Mini-review of inclusive $|V_{ub}|$ measurements

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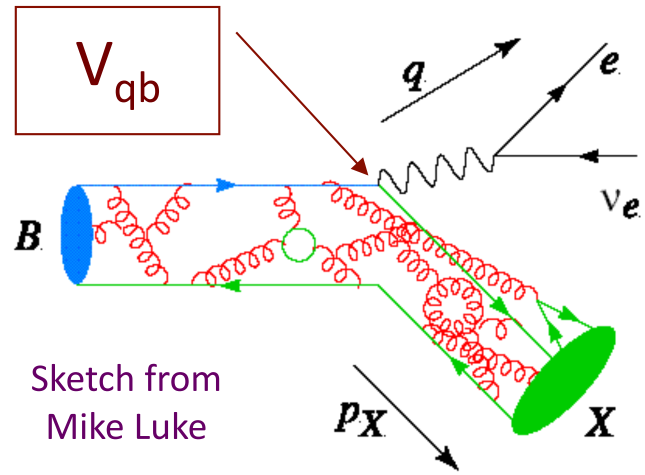
on behalf of the BaBar Collaboration

Semileptonic B decays

- Large BF, only one hadronic current
- Provide access to $|V_{qb}|$
- Inclusive decays $b \rightarrow qe\nu$:
  - Weak decay + QCD corrections
  - Operator Product Expansion in α_s and Λ_{QCD}/m_b
  - For $b \rightarrow ue\nu$ decays, unavoidable complications arise due to the large background from dominant $b \rightarrow ce\nu$ decays
    - kinematic cuts → non-perturbative "shape function" needed
    - universal only at leading order in Λ/m_b



Sketch from Mike Luke

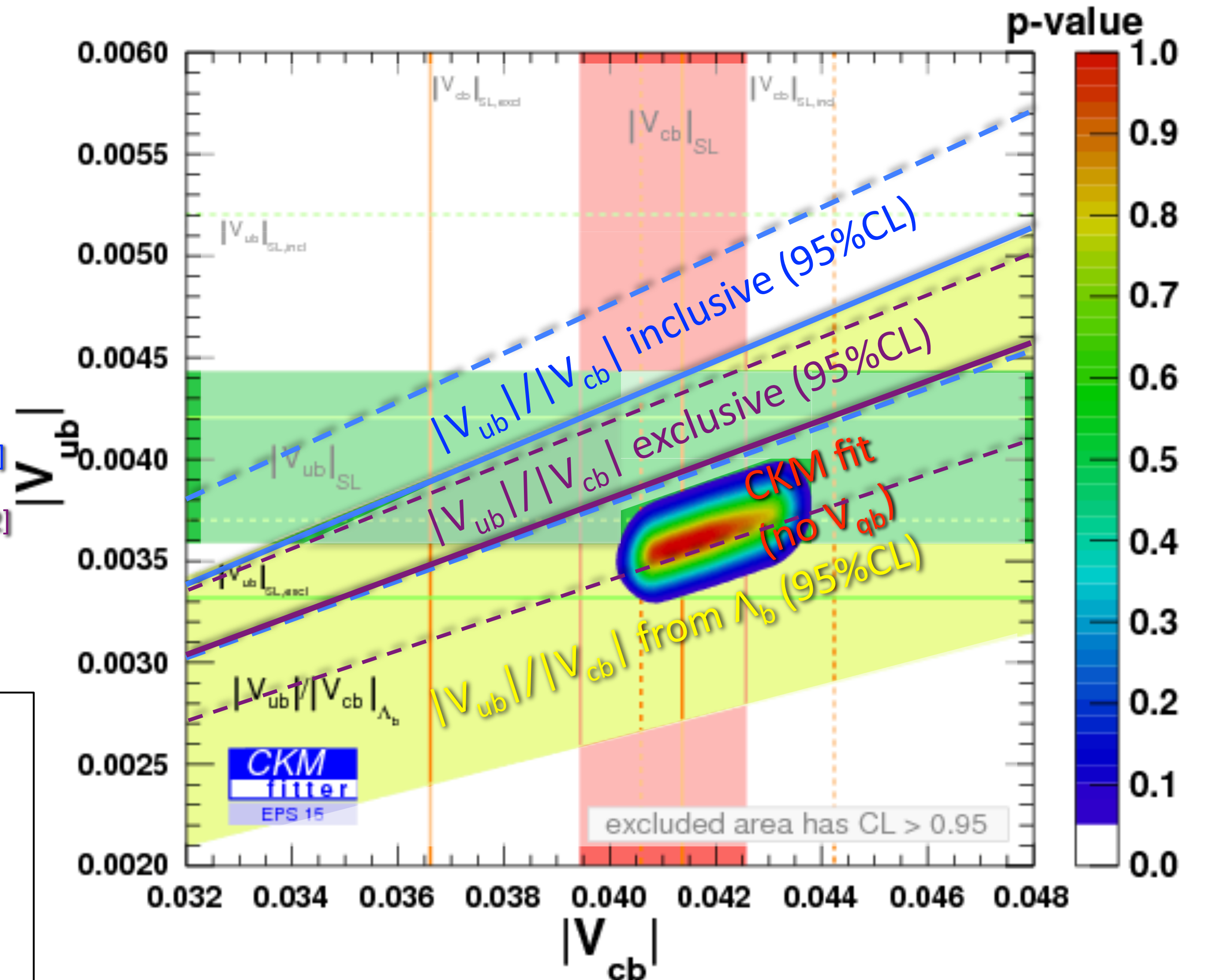
$|V_{qb}|$ – a messy business

- Diagonal bands show $|V_{ub}/V_{cb}|$ from three sets of measurements
- Ellipse is from CKM fit based on other inputs

Incl: $|V_{ub}/V_{cb}|=0.107\pm0.006$[1]

Excl: $|V_{ub}/V_{cb}|=0.095\pm0.005$[2]

Λ_b: $|V_{ub}/V_{cb}|=0.083\pm0.006$

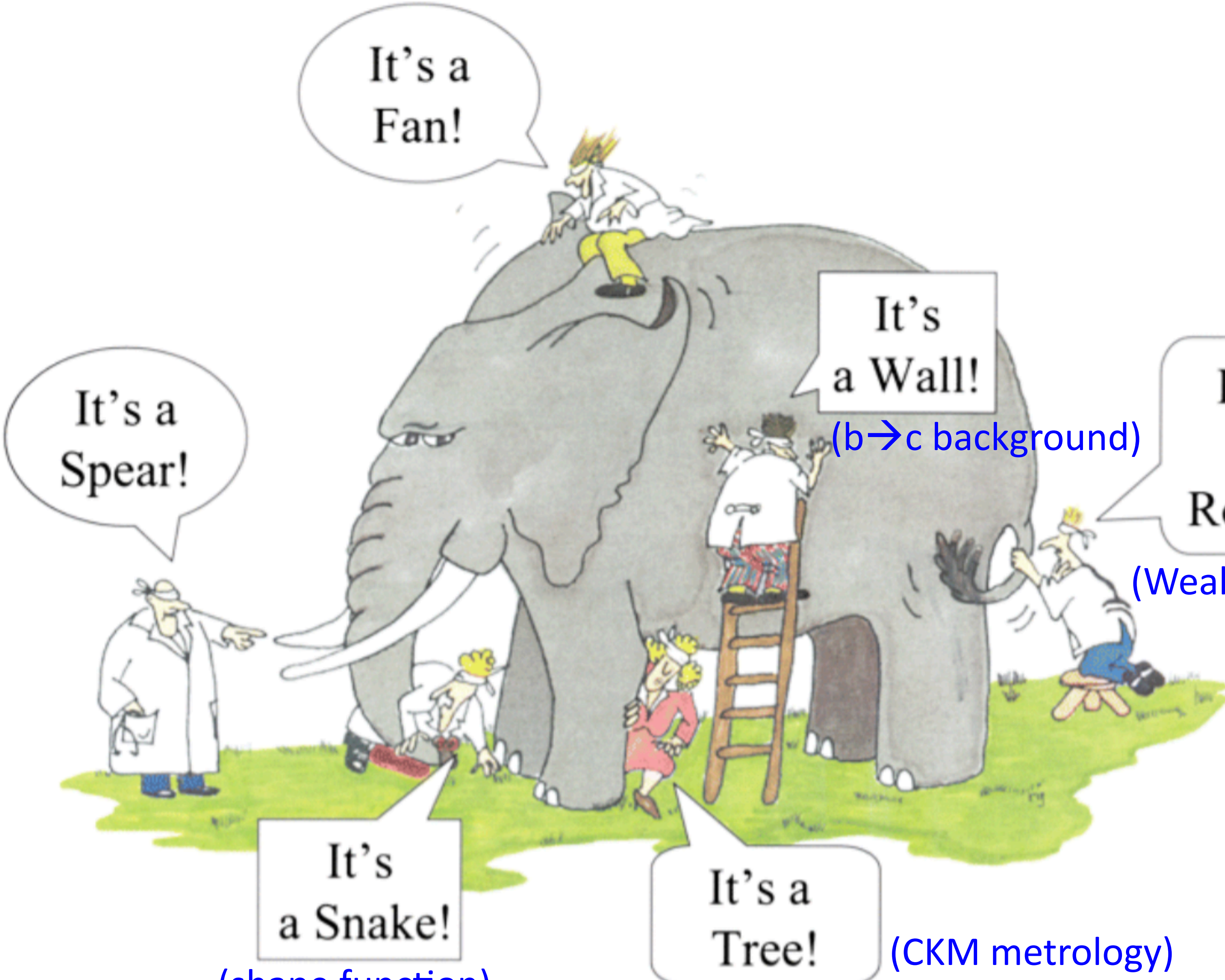


[1] $|V_{ub}| = (4.49\pm0.23)*10^{-3}$
$|V_{cb}| = (42.2\pm0.8)\ *10^{-3}$

[2] $|V_{ub}| = (3.72\pm0.19)*10^{-3}$
$|V_{cb}| = (39.2\pm0.7)\ *10^{-3}$

from PDG 2016 review

The inclusive $b \to u e \nu$ landscape



The inclusive b→uev landscape

This simple story hides the deepest of philosophical ideas:

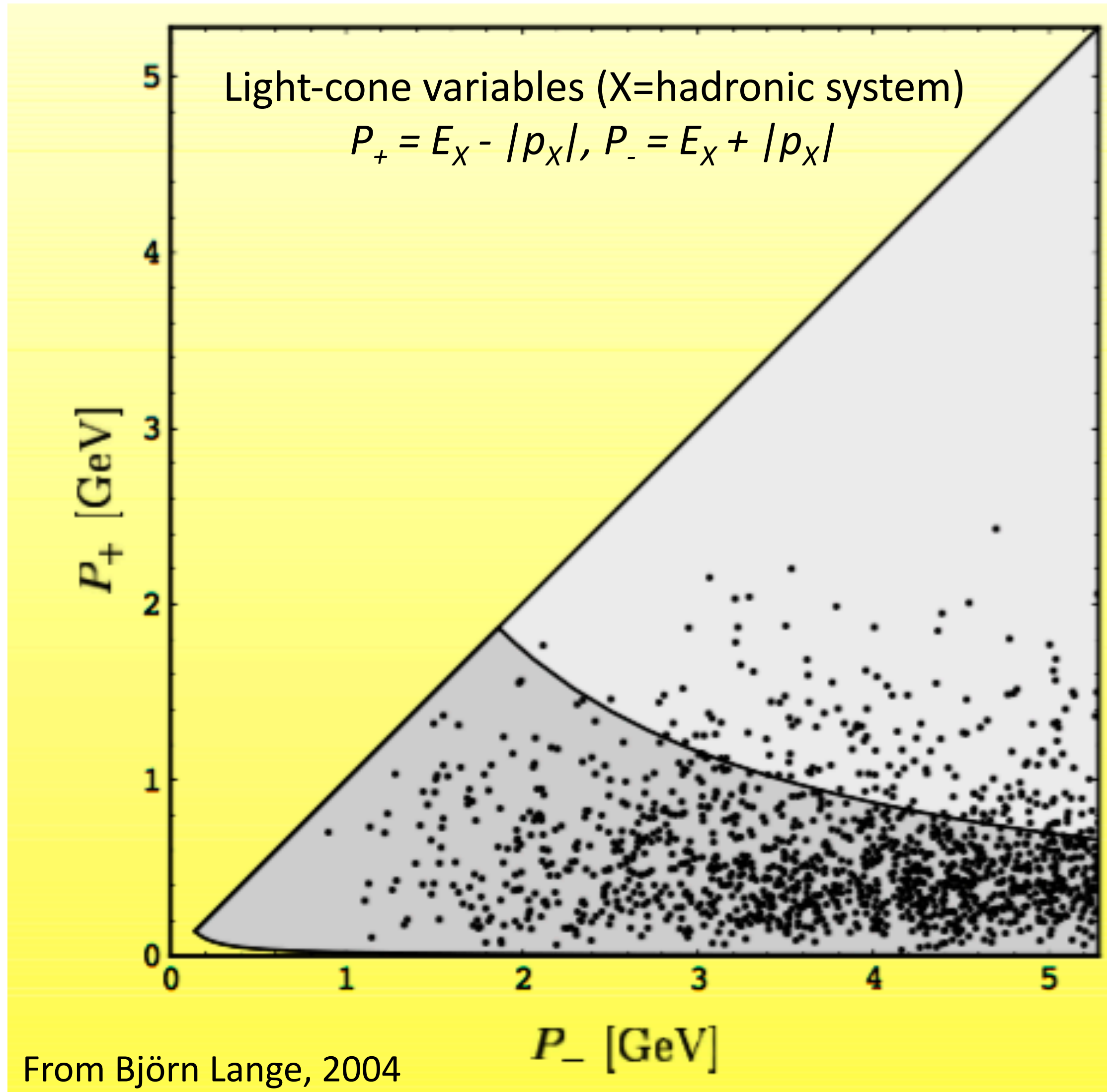
1. **Ignore judgment:** ...
2. **Be careful in giving/receiving advice:** Each of those men made an error in judgment.
3. **Improve your sampling:**
4. **Collaborate:**
5. **World is complex;** don't take shortcuts to a simplistic understanding.

tion)

(shape function)

