$|V_{us}|$ FROM HADRONIC τ DECAYS

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OUTLINE

- V_{us} *from inclusive flavor-breaking sum rules*
  - *Systematics in the conventional implementation and a resolution of the* $> 3\sigma$ *low* $|V_{us}|$ *puzzle*
  - *New implementation strategy results + current experimental limitations*

- *A new lattice+inclusive* us *V+A* τ *data strategy*

CONTEXT

- τ vs. non-τ $|V_{us}|$ determinations: the $> 3\,\sigma$ low inclusive FB τ FESR $|V_{us}|$ puzzle

$\vert V_{us}\vert$	Source
0.2258(9)(?)	3-family unitarity, HT14 $\vert V_{ud}\vert$
$0.2231(4)_{exp}(7)_{latt}$	$K_{\ell 3}$, 2+1+1 lattice $f_+(0)$
$0.2250(4)_{exp}(9)_{latt}$	$\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, lattice f_K/f_π
$0.2176(19)_{exp}(10?)_{th}$	Inclusive FB kinematic wt τ FESR (Passemar CKM14)

- τ result: from "conventional implementation" of more general inclusive FB FESR framework

BASICS: HADRONIC τ DECAYS IN THE SM

- $R_{ij;V/A} \equiv \Gamma[\tau \to \nu_\tau \, \text{hadrons}_{ij;V/A}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$

- With $y_\tau \equiv s/m_\tau^2$, flavor ij decays in SM [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V+A}}{ds} = \frac{12\pi^2 \, |V_{ij}|^2 S_{EW}}{m_\tau^2} \, [1 - y_\tau]^2 \, \tilde{\rho}_{ij;V+A}(s)$$

$$\tilde{\rho}_{ij;V+A}(s) \equiv \left[(1 + 2y_\tau) \, \rho^{(J=1)}_{ij;V+A}(s) + \rho^{(J=0)}_{ij;V+A}(s) \right]$$

kinematic weight : $w_\tau(y) = (1 - y)^2(1 + 2y)$

THE INCLUSIVE FB τ $|V_{us}|$ DETERMINATION

- FESRs for $\Pi^{(J=0+1)}_{ud-us;V+A}(Q^2)$, $\rho^{(J=0+1)}_{ud-us;V+A}(s)$ Cauchy's theorem

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$

experiment OPE

- Experiment: $|V_{ij}|^2\rho^{(0+1)}_{ij;V/A}(s)$ from $dR_{ij;V/A}/ds$

mildly model-dependent continuum J=0 *us* subtraction

- $R^w_{ij;V/A}(s_0)$: re-weighted $R_{ij;V/A}$ analogue

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds\, \frac{dR_{ij;V/A}}{ds}\, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

- FB differences $\delta R^w(s_0) \equiv \frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - \frac{R^w_{us;V+A}(s_0)}{|V_{us}|^2}$

- FESR, OPE for $\delta R^w(s_0)$, input $|V_{ud}| \Rightarrow$

$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

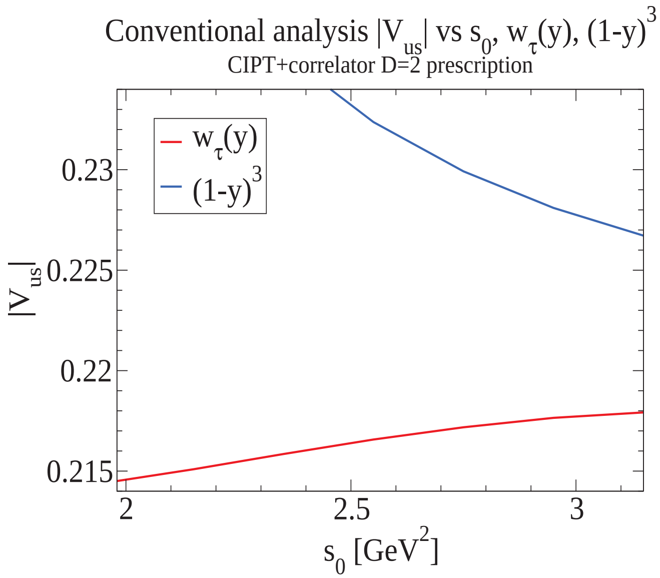
Self-consistency: $|V_{us}|$ independent of s_0, w

- **The conventional implementation** [Gamiz et al. JHEP03(2003)060]
  - $s_0 = m_\tau^2$, $w = w_\tau$ only [spectral integrals from inclusive ud, us BFs, **but no self-consistency tests**]
  - w_τ degree 3 $\Rightarrow$ OPE ($\sum_D C_D/Q^D$) to $D = 8$
  - $D > 4$ **assumptions**: C_6 (VSA, small), C_8 (~ 0)

