School of Mathematics
Tata Institute of Fundamental Research

17 April, 2023

Number Theory Seminar

Speaker	:	***Arindam Jana, TIFR***
Title	:	***Periodicity in filtrations of mod*** p ***representations of*** $\mathrm{GL}_2(\mathbb{F}_q)$
Date & Time	:	***Wednesday, 26 April 2023, 2:30 P.M.***
Venue	:	**Lecture Room (AG-77)**

Abstract

The irreducible mod p representations of $\mathrm{GL}_2(\mathbb{F}_p)$ are exactly the twists of V_r, the r-th symmetric power of the standard representations of $\mathrm{GL}_2(\mathbb{F}_p)$ for small values of r. In this talk, for sufficiently large r, we investigate the periodicity in a filtration of V_r defined by the powers of the polynomial $\theta := X^pY - XY^p$, motivated by a classical result of Glover. Ghate and Vangala studied the periodicity of the higher quotients in the filtration of V_r using generalized dual numbers. We strengthen their result by defining an explicit isomorphism between these quotients of V_r and generalized mod p principal series representations using differential operators, and extend it to $\mathrm{GL}_2(\mathbb{F}_q)$ for $q = p^f, f \geq 1$. In search of a similar periodicity result in case of cuspidal representations, Reduzzi proved that the reduction mod p of a cuspidal representation of $\mathrm{GL}_2(\mathbb{F}_q)$ is isomorphic to the cokernel of a differential operator on V_r defined by Serre. This isomorphism is proved using crystalline cohomology and is not explicit. We define this isomorphism explicitly after tensoring with V_{q-1}. This work is joint with Eknath Ghate.

Milind Pilankar