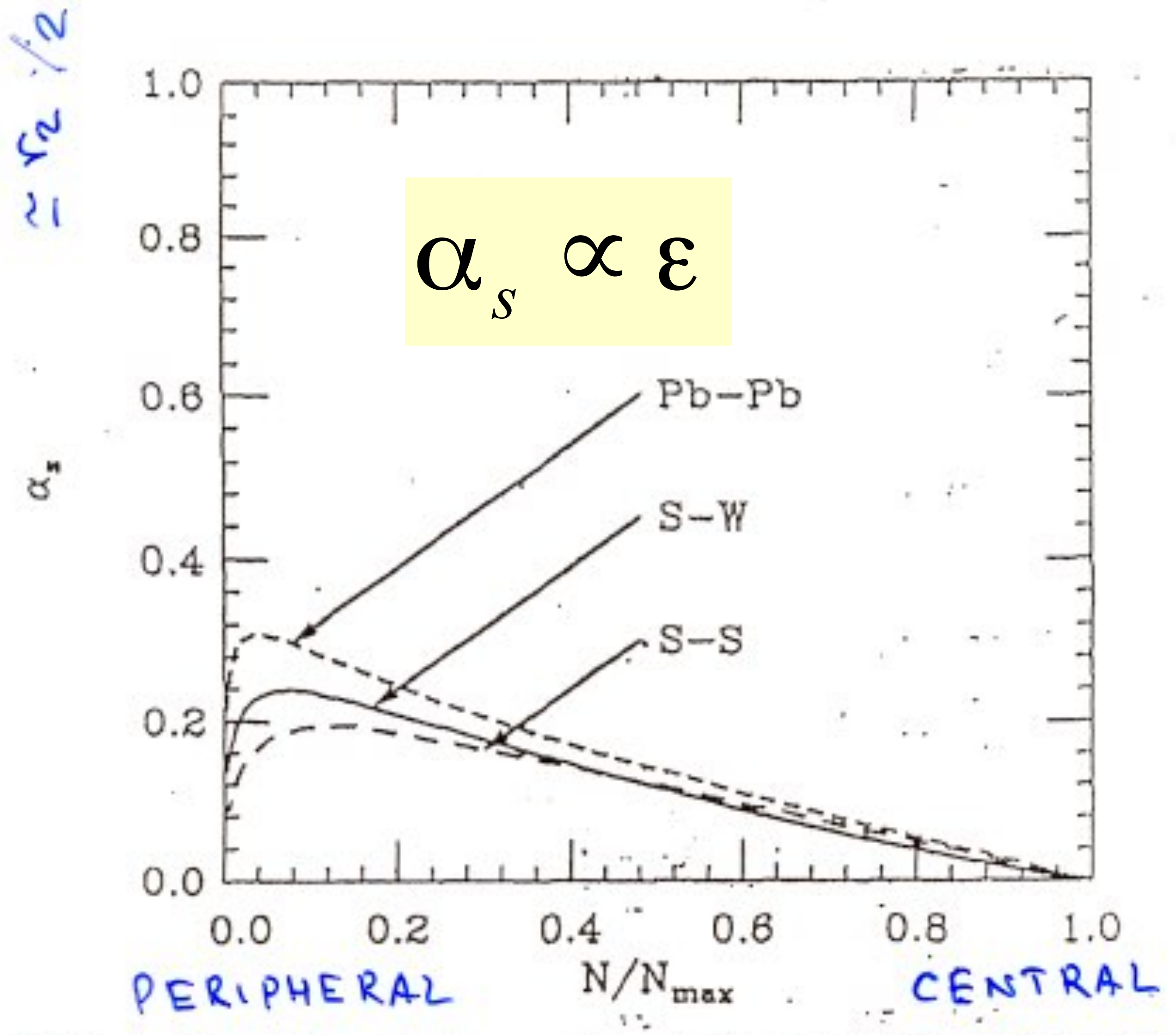


FIG. 3. Spatial anisotropy for various colliding systems. α_s, defined by Eq. (4.18), is plotted against the number of participating nucleons, scaled to its maximum value (reached for a central collision) N_{max}. Short dashes: lead-lead collision ($N_{max} \simeq 395$). Long dashes: sulfur-sulfur collision ($N_{max} \simeq 51$). Solid line: sulfur-tungsten collision ($N_{max} \simeq 121$).

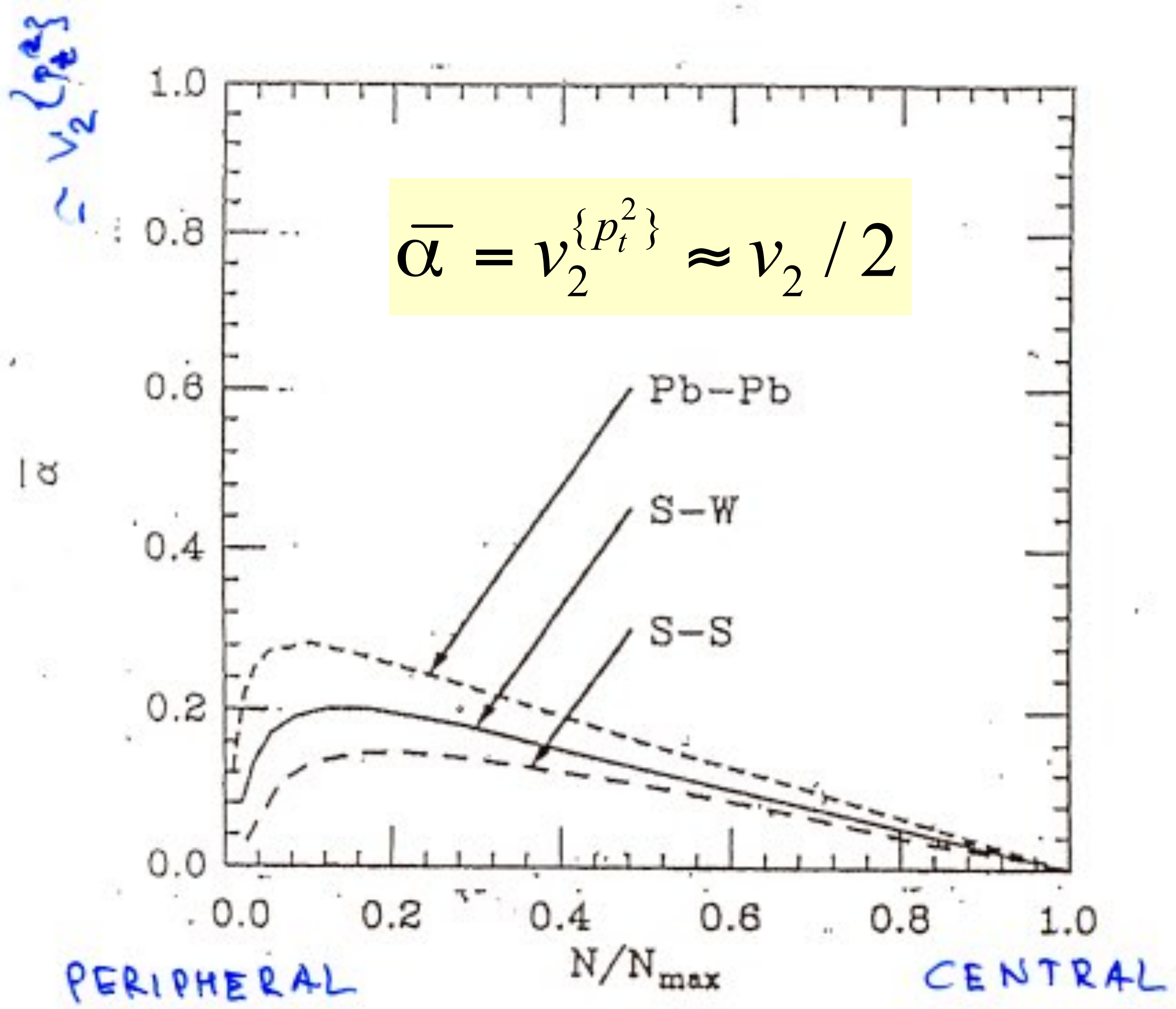
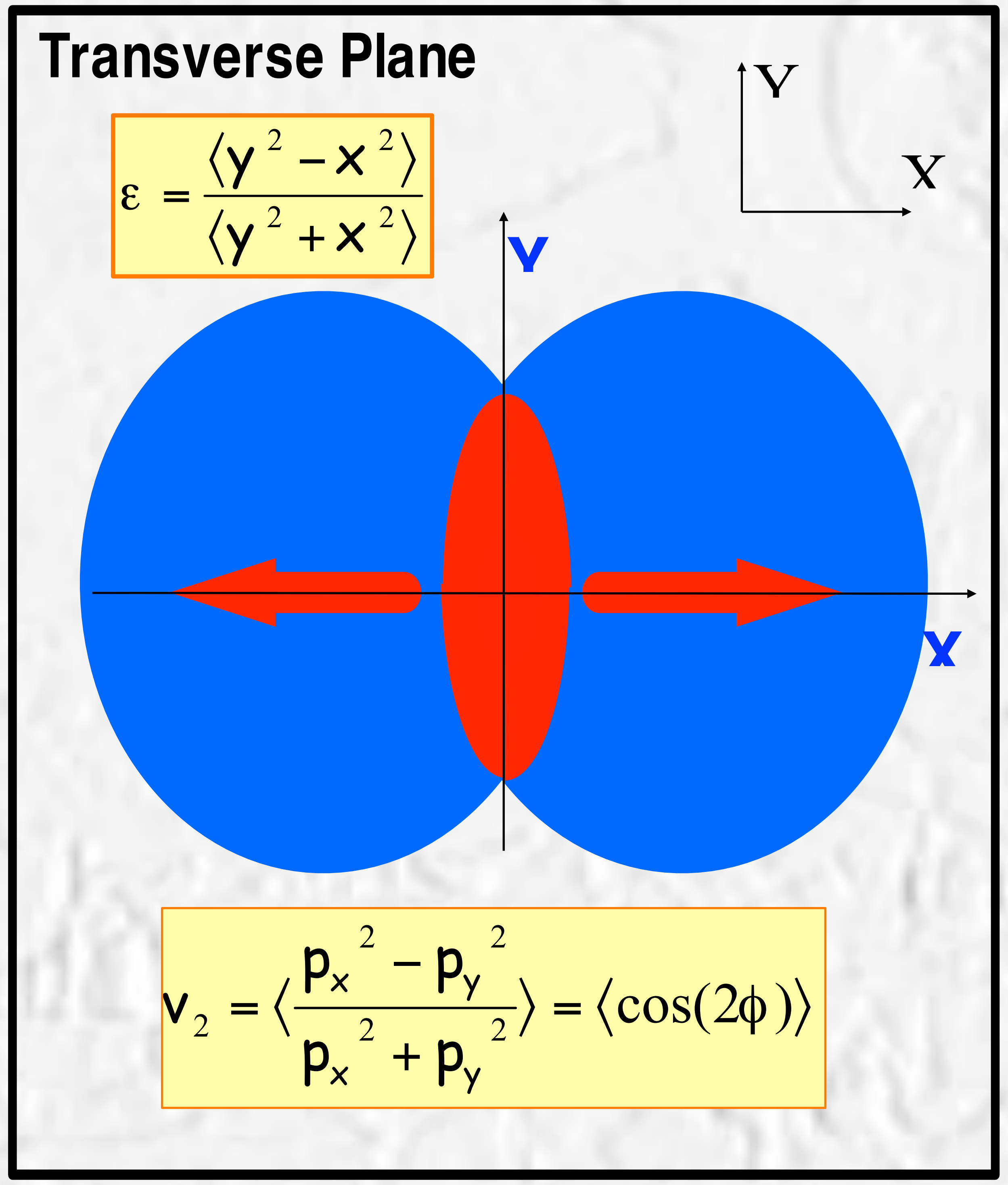


FIG. 6. Comparison between various colliding systems. $\bar{\alpha}$ is plotted against the number of participating nucleons scaled to its maximum value N_{max}, as in Fig. 4. The decoupling temperature is $T_d = 150$ MeV and the initial time $t_0 = $fm/$c$ for the three curves.



The term "elliptic flow" came later< about 1995

Flow vectors, measurements

$$E\frac{d^3n}{d^3p} = \frac{1}{2\pi p_T}\frac{d^2n}{dp_t\,dy}\left(1+\sum_n 2v_n\cos[n(\phi-\Psi_{RP})]\right)$$

$$v_n(p_T, y) = \langle\cos[n(\phi_i - \Psi_{RP})]\rangle$$

n-th harmonic Flow vector

$$Q_{n,x} = \sum_i w_i\cos(n\phi_i) = \mathbf{Q}_n\cos(n\Psi_n),$$

$$Q_{n,y} = \sum_i w_i\sin(n\phi_i) = \mathbf{Q}_n\sin(n\Psi_n),$$

2-particle correlations

$$\langle\cos[n(\phi_i - \phi_j)]\rangle = v_n^2$$

Methods for analyzing anisotropic flow in relativistic nuclear collisions #4

Arthur M. Poskanzer (LBL, Berkeley), S.A. Voloshin (Heidelberg U.) (May, 1998)

Published in: *Phys.Rev.C* 58 (1998) 1671-1678 • e-Print: nucl-ex/9805001 [nucl-ex]

pdf DOI cite claim reference search 1,415 citations

Concise summary of the methods (published previously)

Notations: $v_n\{\text{method}\}$, e.g. $v_n\{2\}$, $v_n\{4\}$, $v_n\{EP\}$

Ideal world:
no other correlations besides flow,
$v_n = \text{const}$

Event plane method

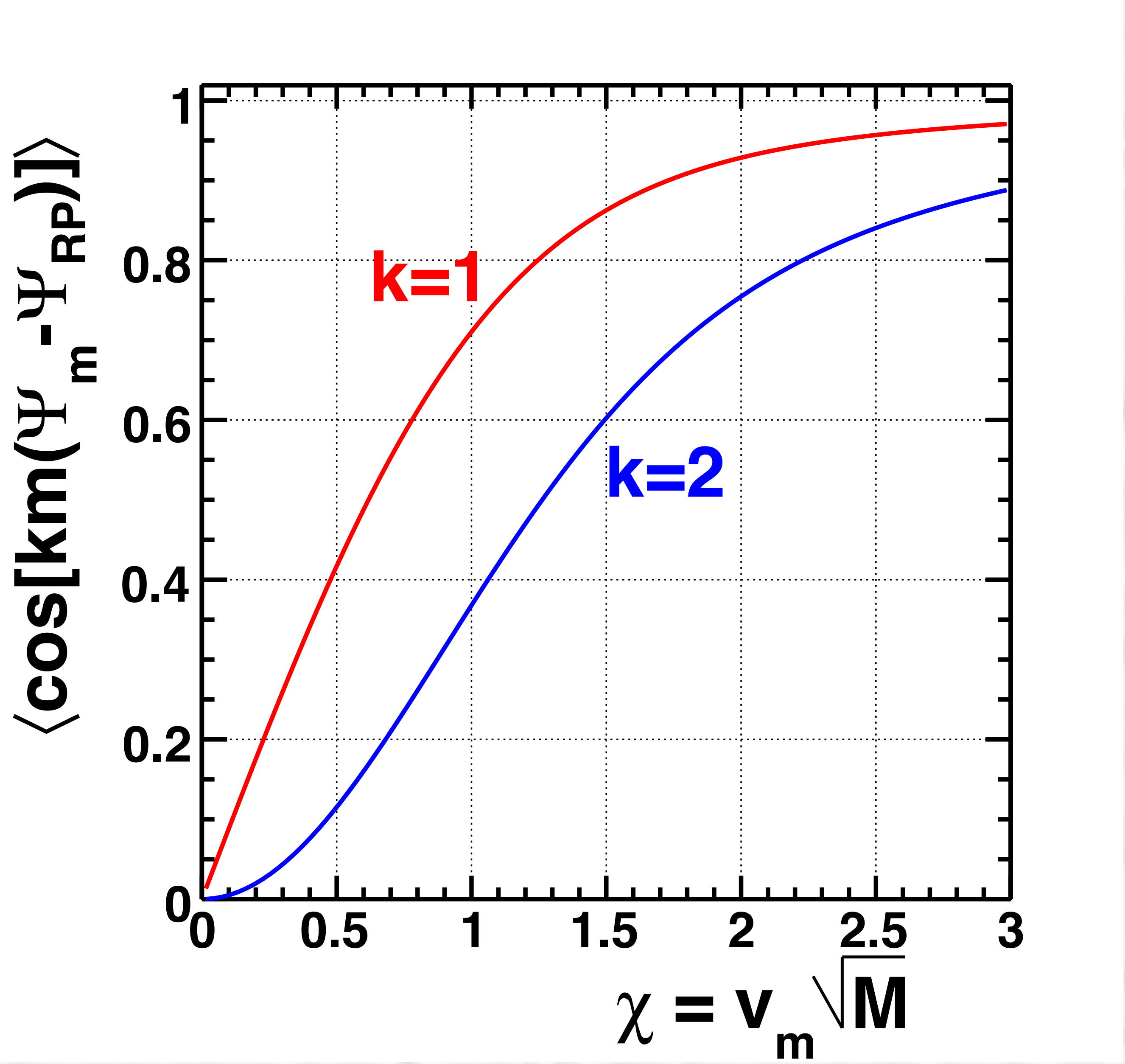
$$v_n^{obs}(p_T, y) = \langle\cos[n(\phi_i - \Psi_n)]\rangle,$$

$$\mathcal{R}_n = \langle\cos[n(\Psi_n - \Psi_{RP})]\rangle,$$

$$\mathcal{R}_{n,sub} = \sqrt{\langle\cos[n(\Psi_n^A - \Psi_n^B)]\rangle},$$

$$\chi = v_n\sqrt{M} \qquad \mathcal{R}_{full} = \mathcal{R}(\sqrt{2}\,\chi_{sub})$$

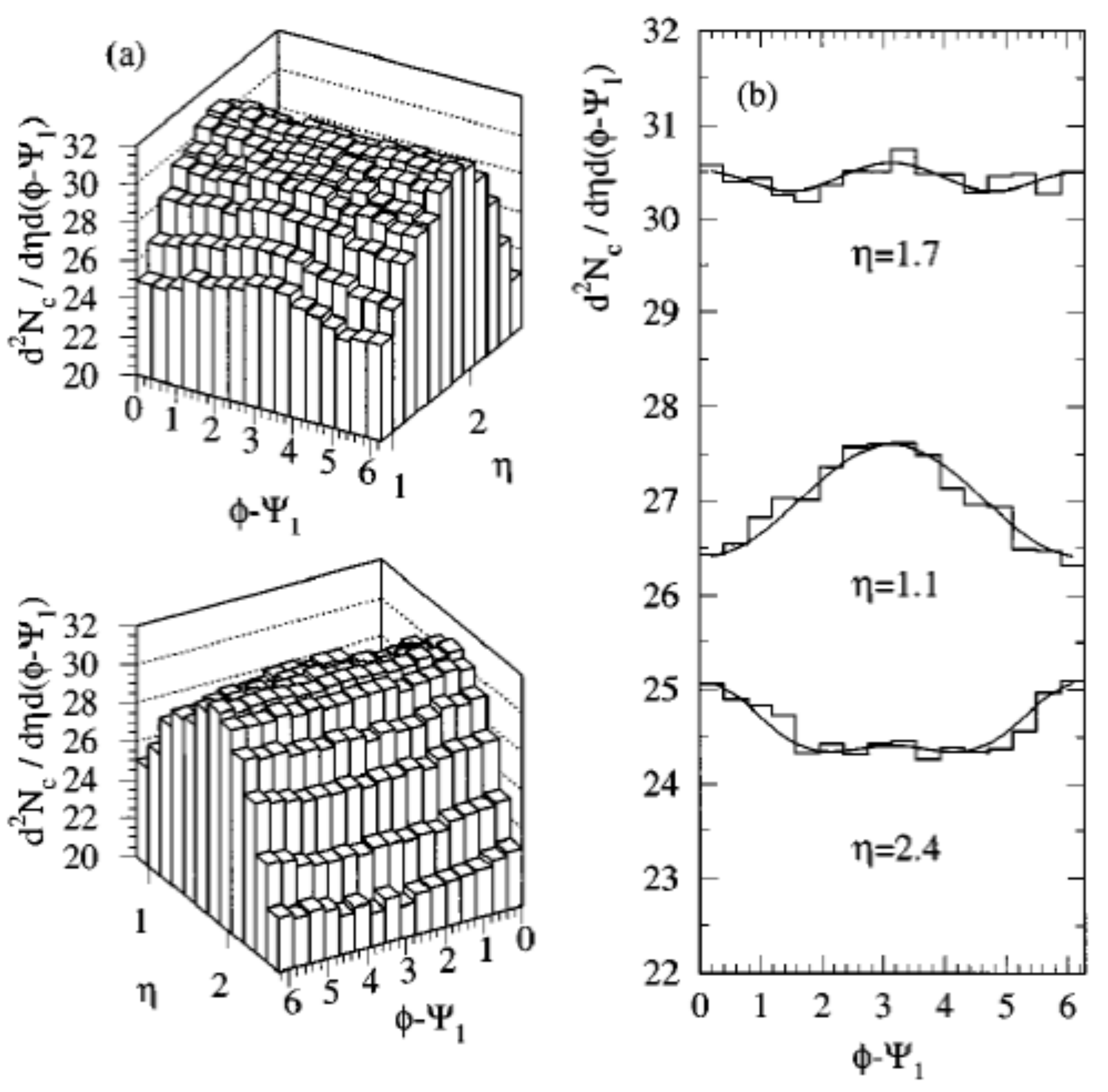
$$v_n = \frac{v_n^{obs}}{\mathcal{R}_n}.$$

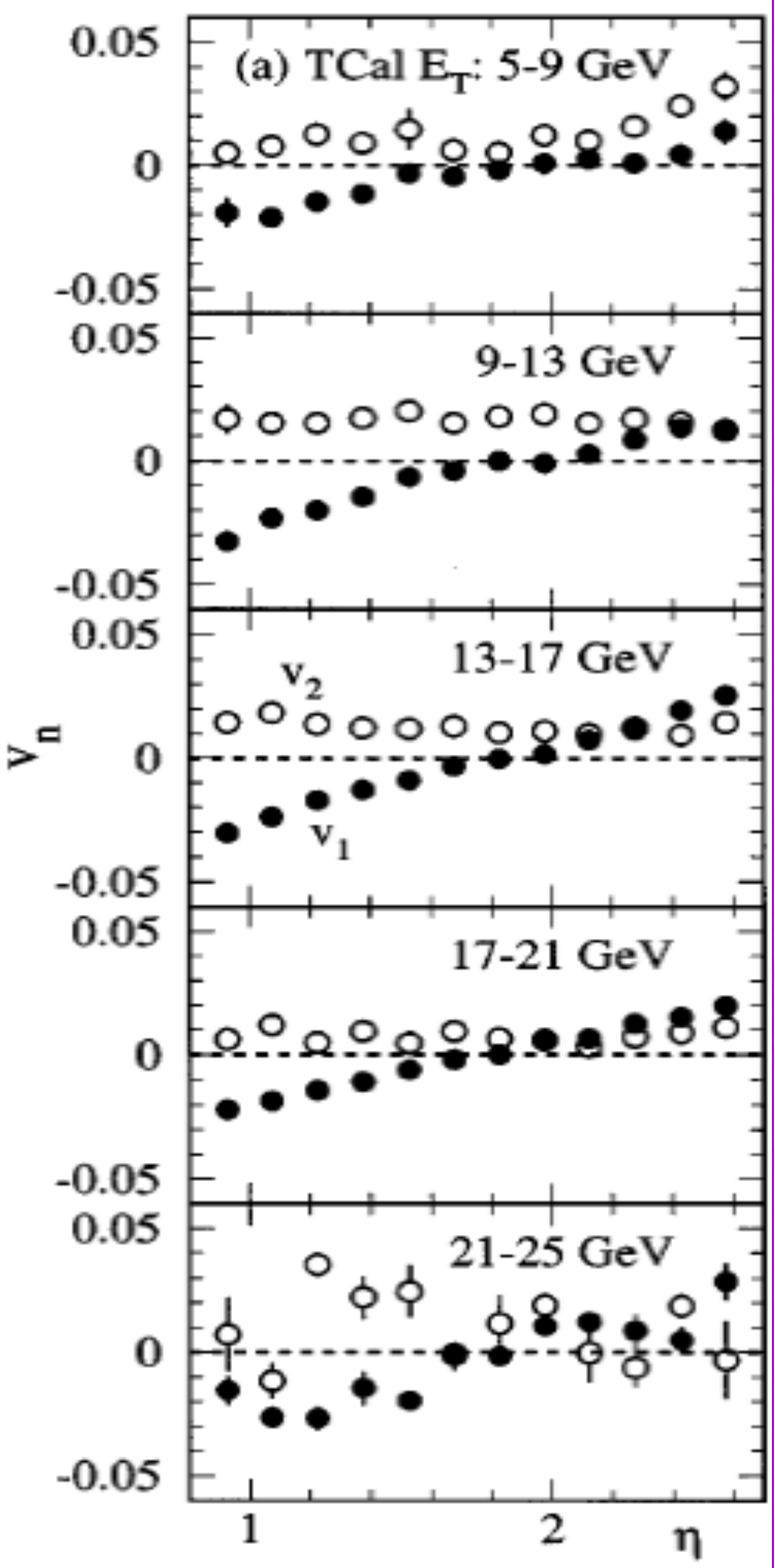


"Differential flow". First observation of $v_2 > 0$

E877, PRC 55 (1997) 1420

Distribution of hits in the silicon pad detector wrt the first order Event Plane determined by calorimeters.





v_1 –filled circles
v_2 – open circles

Low density and "hydro" limits

The physics of the centrality dependence of elliptic flow

S.A. Voloshin [a,b], A.M. Poskanzer [a]

[a] Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
[b] Department of Physics and Astronomy, Wayne State University, Detroit, MI 48201, USA

Received 30 September 1999; received in revised form 13 December 1999; accepted 29 December 1999
Editor: J.-P. Blaizot

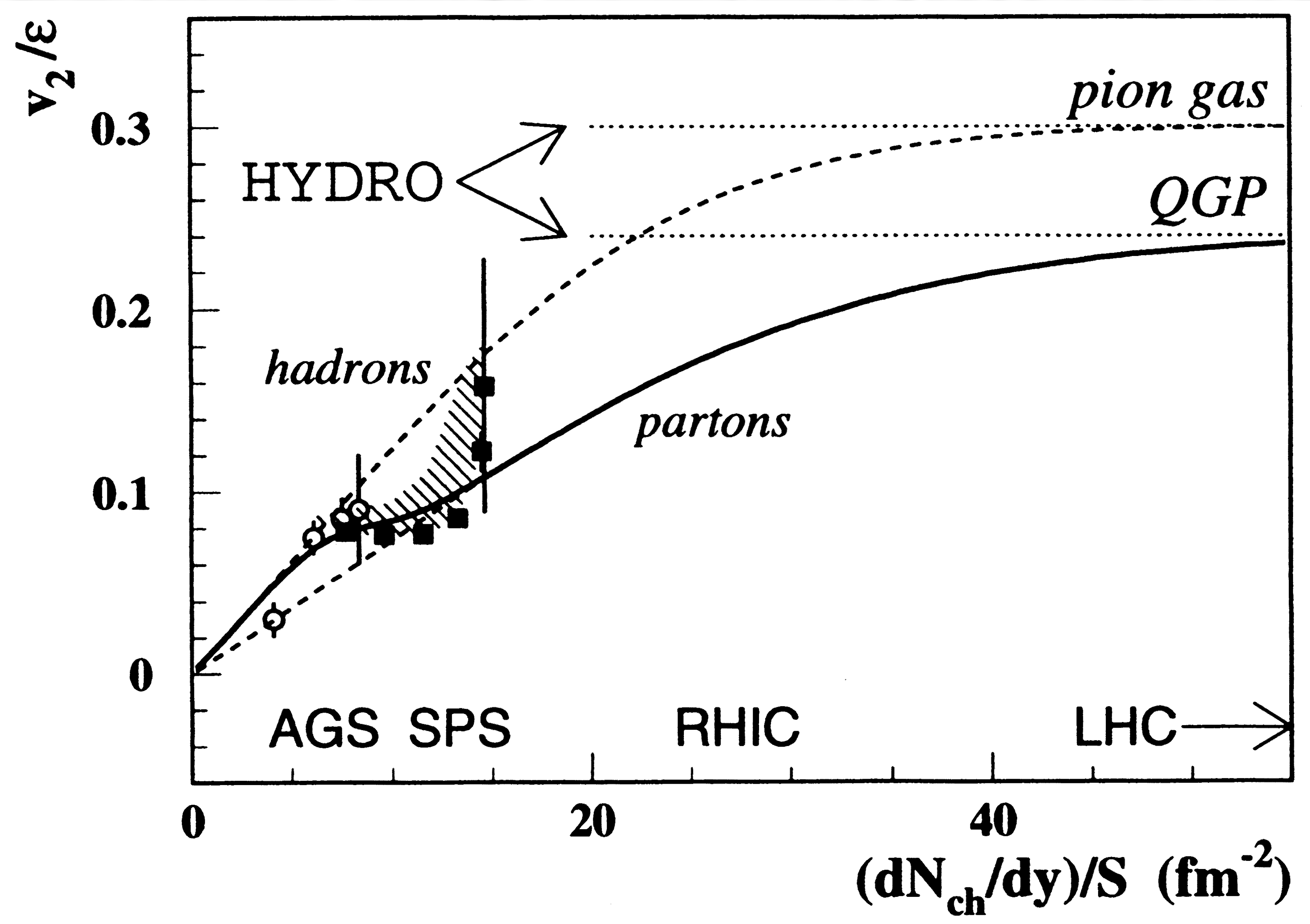


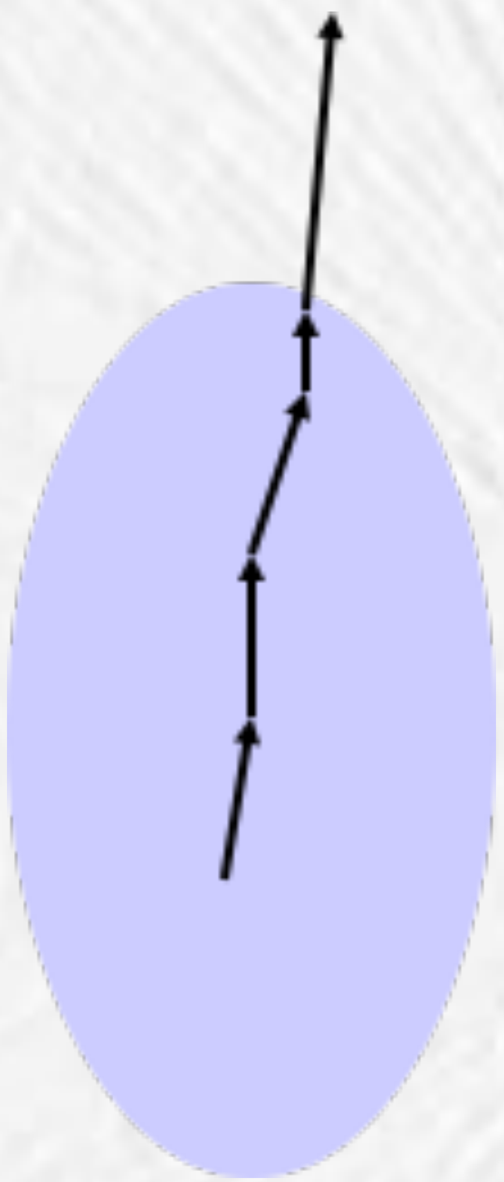
Fig. 3. Elliptic flow divided by the initial space elliptic anisotropy at the AGS (open circles) and the SPS (filled squares). The shaded area shows the uncertainty in the SPS experimental data due to the uncertainty in the centrality determination. See text and footnote for the description of the curves and hydro limits.

Low density limit

$$v_2 \propto \varepsilon_2 \frac{1}{S}\frac{dN}{dy}$$

$$\varepsilon_2 = \frac{\langle y^2 - x^2\rangle}{\langle y^2 + x^2\rangle}$$

$$S = \pi\sqrt{\langle y^2\rangle\langle x^2\rangle}$$

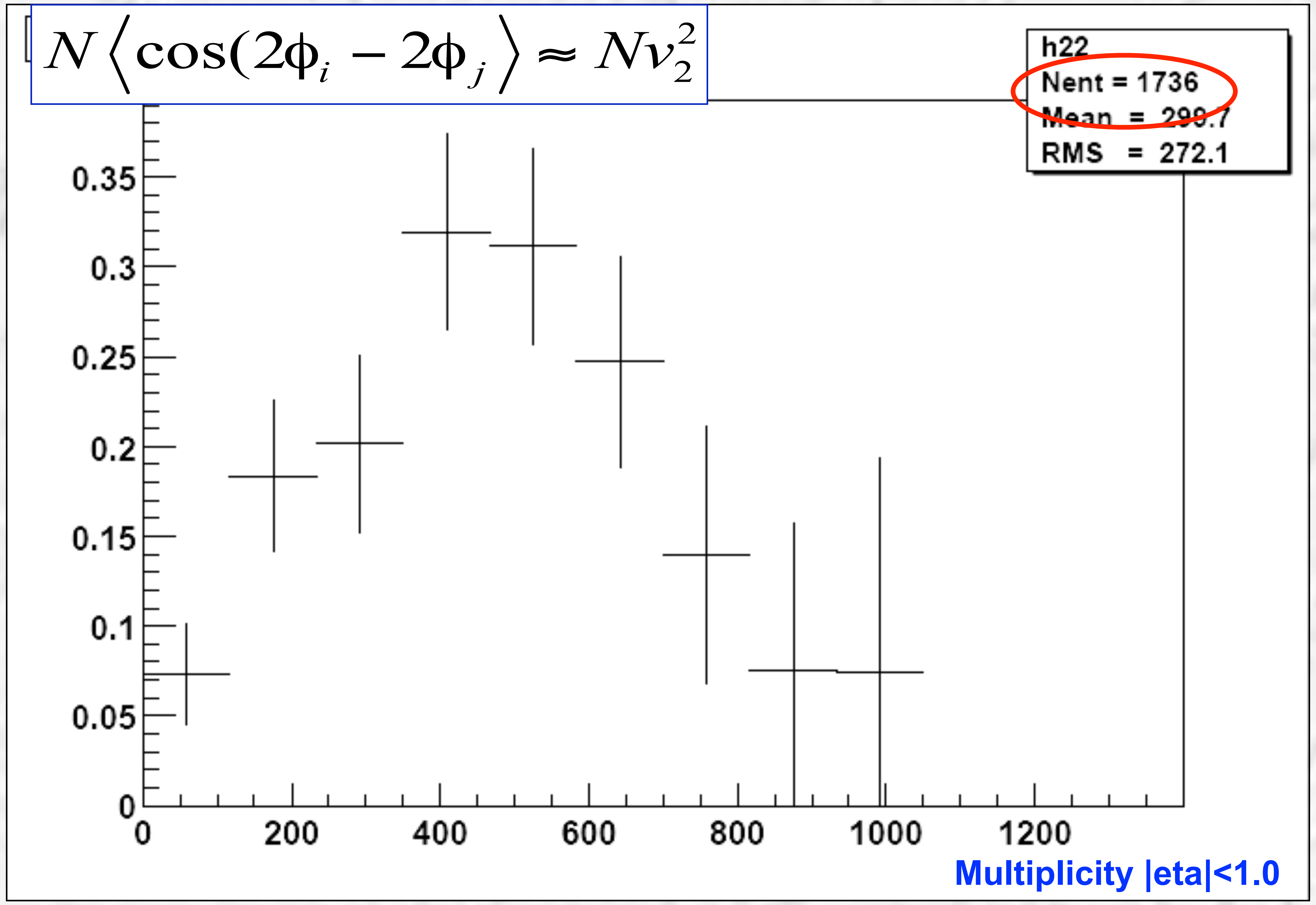


Hydro limit

$$v_2 \propto \varepsilon_2$$

First observation of flow at RHIC

2-particle azimuthal correlations



Results from data taken during the first three hours of RHIC operation

Elliptic flow centrality/energy dependence

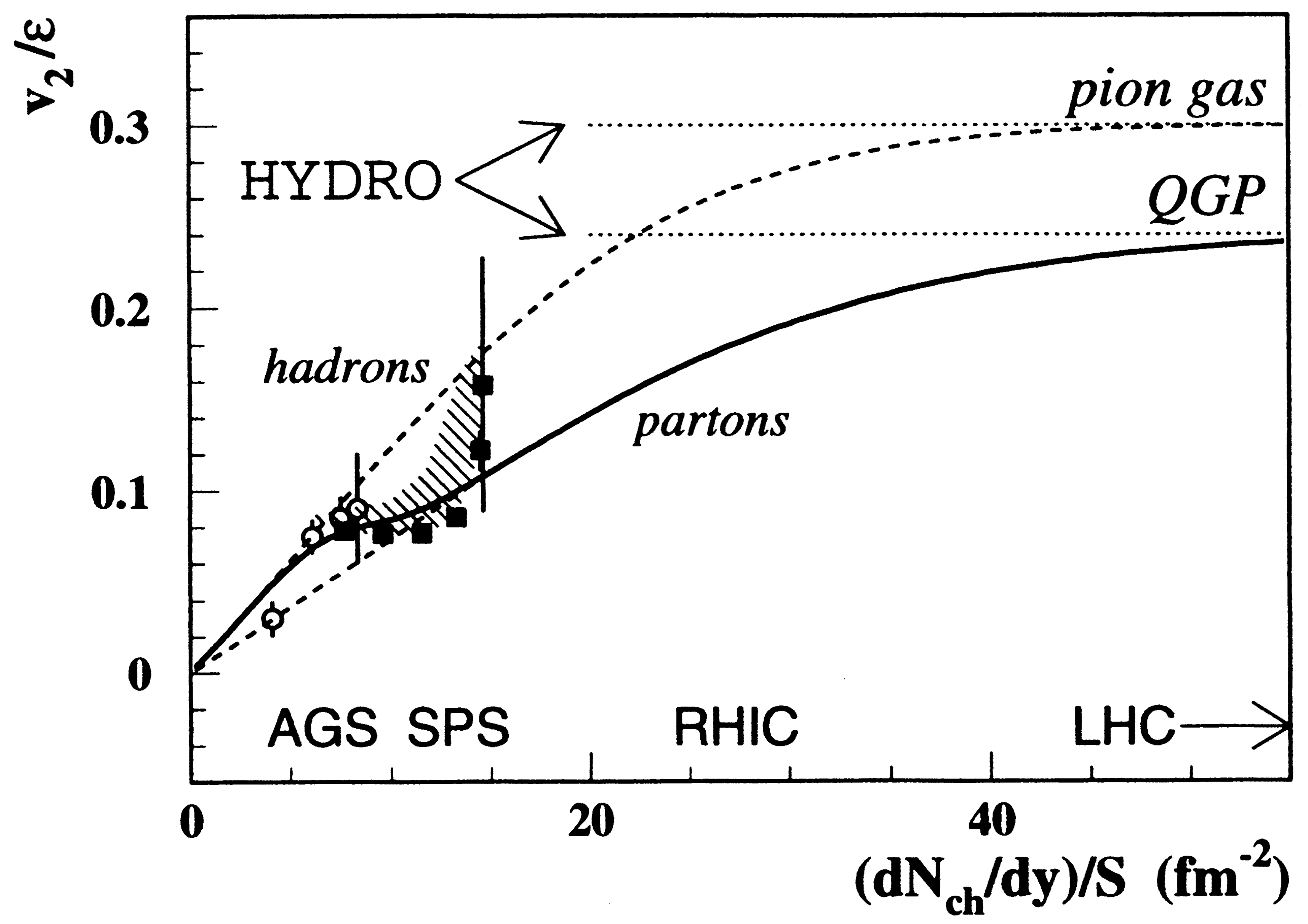
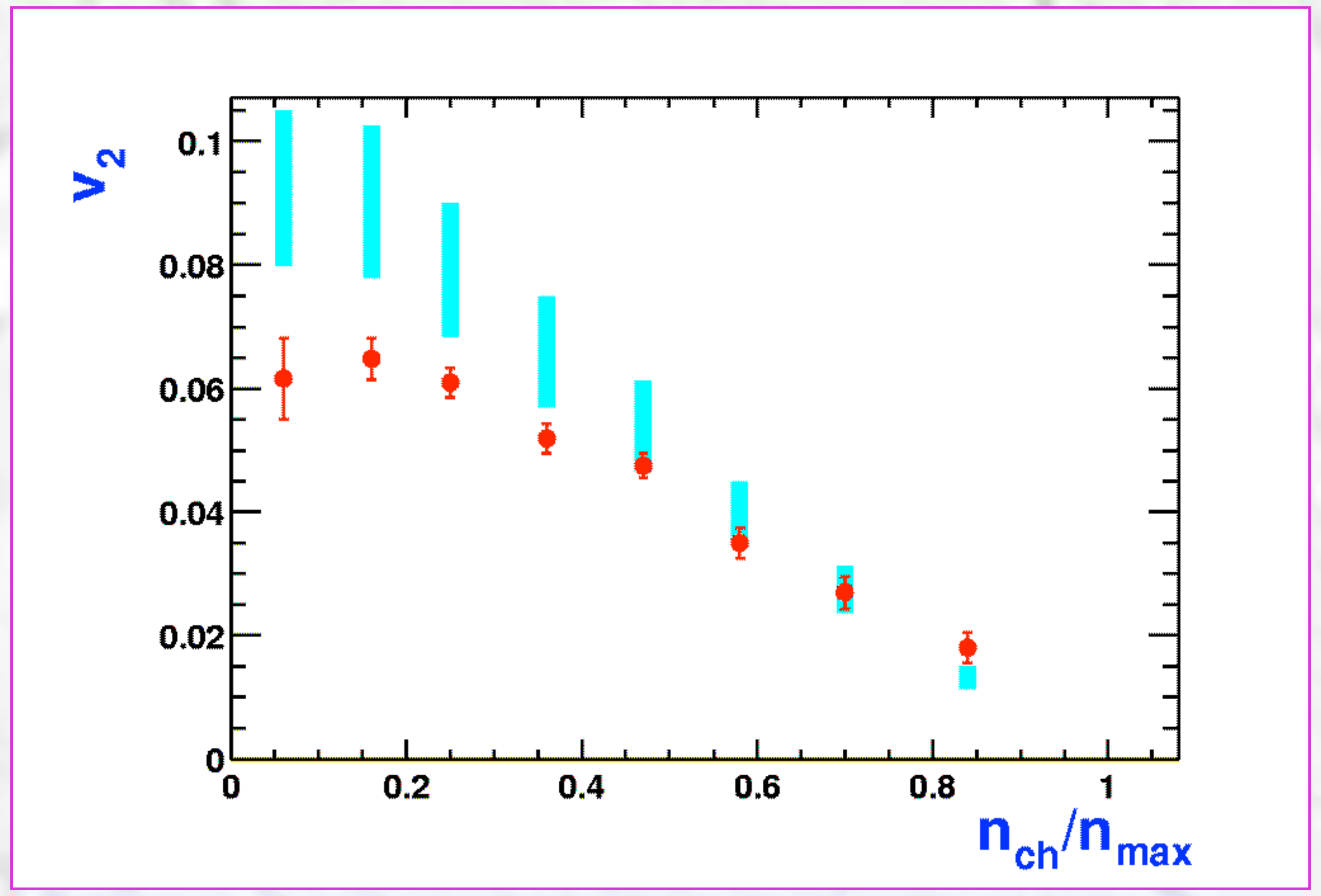


Fig. 3. Elliptic flow divided by the initial space elliptic anisotropy at the AGS (open circles) and the SPS (filled squares). The shaded area shows the uncertainty in the SPS experimental data due to the uncertainty in the centrality determination. See text and footnote for the description of the curves and hydro limits.



