Pion/Kaon Decay in Very Special Relativity

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Very Special Relativity (VSR)

- **The speed of light is a universal constant, same in all frames of reference**
- This does not really imply that the fundamental symmetry group is the Lorentz group
- It only leads to a subgroup, which may be HOM(2), SIM(2) (Cohen and Glashow 2006)

SIM(2)

- **SIM(2): T_1, T_2, J_z, K_z**

$$T_1 = K_x - J_y,\quad T_2 = K_y + J_x$$

J: rotations

K: boosts

$[T_1, T_2] = 0,\qquad [J_z, K_z] = 0$

$[J_z, T_1] = iT_2\quad ,\qquad [J_z, T_2] = -iT_1$

$[K_z, T_1] = iT_1\quad ,\qquad [K_z, T_2] = iT_2$

Translations are preserved in VSR

Transformation to rest frame

- **We can make a HOM(2) (T_1, T_2, K_z) transformation to the rest frame of a particle**
- Several consequences of Lorentz invariance, such as,

Law of velocity addition

time dilation

universal maximal velocity

remain preserved

Relationship with P, T, CP (CT)

- **VSR is possible as long as P, T, CP (or CT) are all violated**
- **If any one of these discrete symmetries are imposed we get back the full Lorentz group**

Dispersion Relations

- Particle Dispersion relations do not get modified **$E^2 = P^2 + M^2$**
- **This is in contrast to other models of Lorentz violation, perhaps arising from quantum gravity effects**
- **Hence many constraints based on such effects do not apply**

Effective Lagrangian approach to VSR

- **We assume that Lorentz violating effects are small**
- **We can express these in terms of effective interactions, added to the Standard Model**

VSR Effective Lagrangian

$$L = \overline{\psi}_L \left(iD_\mu \gamma^\mu - m + im_1^2 \frac{n_\mu \gamma^\mu}{n \cdot D} \right) \psi_L + (L \to R)$$

n^μ = (1,0,0,1) ; D = gauge covariant derivative

This is invariant under VSR transformations. Either n^μ does not change or the change cancels out

The term is non-local. This appears to be necessary

Leads to mass of fermions as well as interactions with gauge bosons

VSR Effective Lagrangian

- Similar VSR terms are also present in gauge and scalar sector

Cohen and Glashow 2006
Dunn and Mehen 2006
Alfaro et al 2015

VSR Mass term

The nonlocal term provides a mass term for neutrinos.

All fermions get a correction term to mass

On-shell: $P^2 = m^2 + m_1^2$

VSR Mass term

Also leads to different masses for left and right handed particles.

Stringent limits on such left-right mass difference for electron.

We impose C invariance to eliminate this mass difference.

Requires fine tuning

Dunn and Mehen 2006
Fan, Goldberger, Skiba 2006

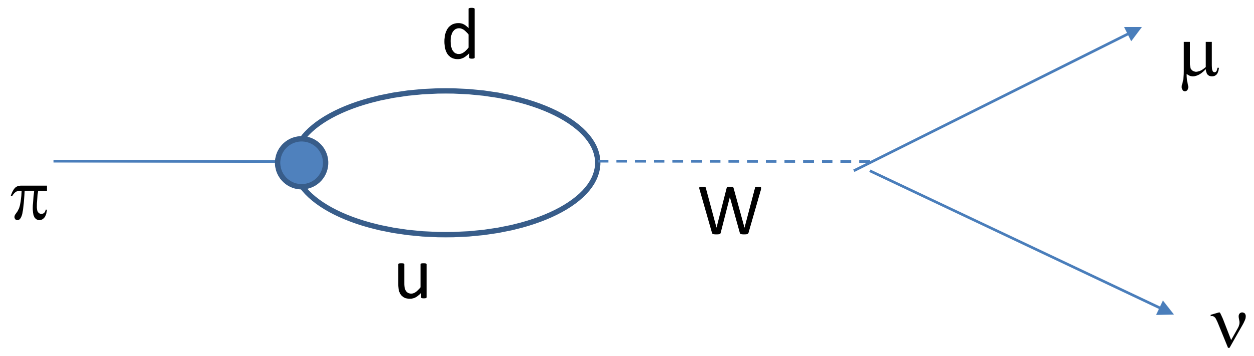
Pion/Kaon Decay

- We consider charged pion/kaon decay within VSR in order to study this phenomena
- Similar effects are expected in all processes

