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Large N limits and the Golden ratio

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Workshop on QCD in the non-perturbative regime, TIFR

Large N gauge theories

$SU(N)$

$N \to \infty$

$\lambda = Ng^2$

't Hooft coupling

- Connection with AdS/CFT & string theory
- Non-perturbative effects $\longrightarrow$ Lattice

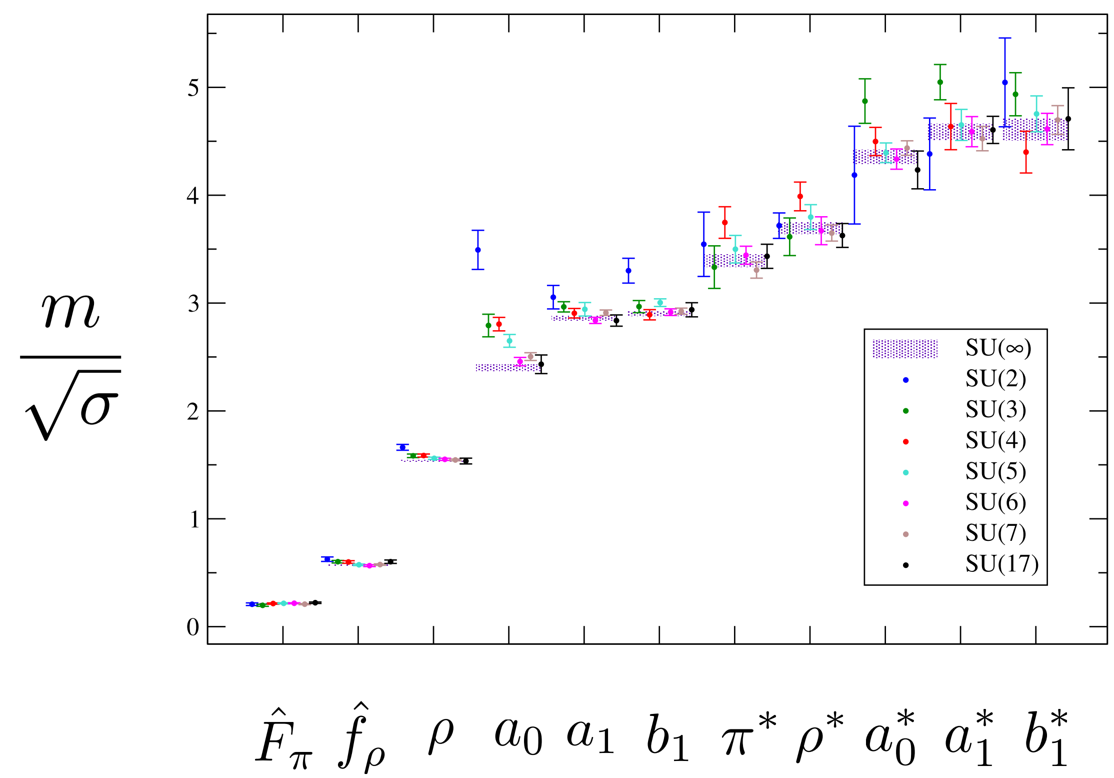
Large N & Lattice - the standard approach

- Lattice simulations at various values of N
- Large N extrapolation + continuum limit

Meson spectrum, decay constans

[Bali et al. arXiv:1304.4437]

Limited values of N



Review - Lucini & Panero 2013

This talk - an alternative approach

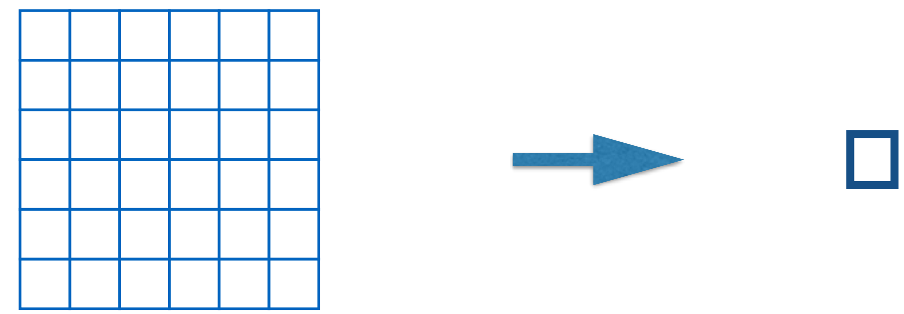
Volume reduction [Eguchi & Kawai 1982]

Revival - several working prescriptions

- Efficient tool for lattice large N studies
- Allows very large N (N=1369, Gonzalez-Arroyo & Okawa)
- Theoretically appealing - new avenues @ lattice

Eguchi-Kawai reduction

U(N) gauge theory at large N



is volume independent

- Exact result based on the equality of the loop equations **provided**

$$\boxed{\mathrm{Tr}\,(\longrightarrow) = 0}$$

Preserved center symmetry Z_N^d

But with **periodic** boundary conditions & **d=4** → Z_N^4 breaking by quantum fluctuations [Bhanot, Heller & Neuberger]

Does the PT vacuum respect the symmetry?

Yang-Mills SU(N) finite temperature

One-loop effective potential

Polyakov loop

$$V_{\text{eff}} = - \frac{2}{\pi^2 l^4} \sum_{n=1}^{\infty} \frac{1}{n^4} |\text{Tr}\Omega^n|^2$$

S[1] period

Minima have $\text{Tr}\Omega \in Z_N$

Z_N breaking by quantum fluctuations

T^4 with PBC [Bhanot, Heller & Neuberger]

[Gonzalez-Arroyo, Jurkiewickz & Korthals-Altes]

Twisted EK

[González-Arroyo, Okawa]

Use twisted boundary conditions **TEK**

- Working prescription - large N lattice simulations
- Continuum- Precursor of Non-commutative gauge theories

  First formulation of NC Feynman rules

  [González-Arroyo & Korthals-Altes]

Other working prescriptions for EK

- Continuum reduction PBC $La > 1/T_c$
  [Kiskis, Narayanan & Neuberger]

- Trace deformations
  [Unsal & Yaffe]

- QCD(Adj) - add massless adjoint fermions with PBC
  [Kotvun, Unsal &Yaffe]

Twisted EK

[González-Arroyo, Okawa]

Pure Yang-Mills on **R^d x T^{2n}** with twisted boundary conditions

$$A_\mu(x + l\hat{\nu}) = \Gamma_\nu A_\mu(x) \Gamma_\nu^\dagger$$

$$\Gamma_\mu \Gamma_\nu = Z_{\mu\nu} \Gamma_\nu \Gamma_\mu$$

't Hooft flux

twist

$$\boxed{Z_{\mu\nu} = \exp\left\{ i\epsilon_{\mu\nu} \frac{2\pi k}{\hat{N}} \right\}}$$

$\hat{N} = N$ **T^2 x R^d**

$\hat{N} = \sqrt{N}$ **T^4**

Does the PT vacuum respect the symmetry?

For the vacuum configuration all loops winding less than $\hat{N}$ in each direction are traceless for:

$$k \;\&\; \hat{N} \quad \text{coprime}$$

$\boxed{Z_{\hat{N}}^{2n} \text{ preserved at zeroth PT order}}$

Does the PT vacuum respect the symmetry?