DIRECTED AND ELLIPTIC FLOW OF CHARGED PIONS . . . PHYSICAL REVIEW C **68**, 034903 (2003)

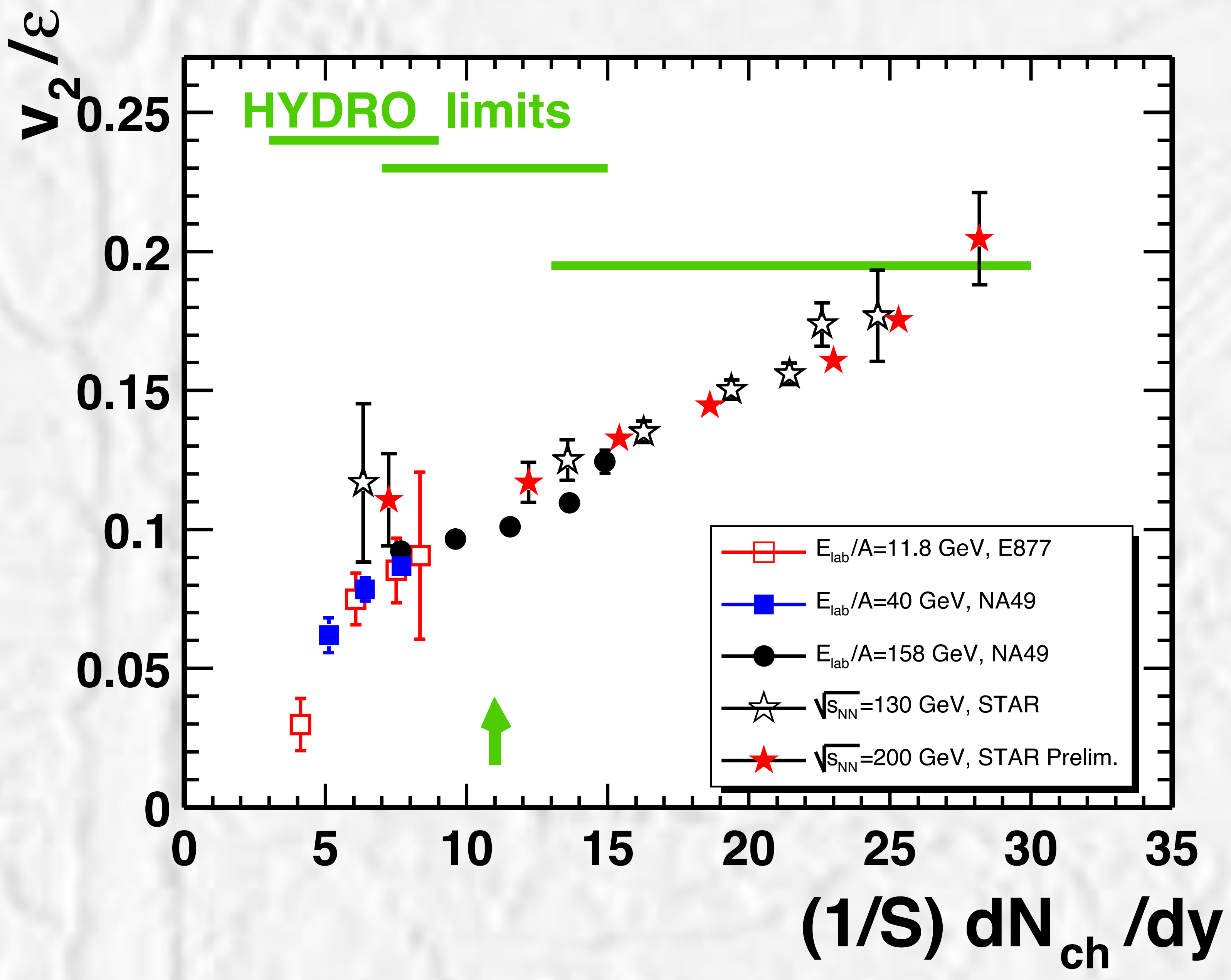
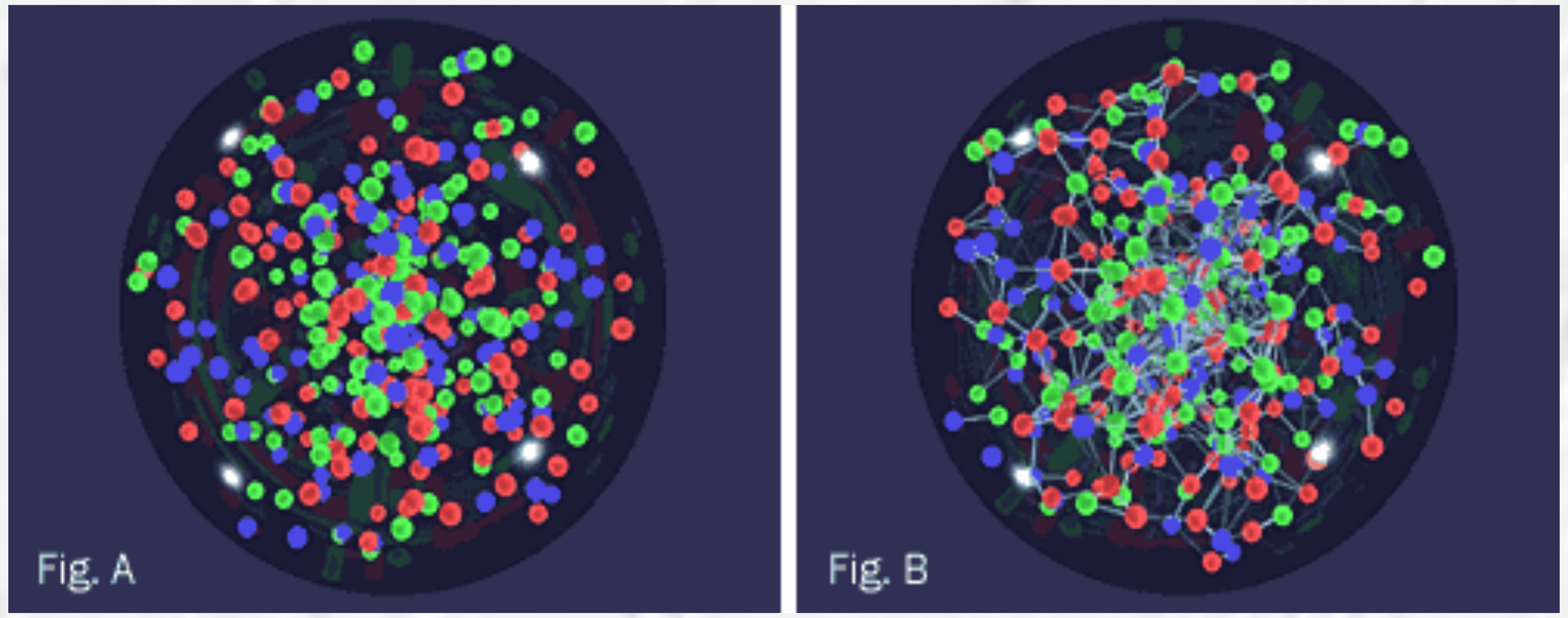
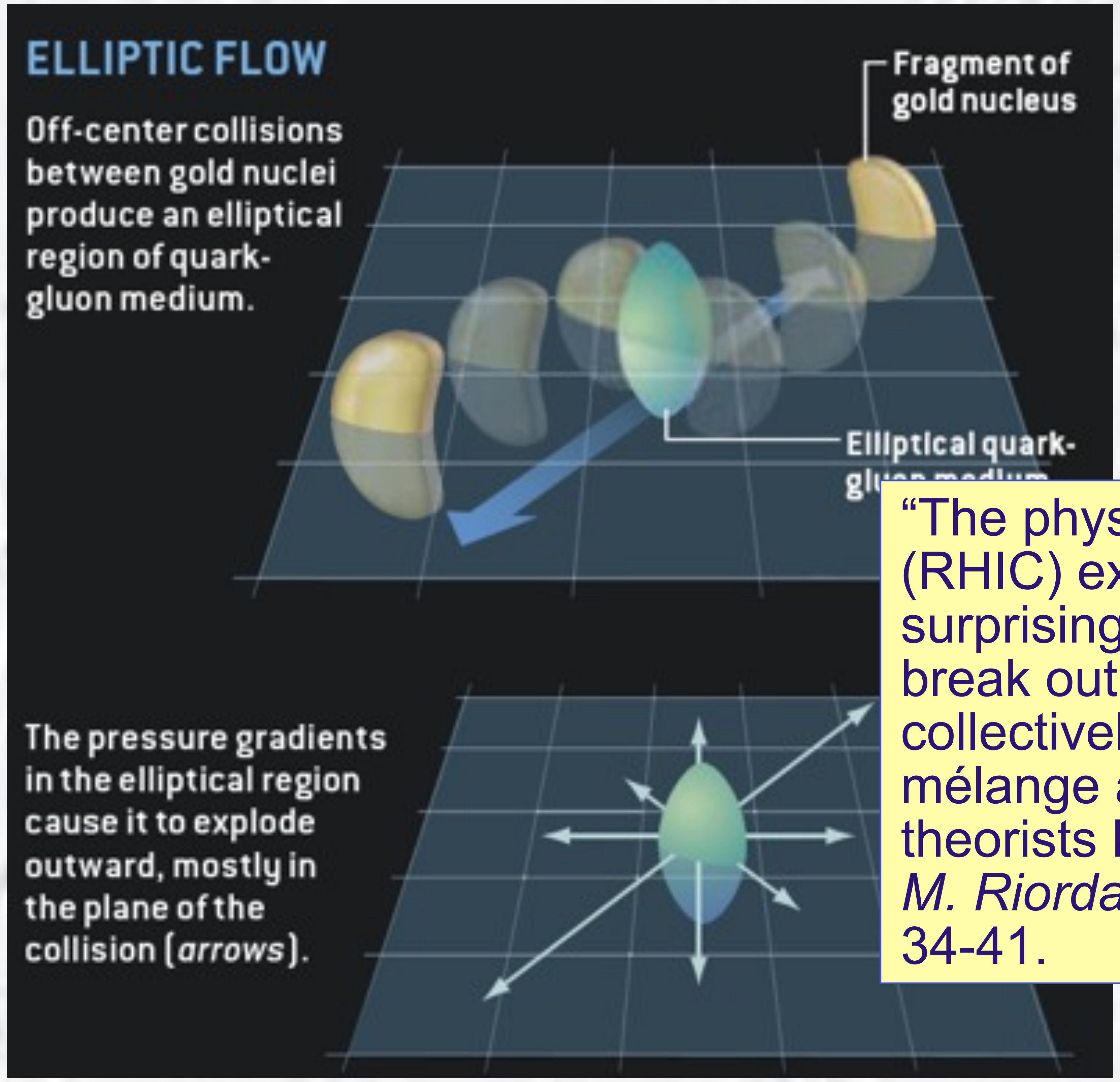


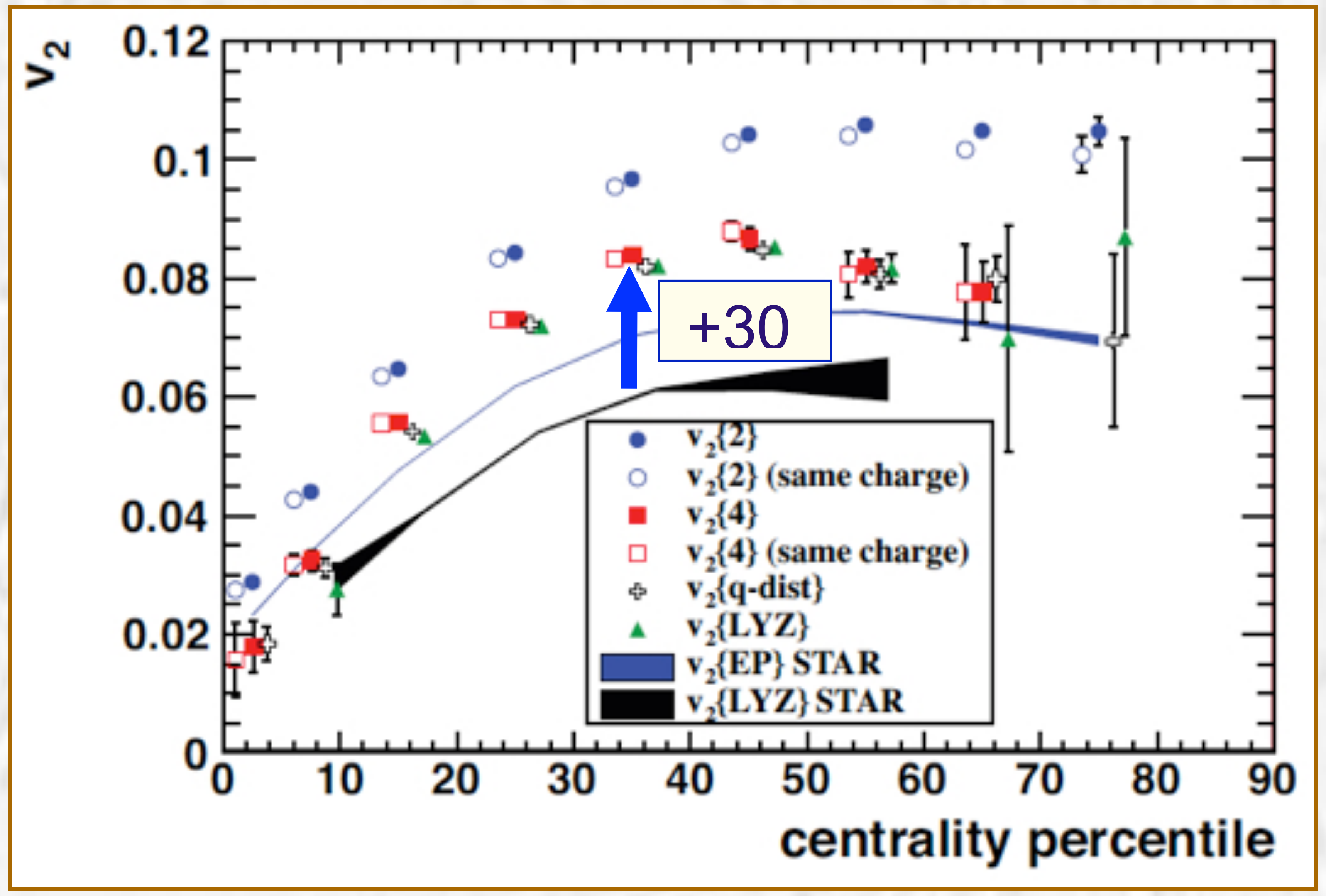
FIG. 25. (Color online) v_2/ϵ as a function of particle density. The v_2 values are for near midrapidity ($0<y<0.6$ for $40A$ GeV and $0<y<0.8$ for $158A$ GeV). The results of NA49 pion v_2 are compared to charged particle v_2 measured by E877 and STAR. The meaning of the horizontal lines (hydro limits) and of the arrow will be discussed in Sec. VI.

QGP - Gas or Liquid?





"The physical picture emerging from the four (RHIC) experiments is consistent and surprising. The quarks and gluons indeed break out of confinement and behave collectively, if only fleetingly. But this hot mélange acts like a liquid, not the ideal gas theorists had anticipated."
M. Riordan, W. Zajc, Sci. Am., May 2006, 34-41.



LHC: Increase in elliptic flow ~30%, in agreement with hydrodynamics

CERN Press release, November 26, 2010:
'confirms that the much hotter plasma produced at the LHC behaves as a very low viscosity liquid (a perfect fluid)..'

QGP - The Perfect fluid

Universe May Have Begun as Liquid, Not Gas

Associated Press
Tuesday, April 19, 2005; Page A05

The Washington Post

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have perva

Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

nature

by Mark Peplow
news@nature.com

The Universe consisted of a perfect liquid in its first momen results from an atom-smashing experiment.

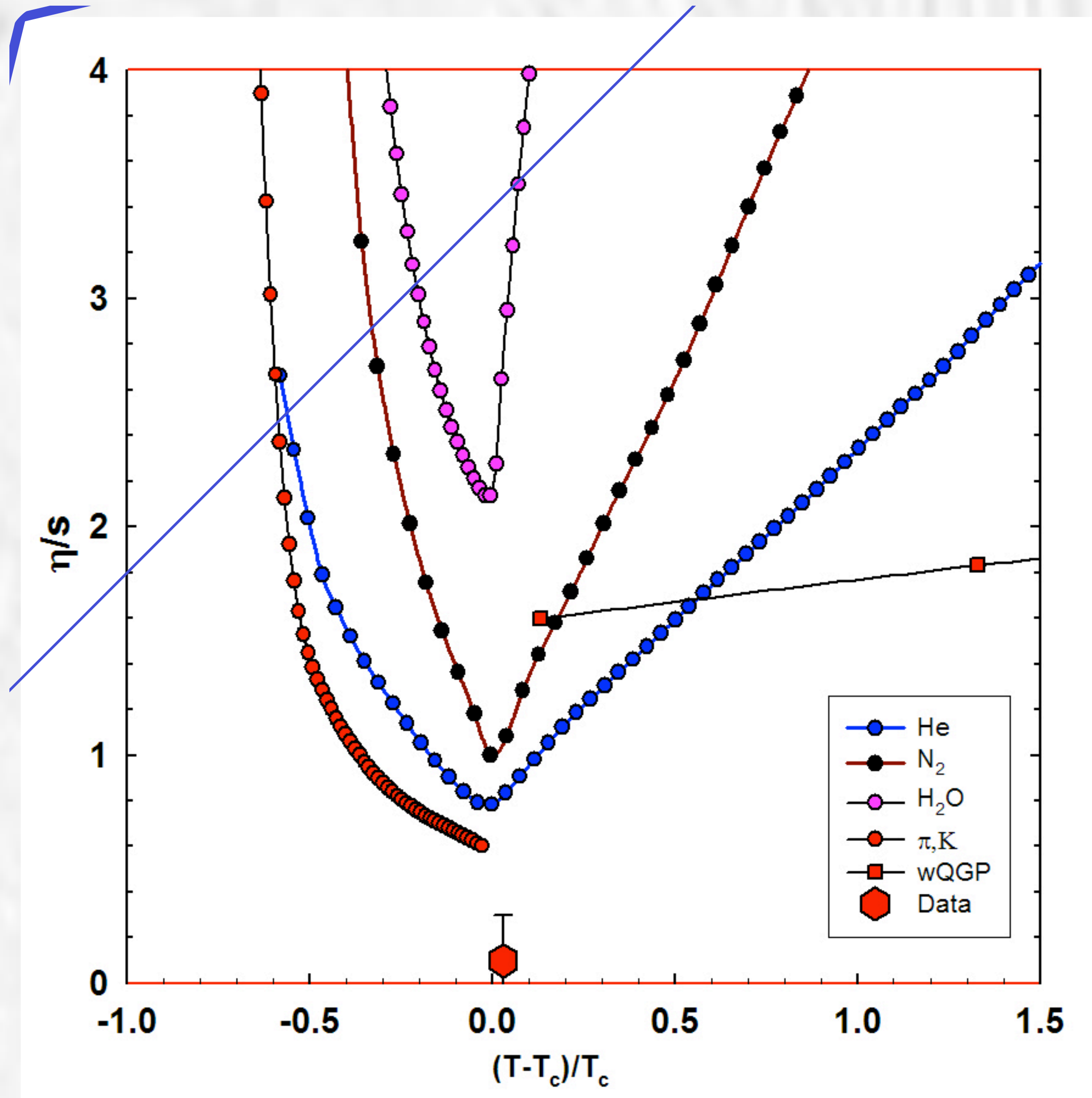
New State of Matter Is 'Nearly Perfect' Liquid

Physicists working at Brookhaven National Laboratory announced today that they have created what appears to be a new state of matter out of the building blocks of atomic nuclei, quarks and gluons. The researchers unveiled their findings--which could provide new insight into the composition of the universe just moments after the big bang--today in Florida at a meeting of the American Physical Society.

SCIENTIFIC AMERICAN

Image: BNL

There are four collaborations, dubbed BRAHMS, PHENIX, PHOBOS and STAR, working at Brookhaven's Relativistic Heavy Ion Collider (RHIC). All of them study what happens when two interacting beams of gold ions smash into one another at great velocities, resulting in thousands of subatomic collisions every second. When the researchers analyzed the patterns of the atoms' trajectories after these collisions, they

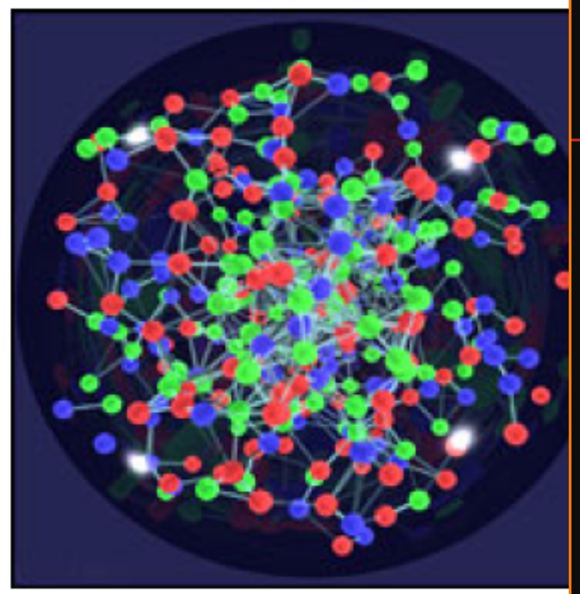


Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.

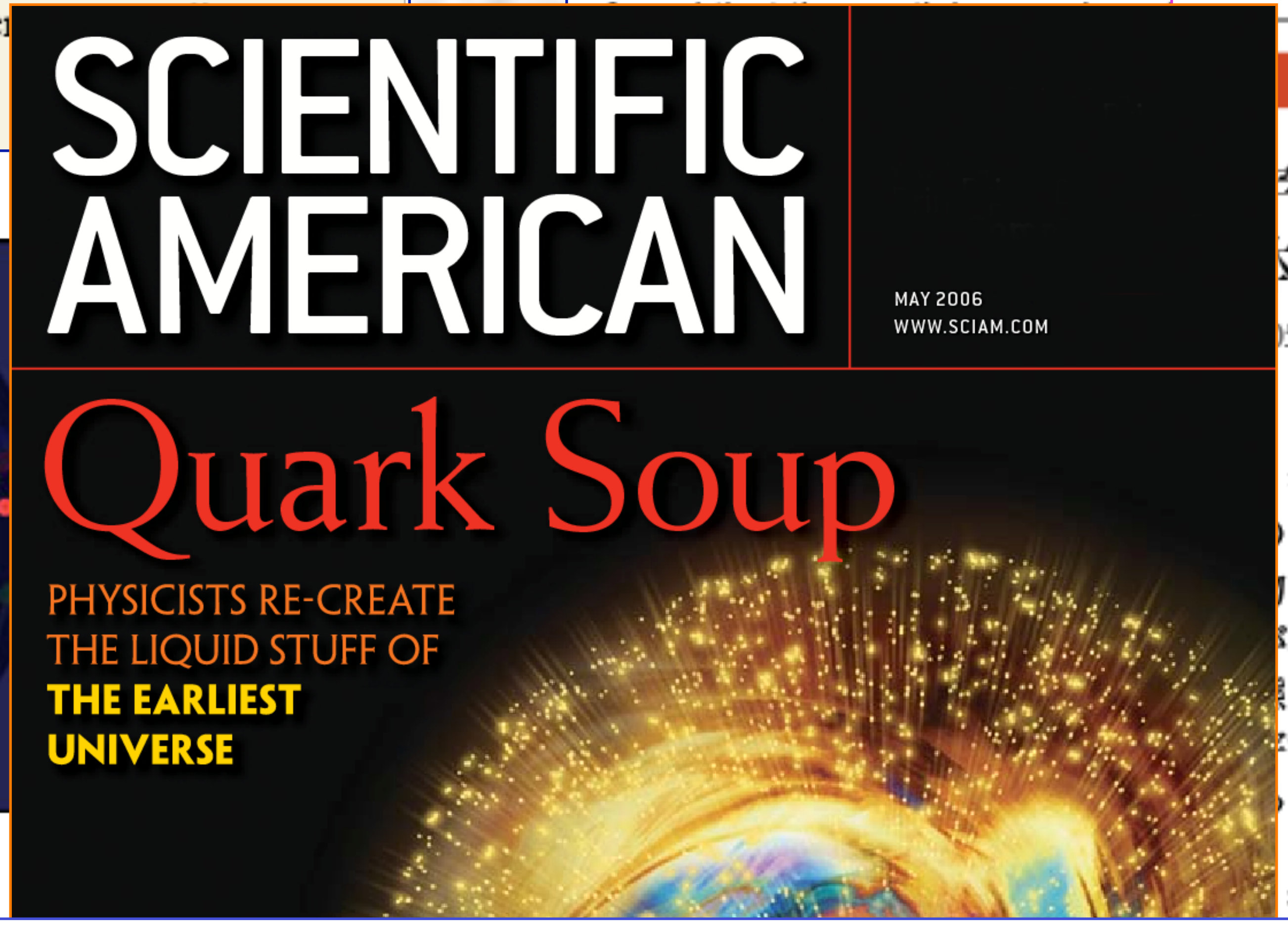
BBC NEWS

The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.



The impression is of matter that is more strongly interacting than predicted

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".



社会 asahi.comトップ> 社会> その他・話題

宇宙の始まりはしずく？　「クオークは液体」と発表

05年04月18日23時34分

宇宙誕生の大爆発「ビッグバン」直後に相当する超高温・高密度の状態を再現する実験をしてきた日米などの国際チームは１８日、物質を形づくる究極の基本粒子クオークは超高温でバラバラになる、気体のように自由に跳び回るのでなく、しずくのような液体状態にあったと考えられる、と発表宇宙や物質のなりたちを説明するる。

基本粒子クオークとそれらをく

Science

Iran Daily April 20, 2005 4

rly Universe Liquid-Like

Aronson, associate ctor for high energy nuclear physics at okhaven National oratory, which is ted on Long Island ut 65 miles east of York city.

tween 2000 and 3 the lab's ativistic Heavy Ion Collider, known as RHIC, repeatedly smashed the nuclei of gold atoms together with such force that their energy briefly generated trillion-degree temperatures. Physicists think of the collider as a time machine, because those extreme temperature conditions last prevailed in the universe less than 100 millionths of a second after the big bang.

Everything was so hot then that quarks and gluons, which are now almost inextricably bound into the protons and neutrons inside atomic nuclei, were thought to have flown around like BBs in a blender.

But by reproducing the conditions of the early universe, RHIC has shown that unconstrained quarks and gluons don't fly away in all directions so much as squirt out in streams.

"The matter that we've formed behaves like a very nearly perfect liquid," Aronson said.

When physicists talk about a perfect liquid, they don't mean the best glass of champagne they ever tasted. The word "perfect" refers to the liquid's viscosity, a friction-like property that affects a flui flow and th to objects try through it. high viscos viscosity is fect liquid ha ity at all, whi sible in realit for theoreti sions.

Theoretical physicists have recently proposed that material swallowed and what goes on when two gold nuclei collide at RHIC.

concept of the early universe, the new discovery offers opportunities to

Society.

"There are a lot of exciting questions," said

Early Universe Went With the Flow

Posted April 18, 2005 5:57PM

Between 2000 and 2003 the lab's Relativistic Heavy Ion Collider repeatedly smashed the nuclei of gold atoms together with such force that their energy briefly generated trillion-degree temperatures. Physicists think of the collider as a time machine, because those extreme temperature conditions last prevailed in the universe less than 100 millionths of a second after the big bang.

Energy dependence: AGS-SPS-RHIC-LHC

version 7, November 7, 2010. Text: new, old, questions

Elliptic flow of charged particles in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

de M. Michel Nostradamus

We report the first measurement of charged particle elliptic flow in Pb+Pb collisions at $\sqrt{s_{NN}}=$ 2.76 TeV with the ALICE detector at the CERN Large Hadron Collider. The measurement is performed in the central pseudorapidity region ($|\eta| < 0.8$) and transverse momentum range $0.25 < p_t < 5$ GeV/c. The elliptic flow signal, v_2, averaged over transverse momentum and pseudorapidity, reaches values of 0.085 for relatively peripheral collisions (40–50% most central). The differential elliptic flow $v_2(p_t)$ reaches a maximum of 0.25 around $p_t = 3$ GeV/c. Compared to RHIC Au+Au collisions at $\sqrt{s_{NN}}=$ 200 GeV, the elliptic flow increases by about 15% in agreement with hydrodynamical model predictions.

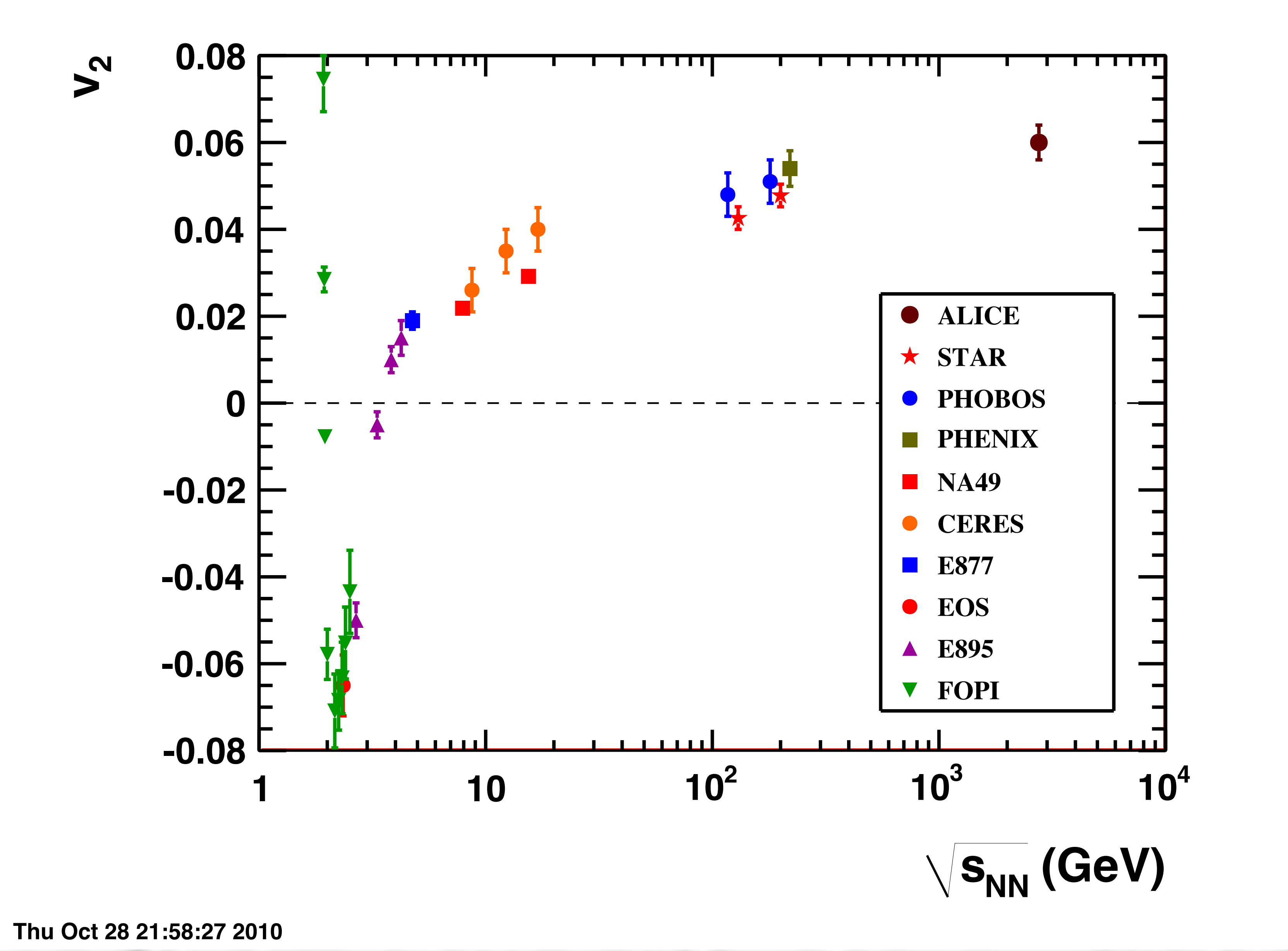


FIG. 4. Integrated elliptic flow in Pb+Pb 20–30% centrality collisions at 2.76 TeV compared with results from lower energies taken at similar centralities. The compilation is taken from [26].

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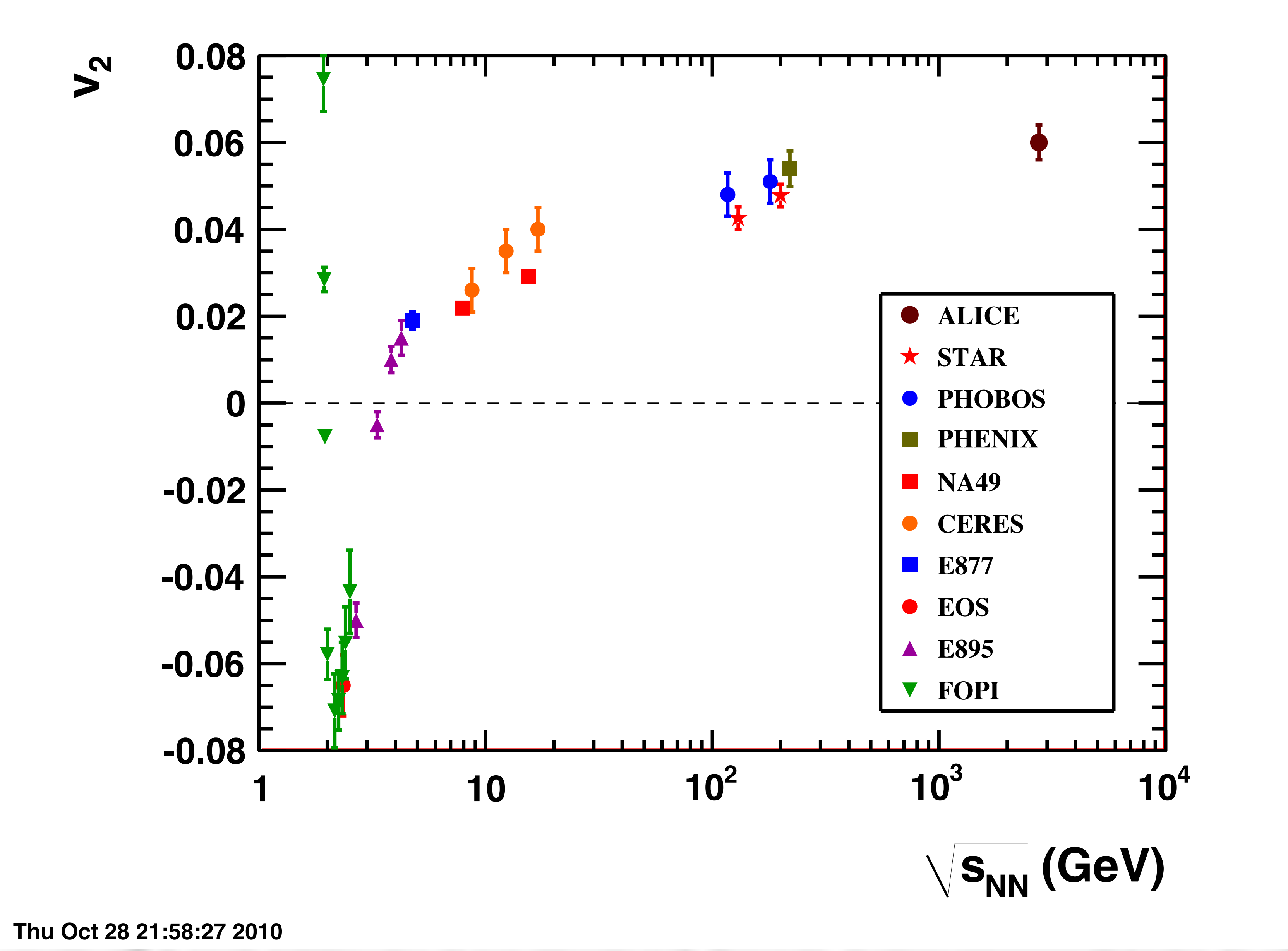

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Selected for a Viewpoint in *Physics*
PRL **105,** 252302 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Elliptic Flow of Charged Particles in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

K. Aamodt *et al.**
(ALICE Collaboration)

(Received 18 November 2010; published 13 December 2010)

We report the first measurement of charged particle elliptic flow in Pb-Pb collisions at $\sqrt{s_{NN}} =$ 2.76 TeV with the ALICE detector at the CERN Large Hadron Collider. The measurement is performed in the central pseudorapidity region ($|\eta| < 0.8$) and transverse momentum range $0.2 < p_t < 5.0$ GeV/c. The elliptic flow signal v_2, measured using the 4-particle correlation method, averaged over transverse momentum and pseudorapidity is 0.087 ± 0.002(stat) $\pm$ 0.003(syst) in the 40%–50% centrality class. The differential elliptic flow $v_2(p_t)$ reaches a maximum of 0.2 near $p_t = 3$ GeV/c. Compared to RHIC Au-Au collisions at $\sqrt{s_{NN}} =$ 200 GeV, the elliptic flow increases by about 30%. Some hydrodynamic model predictions which include viscous corrections are in agreement with the observed increase.

DOI: 10.1103/PhysRevLett.105.252302 PACS numbers: 25.75.Ld, 25.75.Gz, 25.75.Nq

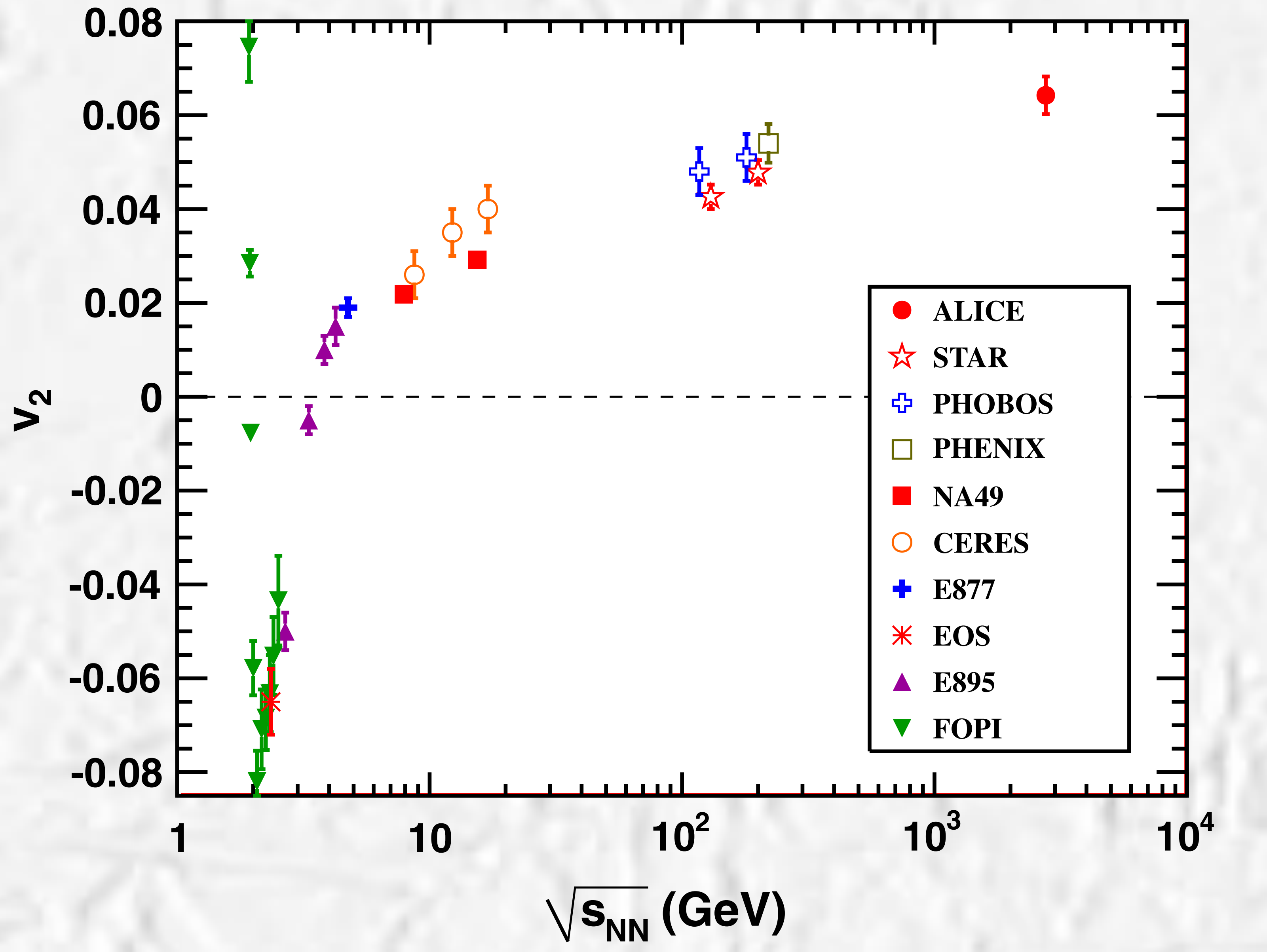
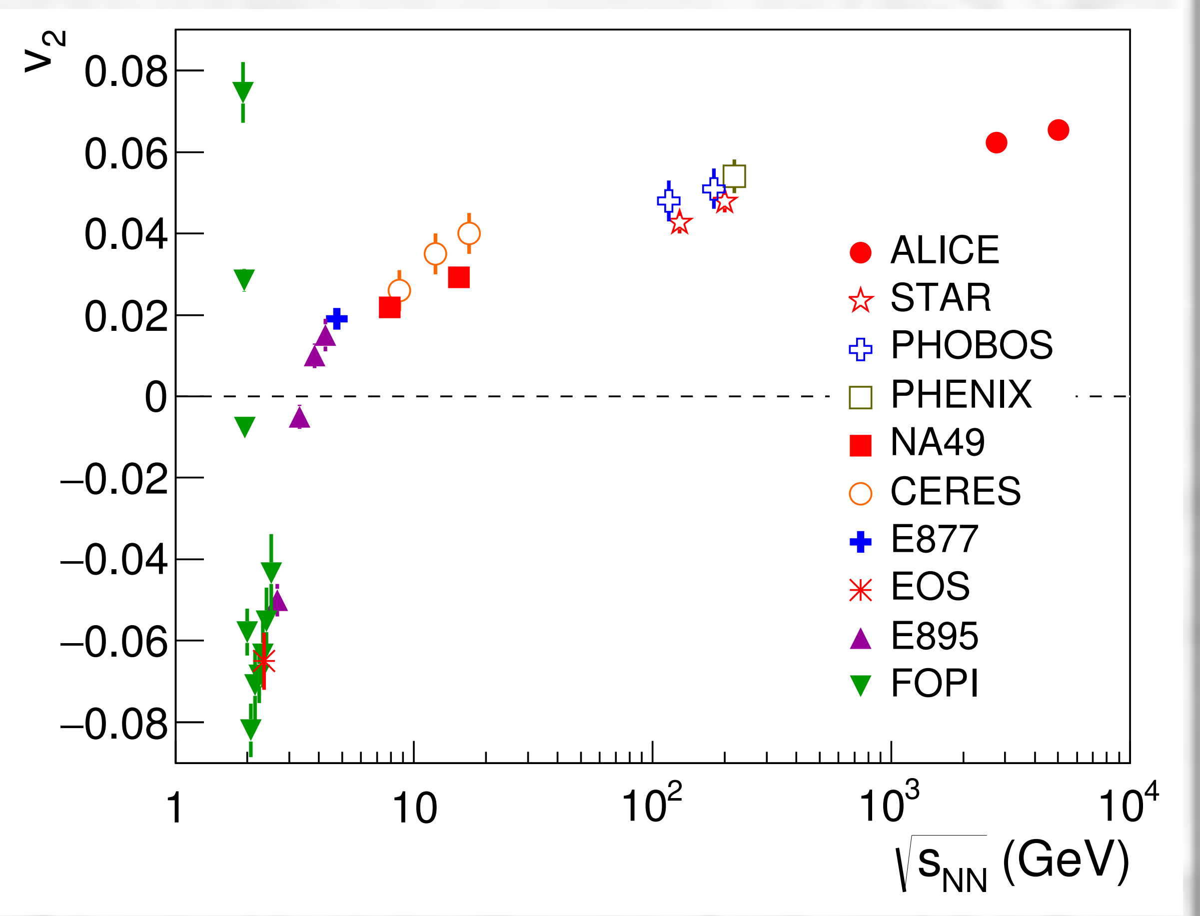
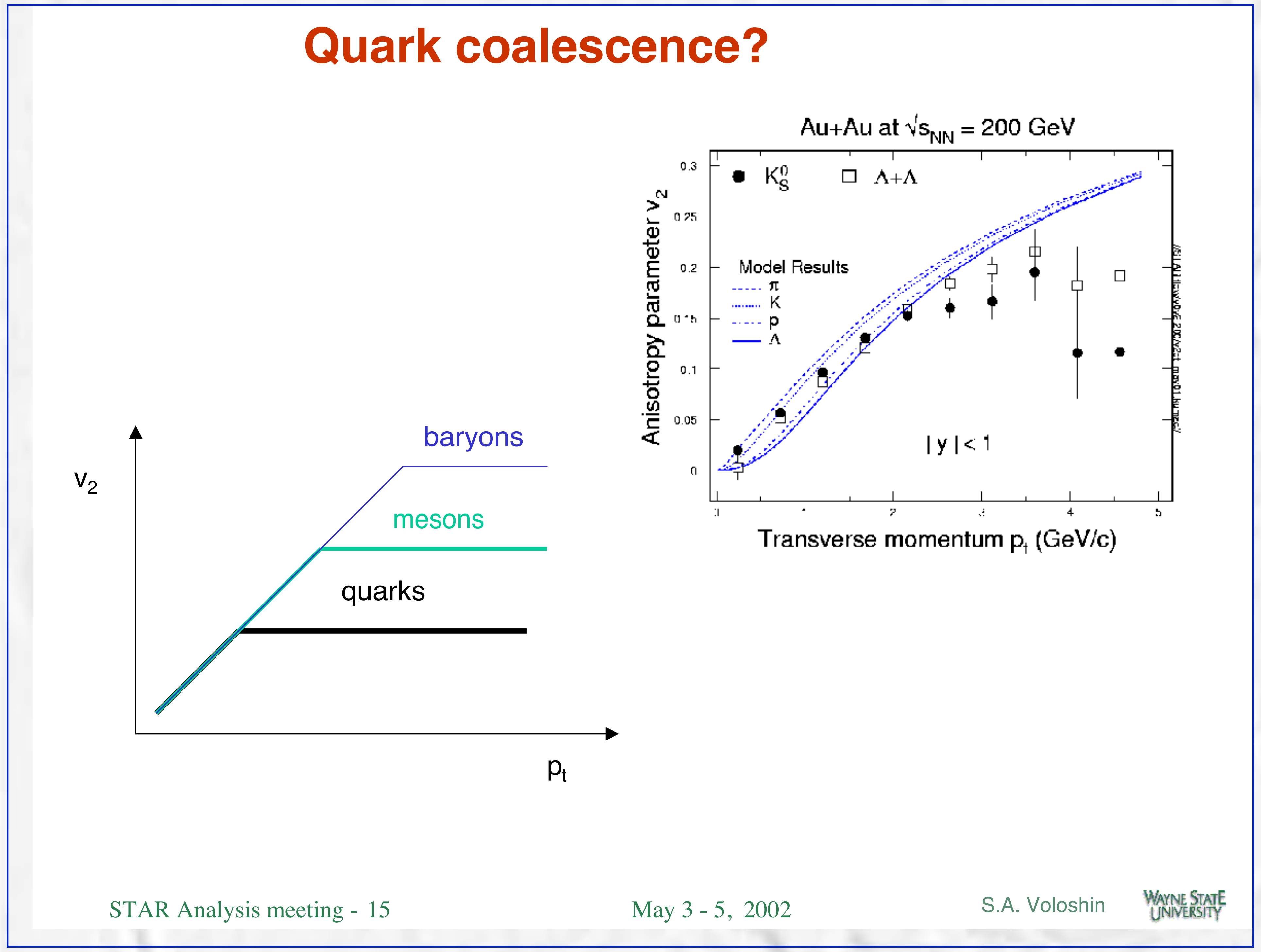


FIG. 4 (color online). Integrated elliptic flow at 2.76 TeV in Pb-Pb 20%–30% centrality class compared with results from lower energies taken at similar centralities [40,43].

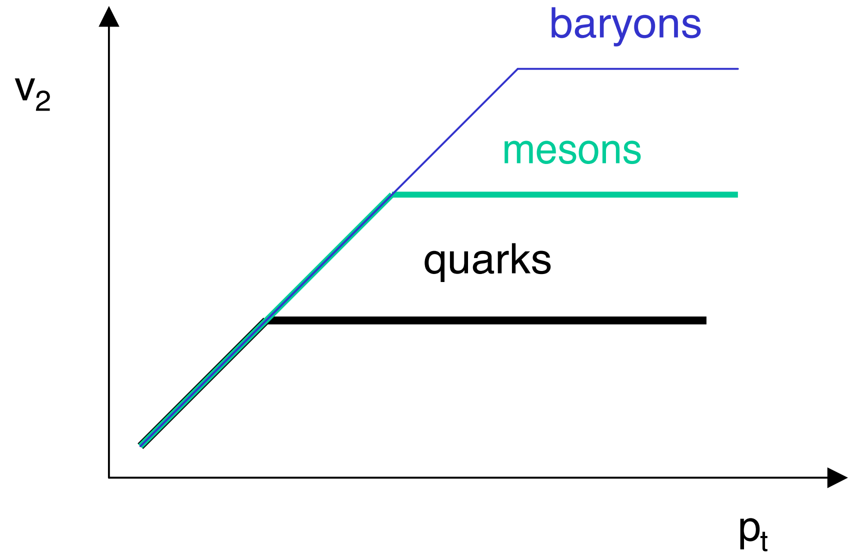


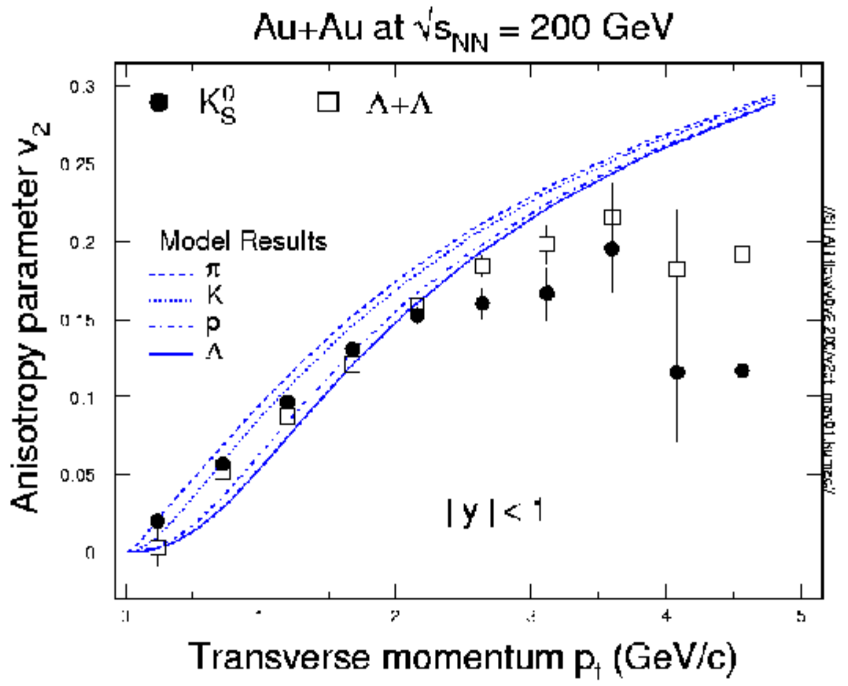
Constituent quark scaling



Constituent quark scaling

Quark coalescence?