- **Conventional implementation tests** [KM et al. arXiv:1511.08514]
  - $|V_{us}|$ stability checks with variable $s_0 \leq m_\tau^2$
  - Targeted $D = 6, 8$ assumptions test: $y = (s/s_0)$,
    $w_\tau(y) = 1 - 3y^2 + 2y^3$ c.f. $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$

D=8

D=6



- Slow $D = 2$ convergence also a potential issue

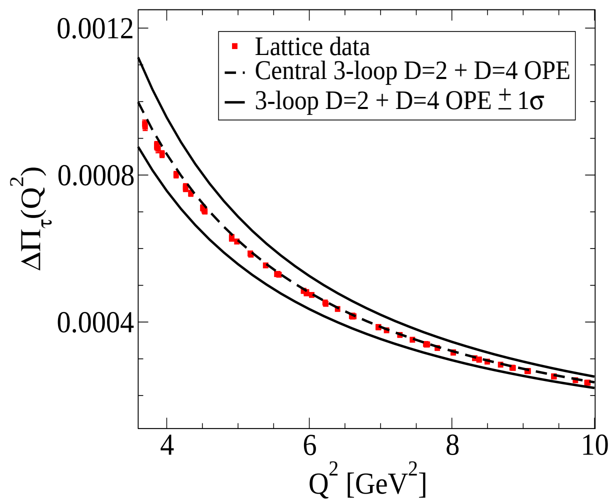
$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\Big[1 + \frac{7}{3}\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + \cdots\Big]$$

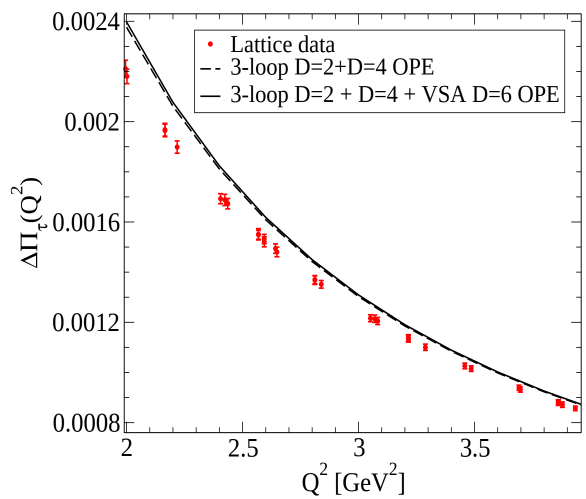
$\overline{MS}$ running $\bar{a} = \frac{\alpha_s(Q^2)}{\pi}$, $\bar{m}_s = m_s(Q^2)$, $\bar{a}(m_\tau^2) > 0.1$

- OPE/lattice $\Pi^{(0+1)}_{ud-us;V+A}(Q^2)$ comparison
  - $n_f = 2{+}1$, $m_\pi \sim 300\ MeV$, $1/a = 2.38\ GeV$, $m_\pi L \sim 4.1$, $32^3 \times 64$ RBC/UKQCD ensemble
  - Tight cylinder cut for continuum correlator behavior
  - Excellent lattice/$D = 2 + 4$ OPE match for fixed scale, 3-loop $D = 2$, $Q^2 \sim 4 - 10\ GeV^2$ [FIG]

- Conventional OPE error estimates VERY conservative despite slow $D=2$ convergence [FIG]

- Confirms non-negligible $D>4$, $Q^2<4\ GeV^2$ [FIG]





AN ALTERNATE FB FESR IMPLEMENTATION

(Mainz workshop talk [HLMZ15, MPLA31 (2016) 1630037] for details)

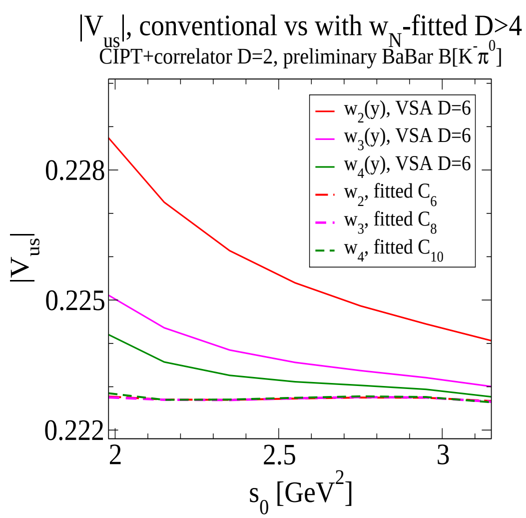
- Theory side
  - No $D > 4$ assumptions: effective condensates $C_{D>4}$ from fits to data (variable s_0 **required**)
  - 3-loop-truncated FOPT $D = 2$, standard $D = 2 + 4$ error estimates [as per comparison to lattice]
  - C_{2N+2}, $|V_{us}|$ from $w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ FESR
  - $|V_{us}|$ from different w_N as self-consistency check

