ELECTROMAGNETIC SHOWERS

Paolo Lipari

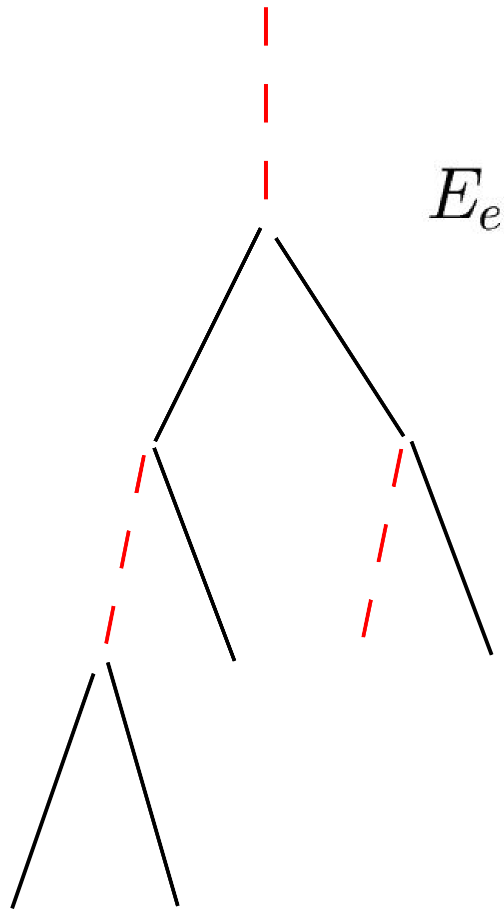
Corsika school
Ooty 17th december 2010

MONTECARLO CALCULATIONS

ANALYTIC CALCULATIONS

PHYSICAL UNDERSTANDING

ELECTROMAGNETIC SHOWERS



$$E_{e^+} = E_\gamma \ u$$

$$E_\gamma = E_e \ v$$

$\psi(u)$ Pair Production

$\varphi(v)$ Brems-strahlung

Radiation Length
(Energy independent)

Vertices :
theoretically understood
(and scaling)

BREMSSTRAHLUNG

Fully ionized free nucleus (approximation of infinite mass)

$$\left.\frac{d\sigma}{d\varepsilon}\right|_{e\to e+\gamma}(v;E) = 4\,Z^2\,\alpha\,r0^2$$

$$\frac{1}{v}\;\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\;\left[\ln\left(\frac{2\;E}{m}\;\frac{v}{1-v}\right)-\frac{1}{2}\right]$$

High Energy Limit (Full screening)

$$v = \frac{E_\gamma}{E_e}$$

$$\left.\frac{d\sigma}{d\varepsilon}\right|_{e\to e+\gamma}(v;E) = 4\,Z^2\,\alpha\,r0^2$$

$$\frac{1}{v}\;\left\{\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\;\ln\left(183\,Z^{-1/3}\right)+\frac{1}{9}(1-v)\right\}$$

PAIR PRODUCTION

Fully ionized free nucleus (approximation of infinite mass)

$$\left.\frac{d\sigma}{du}\right|_{\gamma\to e^+e^-}(u;K) = 4\ Z^2\ \alpha\ r0^2$$

$$\left[u^2 + (1-u)^2 + \frac{2}{3}u\,(1-v)\right]\ \left[\ln\left(\frac{2\ K}{m}\,u(1-u)\right) - \frac{1}{2}\right]$$

High Energy Limit (Full screening)

$$u = \frac{E_{e^+}}{E_\gamma}$$

$$\left.\frac{d\sigma}{du}\right|_{\gamma\to e^+e^-}(u;K) = 4\ Z^2\ \alpha\ r0^2$$

$$\left\{\ \left[u^2 + (1-u)^2 + \frac{2}{3}u\,(1-v)\right]\ \ln\left(183\,Z^{-1/3}\right) - \frac{1}{9}u(1-u)\right\}$$

FUNDAMENTAL PROCESSES In QUANTUM ELECTRODYNAMICS

COMPTON SCATTERING

$$e + \gamma \longrightarrow e + \gamma$$

ELECTRON-POSITRON ANNIHILATION

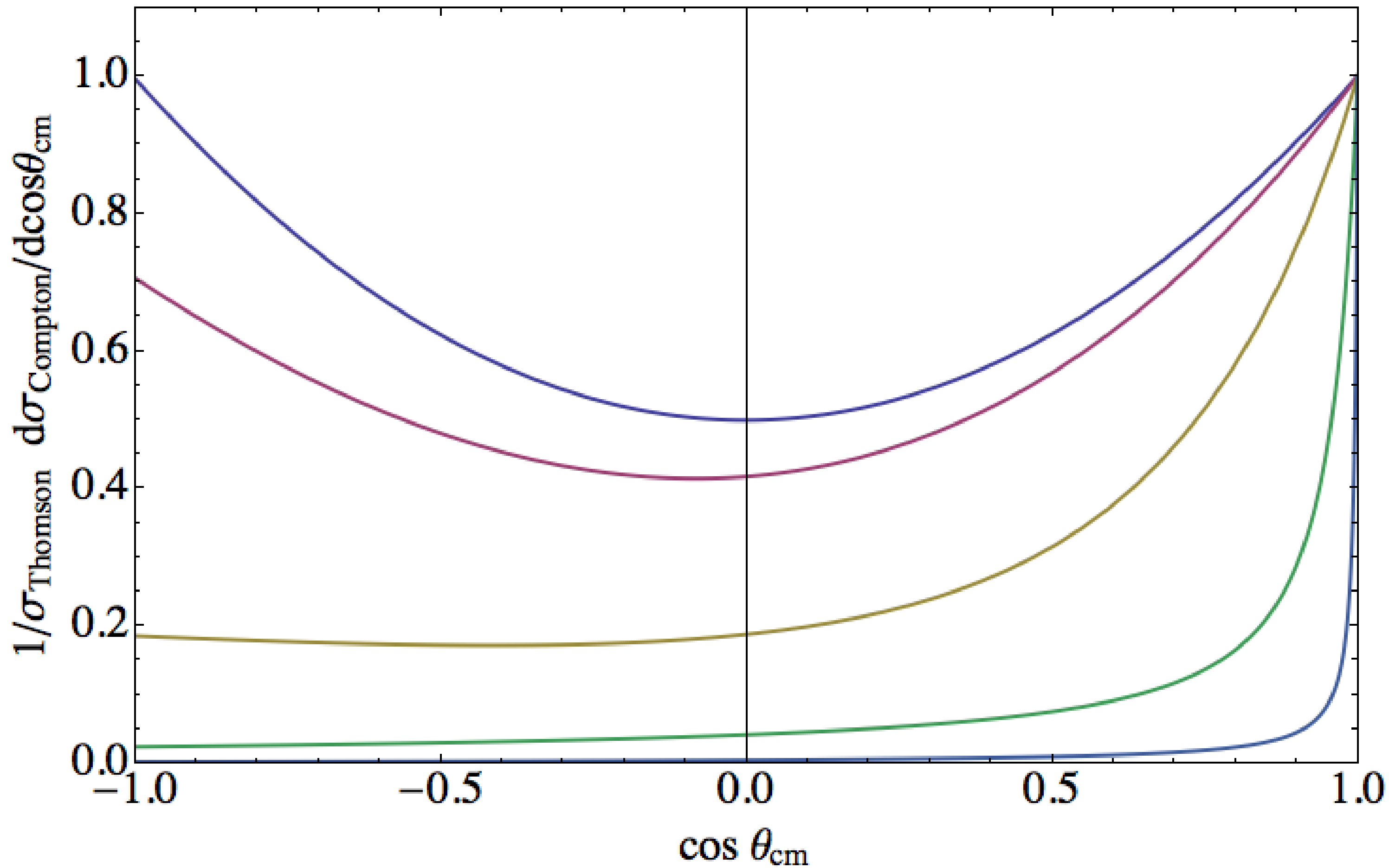
$$e^{+} + e^{-} \longrightarrow \gamma + \gamma$$

ELECTRON-POSITRON CREATION

$$\gamma + \gamma \longrightarrow e^{+} + e^{-}$$

COMPTON scattering $\cos\theta_{\text{rest frame}}$ distribution

```
Plot [{dsigcompt[ct, 0.001], dsigcompt[ct, 0.1],
  dsigcompt[ct, 1], dsigcompt[ct, 10], dsigcompt[ct, 100]},
 {ct , -1, 1 }, PlotStyle → col1, Frame → True]
```



$$\frac{\text{alfa}^2\,(1+\text{ct}^2)}{2\,\text{m}^2}+\frac{\text{alfa}^2\,(-1+\text{ct}-\text{ct}^2+\text{ct}^3)\,\epsilon}{\text{m}^2}+O[\epsilon]^2$$

