Effect of critical point on Λ-hyperon spin polarization

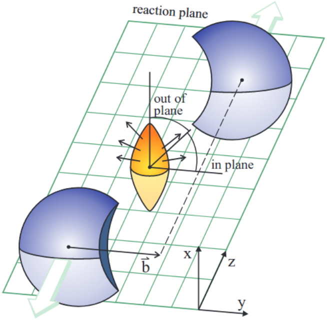
Sushant Kumar Singh
(sushant7557@gmail.com)
Variable Energy Cyclotron Centre, Kolkata

based on arXiv:2110.15604 and 2205.14469

(in collaboration with Prof. Jane Alam)

March 23, 2023

Large OAM in non-central heavy-ion collision

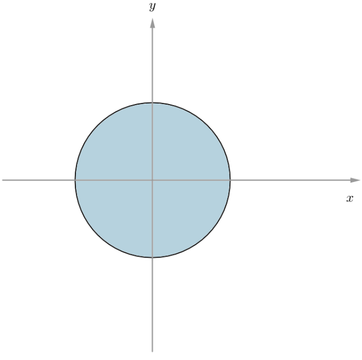


arXiv:0910.4114

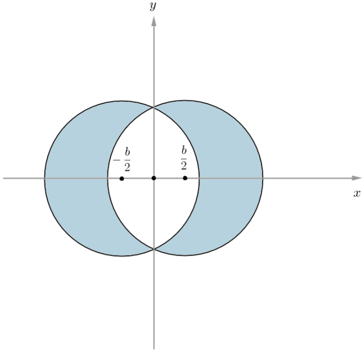
- Nuclei carry a large orbital angular momentum (OAM), $L_0 = pb \simeq A\sqrt{s_{NN}}b/2$.
- e.g. for $\sqrt{s_{NN}} = 200$ GeV and $b = 5$ fm, $L_0 \sim 5 \times 10^5$.
- A fraction of L_0 is transferred to QGP fireball.

Define thickness function (number of nucleons per unit transverse area) as

$$T(x,y) = \int dz \; \rho(x,y,z) \qquad , \qquad \rho \rightarrow \text{Nuclear density distribution}$$



$T(x,y)$



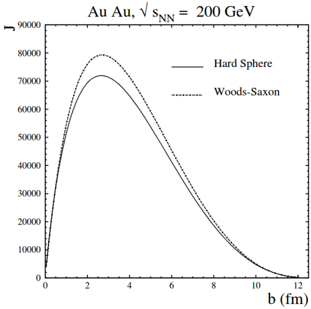
$T(x+b/2,y)$ $T(x-b/2,y)$

$$\frac{dP}{dxdy} = [T(x-b/2,y) - T(x+b/2,y)] \frac{\sqrt{s_{NN}}}{2}$$

PRC77, 024906 (2008)

Initial angular momentum of the fireball is then

$$J_y \sim \int dx \int dy\ x \frac{dP}{dxdy} = \int dx \int dy\ x \left[T(x-b/2,y) - T(x+b/2,y)\right] \frac{\sqrt{s_{NN}}}{2}$$



PRC77, 024906 (2008)

$J_y \sim 0.29 L_0$ at $b = 2.5$ fm

Spin polarization of hadrons

Parton scattering polarizes quarks along the OAM direction due to spin-orbital coupling in QCD, $P_q \sim -0.3$ at RHIC.

PRL **94,** 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]

[1]*Department of Physics, Shandong University, Jinan, Shandong 250100, China*
[2]*Nuclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
(Received 25 October 2004; published 14 March 2005)

Produced partons have a large local relative orbital angular momentum along the direction opposite to the reaction plane in the early stage of noncentral heavy-ion collisions. Parton scattering is shown to polarize quarks along the same direction due to spin-orbital coupling. Such global quark polarization will lead to many observable consequences, such as left-right asymmetry of hadron spectra and global transverse polarization of thermal photons, dileptons, and hadrons. Hadrons from the decay of polarized resonances will have an azimuthal asymmetry similar to the elliptic flow. Global hyperon polarization is studied within different hadronization scenarios and can be easily tested.

One distinctive signature of an OAM would be the polarization of the emitted hadrons. Considering hadronization via quark recombination, $P_\Lambda = P_s$, for example.

Experimental observation of Λ-polarization

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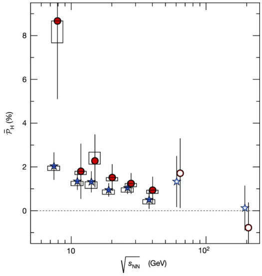
Published: 03 August 2017

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration

Nature **548**, 62–65 (2017) | Cite this article

The $\sqrt{s_{NN}}$-averaged polarizations indicate a vorticity of $\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$, with a systematic uncertainty of a factor of two, mostly owing to uncertainties in the temperature. This far surpasses the vorticity of all other known fluids, including solar subsurface flow[23] ($10^{-7}\,\mathrm{s}^{-1}$); large-scale terrestrial atmospheric patterns[24] (10^{-7}–$10^{-5}\,\mathrm{s}^{-1}$); supercell tornado cores[25] ($10^{-1}\,\mathrm{s}^{-1}$); the great red spot of Jupiter[26] (up to $10^{-4}\,\mathrm{s}^{-1}$); and the rotating, heated soap bubbles ($100\,\mathrm{s}^{-1}$) used to model climate change[27]. Vorticities of up to $150\,\mathrm{s}^{-1}$ have been measured in turbulent flow[28] in bulk superfluid He II, and Gomez *et al.*[29] have recently produced superfluid nanodroplets with $\omega \approx 10^{7}\,\mathrm{s}^{-1}$.