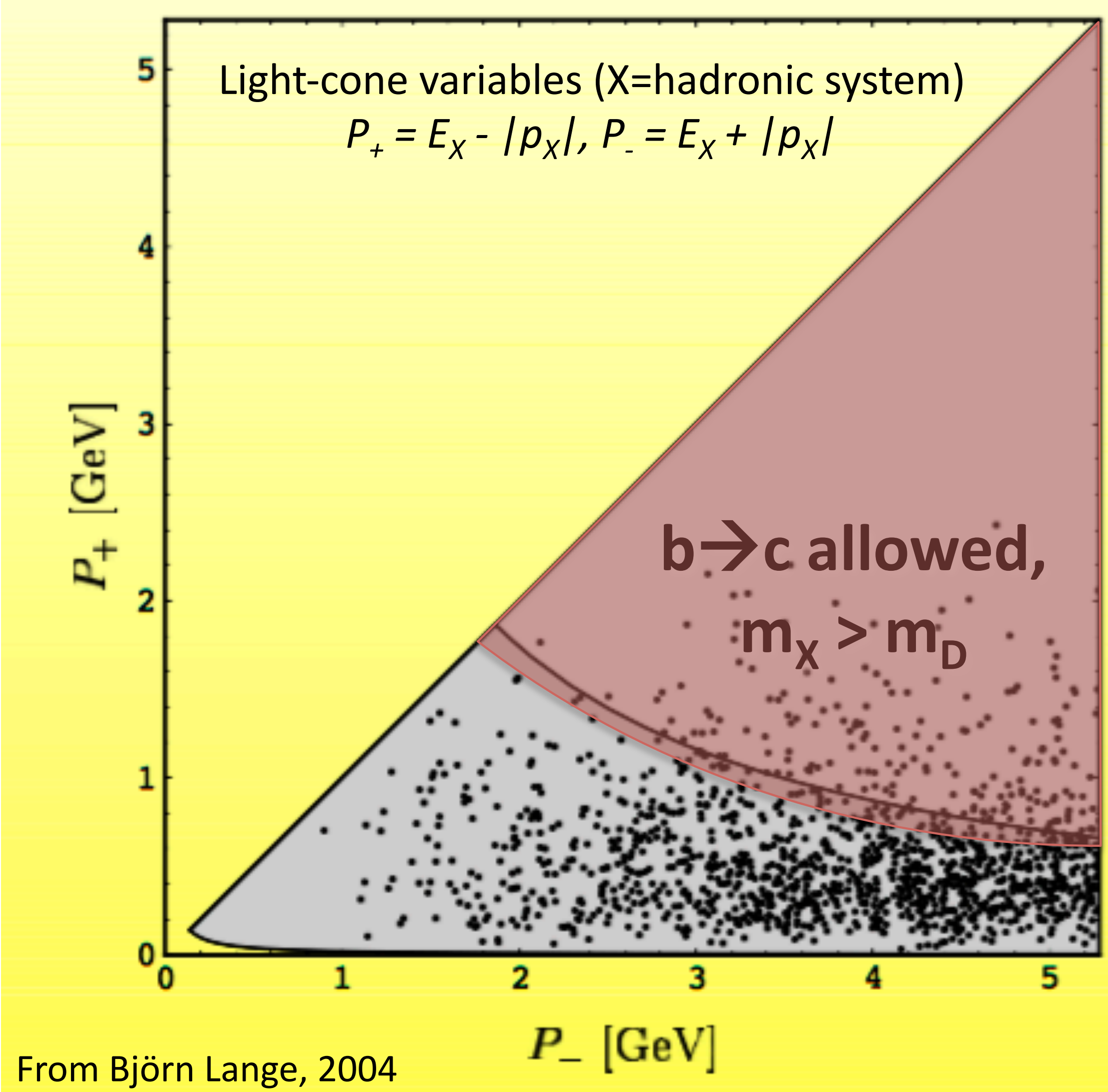
The inclusive b→uev landscape

- Nature – provides PDF for actual decays



The inclusive b→uev landscape

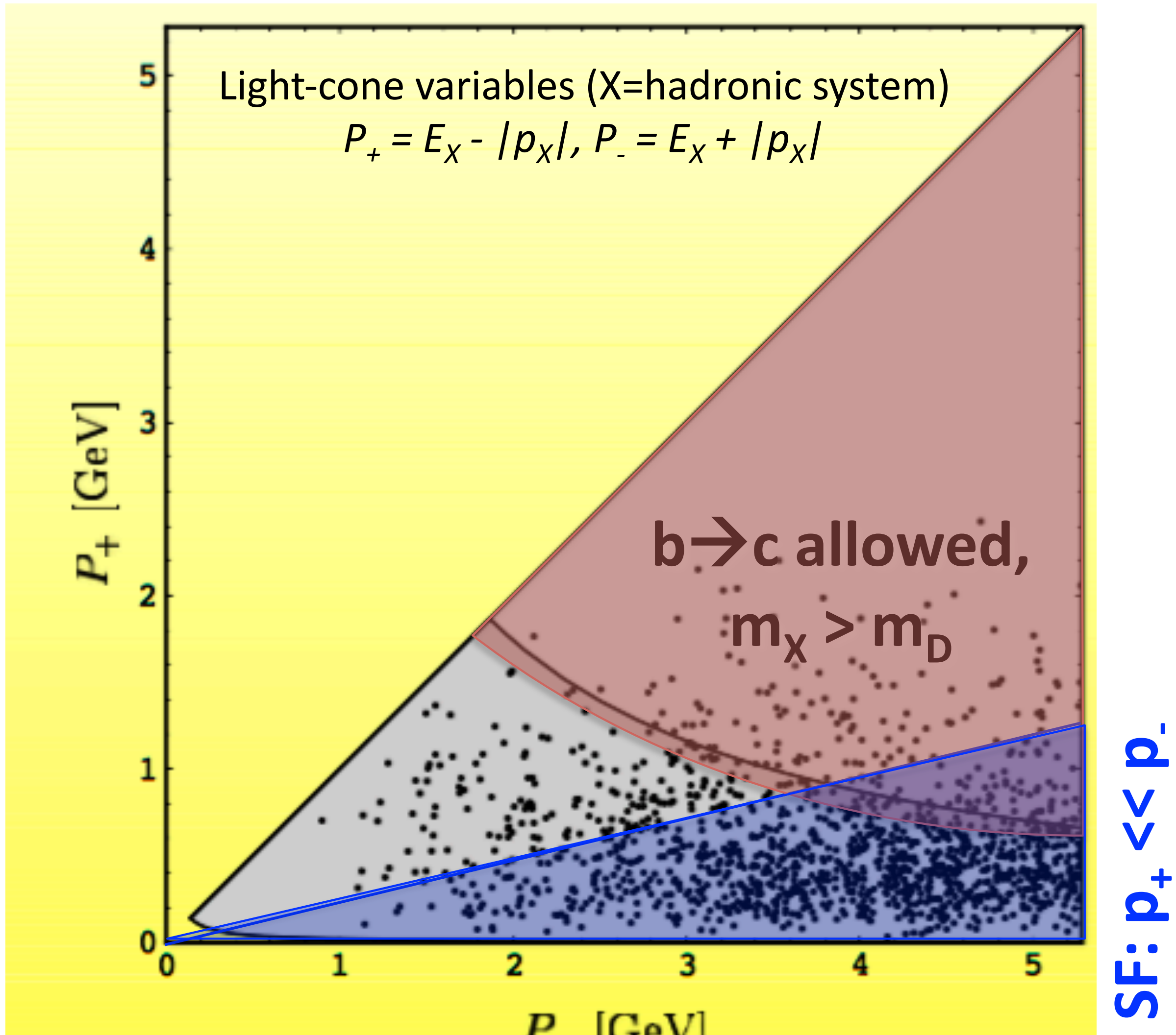
- Nature – provides PDF for actual decays
- Experimenter – avoid b→c background



From Björn Lange, 2004

The inclusive b→uev landscape

- Nature – provides PDF for actual decays
- Experimenter – avoid b→c background
- Theorist – avoid Shape Function region



From Björn Lange, 2004

The inclusive b→uev landscape

- Nature – provides PDF for actual decays
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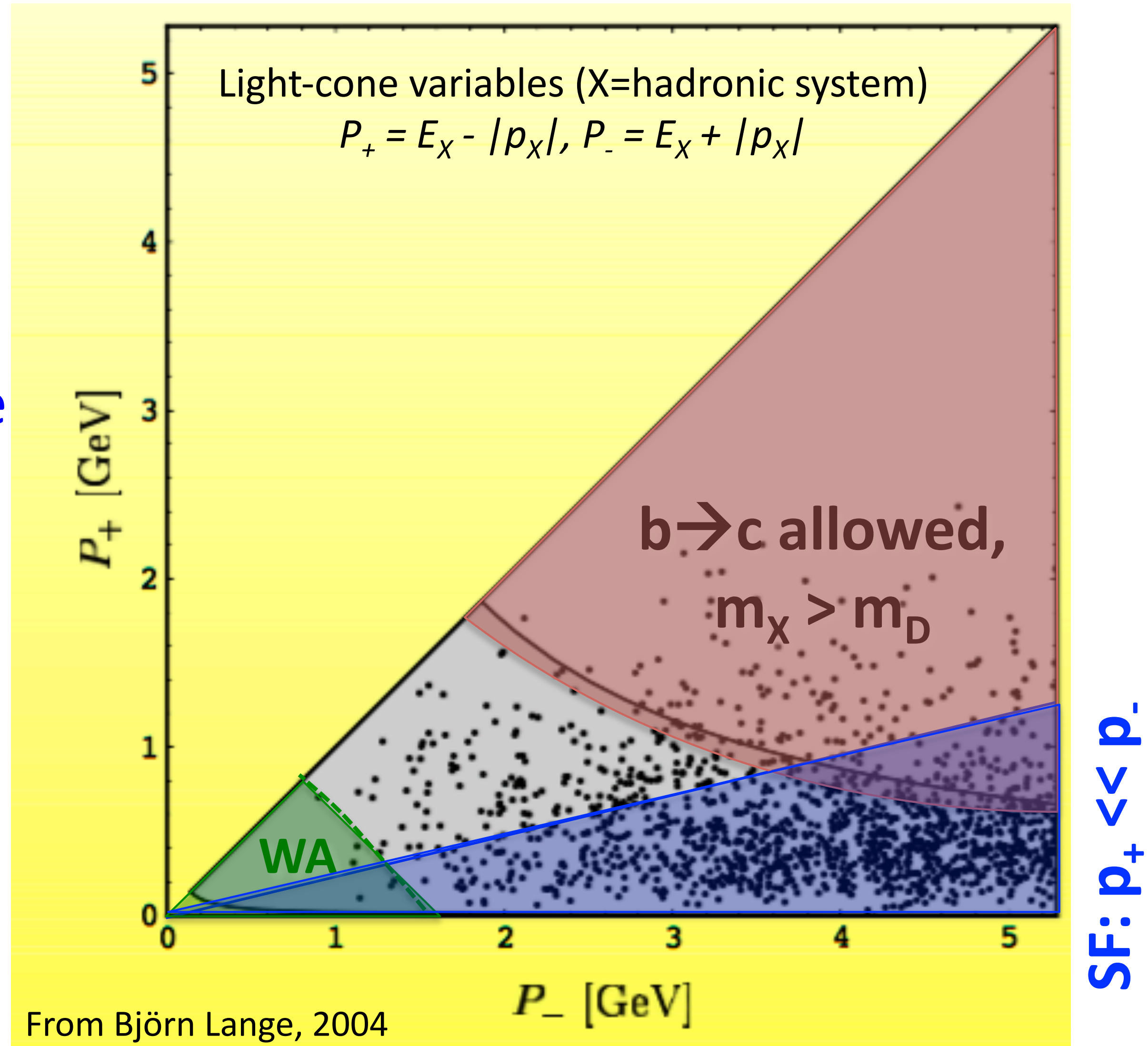
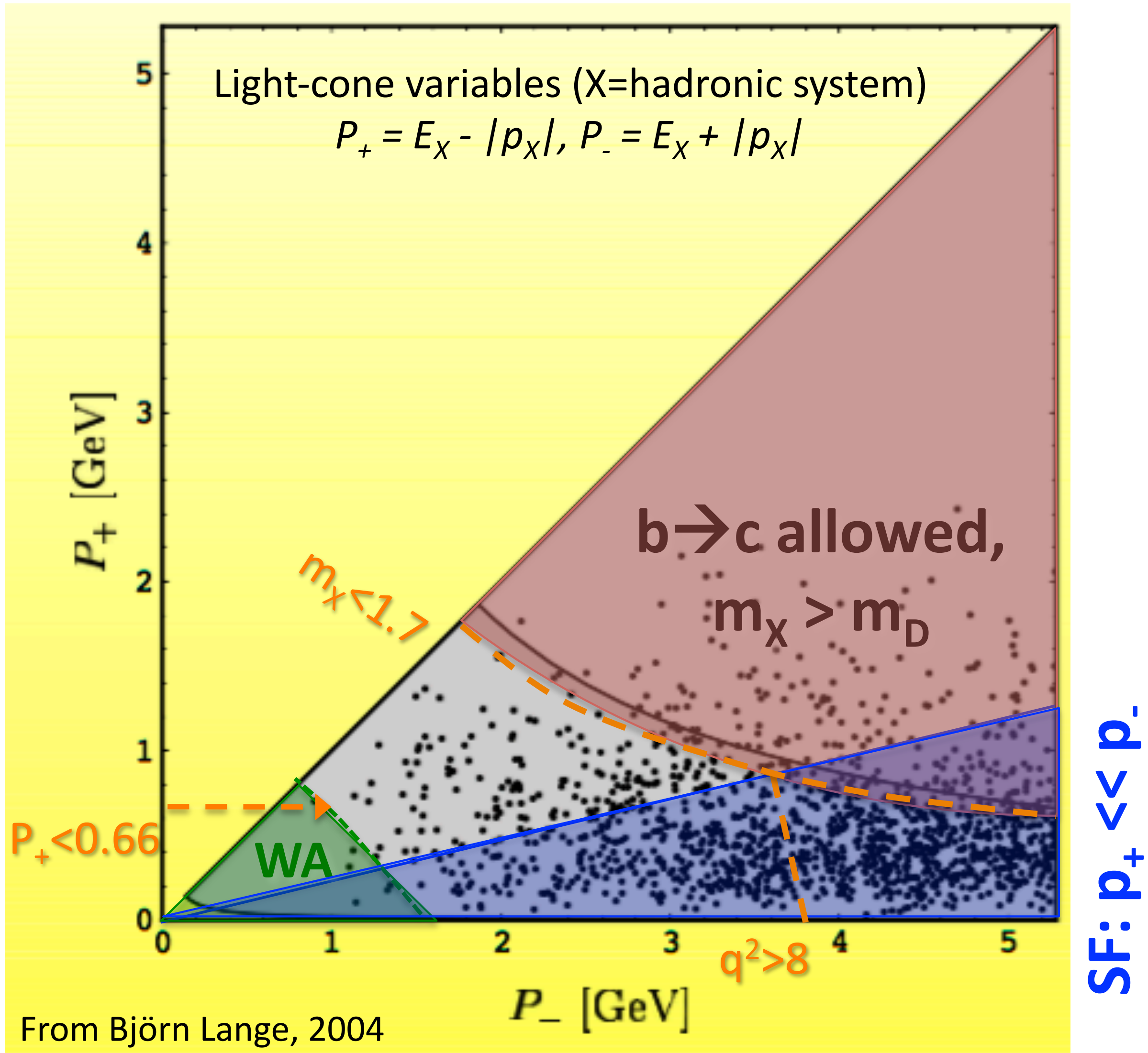
- All – avoid Weak Annihilation