Pion Decay

The hadronic current is proportional to q^{μ} , the pion momentum

$$L_{\pi} = V_{ud} \frac{G_F}{\sqrt{2}} f_{\pi} \partial_{\mu} \pi^{-} \; \overline{\psi}_l \gamma^{\mu} (1 - \gamma^5) \psi_{\nu} + h.c.$$



Leads to standard decay rate

Lorentz Violating VSR term

In VSR the hadronic current can also have an additional piece proportional to n^μ

$$L_{VSR} = g\left(\frac{n_\mu}{n\cdot\partial}\pi^-\right)\overline{\psi}_l\gamma^\mu(1-\gamma^5)\psi_\nu \text{ + h.c.}$$

Such an effective term arises if we assume a VSR quark mass term

Decay Amplitude

- **The dominant contribution comes from the interference between the VSR and the standard term**

$$iM = \cdots + g\ \frac{n_\mu}{n\cdot q}\,\bar{u}(p)\gamma^{\mu}(1-\gamma^{5})v(k)$$

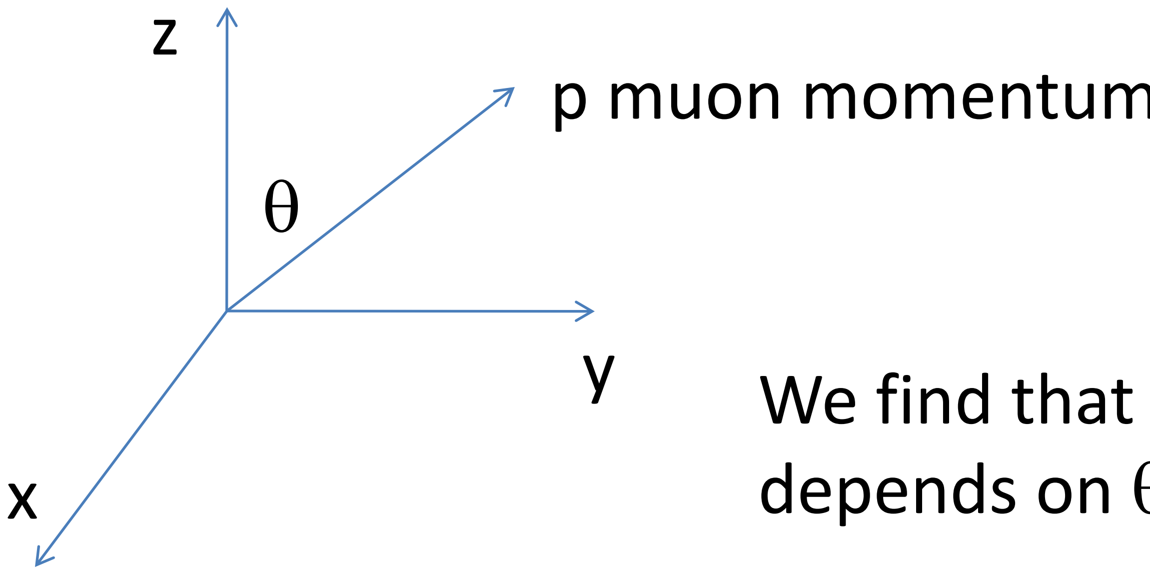
q= pion momentum
p = muon momentum
k = neutrino momentum