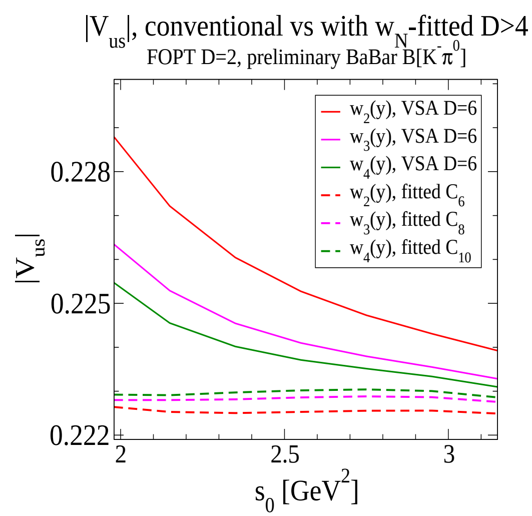
- Experimental input
  - Updated/corrected 2013 ALEPH for ud V+A
  - us V+A from sum over exclusive modes
    - K from $K_{\mu 2}$
    - $K\pi$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$: BaBar, Belle unit-normalized distributions, BFs

      [Note: HFAG $B[K^-\pi^0\nu_\tau] =$ 0.00433(15) (BaBar dominated) c.f. preliminary BaBar (Adametz) thesis 0.00500(15) (recommended by BaBar)]
    - Remaining ("residual modes") from 1999 ALEPH (note: $\sim 25\%$ errors, some MC)

RESULTS

- Unphysical s_0-, $w(y)$-dependence problems solved. E.g., for BaBar (Adametz thesis) $B[K^-\pi^0\nu_\tau]$ input





- $|V_{us}|$ increased by ~ 0.0020 with fitted $C_{D>4}$

- Significant impact of HFAG 2014 $\to$ preliminary BaBar Adametz thesis $B[K^-\pi^0\nu_\tau]$ (3-weight averages)

$$|V_{us}| = 0.2200(23)_{exp}(5)_{th} \quad (HFAG)$$
$$|V_{us}| = 0.2228(23)_{exp}(5)_{th} \quad (Adametz)$$

- Adametz $B[K^-\pi^0\nu_\tau]$ input w-independence example

Weight	$\vert V_{us}\vert$ CIPT+corr $D=2$	$\vert V_{us}\vert$ FOPT $D=2$
w_2	0.2227(23)	0.2225(23)
w_3	0.2227(23)	0.2228(23)
w_4	0.2227(23)	0.2230(23)

- New implementation, updated $B[K^-\pi^0\nu_\tau]$ completely resolves old $> 3\sigma$ low $|V_{us}|$ puzzle

- Very favorable (~ 0.0005) theory error situation

- us spectral integral uncertainty dominates current error

- Error budget, 3-weight, Adametz $B[K^-\pi^0\nu_\tau]$, 3-loop-truncated FOPT $D = 2$ fit

Source	$\delta\lvert V_{us}\rvert$ (w_2 FESR)	$\delta\lvert V_{us}\rvert$ (w_3 FESR)	$\delta\lvert V_{us}\rvert$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00004	0.00004
$\delta m_s(2\ GeV)$	0.00017	0.00019	0.00019
$\delta\langle m_s\bar{s}s\rangle$	0.00035	0.00035	0.00035
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00028	0.00028
us exp	0.00226	0.00227	0.00227

- *Theory error* $\Rightarrow$ *competitive with* $K_{\ell 3}$*,* $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ **with sufficient** us **experimental error improvement**

- us experimental uncertainties currently BF dominated

- BFs more easily improved experimentally than exclusive mode dR/ds distribution contributions

- Near-term low-multiplicity mode progress likely [combined BaBar, Belle (+Belle II) effort on spectral functions from existing B-factory data under way]

- **However** sub-0.5% $|V_{us}|$ needs sub-% $R^w_{us;V+A}$ error

- Exclusive us mode w_N spectral integral contributions

Relative exclusive mode $R^w_{us:V+A}$ contributions

Wt	s_0 $[GeV^2]$	K	$K\pi$	$K\pi\pi$ (B-factory)	Other
w_2	2.15	0.496	0.426	0.062	0.010
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