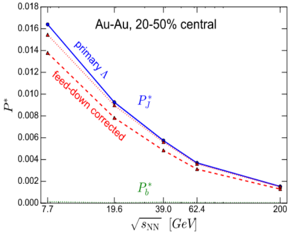
Hydrodynamic simulation for global polarization

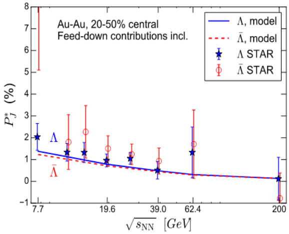
Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics A 967 (2017) 764–767
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

Vorticity in the QGP liquid and Λ polarization at the RHIC Beam Energy Scan

Iurii Karpenko[a,b], Francesco Becattini[a,c]

Initial condition : UrQMD string/hadron cascade, all components of thermal vorticity tensor are initially non-vanishing. Simulation on a constant energy density hypersurface (0.5 GeV/fm^3).





Cooper-Frye formula for particles with spin

Momentum spectrum of of i^{th} hadron is given by

$$E\frac{dN_i}{d^3p} = \int_\Sigma (d\Sigma.p)\, f_i(x,p) \qquad \rightarrow \qquad \text{Cooper-Frye prescription}$$

Polarization vector for spin-1/2 particles

$$P_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau + \mathcal{O}(\varpi^2)$$

where

$$\varpi^{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma) \qquad \text{with} \qquad \beta_\rho = \frac{u_\rho}{T}$$

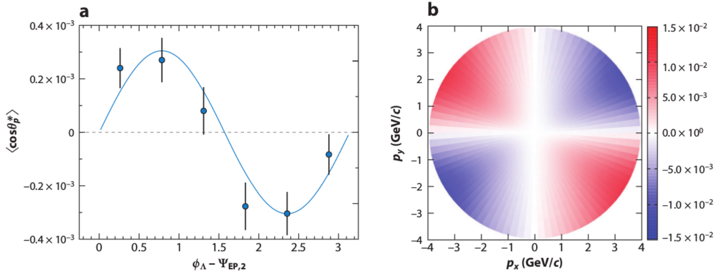
Ann. Phys. 338:32 (2013)

Space-integrated mean polarization vector

$$P_\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, P_\mu(x,p) n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$

Spin sign puzzle

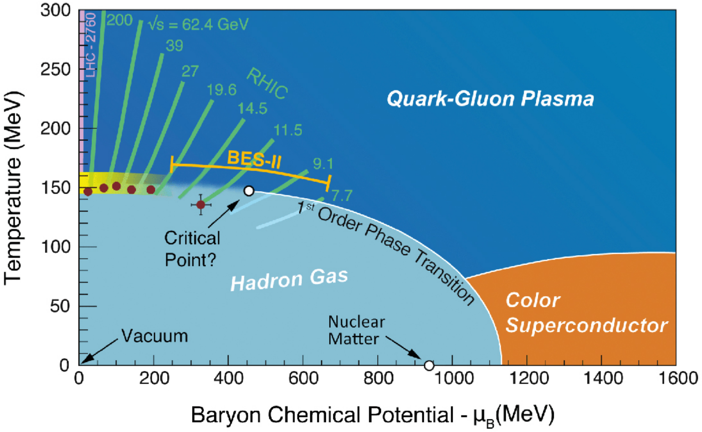
"Hydrodynamic and transport-hybrid calculations predict a negative sign of the longitudinal component of the polarization vector. The magnitude of the effect is significantly larger in the model."



Ann. Rev. Nucl. Part. Sci. 70 (2020) 395

This and several other questions led to the development of relativistic dissipative spin hydrodynamics, still under development. So assume LTE for spin dof.

QCD phase diagram



CERN courier, February 2021

Observed spin polarization has following sources:

- Initial orbital angular momentum.
- Vorticity generated through viscous stresses. This can be seen from non-relativistic vorticity equation:

$$\frac{\partial\vec{\omega}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{\omega}+\theta\vec{\omega}=\left(\vec{\omega}\cdot\vec{\nabla}\right)\vec{v}+\frac{1}{\rho^2}\vec{\nabla}\rho\times\vec{\nabla}p-\frac{1}{\rho^2}\left(\zeta+\frac{1}{3}\eta\right)\vec{\nabla}\rho\times\vec{\nabla}\theta-\frac{\eta}{\rho^2}\vec{\nabla}\rho\times\nabla^2\vec{v}+\frac{\eta}{\rho}\nabla^2\vec{\omega}.$$

Why consider EoS?