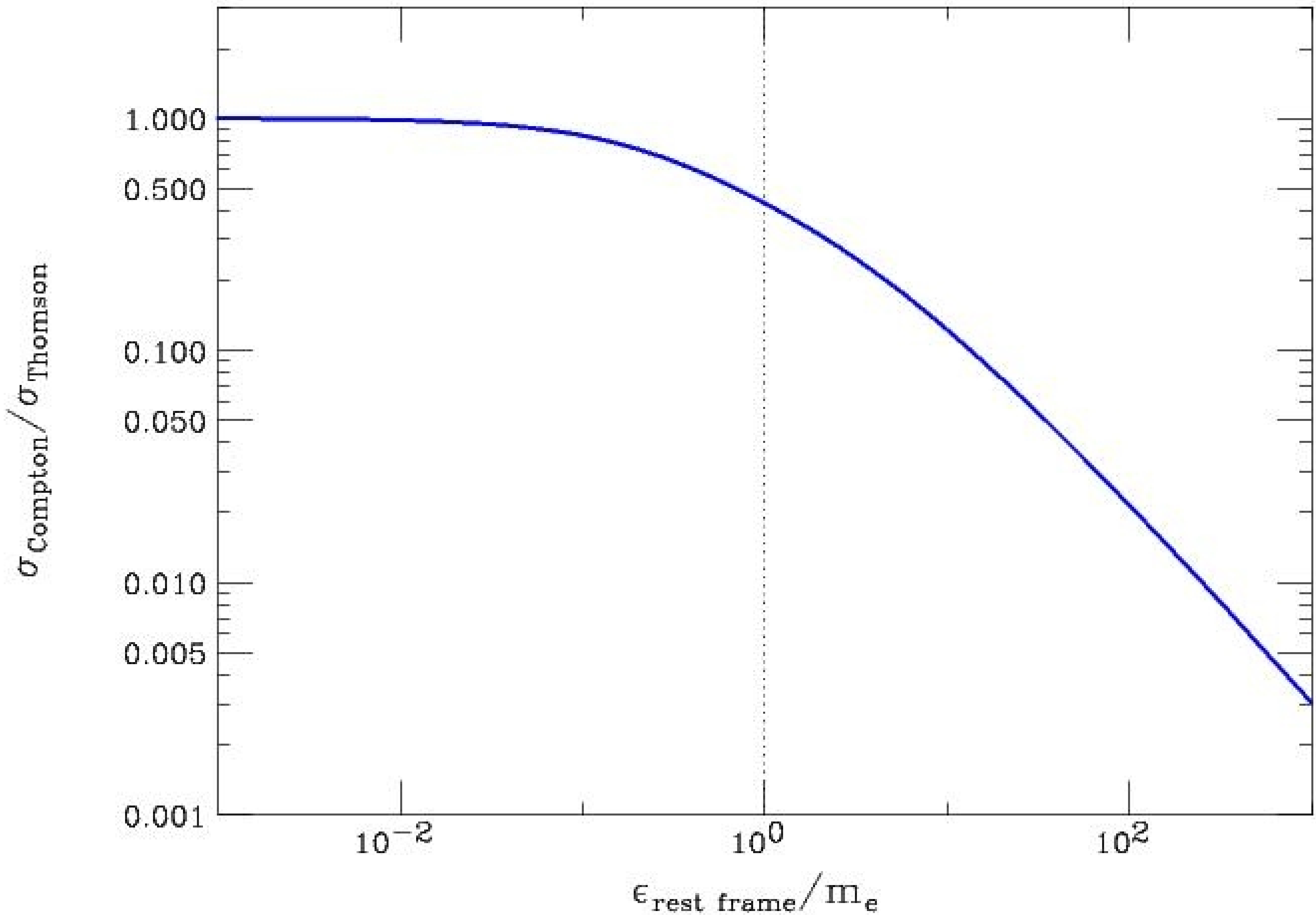
dsigcompt [ct, ϵ] // TraditionalForm

ditionalForm=

$$-\frac{-\epsilon ct^3+(\epsilon^2+\epsilon+1)ct^2-\epsilon(2\epsilon+1)ct+\epsilon^2+\epsilon+1}{2((ct-1)\epsilon-1)^3}$$

Total COMPTON scattering cross section

$$-\frac{1}{m^2\,\epsilon^3}\left(alfa^2\,\pi\left(-\frac{2\,\epsilon\,(2+\epsilon\,(1+\epsilon)\,(8+\epsilon))}{(1+2\,\epsilon)^2}+(2-(-2+\epsilon)\,\epsilon)\,\mathrm{Log}[1+2\,\epsilon]\right)\right)$$



1.000
0.500
0.100
0.050
0.010
0.005
0.001
10^{-2}
10^{0}
10^{2}
$\sigma_{\mathrm{Compton}}/\sigma_{\mathrm{Thomson}}$
$\epsilon_{\mathrm{rest\ frame}}/m_e$

$$\sigma_{\text{Thomson}} = \frac{8\pi}{3} \frac{e^2}{(m_e\, c^2)^2} \quad = \frac{8\pi}{3}\, r_0^2$$

$$\sigma_{\text{Thomson}} \simeq 6.65 \times 10^{-25}\ \text{cm}^2$$

$$r_0 = \frac{e^2}{(m_e\, c^2)^2}$$

Electron “Classical Radius”

$$r_0 = \frac{\alpha}{m_e^2} \left(\frac{\hbar c}{c^4} \right)$$

$$r_0 = 2.81 \times 10^{-13}\ \text{cm}$$

$$a_{\mathrm{Bohr}} = \frac{\hbar^2}{m_e^2\ e^2}$$

Bohr Radius

(dimension of the hydrogen atom)

$$a_{\mathrm{Bohr}} = \frac{r_0}{\alpha^2}$$

$$r_0 = 2.81 \times 10^{-13}\ \mathrm{cm}$$

$$a_{\mathrm{Bohr}} = 0.53 \times 10^{-8}\ \mathrm{cm}$$

Thomson cross section
for the electron scattering

$$e + \gamma \to e + \gamma$$

Larmor Formula:
Radiation of an accelerated charged particle
(system where the velocity is small)

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \dot{v}^2 \sin^2 \Theta$$

$$P = \frac{2}{3} \frac{e^2 \dot{v}^2}{c^3}$$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} \dot{v}^2 \sin^2 \Theta$$

Electromagnetic Plane wave

$$\mathbf{E}(\mathbf{x}, t) = \boldsymbol{\epsilon} E_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$$

Electric field

$$\dot{\mathbf{v}}(t) = \boldsymbol{\epsilon} \frac{e}{m} E_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$$

Acceleration

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} |E_0|^2 \left(\frac{e^2}{mc^2} \right)^2 \sin^2 \Theta$$

$$\frac{d\sigma}{d\Omega} = \frac{\text{Energy radiated/unit time/unit solid angle}}{\text{Incident energy flux in energy/unit area/unit time}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \sin^2 \Theta$$

Linearly Polarized Light
(angle with respect
to the polarization vector)

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \cdot \tfrac{1}{2}(1 + \cos^2 \theta)$$