- **"Other":** 1999 ALEPH data/MC, $\sim$ 25% error

  $\Rightarrow$ **"sufficient improvement" includes experimentally (much) more challenging higher-multiplicity modes**

A PROMISING τ-BASED ALTERNATIVE

- Work with J. Hudspith, T. Izubuchi, R. Lewis, H. Ohki, C. Lehner $+ \cdots$ (RBC/UKQCD)

- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ us V+A polarizations with weights having poles at Euclidean Q^2

  - $\tilde{\Pi}(Q^2)$: polarization sum with spectral function $\tilde{\rho}(s)$ (experimental $dR_{us;V+A}/ds$)

  - Theory: Lattice us 2-point function data (no OPE)

  - Weights tunable, allow suppression of larger-error, higher-multiplicity us spectral contributions

More on the lattice-inclusive us τ approach

- $|V_{us}|^2\, \tilde{\rho}_{us;V+A}(s)$ from experimental $dR_{us;V+A}/ds$

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1 + 2\frac{s}{m_\tau^2}\right) \rho^{(J=1)}_{us;V+A}(s) \,+\, \rho^{(J=0)}_{us;V+A}(s)$$

(no continuum us $J = 0$ subtraction required)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1 - 2\frac{Q^2}{m_\tau^2}\right) \Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)$$

- $\tilde{\rho}_{us;V+A}(s) \sim s$ as $s \to \infty$

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)}$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

$$\int_{th}^{\infty} ds\, w_N(s)\, \tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)}$$

experiment

lattice

  - Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS

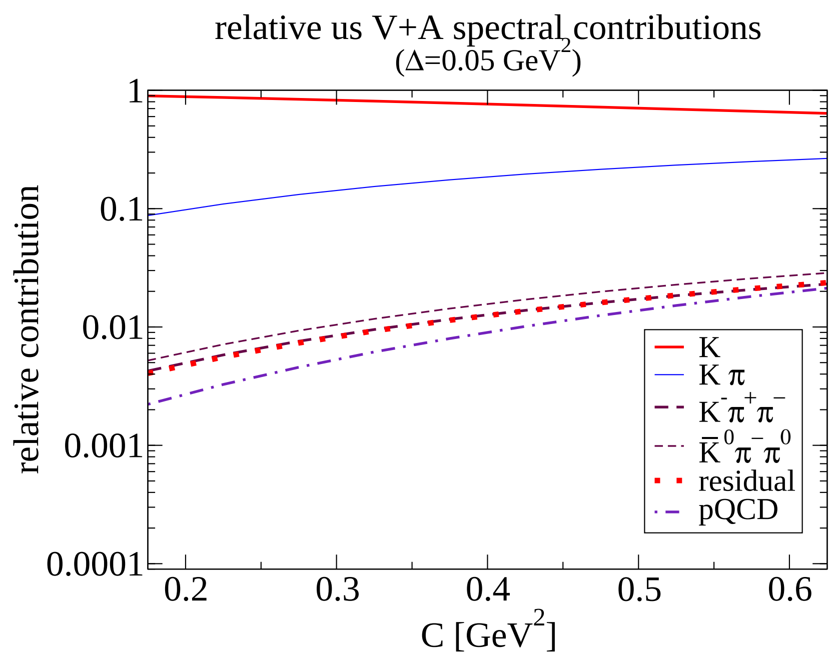
  - LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$

  - $w_N(s)$: rapid fall-off if all $Q_k^2 < 1\ GeV^2$
    $\Rightarrow$ K, $K\pi$ **dominate LHS, near-endpoint multi-particle,** $s > m_\tau^2$ **contributions strongly suppressed**

  - Optimization: increasing $\{Q_k^2\}$ decreases RHS lattice error, increases LHS experimental error

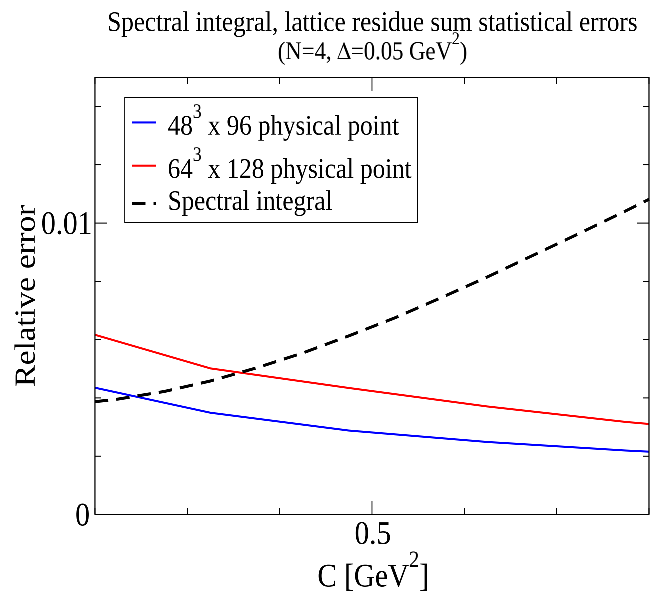
- Below: uniform pole spacing Δ, centroid C