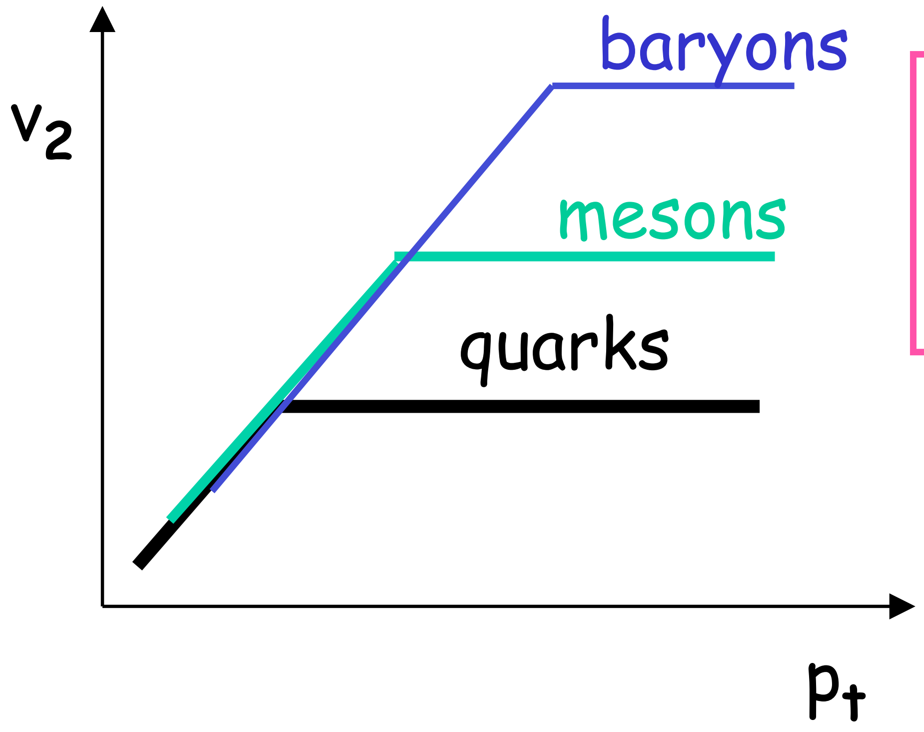


Constituent quark model + coalescence

coalescence — fragmentation

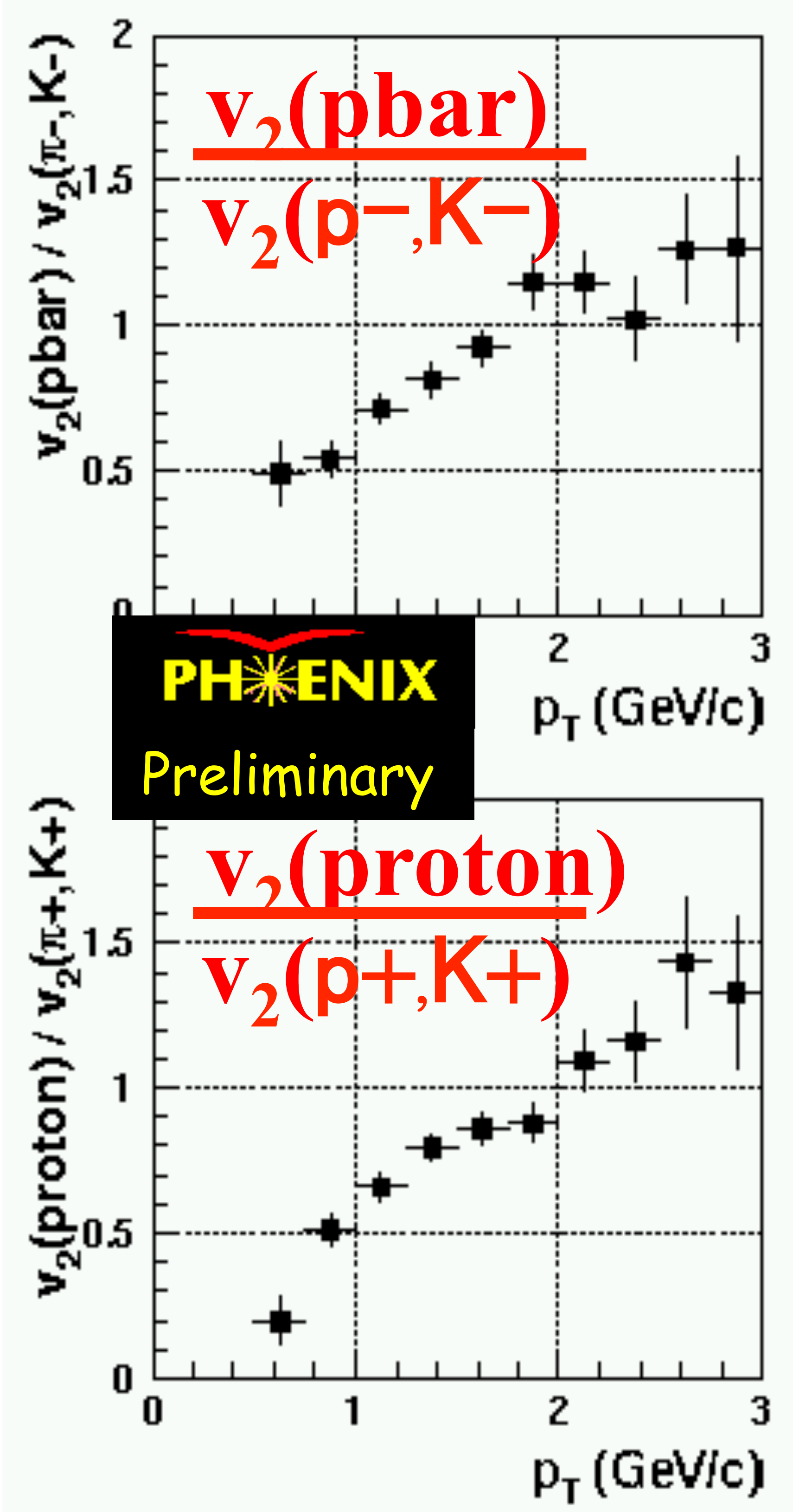
Low p_t quarks — High p_t quarks

Coalescence in the intermediate region (rare products):



$$\frac{d^3 n_M}{d^3 p_M} \propto \left[\frac{d^3 n_q}{d^3 p_q}\left(p_q \approx p_M/2\right)\right]^2$$

Side-notes:
a) more particles produced via coalescence vs parton fragmentation ➔ larger mean p_t...
b) ➔ higher baryon/meson ratio



$\frac{v_2(\text{pbar})}{v_2(p-,K-)}$

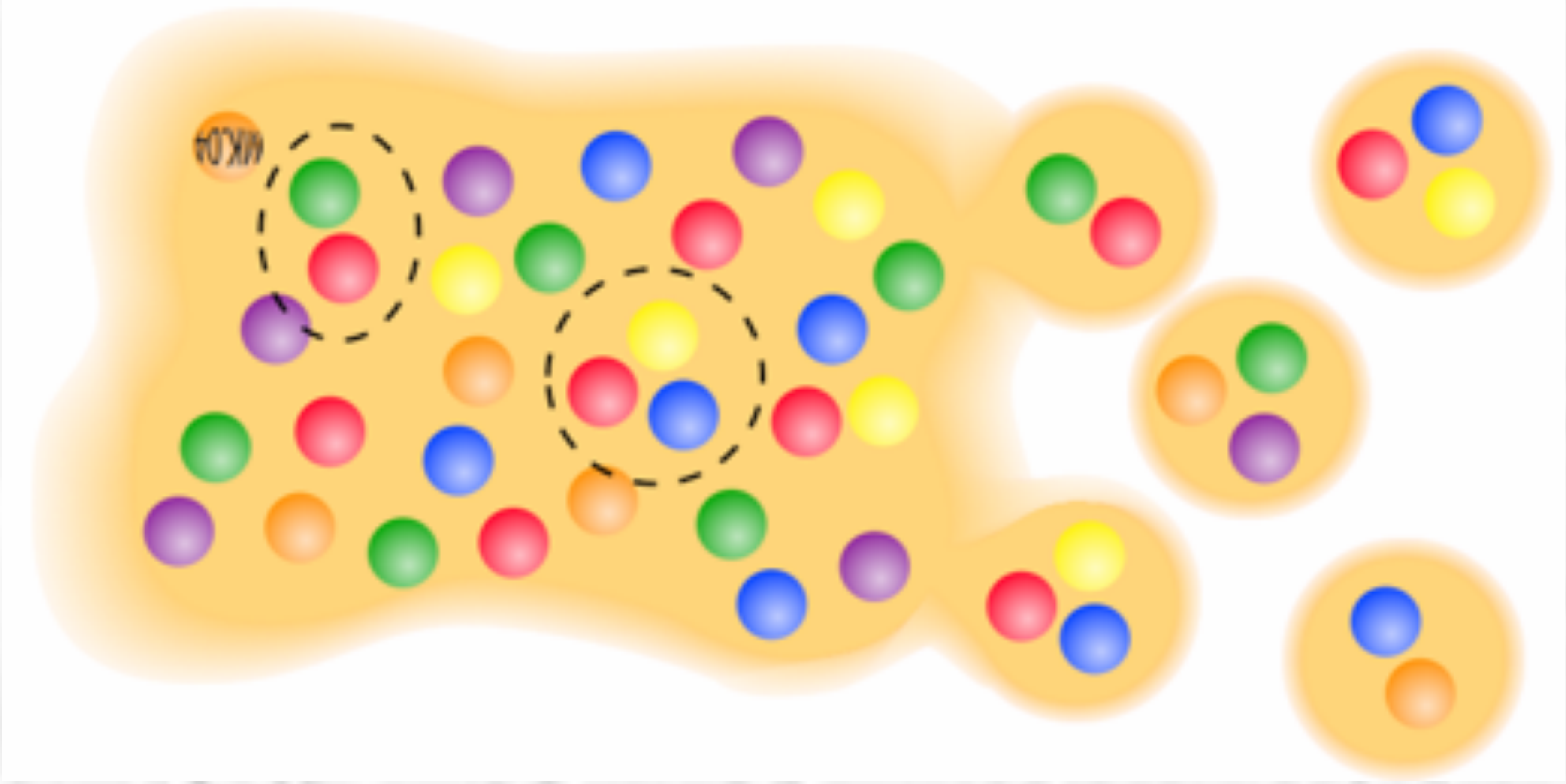
$\frac{v_2(\text{proton})}{v_2(p+,K+)}$

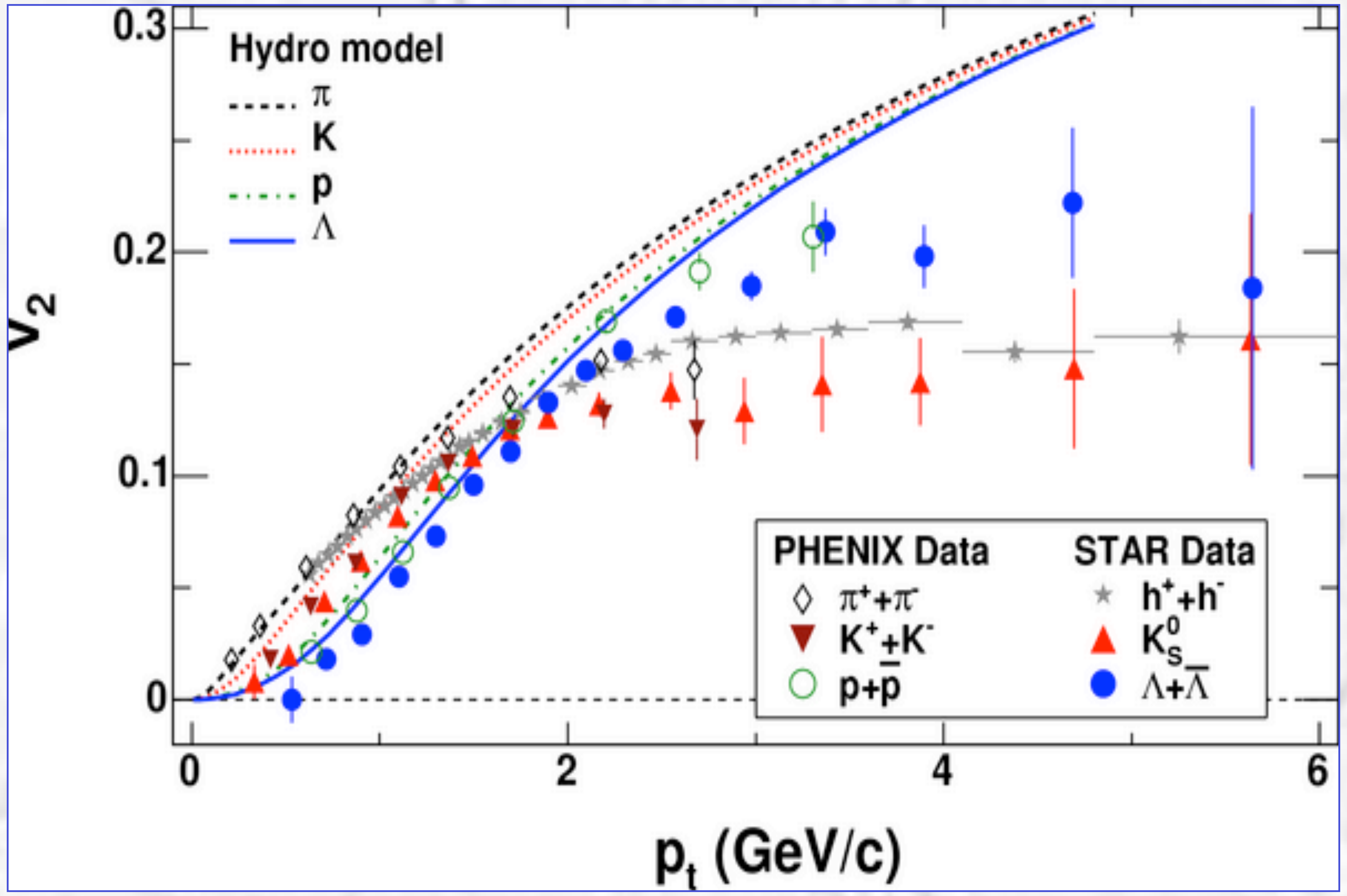
- What is the centrality dependence of the effect?

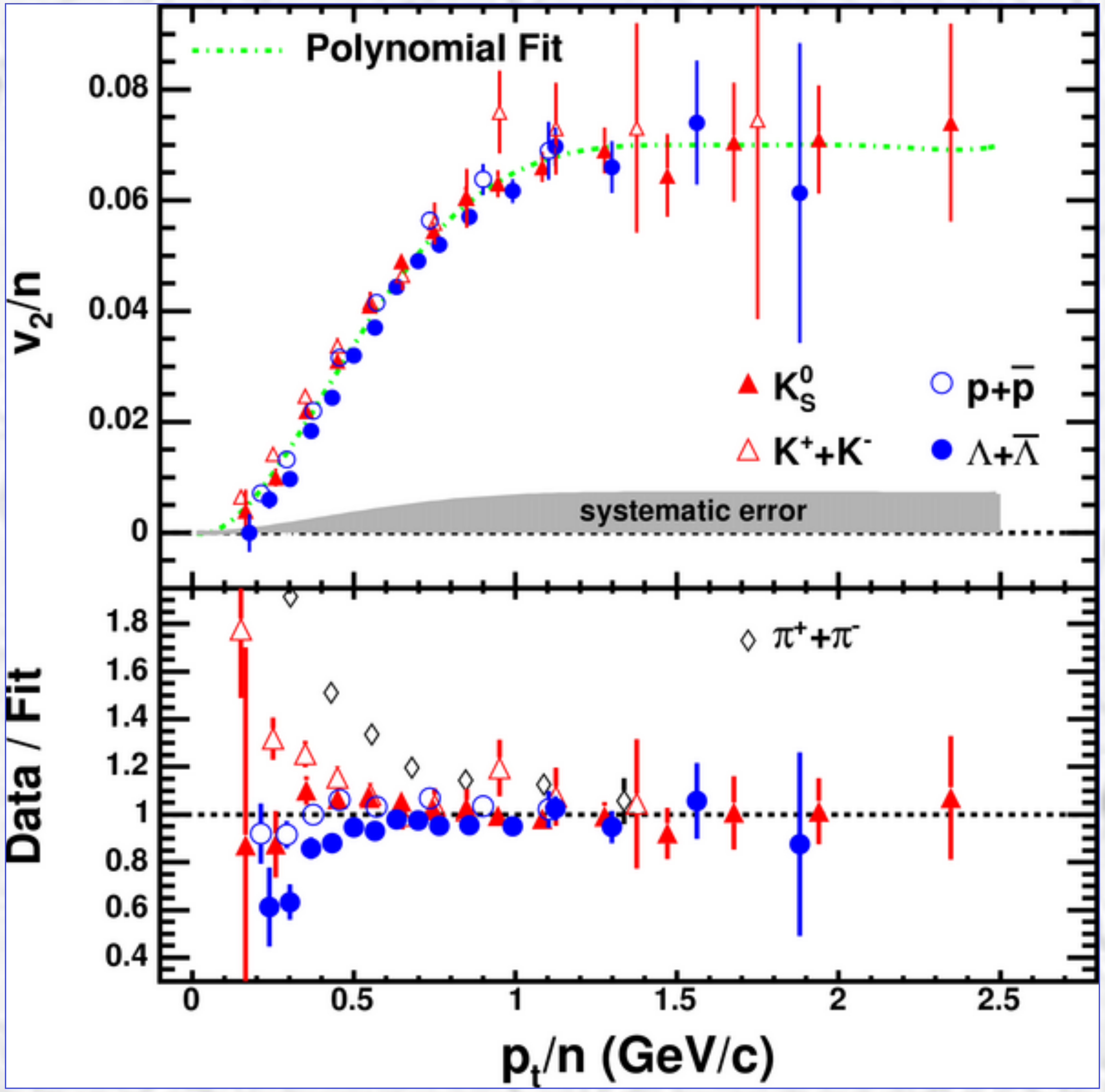
Quarks flow - deconfinement

STAR PRL 92(2004)052302

STAR, Phys Rev C (72), 014904 (2005)







WAYNE STATE UNIVERSITY

Nonflow and flow fluctuation

$$\langle \cos[n(\phi_i - \phi_j)] \rangle = \langle v_n^2 \rangle + \delta_n$$

Flow "non-flow"

Effect of flow fluctuations

$$\langle v_n^2 \rangle = \langle v_n \rangle^2 + \sigma_v^2$$

$$\langle v_{n,a} v_{n,b} \rangle = \langle v_{n,a} \rangle \langle v_{n,b} \rangle + \langle \langle v_{n,a} v_{n,b} \rangle \rangle$$

Nonflow – two- and many-particle azimuthal correlations of any origin other than the common correlation to the reaction plane.

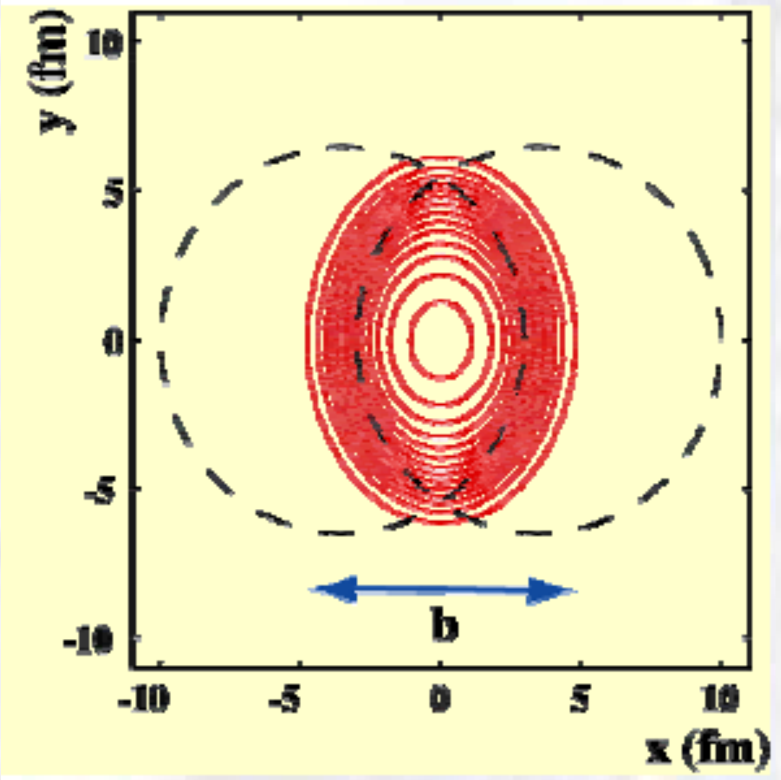
It includes contributions from resonance decay, inter- and intra-jet correlations, etc.

In general, two effects do not factorize; then the equation in a box would serve as a definition of "nonflow", with v_n's defined via single particle spectra.

An example: $v_n \propto \varepsilon_n$

…includes fluctuations in particle density (number of particles, area), etc.

$$\sigma_\varepsilon^2 = \left\langle \varepsilon^2(b) \right\rangle - \left\langle \varepsilon(b) \right\rangle^2$$

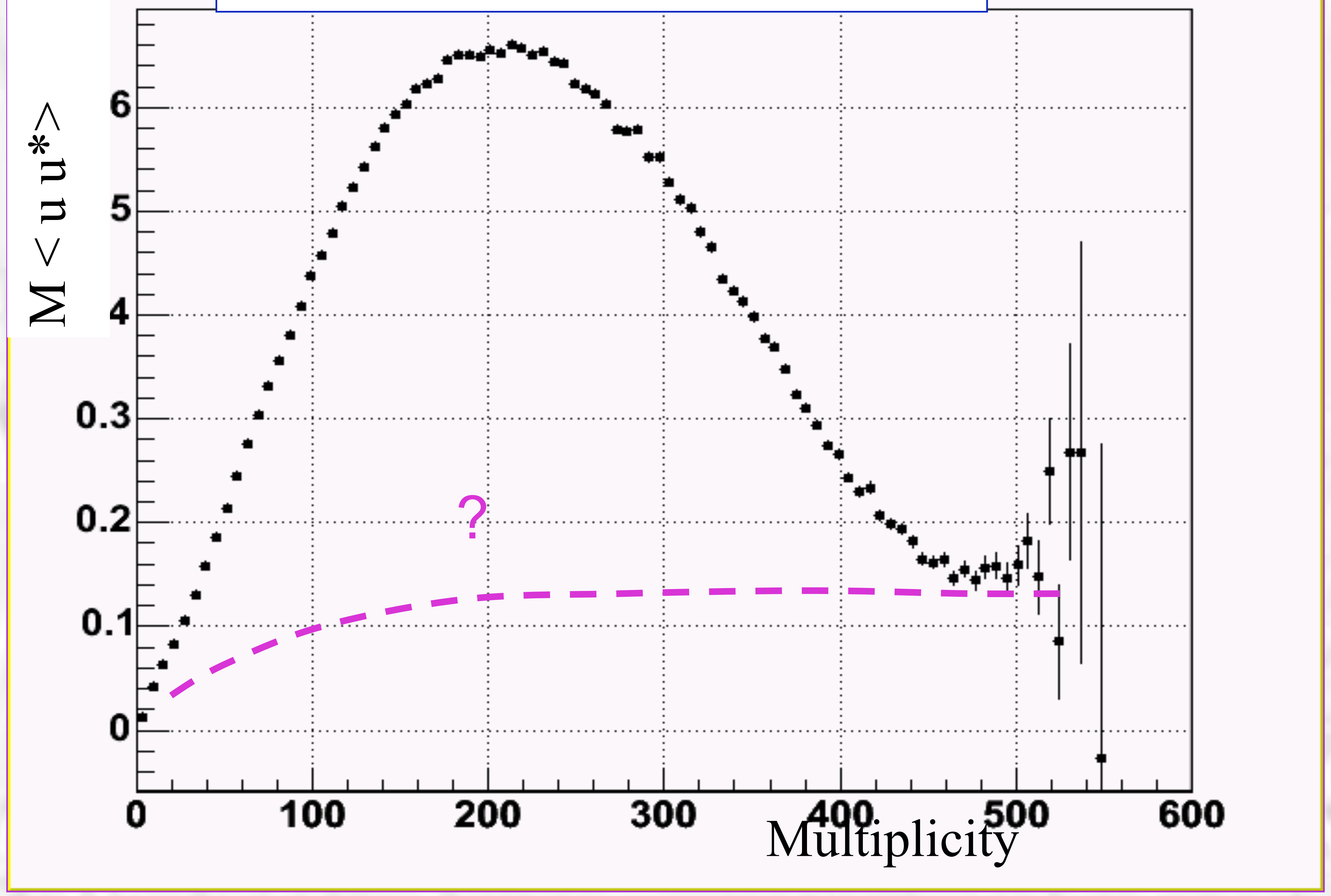


Non-flow estimates. Centrality dependence.

$$\langle v_2^2 \rangle = \langle v_2 \rangle^2 + \sigma_{v_2}^2 + g_2/N$$

Nonflow scales with multiplicity as 1/N, reflecting the probability that the second particle is from the same cluster as the first one.

$$N\left\langle \cos(2\phi_i - 2\phi_j) \right\rangle \approx N v_2^2$$



PHYSICAL REVIEW C **72**, 014904 (2005)

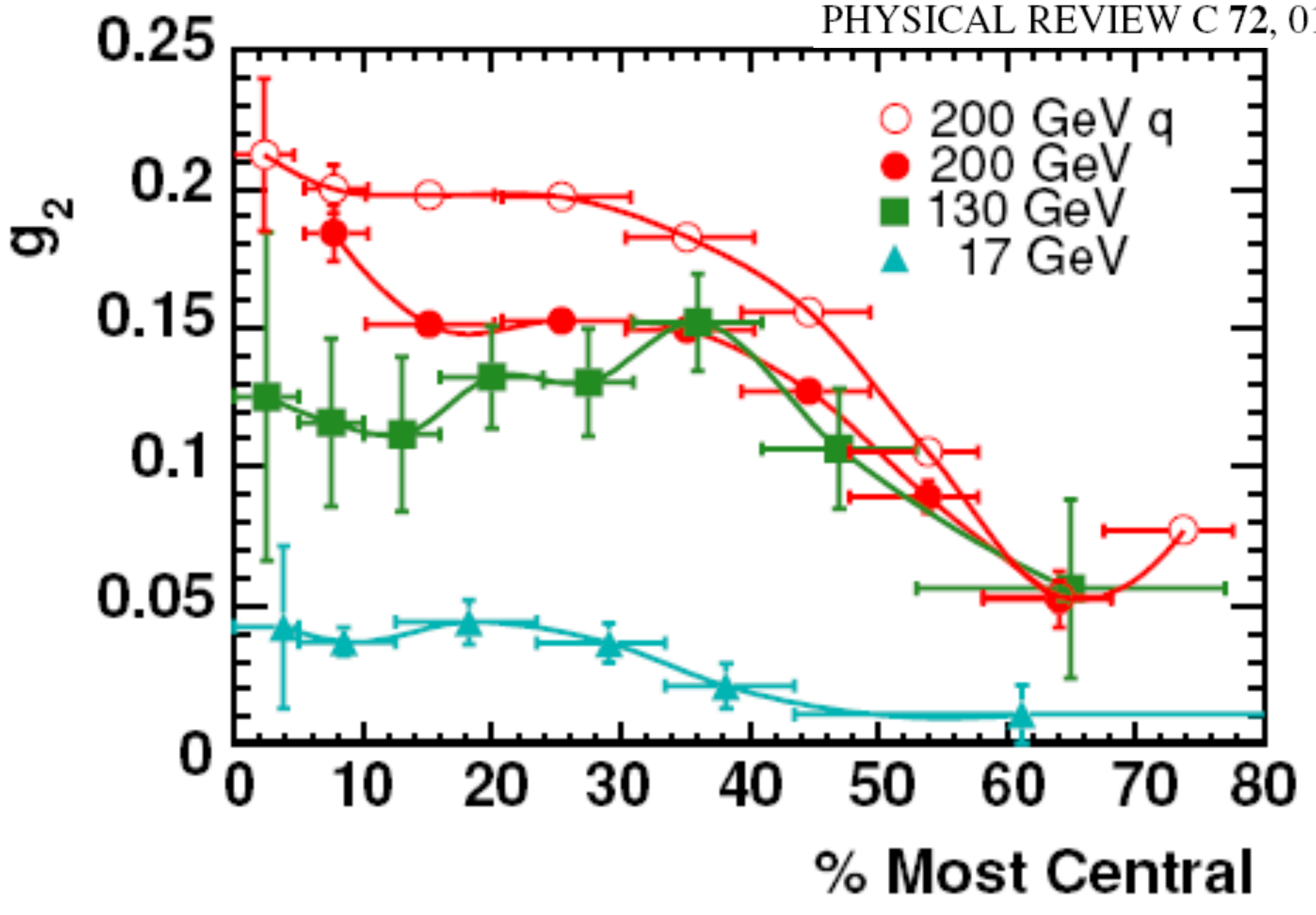


FIG. 31. (Color online) The nonflow parameter, g_2, as a function of centrality. The solid points are from the cumulant method. The open circles are from the q distribution method.

Nonflow: pp vs. AA

$$Q = \sum_{j \in \{b\}} u_j;\; u_j = e^{i\,2\phi_j}$$

$$\langle u_r Q^* \rangle = (v_r v_b + \delta_{rb}^{AA}) M^{AA}$$

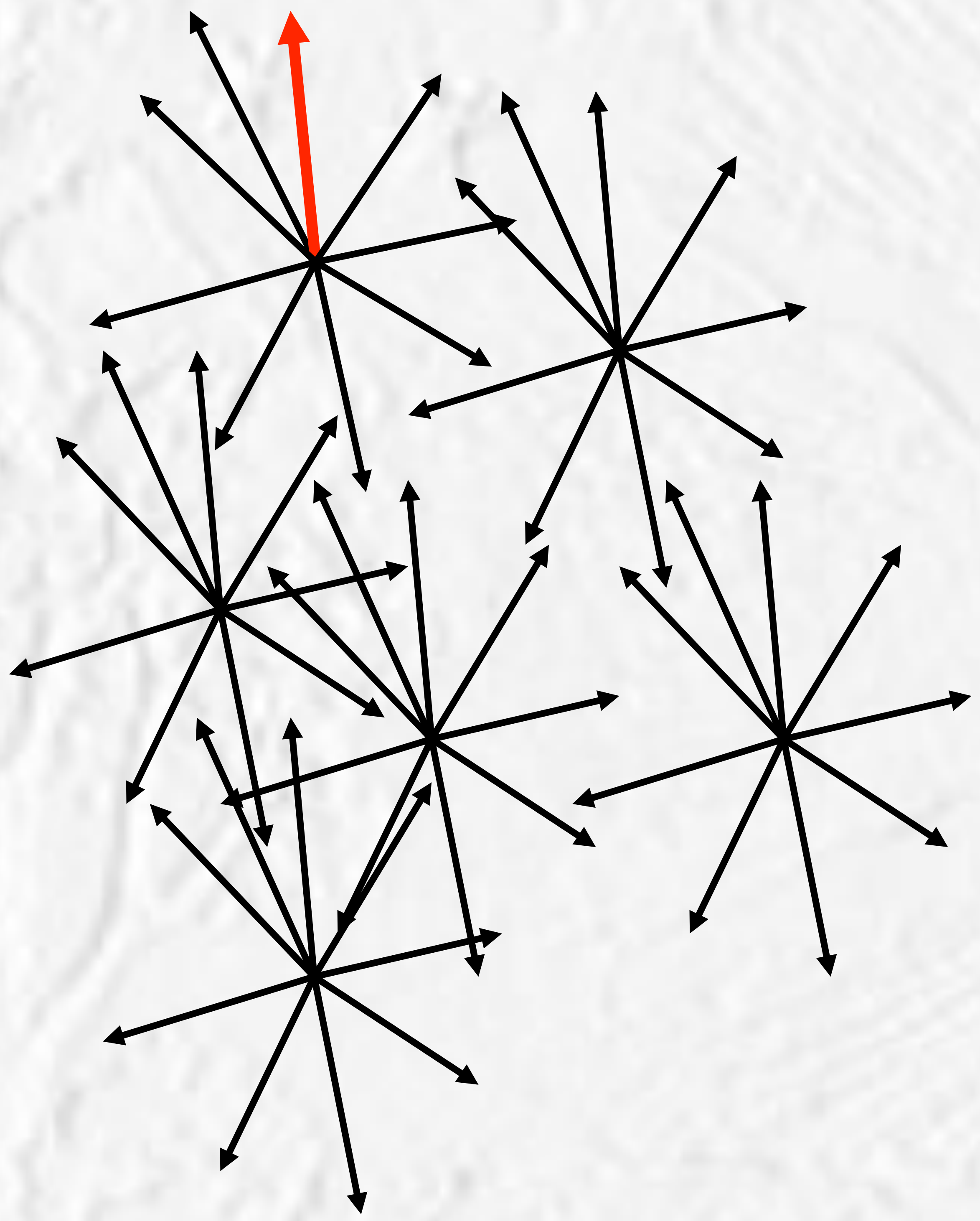
$$\delta_{rb}^{AA} \approx \frac{\delta_{rb}^{pp}}{N_{\rm coll}} \approx \frac{\delta_{rb}^{pp} M^{pp}}{M^{AA}}$$

$$\langle u_r Q^* \rangle^{AA} \approx v_r v_b + \langle u_b Q^* \rangle^{pp}$$

Non-flow looks exactly the same in pp and AA ➔ Results - directly "correctible".

N_{coll} – Number of "independent NN collisions", a la $N_{part}/2$.

Consider correlations of a "red" particle (some momentum "bin") with all other, "black", particles in the event

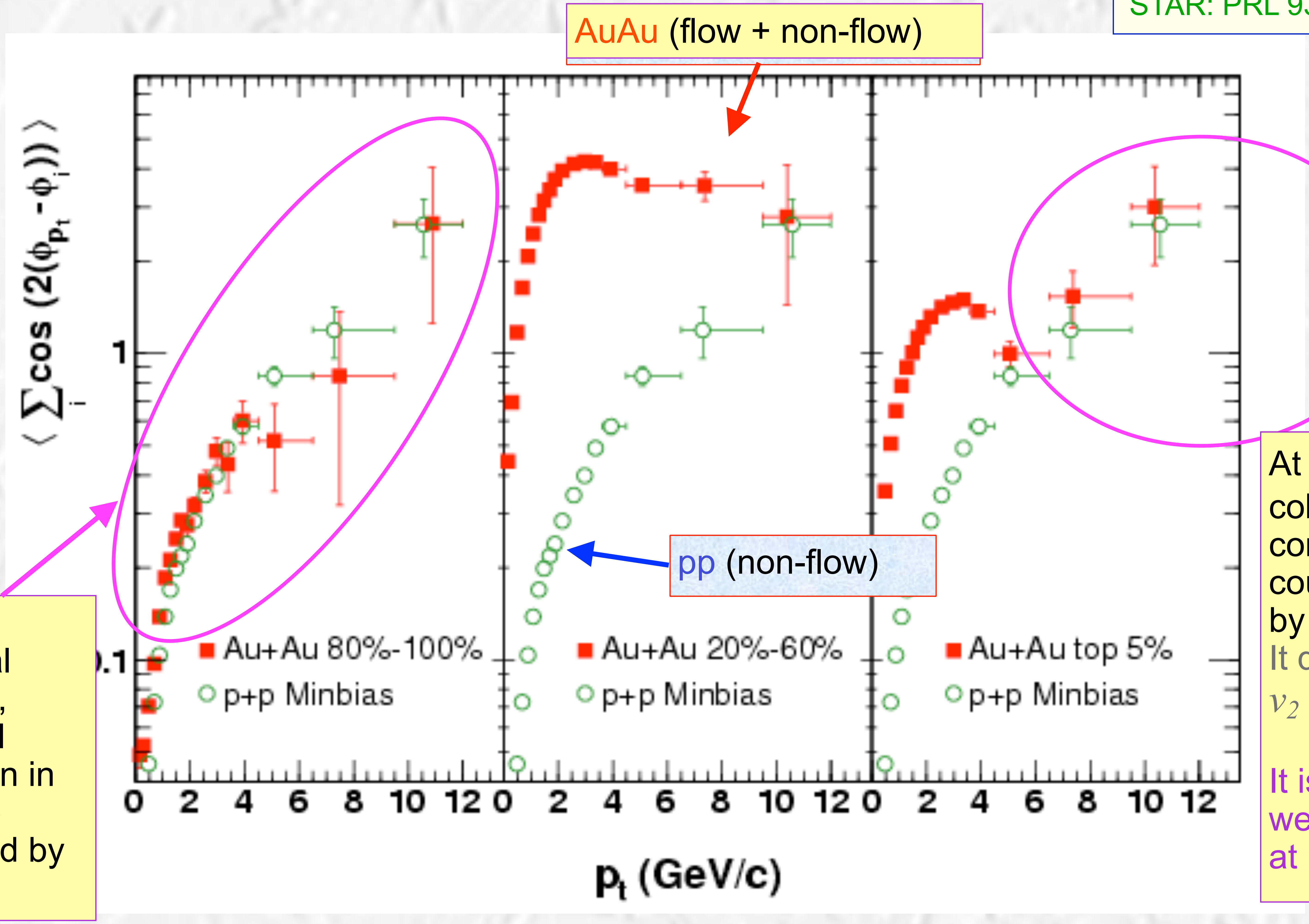


Check if non-flow estimates/measurements reported or *Au+Au* are consistent with measurements in *pp*.
(Expect the difference of the order of factor of <~2.
Extra particles in jets ➔ non-flow contribution increases
B-to-B jet suppression – non-flow goes down)

Use *pp* data to estimate non-flow effects in *Au+Au* when other methods do not work

Comparison: pp & AuAu

STAR: PRL 93(2004)252301



In VERY peripheral collisions, azimuthal correlation in AuAu are dominated by non-flow.

At high p_t in central collisions, azimuthal correlation in AuAu could be dominated by nonflow.
It does not mean that v_2 is zero!

It is remarkable how well they agree at high p_t !

$v_2\{2\}$, $v_2\{4\}$, nonflow, and flow fluctuations

$$\langle u_1 u_2^* \rangle = v_2^2 + \delta; \quad u \equiv e^{i2\phi}$$

$$v_2\{2\} \equiv \langle u_1 u_2^* \rangle^{1/2}$$

$$\langle u_1 u_2 u_3^* u_4^* \rangle = v_2^4 + 2 \cdot 2 v_2^2 \delta + 2\delta^2$$

$$v_2\{4\} \equiv \left(2\langle u_1 u_2^* \rangle^2 - \langle u_1 u_2 u_3^* u_4^* \rangle\right)^{1/4}$$

Assumes $\langle \delta^2 \rangle = \langle \delta \rangle^2$, etc.

$$v_2\{2\} = \sqrt{\langle v_2^2 \rangle + \delta}$$

$$v_2\{4\} = \sqrt[4]{2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle}$$

Several reasons for v_n to fluctuate:

Variation in impact parameter in a centrality bin (easily correctable)

"Real" **flow fluctuations due to fluctuations in the initial conditions or in the system evolution**

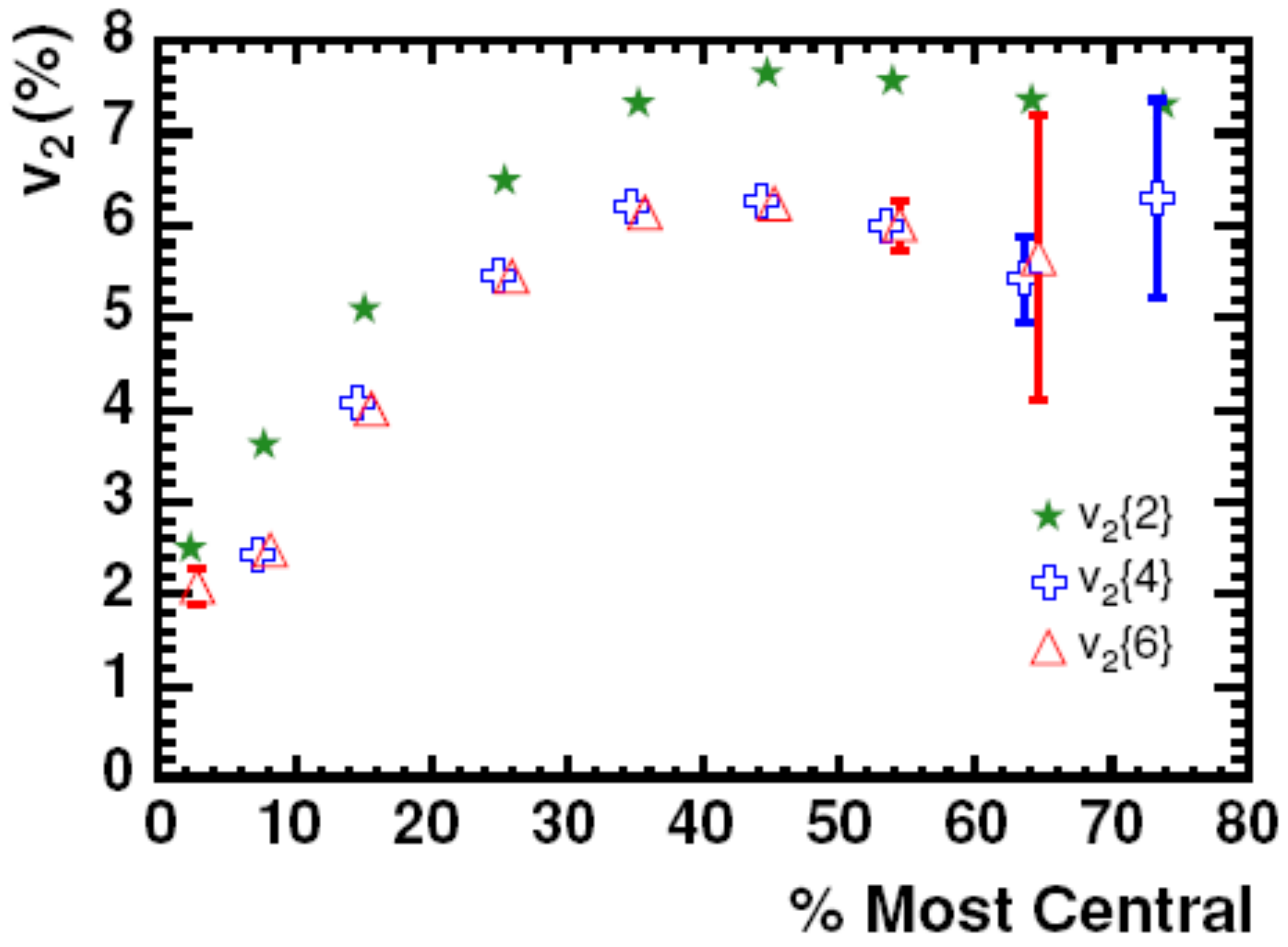
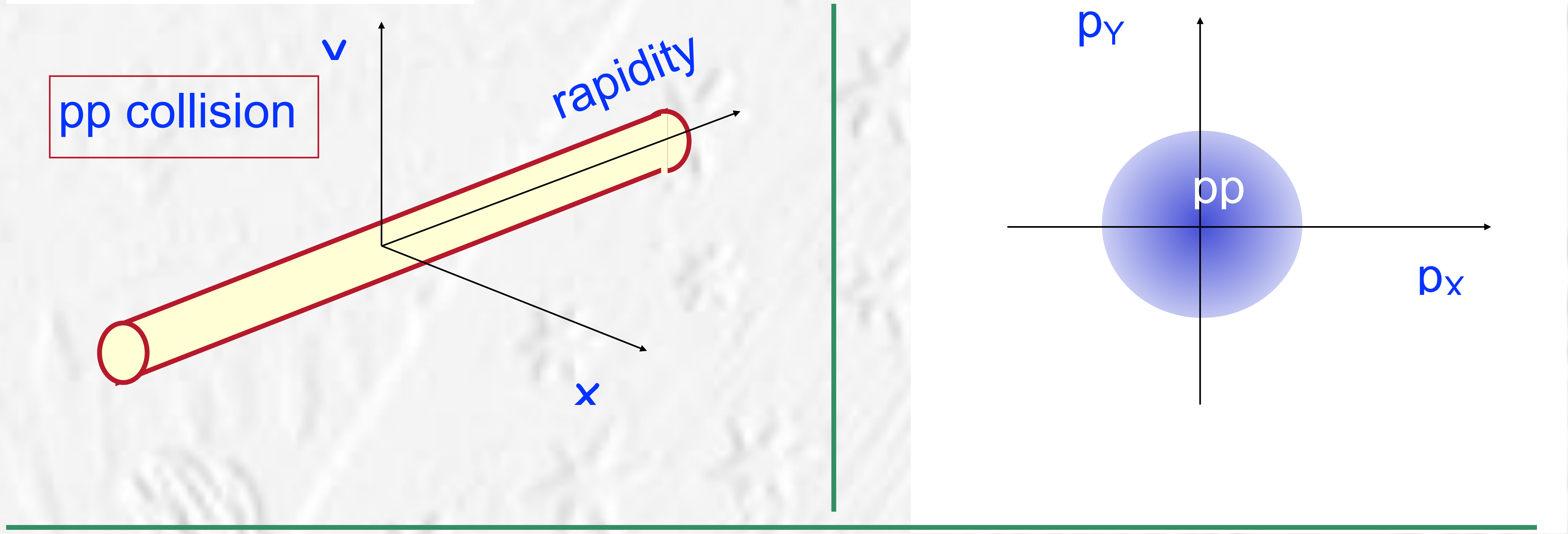


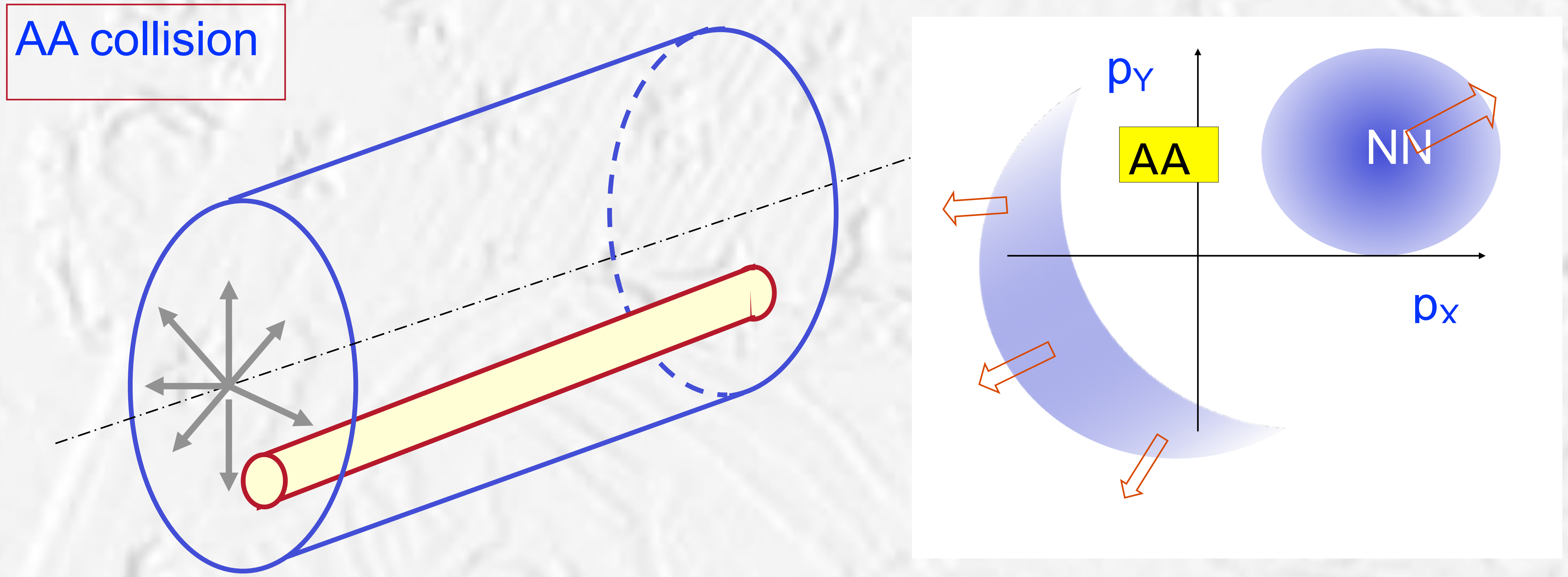
FIG. 30. (Color online) Charged hadron $v_2\{2\}$, $v_2\{4\}$, and $v_2\{6\}$ integrated values as a function of centrality.

$v_2\{2\}$ and $v_2\{4\}$, flow fluctuations or nonflow?

