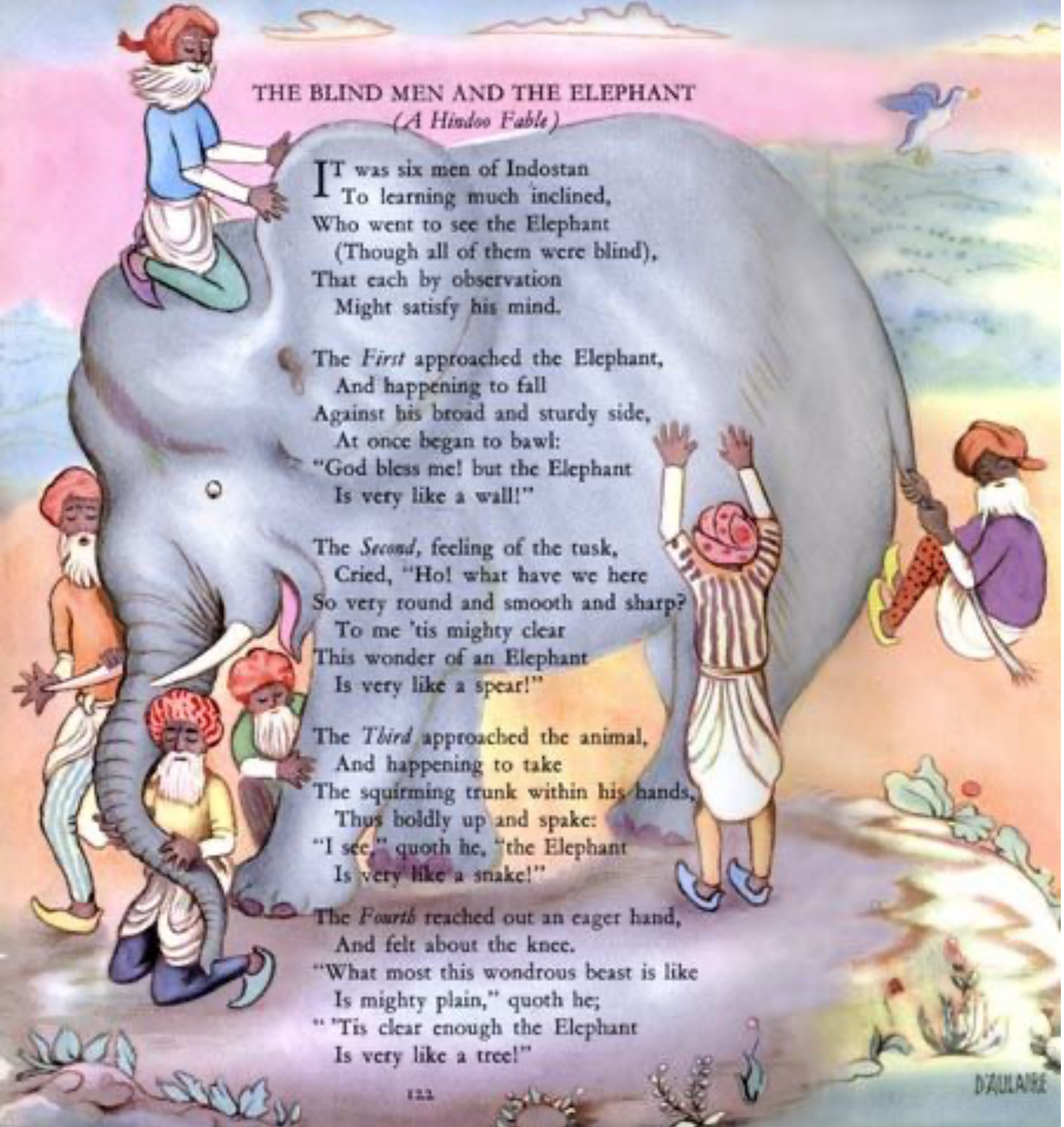
Thank you, Prof. R. V. Gavai, for your dedication and contribution to world science, and also particularly Indian science. Your scientific contribution is an inspiration for us in India…

Correlations between conserved charges: An insight look of e-b-e fluctuation measurements at RHIC

Nihar Ranjan Sahoo
[Arghya Chatterjee (CCNU) and Prithwish Tribedy (BNL)]
Shandong University (SDU), Qingdao, China

Workshop on QCD in the Nonperturbative Regime, TIFR, Bombay
18-20 Nov, 2019

Parable…



THE BLIND MEN AND THE ELEPHANT

(A Hindoo Fable)

It was six men of Indostan
To learning much inclined,
Who went to see the Elephant
(Though all of them were blind),
That each by observation
Might satisfy his mind.

The *First* approached the Elephant,
And happening to fall
Against his broad and sturdy side,
At once began to bawl:
"God bless me! but the Elephant
Is very like a wall!"

The *Second*, feeling of the tusk,
Cried, "Ho! what have we here
So very round and smooth and sharp?
To me 'tis mighty clear
This wonder of an Elephant
Is very like a spear!"

The *Third* approached the animal,
And happening to take
The squirming trunk within his hands,
Thus boldly up and spake:
"I see," quoth he, "the Elephant
Is very like a snake!"

The *Fourth* reached out an eager hand,
And felt about the knee.
"What most this wondrous beast is like
Is mighty plain," quoth he;
" 'Tis clear enough the Elephant
Is very like a tree!"

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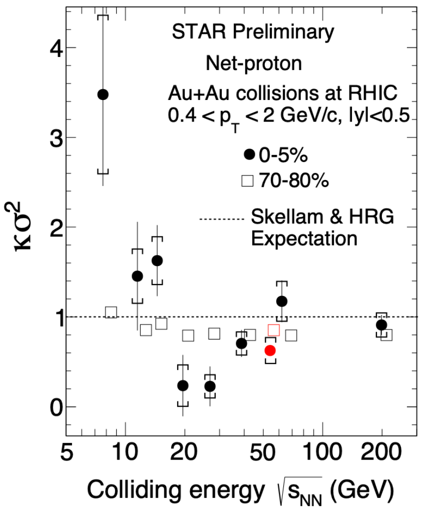
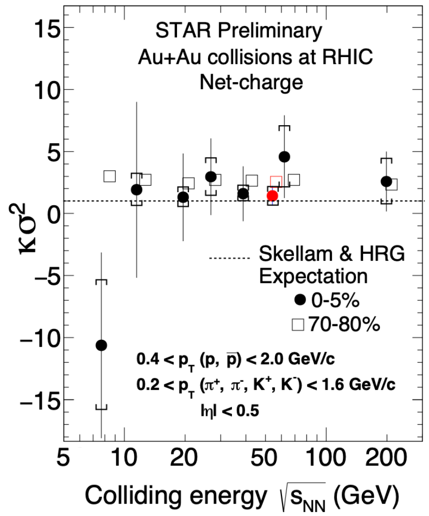
The Blind men and the elephant

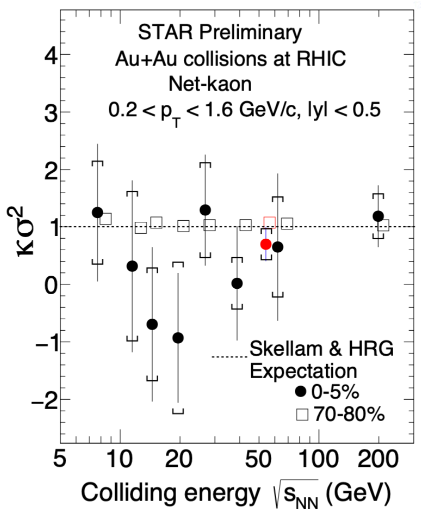
Experiment vs Theory

Net-charge, net-proton and net-kaon fluctuations at RHIC

An attempt by the STAR experiment to search the QCD Critical Point





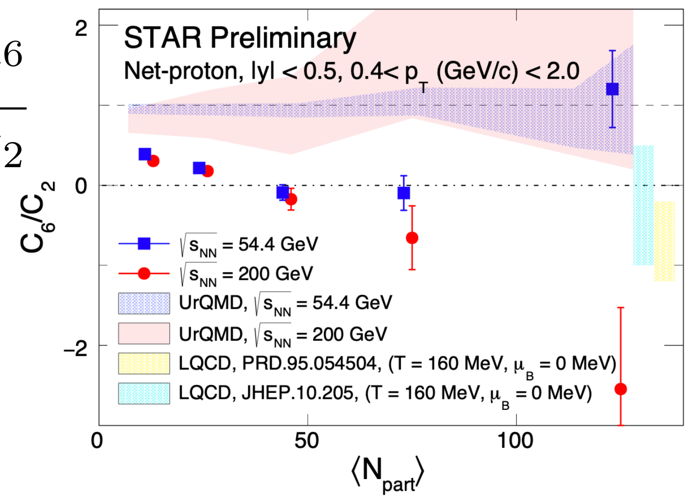


$$\kappa\sigma^2 = \frac{\chi_4}{\chi_2}$$

A. Pandav: QM19

PRL 113, 092301 (2014) [My Ph.D work]
PRL 112, 032302 (2014)
PLB 785 (2018) 551–560

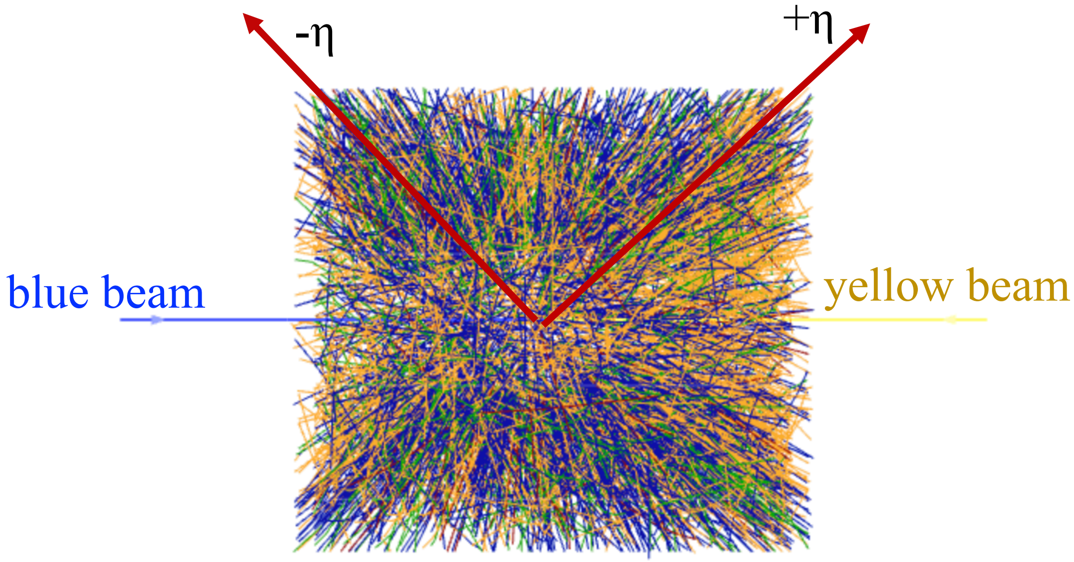
$$\frac{C_6}{C_2} = \frac{\chi^6}{\chi_2}$$



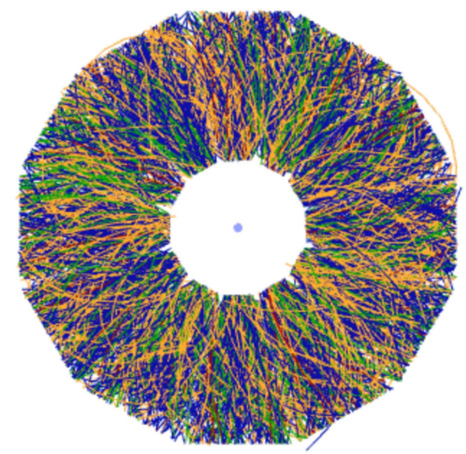
- But, we need a deeper understanding of these fluctuations, and
- how net-Q, net-p, and net-k are correlated among themselves as a function of beam energy?