- "Tuning" impact example [$N = 4$, $\Delta = 0.05\ GeV^2$]



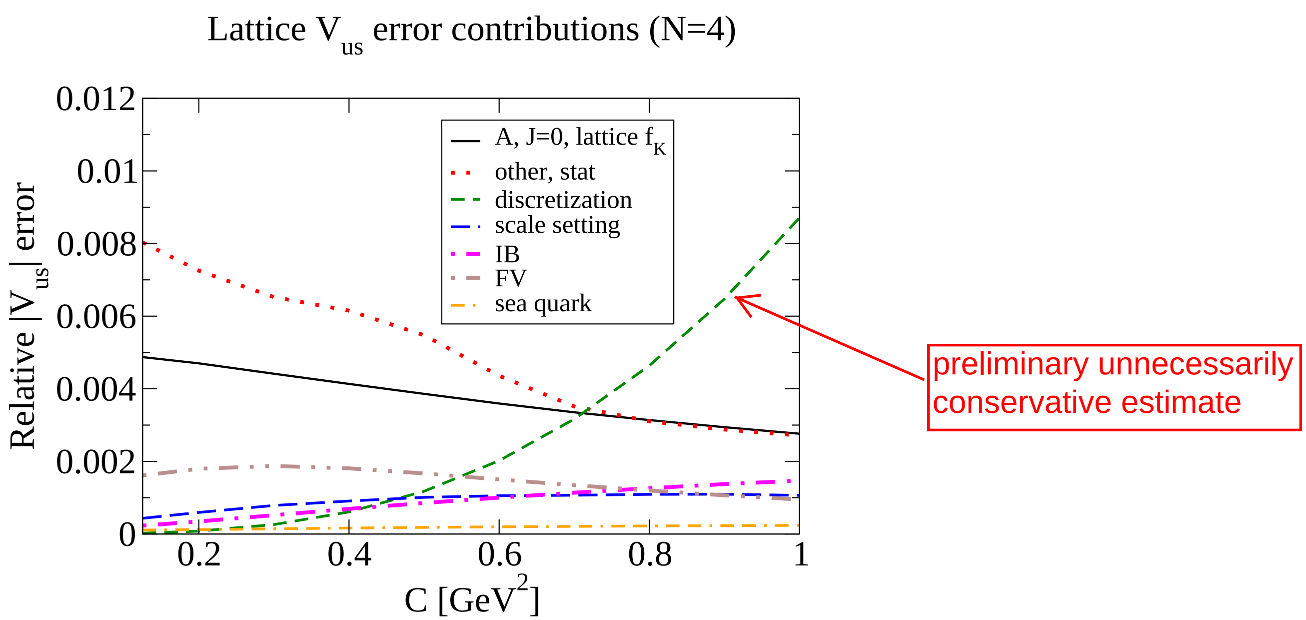
- Sample experimental, lattice statistical errors vs C

(RBC/UKQCD near-physical-point ensembles)