Pion Decay in rest frame

- We cannot make a Lorentz transformation in order to go to the rest frame of pion
- However we do this by making a HOM(2) transformation
- Under this transformation n^{μ} changes by overall constant which cancels out in the amplitude

Pion Decay in rest frame

- We choose a frame such that $n^\mu = (1,0,0,1)$ up to an irrelevant constant



We find that $d\Gamma/d\Omega$ depends on θ

Limit on g

- We first impose a limit on g by demanding that the correction due to VSR is smaller than the error in the total decay rate
- $g < g_0, \quad g_0 = 2.1 \times 10^{-12}\ \text{GeV}$

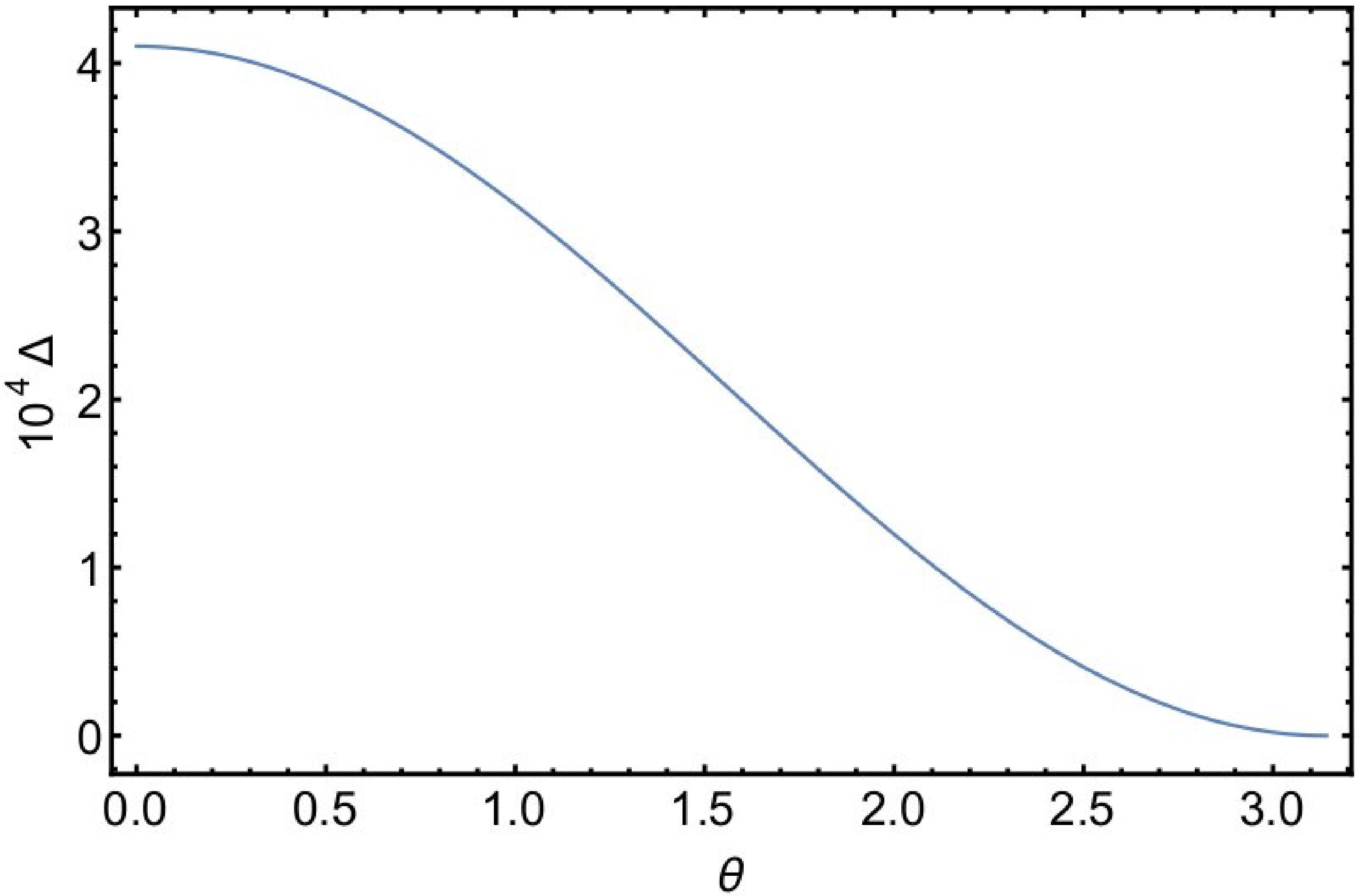
Anisotropic muon distribution

- The final state muon distribution is not isotropic, depends on θ

$$\Delta = \frac{\frac{d\Gamma}{d\Omega}(g \neq 0) - \frac{d\Gamma}{d\Omega}(g=0)}{\frac{d\Gamma}{d\Omega}(g=0)}$$