Experimental challenges

STAR experiment Au+Au 200 GeV collision event display



Finite acceptance



Uniform φ

- Finite phase space ($\Delta\eta$, $\Delta\varphi$ and Δp_T)
- Detector effects (Binominal/Non-binomial assumption)
- Volume fluctuations (Bin-width corrections)
- Statistical uncertainty estimations
- Computational challenges like higher order cumulants and their uncertainties

  Cumulants → Central Moments → Factorial moments $f_{ij} = \epsilon_{+}^{i}\epsilon_{-}^{j}F_{ij}$

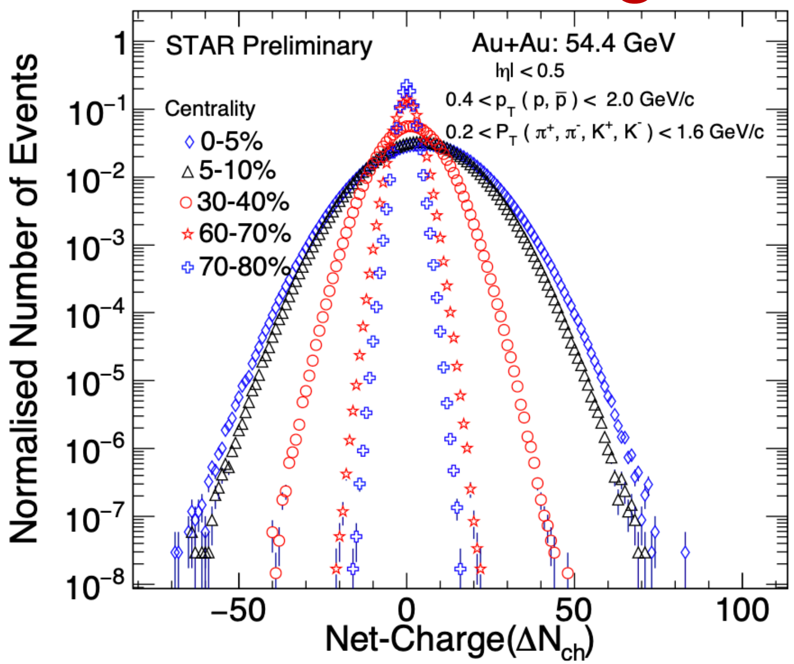
All these challenges have been handled recently.

Event-by-event net-particles distributions

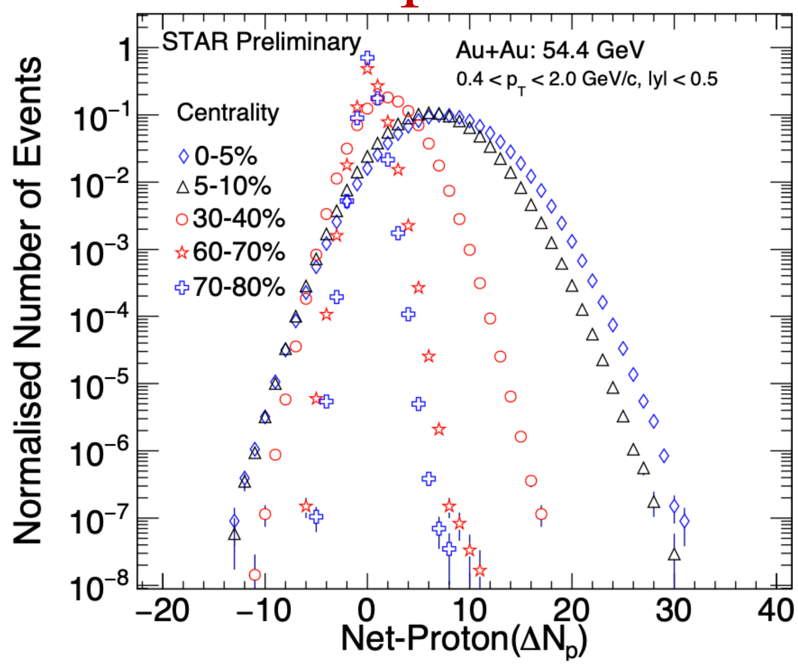
Au+Au 54 GeV collisions

A. Pandav: QM19

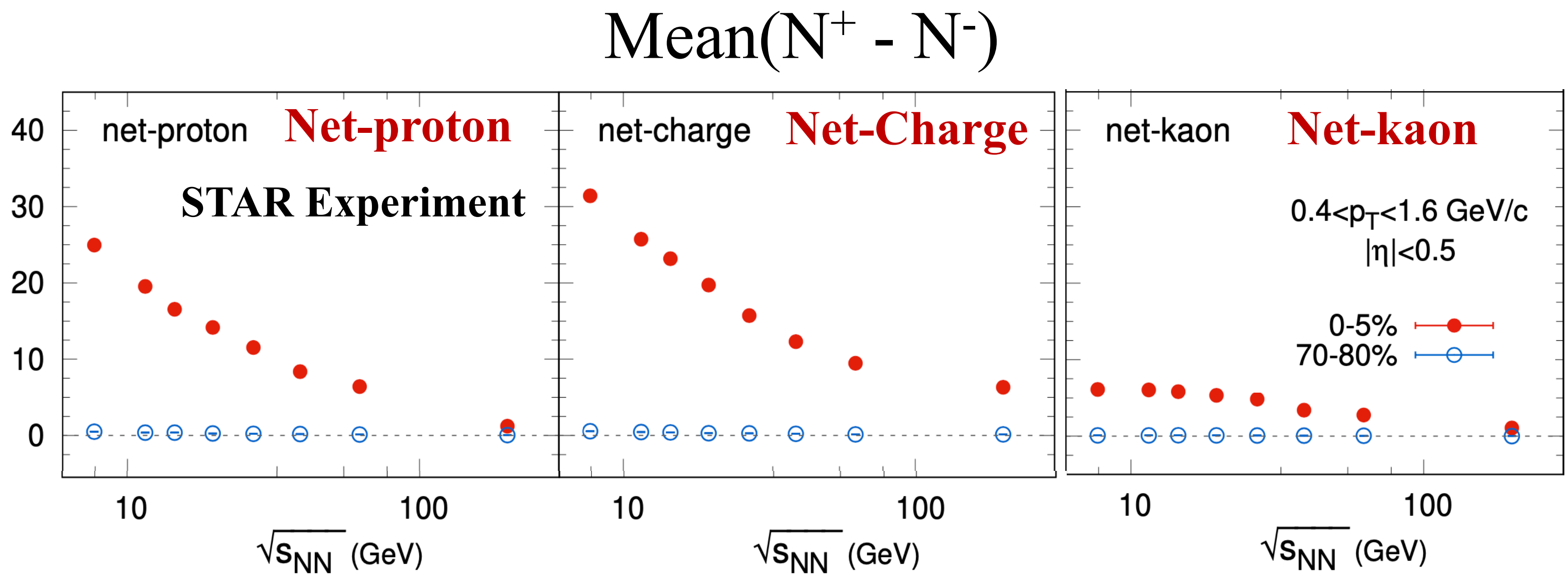
Net-charge



Net-proton



The mean of these fluctuations



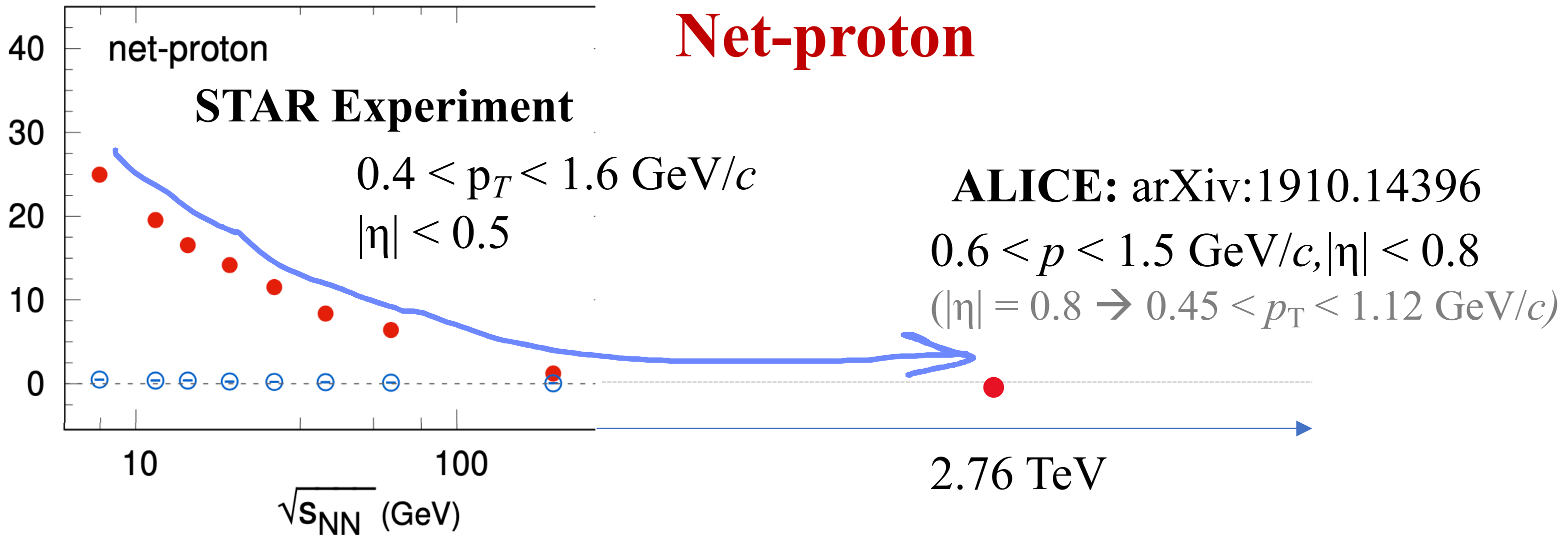
Mean changes with beam energy as per expectation.

Net-charge: PRL 113, 092301 (2014)
Net-proton:PRL 112, 032302 (2014)
Net-kaon:PLB 785 (2018) 551–560
Off-diagonal:PRC 100, 014902 (2019)

The mean of these fluctuations

Mean(N^+ - N^-)

Net-proton



Mean changes with beam energy as per expectation.