Unpolarized light

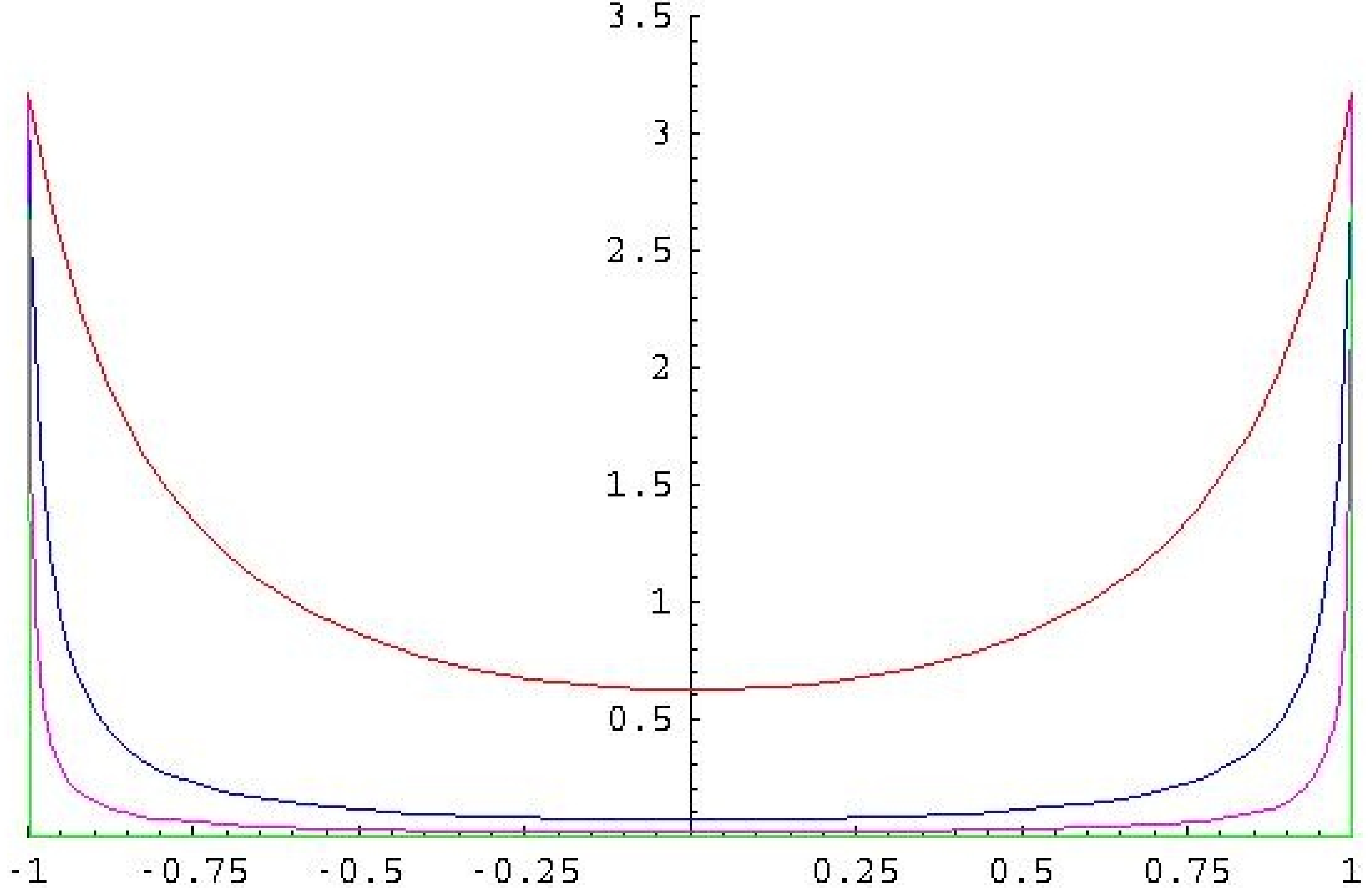
(angle with respect
To photon direction

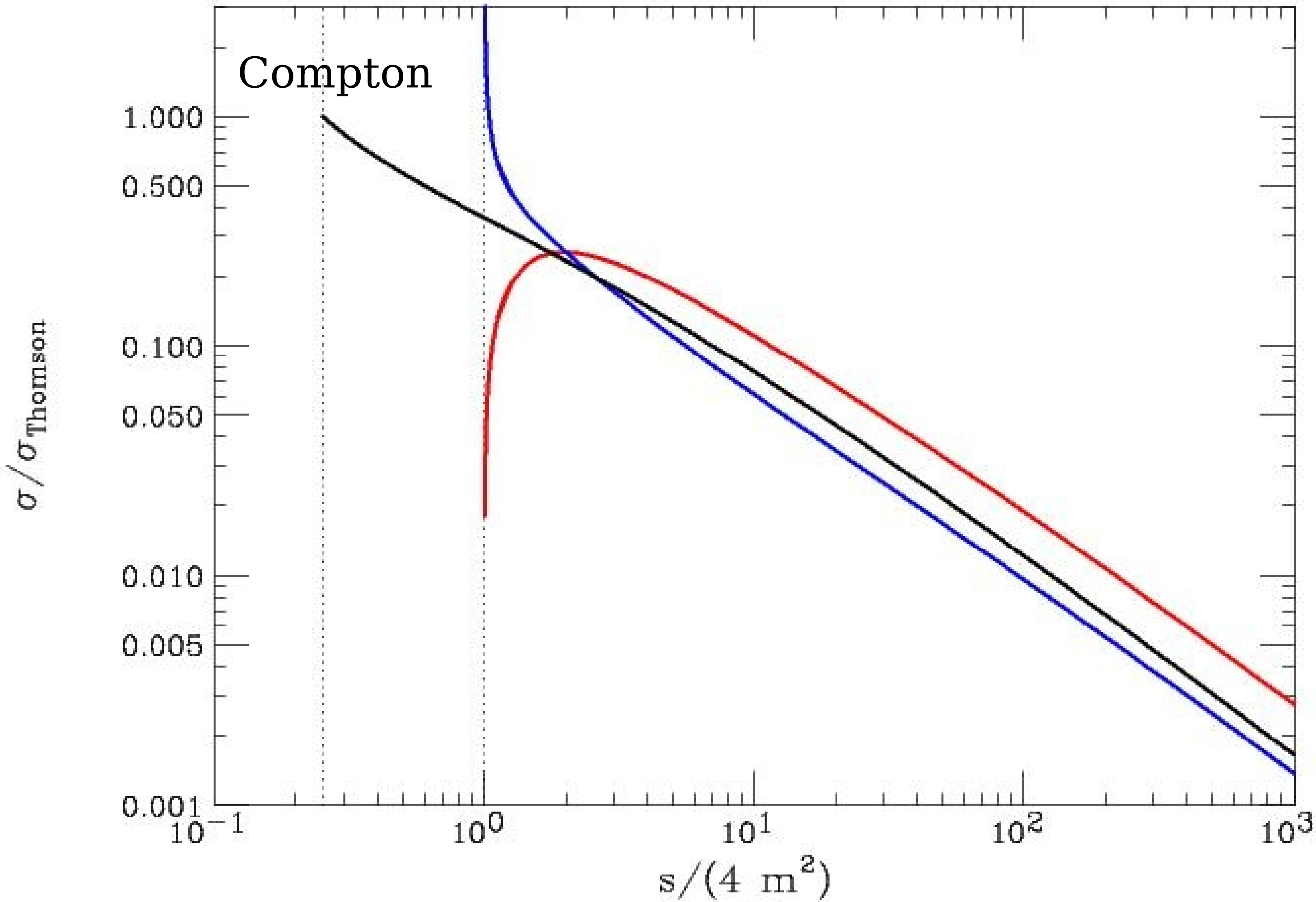
$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$

$$\sigma_{KN} = \left(\frac{e^2}{mc^2}\right)^2 \begin{cases} \dfrac{8\pi}{3}\left(1 - \dfrac{2\hbar\omega}{mc^2} + \cdots\right), & \hbar\omega \ll mc^2 \\ \pi\,\dfrac{mc^2}{\hbar\omega}\left[\ln\left(\dfrac{2\hbar\omega}{mc^2}\right) + \frac{1}{2}\right], & \hbar\omega \gg mc^2 \end{cases}$$

Pair Creation (or Annihilation) $\cos\theta_{cm}$ distribution

```
In[54]:= Plot [{ddsig[ct, 1.005], ddsig[ct, 2], ddsig[ct, 5],
    ddsig[ct, 10], ddsig[ct, 1000]}, {ct, -1, 1},
   PlotStyle → col, PlotRange → {{-1, 1}, {0, 3.5}}]
```





Compton
1.000
0.500
0.100
0.050
0.010
0.005
0.001
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
$s/(4\ m^2)$
$\sigma/\sigma_{\mathrm{Thomson}}$