The spin polarization vector in the rest frame of a hyperon at some point in the fluid is given by (Ann. Rev. Nucl. Part. Sci. 70 (2020) 395)

$$\vec{S}^*(x,p) \propto \frac{\gamma}{T^2}\vec{v}\times\nabla T + \frac{1}{T}\left(\vec{\omega} - (\vec{\omega}\cdot\vec{v})\vec{v}\right) + \frac{1}{T}\gamma\vec{A}\times\vec{v}$$

- $\vec{S}^*$ depends on ∇T, $\vec{\omega} = \nabla\times\vec{v}$ and acceleration of fluid cell.
- In nutshell, gradients of temperature and flow-velocity.
- Gradients depend on the expansion dynamics of the system.
- Expansion depends on the EoS.

Statement of the problem

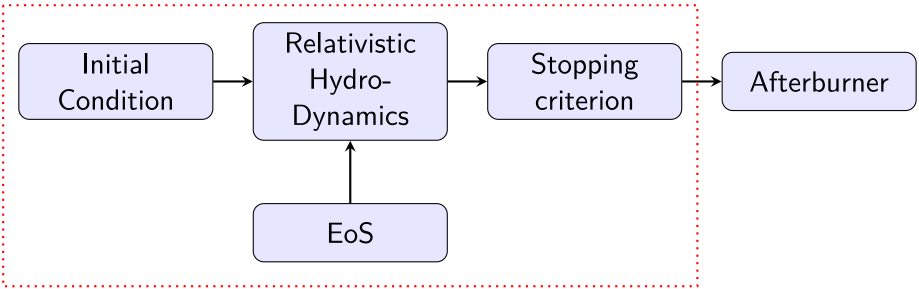
How does thermal vorticity evolve in presence of CP?

Assumptions:

- The dynamical universality class of the QCD critical point is Model H. D. T. Son and M. A. Stephanov, PRD 70 (2004) 056001.
- Hydrodynamics valid near CP. True when not too close to the CP. In that case the back-reaction of critical fluctuations on bulk observables can be neglected. K Rajagopal *et al.*, PRD 102 (2020) 094025
- We assume zero baryon diffusion, for simplicity.

Modeling the fireball evolution

Flow Chart for simulation



- Initial condition : Shifted Glauber
- EoS : BEST model
- Stopping criterion : constant energy density (CORNELIUS)
- Afterburner : UrQMD

Relativistic Hydrodynamics

- Ref : ***SKS** & J. Alam, VECC/IR/2018/04 (VECC Internal Report)*
  ***SKS** & J. Alam, arXiv:2110.15604*
  ***SKS** & J. Alam, arXiv:2205.14469*

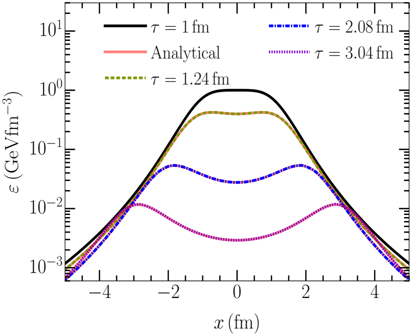
- Hydrodynamic equations

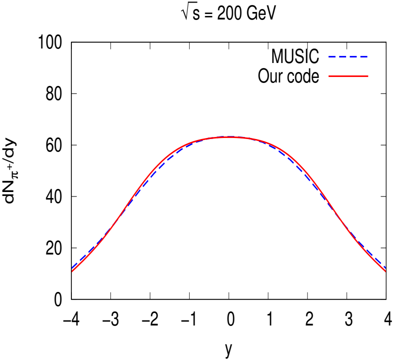
$$D_\mu T^{\mu\nu} = 0$$
$$D_\mu N_B^\mu = 0$$

$$\Delta^{\mu\nu}_{\alpha\beta} u^\gamma D_\gamma \pi^{\alpha\beta} = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu} D_\gamma u^\gamma$$
$$u^\gamma D_\gamma \Pi = -\frac{\Pi - \Pi_{NS}}{\tau_\Pi} - \frac{4}{3}\Pi D_\gamma u^\gamma$$

- We develop the code using relativistic HLLE algorithm and test it against known analytical results and with output from publicly available MUSIC and vHLLE codes.