$$\Delta = \frac{2\sqrt{2}\ g}{f_\pi m_\pi^2 G_f |V_{ud}|}\ (1 + \cos\theta)$$

Anisotropic muon distribution



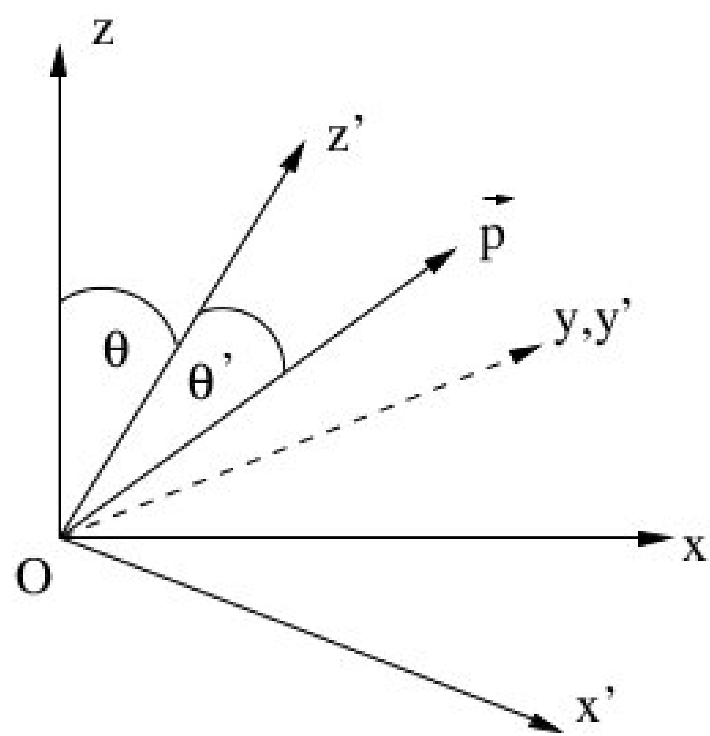
Pion decay in laboratory frame

- We consider a beam of pions moving along z' axis with momentum q

xyz, frame S: In this frame
$n^{\mu} = (1,0,0,1)$

x'y'z', frame S': standard lab frame