Bremsstrahlung

$$e + Z \rightarrow e + \gamma + Z$$

Pair Creation

$$\gamma + Z \rightarrow e^{+} + e^{-} + Z$$

Bremsstrahlung

$$e + Z \rightarrow e + \gamma + Z$$

$$e + \gamma_{\text{virtual}} \rightarrow e + \gamma$$

Pair Creation

$$\gamma + Z \rightarrow e^{+} + e^{-} + Z$$

$$\gamma + \gamma_{\text{virtual}} \rightarrow e^{+} + e^{-}$$

Method of Virtual Photons

$$\vec{E} = q\,\frac{\vec{r}}{r^3}, \qquad \vec{B} = 0$$

Charged particle at rest
Particle at Rest

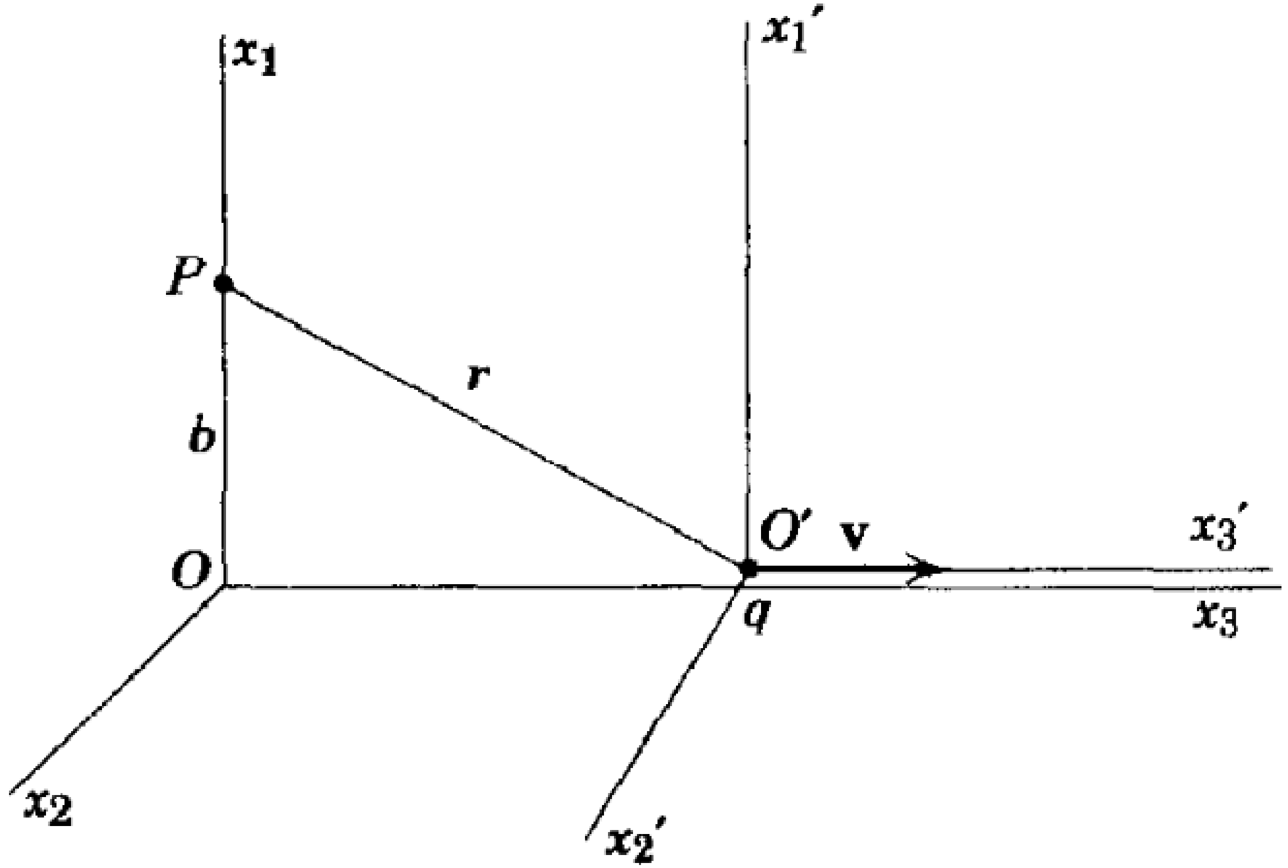
The particle moves with a constant velocity β (Lorentz factor γ) , passing by an observer placed at the observation point P with a minimum distance (impact parameter) b.

Question: What are the electric and magnetic field $(\vec{E}(t),\ \vec{B}(t))$ that the observer at P measures as a function of observer time t?

$$\boxed{(\vec{E}(t),\ \vec{B}(t))}$$

Method of Virtual Photons

E(t)
B(t)
At point P



Particle of charge q
Velocity v

Method of Virtual Photons

$$\vec{E} = q\,\frac{\vec{r}}{r^3}, \qquad \vec{B} = 0$$

Charged particle at rest
Particle at Rest

(primed reference frame, charged particle system)

$$P' = \{x', y', z'\} = \{b, 0, -\beta\, t'\}$$

$$\vec{E}' = \{\frac{q\, b}{(r')^3}, \quad , 0, \quad -\frac{q\, \beta\, t}{(r')^3}\} \qquad \vec{B}' = 0$$

$$\vec{E}' = \{\frac{q\, b}{(b^2 + \beta\, t')^{3/2}}, \quad 0, \quad -\frac{q\, \beta\, t}{(b^2 + \beta\, t')^{3/2}}\}$$

Lorentz Transformation of electromagnetic Field

$$\vec{E}' = \{\gamma\,(E_x - \beta\,B_y),\quad \gamma\,(E_y + \beta\,B_x),\quad E_z\}$$

$$\vec{B}' = \{\gamma\,(B_x + \beta\,B_y),\quad \gamma\,(B_y - \beta\,E_x),\quad B_z\}$$

Lorentz Transformation of electromagnetic Field

$$\vec{E}' = \{\gamma\,(E_x - \beta\,B_y),\quad \gamma\,(E_y + \beta\,B_x),\quad E_z\}$$

$$\vec{B}' = \{\gamma\,(B_x + \beta\,B_y),\quad \gamma\,(B_y - \beta\,E_x),\quad B_z\}$$

$(E'_x,\, E'_y \neq 0)$

$$\boxed{\begin{gathered}\vec{E}' = \{\gamma\,E_x,\quad 0\quad E_z\}\\[2em] \vec{B}' = \{0,\quad -\beta\,\gamma\,E_x,\quad 0\quad\}\end{gathered}}$$

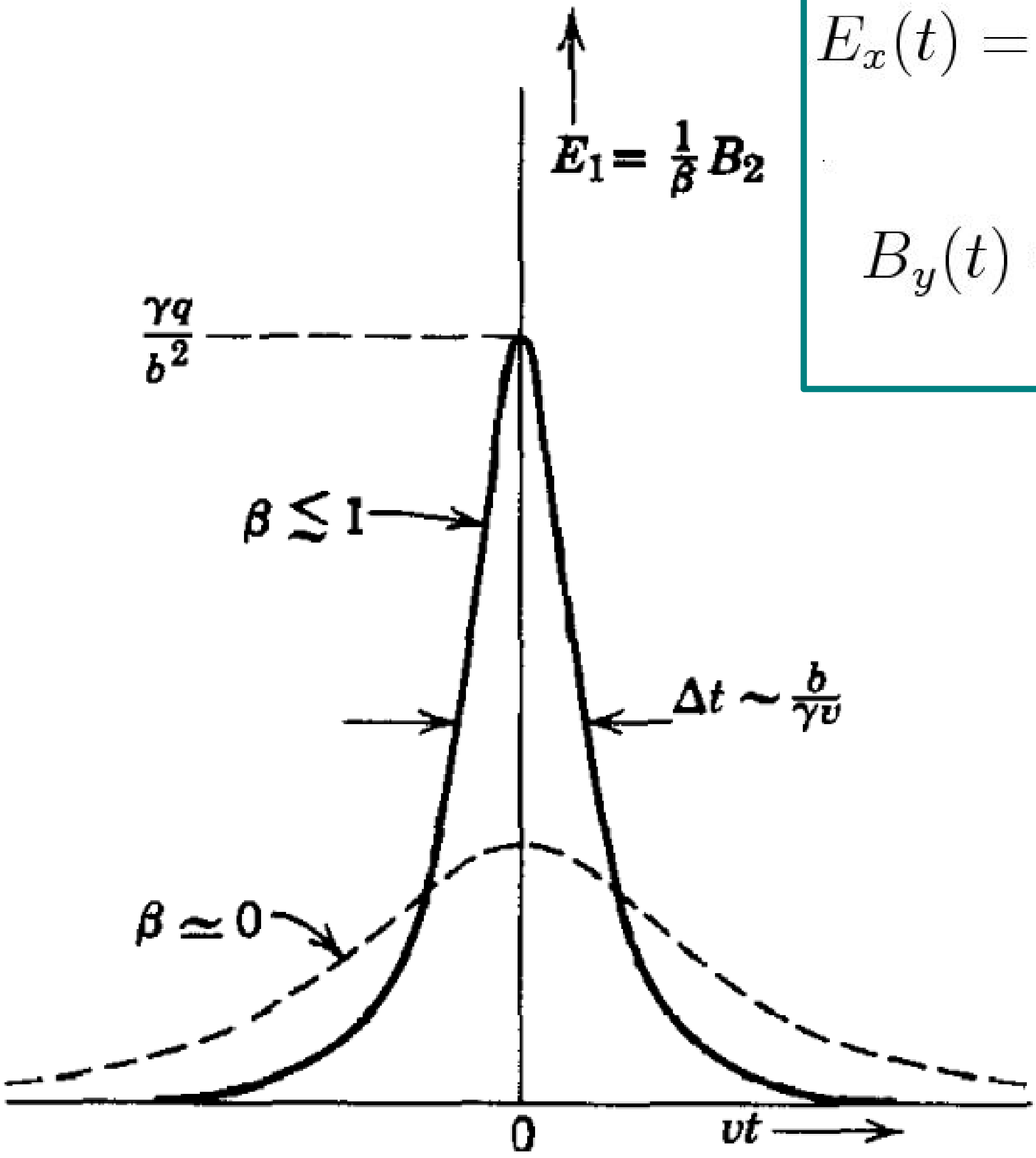
$$\vec{E} = \{\frac{q\,b\,\gamma}{[b^2 + (\beta t')^2]^{3/2}} \quad, 0, \qquad -\frac{q\,(\beta\,t)}{(b^2 + [\beta t')^2]^{3/2}}\}$$