Relativistic Hydrodynamics : Test Results





SKS and J. Alam, arXiv:2110.15604

Description of model

Initial condition

- $S^y \propto [p_\tau \varpi_{\eta x} + p_x \varpi_{\tau\eta} + p_\eta \varpi_{x\tau}]$
- Need an IC with non-zero $\partial_\eta u_x$ or $\partial_x u_\eta$.
- Glauber model for transverse profile along with symmetric rapidity profile for energy density has zero $\varpi_{\eta x}$ at all times.
- We use Glauber model + symmetric rapidity profile + local energy-momentum conservation. C. Shen *et al.* PRC 102 (2020) 014909
- The local collision energy and net longitudinal momentum at a point in the transverse plane are

$$E(x,y) = [n_A(x,y) + n_B(x,y)]\, m_N \cosh(y_{\mathtt{beam}}) = M(x,y) \cosh(y_{\mathtt{CM}})$$
$$P_z(x,y) = [n_A(x,y) - n_B(x,y)]\, m_N \sinh(y_{\mathtt{beam}}) = M(x,y) \sinh(y_{\mathtt{CM}})$$

where

$$M(x,y) = m_N \sqrt{n_A^2 + n_B^2 + 2 n_A n_B \cosh(y_{\texttt{beam}})}$$

$$y_{\texttt{CM}} = \tanh^{-1}\left[\frac{n_A - n_B}{n_A + n_B}\cosh(y_{\texttt{beam}})\right]$$

We must have

$$\int dxdy\ E(x,y) = \int d\Sigma_\mu T^{\mu t}$$

$$\int dxdy\ P_z(x,y) = \int d\Sigma_\mu T^{\mu z}$$

Assuming $u^\tau = 1$ and $u^x = u^y = u^{\eta_s} = 0$, we have $T^{\tau\tau} = \varepsilon(x,y,\eta_s)$ and $T^{\tau\eta} = 0$, so that

$$M(x,y) = \int \tau_0 d\eta_s\ \varepsilon(x,y,\eta_s)\cosh(\eta_s - y_{\texttt{CM}})$$

$$0 = \int \tau_0 d\eta_s\ \varepsilon(x,y,\eta_s)\sinh(\eta_s - y_{\texttt{CM}})$$

If we further assume

$$\varepsilon(x,y,\eta_s) = \mathcal{N}_e(x,y)\exp\left[-\frac{(|\eta_s - y_{\mathtt{CM}}| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s - y_{\mathtt{CM}}| - \eta_0)\right]$$

we get

$$\mathcal{N}_e(x,y) = \frac{M(x,y)}{2\sinh(\eta_0) + \sqrt{\frac{\pi}{2}}\sigma_\eta e^{\sigma_\eta^2/2} C_\eta}$$

with

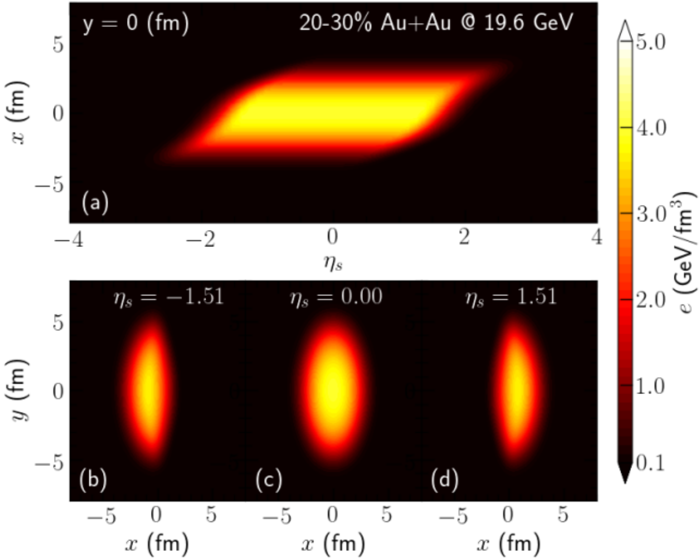
$$C_\eta = e^{\eta_0}\mathrm{erfc}\left(-\sqrt{\frac{1}{2}}\sigma_\eta\right) + e^{-\eta_0}\mathrm{erfc}\left(\sqrt{\frac{1}{2}}\sigma_\eta\right)$$

Initial net baryon density is taken as

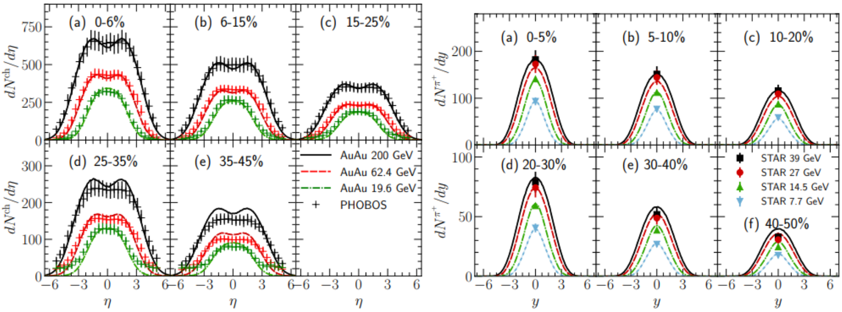
$$n_B(x,y,\eta_s;\tau_0) = \mathcal{N}_B\left[g_A(\eta_s)n_A(x,y) + g_B(\eta_s)n_B(x,y)\right]$$