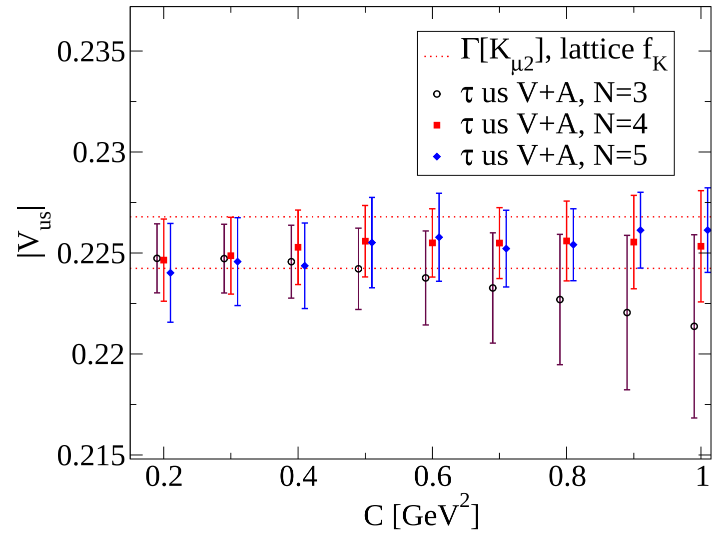


- *Sample $|V_{us}|$ lattice residue error contributions*

$$(N = 4,\ \Delta = 0.067\ GeV^2)$$



- *PRELIMINARY inclusive lattice us V+A results*



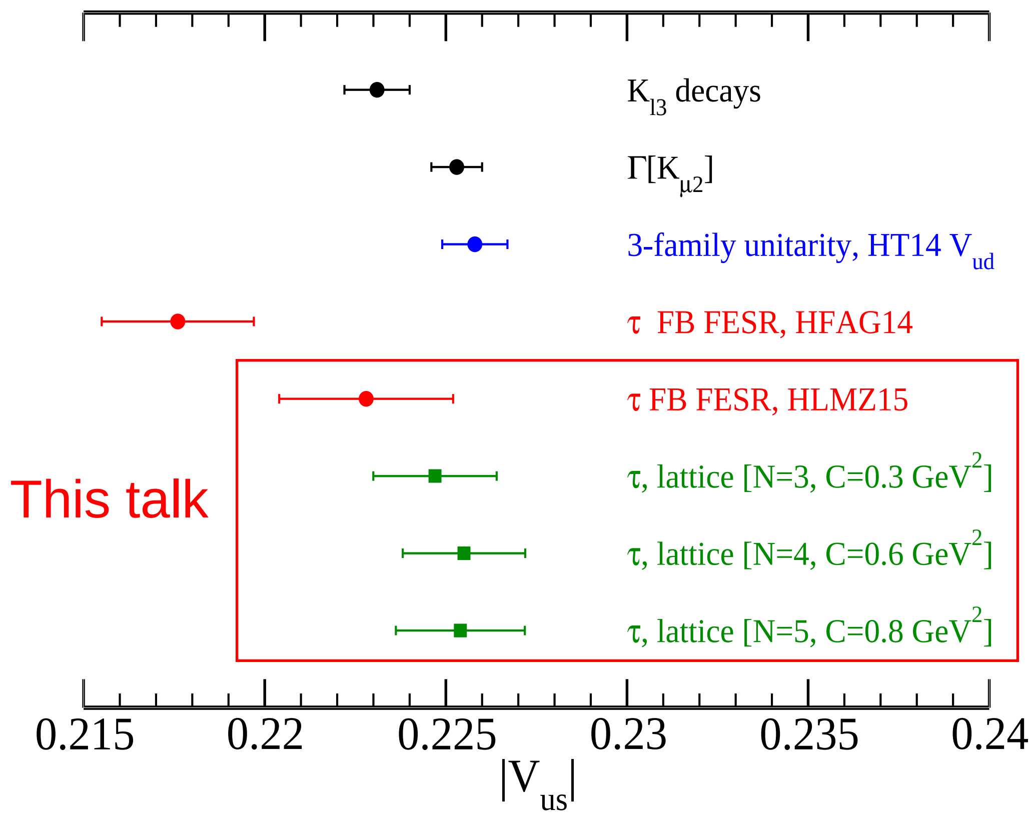
E.g. $N = 3$, $C = 0.3\ GeV^2$: $|V_{us}| = 0.2247(10)_{exp}(14)_{th}$

PRELIMINARY

- Finalizing optimization/systematics (FV, scale setting, continuum limit, IB, $m_{\ell,s}$ mistuning)

- Advantages of lattice-based vs. FB FESR approach
  - K essentially saturates $J = 0$, A contribution $\Rightarrow |V_{us}|$ determinations possible with or without K pole
  - Self-consistency tests via C-, Δ-independence
  - Reduced experimental error (smaller high-s, higher-multiplicity spectral contributions c.f. FB FESR case) *without blowing up theory errors*
  - Theory side: lattice in place of OPE $\Rightarrow$ errors systematically improvable

Comparison to $|V_{us}|$ from other sources



SUMMARY

- Old 3σ low inclusive FB τ FESR $|V_{us}|$ problem resolved
  - Alternate, no-assumptions implementation: $|V_{us}|$ higher by $\sim$ 0.0020, compatible with other determinations
  - Near-term improvements feasible through improvements in us exclusive mode BFs
  - Highly favorable theoretical error situation
  - However, for competitive $|V_{us}|$ need improvements to old ALEPH higher-multiplicity, low-statistics data [unlikely in the near-term]