For the vacuum configuration all loops winding less than $\hat{N}$ in each direction are traceless for:

k & $\hat{N}$ coprime

$Z^{2n}_{\hat{N}}$ **preserv**

The choice of k is critical

k=1 does not work

(Ishikawa&Okawa, Teper&Vairinhos, Azeyanagi et al.)

k scaled with N (see later)

The result is an increased effective volume

Momentum quantization along compact cycles

$$p = \frac{2\pi n}{l_{\text{eff}}}$$

$$\boxed{l_{\text{eff}} = \hat{N} l}$$

$$\hat{N} = N \quad \textbf{T}^{\mathbf{2}}\ \mathbf{x}\ \textbf{R}^{\mathbf{d}}$$

$$\hat{N} = \sqrt{N} \quad \textbf{T}^{\mathbf{4}}$$

The game

$$\boxed{l_{\mathrm{eff}} = \hat{N} l}$$

$$\boxed{x = M \hat{N} l}$$

$l_{\mathrm{eff}} = 1/M$

$x = 1$

Yang-Mills d=4 $\qquad M = \Lambda_{\mathrm{QCD}}$

The game $\begin{aligned} x &= 1 \\ l_{\mathrm{eff}} &= 1/M \end{aligned}$

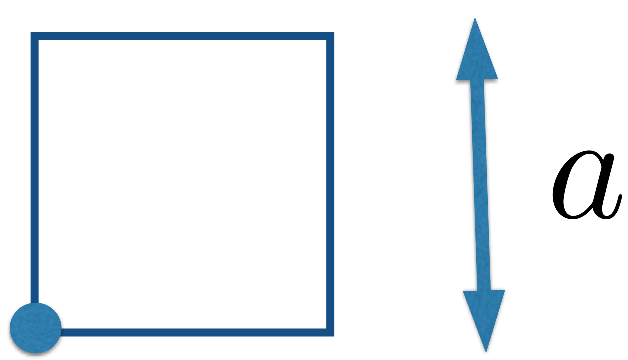
$$\boxed{x = M\hat{N}l}$$

Large N limit

thermodynamic limit

Eguchi Kawai reduction

- **TEK** $l_{\mathrm{eff}} = a\hat{N}$



Thermodynamic limit

at fixed volume

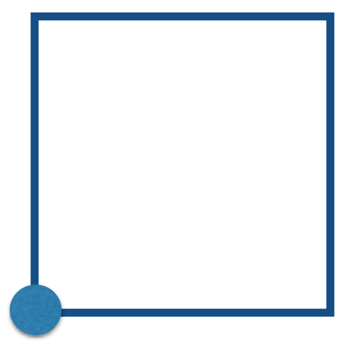
N to infinity

Twisted Eguchi Kawai Reduction on **T**^{4}

't Hooft limit -thermodynamic limit

non-planar suppression

Finite N corrections amount to finite volume effects



TEK $$l_{\text{eff}} = a\hat{N}$$

Implements a lattice with $\hat{N}^4$ sites → play the standard game

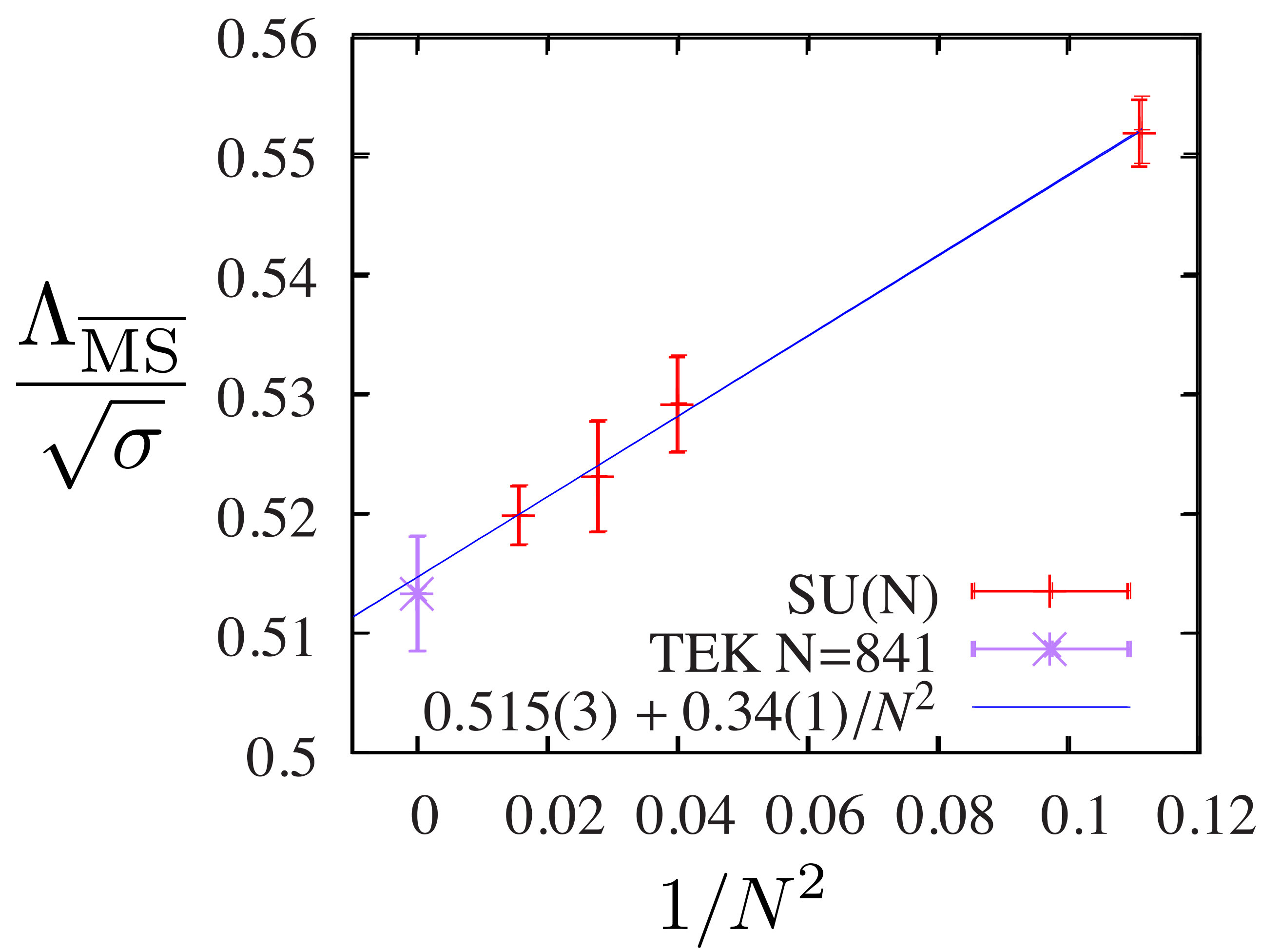
Twisted Eguchi Kawai Reduction on $\mathbf{T^4}$

$L = 1$

Pure Yang-Mills

N=841

String tension



[González-Arroyo & Okawa]