where $\mathcal{N}_B$ is fixed by the condition

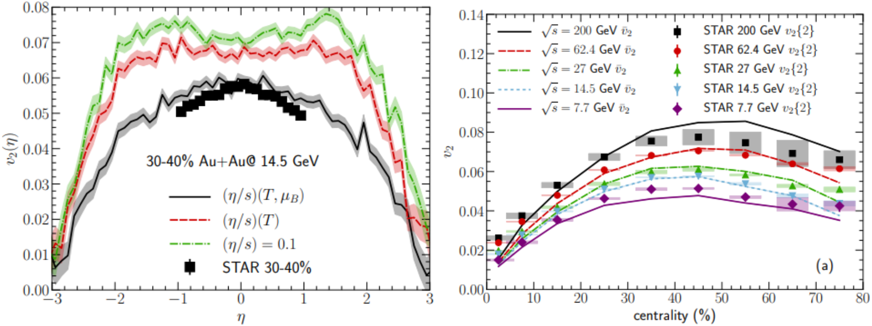
$$\int \tau_0\, dx\, dy\, d\eta_s\;\; n_B(x,y,\eta_s;\tau_0) = N_{\mathtt{part}}$$



PRC 102, 014909 (2020)



PRC 102, 014909 (2020)



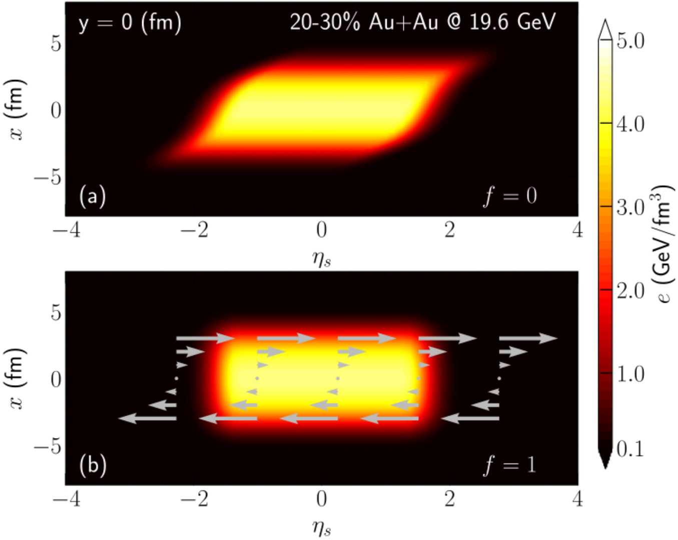
PRC 102, 014909 (2020)

The IC model of C. Shen *et* al. has been generalized to include non-zero initial OAM in S. Ryu *et al.* PRC 104, 054908 (2021). The initial energy-momentum current is assumed to have the following form:

$$T^{\tau\tau} = \varepsilon(x, y, \eta_s)\cosh(y_L)$$
$$T^{\tau\eta_s} = \frac{1}{\tau_0}\varepsilon(x, y, \eta_s)\sinh(y_L)$$

where

$$y_L = f\ y_{\mathtt{CM}} \qquad , \qquad f \in [0, 1]$$



PRC 104, 054908 (2021)

Description of model

Equation of state & transport coefficients

Ref. - PRC 101, 034901 (2020)

- The pressure at non-zero T and μ_B can be obtained through a Taylor series expansion about $\mu_B = 0$ as follows

$$P_{QCD}(\mu_B, T) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

- The presence of CP makes some of the coefficients diverge

$$T^4 c_n(T) \rightarrow T^4 c_n^{\text{Non-Ising}}(T) + T_c^4 c_n^{\text{Ising}}(T)$$

Equivalently,

$$P_{QCD}(\mu_B, T) = P^{\text{reg}}(\mu_B, T) + P^{\text{crit}}(\mu_B, T)$$

- Choose and adjust P^{reg} such that $P_{QCD}(0, T) = P^{\text{LAT}}(T)$.

Description of model

Equation of state & transport coefficients

- Obtain P^{crit} by mapping to 3D-Ising model. The mapping is done as follows:

$$\frac{T - T_C}{T_C} = w\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B - \mu_{BC}}{T_C} = w\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

- The Ising pressure in the critical region is given by

$$P_{\mathtt{Ising}}(R,\theta) = h_0 M_0 R^{2-\alpha}\left[\theta\tilde{h}(\theta) - g(\theta)\right],$$

where

$$h = h_0 R^{\beta\delta}\tilde{h}(\theta) \quad , \quad r = R(1-\theta^2)$$

and

$$\tilde{h}(\theta) = \theta(1+a\theta^2+b\theta^4) \quad , \quad g(\theta) = c_0+c_1(1-\theta^2)+c_2(1-\theta^2)^2+c_3(1-\theta^2)$$

Description of model

Equation of state & transport coefficients

Ref. - PRC 101, 034901 (2020)

- The Ising coefficients are obtained from $P_{\mathtt{Ising}}$ as follows:

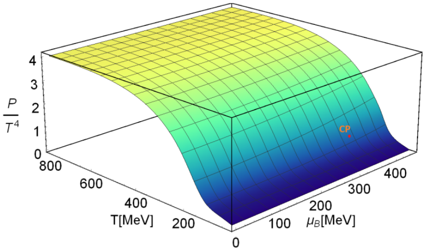
$$c_n^{\text{Ising}}(T) = \frac{1}{n!} T^n \left. \frac{\partial^n P^{\text{Ising}}}{\partial \mu_B^n} \right|_{\mu_B=0}$$

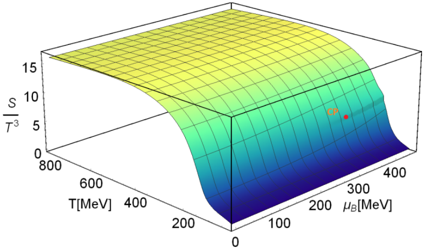
- The "Non-Ising" coefficients are obtained as follows:

$$T^4 c_n^{\text{LAT}}(T) = T^4 c_n^{\text{Non-Ising}}(T) + T_c^4 c_n^{\text{Ising}}(T).$$

- The QCD pressure is then

$$P_{\text{QCD}}(T, \mu_B) = T^4 \sum_n c_{2n}^{\text{Non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^{2n} + T_c^4 P^{\text{Ising}}(R(T, \mu_B), \theta(T, \mu_B))$$





For hydrodynamics we require $p \equiv p(\varepsilon, n_B)$. Discretize $\varepsilon - n_B$ plane:

$$\Delta\varepsilon\,(\mathrm{GeV/fm^3}) = \begin{cases} 0.002 & \text{if} \quad 0.001 \le \varepsilon < 1.001, \\ 0.02 & \text{if} \quad 1.001 \le \varepsilon < 11.001, \\ 0.1 & \text{if} \quad 11.001 \le \varepsilon < 61.001, \\ 0.5 & \text{if} \quad 61.001 \le \varepsilon < 101.001. \end{cases}$$

$$\Delta n_B\,(\mathrm{fm^{-3}}) = \begin{cases} 0.0005 & \text{if} \quad 0 \le n_B < 0.15, \\ 0.001 & \text{if} \quad 0.15 \le n_B < 0.3, \\ 0.01 & \text{if} \quad 0.3 \le n_B < 1, \\ 0.025 & \text{if} \quad 1 \le n_B < 5. \end{cases}$$

Equilibrium Correlation Length, ξ

$$\xi^2 = \frac{1}{H_0}\left(\frac{\partial M(r,h)}{\partial h}\right)_r$$

Assume $H_0 = 1$. M is parameterized in terms of R and θ as

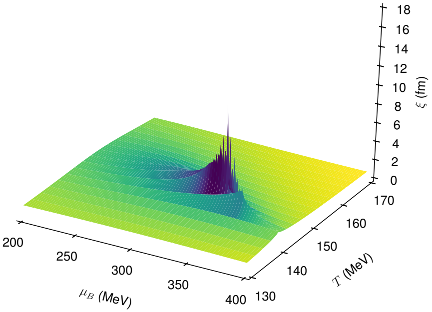
$$M(R,\theta) = M_0 R^\beta \theta$$

We have

$$\left(\frac{\partial M}{\partial h}\right)_r = \left(\frac{\partial M}{\partial R}\right)_\theta \left(\frac{\partial R}{\partial h}\right)_r + \left(\frac{\partial M}{\partial \theta}\right)_R \left(\frac{\partial \theta}{\partial h}\right)_r$$

so that ξ is given by

$$\xi^2 = \frac{M_0}{h_0}\frac{R^{\beta(1-\delta)}}{2\beta\delta\theta\tilde{h}(\theta) + (1-\theta^2)\tilde{h}'(\theta)}\left[1 + (2\beta-1)\theta^2\right]$$



Correlation length, ξ, plotted as a function of μ_B and T.

Near the critical point, the transport coefficients diverge as

$$\zeta \sim \xi^3 \qquad , \qquad \eta \sim \xi^{0.05}$$

The critical behavior of these transport coefficients can be modeled as

$$\zeta = \zeta_0 \left(\frac{\xi}{\xi_0}\right)^3 \qquad , \qquad \eta = \eta_0 \left(\frac{\xi}{\xi_0}\right)^{0.05}$$

Similarly for τ_π and τ_Π, i.e.

$$\tau_\Pi = \tau_\Pi^0 \left(\frac{\xi}{\xi_0}\right)^3 \qquad , \qquad \tau_\pi = \tau_\pi^0 \left(\frac{\xi}{\xi_0}\right)^{0.05}$$

ξ_0 is a parameter for deciding the boundary of the critical region. We choose $\xi_0 = 1.75$ fm. ζ_0 and η_0 taken as