$$\vec{B} = \{0, \quad \frac{q\,b\,\beta\,\gamma}{[b^2 + (\beta t')^2]^{3/2}} \quad, 0\}$$

$$t' = \gamma\,t$$

$$\boxed{\begin{aligned} E_x(t) &= \frac{q\,b\,\gamma}{[b^2 + (\beta\,\gamma\,t)^2]^{3/2}} \\ B_y(t) &= \frac{q\,\beta\,\gamma\,t}{[b^2 + (\beta\,\gamma\,t)^2]^{3/2}} \end{aligned}}$$

$$\vec{E} \perp \vec{B}$$
$$\vec{E} \perp \hat{v}$$
$$\vec{B} \perp \hat{v}$$



$$E_x(t) = \frac{q\, b\, \gamma}{[b^2 + (\beta\, \gamma\, t)^2]^{3/2}}$$

$$B_y(t) = \frac{q\, \beta\, \gamma\, t}{[b^2 + (\beta\, \gamma\, t)^2]^{3/2}}$$

The observer at P, that sees a relativistic particle "zipping by" at relativistic speeed ($\beta \simeq 1, \quad \gamma \gg 1$) sees an electromagnetic field that is undistinguishable from an electromagnetic plane wave propagating along the z direction. $\vec{E}$, $\vec{B}$, $\hat{v}$ mutually perpendicular.

Fourier analysis of the electromagnetic wave

$$E_\omega = \frac{1}{2\pi} \int dt\; e^{i\omega t}\; E(t)$$

$$\boxed{E_x(t) = \frac{q\, b\, \gamma}{[b^2 + (\beta\, \gamma\, t)^2]^{3/2}}}$$

$$E_\omega = \frac{1}{2\pi} \int dt\; e^{i\omega t}\; \frac{q\, \gamma\, b}{(b^2 + \gamma^2\, t^2)^{3/2}}$$

$$E_\omega = \frac{q}{2\pi}\, \frac{b}{b^2} \int dt\; e^{i\omega t}\; \frac{1}{[1 + ((\gamma/b)\, t)^2]^{3/2}}$$

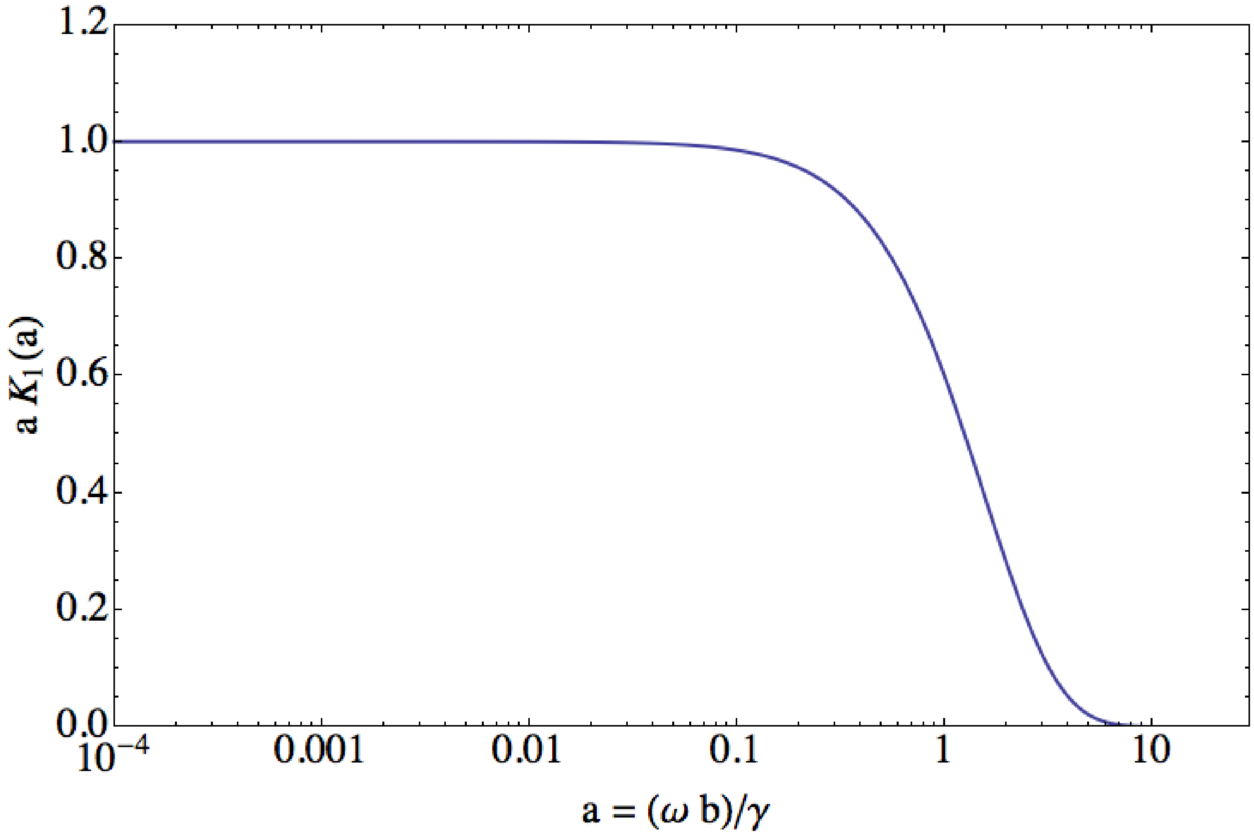
$$x = \frac{\gamma}{b}\, t$$

$$\boxed{E_\omega = \frac{q}{2\pi\, b} \int dx\; e^{i\,(\omega\, b/\gamma)\, x}\; \frac{1}{(1 + x^2)^{3/2}}}$$

$$\boxed{E_\omega = \frac{q}{2\,\pi\,b}\int dx\ e^{i\,(\omega\,b/\gamma)\,x}\ \frac{1}{(1+x^2)^{3/2}}}$$

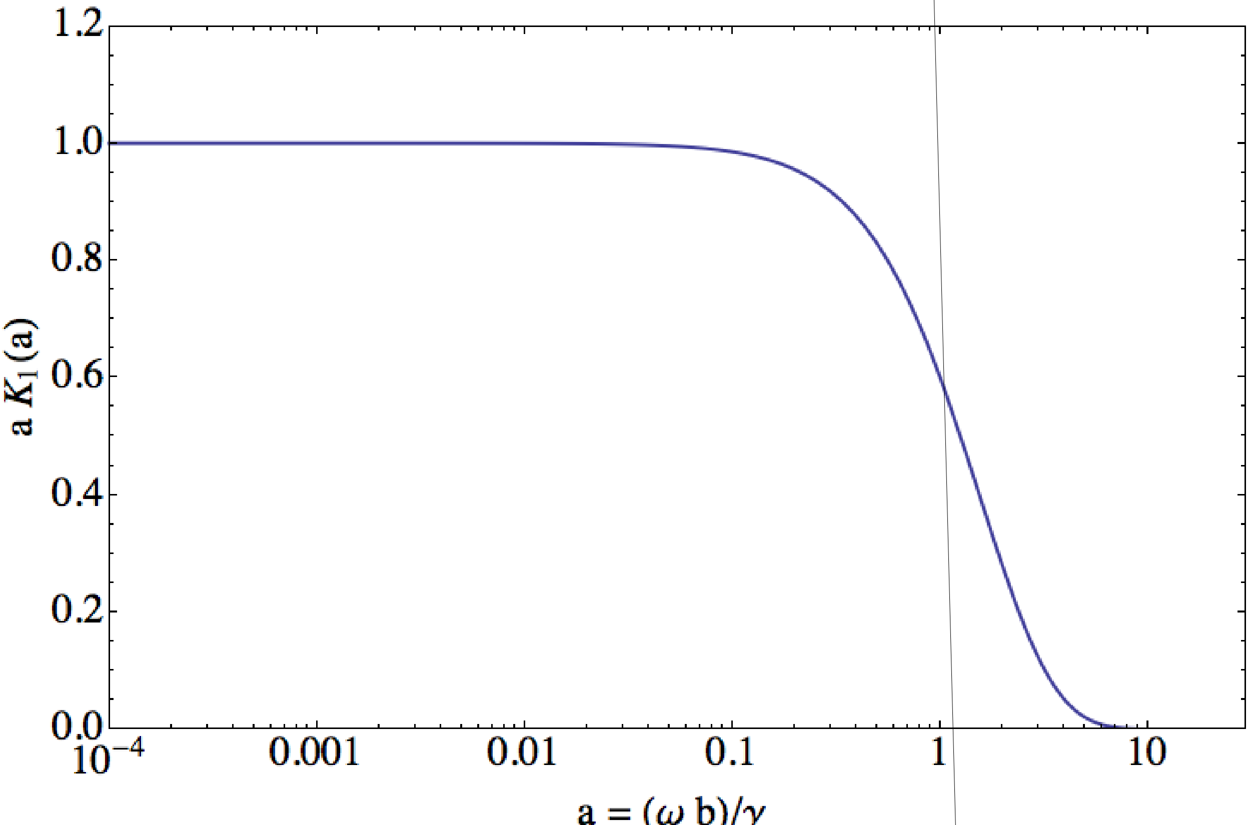
$$\int dt\ e^{i\,a\,x}\ \frac{1}{(1+x^2)^{3/2}} = 2\ a\ K_1(a)$$