Twisted Eguchi Kawai Reduction on $\mathbf{T^4}$

Meson spectrum

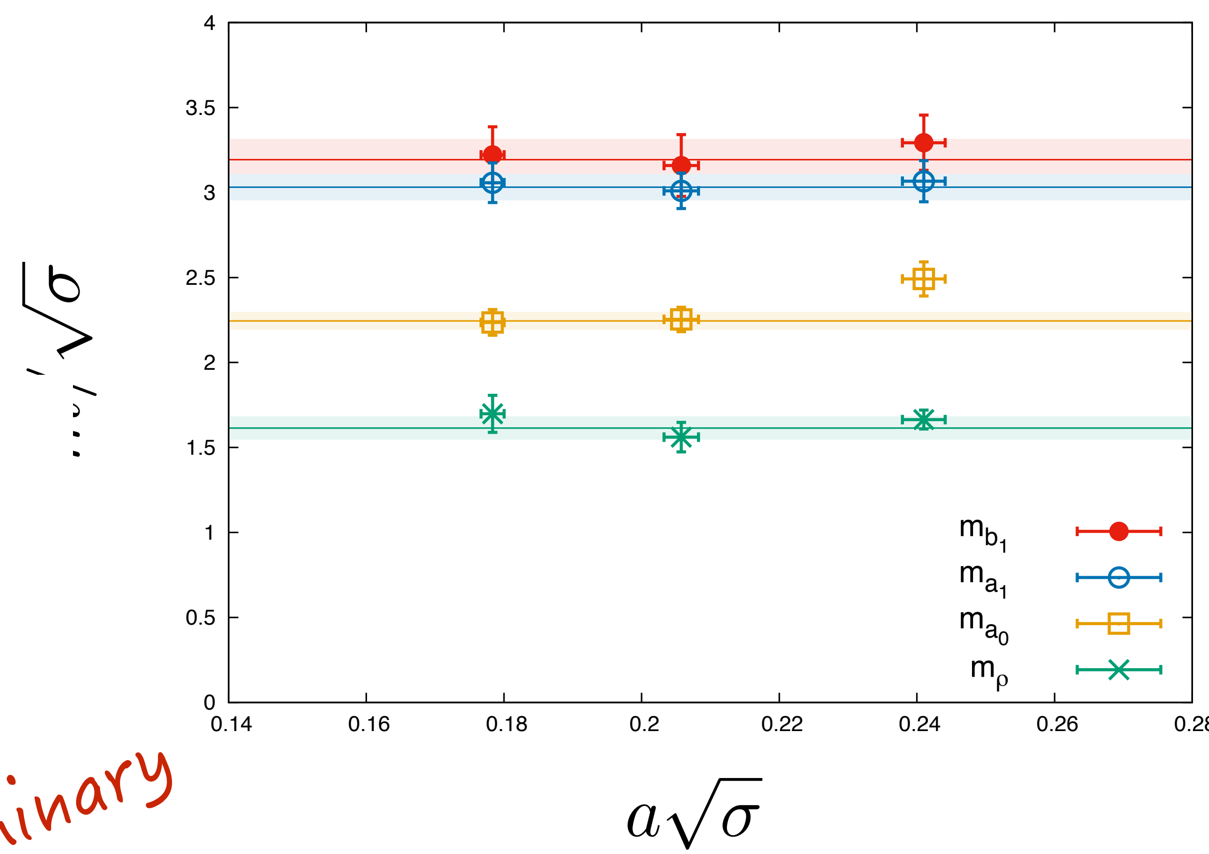
$$N_f/N \to 0$$

Fundamental fermions live on a $\hat{N}^3 \times l_0 \hat{N}$ lattice

N=289

chiral limit

	mass $/\sqrt{\sigma}$	Bali et al
ρ	1.66(7)(5)	1.538(7)
a_0	2.20(5)(4)	2.40(4)
a_1	2.99(8)(2)	2.86(2)
b_1	3.20(12)(18)	2.90(2)



Preliminary

[MGP, González-Arroyo & Okawa]