From Björn Lange, 2004

The inclusive b→uev landscape

- Nature – provides PDF for actual decays
- Experimenter – avoid b→c background
- Theorist – avoid Shape Function region
- All – avoid Weak Annihilation
- Plan – look in multiple regions, assess consistency



Inclusive $|V_{ub}|$ status

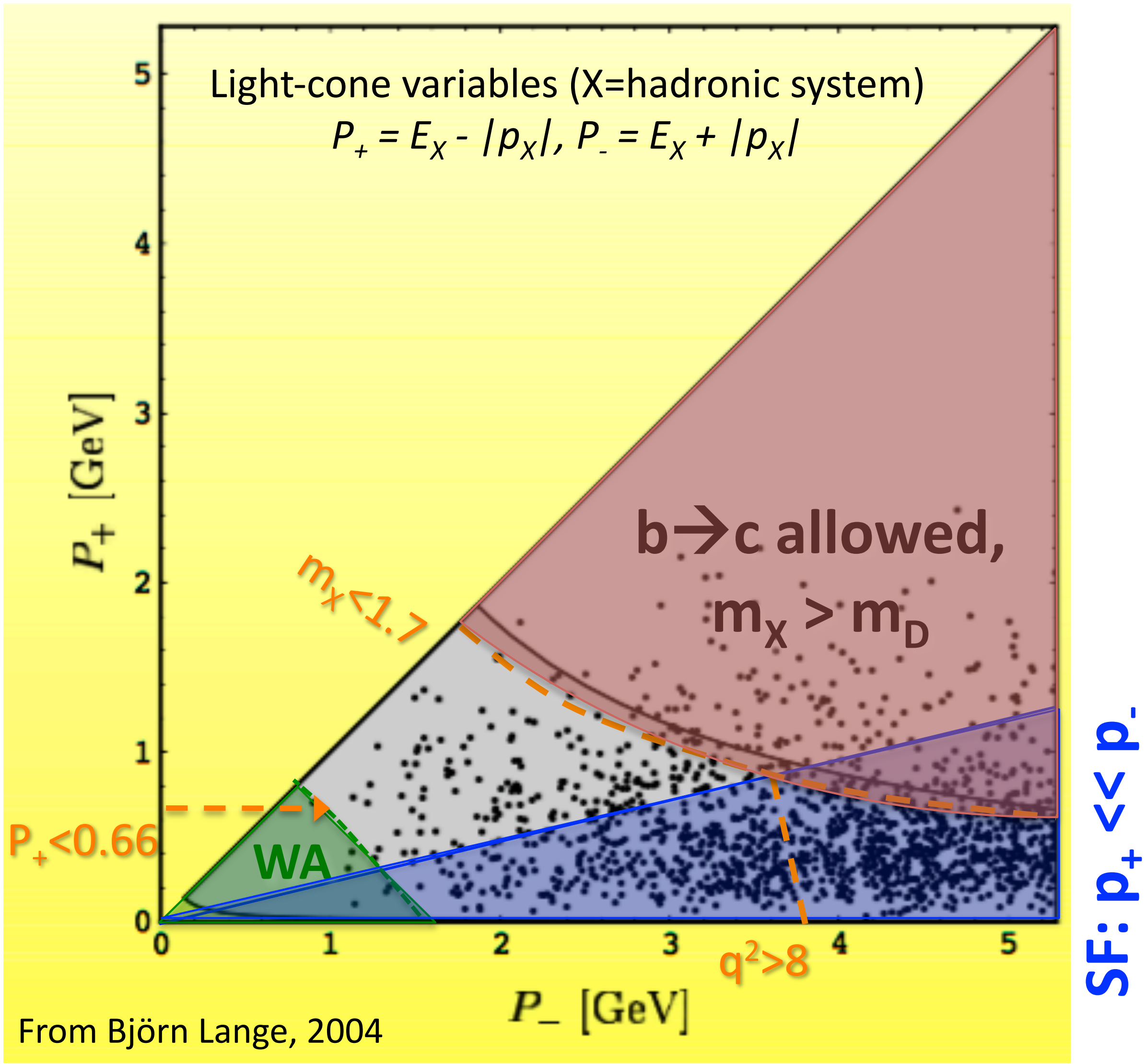
- HFAG/PDG 2016 summary[1] → (CLEO, BaBar, Belle)
- Different acceptance regions are consistent
- **Calculations agree with each other**
- N.b. – for all entries, $|V_{ub}|$ is recalculated from a partial rate measured with just one model
- Correlated uncertainties from
  - HQE parameters: from fits to b→c moments, m_c input, (or b→sγ)
  - Common experimental tools: EvtGen, JETSET hadronisation of X_u, b→cev background, GEANT4

Ref.	cut (GeV)	BLNP	GGOU	DGE
[108]	$E_e > 2.1$	$428 \pm 50\,^{+31}_{-36}$	$421 \pm 49\,^{+23}_{-33}$	$390 \pm 45\,^{+26}_{-28}$
[111]	$E_e - q^2$	$453 \pm 22\,^{+33}_{-38}$	not available	$417 \pm 20\,^{+28}_{-29}$
[110]	$E_e > 2.0$	$454 \pm 26\,^{+27}_{-33}$	$450 \pm 26\,^{+18}_{-25}$	$434 \pm 25\,^{+23}_{-25}$
[109]	$E_e > 1.9$	$493 \pm 46\,^{+27}_{-29}$	$493 \pm 46\,^{+17}_{-22}$	$485 \pm 45\,^{+21}_{-25}$
[113]	$q^2>8$, $m_X<1.7$	$430 \pm 23\,^{+26}_{-28}$	$432 \pm 23\,^{+27}_{-30}$	$427 \pm 22\,^{+20}_{-20}$
[113]	$P_+ < 0.66$	$415 \pm 25\,^{+28}_{-27}$	$424 \pm 26\,^{+32}_{-32}$	$424 \pm 26\,^{+37}_{-32}$
[113]	$m_X < 1.55$	$430 \pm 20\,^{+28}_{-27}$	$429 \pm 20\,^{+21}_{-22}$	$453 \pm 21\,^{+24}_{-22}$
[113]	$E_\ell > 1$	$432 \pm 24\,^{+19}_{-21}$	$442 \pm 24\,^{+9}_{-11}$	$446 \pm 24\,^{+13}_{-13}$
[115]	$E_\ell > 1$	$449 \pm 27\,^{+20}_{-22}$	$460 \pm 27\,^{+10}_{-11}$	$463 \pm 28\,^{+13}_{-13}$
HFAG average		$445 \pm 16\,^{+21}_{-22}$	$451 \pm 16\,^{+12}_{-15}$	$452 \pm 16\,^{+15}_{-16}$

[1] latest preliminary HFAG results in backup slides

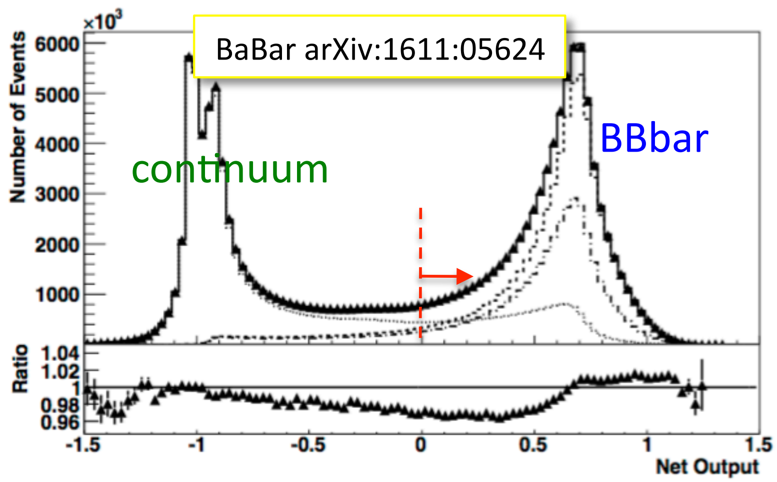
Plan for this talk

- Can we really push into the b→c region?
  - consider new BaBar endpoint analysis as an illustrative example
- Should we embrace the shape function?
  - plug for SIMBA, plea for results
- What can we improve using the large Belle II dataset?
  - need many detailed measurements
  - B tagging not a panacea