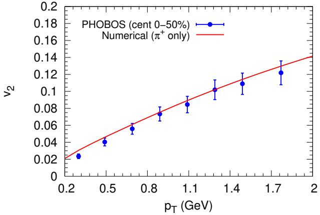
$$\eta_0(\mu_B, T) = 0.08 \left(\frac{\varepsilon + p}{T}\right) \quad , \quad \zeta_0(\mu_B, T) = 15\eta_0(\mu_B, T)\left(\frac{1}{3} - c_s^2\right)^2$$

Stopping criterion

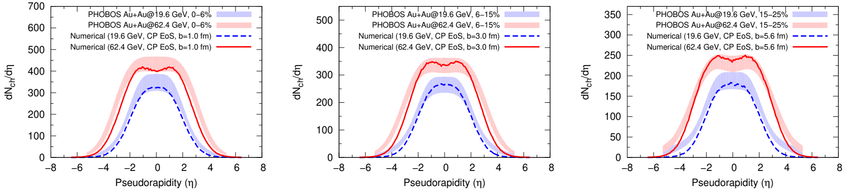
- Constant energy density, $\varepsilon = 0.3$ GeV/fm^3. Close to transition line.
- The surface is found using the CORNELIUS code.
- The surface is input to the UrQMD.
- The spin polarization analysis is done on this surface

Fixing the parameters

Comparison with experimental data

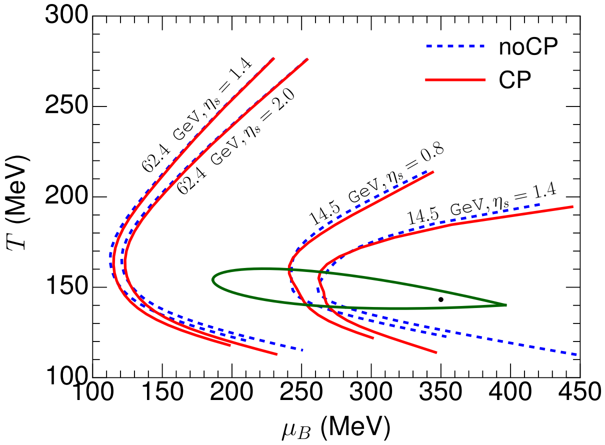


SKS and J. Alam, arXiv:2110.15604



SKS and J. Alam, arXiv:2110.15604

Hydrodynamic trajectories in phase diagram



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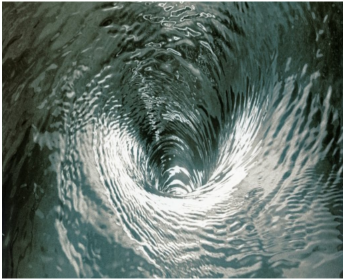

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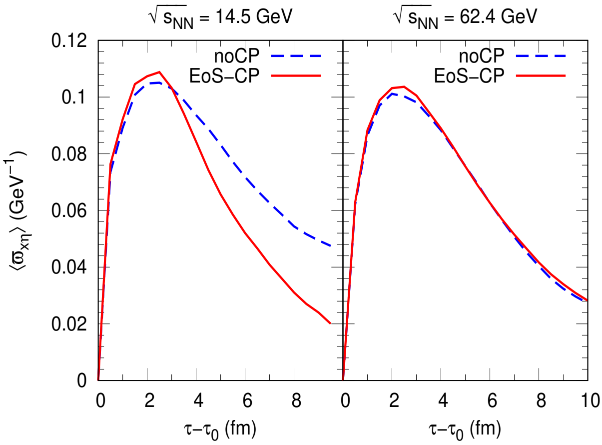
Case I : Zero initial angular momentum

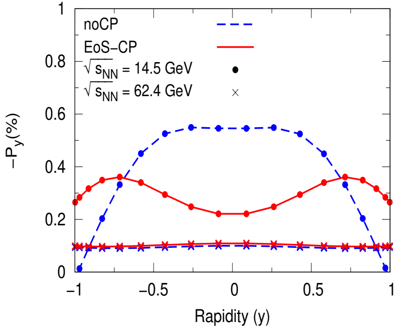
Evolution of thermal vorticity

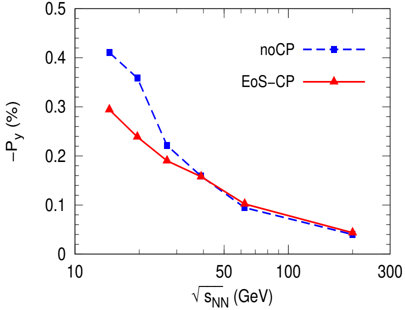


SKS and J. Alam, arXiv:2110.15604

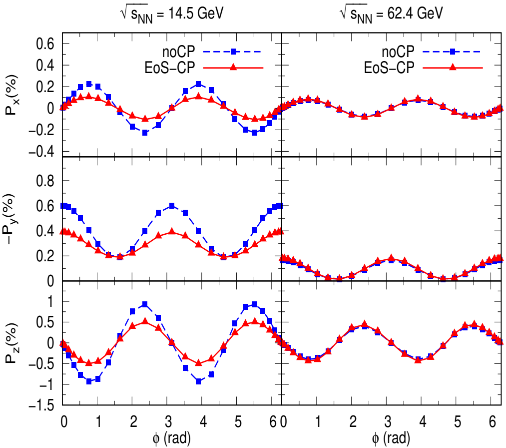
Suppression of spin polarization of Λ-hyperons

- Polarization calculated on constant energy density hypersurface 0.3 GeV/fm^3. No afterburner.