Use rotational invariance about z to align x' in x-z plane



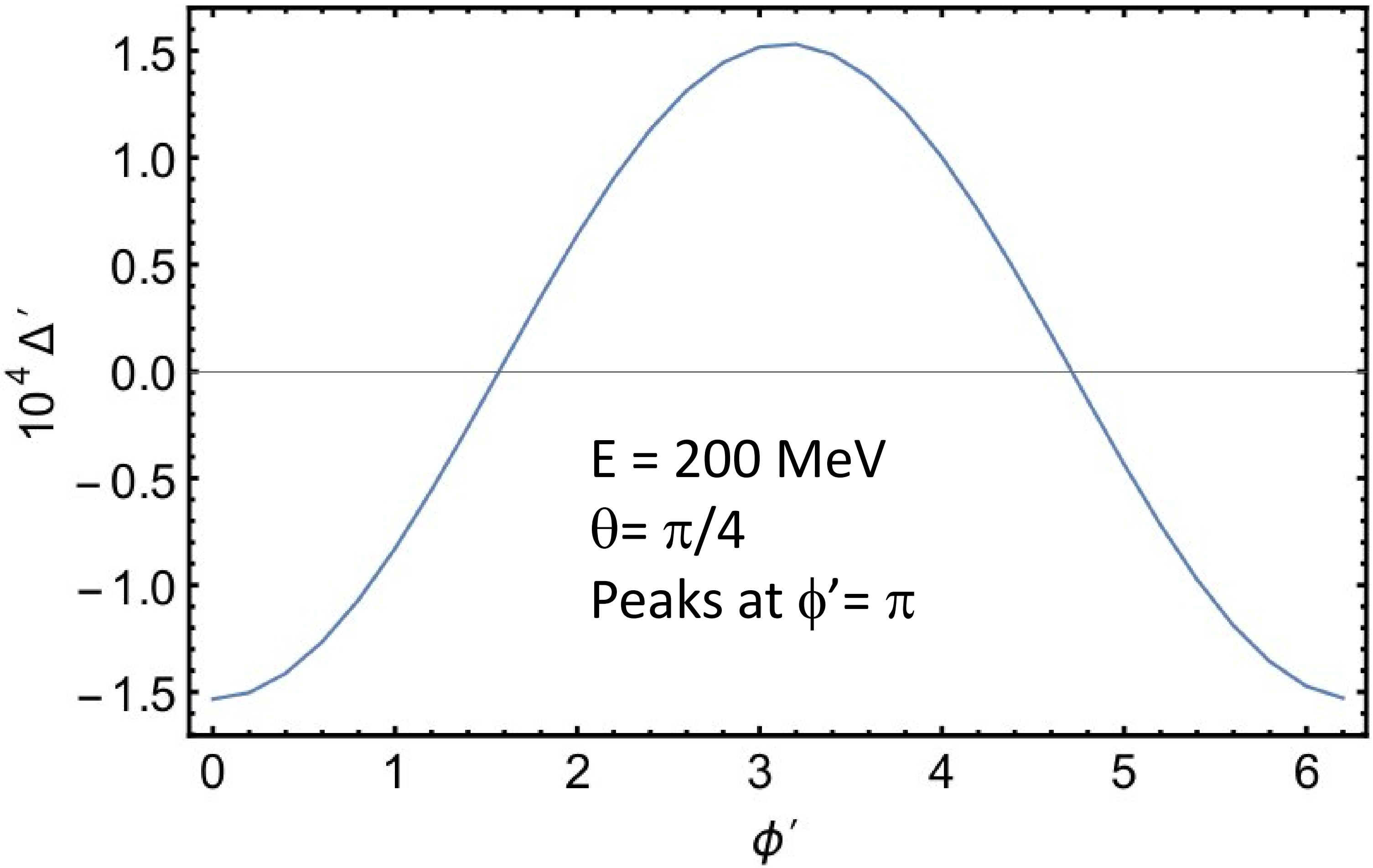
Azimuthal dependence In lab frame

- Final state muon distribution picks up an azimuthal ϕ' dependence in lab frame
- Define

$$\Delta' = \frac{(d\Gamma/d\phi') - \Gamma_{avg}}{\Gamma_{avg}}$$

$$\Gamma_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{d\Gamma}{d\phi'}\, d\phi'$$