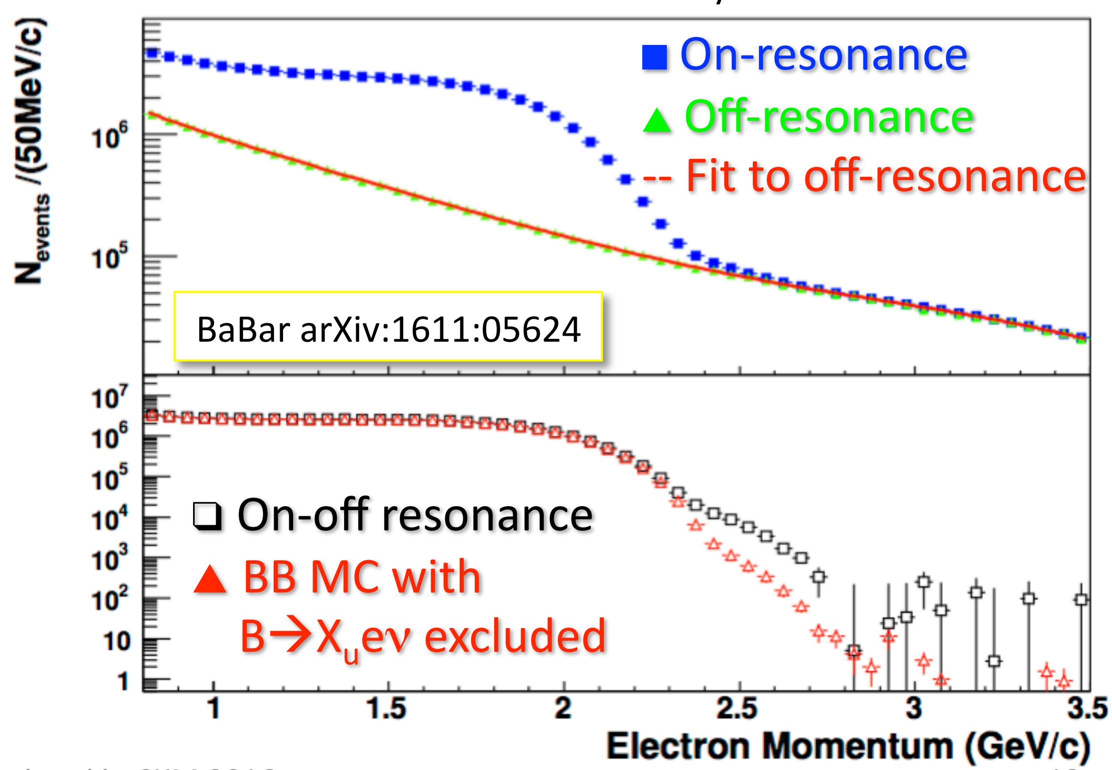


New BaBar result for inclusive $|V_{ub}|$

- BaBar $|V_{ub}|$ from the inclusive electron spectrum [arXiv:1611:05624]
  - full data set: 467M BBbar, 44.4fb^{-1} continuum; supercedes 2006 result
  - updated HQE parameter values as per HFAG 2013
  - fit to $b \to X_u e\nu$, continuum, and 6 B background contributions to determine partial BF for $E_e > p_{min}$, p_{min} as low as 0.8 GeV
- Simulated (MC) BBbar decays generated using EvtGen + JETSET
- Simple NN to reject continuum

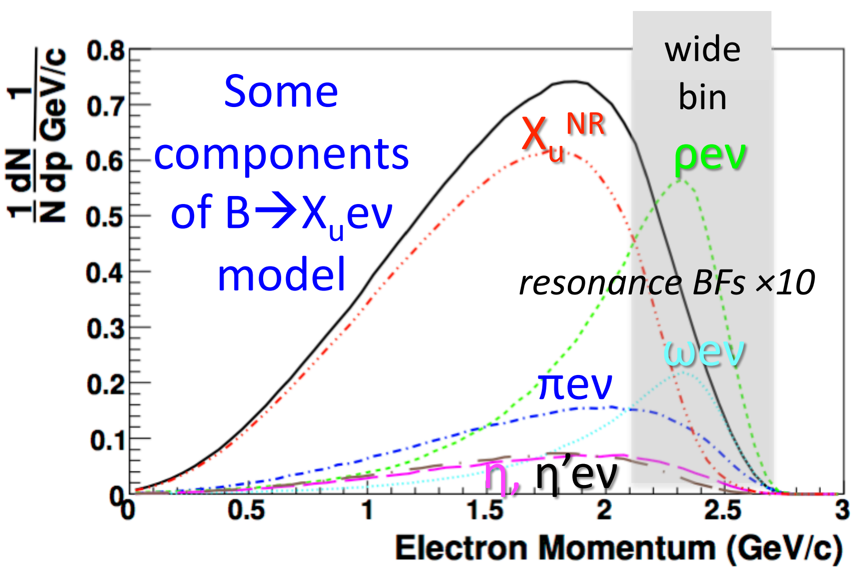


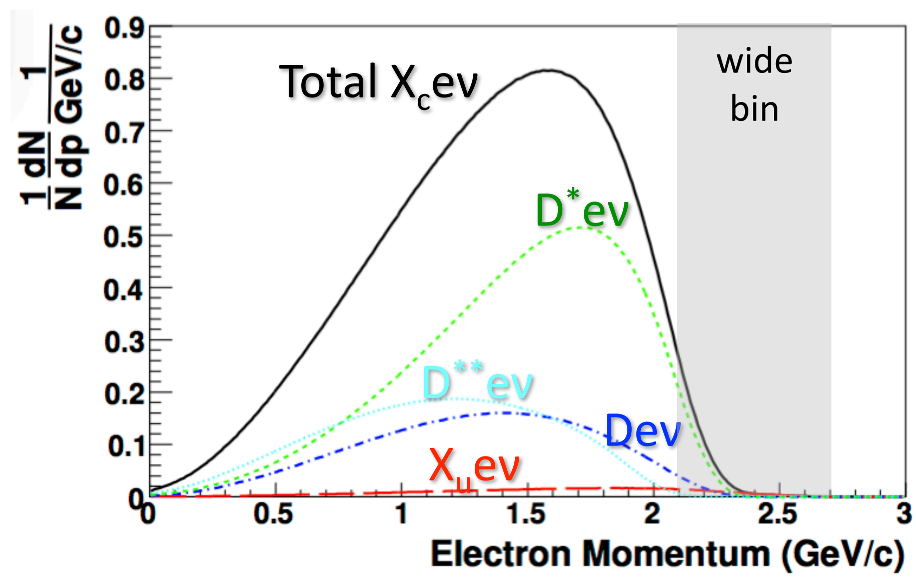
Large statistics: >10^6 events / 50 MeV bin; statistical uncertainties dominated by continuum subtraction



Analysis strategy

- Fit on-Υ(4S) and off-Υ(4S) data simultaneously; separate Υ(4S) contributions into 5 separate b→c components, secondary electrons, b→ueν
- Fit spectrum over a range [p_{min}, 2.7] GeV; with p_{min} from 0.8 – 2.1 GeV
- Avoid details of b→ueν in the endpoint (SF) region by using a wide bin from 2.1-2.7 GeV
- Consider 4 different calculations of b→ueν inclusive spectrum; mix in exclusive final states ("hybrid" model)