Radial expansion → nonflow

[arXiv:nucl-th/0312065]

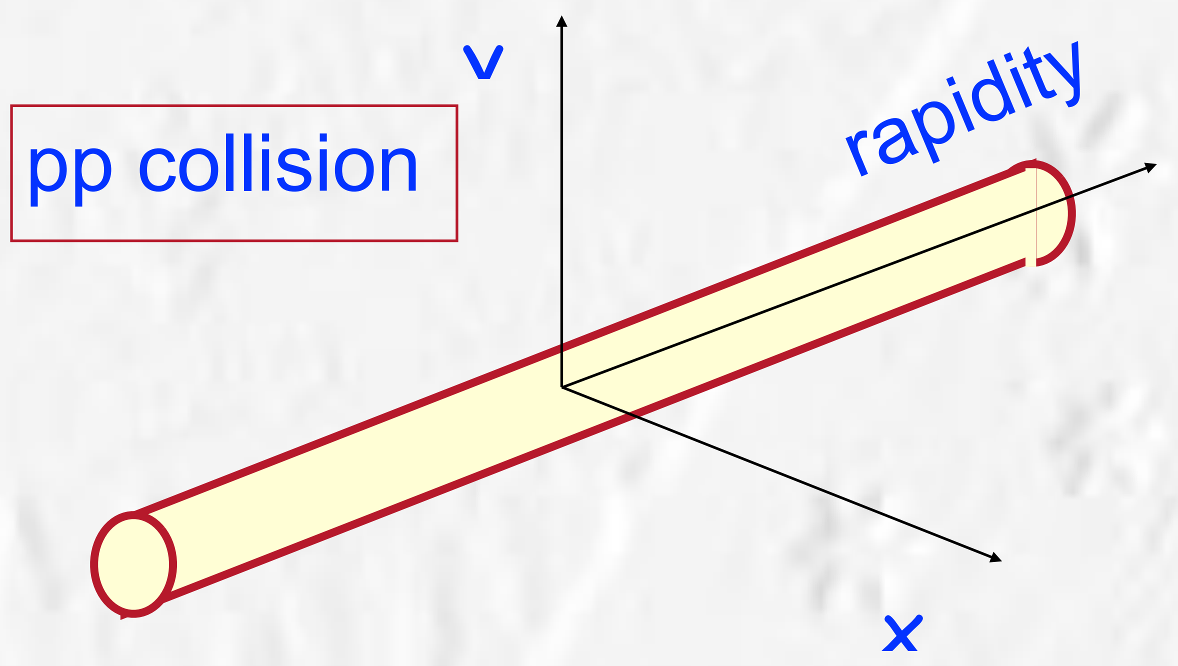


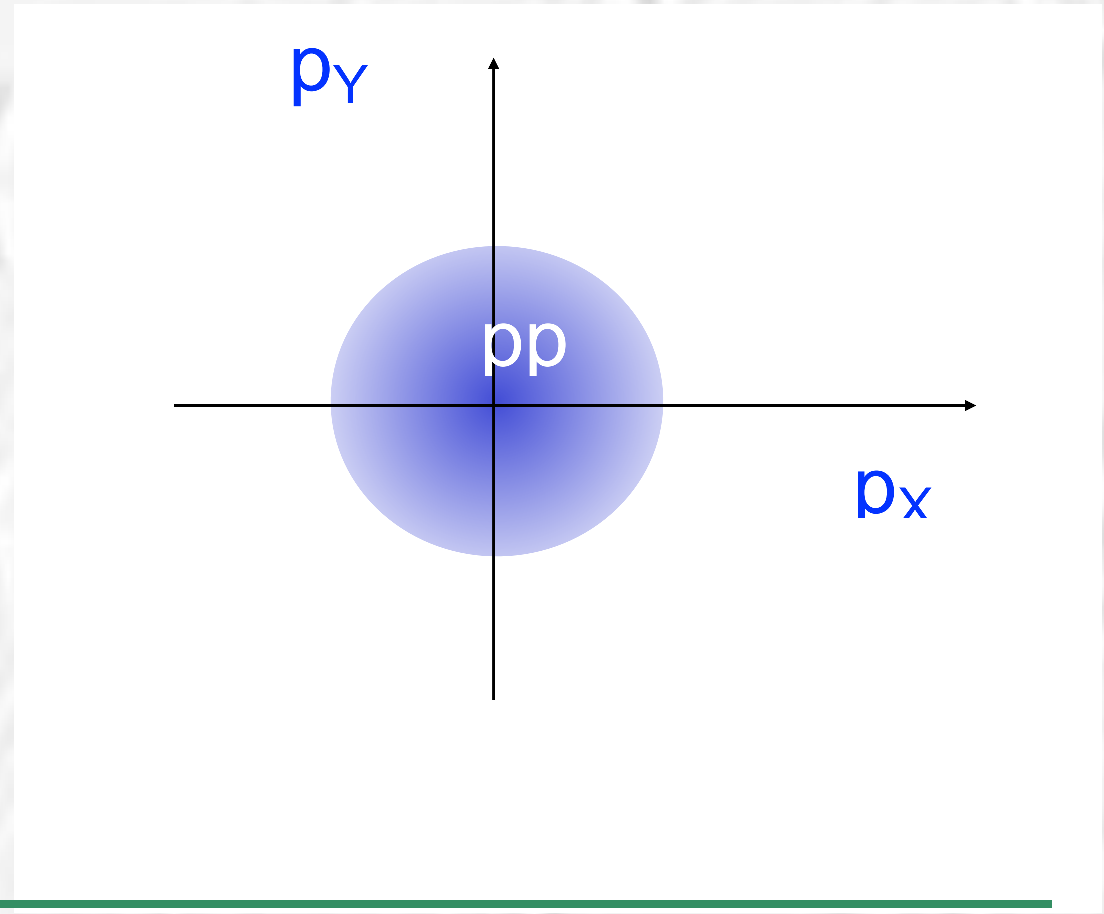


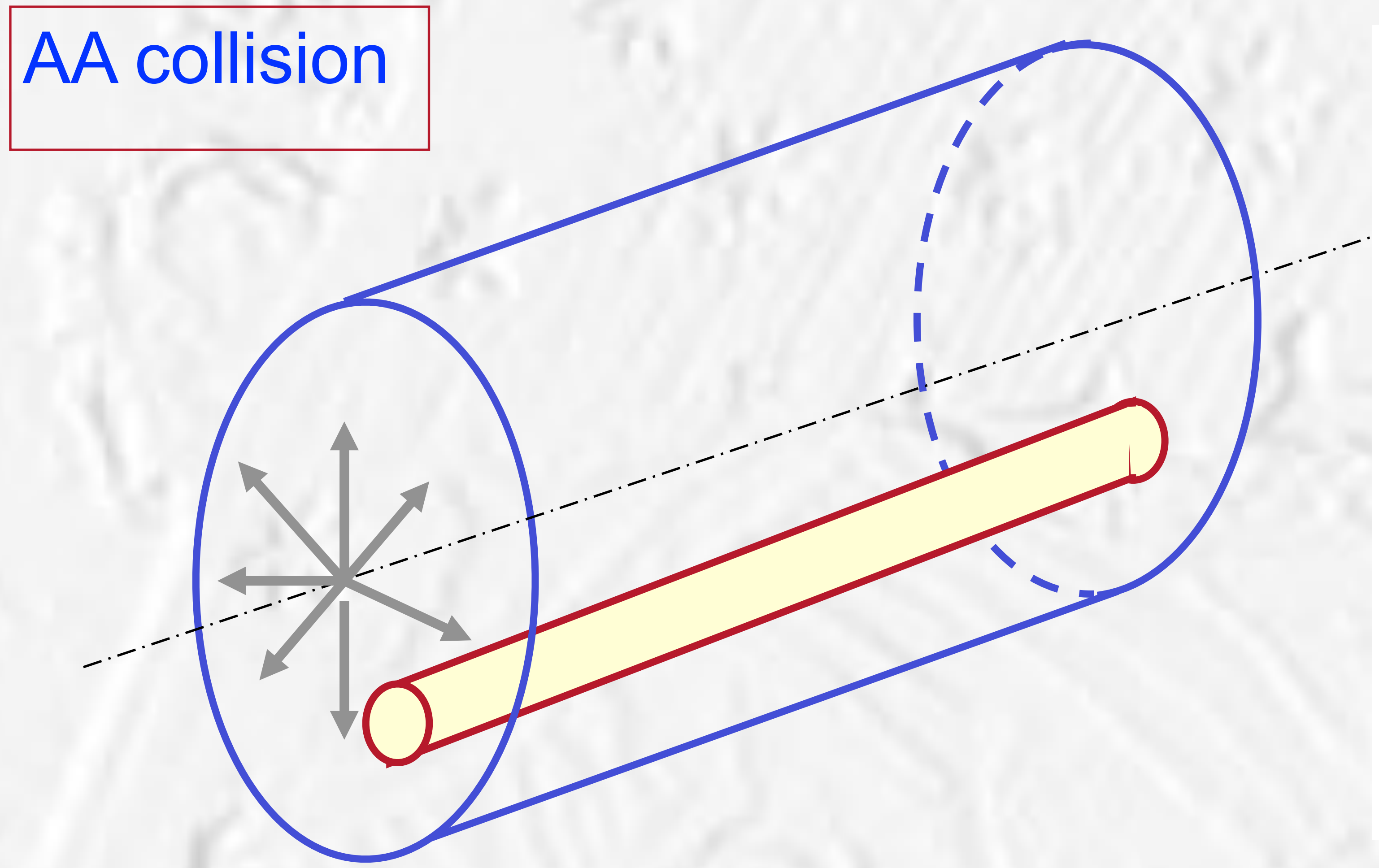
Nonflow specific only for AA?

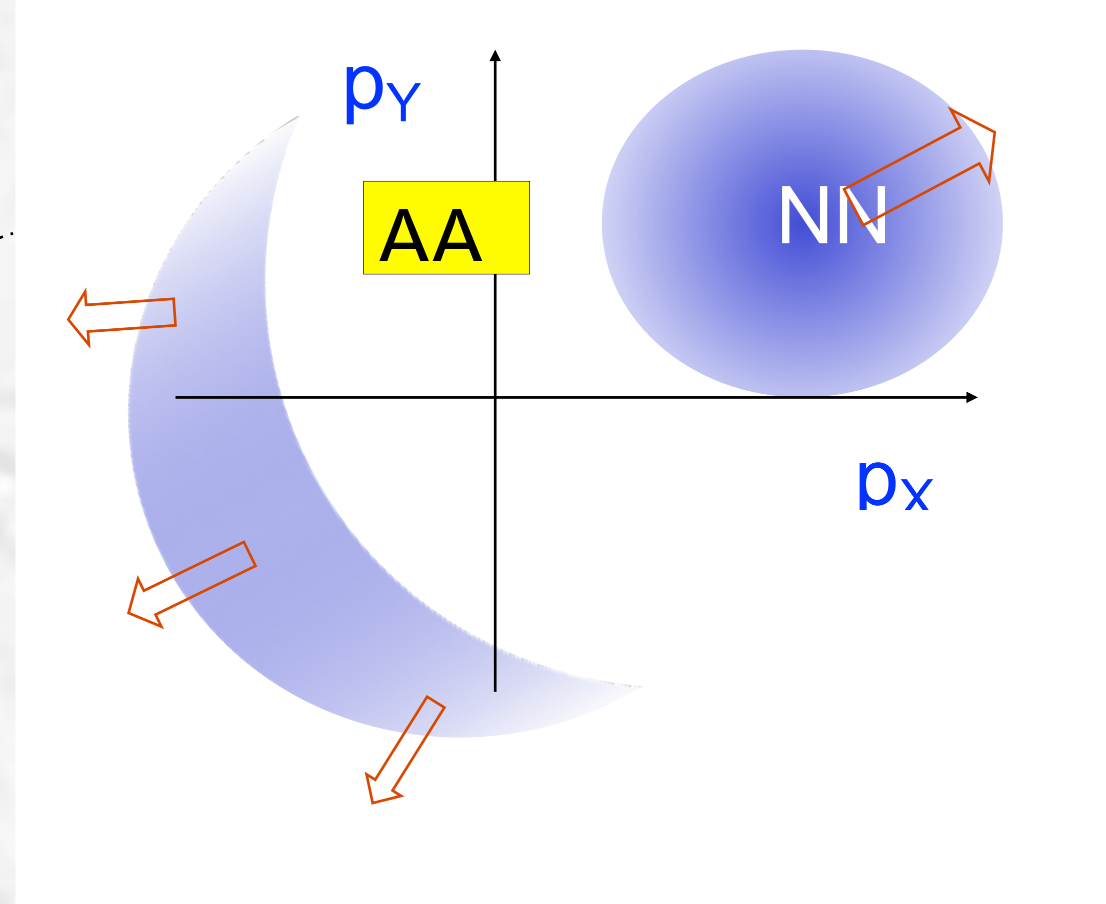
Radial expansion → nonflow

[arXiv:nucl-th/0312065]



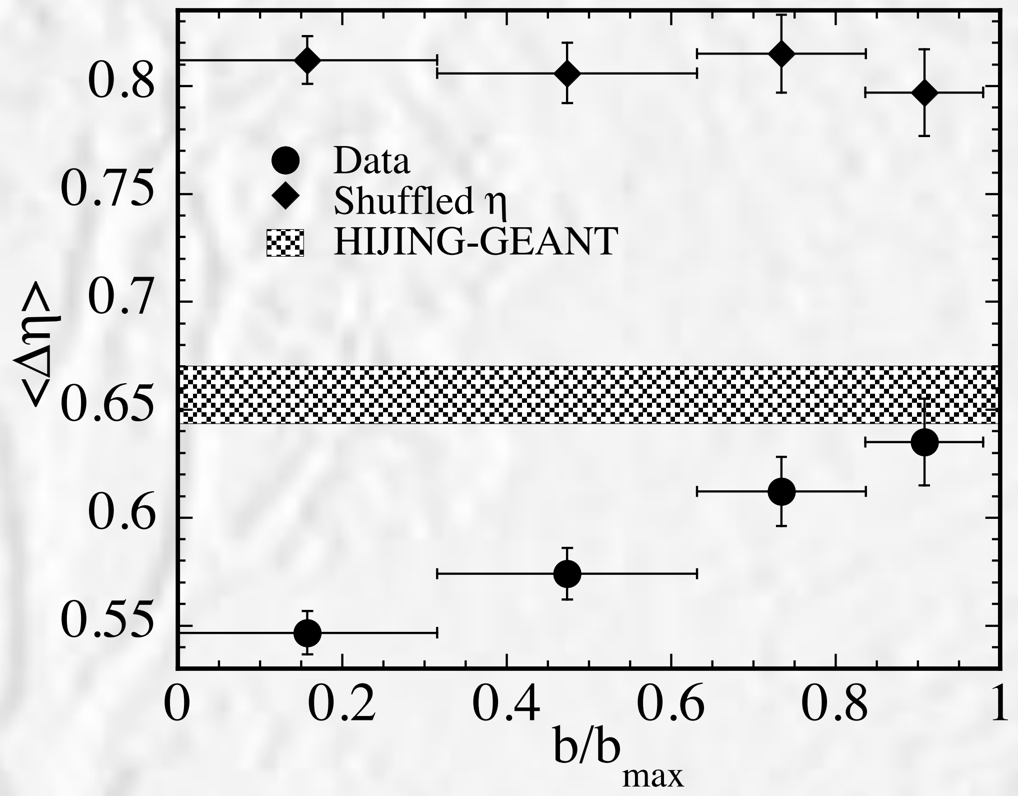






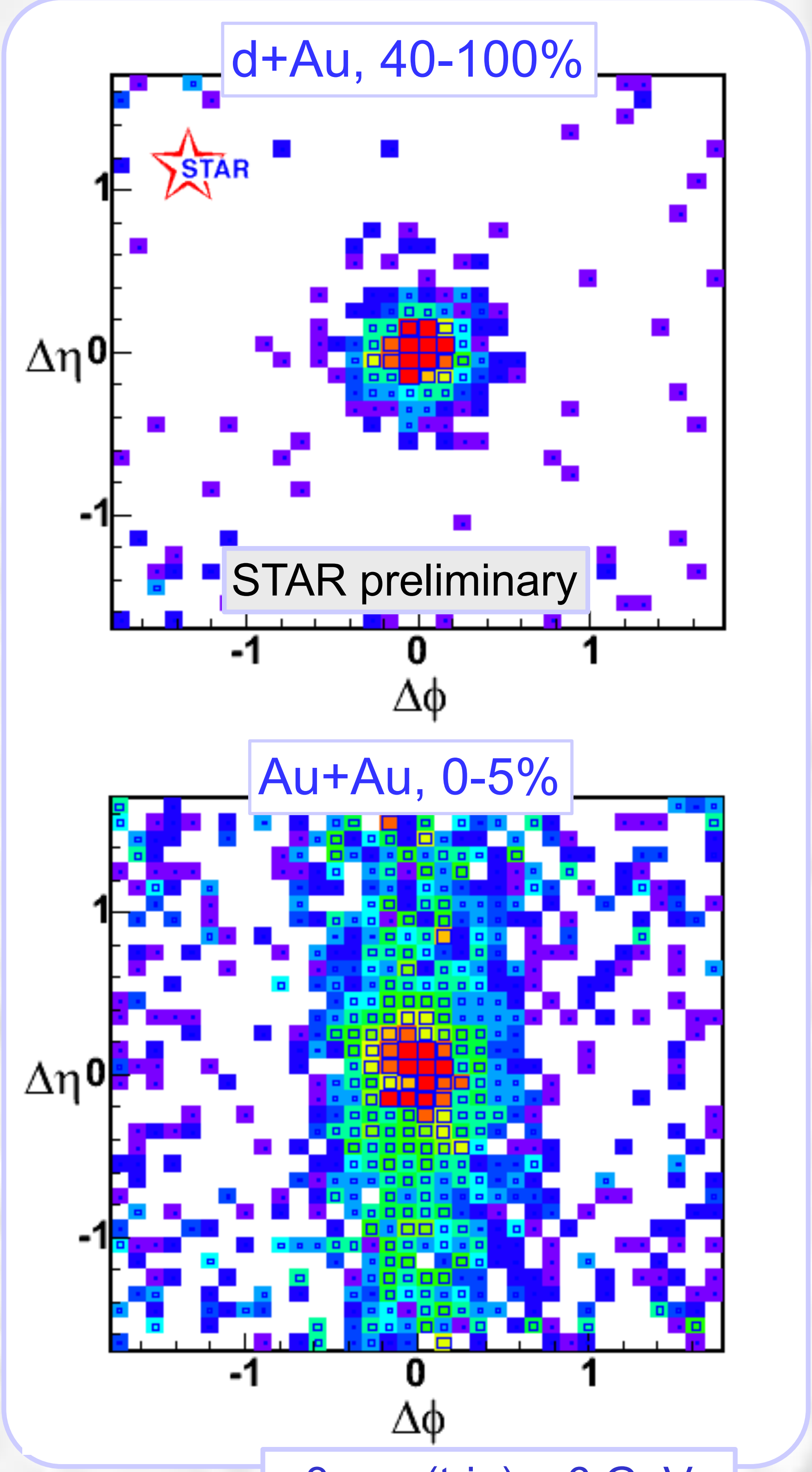
Nonflow specific only for AA?

arXiv:nucl-ex/0301014v1 24 Jan 2003



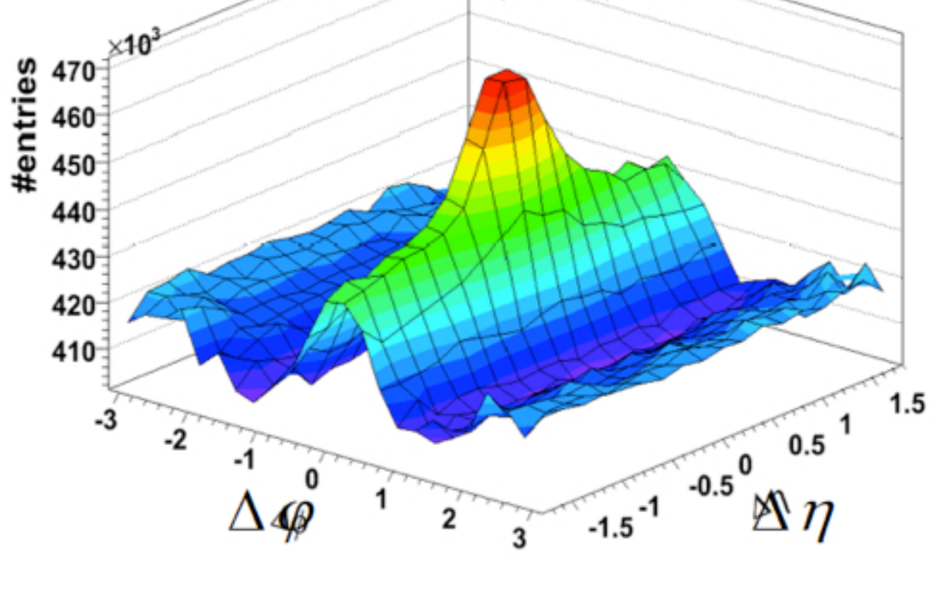
'IG. 2: The width of the balance function for charged par-icles, $\langle\Delta\eta\rangle$, as a function of normalized impact parameter b/b_{max}). Error bars shown are statistical. The width of he balance function from HIJING events is shown as a band whose height reflects the statistical uncertainty. Also shown re the widths from the shuffled pseudorapidity events.

D. Magestro (STAR) – Hard Probes 2004



$3 < p_T(\text{trig}) < 6$ GeV
$2 < p_T(\text{assoc}) < p_T(\text{trig})$

B. I. Abelev et al.[STAR Collaboration], Phys. Rev. C 80 (2009) 6491



Radial expansion → nonflow, cont'd

S.A. Voloshin / Physics Letters B 632 (2006) 490–494 493

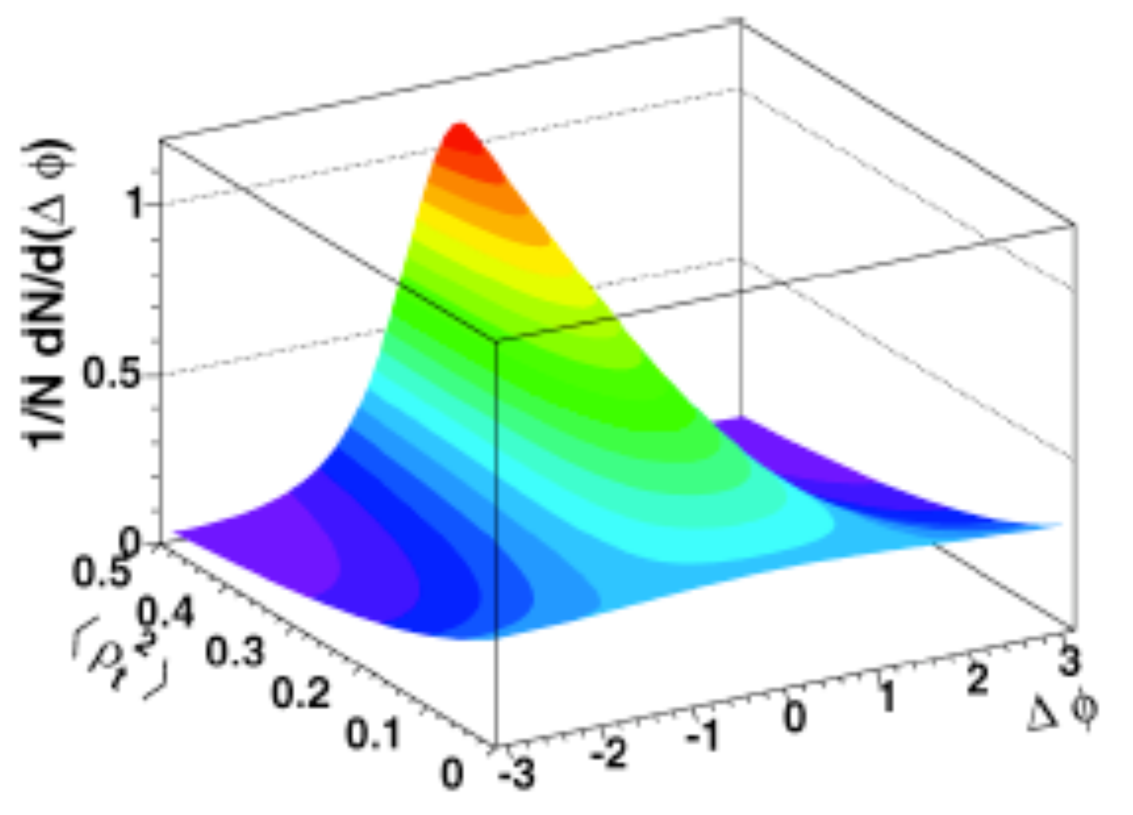


Fig. 3. (Color online.) Two pion $\Delta\phi$ distribution as function of $\langle\rho_t^2\rangle$ in the blast wave model. Linear velocity profile and $T = 110$ MeV have been assumed.

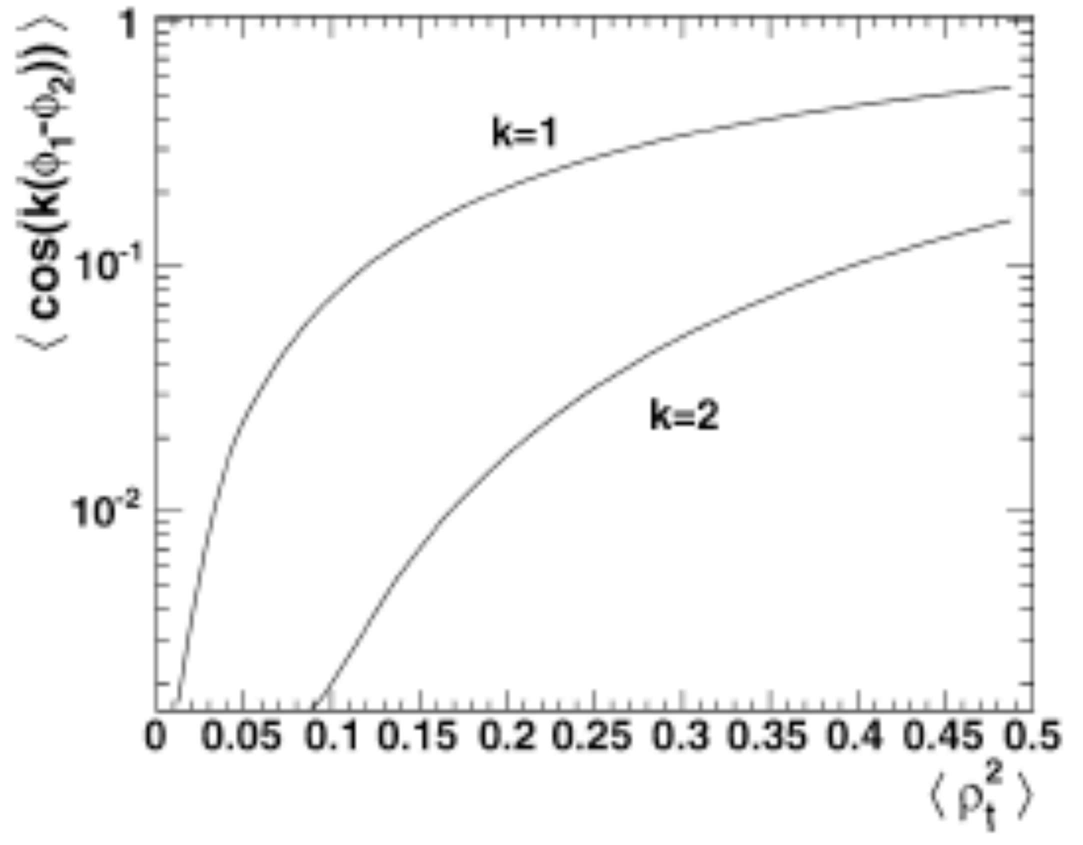
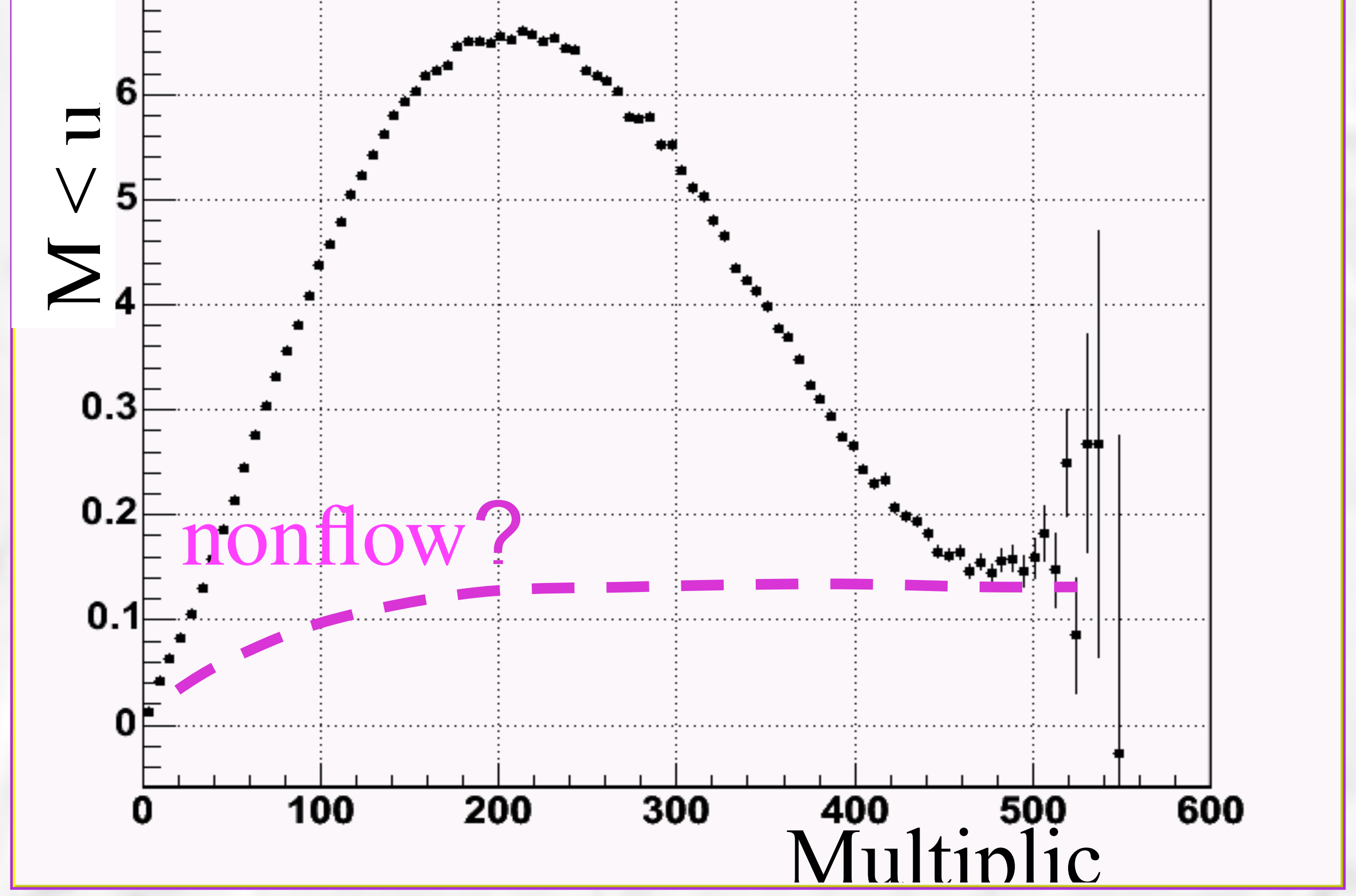


Fig. 4. The average values of $\cos(\Delta\phi)$ and $\cos(2\Delta\phi)$ for the distribution shown in Fig. 3.

Values of transverse flow $\langle\rho_t^2\rangle > 0.25$
(a) would contradict nonflow estimates
(b) oversaturate the difference $v_2\{2\} - v_2\{4\}$

$N\left\langle\cos(2\phi_i - 2\phi_j\right\rangle \approx N v_2^2$



PHYSICAL REVIEW C 72, 014904 (2005)

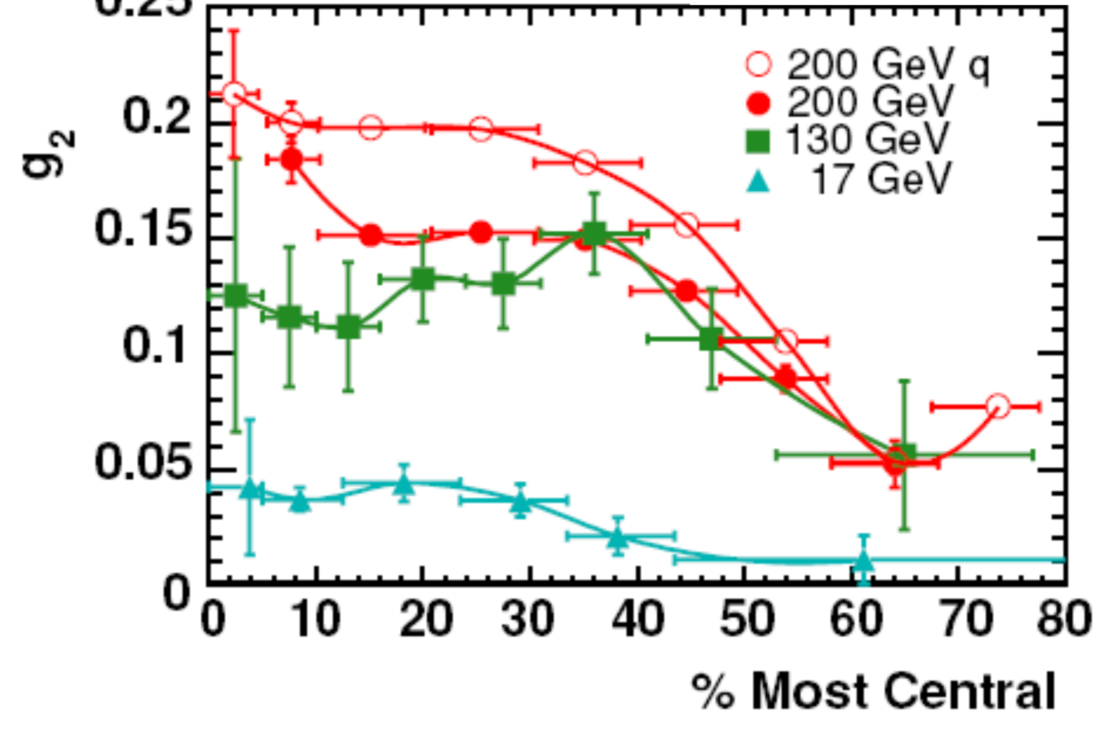
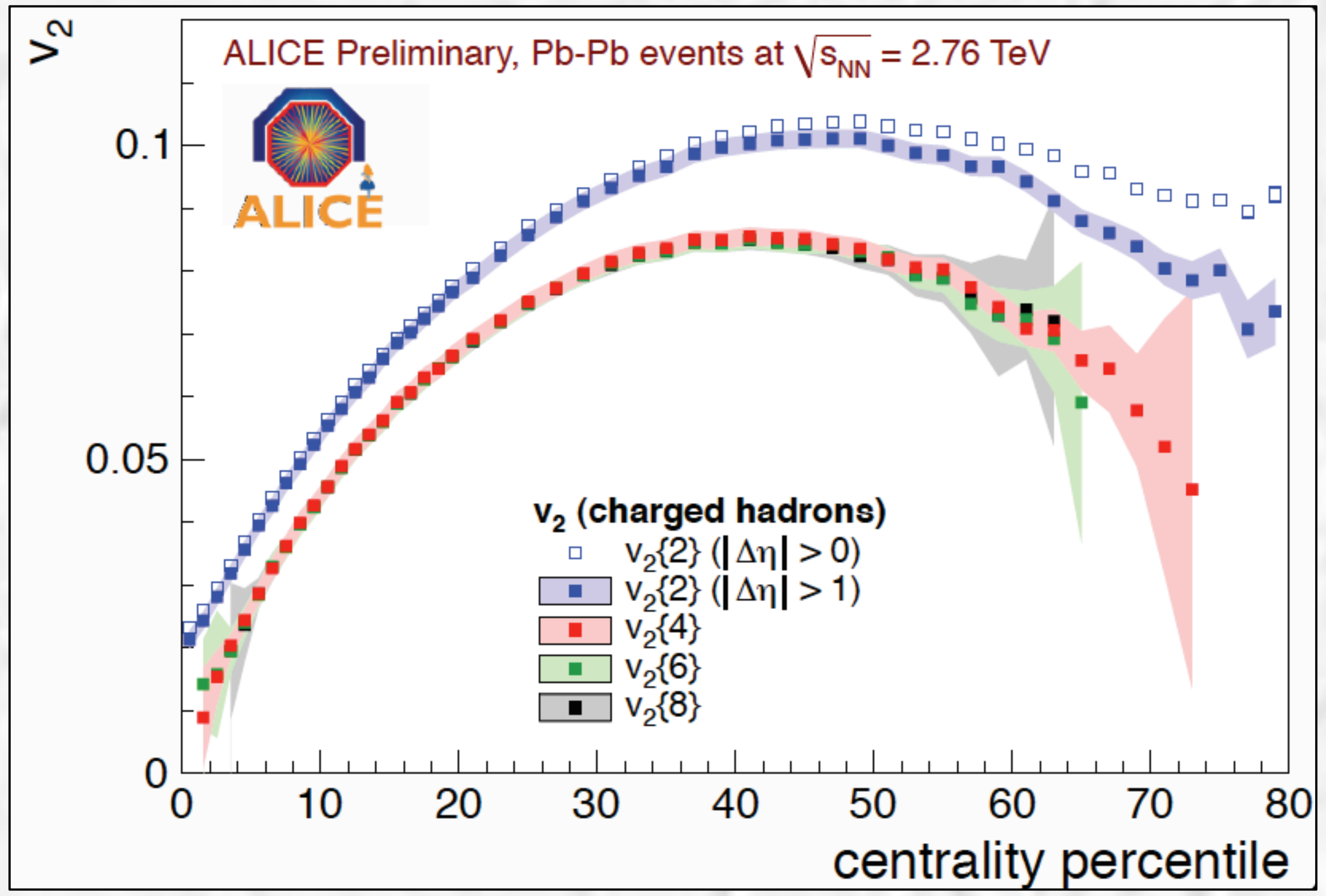
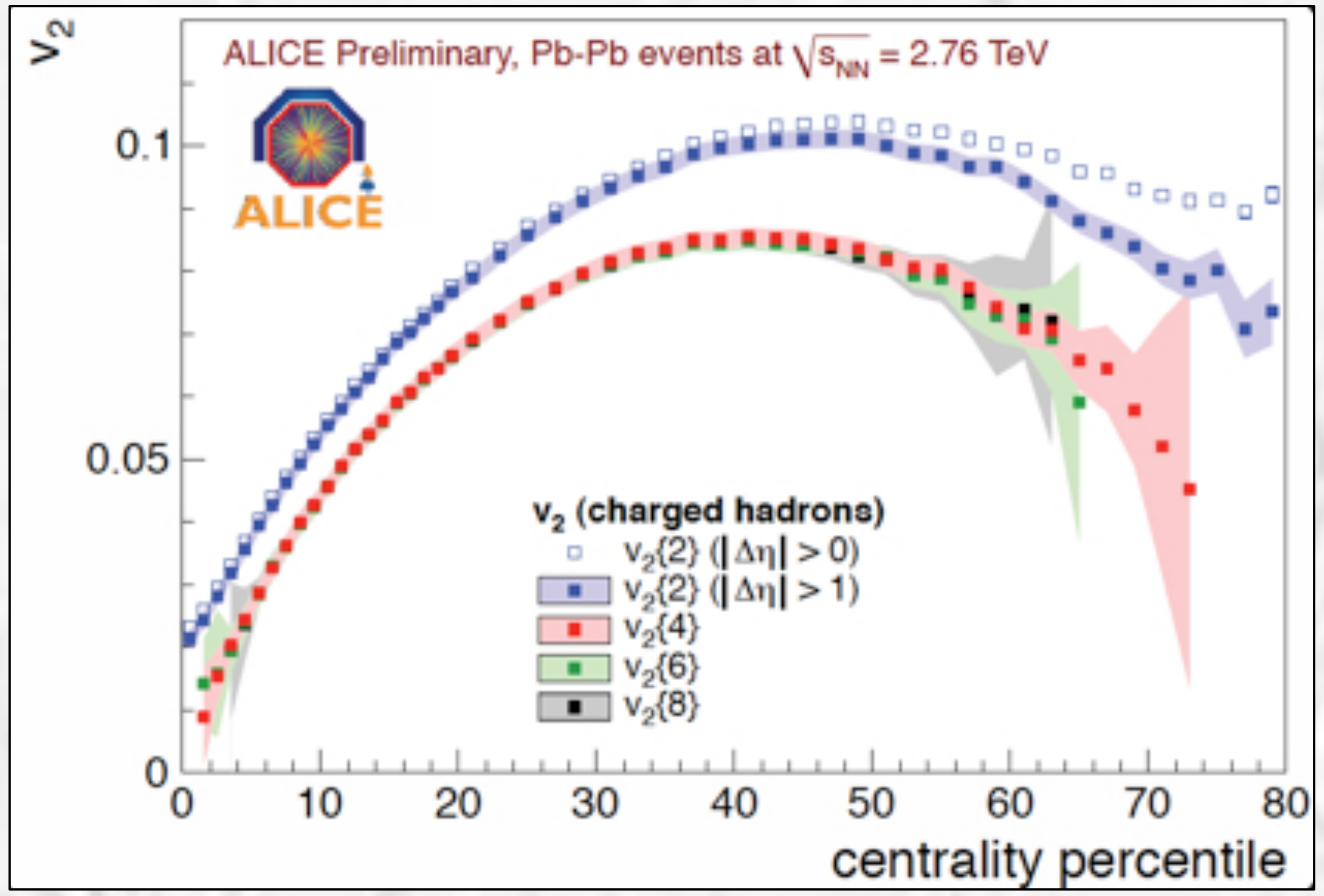


FIG. 31. (Color online) The nonflow parameter, g_2, as a function of centrality. The solid points are from the cumulant method. The open circles are from the q distribution method.



Flow fluctuations



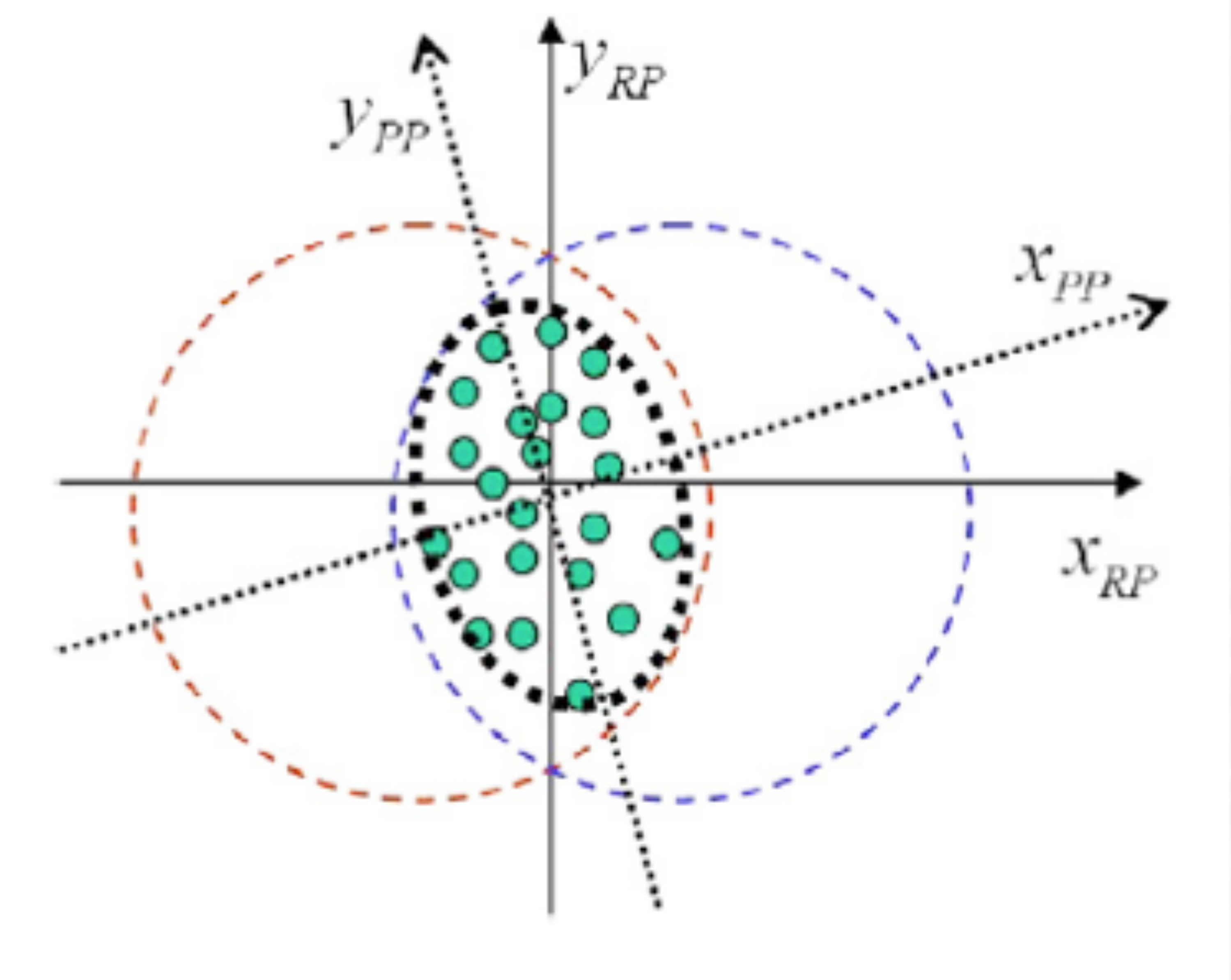


Fig. 1. The definitions of the *RP* and *PP* coordinate systems.

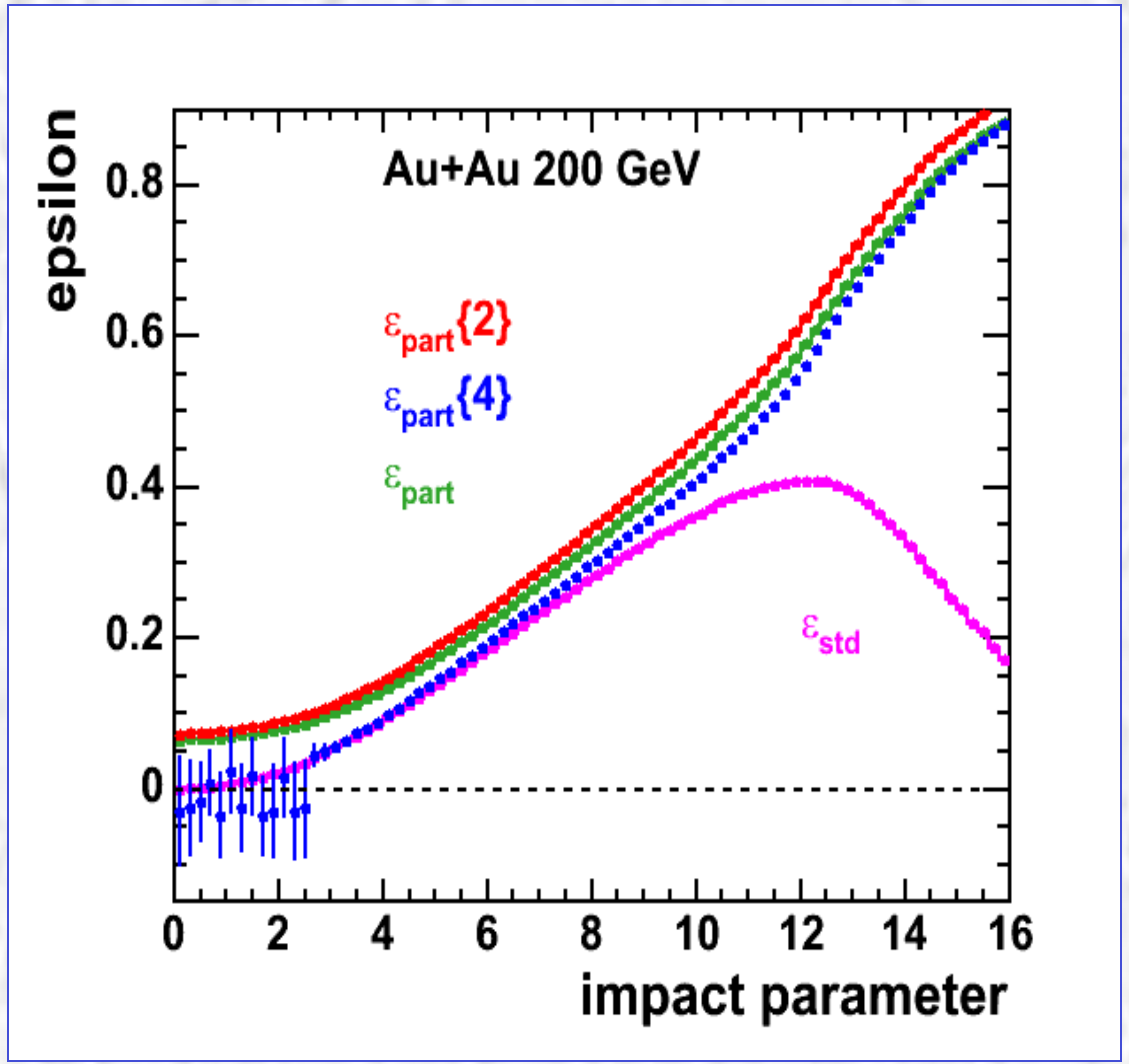
Elliptic flow in the Gaussian model of eccentricity fluctuations *Physics Letters B 659 (2008) 537–541*

Sergei A. Voloshin[a,*], Arthur M. Poskanzer[b], Aihong Tang[c], Gang Wang[d]

$$\langle v_2^2 \rangle = \langle v_2 \rangle^2 + \sigma_{v_2}^2 + g_2/N$$

The difference between two-particle and many-particle correlation results are due to flow fluctuations and nonflow.

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



The difference between $v_2\{2\}$ and $v_2\{4\}$ is almost fully saturated by eccentricity fluctuations according to nucleon participant Glauber MC.