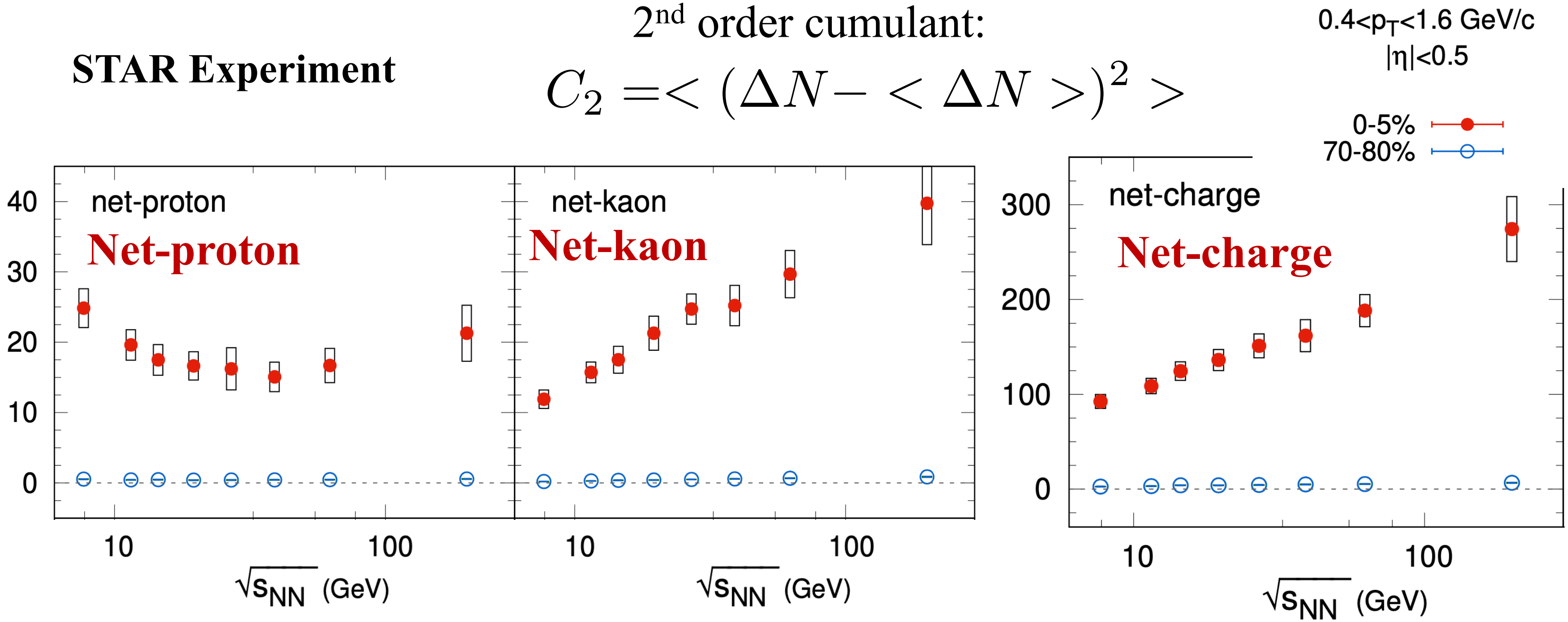
Let's see the variance of these fluctuations...

Let's see the variance of these fluctuations

STAR Experiment

2nd order cumulant:

$$C_2 = \langle (\Delta N - \langle \Delta N \rangle)^2 \rangle$$



- Net-proton shows different trend at lower energy
- Net-charge and net-kaon show the same trends

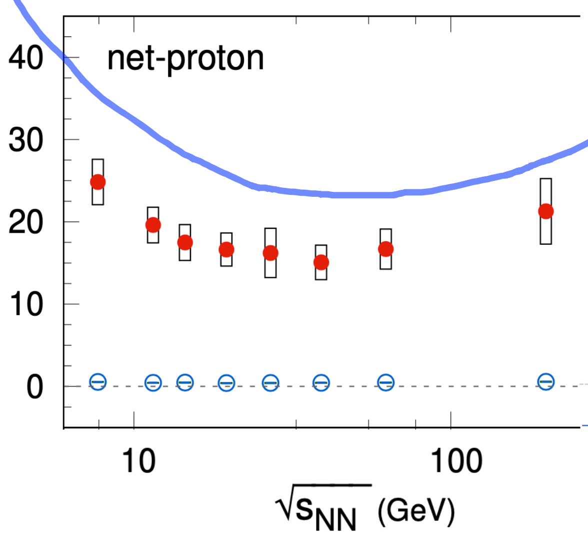
STAR Off-diagonal: PRC 100, 014902 (2019)

Let's see variance of these fluctuations

2nd order cumulant

$$C_2 =< (\Delta N- < \Delta N >)^2 >$$

STAR Experiment



ALICE: arXiv:1910.14396

● $0.6 < p < 1.5$ GeV/c, $|\eta| < 0.8$

($|\eta| = 0.8 \rightarrow 0.45 < p_T < 1.12$ GeV/c)

2.76 TeV

High energy limit

Low energy limit
(STAR FXT/
CBM/NICA)

- Net-proton shows different trend at lower energy
- Net-charge and net-kaon show the same trends

It seems more proton accumulation due to baryon stopping at low energy.
How all these fluctuations are correlated among themselves?

STAR Off-diagonal: PRC 100, 014902 (2019)

Math. expressions to understand the correlations

Relation between susceptibility and cumulants: $\chi_{\alpha}^{2} = \frac{1}{VT^3}\kappa_{\alpha}^{2}, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3}\kappa_{\alpha,\beta}^{1,1}$

2nd order cumulant: $\kappa_{\alpha}^{2} = \sigma_{\alpha}^{2} = \langle(\delta N_{\alpha} - \langle\delta N_{\alpha}\rangle)^2\rangle$

2nd order off-diagonal cumulant: $\kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle(\delta N_{\alpha} - \langle\delta N_{\alpha}\rangle)(\delta N_{\beta} - \langle\delta N_{\beta}\rangle)\rangle$

Math. expressions to understand the correlations

Relation between susceptibility and cumulants: $\chi^2_\alpha = \frac{1}{VT^3}\kappa^2_\alpha, \quad \chi^{1,1}_{\alpha,\beta} = \frac{1}{VT^3}\kappa^{1,1}_{\alpha,\beta}$

2nd order cumulant: $\kappa^2_\alpha = \sigma^2_\alpha = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)^2\rangle$

2nd order off-diagonal cumulant: $\kappa^{1,1}_{\alpha,\beta} = \sigma^{1,1}_{\alpha,\beta} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$

In matrix form: $$\sigma^2 = \begin{pmatrix} \sigma^2_Q & \sigma^{1,1}_{Q,p} & \sigma^{1,1}_{Q,k} \\ \sigma^{1,1}_{p,Q} & \sigma^2_p & \sigma^{1,1}_{p,k} \\ \sigma^{1,1}_{k,Q} & \sigma^{1,1}_{k,p} & \sigma^2_k \end{pmatrix}$$

Math. expressions to understand the correlations

Relation between susceptibility and cumulants: $\chi^2_\alpha = \frac{1}{VT^3}\kappa^2_\alpha, \quad \chi^{1,1}_{\alpha,\beta} = \frac{1}{VT^3}\kappa^{1,1}_{\alpha,\beta}$

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2nd order off-diagonal cumulant: $\kappa^{1,1}_{\alpha,\beta} = \sigma^{1,1}_{\alpha,\beta} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$

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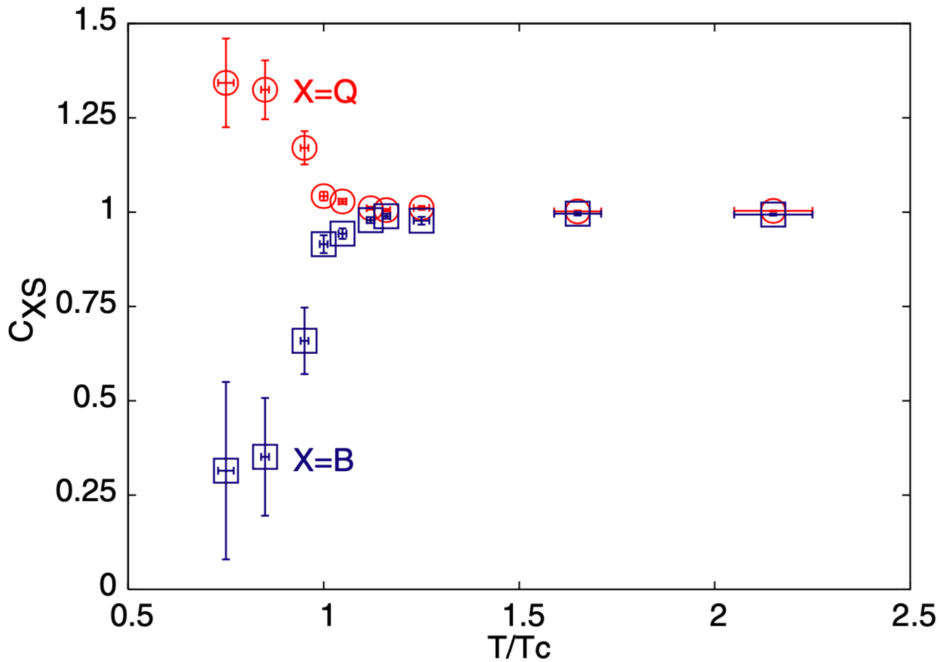
$$\sigma^{1,1}_{X,Y} = \sigma^{1,1}_{Y,X}$$

Similarly one can go to higher order off-diagonal cumulant:

$$\kappa(X_1,\ldots,X_n) = \sum_\pi (|\pi|-1)!(-1)^{|\pi|-1} \prod_{B\in\pi} E\left(\prod_{i\in B} X_i\right)$$

Lattice QCD prediction

R. V. Gavai and S. Gupta: PRD 73, 014004 (2006)



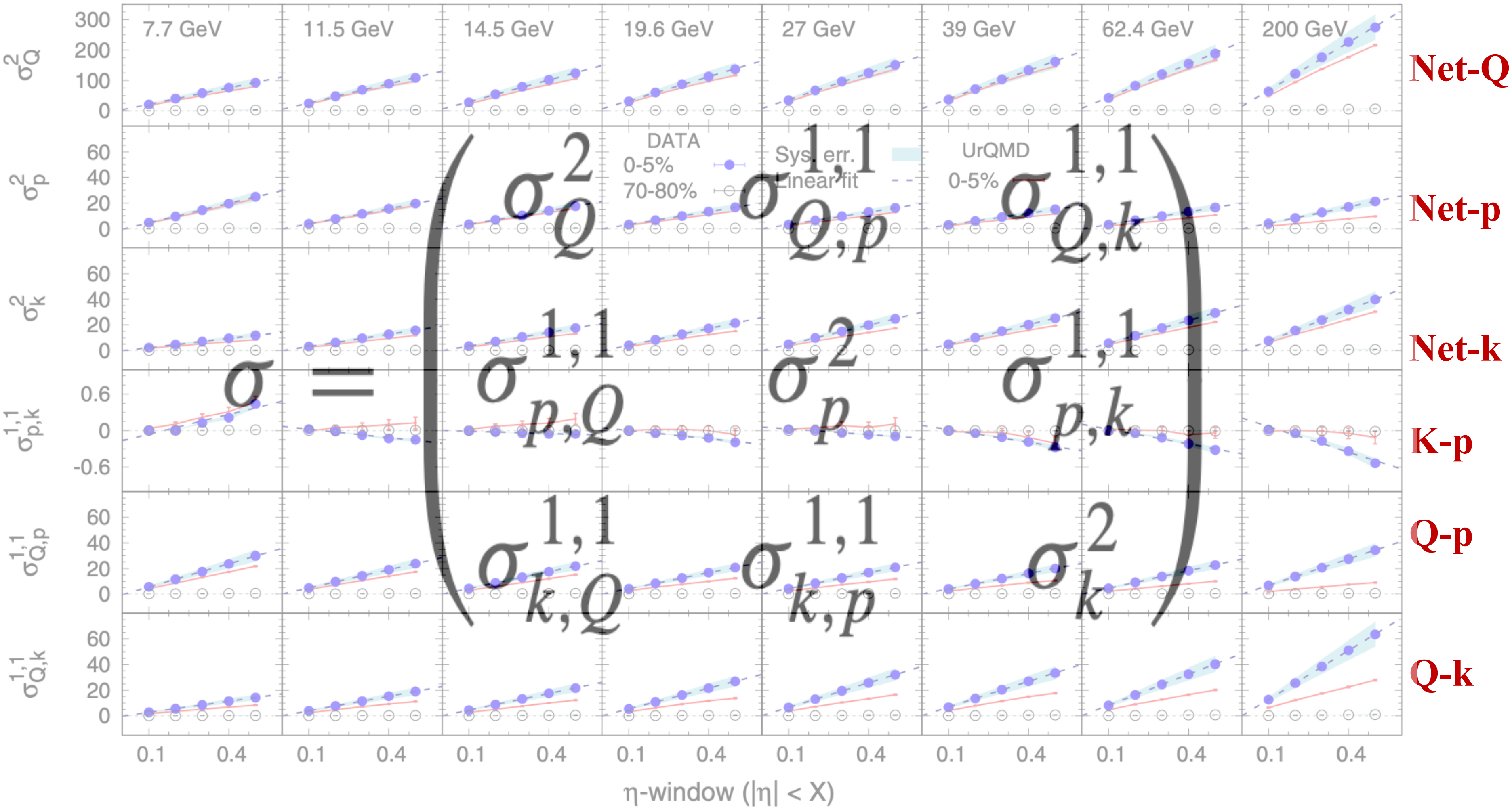
Lattice QCD result:
Strangeness carrier can give information about phase transition at T_c ($\mu_B = 0$)