$$\int dx\ \frac{1}{(1+x^2)^{3/2}} = 2$$



$$E_\omega \simeq \frac{q}{\pi\ b}$$

$$E_\omega \simeq 0$$



Energy Fluence : Energy/(unit Area)

$$\int d\omega\ |E(\omega)|^2 = \int dt\ |E(t)|^2$$

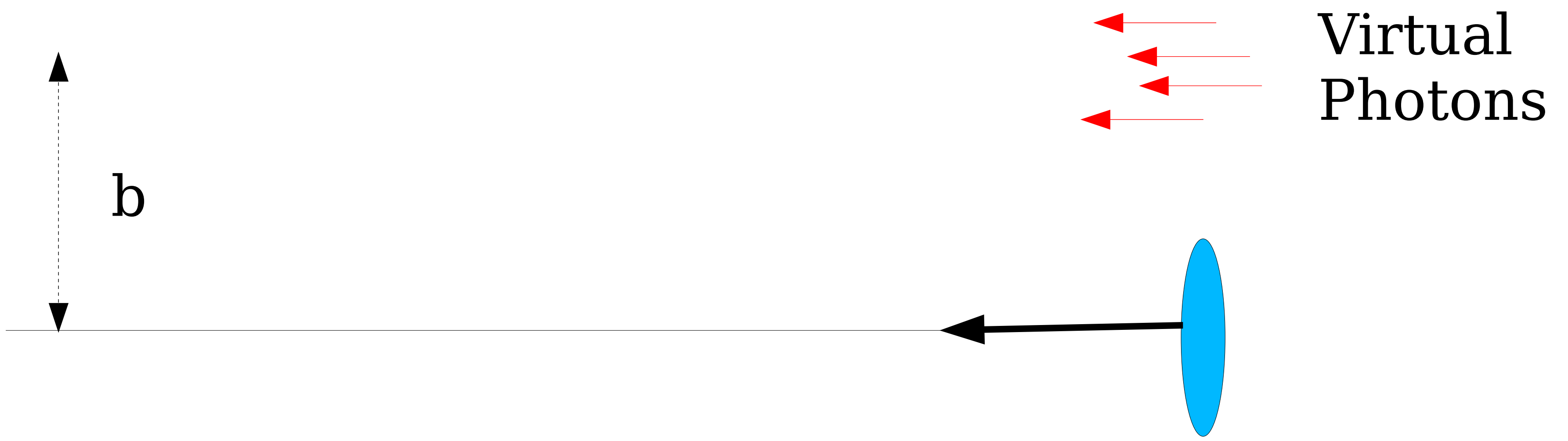
$$|E(\omega)|^2 = \frac{d\mathcal{E}}{d\omega\ dA} = \frac{dN_\gamma^{\rm virtual}}{d\omega\ dA}\ (\hbar\ \omega)$$

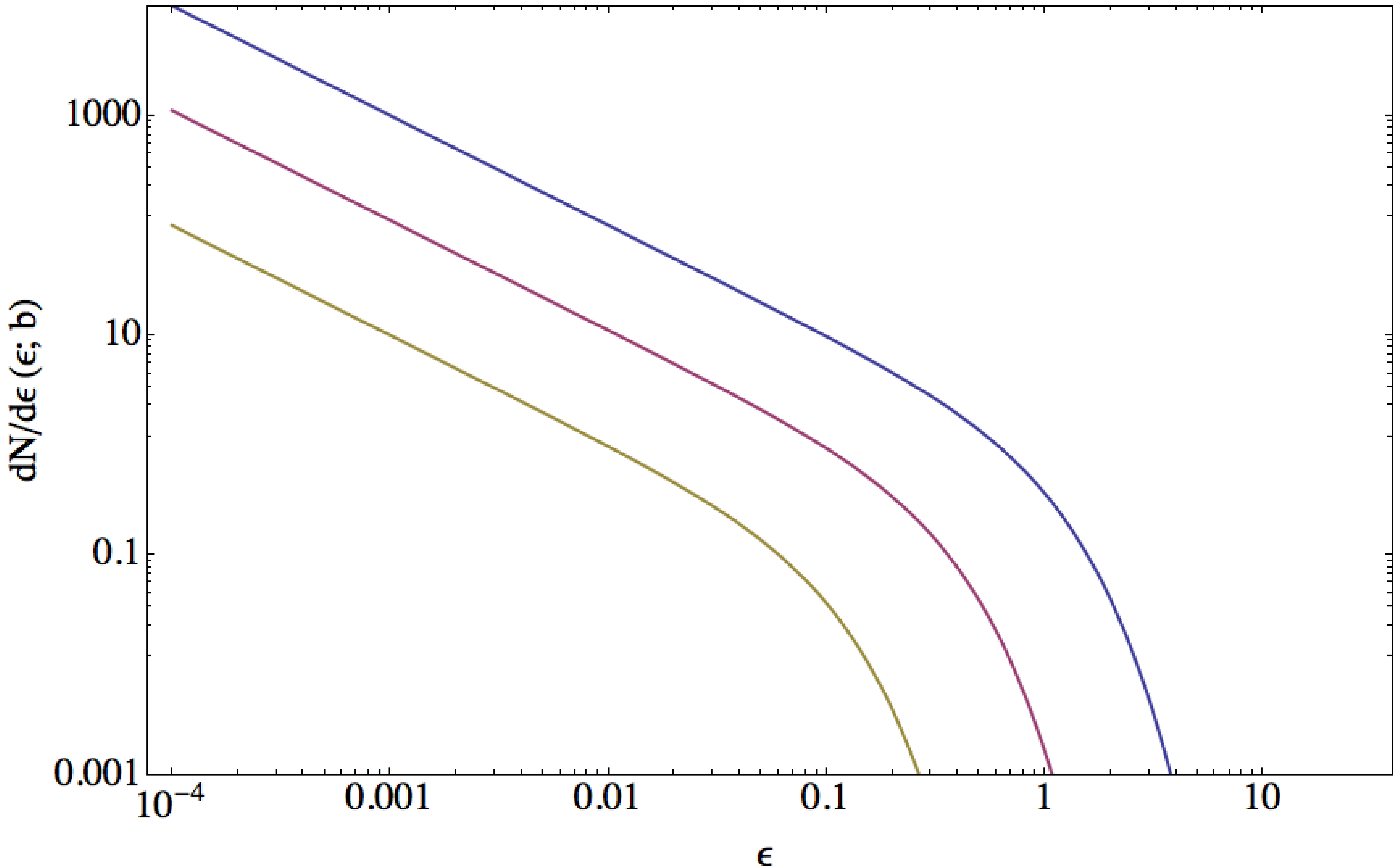
$$\boxed{\frac{dN_\gamma^{\rm virtual}}{d\omega\ dA} = \frac{q^2}{\pi^2}\frac{1}{b^2}\ \frac{1}{\omega}}$$

$$\frac{dN_\gamma^{\rm virtual}}{d\omega\ dA} \simeq 0$$

Electron/Photon interaction with a nucleus.