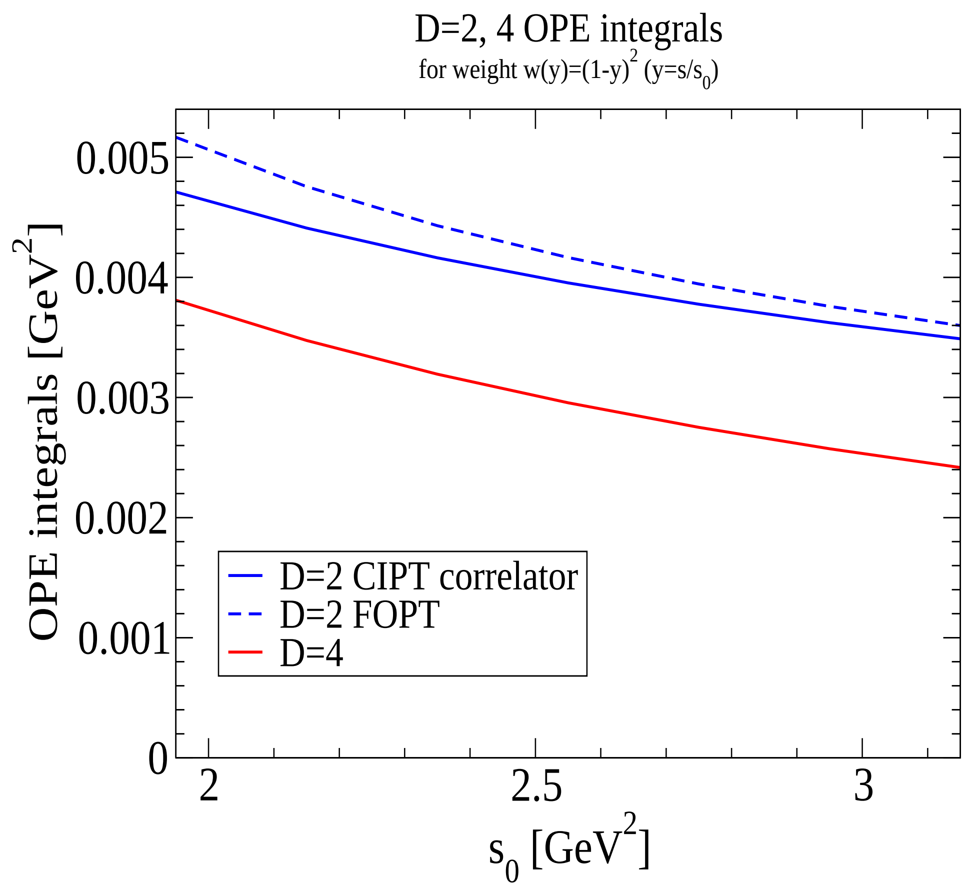
- Advantage of new lattice-inclusive us $V+A$ τ approach
  - Theory:
    - Lattice in place of OPE; no us $J=0$ subtraction; improvement through increased statistics
    - Parasitic on lattice a_μ effort (major effort in lattice community)
  - Spectral integrals:
    - Theory errors still small for weights strongly suppressing higher multiplicity contributions
    - Strong K, $K\pi$ dominance of spectral integral
    - Significant experimental improvements possible through just improved $K\pi$ BFs, distributions

BACKUP SLIDES

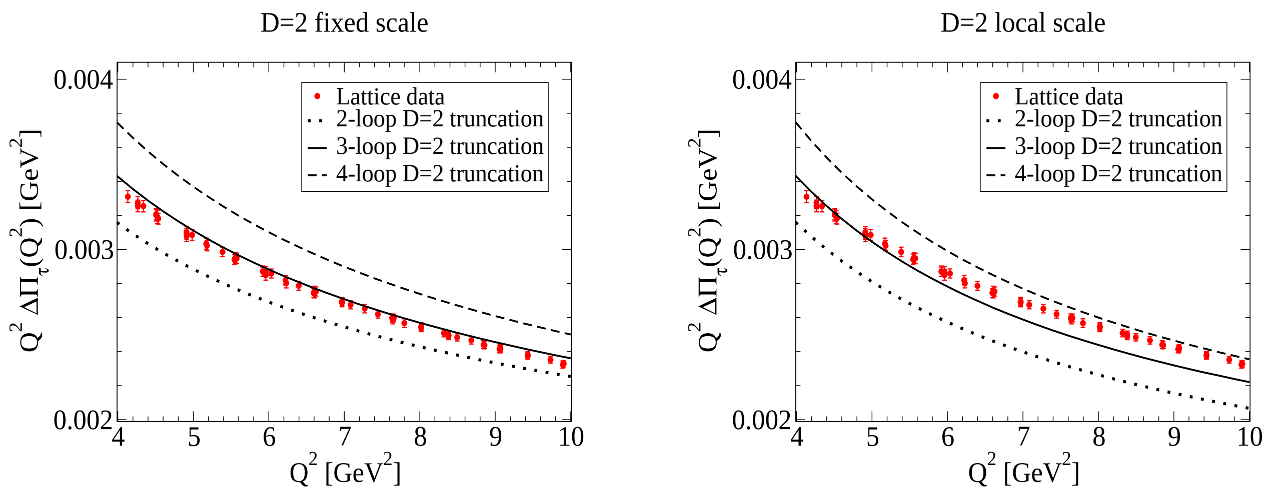
Are $D = 6, 8$ OPE contributions likely to be small for the conventional inclusive FB FESRs?