$$C_{BS} = -3C_{(BS)/S} = -3\frac{\chi_{BS}}{\chi_S} = 1 + \frac{\chi_{us} + \chi_{ds}}{\chi_s}$$
$$= 1 + C_{(us)/s} + C_{(ds)/s} = 1 + 2C_{(us)/s}$$

$$C_{QS} = 3C_{(QS)/S} = 1 - \frac{2\chi_{us} - \chi_{ds}}{\chi_s}$$

V. Koch, A. Majumder, and J. Randrup, Phys. Rev. Lett. 95, 182301 (2005)
A. Majumder and B. Muller, Phy. Rev. C 74, 054901 (2006)
M. Cheng, et al, Phy. Rev. D 79, 074505 (2009)

2^{nd} order cumulant matrix information for different η-window

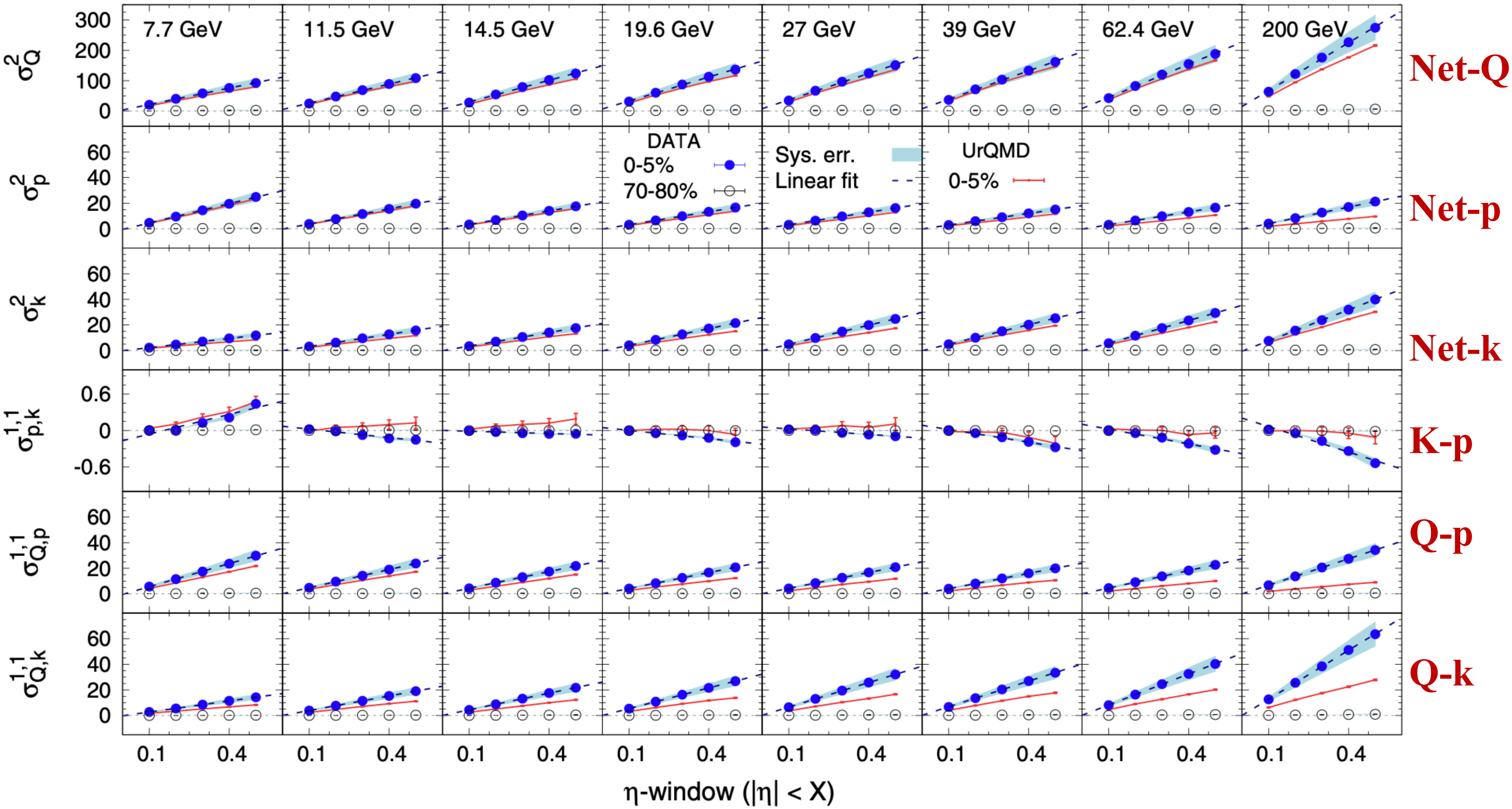


$$\sigma = \begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{1,1} & \sigma_{Q,k}^{1,1} \\ \sigma_{p,Q}^{1,1} & \sigma_p^2 & \sigma_{p,k}^{1,1} \\ \sigma_{k,Q}^{1,1} & \sigma_{k,p}^{1,1} & \sigma_k^2 \end{pmatrix}$$

η window

STAR Off-diagonal: PRC 100, 014902 (2019)

2^{nd} order cumulant matrix information for different η-window

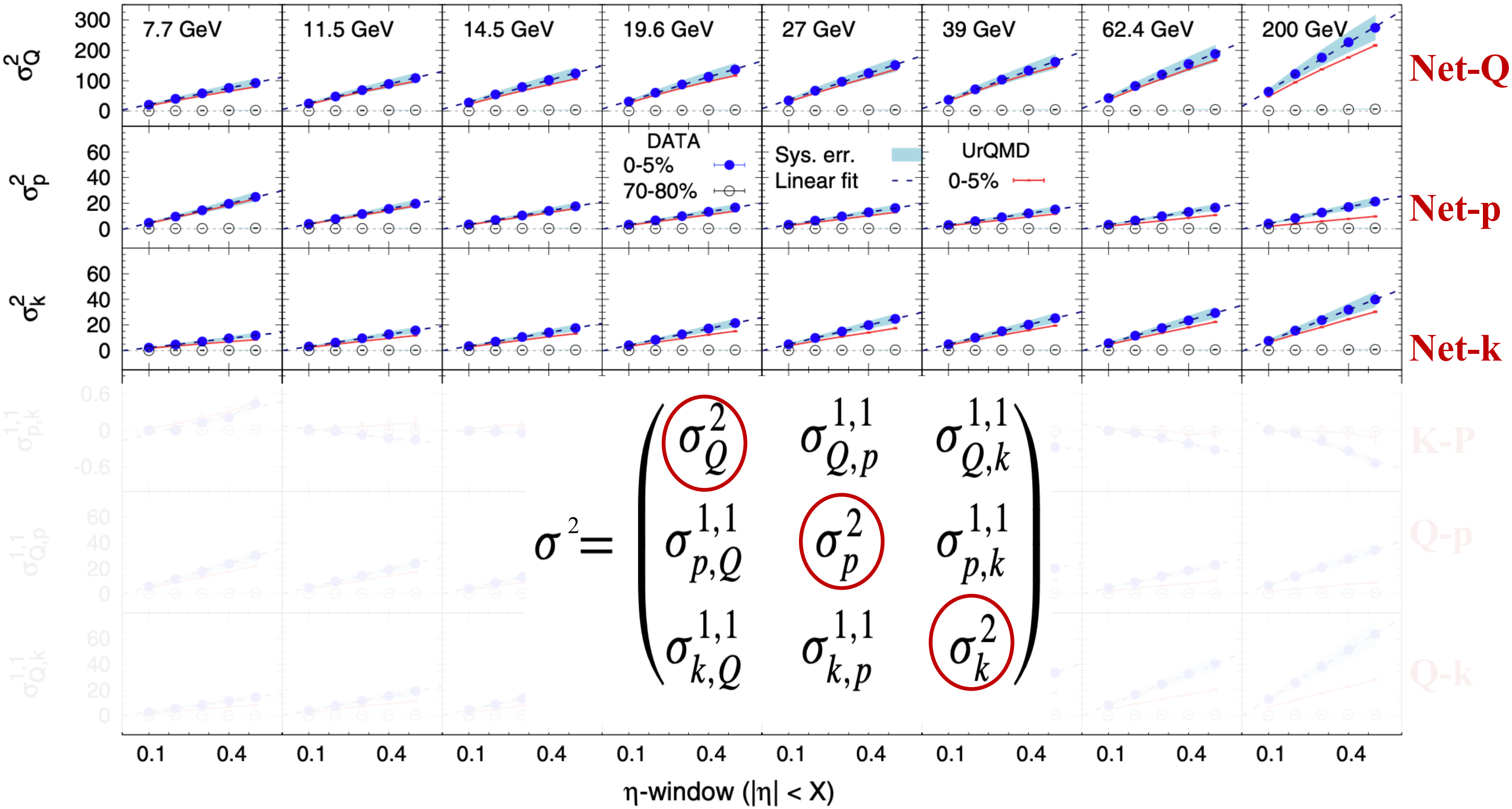


$$\sigma^2 = \begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{1,1} & \sigma_{Q,k}^{1,1} \\ \sigma_{p,Q}^{1,1} & \sigma_p^2 & \sigma_{p,k}^{1,1} \\ \sigma_{k,Q}^{1,1} & \sigma_{k,p}^{1,1} & \sigma_k^2 \end{pmatrix}$$

η window

STAR Off-diagonal: PRC 100, 014902 (2019)