Twisted Eguchi Kawai Reduction on $\mathbf{T^4}$

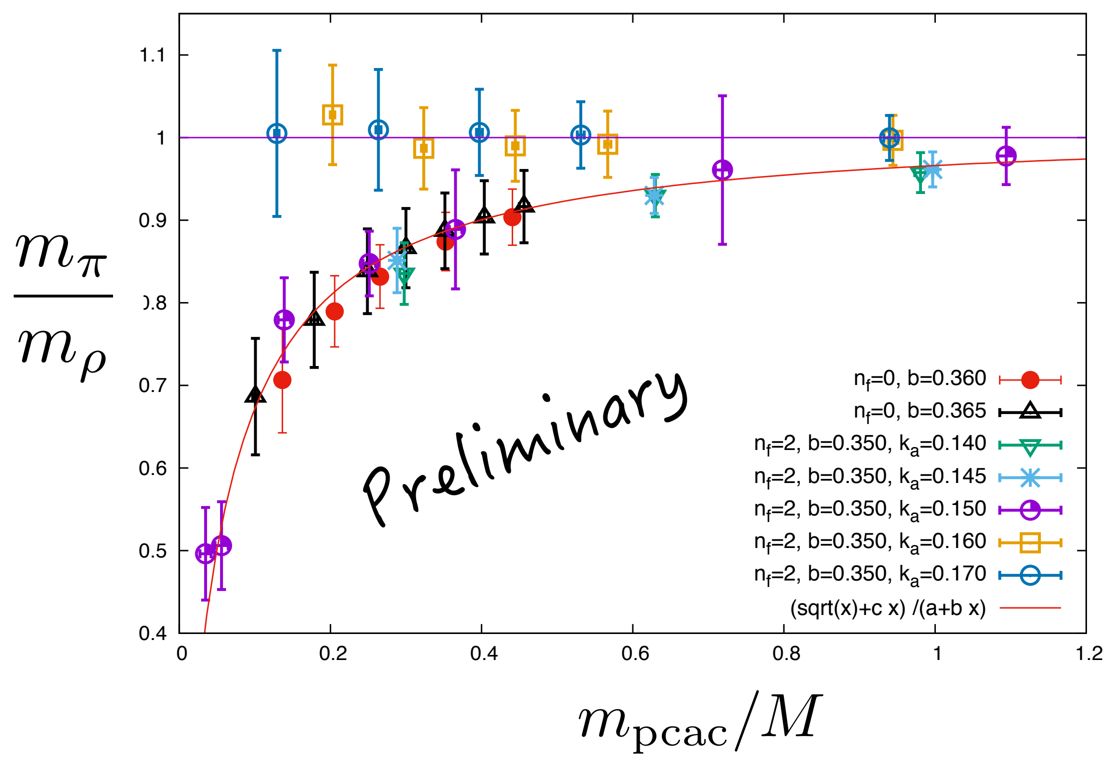
With adjoint fermions N_f=2

N=289

Mass anomalous dimension

$$\gamma_* = 0.269 \pm 0.002 \pm 0.05$$

IR conformality



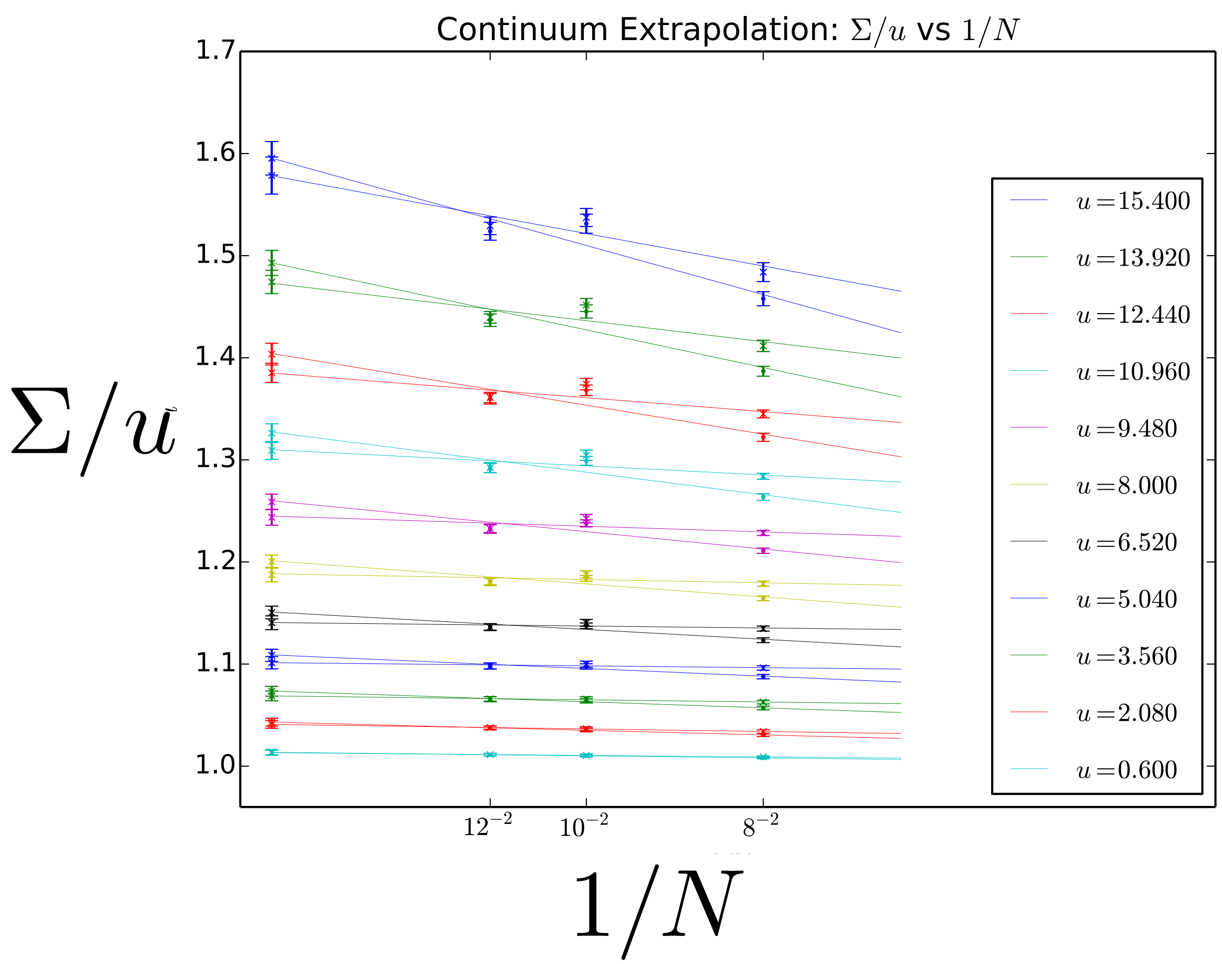
Meson spectrum with fermions in the fundamental

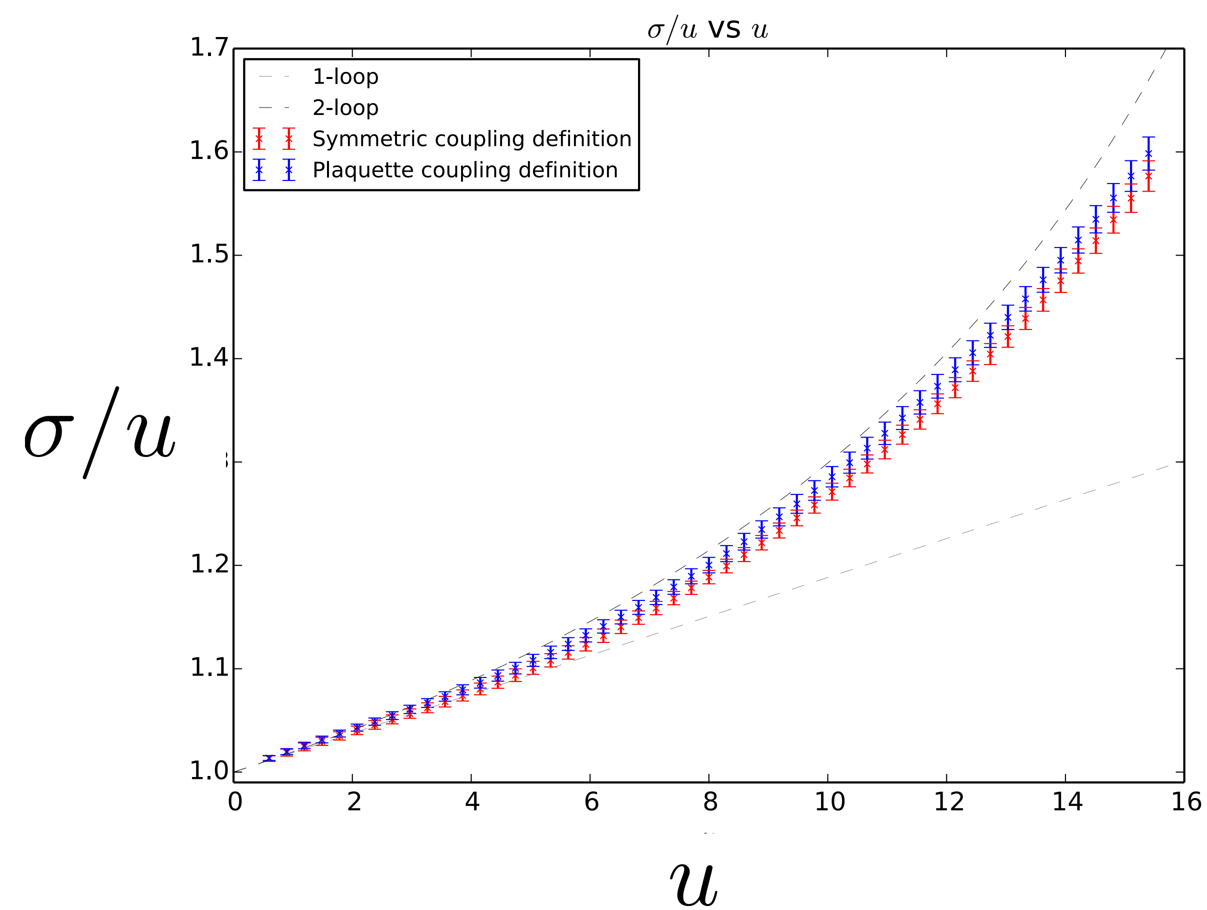
[MGP, Keegan, González-Arroyo & Okawa]