SKS and J. Alam, arXiv:2110.15604

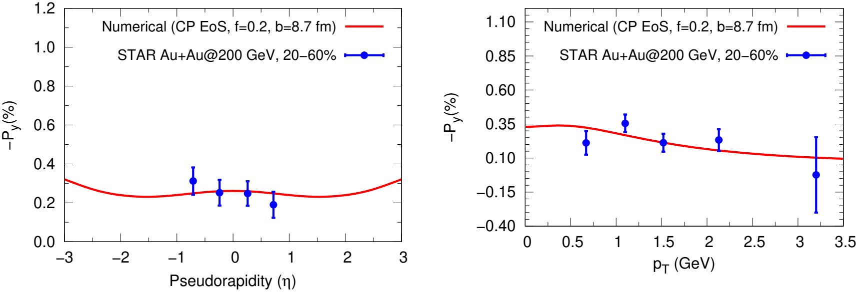


x, y and z components of Λ-polarization are plotted respectively in the upper, middle and the lower panels as a function of azimuthal angle in momentum space for $\sqrt{s_{NN}} = 14.5$GeV and 62.4GeV. Ref: SKS and J. Alam, arXiv:2110.15604

Case II : Non-zero initial angular momentum

Comparison with experimental data at $\sqrt{s_{NN}} = 200$ GeV

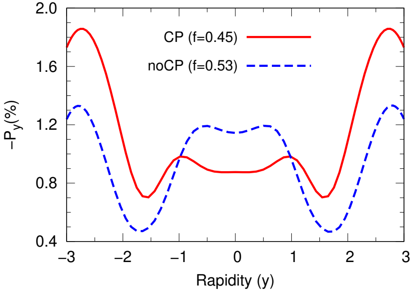
- In S. Ryu *et al.*, PRC 104, 054908 (2021), non-zero initial vorticity is obtained by introducing a parameter f that controls the fraction of longitudinal momentum that can be attributed to the flow velocity.

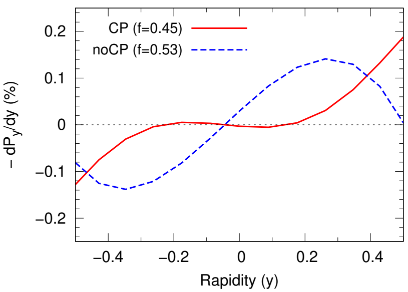


SKS and J. Alam, arXiv:2110.15604

Prediction near critical point

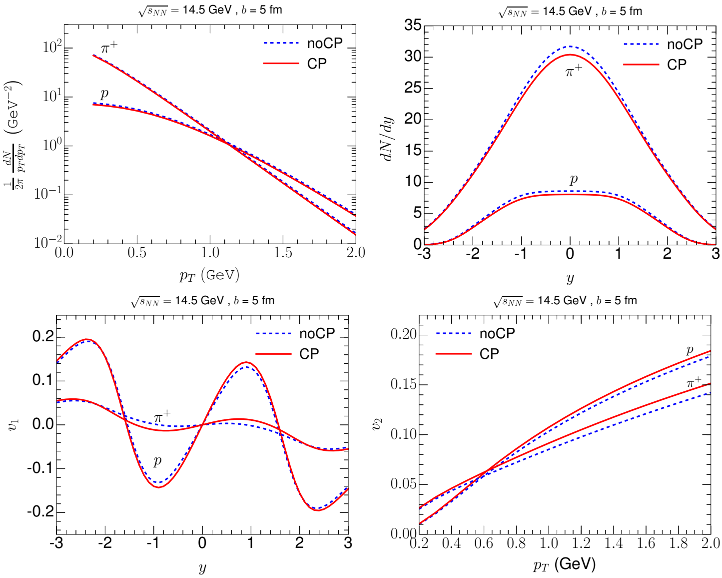
Au+Au collisions at $\sqrt{s_{NN}} = 14.5$ GeV with $b = 5.6$ fm





SKS and J. Alam, arXiv:2110.15604

- We also find that the other bulk observables like elliptic flow, p_T-spectra etc. are not much affected due to the CP. SKS and J. Alam, arXiv:2205.14469.



SKS and J. Alam, arXiv:2205.14469

Summary

- Observables dependent on gradients are more sensitive to the EoS.
- We observe a suppression in thermal vorticity and hence, polarization of Λ-hyperons, as the CP is approached.
- Suppression in the rapidity distribution of spin polarization may be useful for locating CP. Further study needed.

Thank You !!!