2^{nd} order cumulant matrix information for different η-window

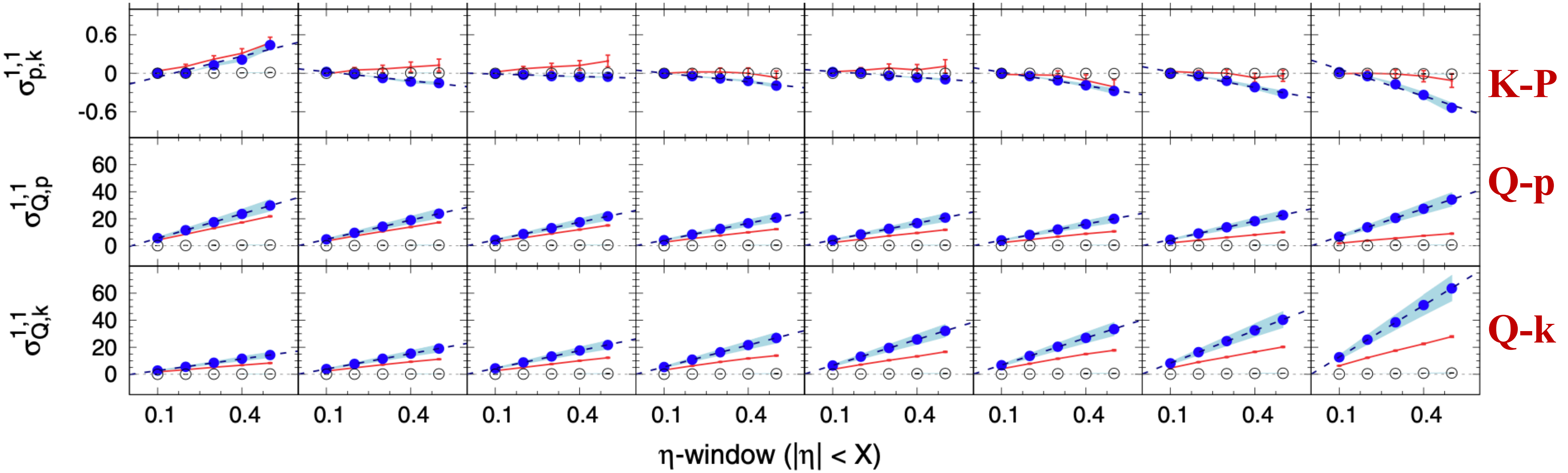


$$\sigma^2 = \begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{1,1} & \sigma_{Q,k}^{1,1} \\ \sigma_{p,Q}^{1,1} & \sigma_p^2 & \sigma_{p,k}^{1,1} \\ \sigma_{k,Q}^{1,1} & \sigma_{k,p}^{1,1} & \sigma_k^2 \end{pmatrix}$$

STAR Off-diagonal: PRC 100, 014902 (2019)

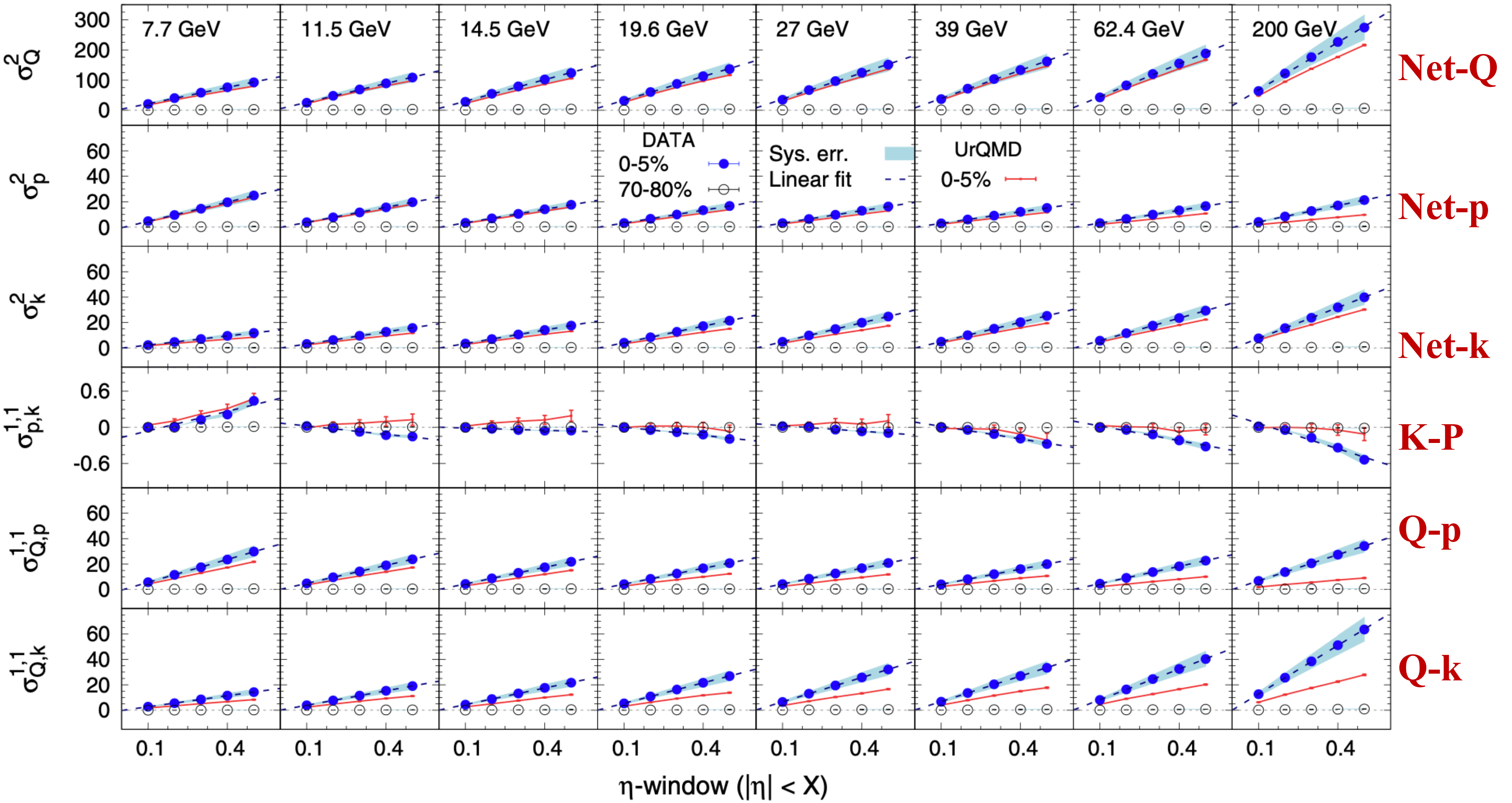
2nd order cumulant matrix information for different η-window

$$\sigma^2 = \begin{pmatrix} \sigma_Q^2 & \sigma_{Q,p}^{1,1} & \sigma_{Q,k}^{1,1} \\ \sigma_{p,Q}^{1,1} & \sigma_p^2 & \sigma_{p,k}^{1,1} \\ \sigma_{k,Q}^{1,1} & \sigma_{k,p}^{1,1} & \sigma_k^2 \end{pmatrix}$$



STAR Off-diagonal: PRC 100, 014902 (2019)

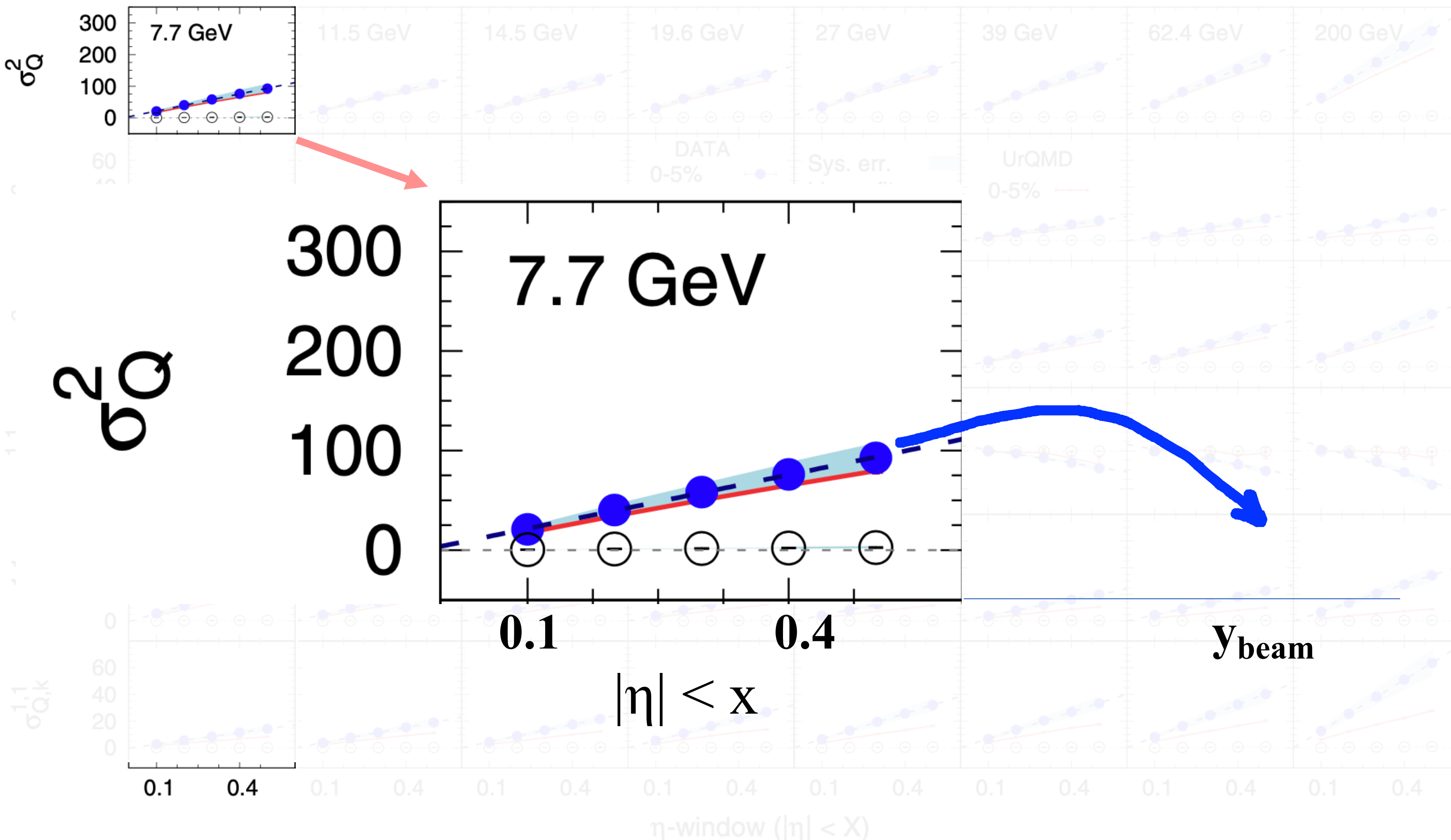
2^{nd} order cumulant matrix information for different η-window