Initial eccentricity: “optical”, “standard”, ”participant”

“Monte-Carlo” Glauber model:

M. Miller and R. Snellings, nucl-ex/0312008.

$$\varepsilon_{Std} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

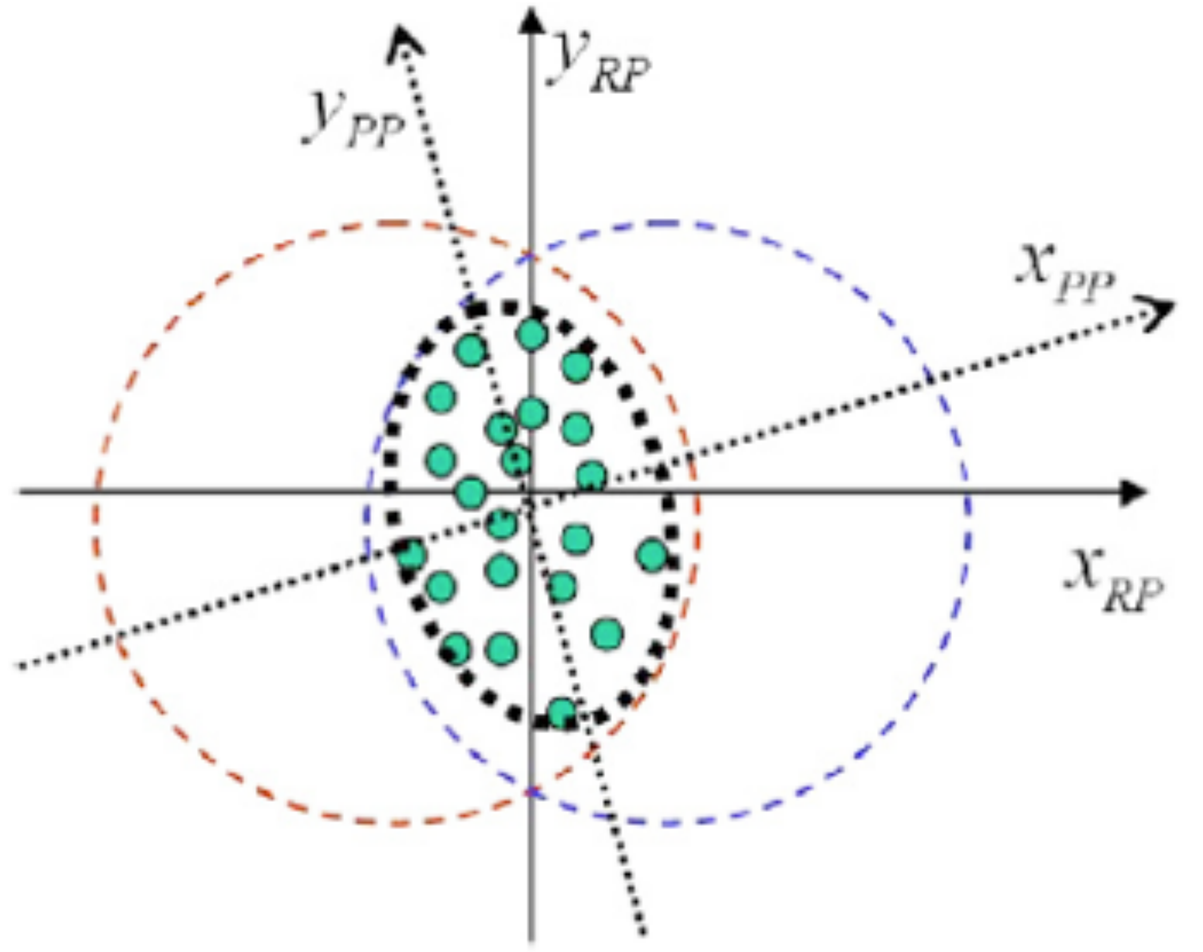


Fig. 1. The definitions of the *RP* and *PP* coordinate systems.

“New” coordinate system – rotated, shifted

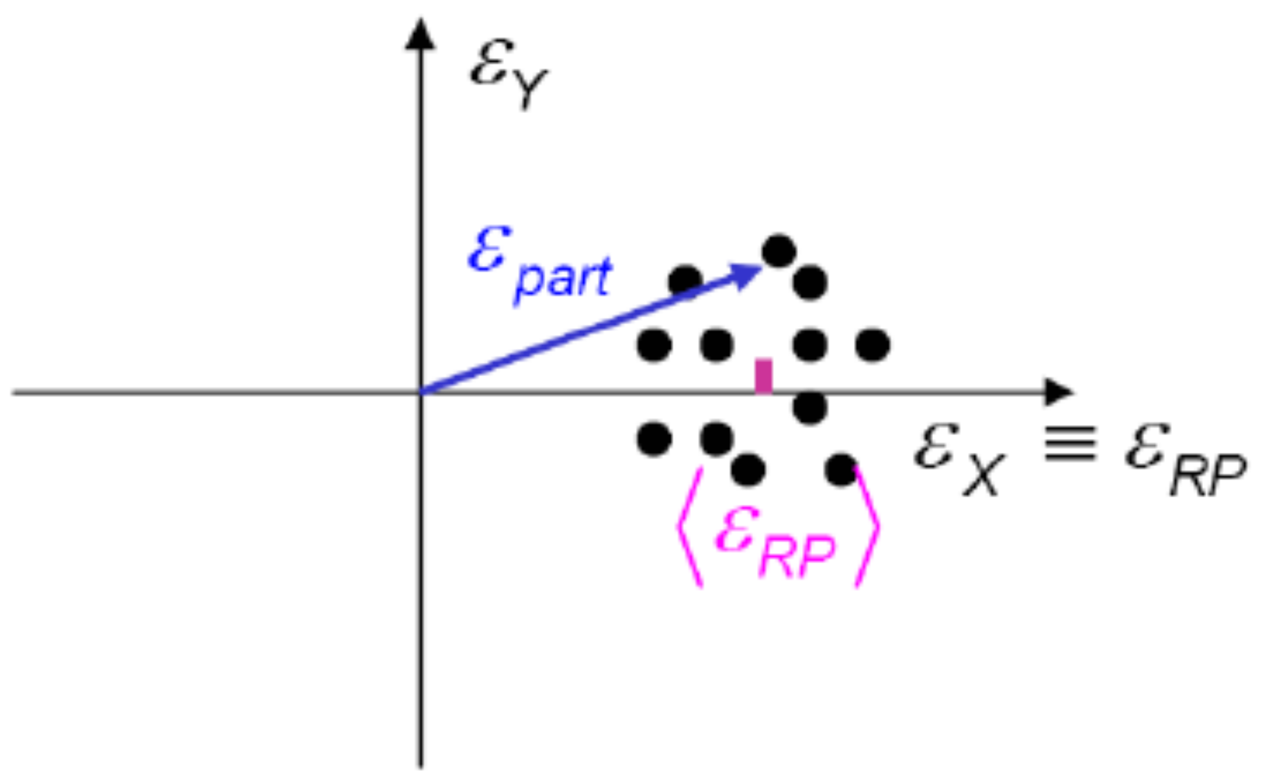


FIG. 3: Definition of ε_{part}.

$$\varepsilon_{part} = \frac{\langle y'^2 - x'^2 \rangle}{\langle y'^2 + x'^2 \rangle}$$

Reaction, “participant”, and event (flow vector) planes

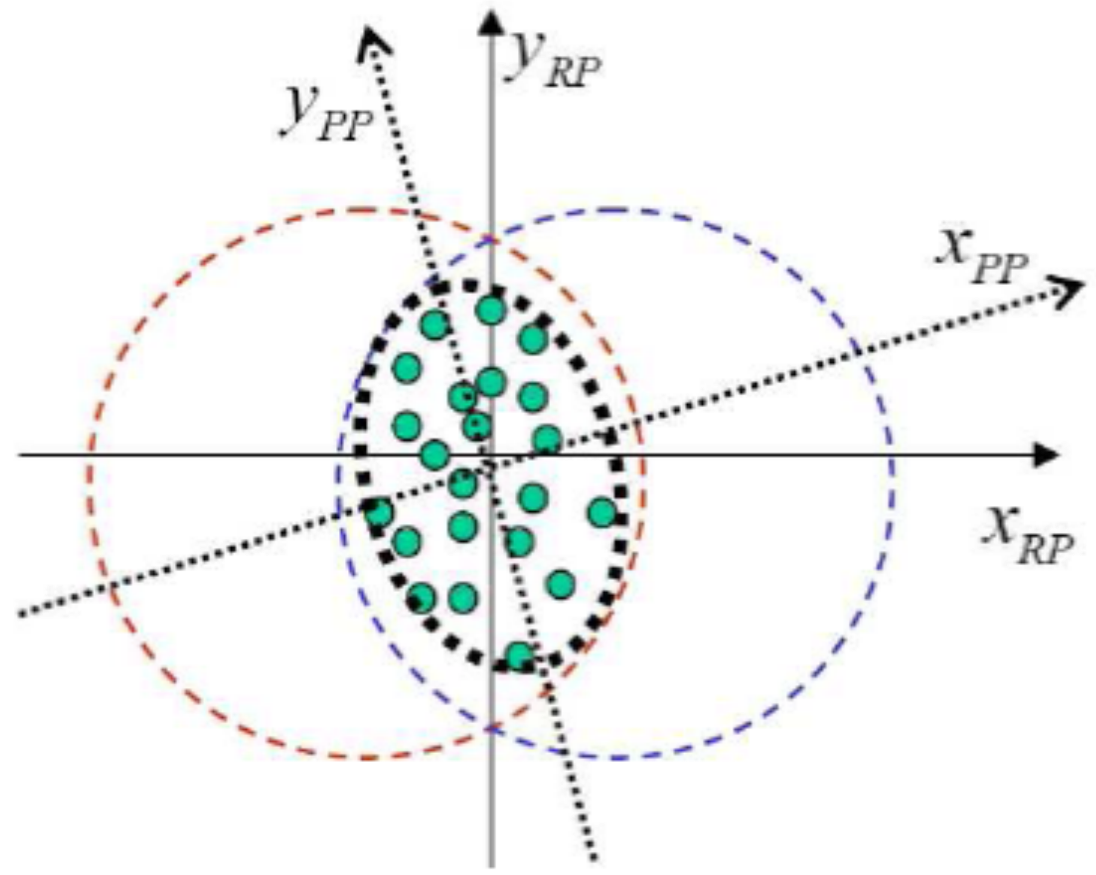


FIG. 1: The definition of the RP and PP coordinate systems.

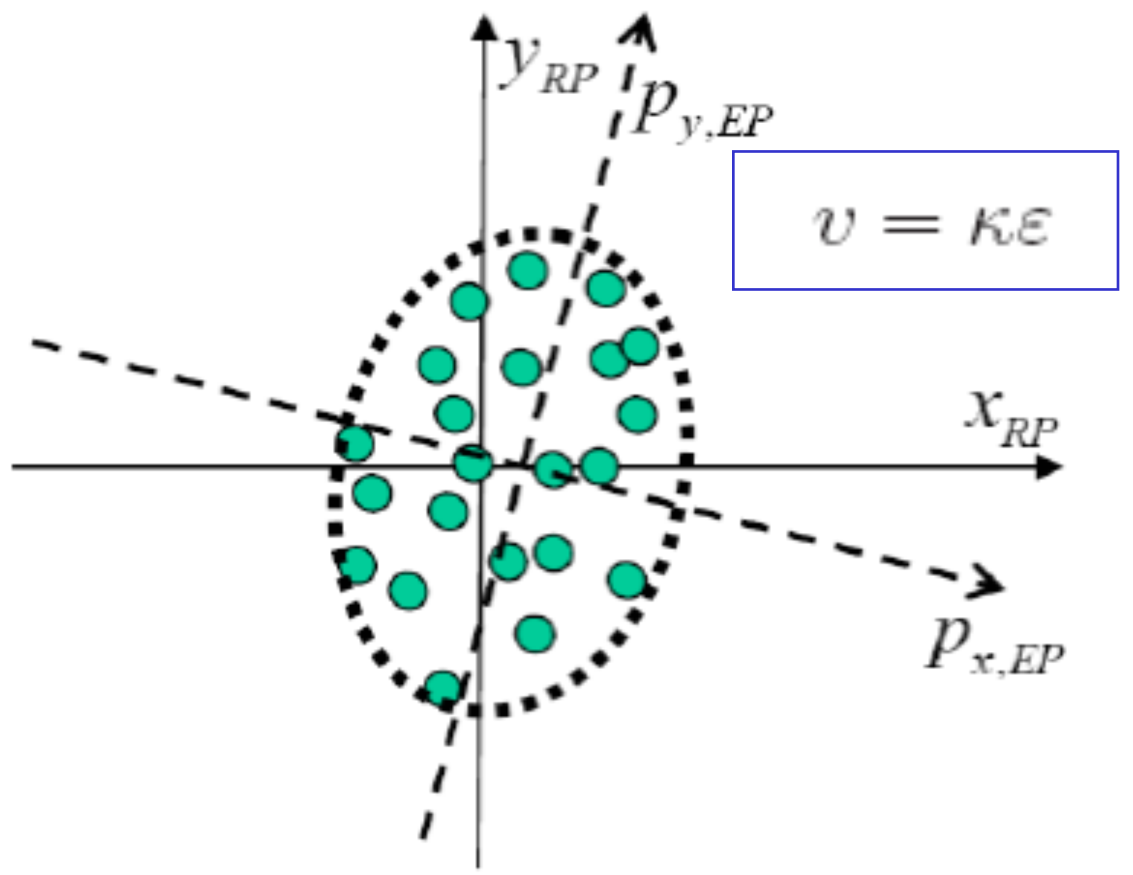


FIG. 2: The definitions of the EP coordinate system

$$\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part} \right\}$$

Physics Letters B 659 (2008) 537–541

Elliptic flow in the Gaussian model of eccentricity fluctuations

Sergei A. Voloshin [a,*], Arthur M. Poskanzer [b], Aihong Tang [c], Gang Wang [d]

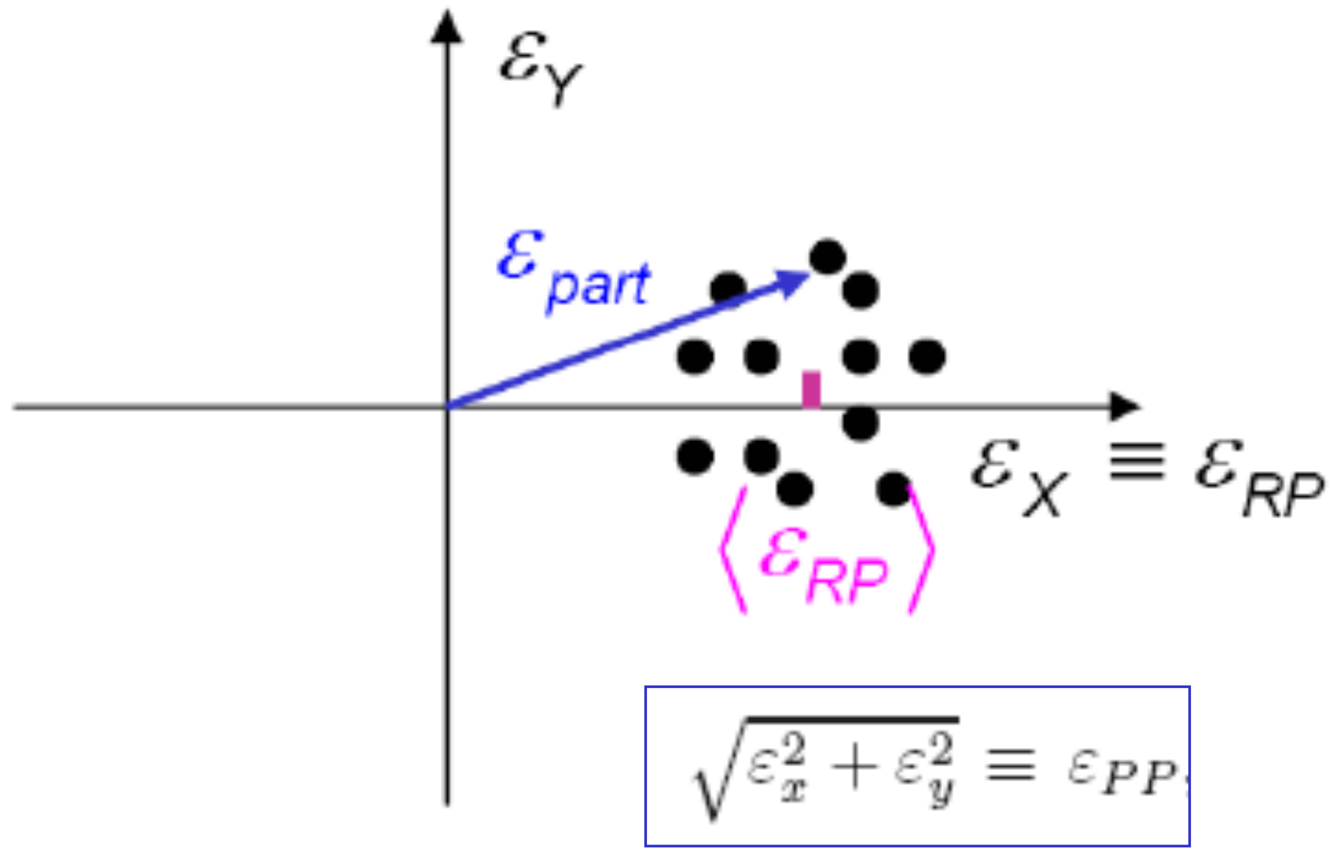


FIG. 3: Definition of ε_{part}.

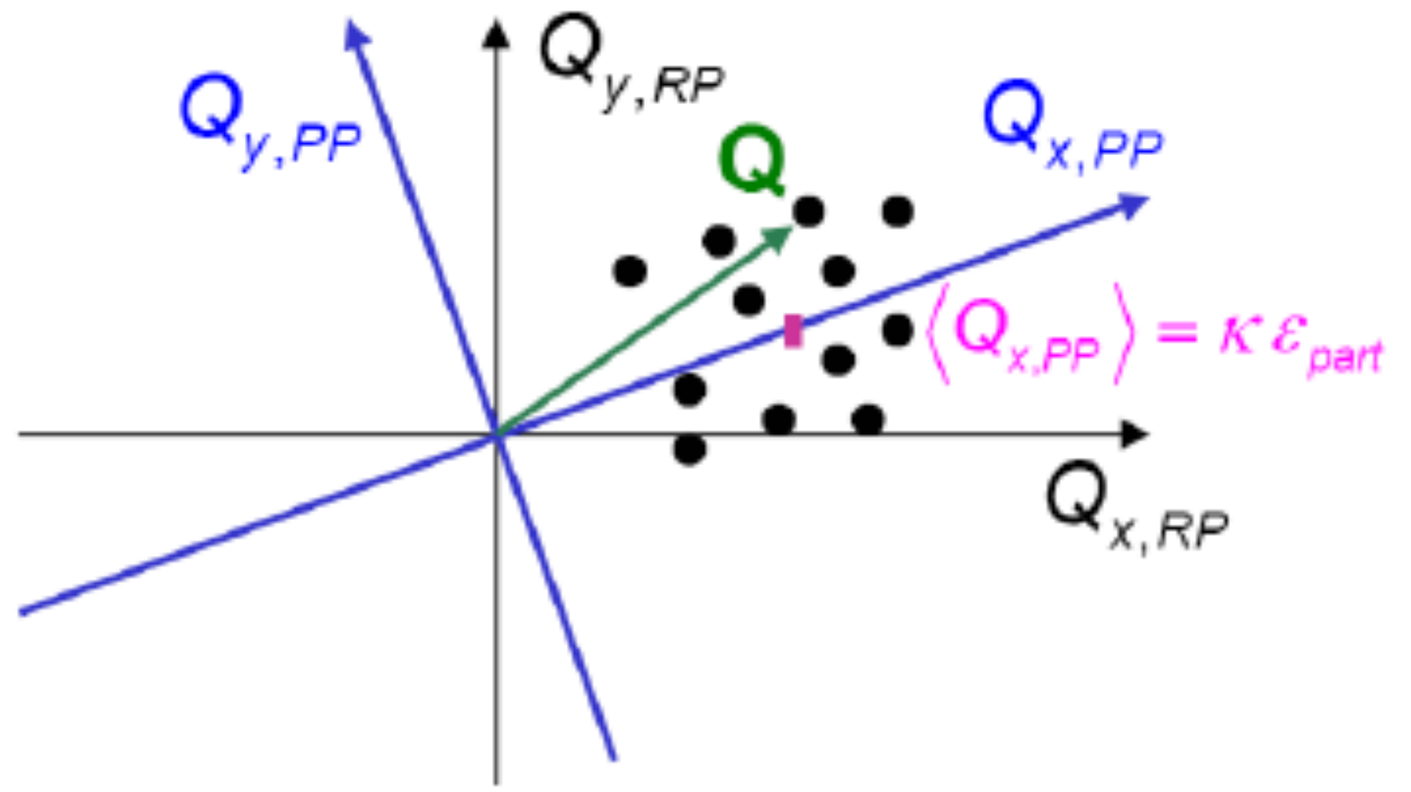


FIG. 4: Flow vector distribution in events with fixed ε

PHYSICAL REVIEW C **105**, 024902 (2022)

Flow-plane decorrelations in heavy-ion collisions with multiple-plane cumulants

Zhiwan Xu [1,*] Xiatong Wu [1] Caleb Sword [2] Gang Wang [1,†] Sergei A. Voloshin [2] and Huan Zhong Huang [1,3]

(i) The reaction plane (RP) is the plane spanned by the beam direction and the impact parameter vector. This plane is unique for every collision.

(ii) The participant plane (PP) is defined by the initial density distribution. Subtle differences may exist, depending on, e.g., whether entropy or energy density is used as a weight, but these potentially small differences are not discussed in this paper. We assume that the properly constructed PPs define the development of anisotropic flow.

(iii) The flow symmetry plane or the flow plane (FP) determines the orientation of the corresponding harmonic anisotropic flow. It is assumed that the FP coincides with the PP of the same harmonic (linear flow mode) or a proper combination of the lower harmonic PPs (nonlinear flow mode). With the nonlinear flow modes being neglected, FP and PP are often used interchangeably.

(iv) The event plane (EP) estimates the FP by analyzing the particle azimuthal distribution in a particular kinematic region. Owing to the finite number of particles involved in such an estimate, the EP is subject to statistical fluctuations. The measurements obtained with the EP have to be corrected for the event-plane resolution [2], characterized by $\langle \cos[n(\Psi_n^{EP} - \Psi_n^{FP})] \rangle$. Ψ^{EP} is the azimuthal angle of the reconstructed nth-harmonic flow vector $\mathbf{Q}_n = [\sum_i^N w_i \cos(n\varphi_i), \sum_i^N w_i \sin(n\varphi_i)]$, where w_i is the weight for each particle. For simplicity, we use unity weights in the event-plane calculation.

(v) The spectator plane (SP) is determined by a sideward deflection of spectator nucleons and is regarded as a better proxy for RPs than FPs (determined by participants).

Gaussian model of eccentricity fluctuations

Sergei A. Voloshin [a,*], Arthur M. Poskanzer [b], Aihong Tang [c], Gang Wang [d] Physics Letters B 659 (2008) 537–541

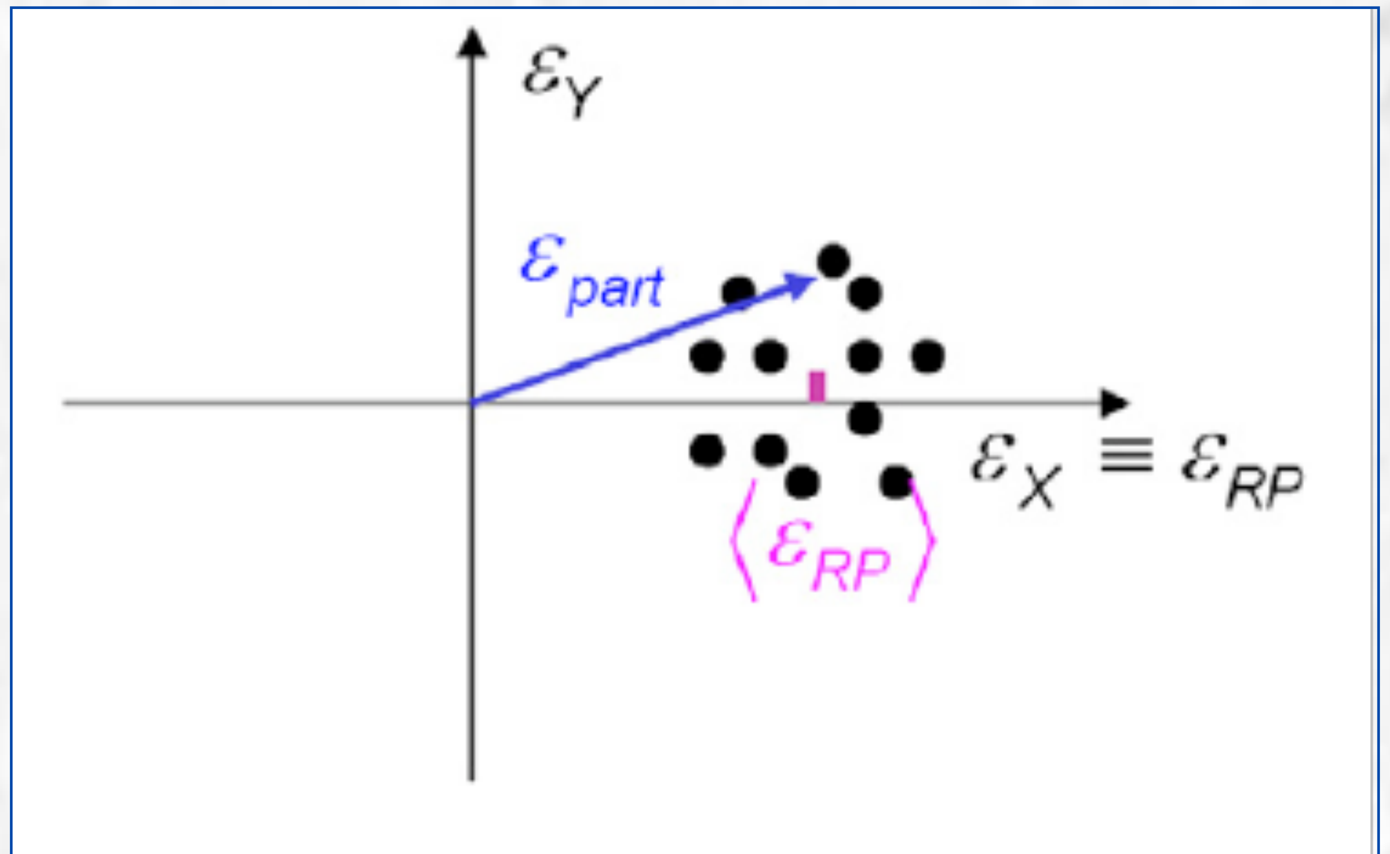
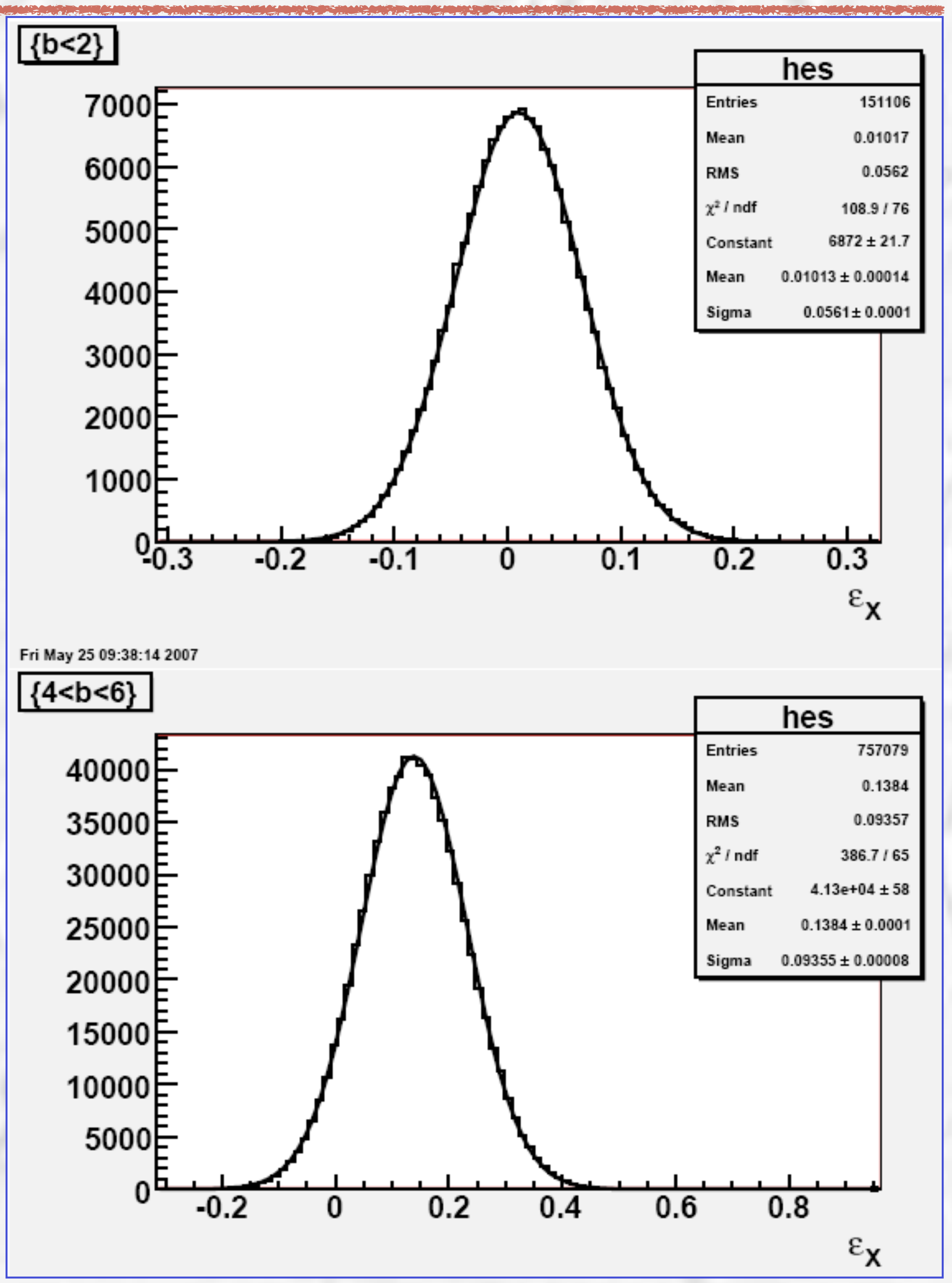
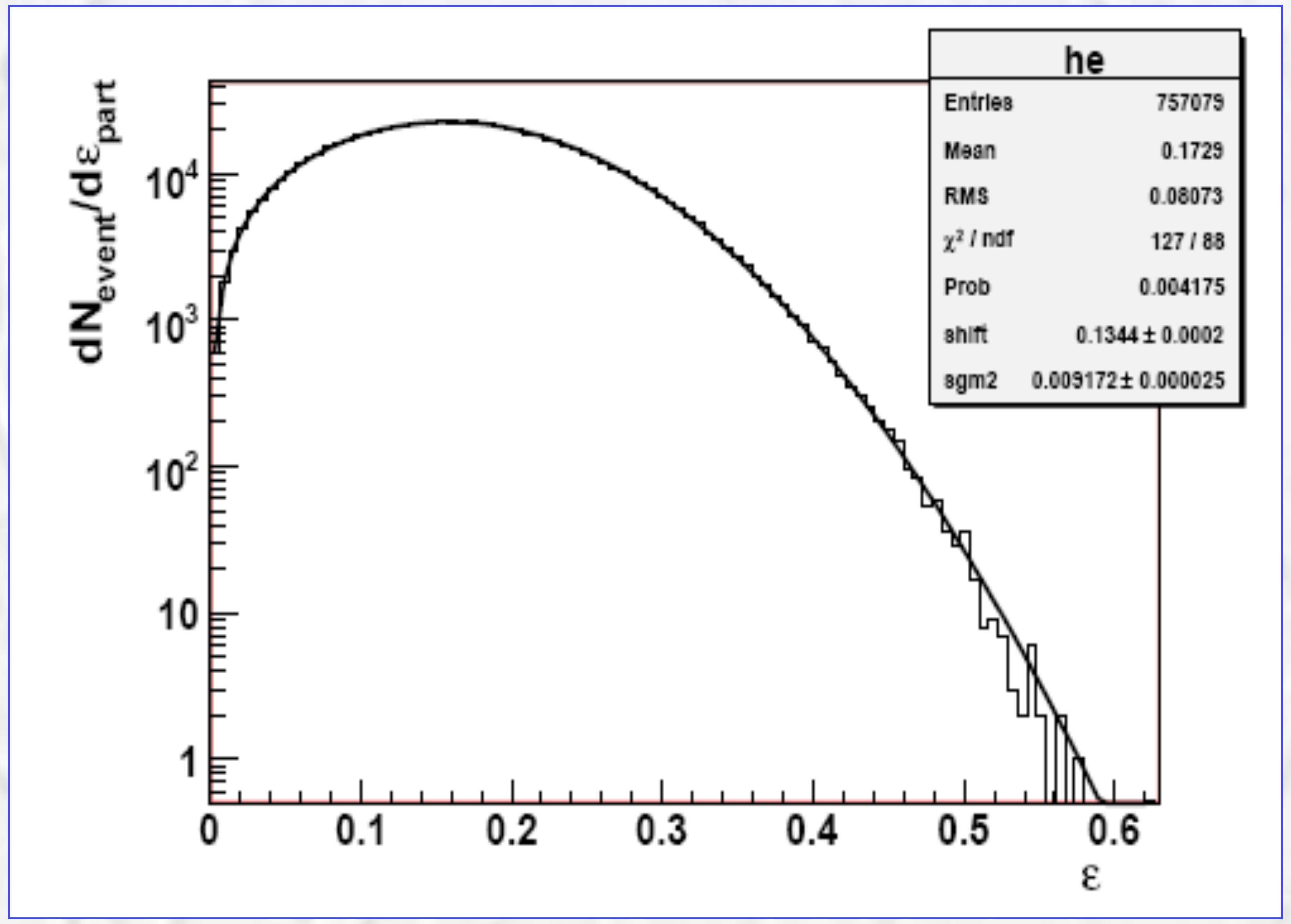


Fig. 3. Definition of $\varepsilon_{\rm part}$.

Model assumes a Gaussian form for the distributions in ε_x and ε_y, (a very good approximation of MC Glauber calculations).



$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle\varepsilon_{RP}\rangle}{\sigma_\varepsilon^2}\right)\exp\left(-\frac{\varepsilon_{part}^2 + \langle\varepsilon_{RP}\rangle^2}{2\sigma_\varepsilon^2}\right) \equiv \mathrm{BG}(\varepsilon_{part};\bar{\varepsilon},\sigma_\varepsilon),$$



$$\boldsymbol{\varepsilon} = \{\varepsilon_x, \varepsilon_y\} = \left\{\left\langle\frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2}\right\rangle_{\rm part}, \left\langle\frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2}\right\rangle_{\rm part}\right\}$$

$$v_2\{2\}^2 = \kappa^2\left(\langle\varepsilon_{RP}\rangle^2 + 2\sigma_\varepsilon^2\right) + \delta = \langle v_{RP}\rangle^2 + 2\sigma_{vx}^2 + \delta$$

$$v_2\{4\}^4 = 2\langle v_2^2\rangle^2 - \langle v_2^4\rangle = \bar{v}_2^4 = \langle v_{RP}\rangle^4$$

$$v_2\{6\}^6 = \left(\langle v_2^6\rangle - 9\langle v_2^4\rangle\langle v_2^2\rangle + 12\langle v_2^2\rangle^3\right)/4 = \langle v_{RP}\rangle^6$$

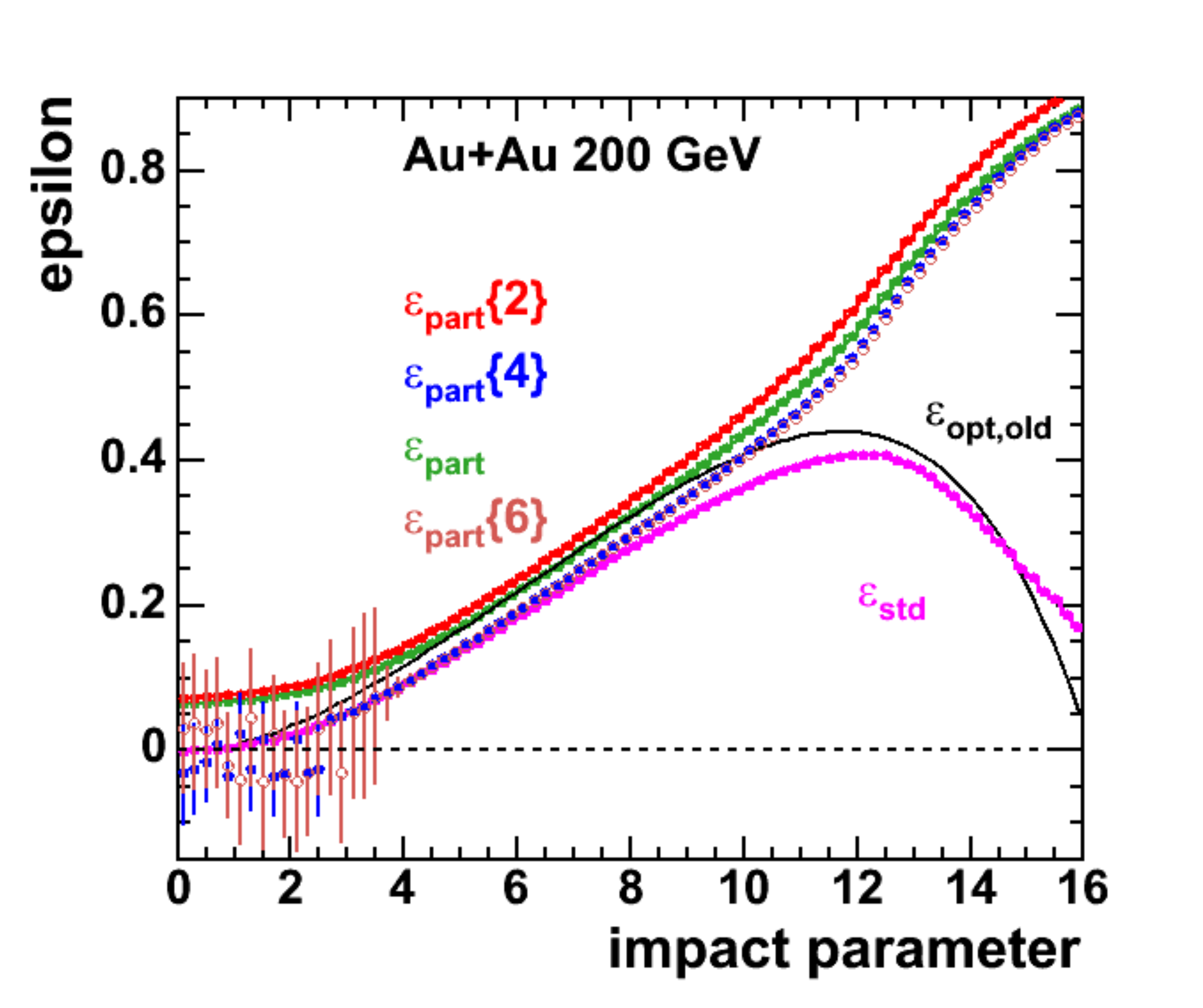
➔ $v_2\{4\}$ (and higher cumulants) coincides with $v_2\{\mathrm{RP}\}$

Eccentricity fluctuations: 'Standard' vs 'Participant'

Main idea: use proper $\varepsilon\{n\}$ to rescale corresponding $v_2\{n\}$:

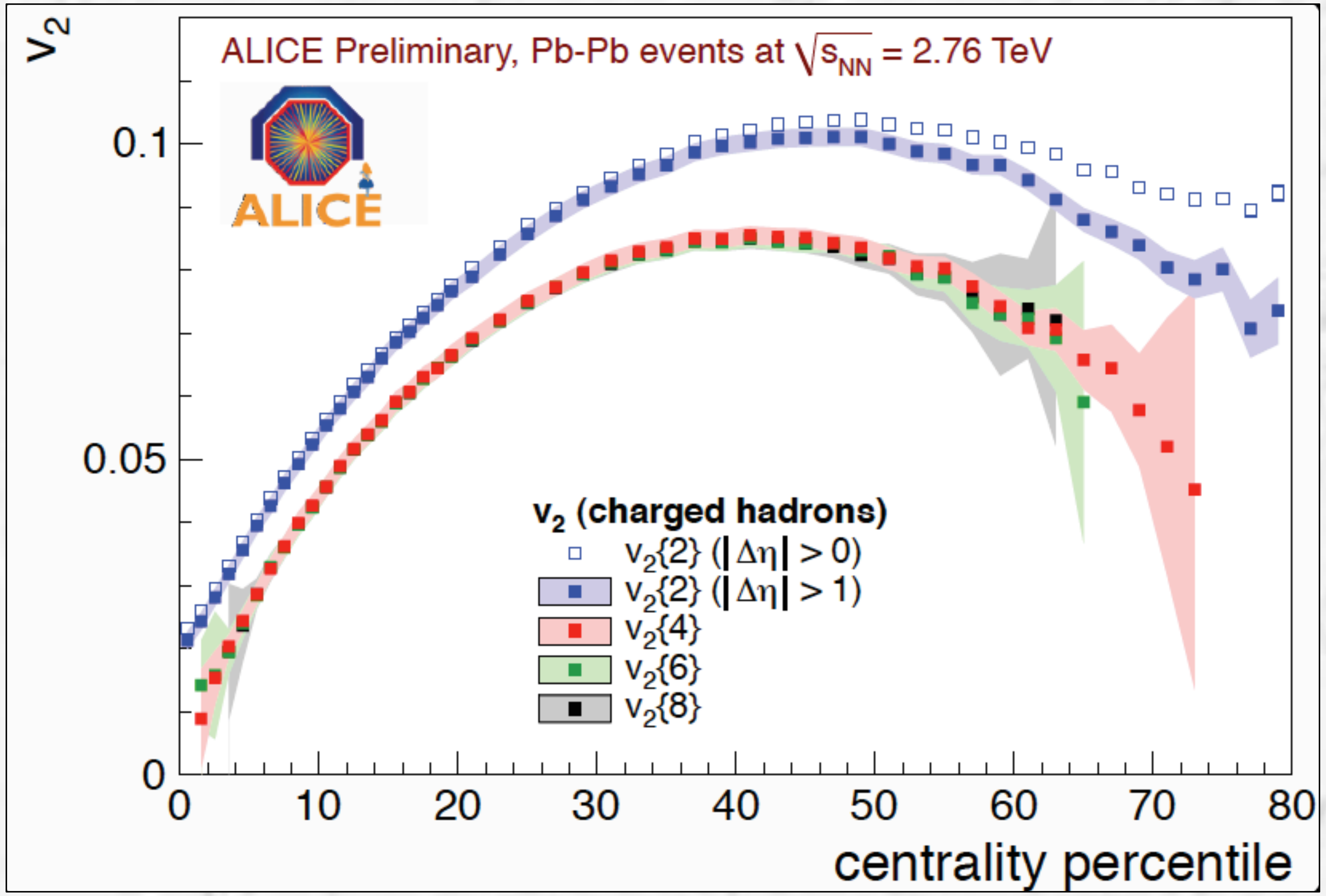
$$v\{n\} = \langle v \rangle \, \varepsilon\{n\} / \langle \varepsilon \rangle$$

$$\varepsilon^2\{2\} \equiv \langle \varepsilon^2 \rangle = \langle \varepsilon \rangle^2 + \sigma_\varepsilon^2$$
$$\varepsilon^4\{4\} \equiv 2\langle \varepsilon^2 \rangle^2 - \langle \varepsilon^4 \rangle$$



Note:
"participant" eccentricity values are larger compared to "standard"
$\varepsilon_{std} \approx \varepsilon_{part} \cos(\Delta\Psi)$.
- higher cumulant results are very close to "standard" ones for midcentral collisions

v2{2} vs v2{4}, flow fluctuations, or nonflow?



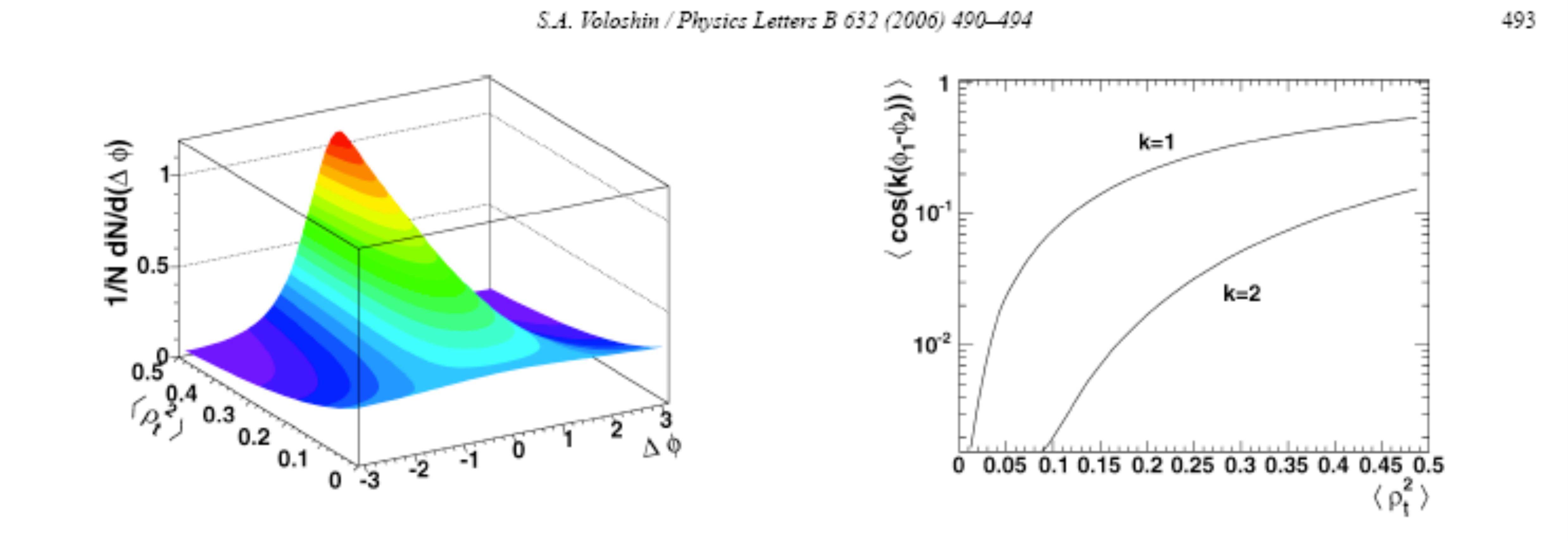
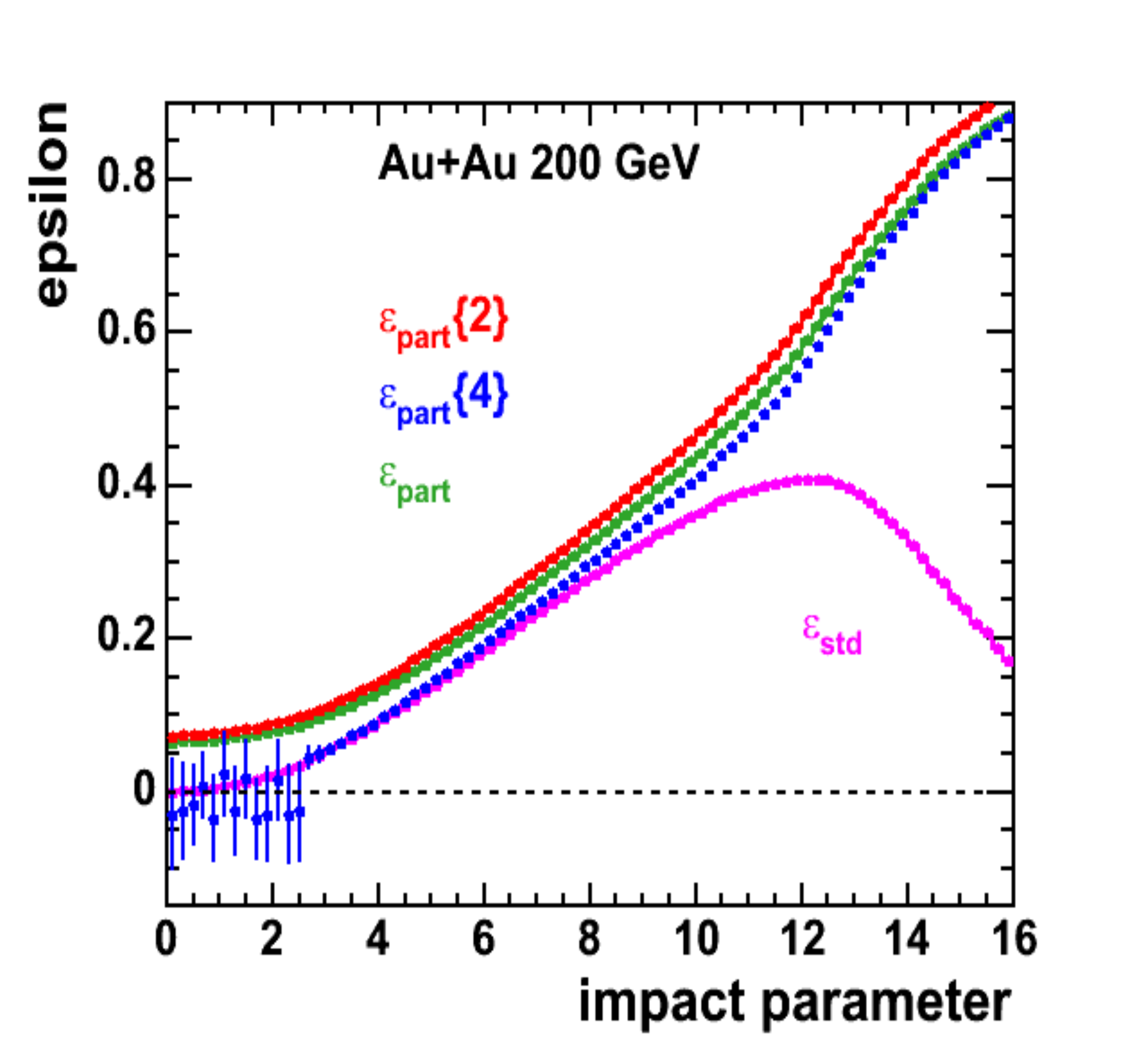


Fig. 3. (Color online.) Two pion $\Delta\phi$ distribution as function of $\langle\rho_t^2\rangle$ in the blast wave model. Linear velocity profile and $T = 110$ MeV have been assumed.