Twisted Eguchi Kawai Reduction on $\mathbf{T^4}$

Yang Mills running coupling

Scaling with the rank of the group $\quad l_{\text{eff}} = a\hat{N} \Rightarrow \lambda(l_{\text{eff}})$



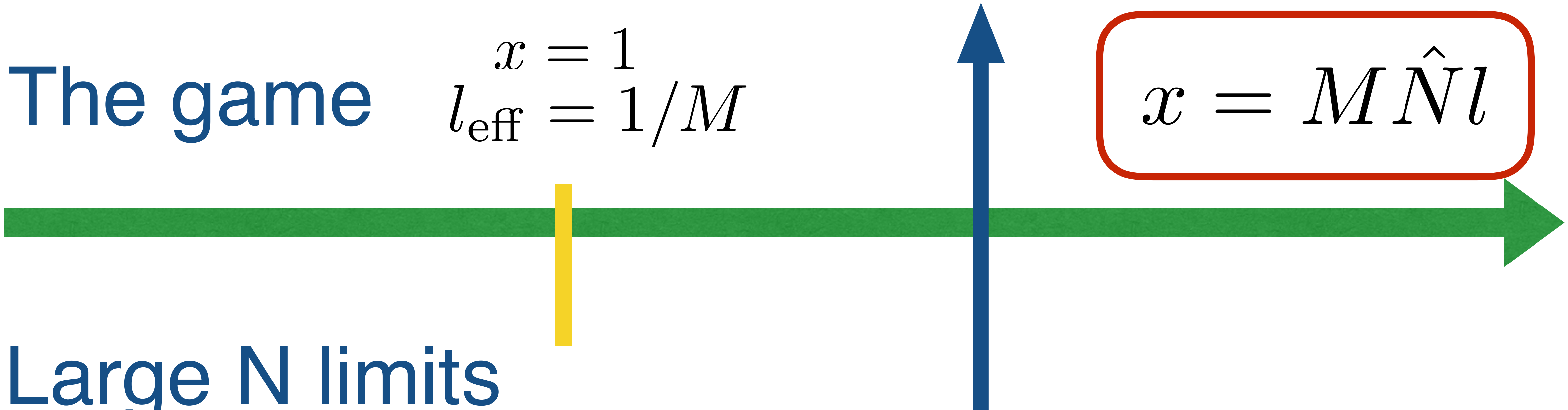


[MGP, Keegan, González-Arroyo & Okawa]

The game

$$x = 1$$
$$l_{\text{eff}} = 1/M$$

$$\boxed{x = M\hat{N}l}$$



Large N limits

Finite x **non-planar diagrams survive**

Singular large N limit

[Alvarez-Gaumé & Barbón]

[Guralnik]

[Griguolo, Seminara & Valtancoli]

Singular large N limit

N to infinity

Volume to zero

x fixed

Non-commutative gauge theory

Going to large N - Singular large N limits

T^2 x R with twist SU(N)

$$x = \lambda N l/(4\pi)$$

$$\tilde{\theta} = 2\pi \bar{k}/N$$

Non-commutative U(1)

$$l_{\text{eff}} = Nl$$

$$\Theta = \frac{l_{\text{eff}}^2}{(2\pi)^2}\,\tilde{\theta}$$

$$k\bar{k} = 1(\text{mod} N)$$

Morita duality

- First formulation of NC Feynman rules

[González-Arroyo & Korthals-Altes]

- Possible non-perturbative regulator of NC gauge theories

[Ambjorn, Makeenko, Nishimura & Szabo]

Going to large N - Singular large N limits

- Obtain irrational $\dfrac{\tilde{\theta}}{2\pi}$ from a limiting sequence

$$\lim_{i \longrightarrow \infty} \frac{\bar{k}_i}{N_i} = \frac{\tilde{\theta}}{2\pi} \qquad \Theta = \frac{l_{\text{eff}}^2}{(2\pi)^2}\,\tilde{\theta} \qquad l_{\text{eff}} = N_i l_i$$

BUT

Tachyonic instabilities at one-loop

[Guralnik, Landsteiner, López]

Spectrum for $T^2 \times R$

$\boxed{\text{Mass Gap in PT}}$ $\quad \frac{2\pi|\vec{n}|}{Nl} \quad \vec{n} \neq \vec{0} \pmod{N}$

one-gluon states $\longrightarrow$ electric flux $\boxed{e_i = -\bar{k}\epsilon_{ij}n_j}$

Lowest state has flux $\bar{k}$ $\quad \boxed{\frac{\mathcal{E}_1}{\lambda} = \frac{1}{2x}}$

$\boxed{\text{Glueball mass in PT}}$ $\quad \boxed{\frac{\mathcal{E}_G}{\lambda} = \frac{1}{x}}$

Tachyonic instabilities at one-loop

$$\tilde{\theta} = 2\pi \bar{k}/N$$

momentum

electric flux/N

$$\mathcal{E}^2_{\vec{n}}\left(x, \tilde{\theta}\right) = \frac{|\vec{n}|^2}{4x^2} - G\left(\frac{\tilde{\theta}\vec{n}}{2\pi}\right)\frac{1}{x}$$

Energy of electric flux (in units of λ)

Gluon self-energy

$$G\left(z\right) \propto \left|\left|z\right|\right|^{-1}$$

[MGP, González-Arroyo & Okawa]