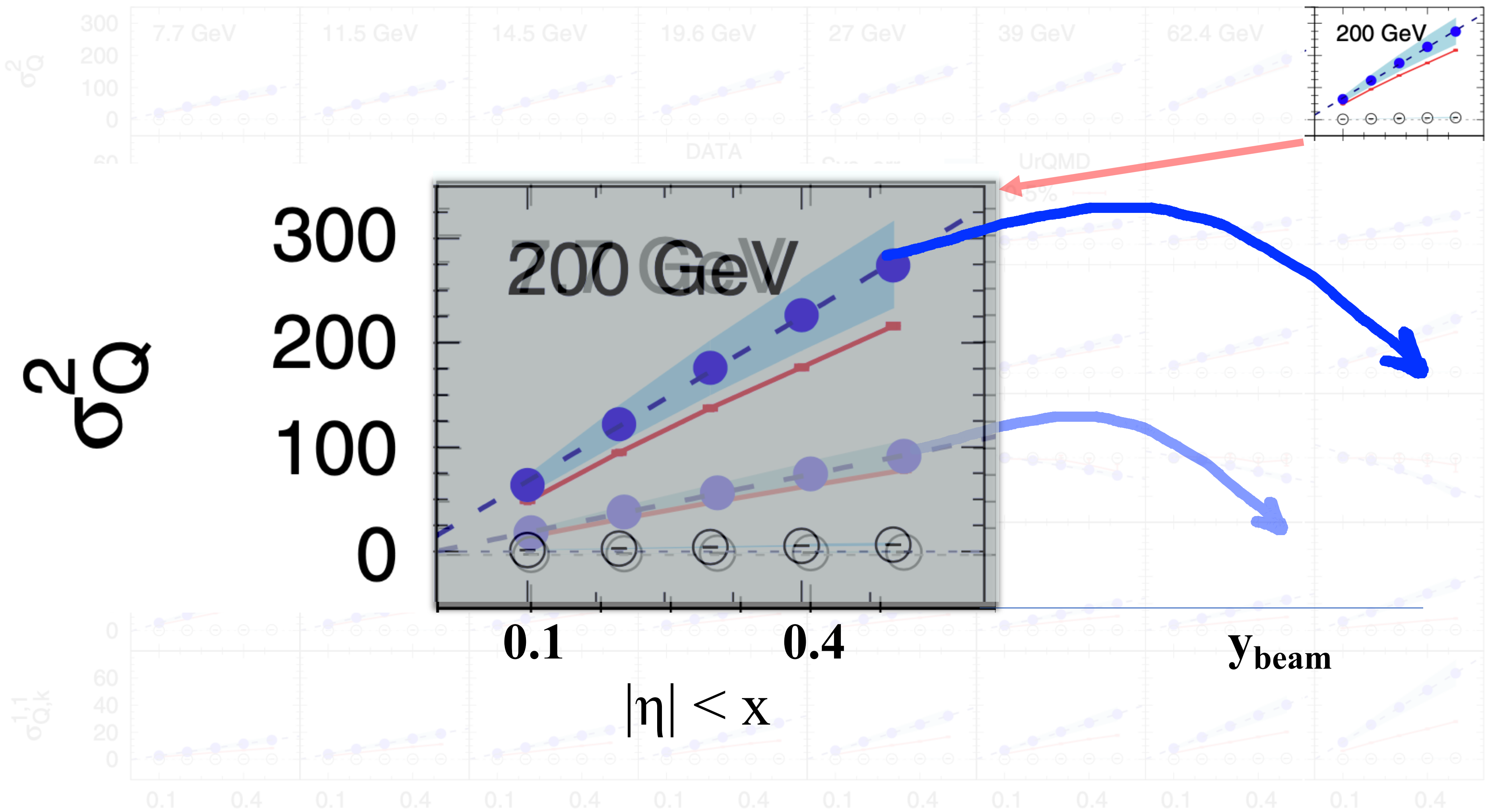
Now let’s see how these fluctuations evolve with η-window.

STAR Off-diagonal: PRC 100, 014902 (2019)

Net-charge for different η-windows

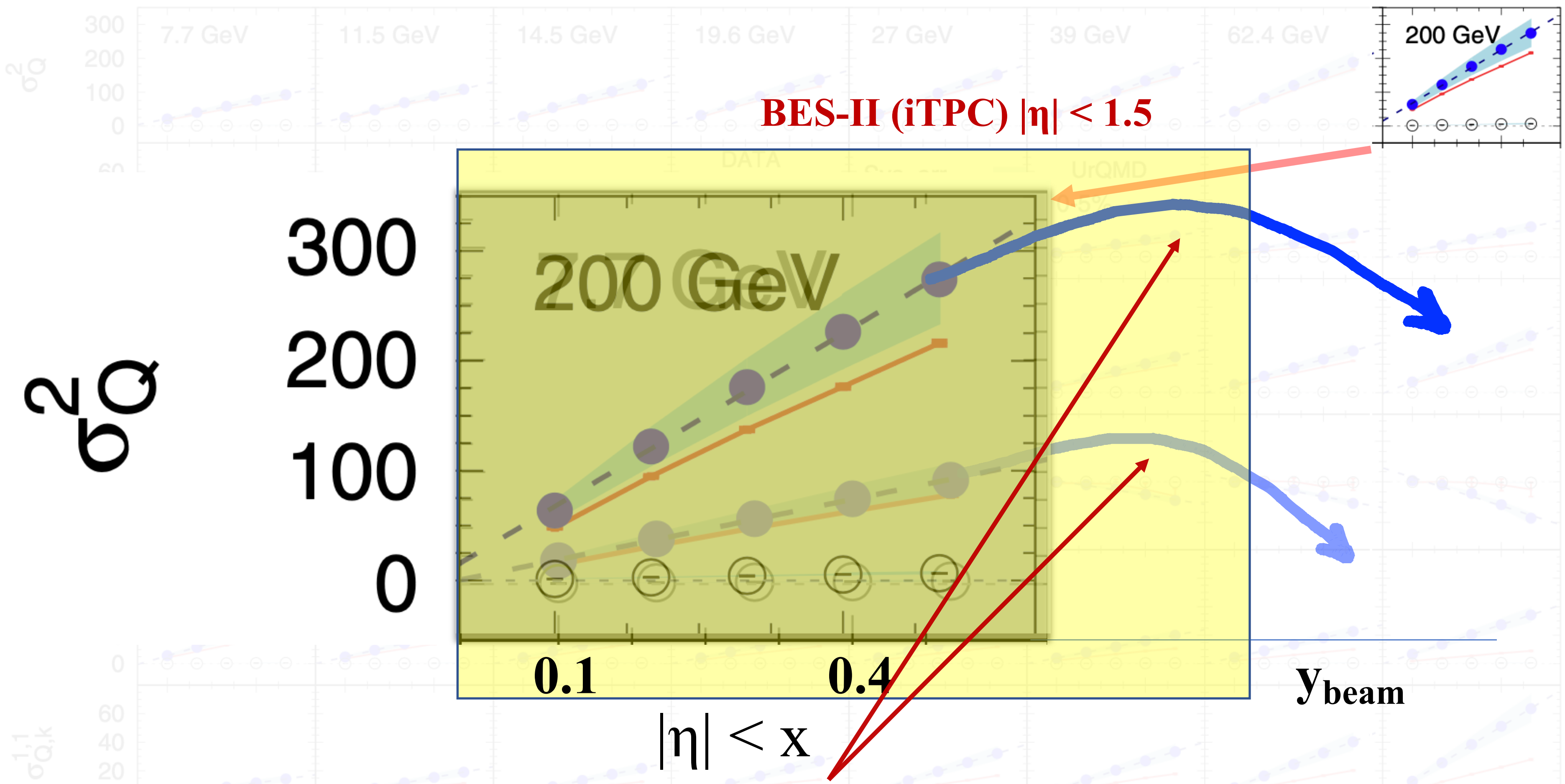


Net-charge for different η-windows



Other higher cumulants can be calculated.

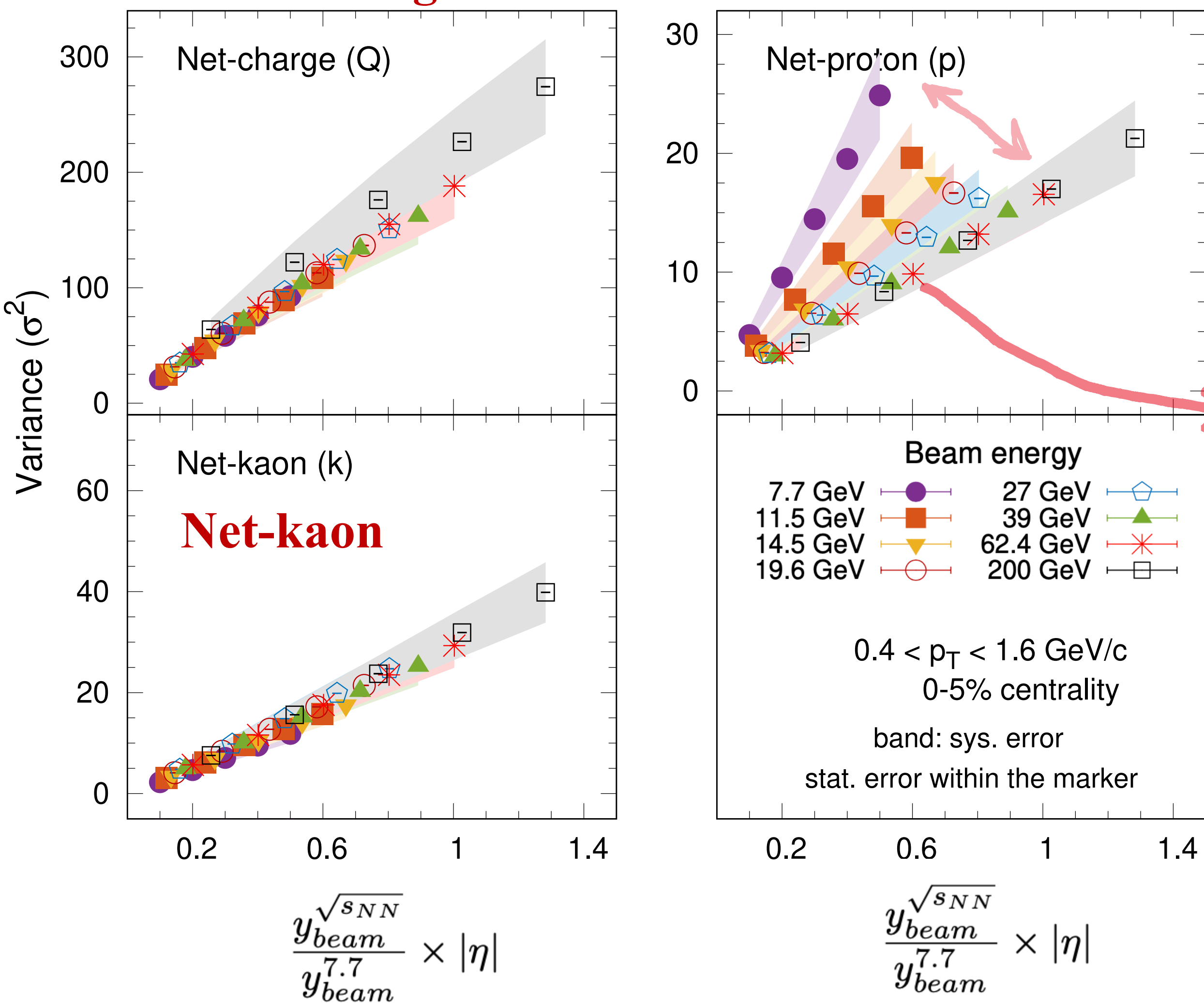
Net-charge for different η-windows



- Then, we need to ask what is the optimum range of η-window we are interested in?
- What physics message we really need to know?
- At what phase space, lattice results and experimental data are comparable?