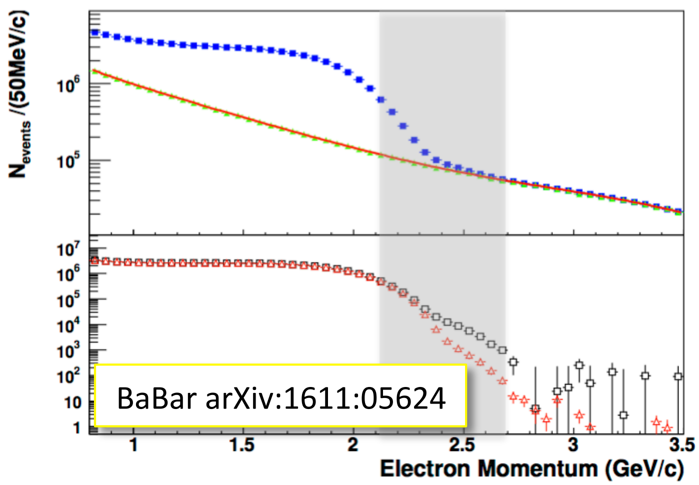
Theoretical models, external input

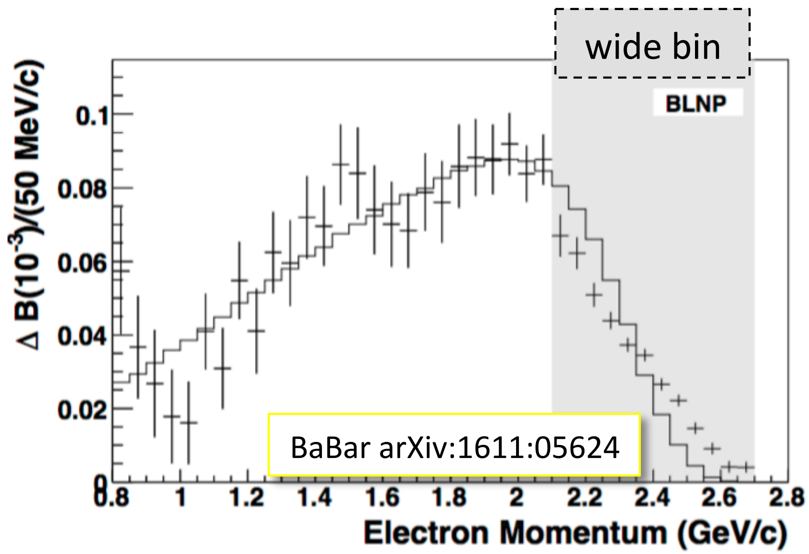
- Calculations based on OPE + SF modeling
  - DN DeFazio, and Neubert, JHEP 9906, 017 (1999), (superceded by BLNP)
  - BLNP Bosch, Lange, Neubert, Paz, Nucl. 894 Phys. B 699, 335 (2004)
  - GGOU Gambino, Giordano, Ossola, Uraltsev, JHEP 908 10, 058 (2007)
    - BLNP and GGOU include perturbative and non-perturbative effects in an expansion in powers of $1/m_b$. SFs used depend on externally-determined parameters
  - DGE Andersen, Gardi, JHEP 0601, 097 (2006), calculates SF using Sudakov resummation
- Parametric input:
  - Parametric input comes from global fits to hadronic mass and lepton energy moments in $B \rightarrow X_c e\nu$ decays, combined with either a constraint on m_c or E_γ moments from $B \rightarrow X_s\gamma$
  - Translation to other schemes as needed

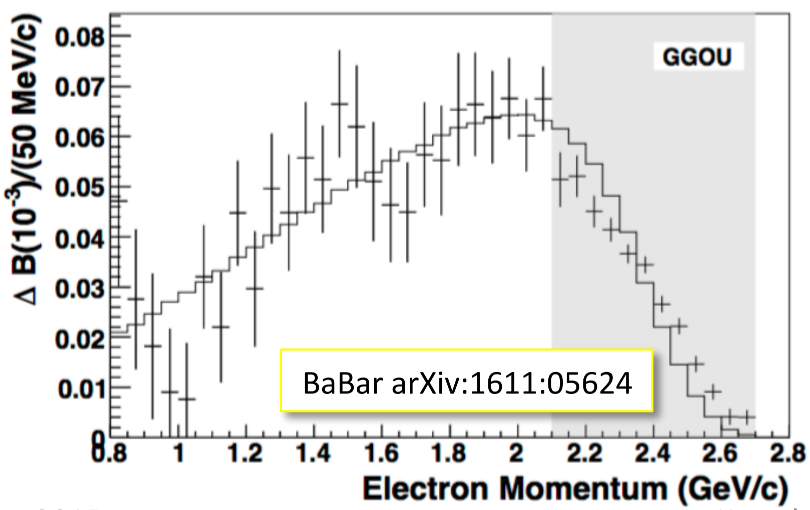
HFAG 2013 kinetic scheme	$X_c e\nu + m_c$ constraint	$X_c e\nu + X_s\gamma$
m_b [GeV]	4.560±0.023	4.574±0.032
μ_π^2 [GeV^2]	0.453±0.036	0.459±0.037

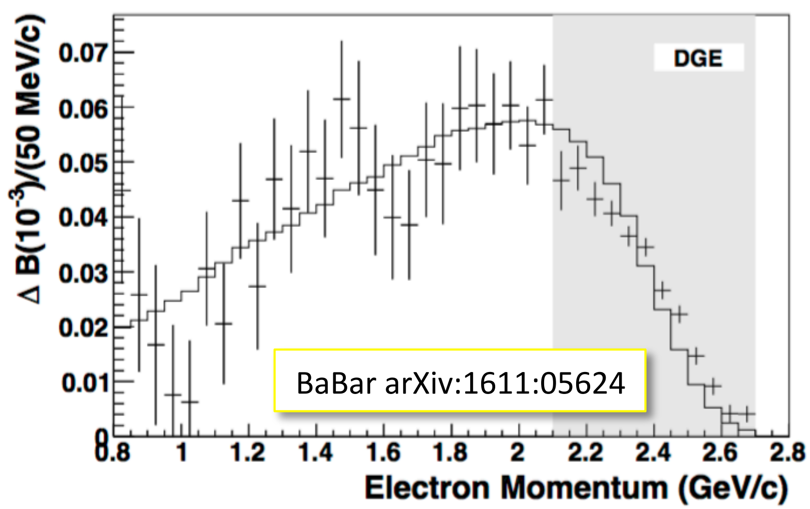
Fitted spectra in Υ(4S) frame

- $B \rightarrow X_u e\nu$ electron spectra for $p_e > 0.8$ GeV after $b \rightarrow c$ and continuum subtraction based on fit









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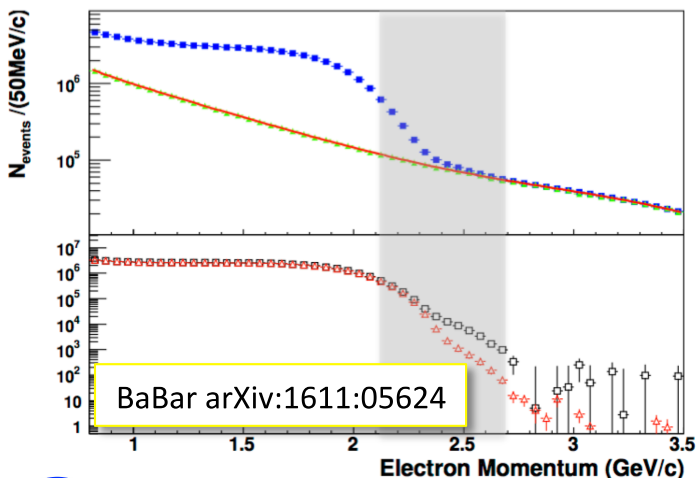
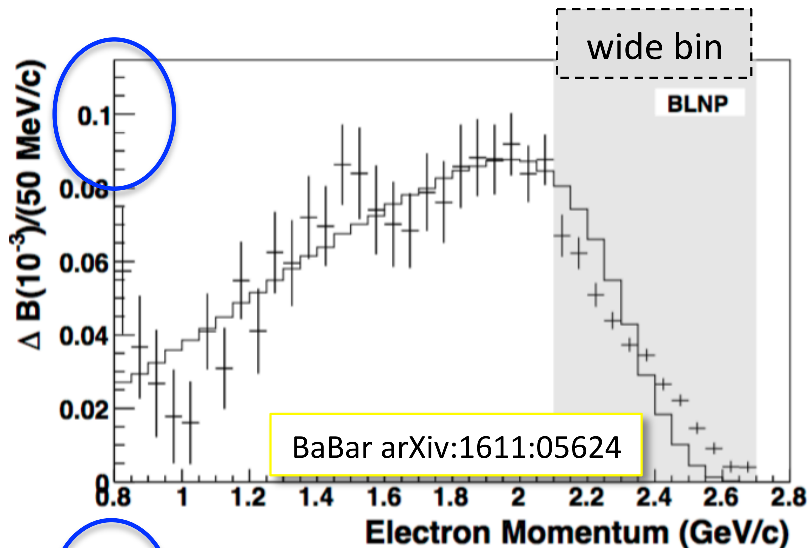
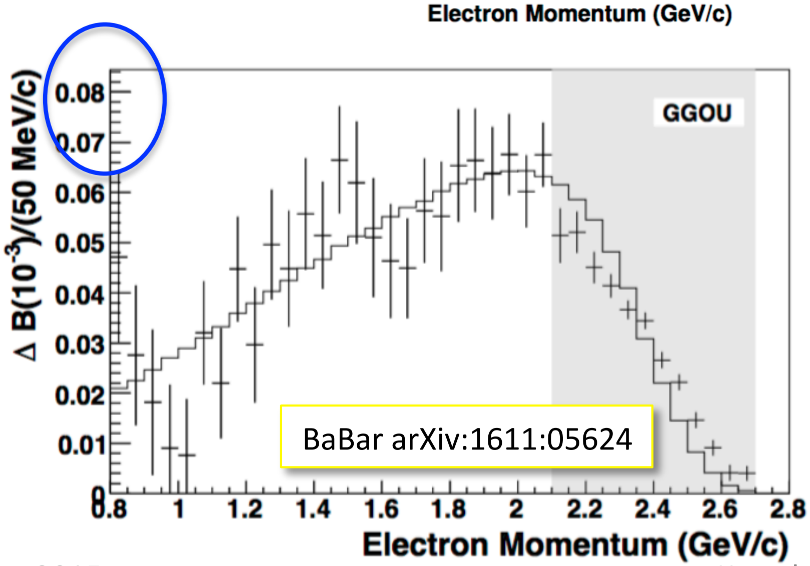
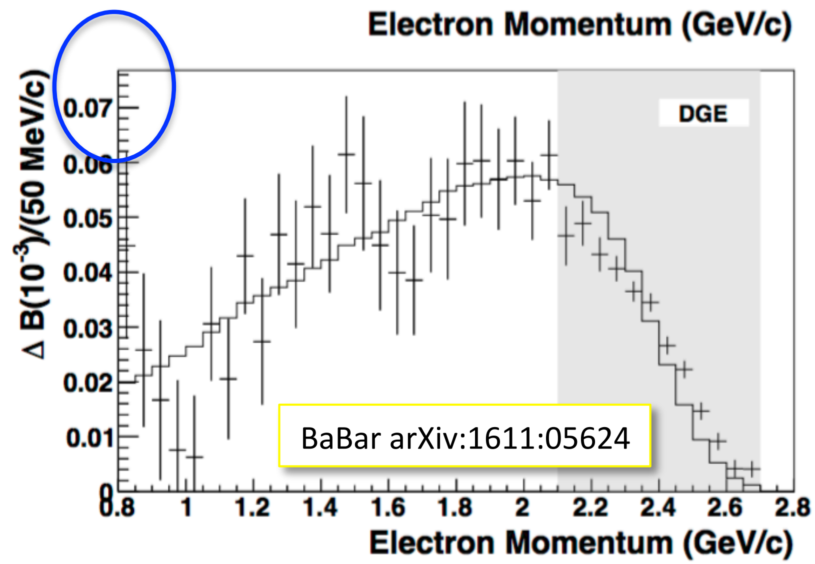