Beyond PT

Confinement

String picture

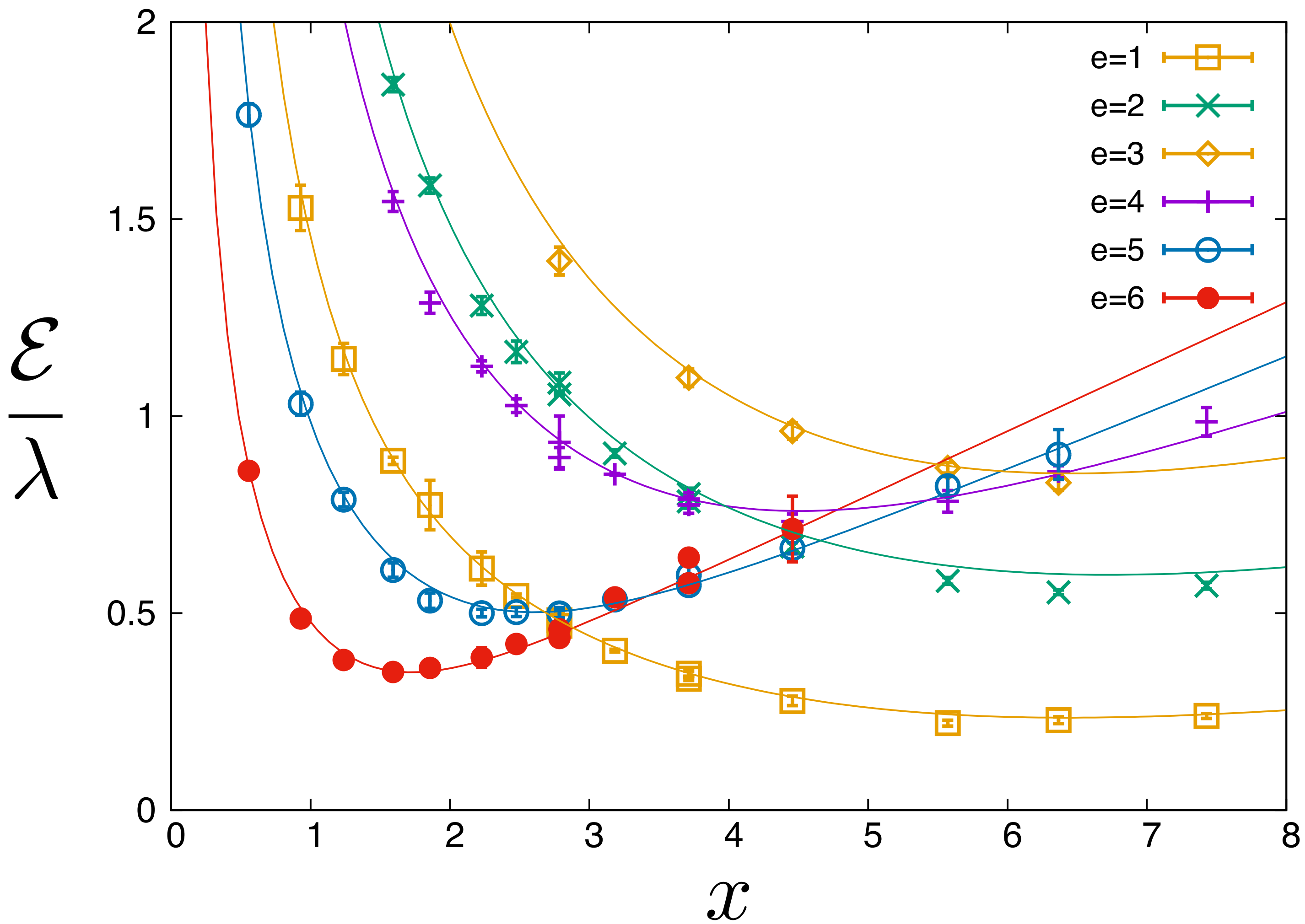
$$\frac{\mathcal{E}_e}{\lambda} = \frac{\sigma_e}{\lambda}\, l = \frac{\sigma}{\lambda^2}\phi\left(\frac{e}{N}\right)\, x$$

Linear growth with x

Going to large N - Singular large N limits

Combined analytic and numerical analysis for the electric flux spectrum at various N

N=17, k=3

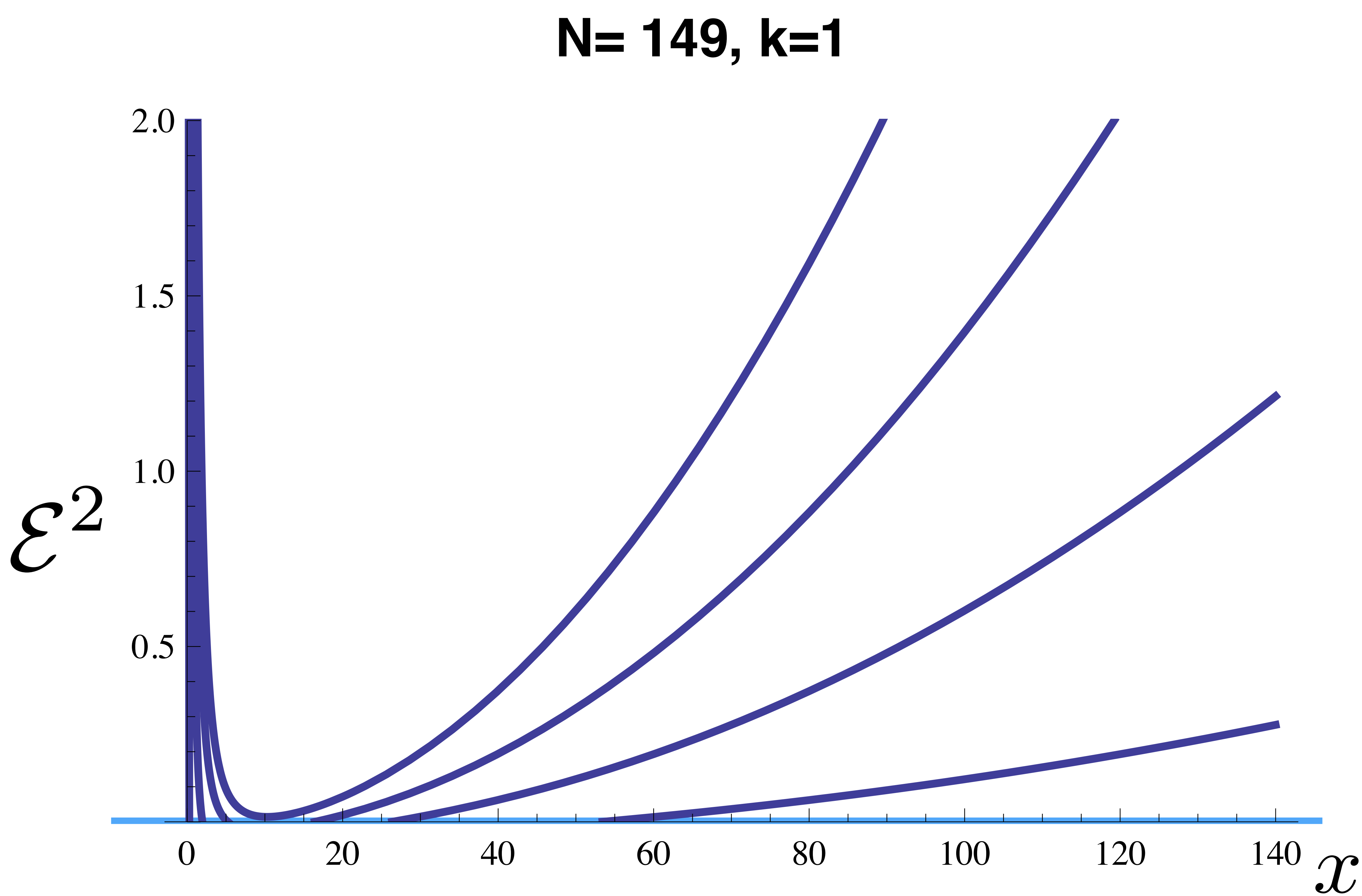


$$\mathcal{E}^2 = \mathcal{E}^2_{1-\mathrm{loop}} + \mathcal{E}^2_{NG}$$

Nambu-Goto action

[MGP, Koren, González-Arroyo & Okawa]

$$\mathcal{E}^2 = \mathcal{E}^2_{1-\mathrm{loop}} + \mathcal{E}^2_{\mathrm{Nambu-Goto}}$$



Going to large N - Singular large N limits

The absence of tachyonic behaviour in the electric flux spectrum requires $\mathcal{E}^2 > 0$

$$\boxed{Z_{\min}(N,k) \equiv \min_{e \perp N} e \left|\left| \frac{ke}{N} \right|\right| \gtrsim 0.1}$$

[González-Arroyo & Chamizo]

Is it possible for any N to choose an appropriate k?