Scaling of 2nd order cumulants

Net-charge **Net-proton**



$$\boxed{\frac{y_{beam}^{\sqrt{s_{NN}}}}{y_{beam}^{7.7}} \times |\eta| \sim \text{constant}}$$

It seems net-kaon and net-charge follow some scaling; not net-proton.

Why this scaling doesn't work for net-proton?

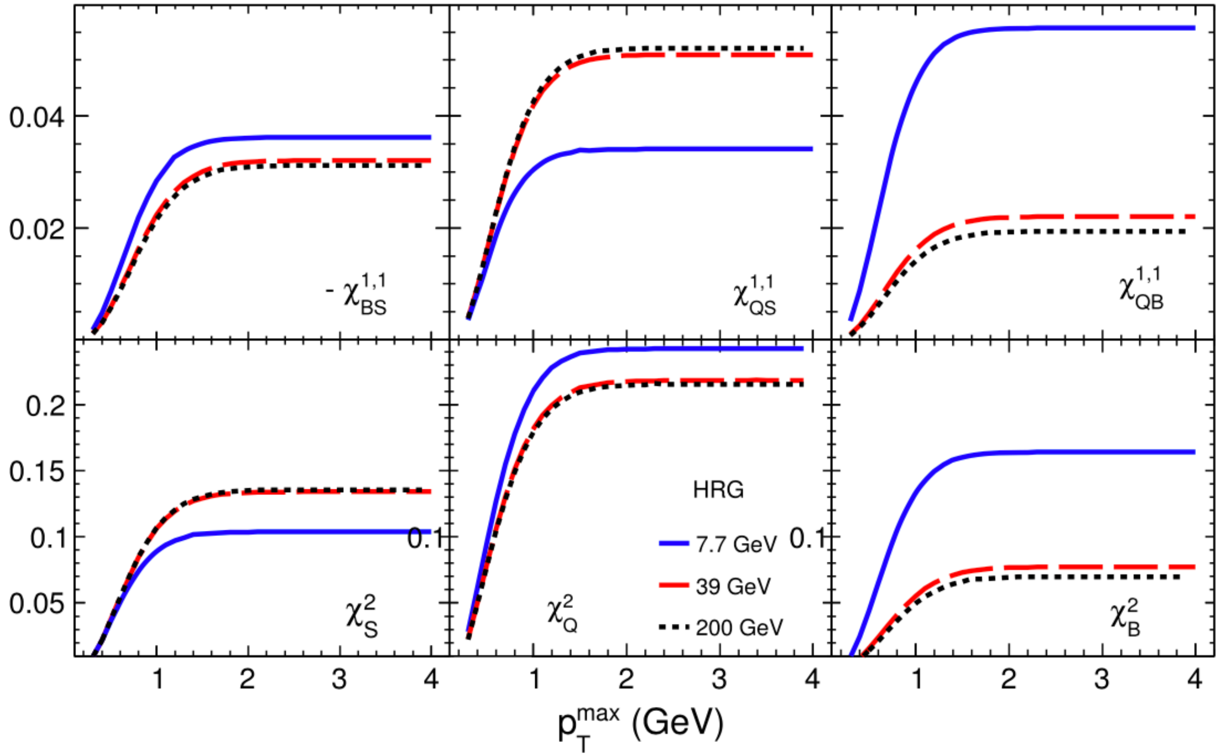
Probably, produced particles are properly scaled, like net-kaon and net-charge

Evolution of fluctuations as a function of p_T

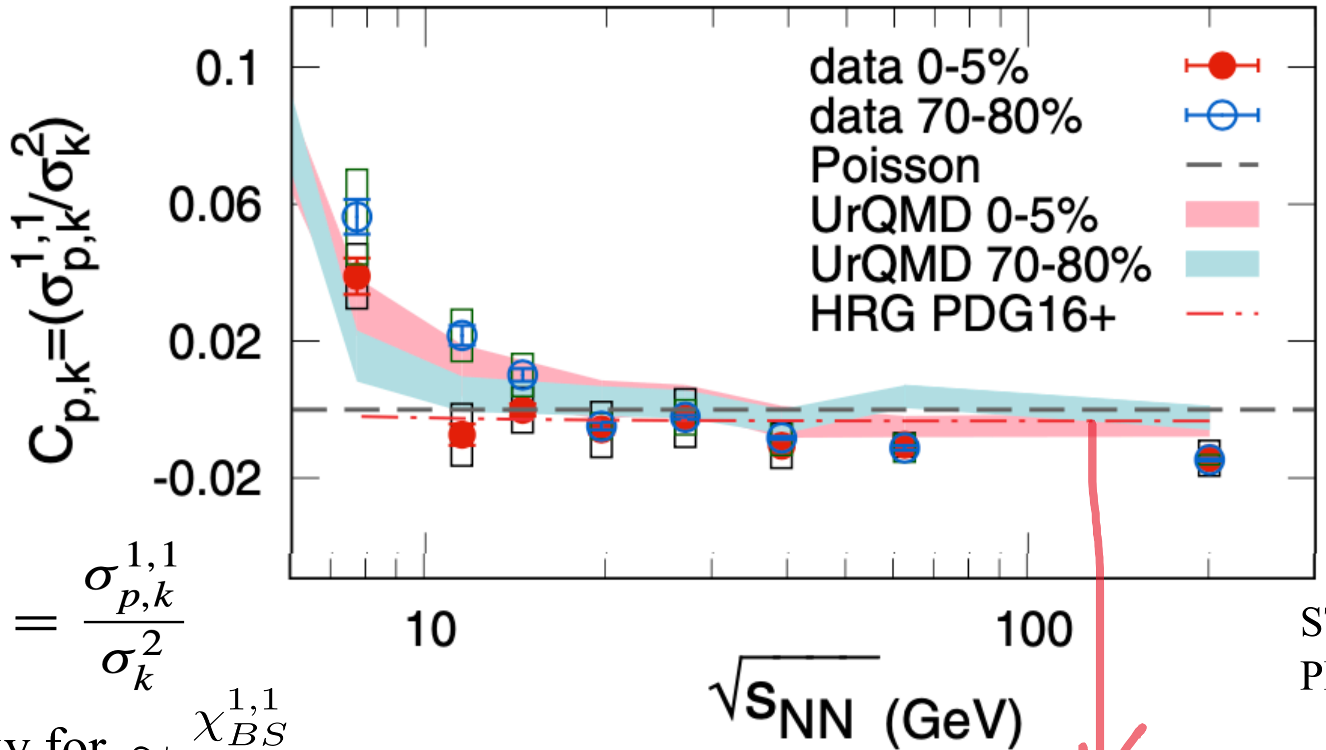
- E-b-e fluctuations grow with p_T
- Contribution from the thermal and non-thermal components
- Need to understand how non-thermal component contributes to cumulant analysis

Hadron Resonance Gas Model

A Chatterjee et al, JPG 43 (2016) 125103



Off-diagonal cumulant ratio



$$C_{p,k} = \frac{\sigma^{1,1}_{p,k}}{\sigma^2_k}$$

Proxy for $\sim \dfrac{\chi^{1,1}_{BS}}{\chi^2_S}$

STAR Off-diagonal:
PRC 100, 014902 (2019)

UrQMD: consistent with zero or positive
39-200 GeV

Hadronic decay processes can not explain these results at high energy:

$$\Lambda(1520) \rightarrow p + K^-$$

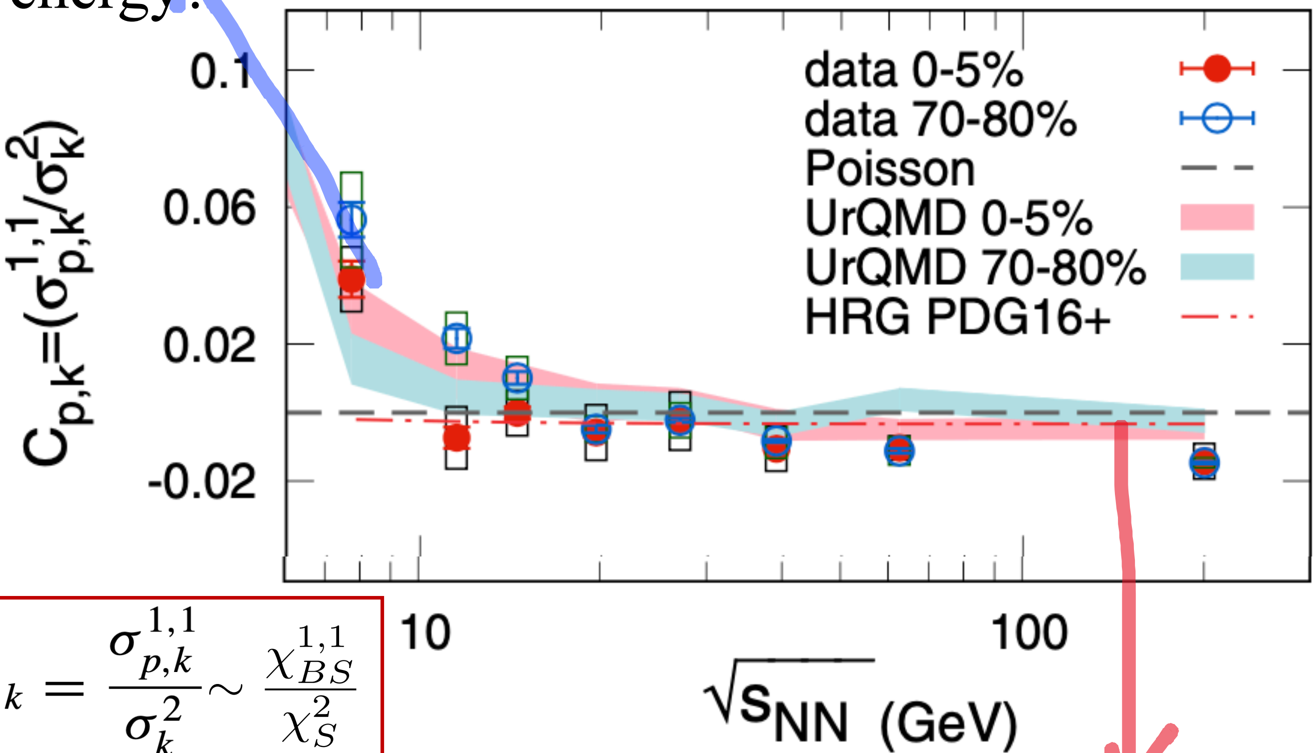
$$pp \rightarrow p\Lambda(1115)K^+$$

$$\Lambda(1115) \rightarrow p + \pi^-$$

Negative correlation between p-k at high energy could come from QGP phase.

Off-diagonal cumulant ratio

Low energy?



LHC energy?

STAR Off-diagonal:
PRC 100, 014902 (2019)

$$\boxed{C_{p,k} = \frac{\sigma^{1,1}_{p,k}}{\sigma^2_k} \sim \frac{\chi^{1,1}_{BS}}{\chi^2_S}}$$

Hadronic processes can't explain at all energy :

$$\Lambda(1520) \rightarrow p + K^-$$

$$pp \rightarrow p\Lambda(1115)K^+$$

$$\Lambda(1115) \rightarrow p + \pi^-$$

UrQMD: consistent with zero or positive 39-200 GeV
HRG is consistent with zero.