Fig. 4. The average values of $\cos(\Delta\phi)$ and $\cos(2\Delta\phi)$ for the distribution shown in Fig. 3.



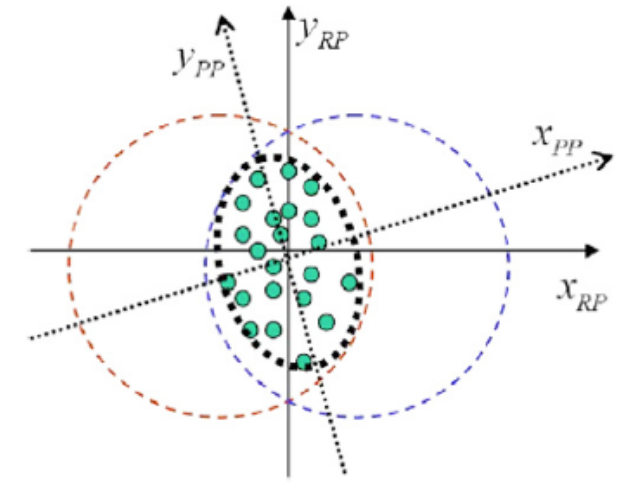
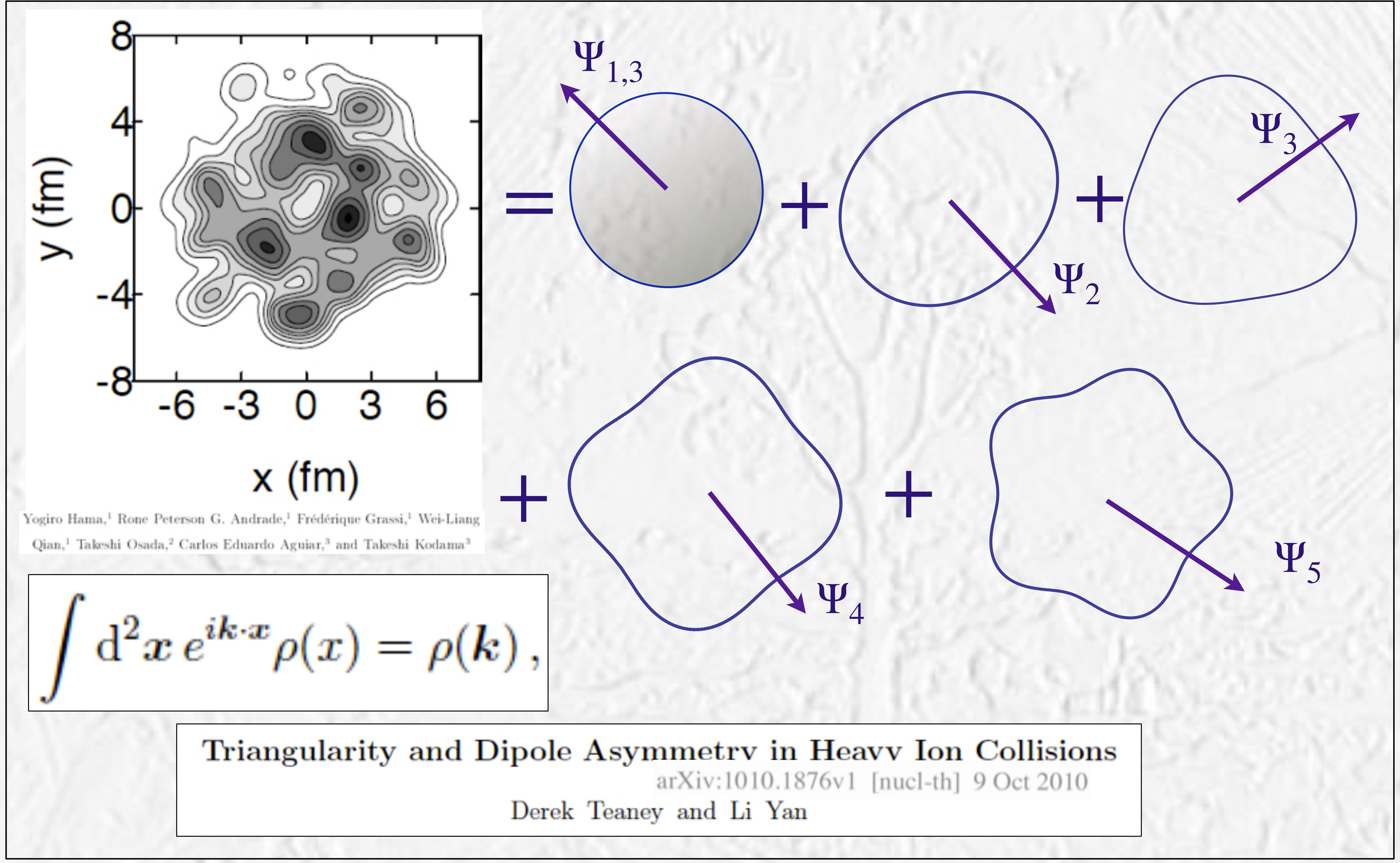


Fig. 1. The definitions of the RP and PP coordinate systems.

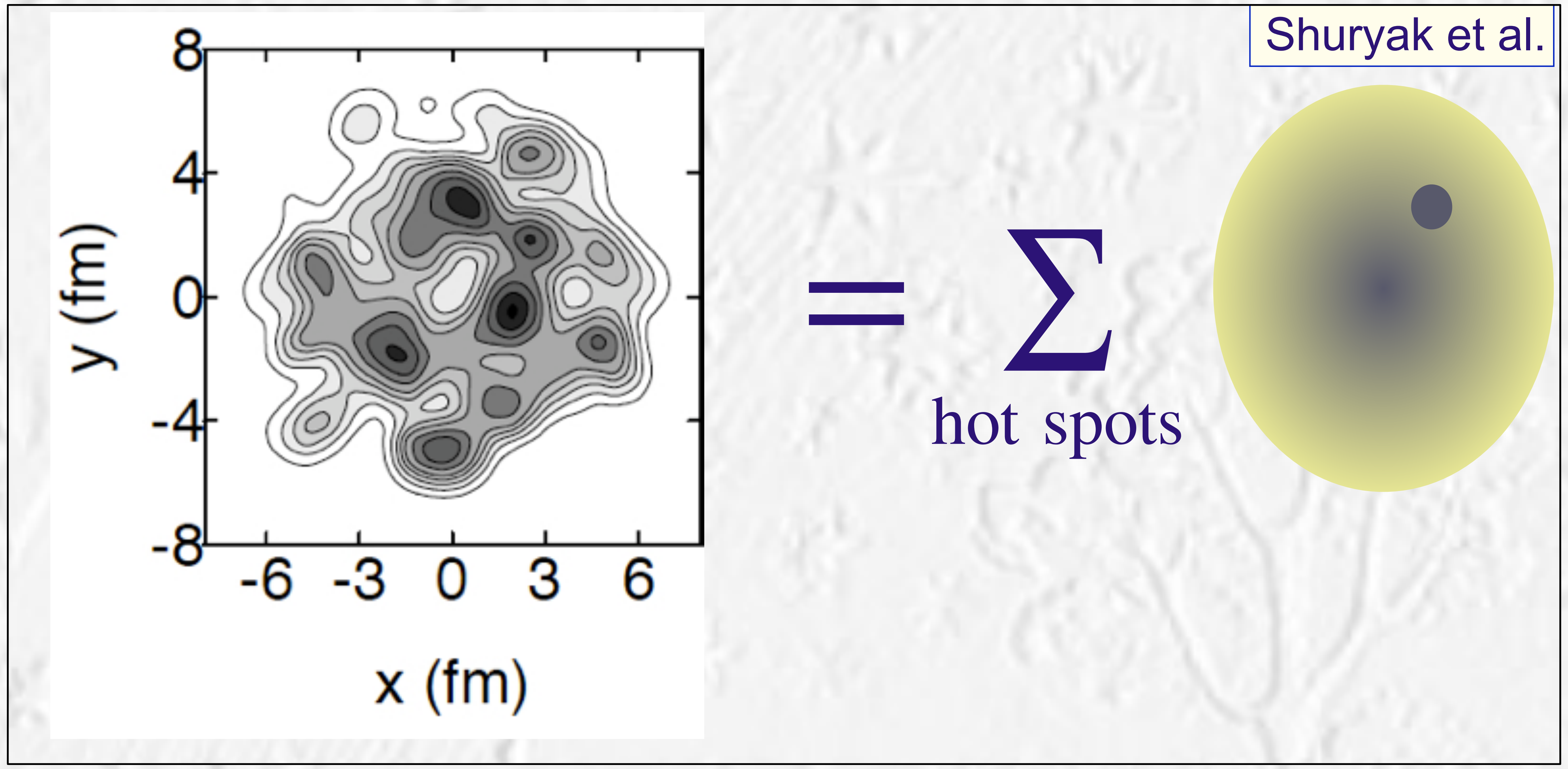
Any of two could "explain" the entire difference between $v_2\{2\}$ and $v_2\{4\}$

Flow fluctuations = nonflow (radial expansion)

2d Fourier density decomposition



In the linear approximation:



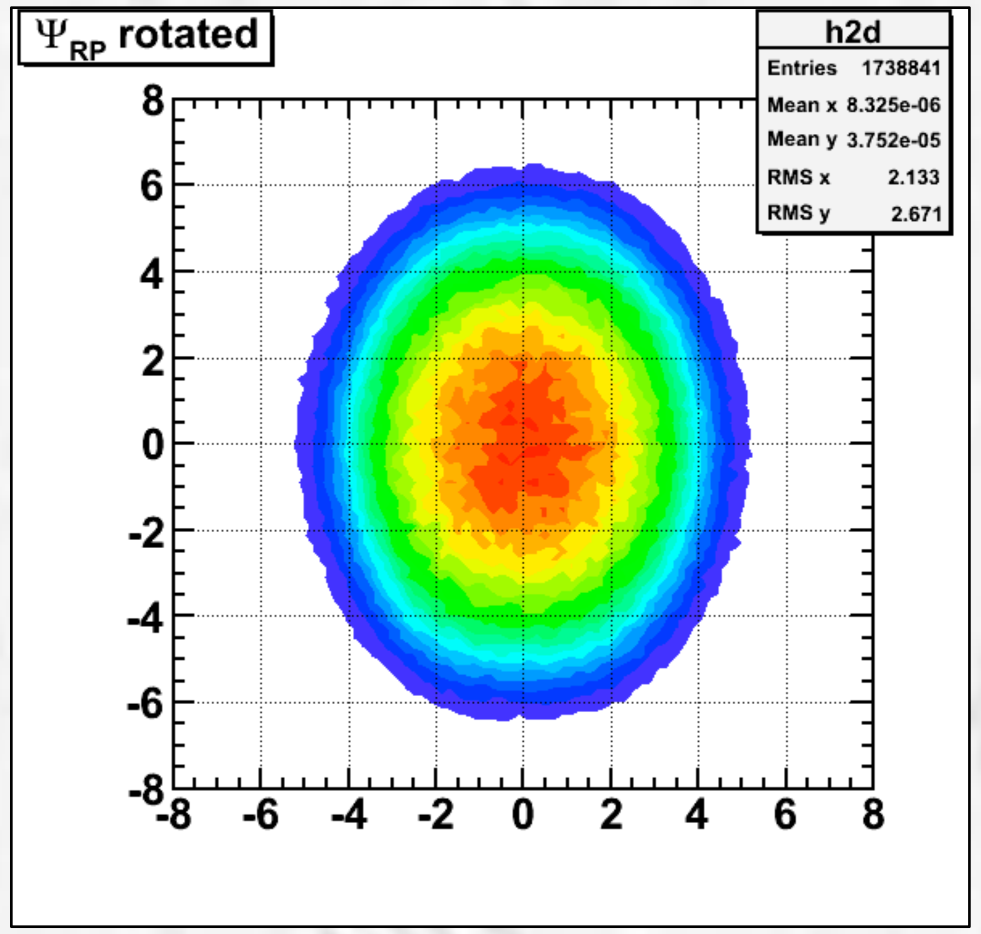
$$\varepsilon_n e^{in\Phi_n} \equiv -\frac{\int r\,dr\,d\phi\, r^n e^{in\phi}\, e(r,\phi)}{\int r\,dr\,d\phi\, r^n e(r,\phi)} \quad (n>1),$$

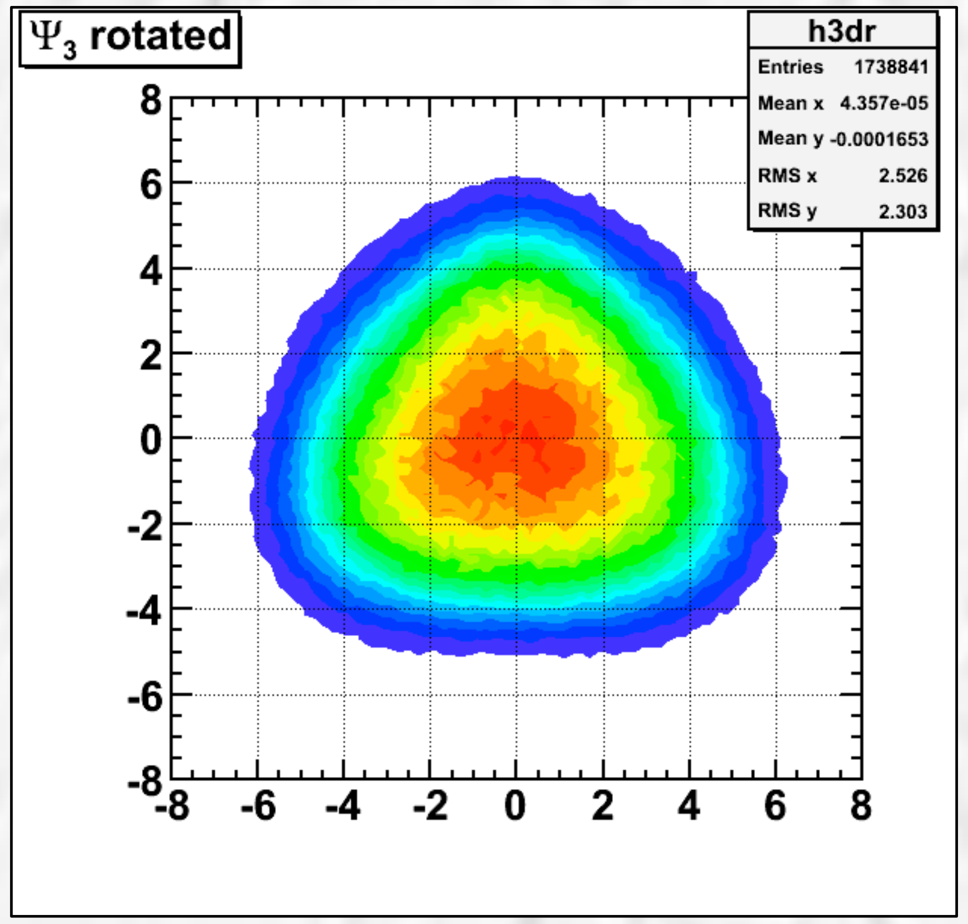
$$\varepsilon_1 e^{i\Phi_1} \equiv -\frac{\int r\,dr\,d\phi\, r^3 e^{i\phi}\, e(r,\phi)}{\int r\,dr\,d\phi\, r^3 e(r,\phi)}$$

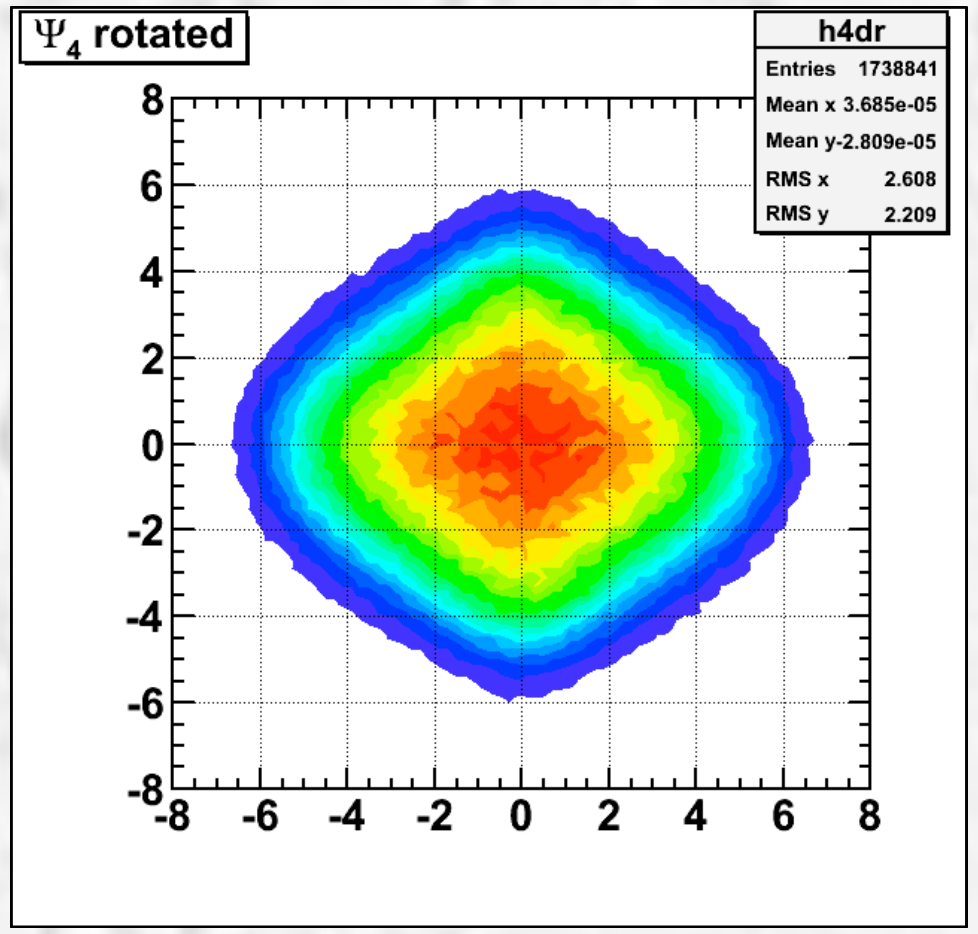
$v_n\{RP\}$ - "hot spots" correlations = nonflow
$v_n\{PP\}$ - flow vs "participant planes" - "hot spot" correlations = part of flow fluctuations

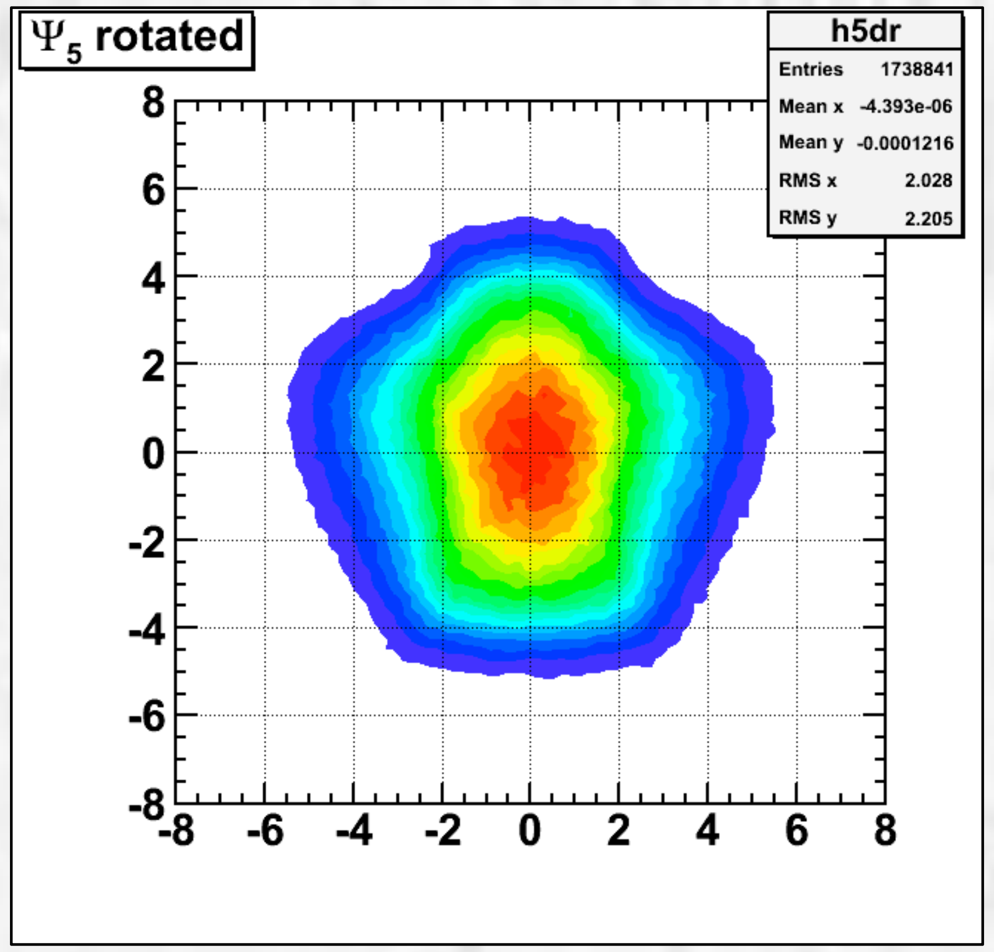
Density distributions

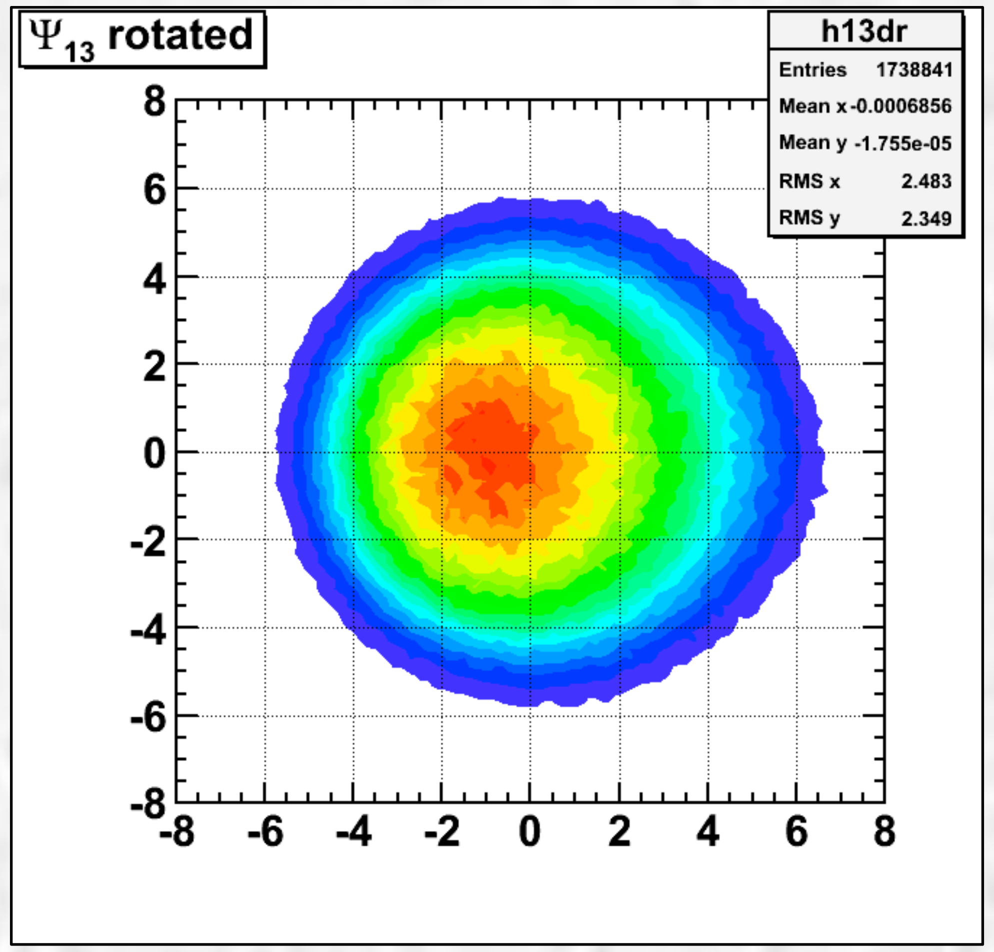
10k Pb+Pb events, b=8 fm











$$E\frac{d^3n}{d^3p} = \frac{1}{2\pi p_T}\frac{d^2n}{dp_t\,dy}\left(1 + \sum_n 2\overline{v}_n \cos[n(\phi - \overline{\Psi}_n)]\right)$$

$$\overline{v}_n(p_T, y) = \langle \cos[n(\phi_i - \overline{\Psi}_n)] \rangle$$

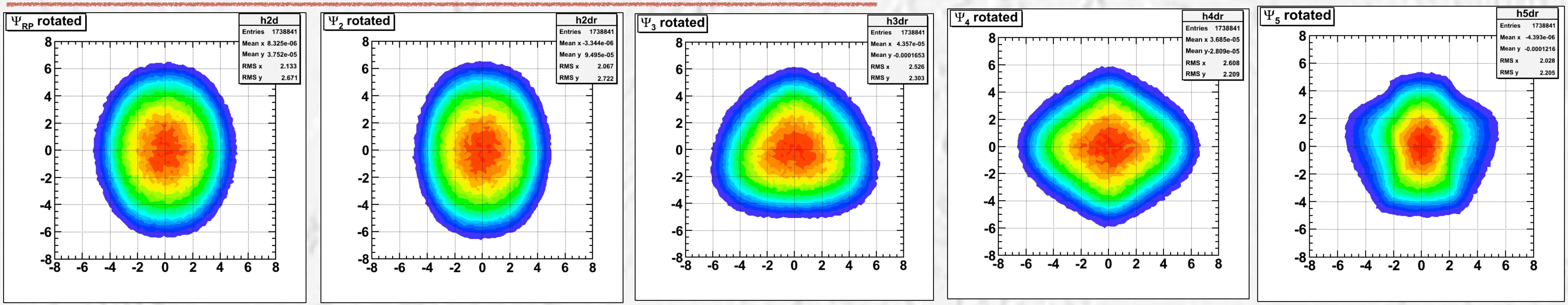
Note the difference in definitions of v_n.
$\overline{\Psi}_n$ accounts for contribution from non-linear modes (see below)

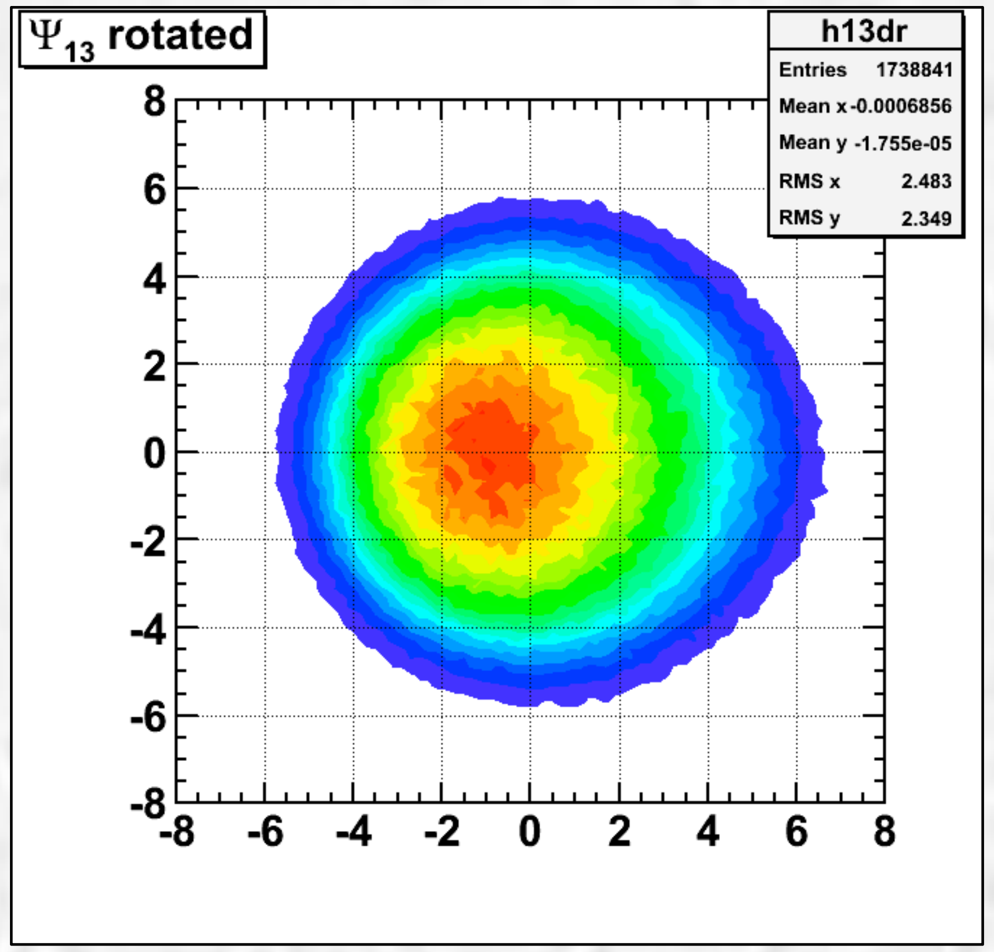
Usually all "overlines" are omitted

Note, that in such a picture, the flow plane (orientation of anisotropic flow) can depend on particle momentum (rapidity and p_T), on the particle type, strength of another harmonic, etc.

Density distributions

10k Pb+Pb events, b=8 fm





$$E\frac{d^3n}{d^3p} = \frac{1}{2\pi\, p_T}\frac{d^2n}{dp_t\, dy}\left(1 + \sum_n 2\overline{v}_n \cos[n(\phi - \overline{\Psi}_n)]\right)$$

$$\overline{v}_n(p_T, y) = \langle \cos[n(\phi_i - \overline{\Psi}_n)]\rangle$$

Note the difference in definitions of v_n.
$\overline{\Psi}_n$ accounts for contribution from non-linear modes (see below)

Usually all “overlines” are omitted

Note, that in such a picture, the flow plane (orientation of anisotropic flow) can depend on particle momentum (rapidity and p_T), on the particle type, strength of another harmonic, etc.

Flow correlations and decorrelations

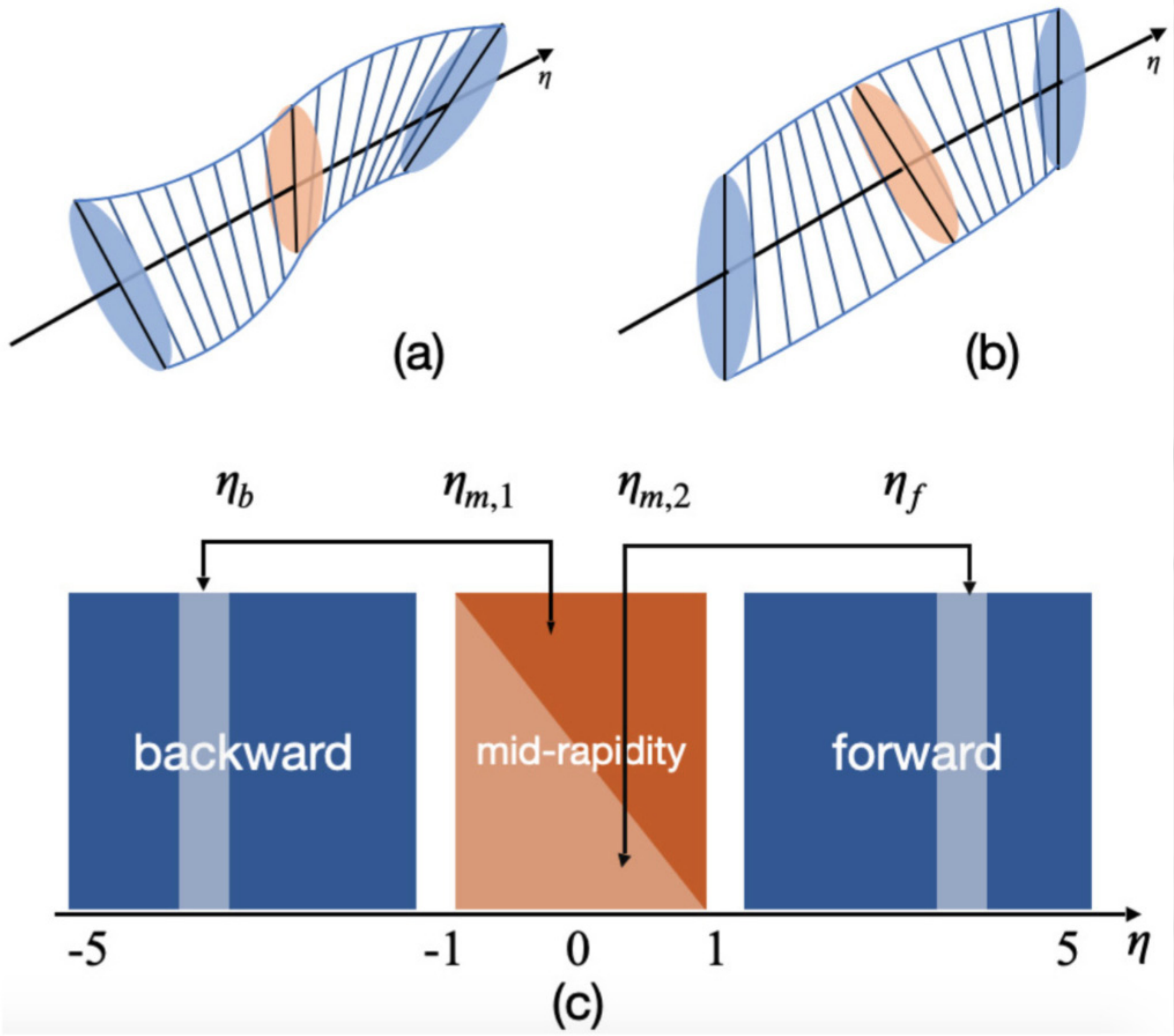


FIG. 2. Schematic view of (a) the "torque" or S-shaped decorrelation patterns and (b) the bow or C-shaped decorrelation patterns in the longitudinal distribution of the flow plane angle. The ellipses indicate the transverse momentum distributions of the final-state particles. Panel (c) delimits the kinematic regions for the particles at mid, forward, and backward pseudorapidities.

V. Khachatryan *et al.* (CMS Collaboration), Phys. Rev. C **92**, 034911 (2015).

$$r_2^{(\Psi)} = 1 - 2F_2^{(\Psi)}\eta.$$

$$r_2^{\Psi}(\eta) = \frac{\langle\cos[2(\Psi_{-\eta} - \Psi_{\rm f})]\rangle}{\langle\cos[2(\Psi_{\eta} - \Psi_{\rm f})]\rangle}.$$

Measures a decrease in flow due to plane decorrelations over the "distance" 2η

Independence of r_2 on η_f indicates a "random walk" variation of Ψ with rapidity

PHYSICAL REVIEW C **105**, 024902 (2022)

Flow-plane decorrelations in heavy-ion collisions with multiple-plane cumulants

Zhiwan Xu[1,*] Xiatong Wu[1] Caleb Sword[2] Gang Wang[1,†] Sergei A. Voloshin[2] and Huan Zhong Huang[1,3]

$$T_2 = \frac{\langle\langle\sin 2(\Psi_{\rm f} - \Psi_{\rm m,1})\sin 2(\Psi_{\rm b} - \Psi_{\rm m,2})\rangle\rangle}{{\rm Res}(\Psi_{\rm f}){\rm Res}(\Psi_{\rm m,1}){\rm Res}(\Psi_{\rm b}){\rm Res}(\Psi_{\rm m,2})},$$

Symmetric cumulants

$$SC(k,l) \equiv \langle v_k^2 v_l^2\rangle - \langle v_k^2\rangle\langle v_l^2\rangle \qquad sc(n,m) \equiv \frac{\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle}{\langle v_n^2\rangle\langle v_m^2\rangle}.$$

$$\begin{aligned}\langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c &= \langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle \\ &\quad - \langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle\langle\langle\cos[n(\varphi_1 - \varphi_2)]\rangle\rangle \\ &= \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle\end{aligned}$$

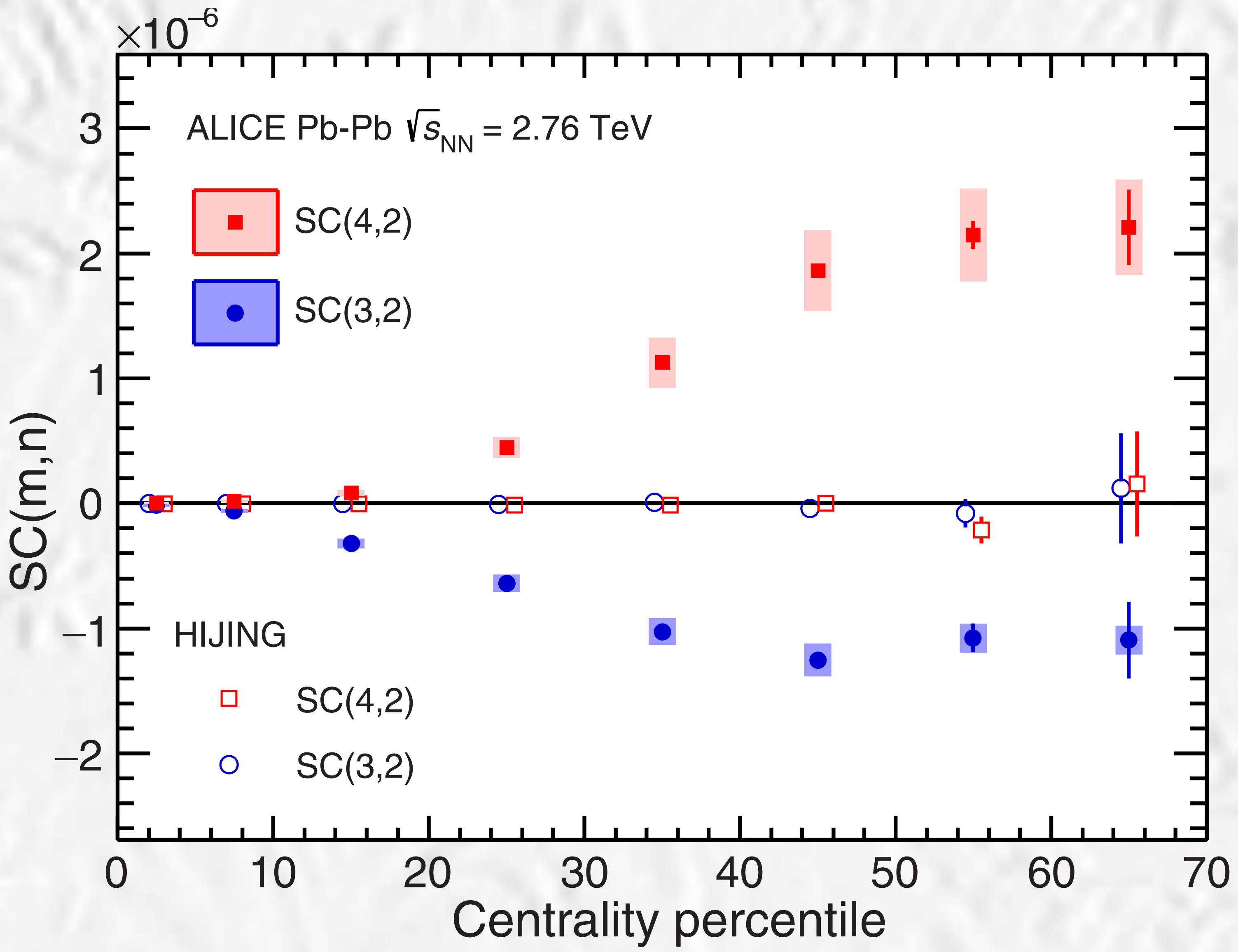
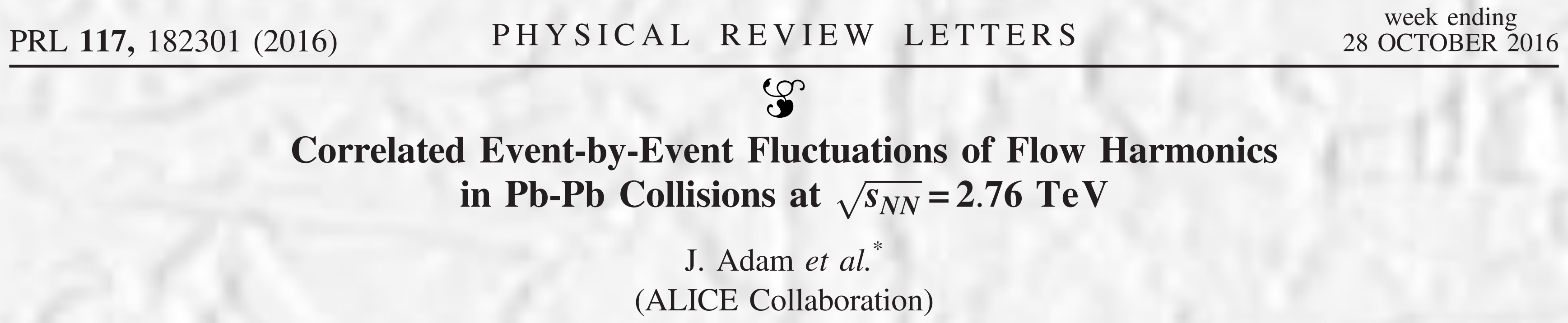


FIG. 1. Centrality dependence of the observables SC(4,2) (red filled squares) and SC(3,2) (blue filled circles) in Pb-Pb collisions at 2.76 TeV. Systematic errors are represented with boxes. The results for the HIJING model are shown with hollow markers.


PRL **117**, 182301 (2016) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2016

Correlated Event-by-Event Fluctuations of Flow Harmonics in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

J. Adam *et al.**
(ALICE Collaboration)

Linear and nonlinear flow modes

[34] L. Yan, J.-Y. Ollitrault, $\nu_4, \nu_5, \nu_6, \nu_7$: nonlinear hydrodynamic response versus LHC data, Phys. Lett. B 744 (2015) 82–87, arXiv:1502.02502 [nucl-th].

$$V_n = v_n e^{in\Psi_n}$$

$$V_4 = V_4^{NL} + V_4^{L} = \chi_{4,22}(V_2)^2 + V_4^{L},$$
$$V_5 = V_5^{NL} + V_5^{L} = \chi_{5,32} V_2 V_3 + V_5^{L},$$
$$V_6 = V_6^{NL} + V_6^{L}$$
$$= \chi_{6,222}(V_2)^3 + \chi_{6,33}(V_3)^2 + \chi_{6,42} V_2 V_4^{L} + V_6^{L}.$$

$$v_{4,22}^{A}(p_T) = \frac{d_{4,22}^{A}(p_T)}{\sqrt{c_{22,22}}} = \frac{\langle\langle \cos(4\varphi_1^A(p_T) - 2\varphi_2^B - 2\varphi_3^B)\rangle\rangle}{\sqrt{\langle\langle \cos(2\varphi_1^A + 2\varphi_2^A - 2\varphi_3^B - 2\varphi_4^B)\rangle\rangle}},$$

$$v_{4,22}(p_T) = \frac{\langle v_4(p_T) v_2^2 \cos(4\Psi_4 - 4\Psi_2)\rangle}{\sqrt{\langle v_2^4\rangle}} \approx \langle v_4(p_T)\cos(4\Psi_4 - 4\Psi_2)\rangle$$

NCQ scaling in nonlinear flow modes

JHEP
PUBLISHED FOR SISSA BY SPRINGER
RECEIVED: *February 10, 2020*
REVISED: *May 11, 2020*
ACCEPTED: *June 7, 2020*
PUBLISHED: *June 24, 2020*

Non-linear flow modes of identified particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

ALICE
The ALICE collaboration

$$V_n = v_n e^{in\Psi_n}$$

$$V_4 = V_4^L + V_4^{NL} = V_4^L + \chi_{4,22}(V_2)^2,$$

Does the ratio of baryon and meson flow in nonlinear modes equals to the square or first power of that in linear parts?

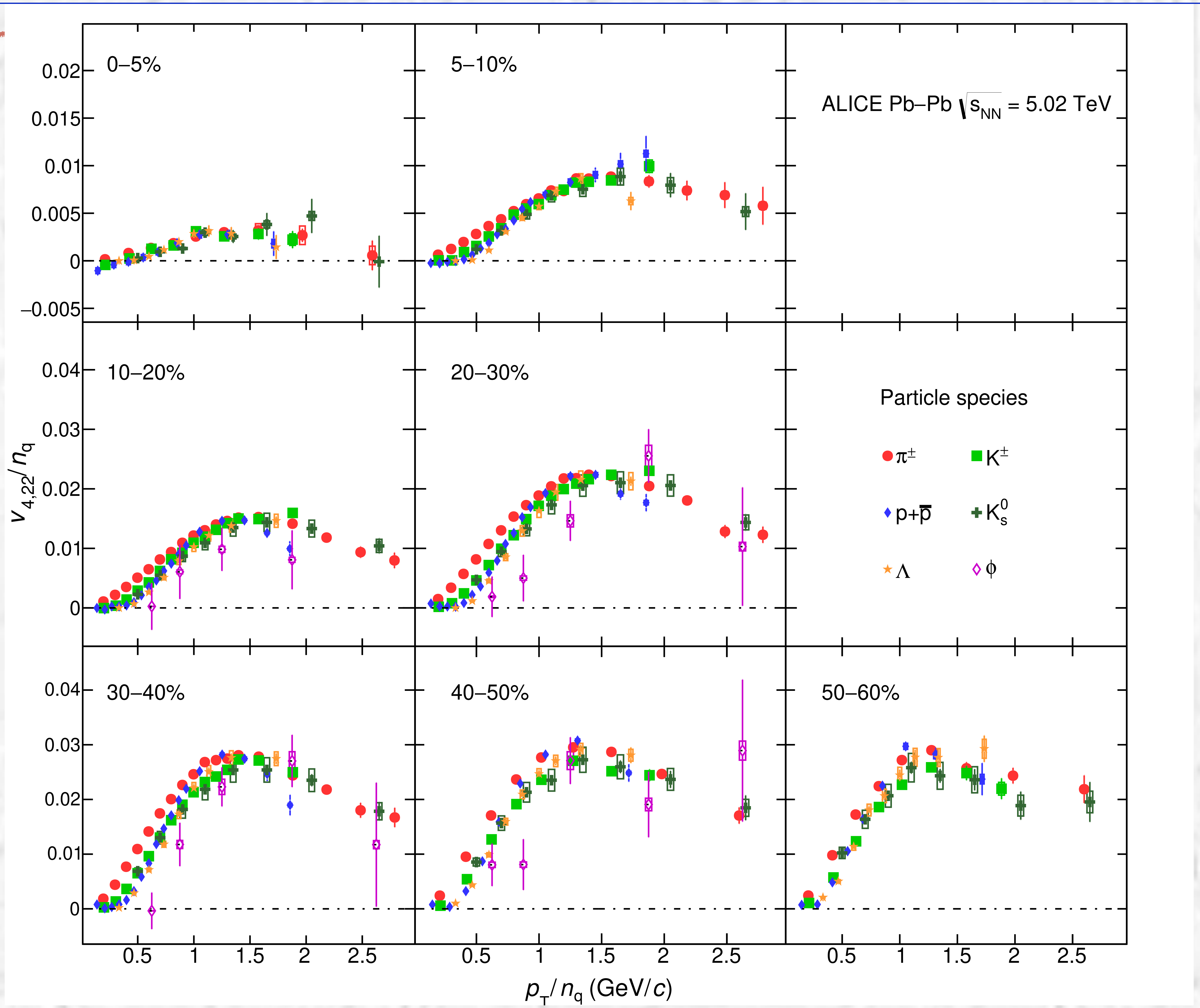


Figure 10. The p_T/n_q-dependence of $v_{4,22}/n_q$ for different particle species grouped into different centrality intervals of Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Statistical and systematic uncertainties are shown as bars and boxes, respectively.