Azimuthal dependence

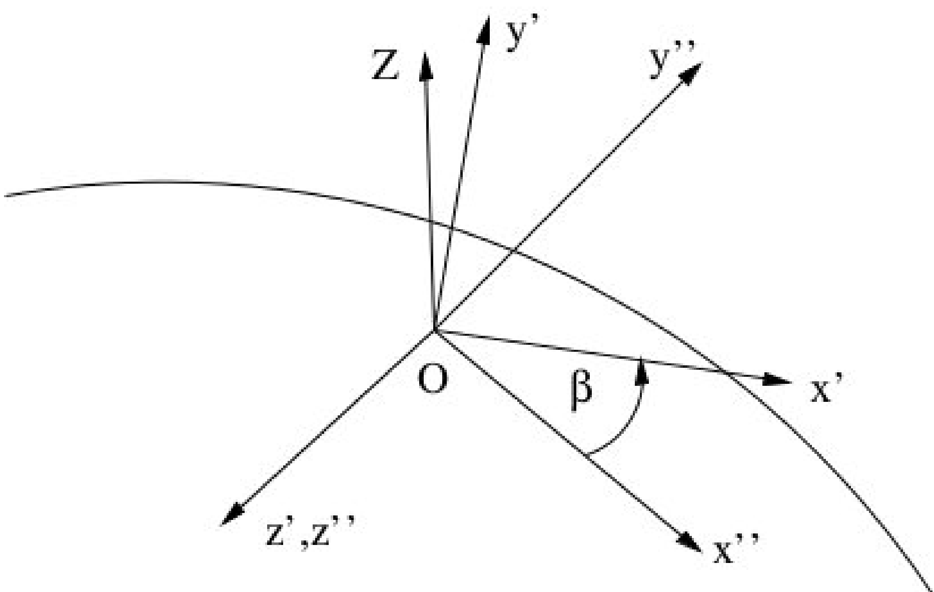


Daily Variation

- The angle θ between preferred axis and beam changes with time due to Earth's rotation
- $\Rightarrow$periodic variation of $d\Gamma/d\Omega$ with a time period of 1 sidereal day
- Sidereal day is a day relative to fixed stars rather than the Sun. It is a little shorter than solar day