Unproven Zaremba's conjecture

It holds for almost all values of N [Huang]

The Golden Ratio and Non-Commutative gauge theory

A different question - singular large N limits

Can we reach any value of the NC parameter at large N avoiding tachyonic instabilities?

$$\lim_{i \longrightarrow \infty} \frac{\bar{k}_i}{N_i} = \frac{\tilde{\theta}}{2\pi}$$

NO, only for an uncountable set of measure zero [Huang]

Optimal cases

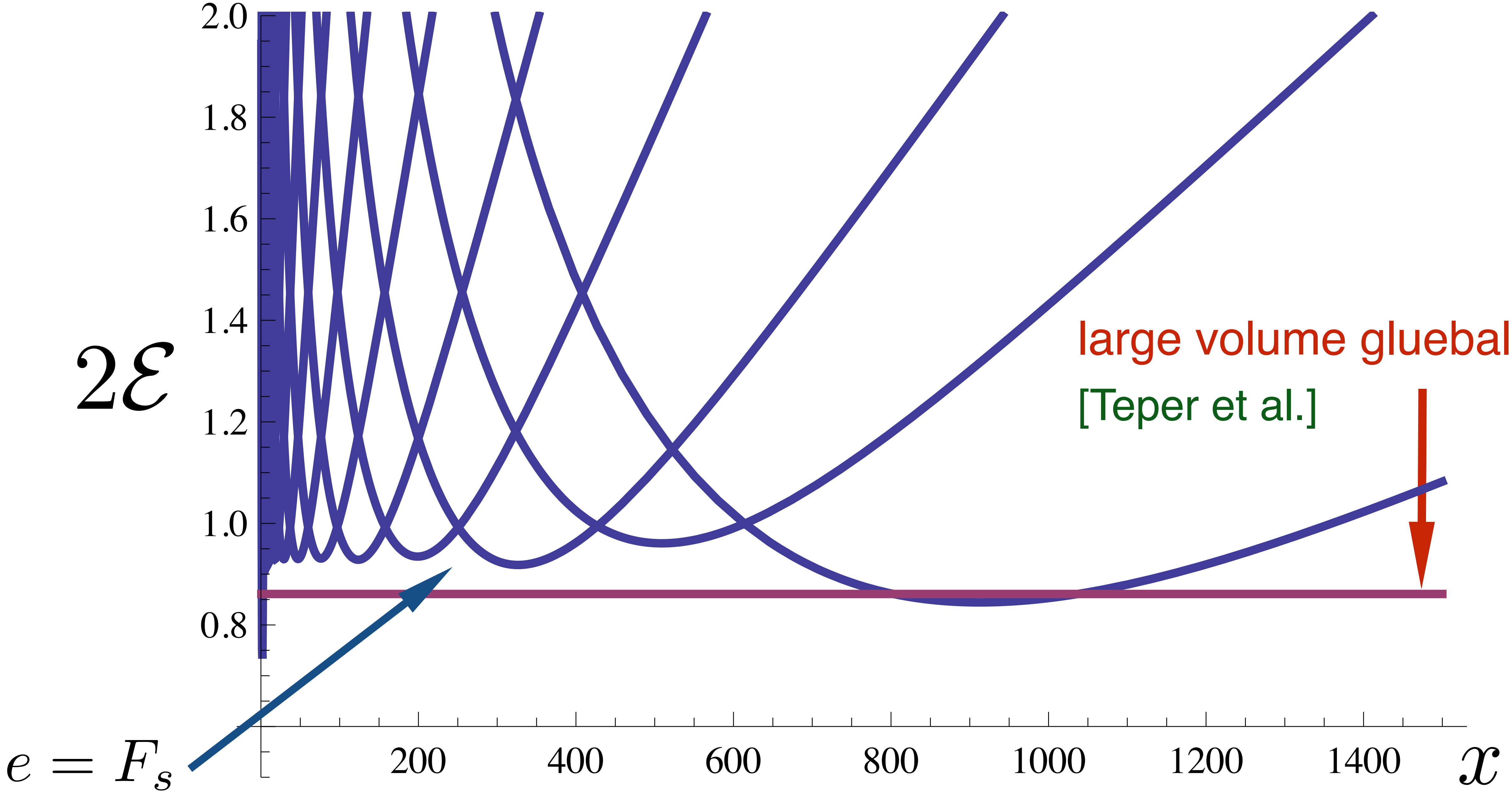
$$\frac{\bar{k}_i}{N_i} = \frac{F_{i-2}}{F_i} \longrightarrow \frac{3-\sqrt{5}}{2} \qquad Z_{\min} = \frac{\bar{k}_i}{N_i} \to 0.381966$$