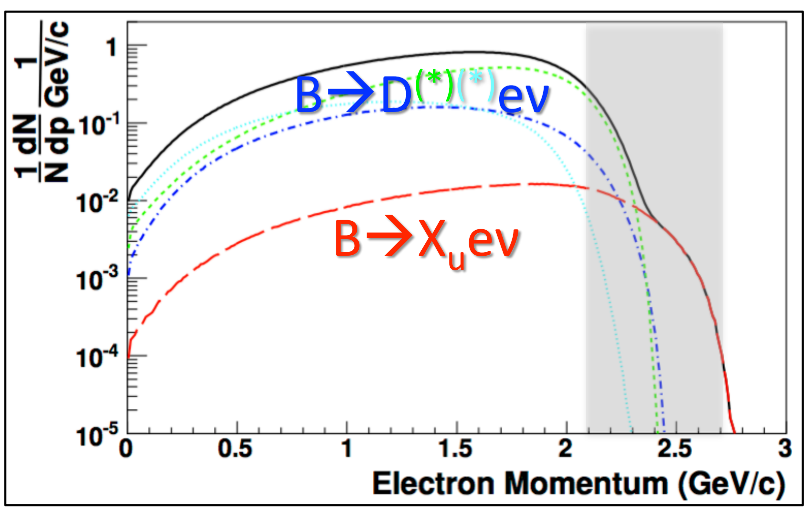


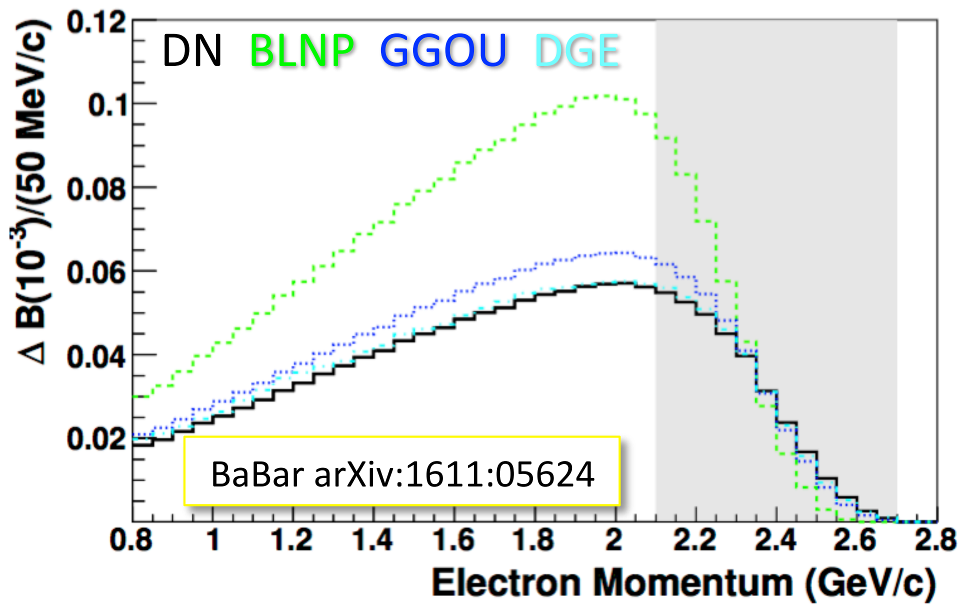


Understanding the fit results

- The region with good sensitivity to $B \rightarrow X_u e\nu$ is in the wide 2.1-2.7 GeV bin
  - Models differ in their predictions for the fractional rate in this bin
  - The normalization of the predicted $B \rightarrow X_u e\nu$ spectrum, and thus of its BF, is largely fixed by this bin

⚠ This dependence of the total rate and/or $|V_{ub}|$ on the $b \rightarrow ue\nu$ model impacts *any* measurement that ventures into the $b \rightarrow ce\nu$ – allowed region





Fit results for B → $X_u e\nu$ partial BF

- Partial BF results for $p_{min} < E_e < 2.7$ GeV:

BaBar arXiv:1611:05624

ΔBF×10³\model	DN	DGE	GGOU	BLNP
p_{min} = 0.8	$1.40 \pm 0.08^{+0.21}_{-0.15}$	1.43 ± 0.08	$1.55 \pm 0.08^{+0.10}_{-0.09}$	$2.27 \pm 0.13^{+0.19}_{-0.16}$
p_{min} = 2.1	$0.33 \pm 0.02^{+0.01}_{-0.01}$	0.33 ± 0.02	$0.34 \pm 0.02^{+0.01}_{-0.01}$	$0.40 \pm 0.02^{+0.01}_{-0.01}$
	exp SF	exp	exp SF	exp SF

- HQE parameter input from $X_c e\nu$ moments + m_c constraint
  - $m_b = 4.560 \pm 0.023$, $\mu_\pi^2 = 0.453 \pm 0.036$
  - results for other inputs given in arXiv:1611.05624
- Quoted values are corrected for final-state radiation
- Note dependence of partial BF on model (especially large for p_{min} = 0.8 GeV, but still present for p_{min} = 2.1 GeV)

BaBar

Systematic uncertainties

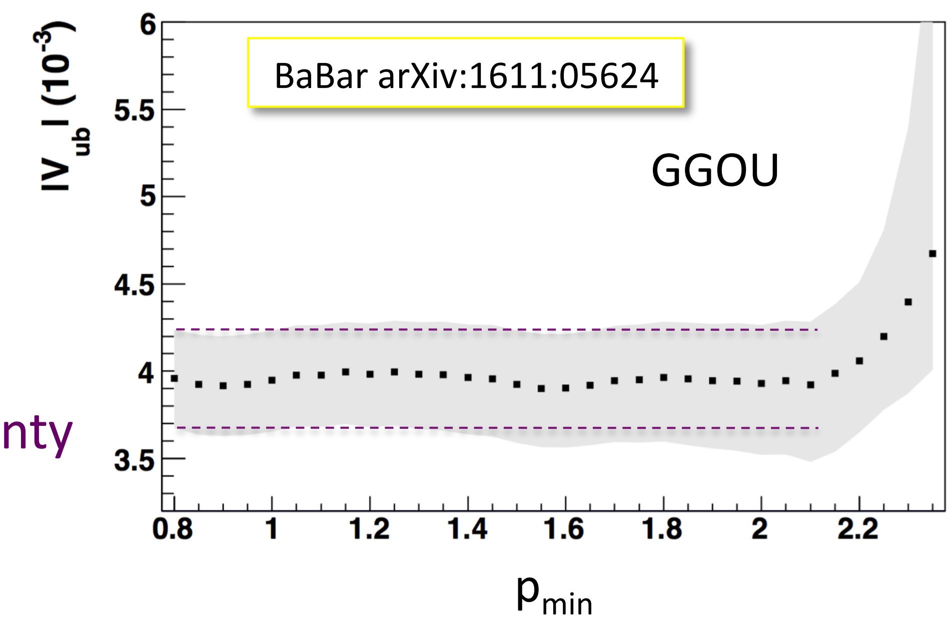
- Experimental systematic uncertainties due to
  - luminosity; event selection; form factors of D, D*, D**; composition of P-wave and heavier charm states; J/ψ, τ and fake backgrounds; bremsstrahlung; CM energy; wide bin width; non-BB backgrounds

☞ Total experimental error on $|V_{ub}|$ below 4% for $p_{min} < 2.4$ GeV

- Uncertainties in calculated rate due to
  - parametric input (m_b, μ_π^2)
  - QCD matching scales, weak annihilation, α_s