Coordinates

Assume observer at latitude λ



z', z'' : Beam axis

x"y"z": Lab coordinates

y" = local normal

x'y'z': also lab coordinates

with x' in z-z' plane

Z = rotation axis

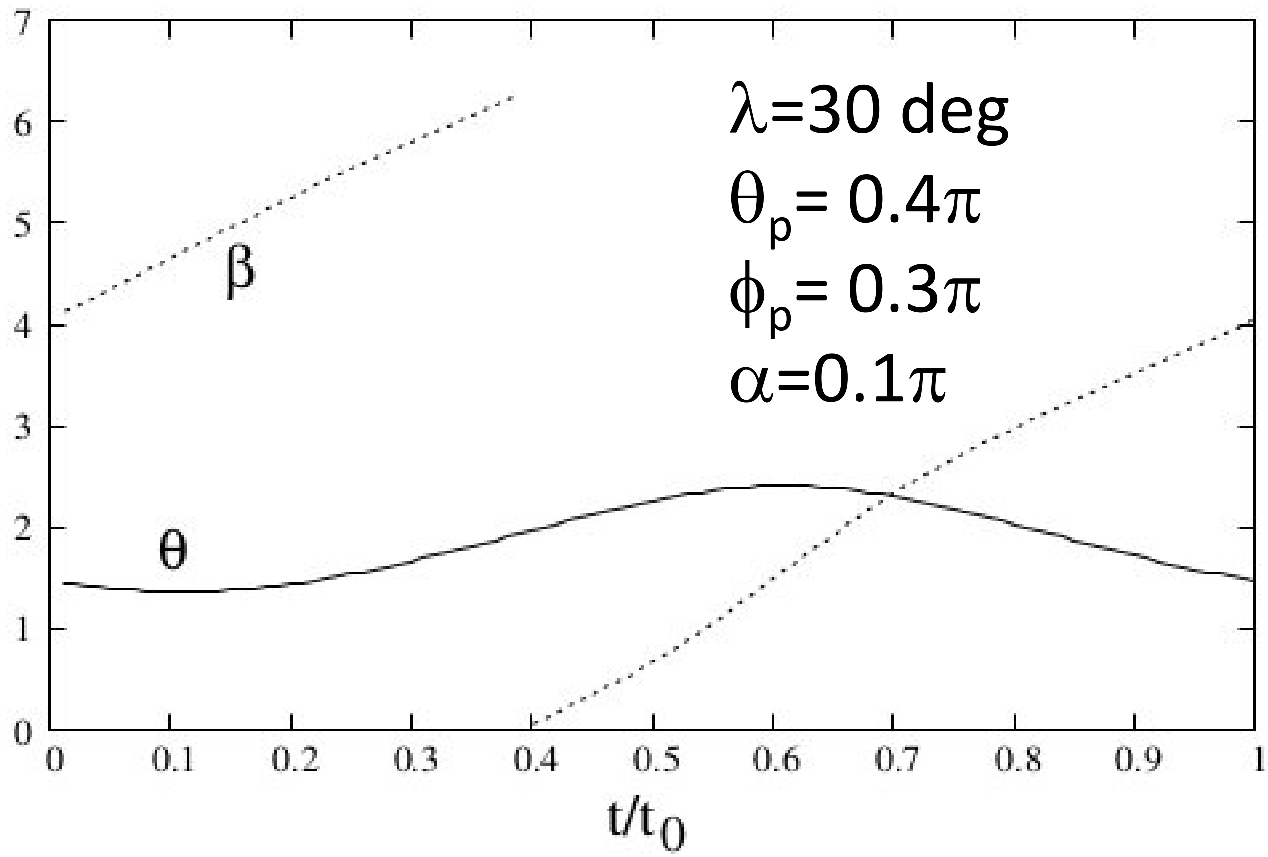
z= preferred axis

$$\hat{z} = \cos\theta\,\widehat{z''} - \sin\theta\left(\cos\beta\widehat{x''} + \sin\beta\widehat{y''}\right)$$

Daily Variation

- **Due to change in θ the magnitude of $d\Gamma/d\Omega$ will change periodically with time**
- **Due to change in orientation of the beam relative to preferred axis, the peak position in the azimuthal distribution would also show a correlated change with time**

Time dependence of θ and β



t_0 = 1 sidereal day

Experimental Proposal

- We propose to test the angular dependences and daily variation experimentally
- These will arise in many processes, both involving decay as well as collision

Experimental Proposal

- We divide a sidereal day into several bins
- For each time bin, divide data into bins in azimuthal angle
- Collect data in each bin over many days
- Determine the azimuthal dependence of final state particles for each time bin
- The peak position should show time dependence with a period of 1 sidereal day
- Correlated with this the amplitude should also vary with the same period

Experimental Proposal

- Furthermore polar angle dependence should also time dependence with a period of a sidereal day

Conclusions

- VSR is an interesting theoretical proposal
- Here we have shown that it leads to time and azimuthal angle dependence of final state particles which can be tested experimentally

Collaborators

- Alekha Nayak
- And earlier: Ravindra Verma
- Subhadip Mitra

Azimuthal distribution

- In x'y'z' system the peak in ϕ' distribution occurs at π
- the lab x"y"z" is related to x'y'z' system by a rotation $-\beta$ about the common z', z" axis
- Hence the peak in this system occurs at $\phi''=\pi-\beta$
- We need the time dependence of θ and β

Time dependence of θ and β

- We use the astronomical equatorial coordinate system XYZ
- Relate the preferred coordinates xyz to XYZ, assume preferred axis at θ_p , ϕ_p
- Also relate lab coordinates x”y”z” to XYZ. This gives us the variation of lab coordinates with time
- Hence we can find time dependence of θ and β