- $D = 4 << D = 2$ for $w_\tau(y) = 1 - 3y^2 + 2y^3$, $y = s/s_0$, "accidental" [$O(\alpha_s^2)$ suppression due to absence of term linear in y in $w_\tau(y)$]

- Comparison of $D = 2, 4$ OPE contributions for $w(y) = (1-y)^2$ (a case without this suppression) to see natural relative sizes


D=2, 4 OPE integrals
for weight w(y)=(1-y)^2 (y=s/s_0)
OPE integrals [GeV^2]
0.005
0.004
0.003
0.002
0.001
0
2
2.5
3
s_0 [GeV^2]
D=2 CIPT correlator
D=2 FOPT
D=4

Fixed- vs local-scale $D = 2$ series treatment



- Higher Q^2: best (excellent) lattice vs $D = 2 + 4$ OPE match for 3-loop-truncated, fixed-scale $D = 2$

- Fixed scale suggests FOPT for FESR $D = 2$

OPE, SPECTRAL INPUT

- PDG, FLAG, HPQCD input for $D = 2, 4$ OPE

- ud V+A spectral data from ALEPH 2013

- us V+A spectral data from sum over exclusive modes [$>$ 90% of B_{us}^{TOT} from $K_{\ell 2}$, Belle, BaBar $K\pi$, $K\pi\pi$, $3K$ results; residual: 1999 ALEPH]

- Favored $K\pi$ normalization: including preliminary BaBar $B[\tau \to K^- \pi^0 \nu_\tau]$ update (Adametz thesis)

MORE ON THE us DATA

- K pole via $f_K|V_{us}|$ from $K_{\ell 2}$

- Rather precise unit-normalized $K^-\pi^0$, $\bar{K}^0\pi^-$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$, $3K$ distributions from Belle, BaBar (main uncertainties from BFs)

- K, B-factory modes over 90% of B_{us}^{TOT}

- Residual us exclusive mode contributions (1999 ALEPH data, covariances) involves significant MC input

THE EXPERIMENTAL $K\pi$ BF SITUATION

- HFAG 2014 $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0126$

- HFAG 2014 $B[K^-\pi^0\nu_\tau] = 0.00433(15)$ value $\rightarrow$ preliminary BaBar (Adametz thesis) result 0.00500(15) yields $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0134$

- Central $B[(K^-\pi^0+\bar{K}^0\pi^-)\nu_\tau]$ from $K_{\ell 3}$, dispersive analysis expectations [ACLP13]: 0.0133

- 0.07% BF difference "small" but represents $\sim 2.4\%$ of B_{us}^{TOT}, hence $\sim 1.2\%$ increase in $|V_{us}|$

Results for $|V_{us}|$ for current HFAG 2014 $K\pi$ BFs

Weight	$\|V_{us}\|$ CIPT+corr $D=2$	$\|V_{us}\|$ FOPT $D=2$
w_2	0.21985(230)	0.21966(230)
w_3	0.21985(231)	0.21966(231)
w_4	0.21985(231)	0.22009(231)

Error budget, existing $K\pi$ BFs

Source	$\delta\|V_{us}\|$ (w_2 FESR)	$\delta\|V_{us}\|$ (w_3 FESR)	$\delta\|V_{us}\|$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00003	0.00005
$\delta m_s(2\ GeV)$	0.00017	0.00018	0.00020
$\delta\langle m_s\bar{s}s\rangle$	0.00034	0.00034	0.00034
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00027	0.00027
us exp	0.00229	0.00229	0.00230

Stability of $|V_{us}|$ with fitted C_{2N+2} input, existing $K\pi$ BF normalization