NCQ scaling in nonlinear flow modes

JHEP
Published for SISSA by Springer
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Non-linear flow modes of identified particles in Pb-Pb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV

ALICE
The ALICE collaboration

$$V_{\rm n} = v_{\rm n} e^{in\Psi_{\rm n}}$$

$$V_4 = V_4^{\rm L} + V_4^{\rm NL} = V_4^{\rm L} + \chi_{4,22}(V_2)^2,$$

Does the ratio of baryon and meson flow in nonlinear modes equals to the square or first power of that in linear parts?

Might be better to define
$V_4 = \kappa_4^L \mathscr{E}_4 + \kappa_4^{NL} \mathscr{E}_2^2$

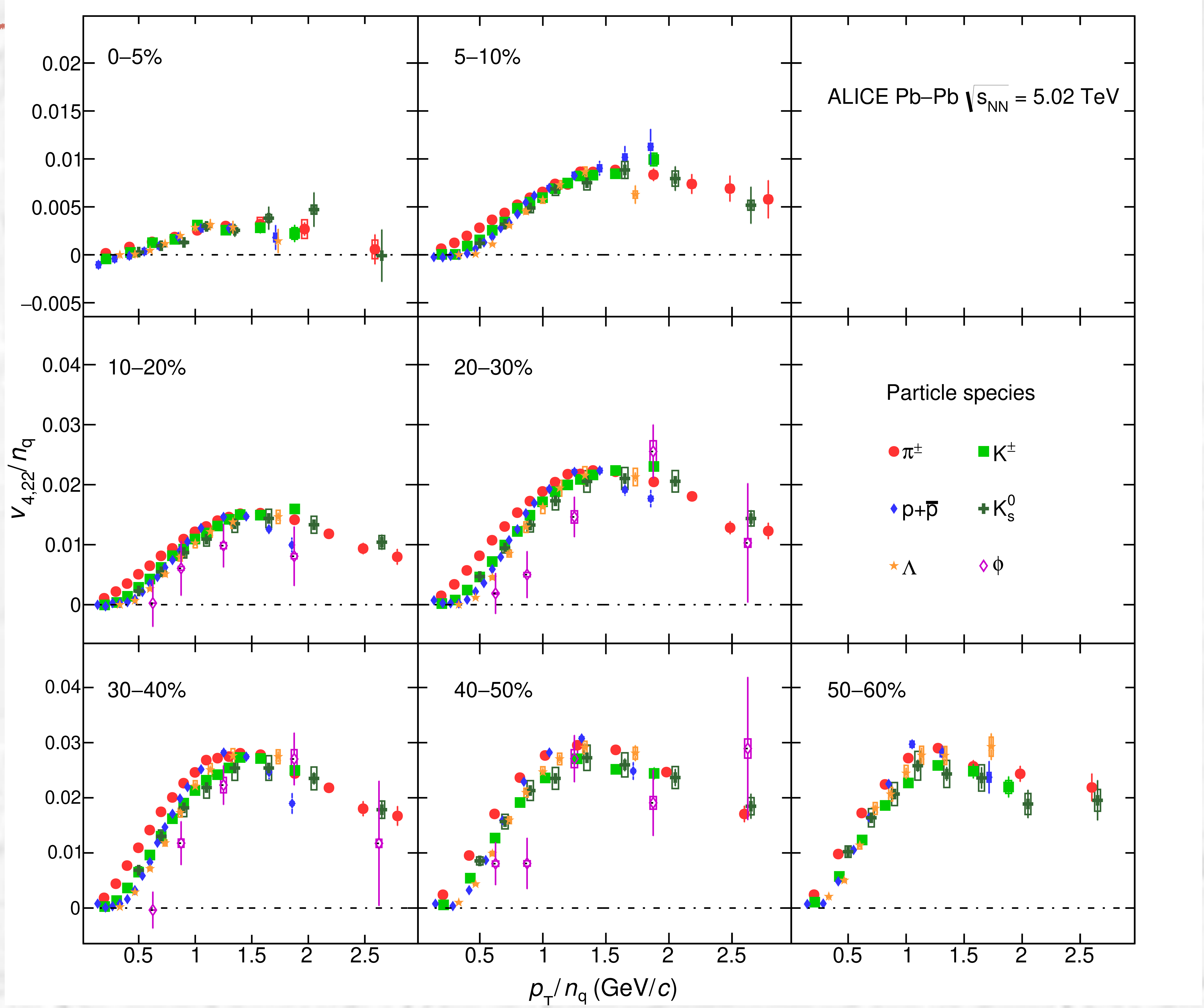


Figure 10. The $p_{\rm T}/n_q$-dependence of $v_{4,22}/n_q$ for different particle species grouped into different centrality intervals of Pb-Pb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV. Statistical and systematic uncertainties are shown as bars and boxes, respectively.

Summary

Measurements of anisotropic flow, flow fluctuations, correlations between flow of different harmonics or the same harmonic at different momentum ranges are sensitive to many details of the initial conditions and the sytem evolution.

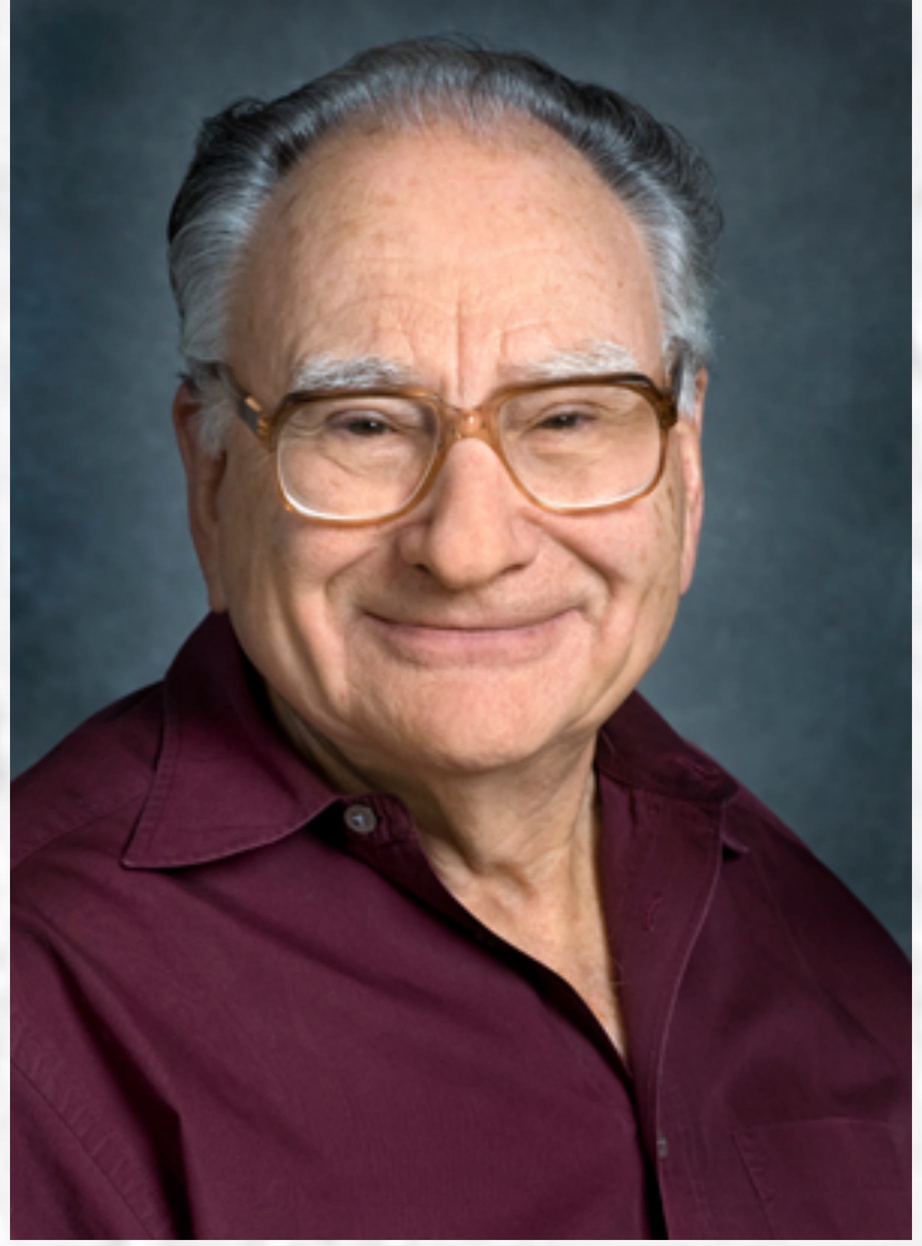
PAY ATTENTION TO DETALS: references, definitions and terminology, clearly define physical goals and corresponding measurements/observables

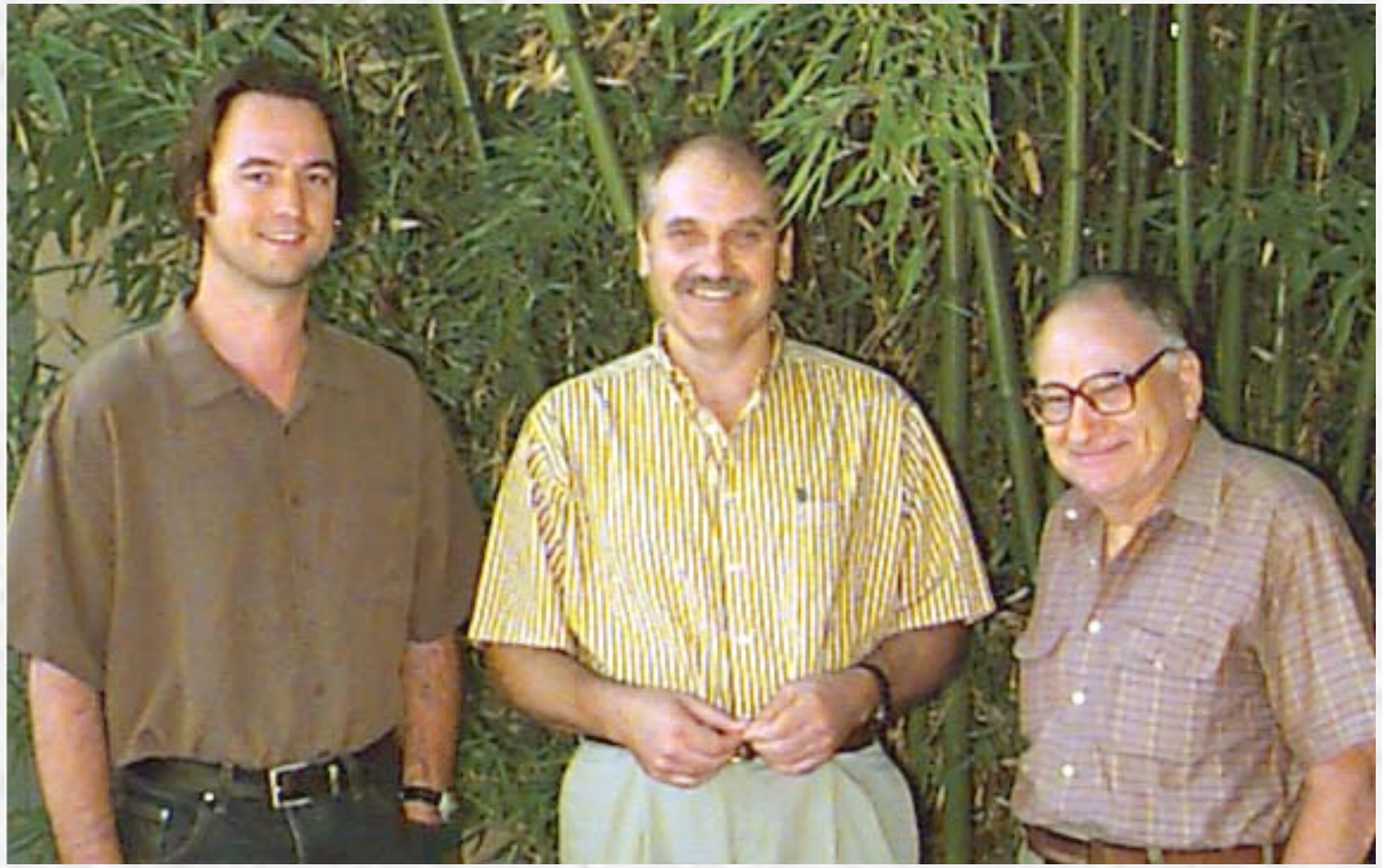
In memory of Art Poskanzer

Art: physics, inspiration, and much more

Flow, as a truly ideal fluid, has interpenetrated
all parts of heavy ion physics, it brings
new discoveries and contributes greatly
to our understanding of strongly interacting matter.

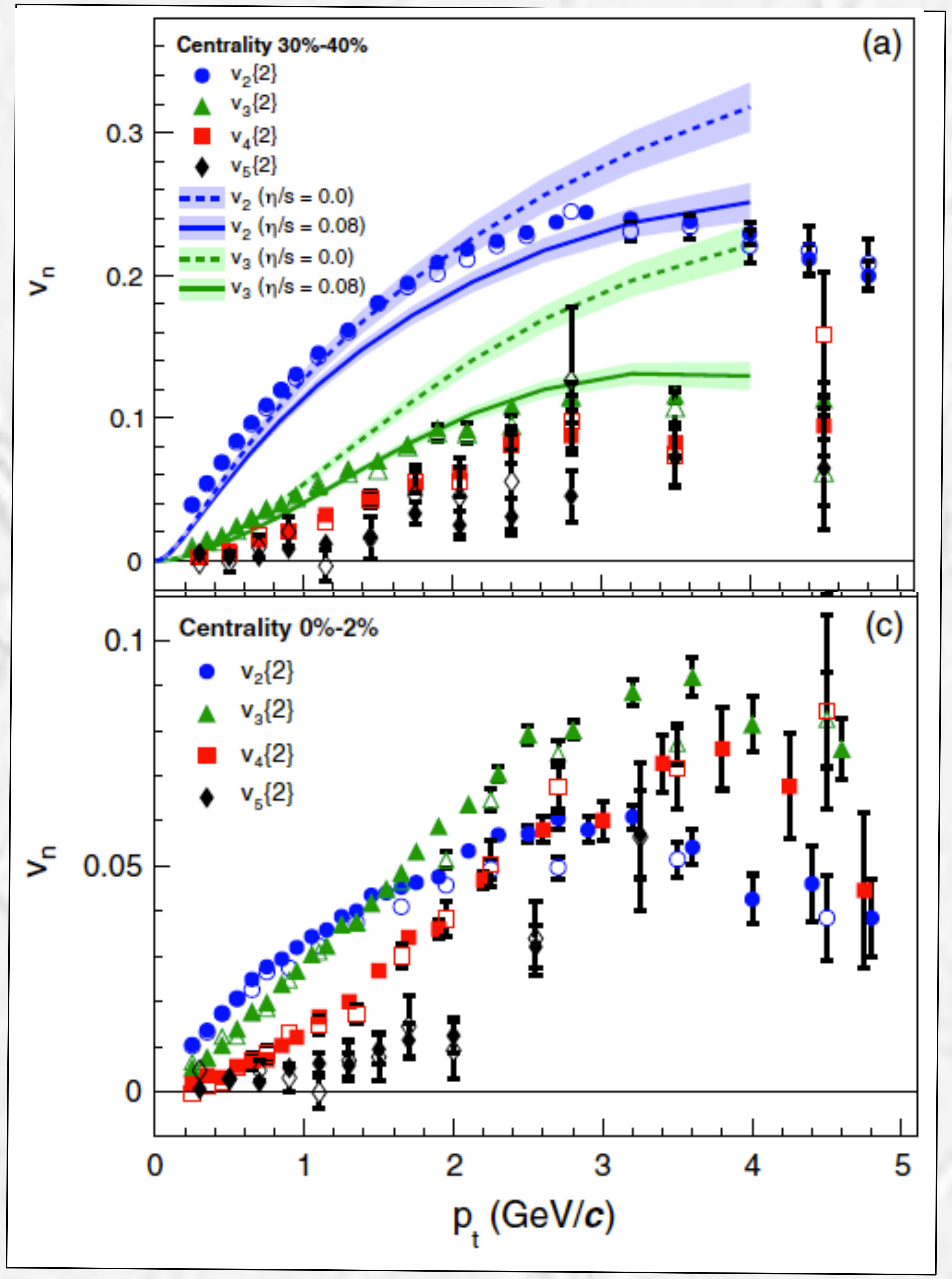
We are grateful to Art, who made it works.





EXTRA SLIDES

Flow fluctuations - “ridge” duality



PRL **107**, 032301 (2011)

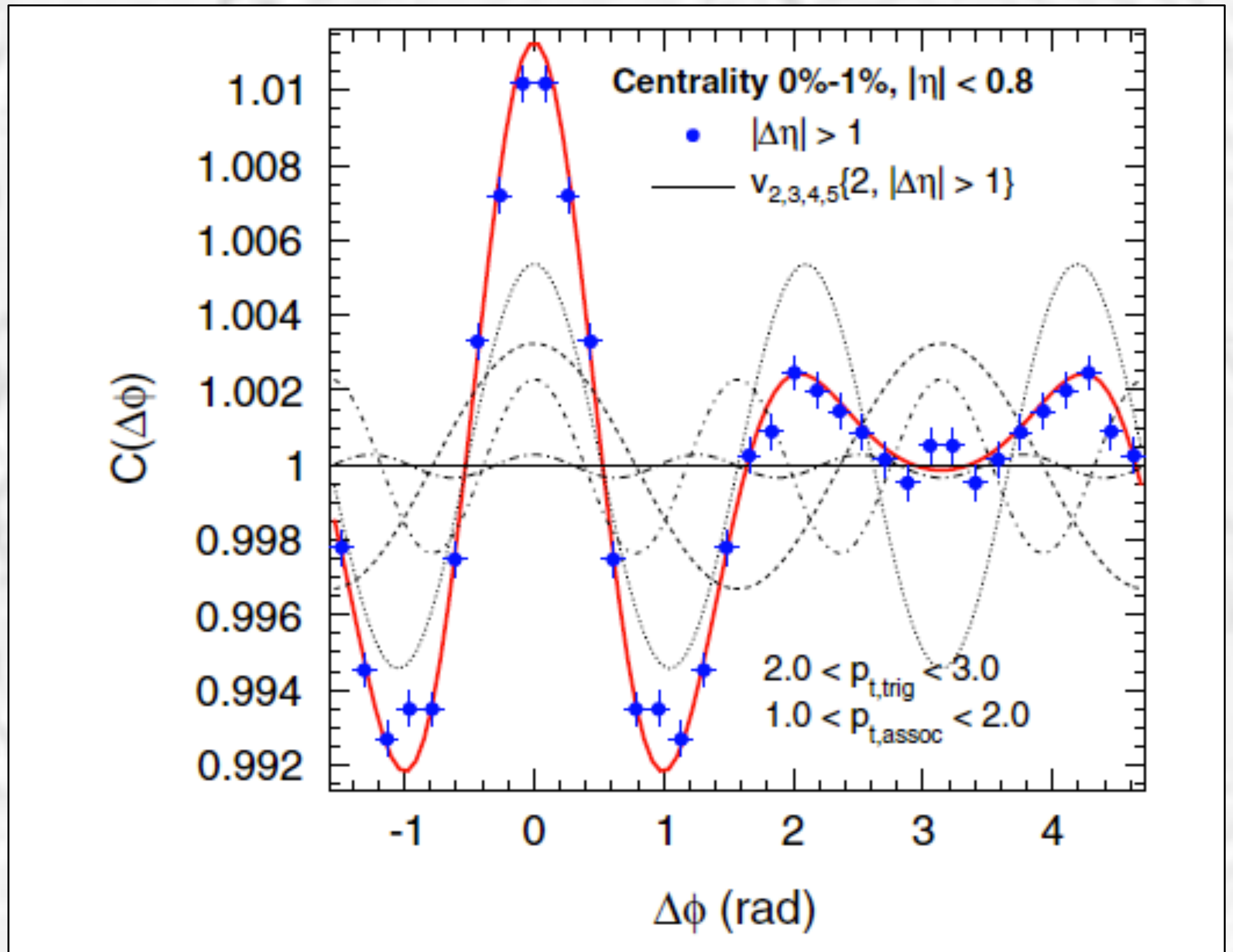
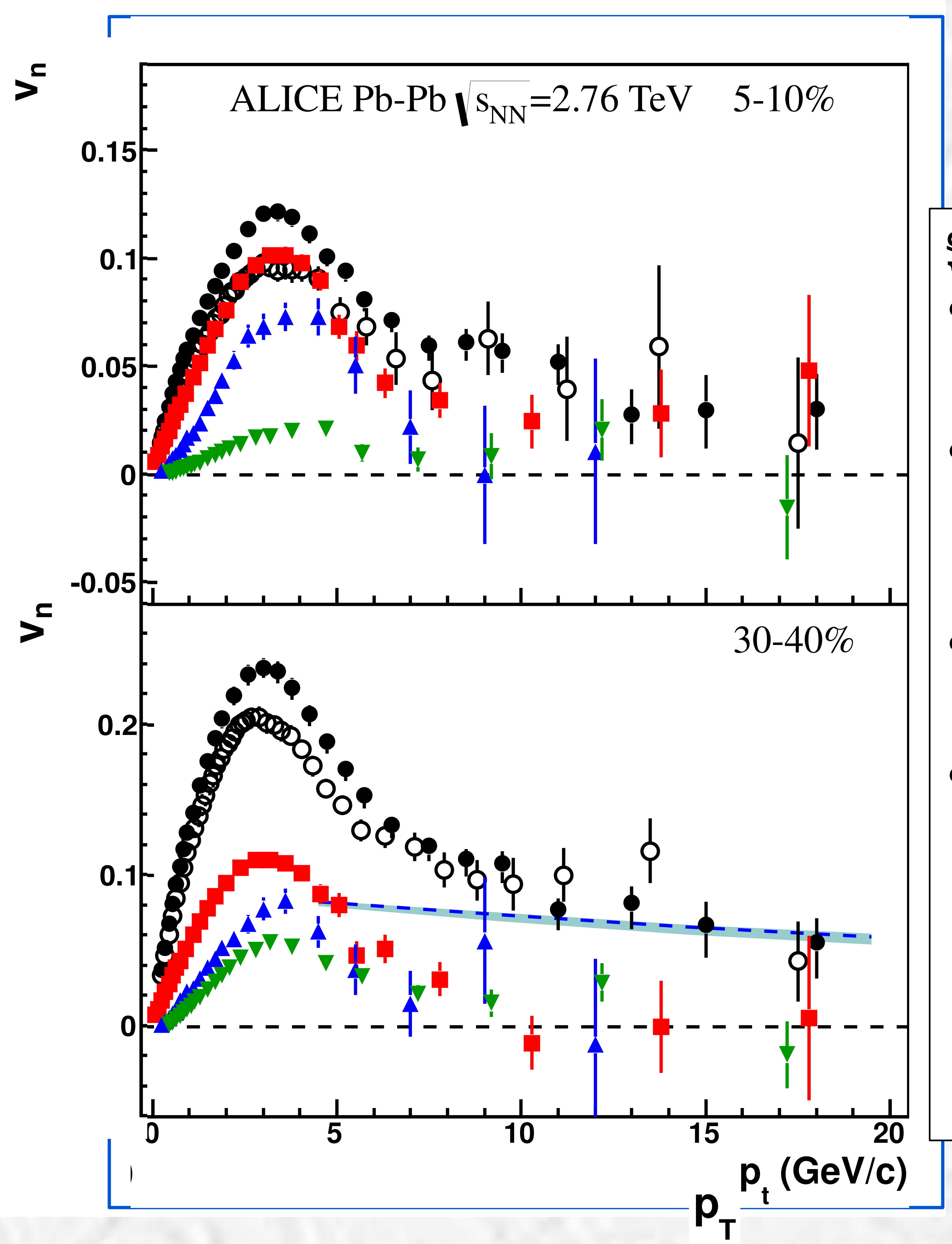


FIG. 4 (color online). The two-particle azimuthal correlation, measured in $0 < \Delta\phi < \pi$ and shown symmetrized over 2π, between a trigger particle with $2 < p_t < 3$ GeV/c and an associated particle with $1 < p_t < 2$ GeV/c for the 0%–1% centrality class. The solid red line shows the sum of the measured anisotropic flow Fourier coefficients v_2, v_3, v_4, and v_5 (dashed lines).

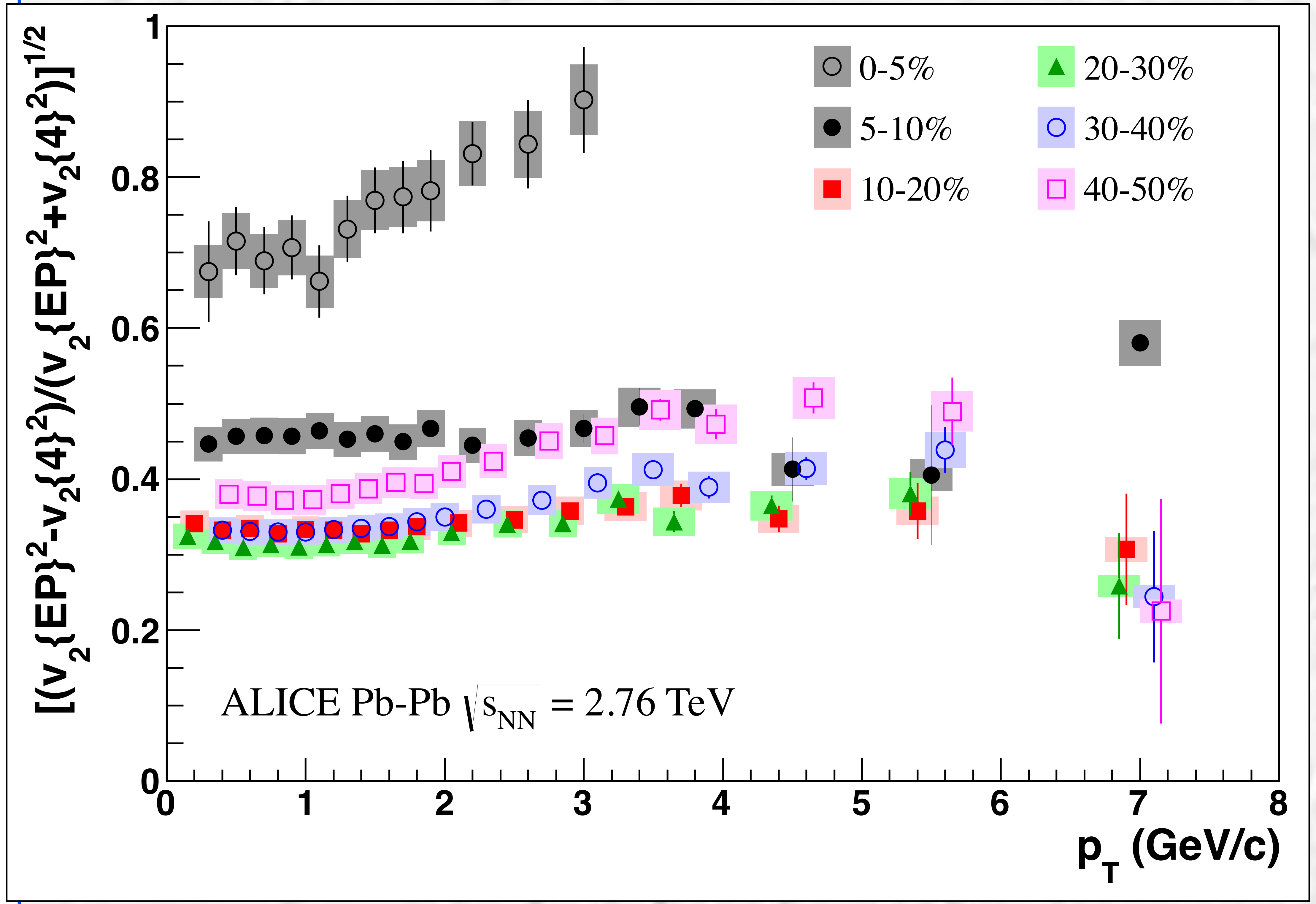
$$C(\Delta\phi) \equiv \frac{N_{\rm mixed}}{N_{\rm same}} \frac{dN_{\rm same}/d\Delta\phi}{dN_{\rm mixed}/d\Delta\phi},$$

Fluctuations vs p_T

ALICE: arXiv:1205.5761



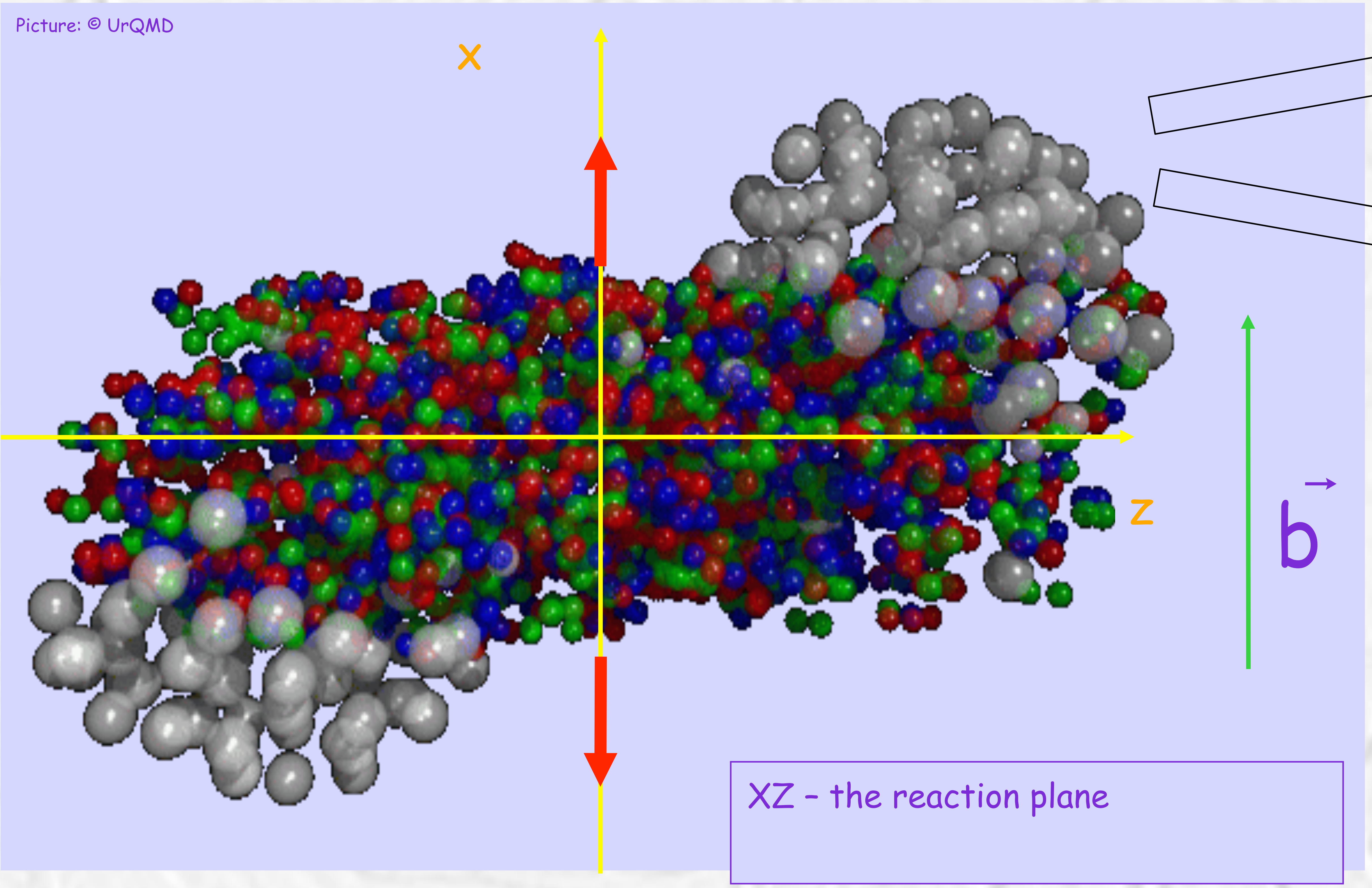
$((v_2\{EP\}^2 - v_2\{4\}^2)/(v_2\{EP\}^2 + v_2\{4\}^2))^{1/2}$



Fluctuations extend up to $p_T \sim 8$ GeV/c with very similar magnitude

Note that v_4 measured wrt Ψ_2 and Ψ_4 becomes very similar at the same p_T

Where do spectators flow?

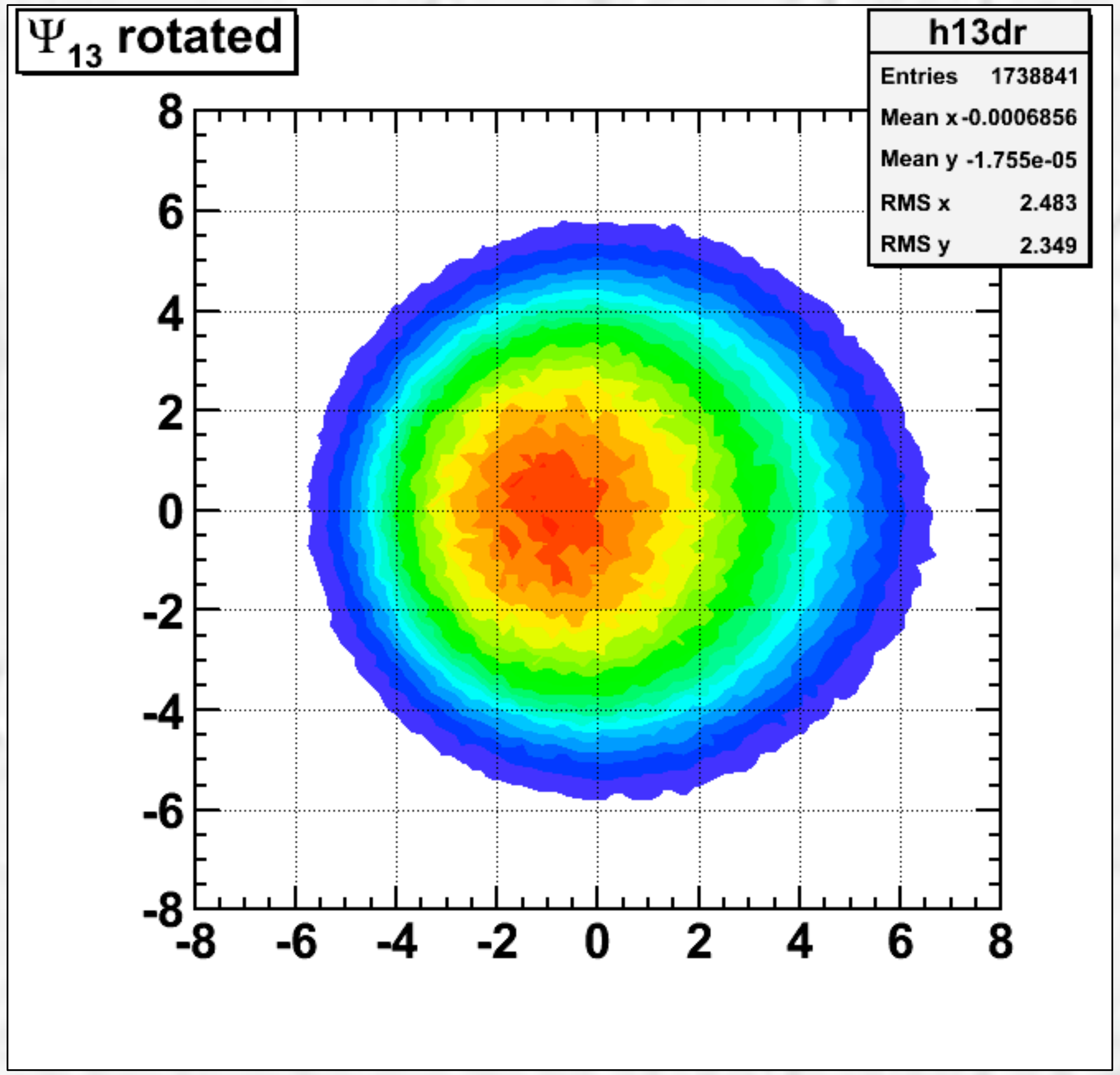


Here

or

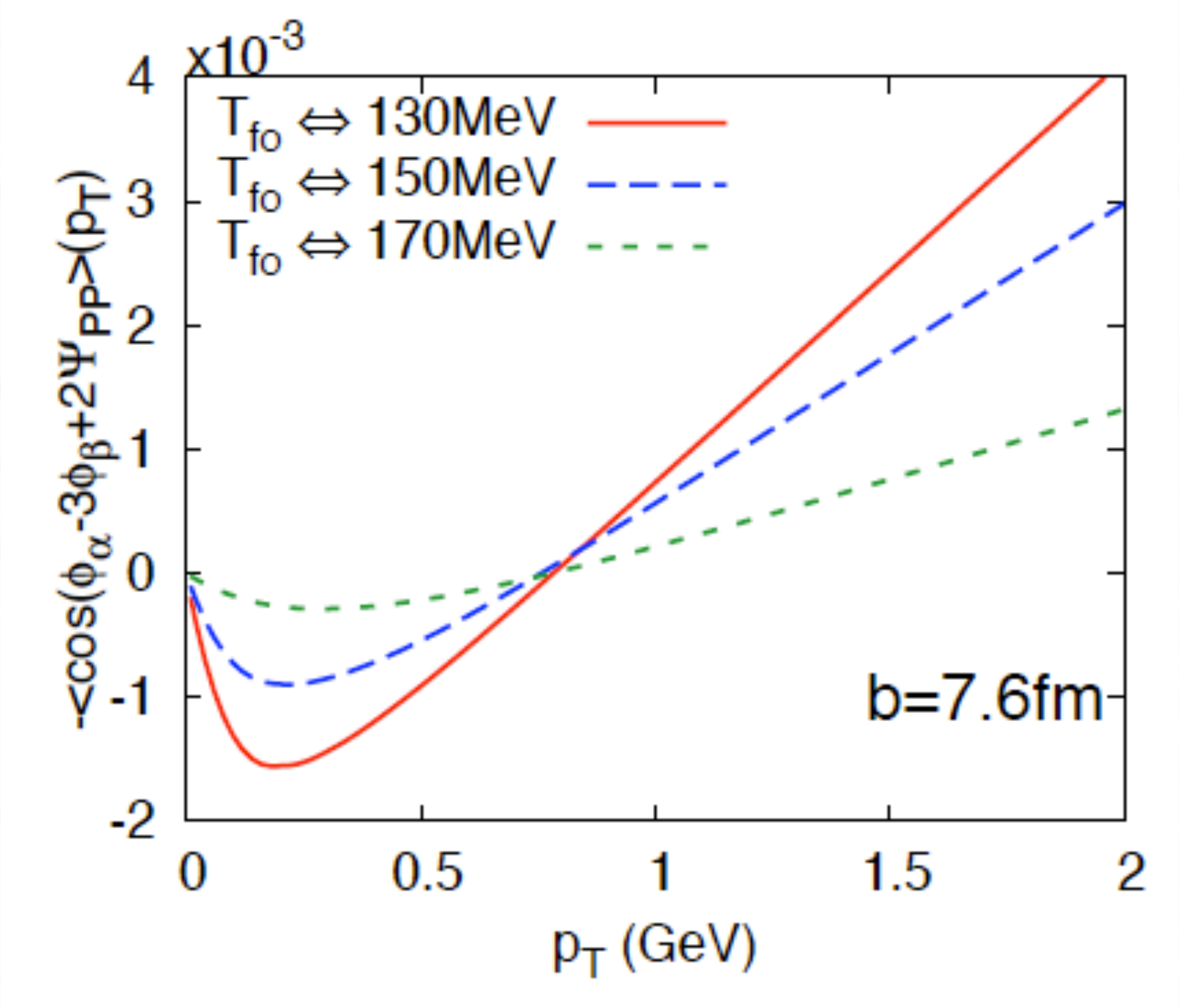
There?

$v_1(p_t)$



Triangularity and Dipole Asymmetry in Heavy Ion Collisions

Derek Teaney and Li Yan



$$\varepsilon_n e^{in\Phi_n} \equiv -\frac{\int r\,dr\,d\phi\, r^n e^{in\phi}\, e(r,\phi)}{\int r\,dr\,d\phi\, r^n e(r,\phi)} \quad (n>1),$$

$$\varepsilon_1 e^{i\Phi_1} \equiv -\frac{\int r\,dr\,d\phi\, r^3 e^{i\phi}\, e(r,\phi)}{\int r\,dr\,d\phi\, r^3 e(r,\phi)}$$

Dipole flow, circa 2004

arXiv:nucl-th/0403044v1 15 Mar 2004

Rapidity dependent momentum anisotropy at RHIC

Ulrich Heinz† and Peter F Kolb‡

† Department of Physics, The Ohio State University, Columbus, OH 43210, USA
‡ Physik Department, TU München, D-85747 Garching, Germany

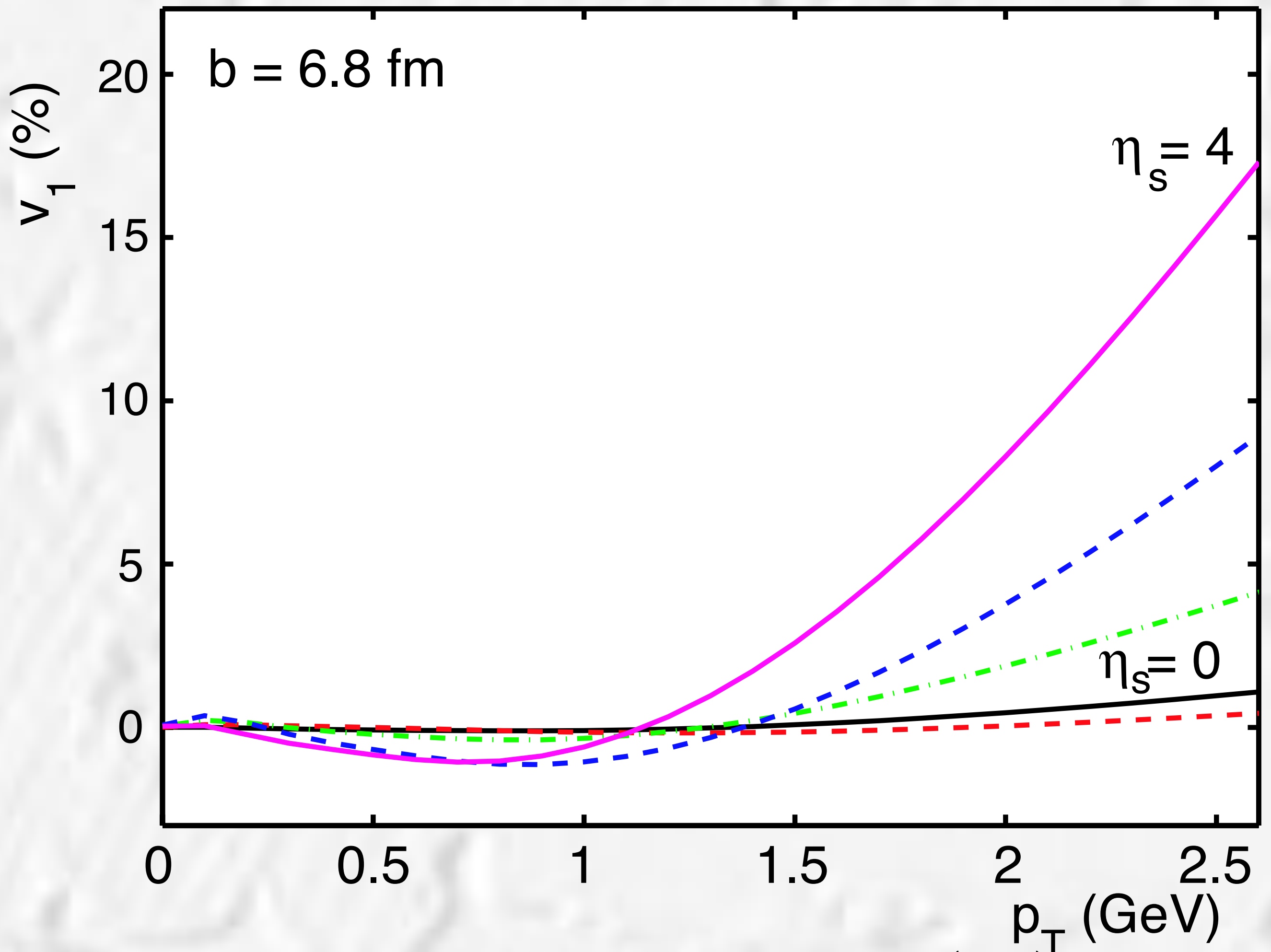
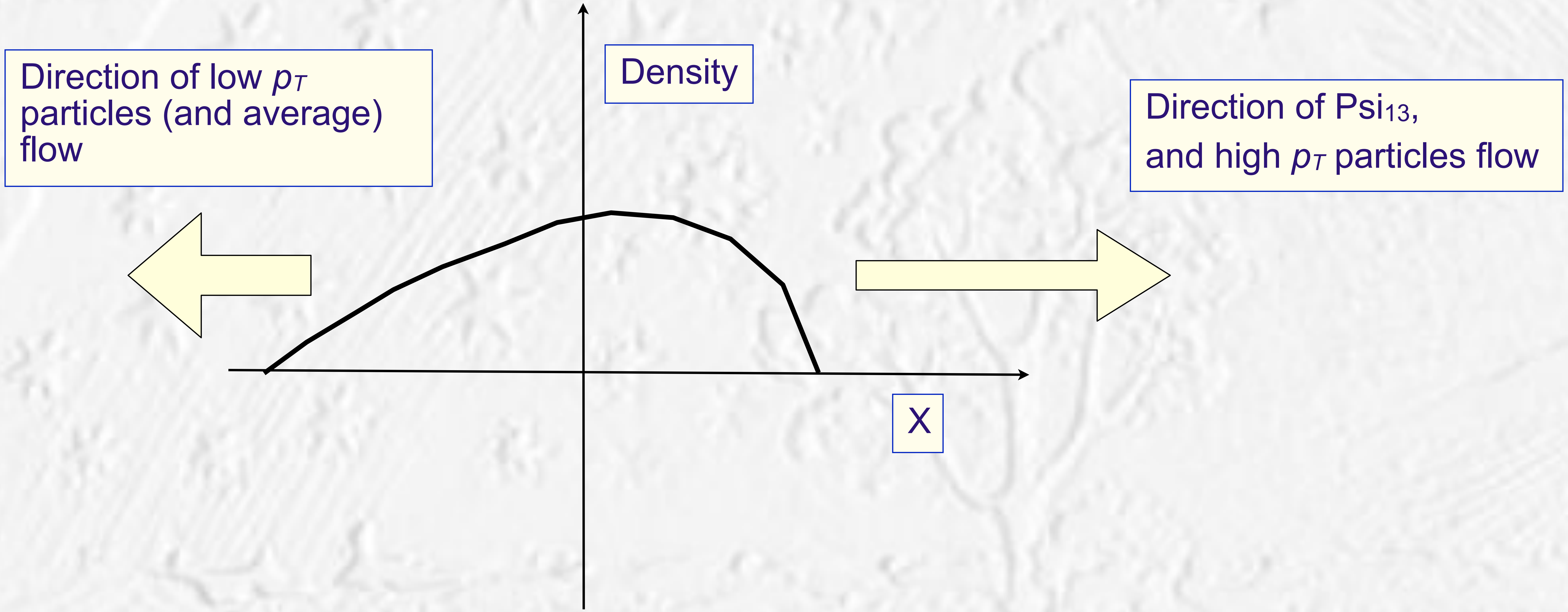


Figure 2. *Left:* Differential directed flow $v_1(p_\perp)$ of directly emitted pions (no resonance decays) for $\eta_s = y = 0, 1, 2, 3, 4$. Except for a region of positive v_1 at $0 < p_\perp < 0.5$ GeV and a shift of the rest of the curves by about 0.5 GeV to larger $p_\perp$, the curves for direct protons look similar. *Right:* $p_\perp$-integrated elliptic flow

At forward rapidities the transverse overlap region becomes asymmetric and is shifted sidewards in the x (or impact parameter) direction. This turns out to give rise to a non-zero directed flow signal $v_1(p_\perp)$ which increases with $|\eta_s|$ (left panel in Fig. 2). Of course, since the colliding matter receives no overall transverse kick, the $p_\perp$-integrated directed flow is zero.