Can be seen as an interaction with a virtual photon of energy ε





1000
10
0.1
0.001
dN/dε (ε; b)
10−4
0.001
0.01
0.1
1
10
ε

$$\boxed{\frac{dN_\gamma^{\mathrm{virtual}}}{d\omega\ dA} = \frac{q^2}{\pi^2}\frac{1}{b^2}\ \frac{1}{\omega}}$$

Extends only up to a maximum impact parameter

$$\boxed{b_{\mathrm{max}} \simeq \min[\frac{1}{2\ \varepsilon}, \mathrm{R}_{\mathrm{atom}}]}$$

$$E_\omega = \frac{q}{2\,\pi\,b} \int dx \; e^{i\,(\omega\,b/\gamma)\,x} \; \frac{1}{(1+x^2)^{3/2}}$$

$$e^{i\,(\omega\,b/\gamma)\,x}$$

$$b_{\mathrm{max}} = \frac{\gamma}{\omega}$$

Considering the “Target Rest Frame

$$\varepsilon = \frac{\omega}{2\,\gamma}$$

$$b_{\mathrm{max}} = \frac{1}{2\,\varepsilon}$$

$$b_{\mathrm{max}} \simeq \min[\frac{1}{2\,\varepsilon}, \mathrm{R}_{\mathrm{atom}}]$$

$$\boxed{\langle r_{\mathrm{atomo}} \rangle \propto \frac{\xi}{m\,\alpha}\; Z^{-1/3} \propto \underline{183\;\; Z^{-1/3}}}$$

m

BREMSSTRAHLUNG

Fully ionized free nucleus (approximation of infinite mass)

$$\left.\frac{d\sigma}{d\varepsilon}\right|_{e\to e+\gamma}(v;E) = 4\,Z^2\,\alpha\,r0^2$$

$$\frac{1}{v}\;\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\;\left[\ln\left(\frac{2\;E}{m}\;\frac{v}{1-v}\right)-\frac{1}{2}\right]$$

High Energy Limit (Full screening)

$$\left.\frac{d\sigma}{d\varepsilon}\right|_{e\to e+\gamma}(v;E) = 4\,Z^2\,\alpha\,r0^2$$

$$\frac{1}{v}\;\left\{\left[1+(1-v^2)-\frac{2}{3}(1-v)\right]\;\ln\left(183\,Z^{-1/3}\right)+\frac{1}{9}(1-v)\right\}$$

PAIR PRODUCTION

Fully ionized free nucleus (approximation of infinite mass)

$$\left.\frac{d\sigma}{du}\right|_{\gamma\rightarrow e^+e^-}(u;K) = 4\ Z^2\ \alpha\ r0^2$$

$$\left[u^2 + (1-u)^2 + \frac{2}{3}u\,(1-v)\right]\ \left[\ln\left(\frac{2\ K}{m}\,u(1-u)\right) - \frac{1}{2}\right]$$

High Energy Limit (Full screening)

$$\left.\frac{d\sigma}{du}\right|_{\gamma\rightarrow e^+e^-}(u;K) = 4\ Z^2\ \alpha\ r0^2$$

$$\left\{\ \left[u^2 + (1-u)^2 + \frac{2}{3}u\,(1-v)\right]\ \ln\left(183\,Z^{-1/3}\right) - \frac{1}{9}u(1-u)\right\}$$

Total Cross section for Pair Production (full screening)

$$\sigma_{\mathrm{pair}}^{\mathrm{tot}} = \int_0^1 du\, \frac{d\sigma_{\mathrm{pair}}}{du}(u) = 4\,\alpha\, r_0^2\, Z^2 \left\{ \ln[183\, Z^{-1/3}]\, \frac{7}{9} - \frac{1}{54} \right\}$$

Total Pair Production Cross section

$$\sigma_{\gamma \rightarrow e^+ e^-}(K) = \int d^2 b \int d\varepsilon \, \frac{dN_\gamma^{\mathrm{virtual}}}{dA \, d\omega} \, \sigma_{\gamma\gamma \rightarrow e^+ e^-}(\hat{s})$$

$$\hat{s} = 4 \, K \, \varepsilon$$

Total Pair Production Cross section

$$\sigma_{\gamma\to e^+e^-}(K) = \int d^2b \int d\varepsilon \, \frac{dN_\gamma^{\rm virtual}}{dA\, d\omega} \, \sigma_{\gamma\gamma\to e^+e^-}(\hat{s})$$

$$(2\,\pi) \int db\, b \int d\varepsilon \left\{ \frac{\alpha\, Z^2}{\pi^2\, b^2} \, \frac{1}{\varepsilon} \, \Theta[b_{\rm max}(\varepsilon) - b] \right\} \quad \sigma_{\gamma\gamma}(4\, K\, \varepsilon)$$

$$b_{\rm max}(\varepsilon) \simeq \frac{1}{2\,\varepsilon}$$

$$b_{\rm max} \simeq R_{\rm atom}$$

$$\sigma_{\gamma \to e^+ e^-}(K) = \frac{\alpha \; Z^2}{\pi^2} \; \left[(2\,\pi) \; \int_{b_{\min}}^{b_{\max}} \frac{db}{b} \right]$$

$$\int_{4\,m^2/K}^{\infty} \frac{d\varepsilon}{\varepsilon} \; \sigma_{\gamma\gamma}(4\,K\,\varepsilon)$$

$$\sigma_{\gamma\to e^+e^-}(K) = \frac{\alpha\ Z^2}{\pi^2}\ \left[(2\ \pi)\ \int_{b_{\min}}^{b_{\max}} \frac{db}{b}\right]$$

$$\int_{4\,m^2/K}^{\infty} \frac{d\varepsilon}{\varepsilon}\ \sigma_{\gamma\gamma}(4\ K\ \varepsilon)$$

$$\frac{2\,\alpha\ Z^2}{\pi}\ \log\left[\frac{b_{\max}}{b_{\min}}\right]\ \int_{4\,m^2}^{\infty} \frac{ds}{s}\ \sigma_{\gamma\gamma}(s)$$

$$\boxed{\int_{4\,m^2}^{\infty} \frac{ds}{s}\sigma_{\gamma\gamma\to e^+e^-}(s) = (2\pi\ r_0^2)\ \frac{7}{9}}$$

$$\sigma_{\gamma \to e^+ e^-}(K) = \frac{2\,\alpha\ Z^2}{\pi}\ \log\left[\frac{b_{\rm max}}{b_{\rm min}}\right]\quad (2\pi\ r_0^2)\ \frac{7}{9}$$

Estimate of bmax and bmin

$$b_{\rm min} \simeq \frac{\hbar}{m_e\,c}$$

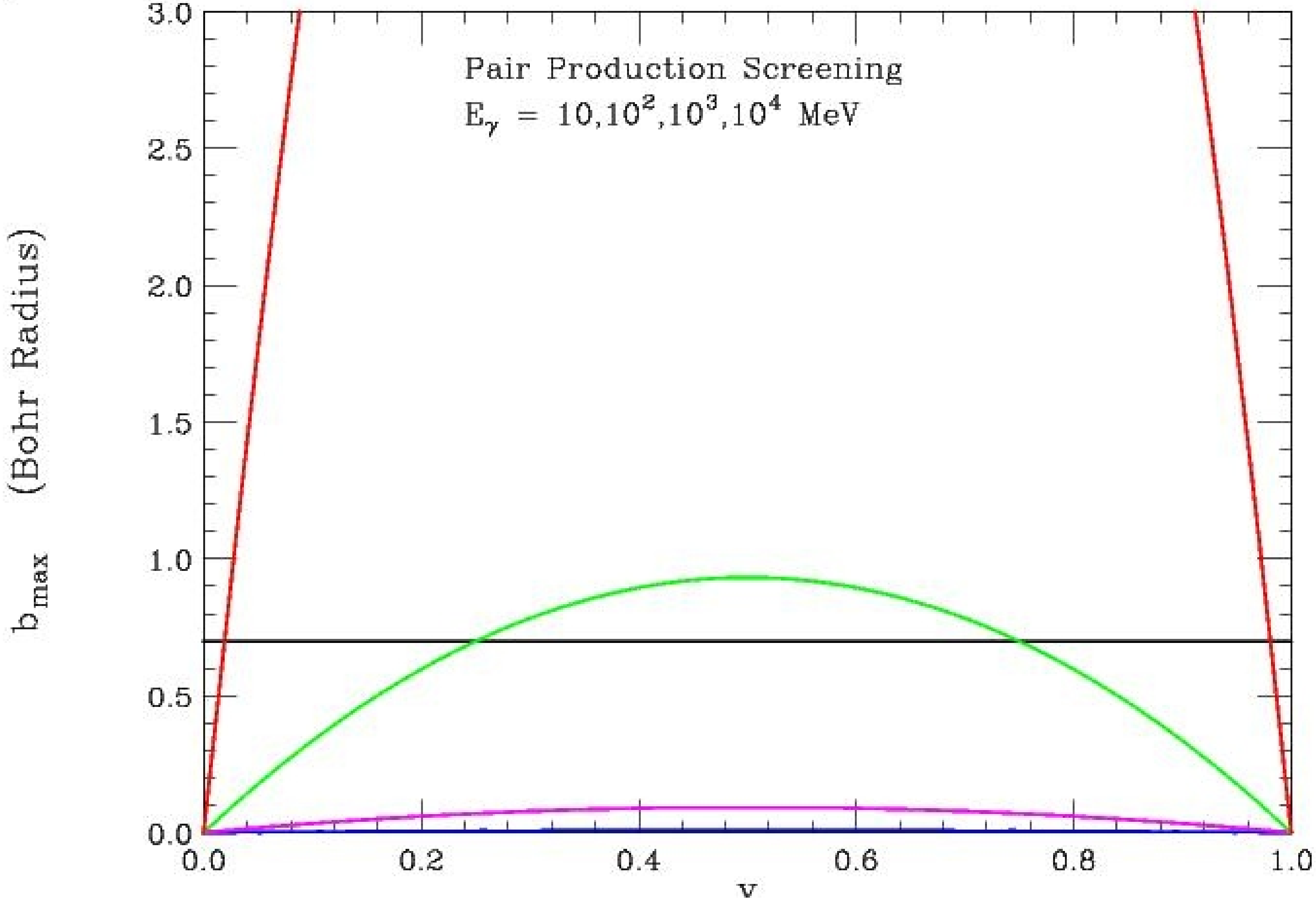
Quantum mechanical
Effect
(uncertainty principle)