Fibonacci numbers

The Golden Ratio and Non-Commutative gauge theory

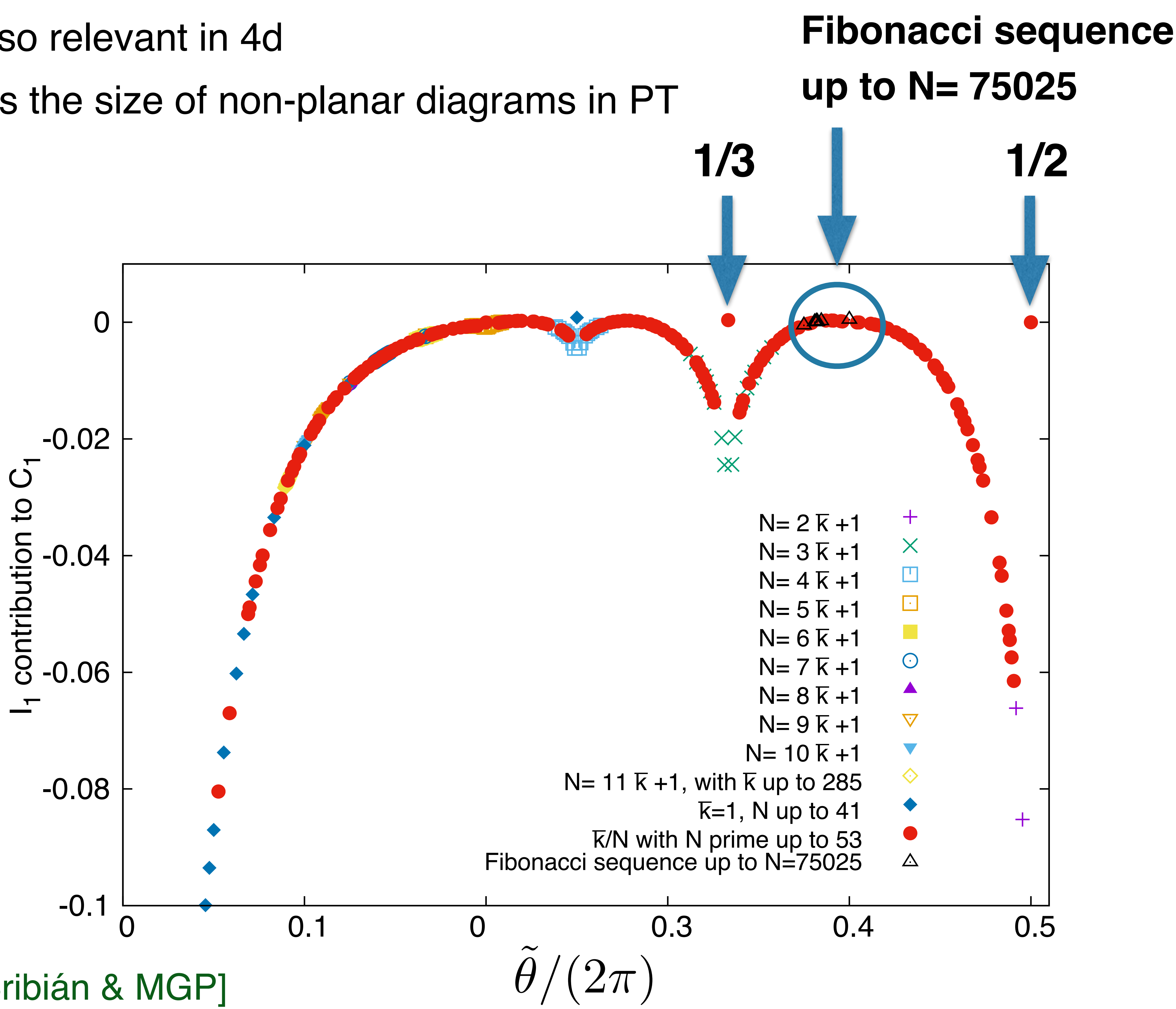
Glueball mass

N = 1597, k = 610

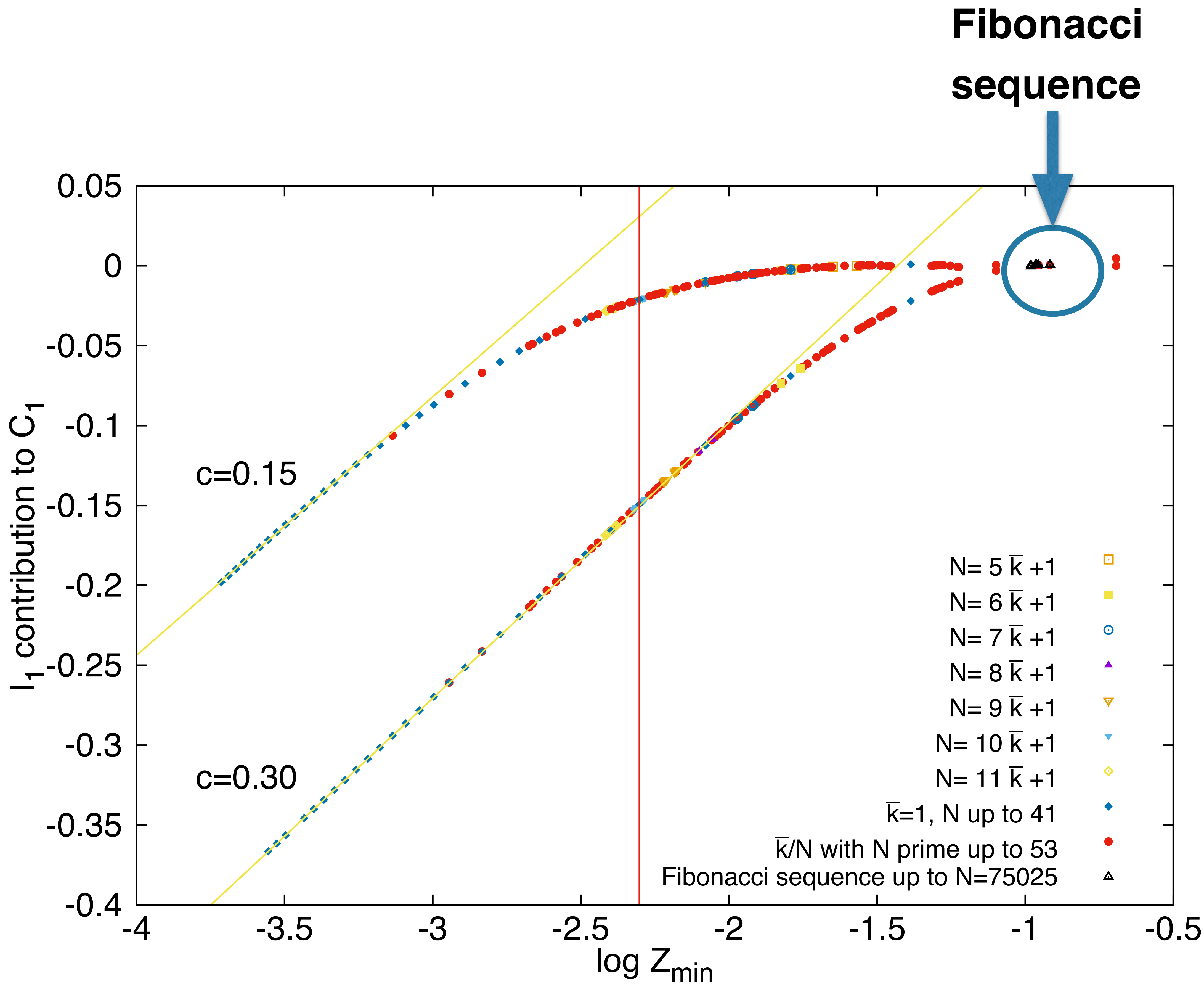


Z_{min} also relevant in 4d

Controls the size of non-planar diagrams in PT



[E. Bribián & MGP]



Fibonacci sequence
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
log Z_{min}
I_1 contribution to C_1
c=0.15
c=0.30
N= 5 $\bar{k}$ +1
N= 6 $\bar{k}$ +1
N= 7 $\bar{k}$ +1
N= 8 $\bar{k}$ +1
N= 9 $\bar{k}$ +1
N= 10 $\bar{k}$ +1
N= 11 $\bar{k}$ +1
$\bar{k}$=1, N up to 41
$\bar{k}$/N with N prime up to 53
Fibonacci sequence up to N=75025

Summary

- Large N reduction - useful method to determine large N observables
- At work for

$$\boxed{Z_{\min}(N,k) \equiv \min_{e \perp N} e \left|\left| \frac{ke}{N} \right|\right| \gtrsim 0.1}$$

- Define NC at some irrational values of $\tilde{\theta}$ as a singular large N limit

Example: choose k & N in the Fibonacci sequence

$$\mathbf{k = F_{i-2}} \qquad \mathbf{N = F_i}$$

[González-Arroyo & Chamizo]

$$\frac{\bar{k}}{N} = [a_0; a_1, a_2, \ldots, a_M] := a_0 + 1/\Big(a_1 + 1/\big(a_2 + 1/(a_3 + \ldots)\big)\Big)$$

$$A_{\text{max}}(N, \bar{k}) = \text{max}_i a_i$$

$$\frac{1}{2 + A_{\text{max}}} < Z_{\text{min}} < \frac{1}{A_{\text{max}}}$$

$$Z_{\text{min}} > 0.1 \quad \longrightarrow \boxed{A_{\text{max}} < 10}$$

The set of limiting irrationals is uncountable and has measure zero

[Huang]