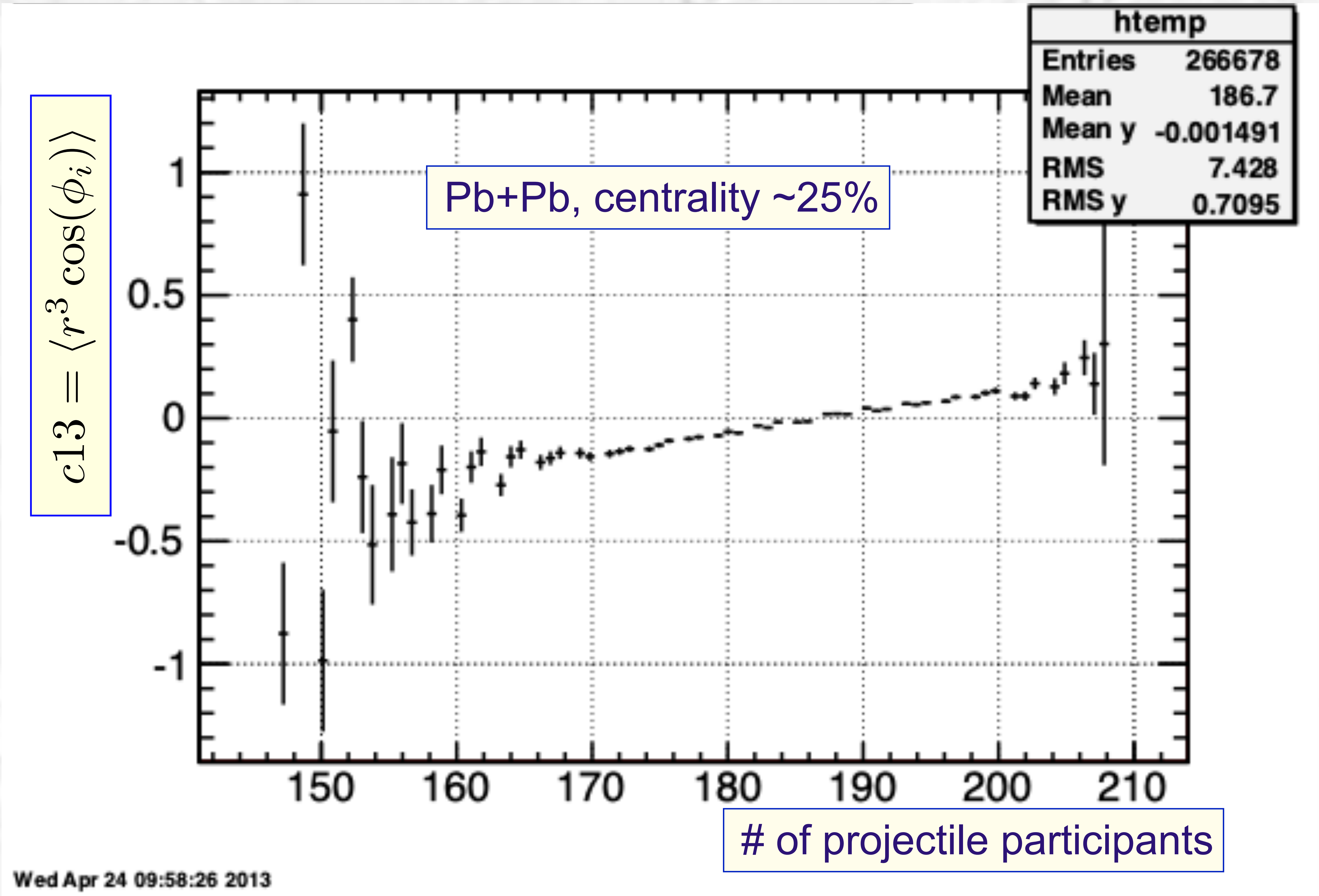
Similar observations has been made later by Teaney and Yan, Retinskaya et.al., in relation to effects of initial density fluctuations

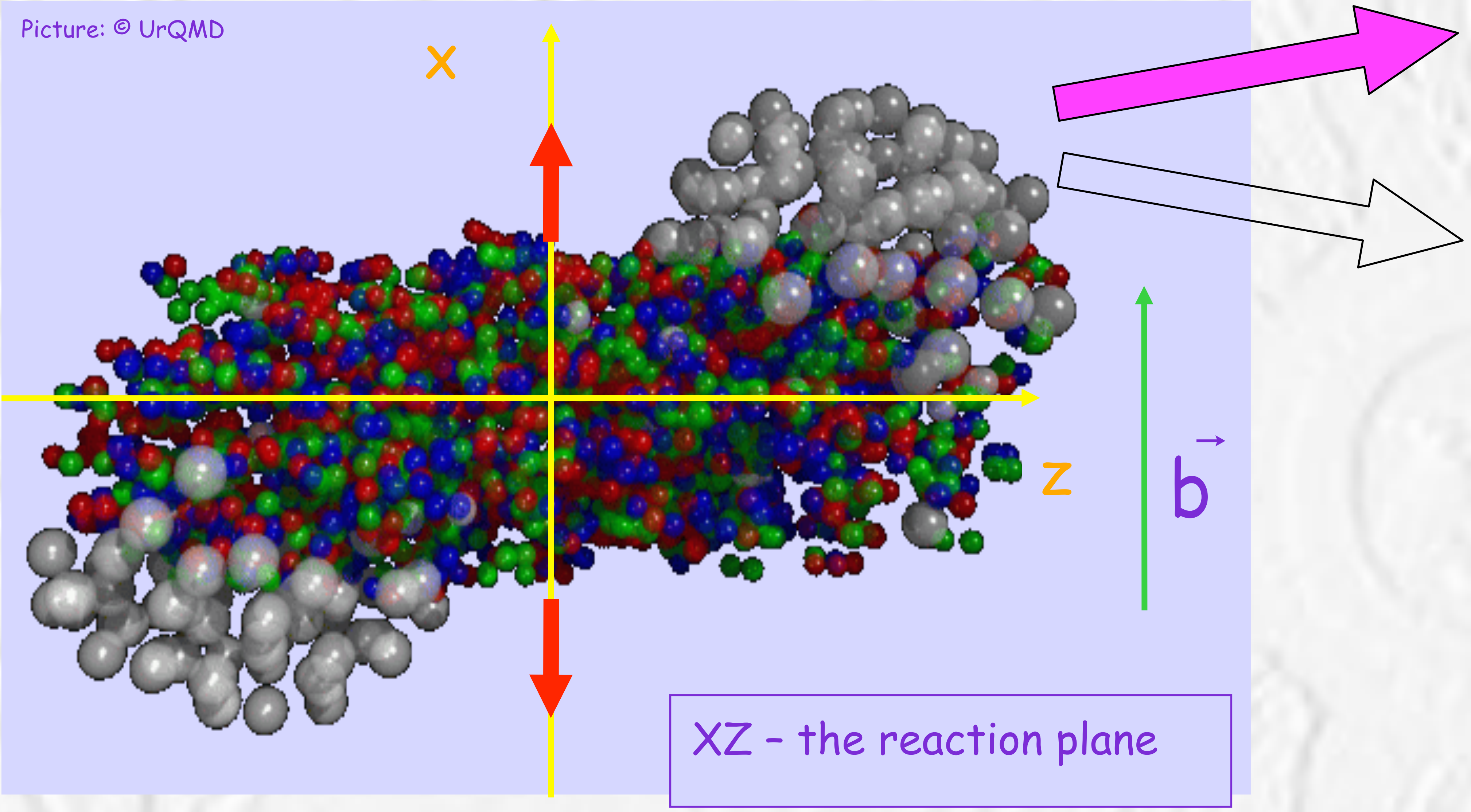
Dipole flow direction



$$\langle x \rangle = 0, \quad \langle x^3 \rangle < 0$$

MC Glauber model





More projectile spectators → More projectile participants

↓

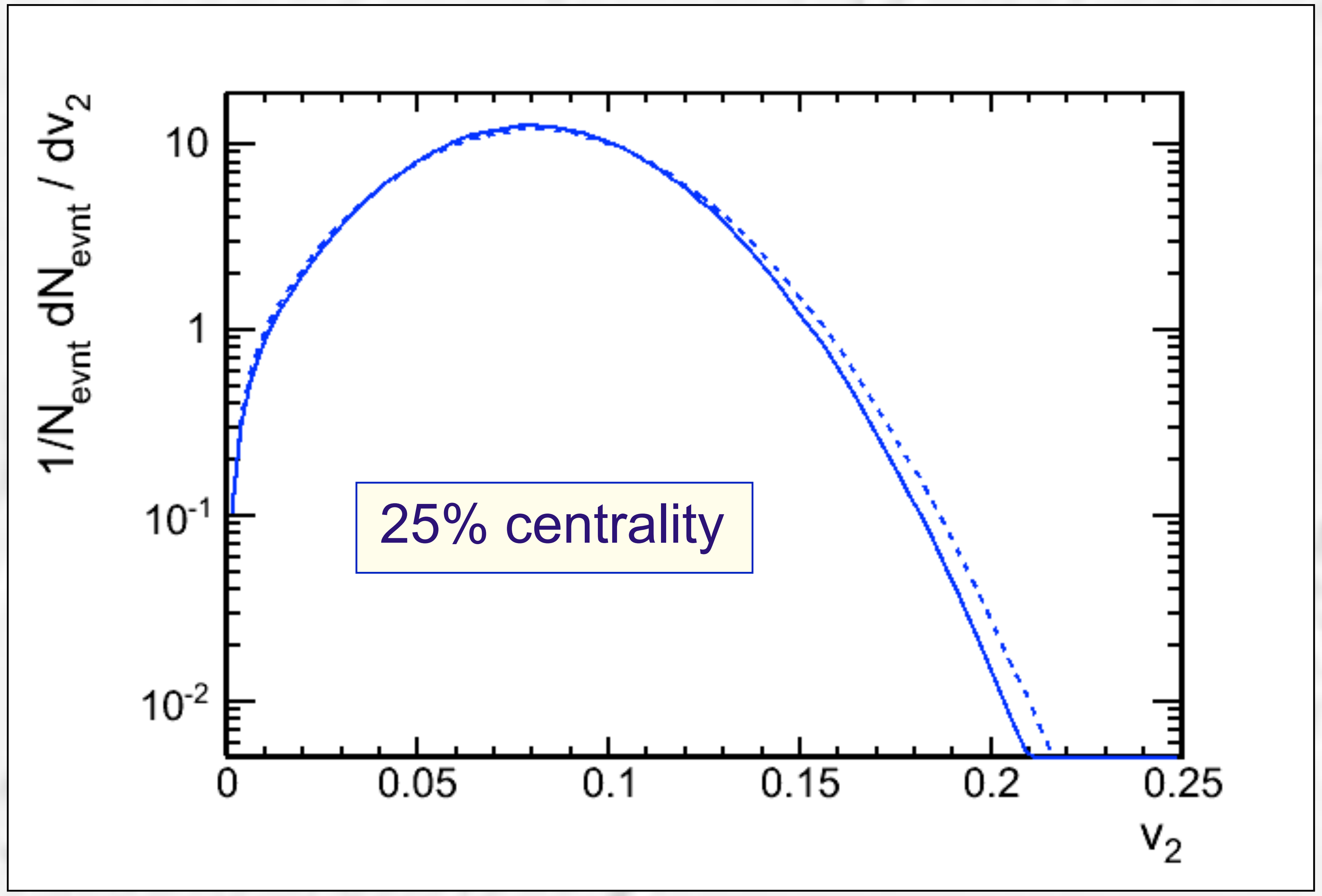
c13 < 0

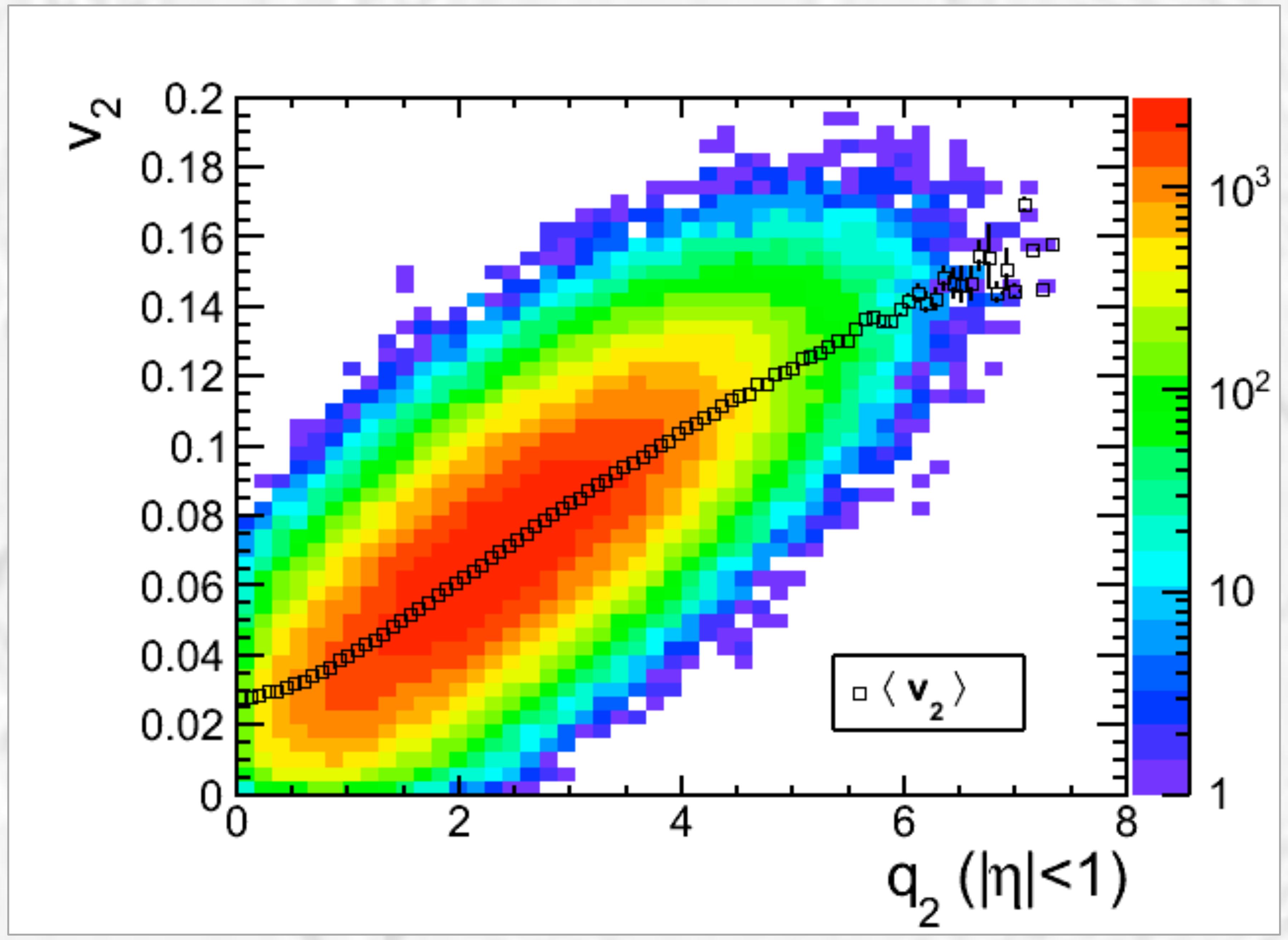
↓

v1<0

Event shape engineering

J. Schukraft, A. Timmins and *S. A. Voloshin*, Phys. Lett. B **719**, 394 (2013)





Based on the use of flow vector as discussed in *S. A. Voloshin*, Phys. Rev. Lett. **105**, 172301 (2010)

Event shape engineering (ESE) - selection of events corresponding to either large or small flow

$$Q_{n,X} = \sum_{i=1}^{M} \cos(n\phi_i)$$

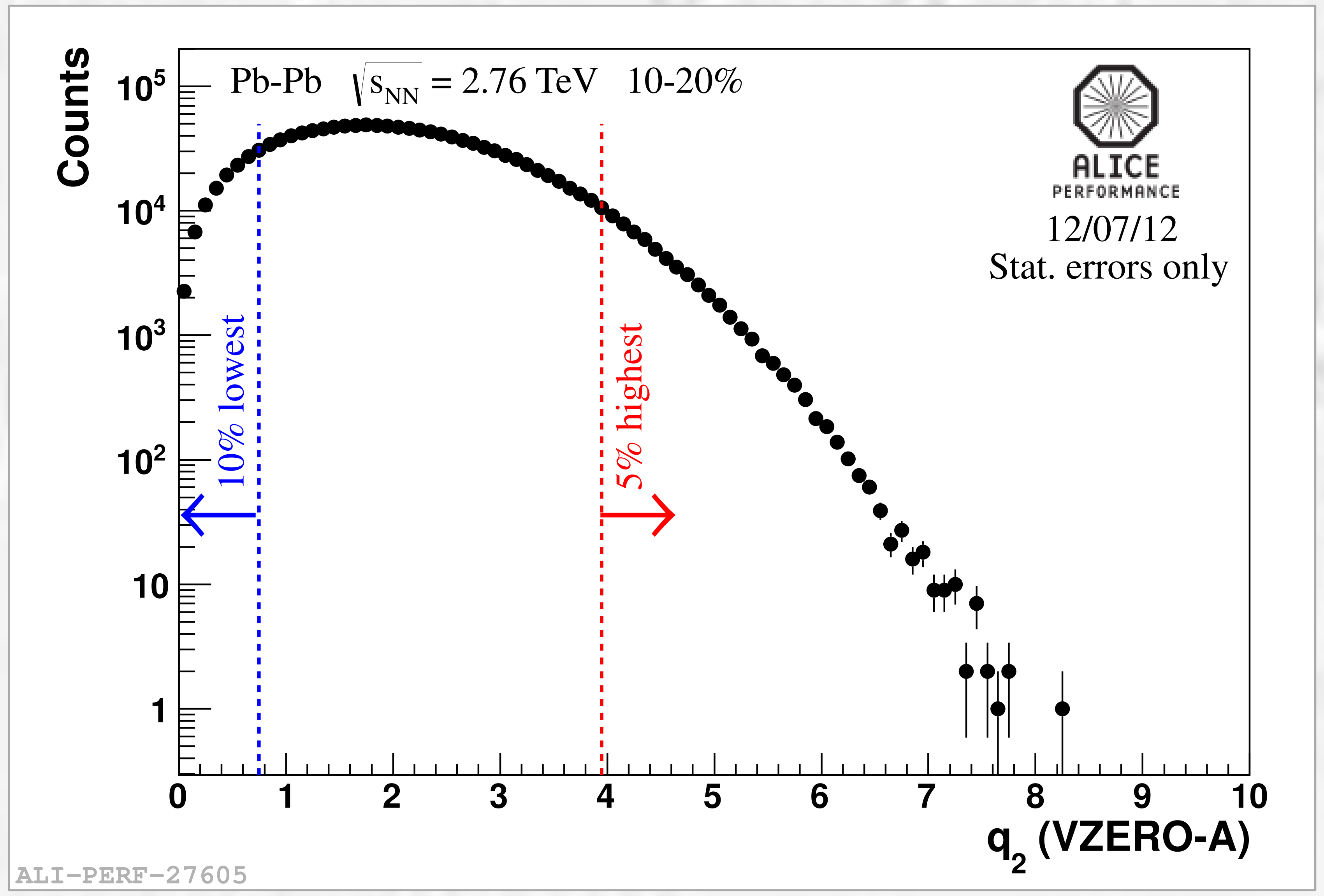
$$Q_{n,Y} = \sum_{i=1}^{M} \sin(n\phi_i)$$

$$q_n = Q_n/\sqrt{M}$$

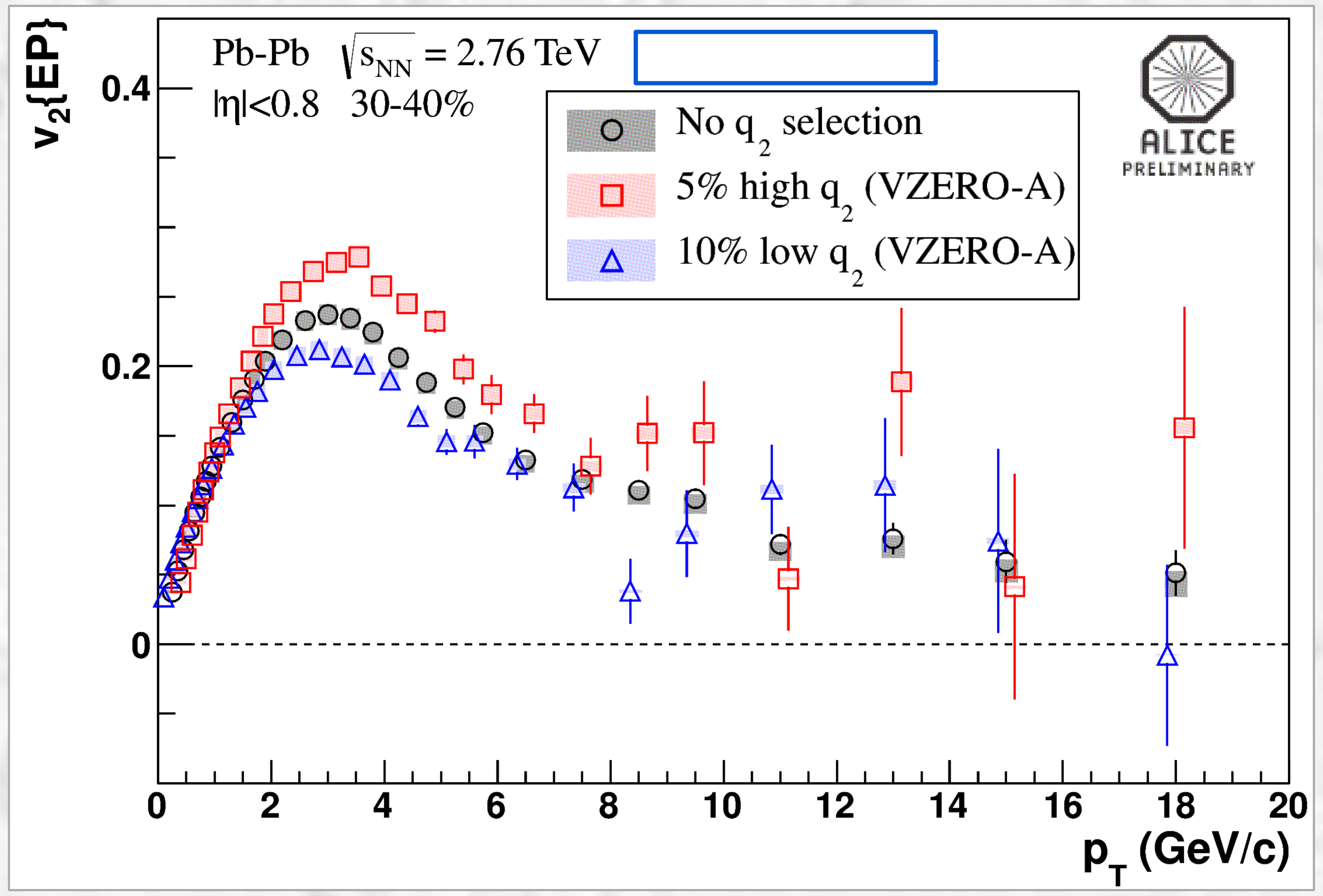
Flow in SE events: p_T dependence

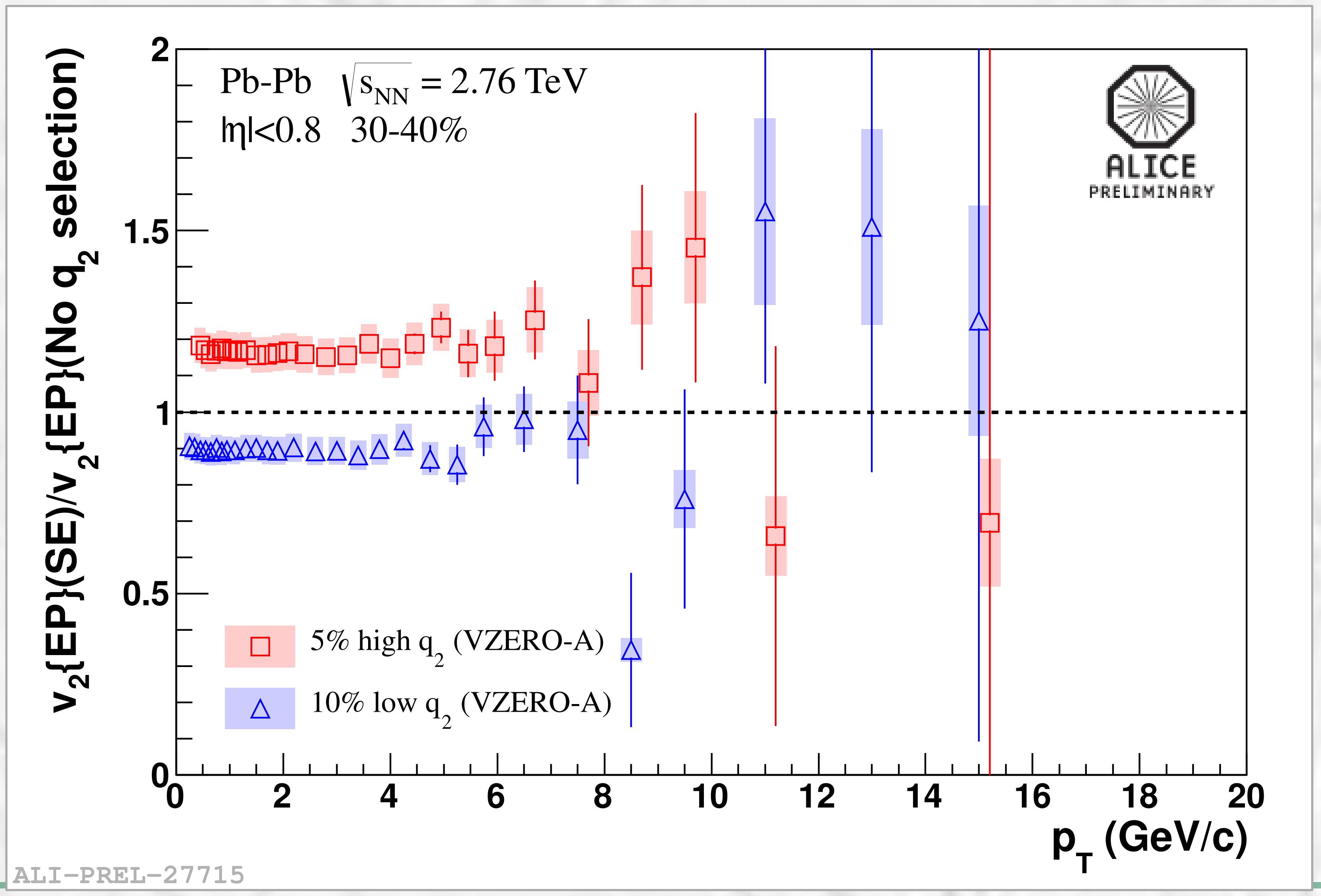
A. Dobrin [ALICE], Quark Matter 2012, Washington DC, August 2012.

event selection q_2 vector: $2.8 < \eta < 5.1$



analysis: $|\eta| < 0.8$

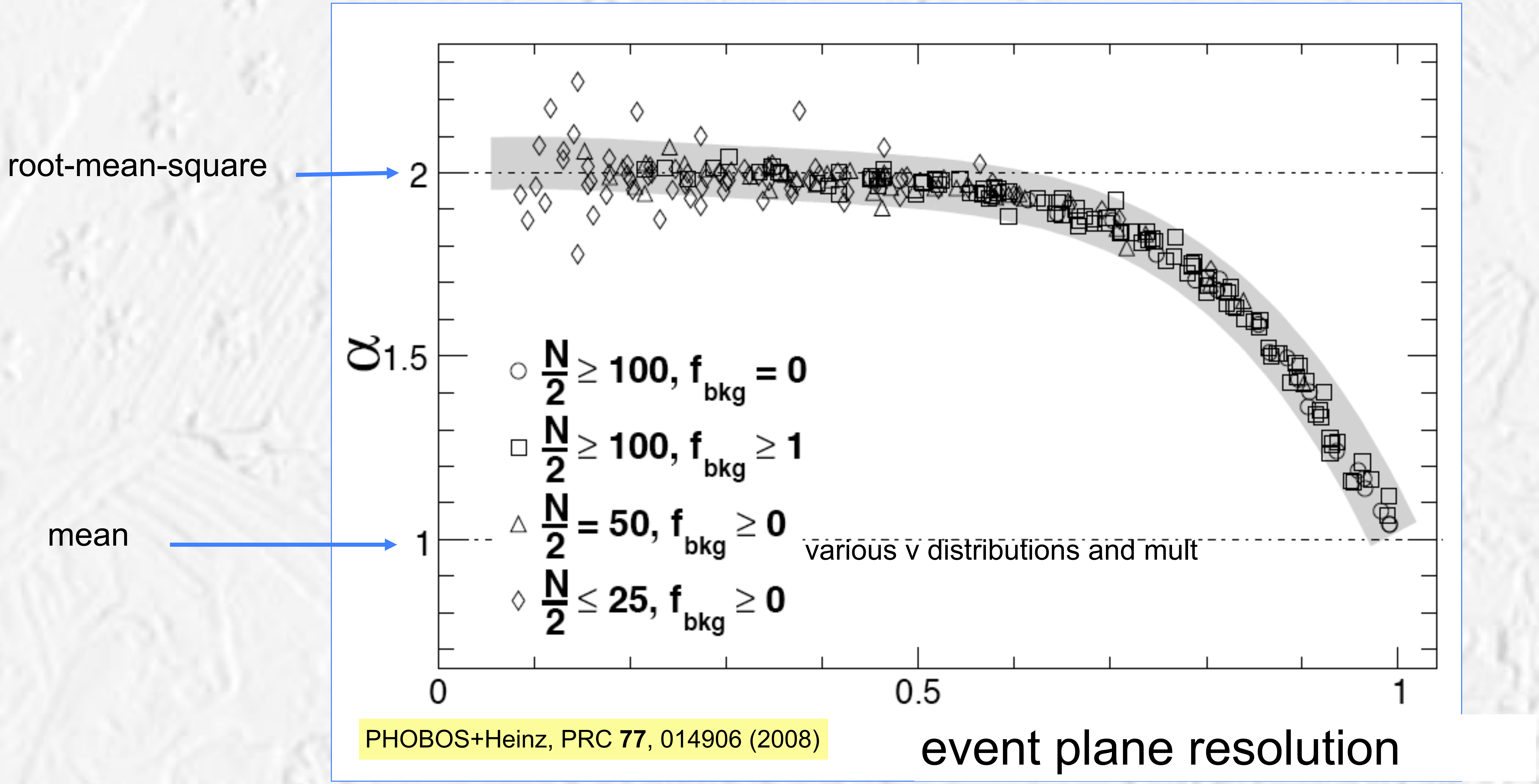




Initial shape fluctuation effect
is very similar up to $p_T \sim 6$ GeV/c

PHOBOS Simulations

$$v_2\{\ \} = \langle v^{\alpha} \rangle^{1/\alpha}$$

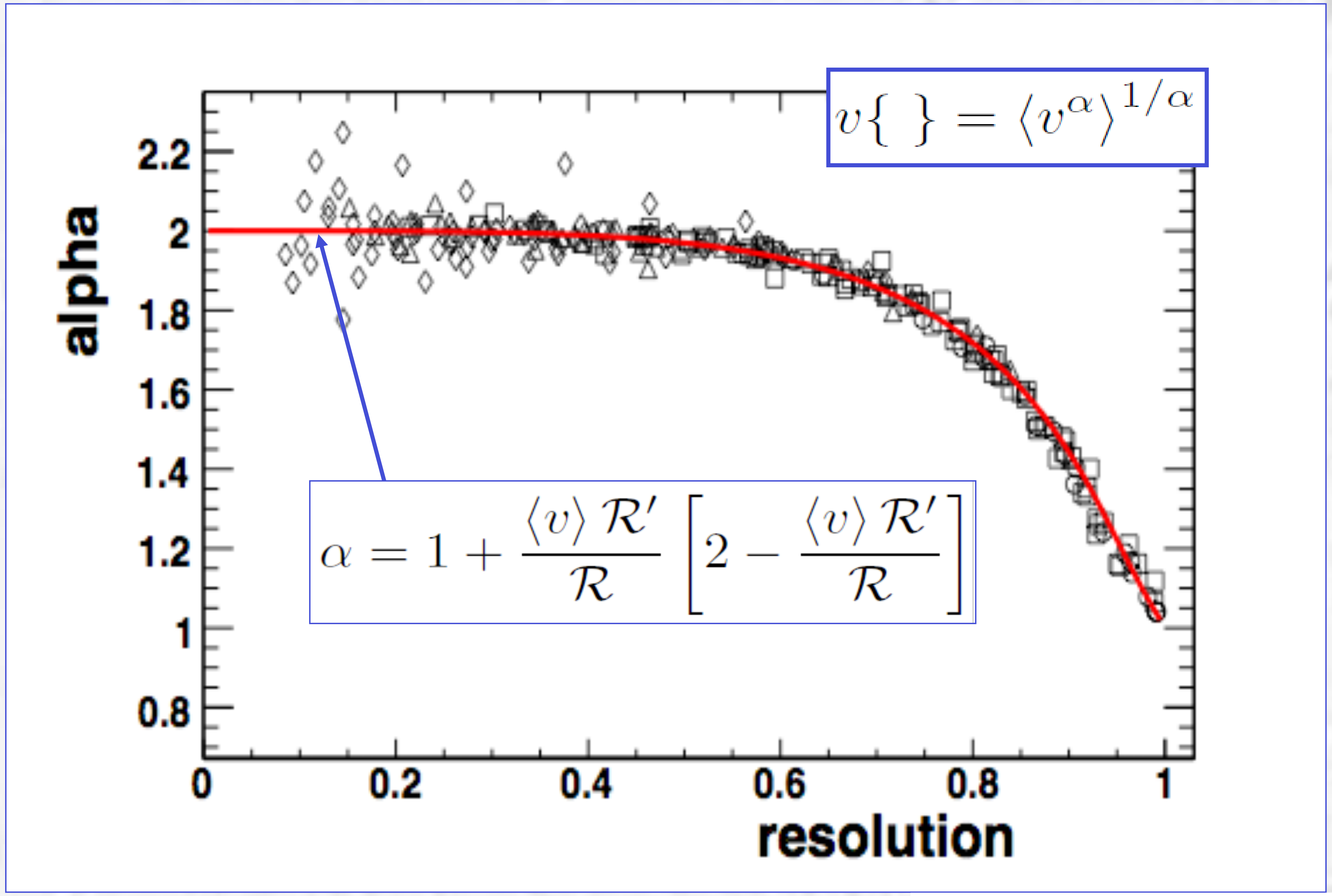


PHOBOS+Heinz, PRC **77**, 014906 (2008)

v{EP}

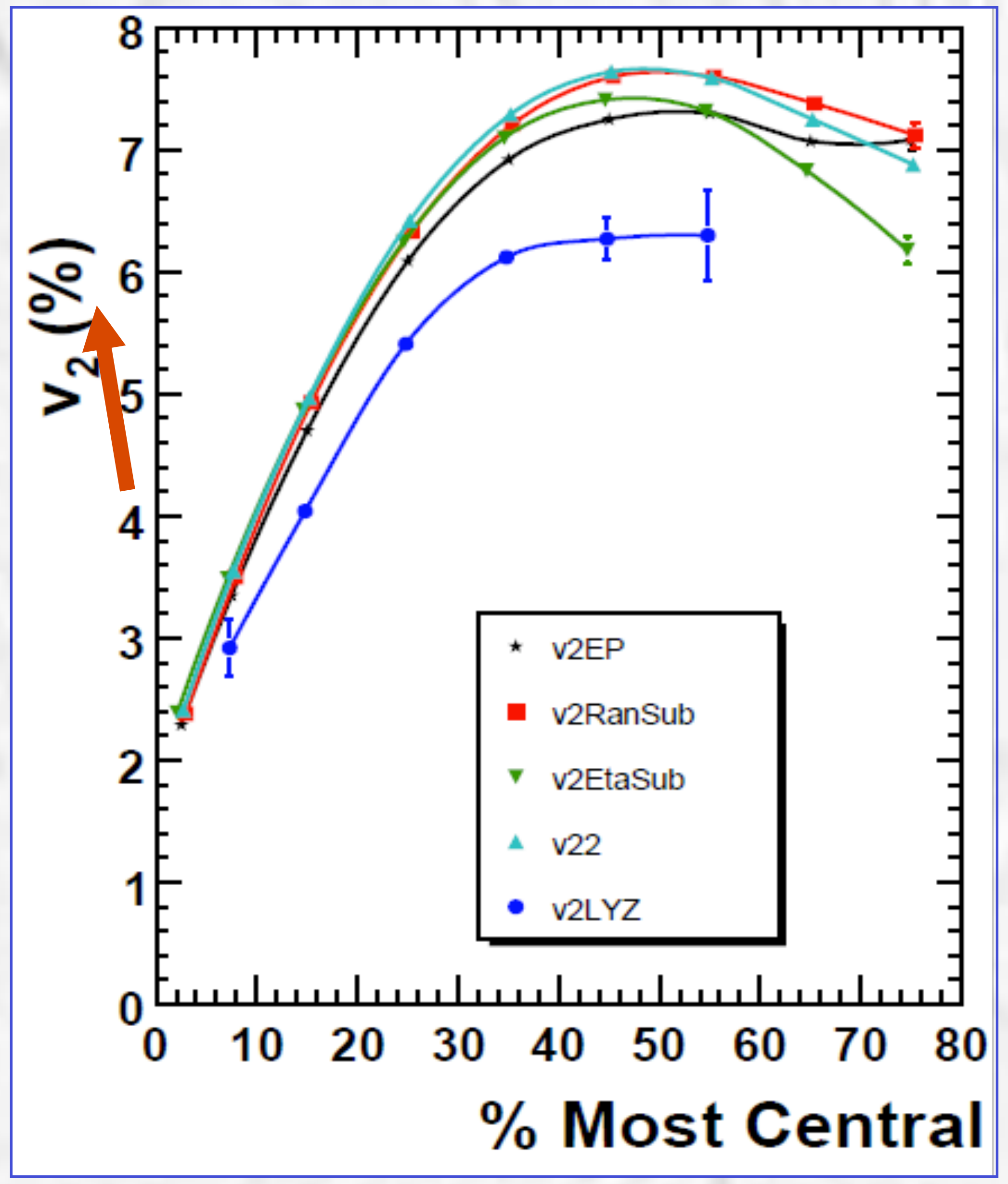
PHYSICAL REVIEW C 80, 014904 (2009)

Jean-Yves Ollitrault,[1] Arthur M. Poskanzer,[2] and Sergei A. Voloshin[3]



$$v\{\ \} = \langle v^{\alpha} \rangle^{1/\alpha}$$

$$\alpha = 1 + \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \left[2 - \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \right]$$



$$v\{\mathrm{EP}\} \equiv \frac{\langle \cos(\phi - \Psi_R) \rangle}{R}$$

$$R = \sqrt{\langle \cos(\Psi_A - \Psi_B) \rangle}$$

$$\mathbf{q} \cos \Psi_R = \frac{\mathbf{Q}}{\sqrt{N}} \cos \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \cos \phi_j$$

$$\mathbf{q} \sin \Psi_R = \frac{\mathbf{Q}}{\sqrt{N}} \sin \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sin \phi_j$$

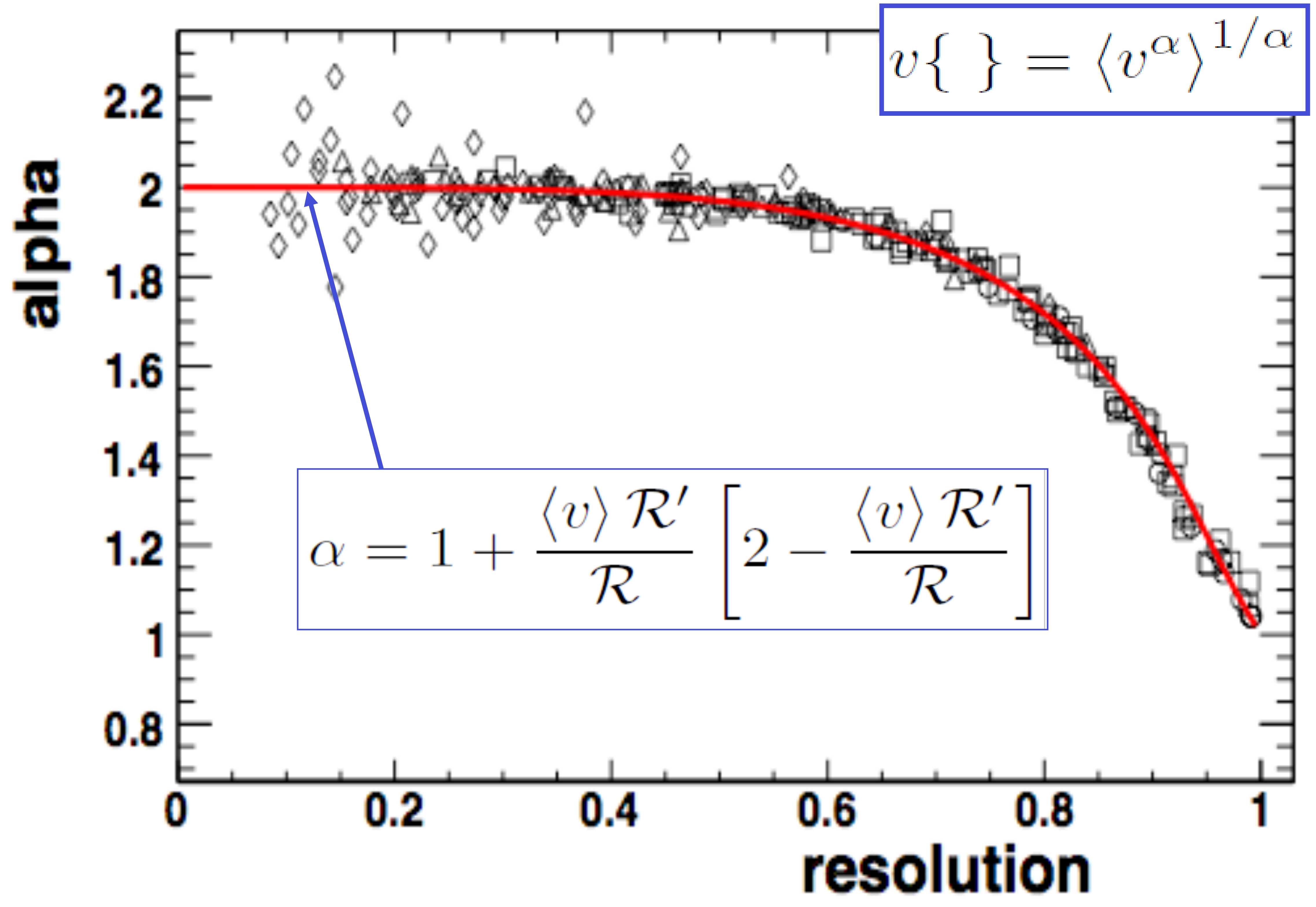
$$\mathcal{R}(\chi) = \frac{\sqrt{\pi}}{2} e^{-\chi^2/2} \chi \left[I_0\left(\frac{\chi^2}{2}\right) + I_1\left(\frac{\chi^2}{2}\right) \right]$$

$$\chi_s = v\sqrt{N_s}$$

v{EP}

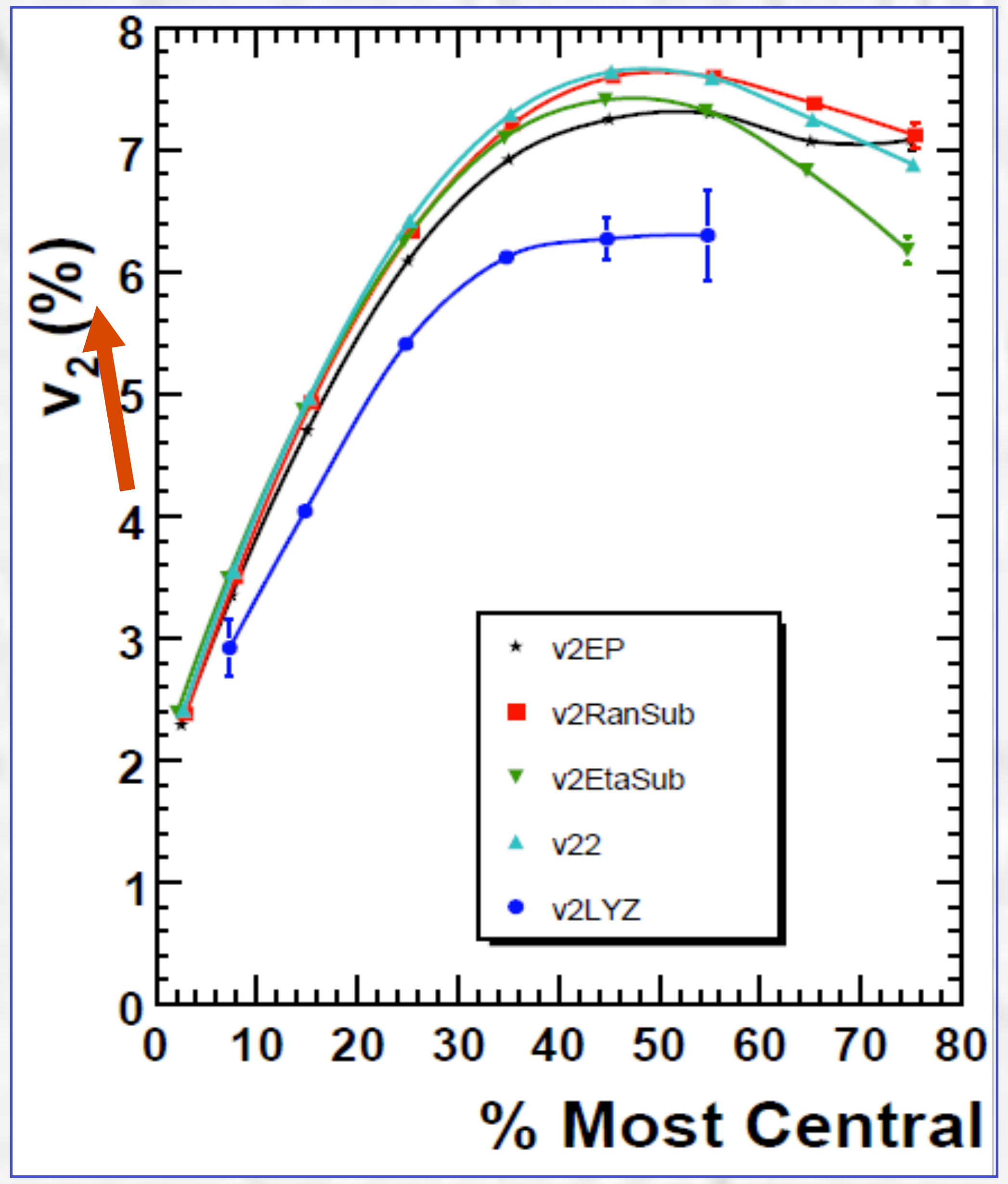
PHYSICAL REVIEW C 80, 014904 (2009)

Jean-Yves Ollitrault,[1] Arthur M. Poskanzer,[2] and Sergei A. Voloshin[3]



$$v\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$$

$$\alpha = 1 + \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \left[2 - \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \right]$$



$$v\{\mathrm{EP}\} \equiv \frac{\langle \cos(\phi - \Psi_R) \rangle}{R}$$

$$R = \sqrt{\langle \cos(\Psi_A - \Psi_B) \rangle}$$

$$\mathbf{q} \cos \Psi_R = \frac{\mathbf{Q}}{\sqrt{N}} \cos \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \cos \phi_j$$

$$\mathbf{q} \sin \Psi_R = \frac{\mathbf{Q}}{\sqrt{N}} \sin \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sin \phi_j$$

$$\mathcal{R}(\chi) = \frac{\sqrt{\pi}}{2} e^{-\chi^2/2} \chi \left[I_0\left(\frac{\chi^2}{2}\right) + I_1\left(\frac{\chi^2}{2}\right) \right]$$

$$\chi_s = v\sqrt{N_s}$$

… but it is still not possible to separate the effect of fluctuations from non-flow…