$$\frac{d\sigma_{\text{brems}}}{dv}(v, E_e) = \int_{\varepsilon_{\min(v,E_e)}}^{\infty} d\varepsilon \; [\ldots]$$

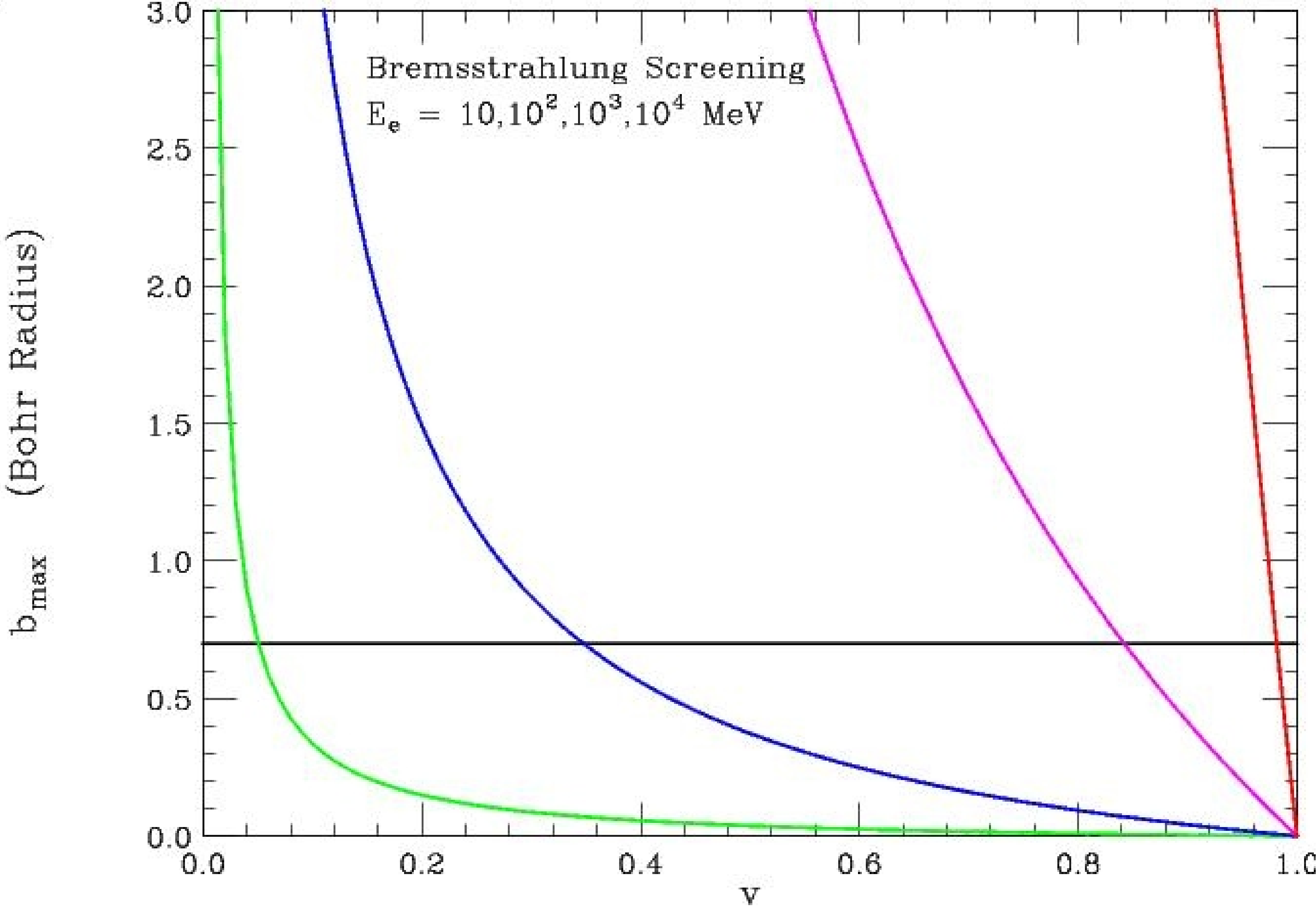
$$\frac{d\sigma_{\text{pair}}}{du}(u, E_\gamma) = \int_{\varepsilon_{\min(u,E_e)}}^{\infty} d\varepsilon \; [\ldots]$$

$$\varepsilon_{\min}(v, E_e) = \frac{m^2}{4\,E_e}\,\frac{v}{1-v}$$

$$\varepsilon_{\min}(u, E_\gamma) = \frac{m^2}{4\,E_e\,u(1-u)}$$



Pair Production Screening
E_γ = 10,10^2,10^3,10^4 MeV
b_{max} (Bohr Radius)
v
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.2
0.4
0.6
0.8


Bremsstrahlung Screening
$E_e = 10, 10^2, 10^3, 10^4$ MeV
b_{max} (Bohr Radius)
v
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Total Cross section for Pair Production (full screening)

$$\sigma_{\text{pair}}^{\text{tot}} = \int_0^1 du \, \frac{d\sigma_{\text{pair}}}{du}(u) = 4\,\alpha\, r_0^2 \, Z^2 \left\{ \ln[183\, Z^{-1/3}]\, \frac{7}{9} - \frac{1}{54} \right\}$$

Bremsstrahlung

$$\sigma_{\text{brems}}^{\text{tot}} \to \infty$$

$$\int_0^1 dv\; v\, \frac{d\sigma_{\text{brems}}}{dv}(v) = 4\,\alpha\, r_0^2 \, Z^2 \left\{ \ln[183\, Z^{-1/3}] + \frac{1}{18} \right\}$$

$$\left.\frac{dE}{dX}\right|_{\mathrm{brems}} = \frac{N_A}{A} \int dE_\gamma \; E_\gamma \; \frac{d\sigma}{dE_\gamma}(E_\gamma, E_e)$$

$$\frac{dE}{dX} = \left\{ \frac{N_A}{A} \int_0^1 dv \; v \; \frac{d\sigma}{dv}(v) \right\} \; E = \frac{E}{\lambda_{\mathrm{rad}}}$$

$$\left.\frac{dE}{dX}\right|_{\text{brems}} = \frac{N_A}{A} \int dE_\gamma\ E_\gamma\ \frac{d\sigma}{dE_\gamma}(E_\gamma, E_e)$$

$$\frac{dE}{dX} = \left\{\frac{N_A}{A}\ \int_0^1 dv\ v\ \frac{d\sigma}{dv}(v)\right\}\ E = \frac{E}{\lambda_{\text{rad}}}$$

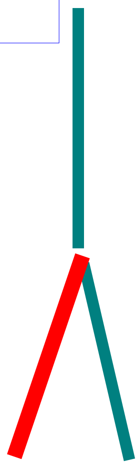
$$\lambda_{\text{rad}} = \frac{N_A}{A}\ 4\,\alpha\ r_0^2\ \left\{\ln[183\ Z^{-1/3}] + \frac{1}{18}\right\}$$

$$\boxed{\lambda_{\text{rad}} = \frac{N_A}{A}\ 4\,\alpha\ r_0^2\ \ln[183\ Z^{-1/3}]}$$

Radiation Length

The "SPLITTING FUNCTIONS"

$$\varphi(v) = \left[\frac{d\sigma}{dv}(v)\right]_{\rm brems} \left(\frac{N_A}{A}\,\lambda_{\rm rad}\right)$$



$$\psi(u) = \left[\frac{d\sigma}{du}(u)\right]_{\rm pair} \left(\frac{N_A}{A}\,\lambda_{\rm rad}\right)$$