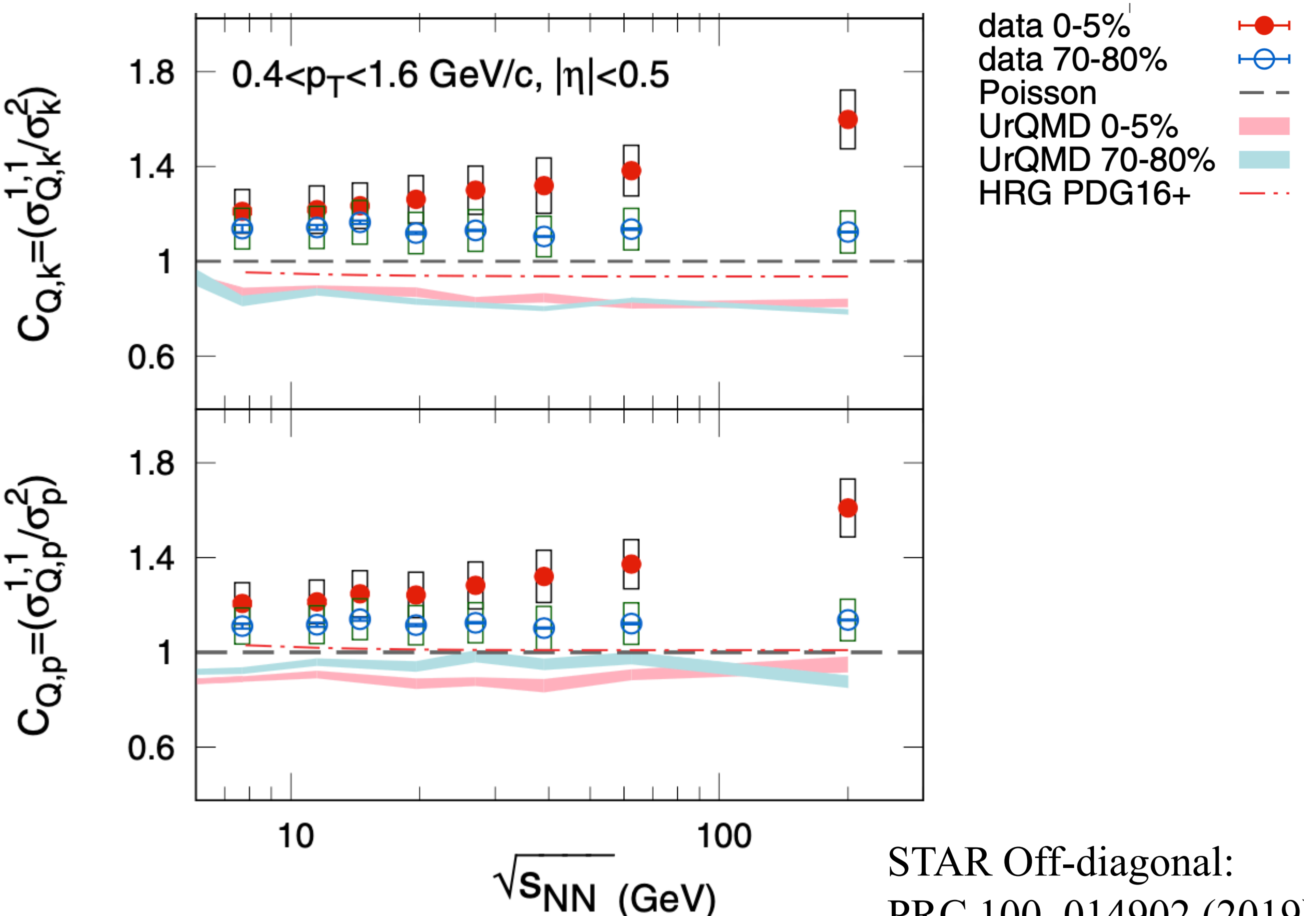
Negative correlation between p-k at high energy could come from QGP phase.

Off-diagonal cumulant ratio

$$C_{Q,k} = \boxed{\frac{\sigma^{1,1}_{Q,k}}{\sigma^2_k}} \sim \frac{\chi^{1,1}_{QS}}{\chi^2_S}$$

$$C_{Q,p} = \boxed{\frac{\sigma^{1,1}_{Q,p}}{\sigma^2_p}} \sim \frac{\chi^{1,1}_{QB}}{\chi^2_B}$$

Excess correlation between Q-p and Q-k



STAR Off-diagonal:
PRC 100, 014902 (2019)

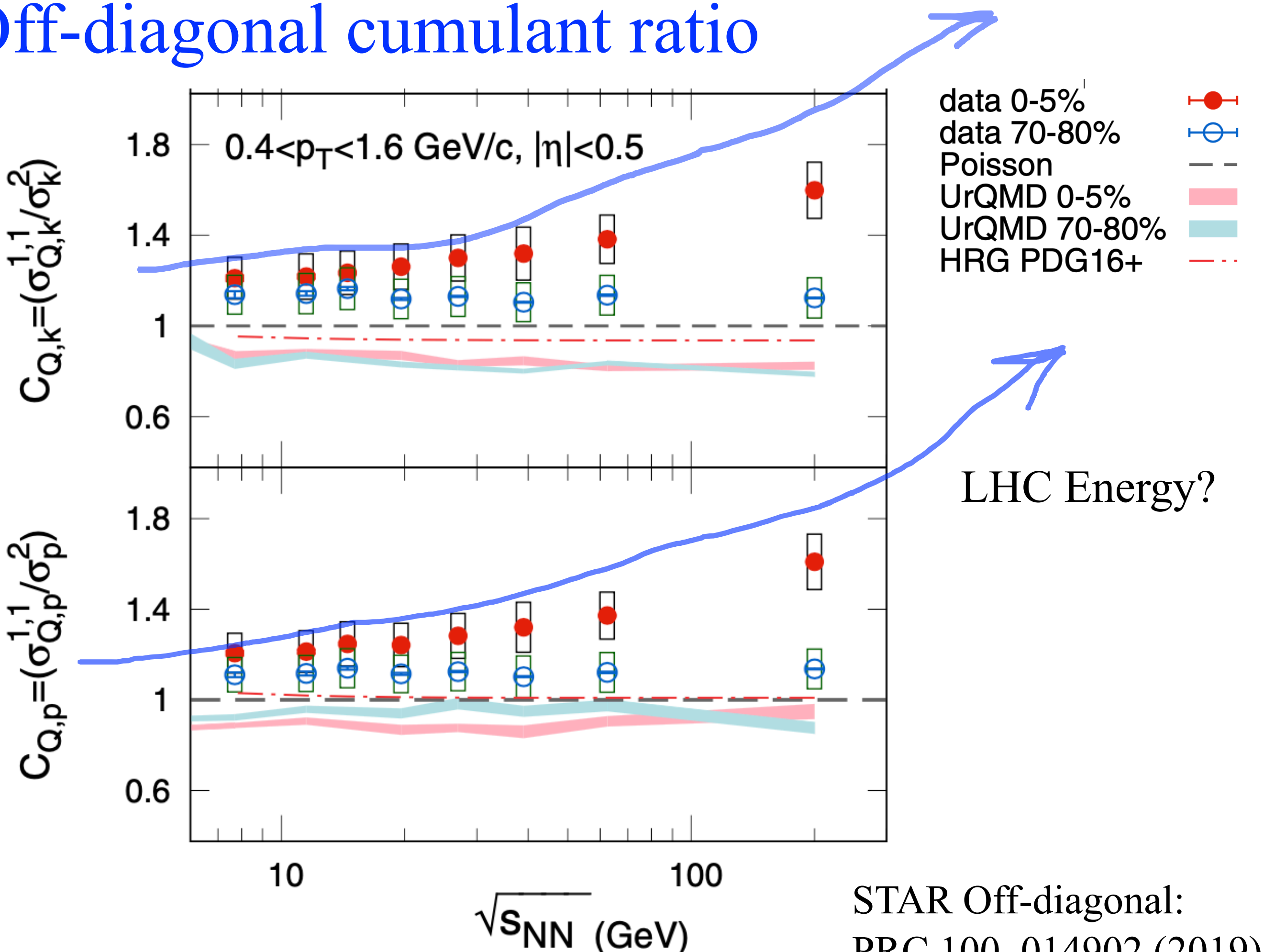
- The excess correlation increase with energy
- Non of the hadronic models explain the data
- A model calculation that includes QGP may explain this rise of correlation between Q-p and Q-k with energy.

Off-diagonal cumulant ratio

$$C_{Q,k} = \boxed{\frac{\sigma^{1,1}_{Q,k}}{\sigma^2_k}} \sim \frac{\chi^{1,1}_{QS}}{\chi^2_S}$$

$$C_{Q,p} = \boxed{\frac{\sigma^{1,1}_{Q,p}}{\sigma^2_p}} \sim \frac{\chi^{1,1}_{QB}}{\chi^2_B}$$

Excess correlation between Q-p and Q-k



STAR Off-diagonal: PRC 100, 014902 (2019)

- The excess correlation increase with energy
- Non of the hadronic models explain the data
- A model calculation that includes QGP may explain this rise of correlation between Q-p and Q-k with energy.

Higher order off-diagonal cumulants

$$C(X_1, X_2, \ldots, X_n) = \sum_{\pi} (|\pi|-1)!(-1)^{(|\pi|-1)} \prod_{C\in\pi, |C|\geq 2} E\left(\prod_{i\in C} \delta X_i\right)$$

$$C(S,S) = C_2^S = \left\langle (\delta S)^2 \right\rangle$$

$$C(B,S) = C_{11}^{BS} = \langle \delta B \delta S \rangle$$

$$C(S,S,S,S) = C_4^S = \left\langle (\delta S)^4 \right\rangle - 3\left\langle (\delta S)^2 \right\rangle^2$$

$$C(B,S,S,S) = C_{13}^{BS} = \left\langle \delta B (\delta S)^3 \right\rangle - 3\langle \delta B \delta S \rangle \left\langle (\delta S)^2 \right\rangle$$

$$C(B,B,S,S) = C_{22}^{BS} = \left\langle (\delta S)^2 (\delta S)^2 \right\rangle - 2\langle \delta B \delta S \rangle^2 - \left\langle (\delta B)^2 \right\rangle \left\langle (\delta S)^2 \right\rangle$$

$$C(B,B,B,S) = C_{31}^{BS} = \left\langle (\delta B)^3 \delta S \right\rangle - 3\langle \delta B \delta S \rangle \left\langle (\delta B)^2 \right\rangle$$

$$C_{\mathrm{mn}}^{BS} = VT^3 \chi_{mn}^{BS}(t,\mu)$$

$$C_{pk}^{21}, C_{pk}^{12}$$
$$C_{Qp}^{12}, C_{QP}^{21}$$
$$C_{Qk}^{12}, C_{Qk}^{21}$$
$$C_{pk}^{31}, C_{Qp}^{13}$$
$$\cdots$$

- Plan to estimate these higher order off-diagonal cumulants.
- But it has computational challenges and high statistics hungry observables.

Summary and outlook

- Choice of phase-space window for e-b-e fluctuation measurement is crucial to understand physics
  - η- and p_T-window dependence

- Now, RHIC provides both diagonal and off-diagonal cumulants information in a differential way to constrain various models, particularly FO T and μ_B.

- BES-II dataset with recent detector upgrade can provide deeper understanding of event-by-event fluctuation at large phase space
- We need to be careful while comparing with experimental data with theory

Thank you!
Stay tuned…