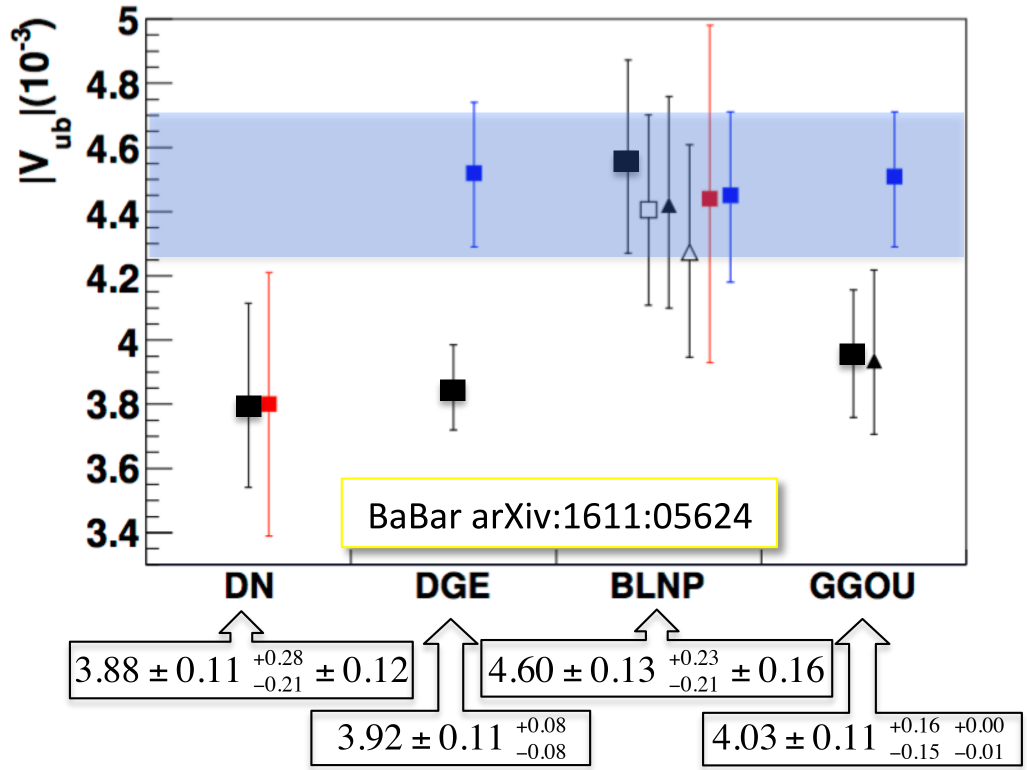
☞ Theory + parameter uncertainty estimates on $|V_{ub}|$ are ~4-7% for $p_{min} < 2.2$ GeV

- Current estimates of theory uncertainty fall with p_{min}; is this ok?



Electron spectrum $|V_{ub}|$ results

- Results are lower than most previous measurements (but not for BLNP)
- Consistent with previous BaBar analysis (2006)
- Band shows PDG 2016 inclusive average
- N.b. – the most precise measure-ments in the average are for E_e>1.0 GeV (B-tagged), i.e. deep into b→c allowed region. The partial BFs in those measurements were not done separately for each model



■ solid squares – parameters from X_c fit with m_c constraint

▲ solid triangles – parameters from $X_c + X_s\gamma$ fit

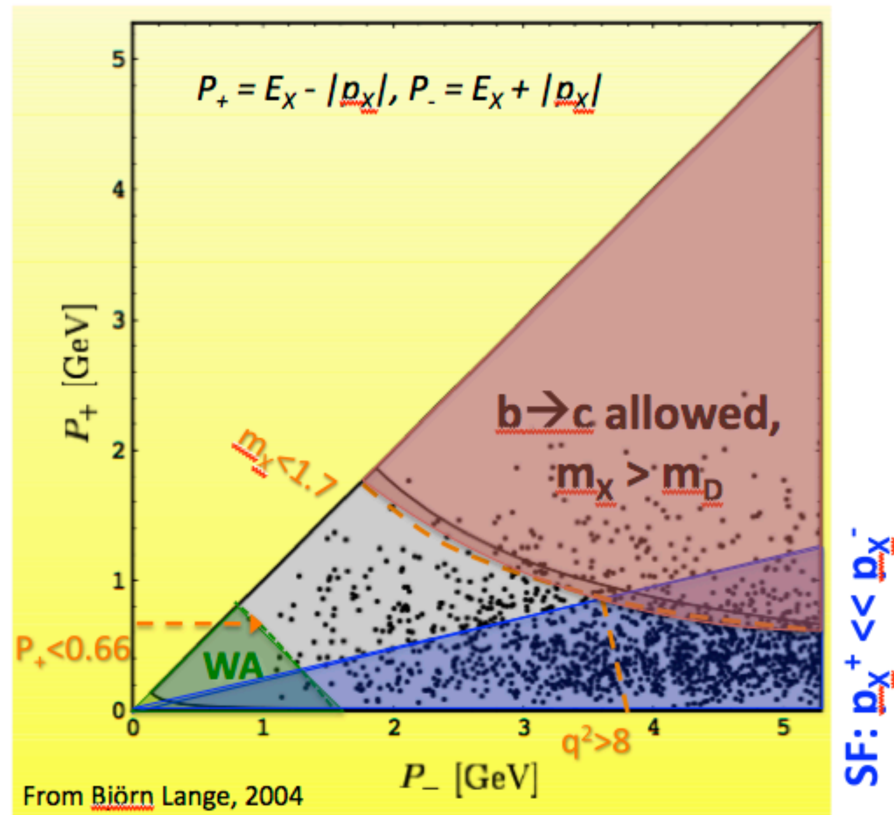
❑ open symbols – translation from "kinetic" to "shape-function" scheme using μ = 1.5 GeV (default is μ = 2.0 GeV)

■ Previous BaBar electron spectrum result (2006)

■ HFAG averages of all inclusive measurements

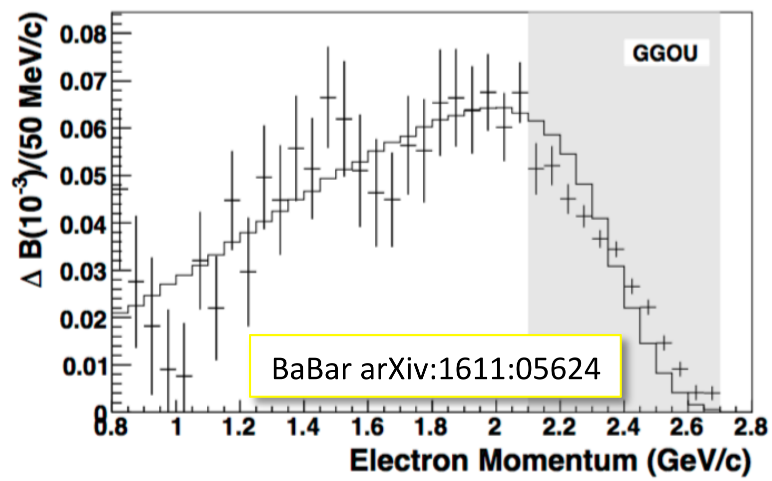
What improves with B tagging?

- B tagging removes most continuum, allows determination (with tails) of q^2, P_+, hadronic variables m_X, …
- Measurements in b→c allowed region still have huge background; K rejection, D* partial reco help, but can't overcome the factor of 50 in the ratio of rates
- The experimentally constraining information is in the b→c forbidden region; this will be true even with Belle II statistics



Embrace the shape function region

- The data in the lepton endpoint region carry a lot of information – it is currently being thrown away
- SIMBA authors have talked about using this – I'd love to see results!



- In B-tagged measurements, statistics are lower but one can use P_+ (or, in the language of PLB **541**, 29 (2002), "$E_W+|P_W|$"), which carries more direct information about the SF
- A promising path: combine inclusive Belle II measurements in $b \rightarrow c$ forbidden region with SIMBA

Other measurements for Belle II

- A number of items need improvement
  - The composition of the $B \to X_c e\nu$ background, in particular if one continues to push into the $b \to c e\nu$ – allowed region
  - ☞ Measure exclusive $B \to X_c e\nu$ final states/FFs for $D(*)n\pi$, $D(*)\eta$, etc.
  - The modeling of $B \to X_u e\nu$ decays
    - the incorporation of both non-resonant and resonant states into MC generators is an ad-hoc procedure without tight constraints from data
    - ☞ Measure exclusive $B \to n\pi e\nu$ final states and higher-mass charmless resonances
- Further supporting measurements:
  - Weak annihilation
  - ssbar production in X_u hadronisation
  - …
- Perhaps use "sum-of-exclusives" approach of $b \to s\gamma$, as suggested by P. Urquijo in Mainz last year

Summary

- Inclusive $|V_{ub}|$ results remain a puzzle; internally consistent, but above CKM fit and exclusive results
- New BaBar electron spectrum result for $|V_{ub}|$ uses CMS momenta p* as low as 0.8 GeV, but sensitivity to b→ueν is primarily above 2.1 GeV
- Partial rates that include the b→c allowed region depend on the b→u model. *It's essential to account for this and use the same model when deriving a partial rate and $|V_{ub}|$*
- Theory/parameter uncertainties currently dominate – we need more information to constrain SF uncertainties
- The use of high-quality experimental spectra in the b→c forbidden region in a global fit (e.g. with SIMBA) seems like the way to go – what are the limitations to this approach?

Backup

BABAR

Fit results for B → $X_u e\nu$ partial BF

BFs in percent
Errors are statistical

BaBar arXiv:1611:05624

	DN	$BLNP_{\mu_i=2.0\,\mathrm{GeV}}$ m_c constraint	GGOU m_c constraint	DGE
$X_u e\nu$	0.149 ± 0.005	0.240 ± 0.008	0.166 ± 0.006	0.153 ± 0.005
$De\nu$ Dev includes Gaussian constraint	2.233 ± 0.090	2.197 ± 0.088	2.226 ± 0.089	2.230 ± 0.089
$D^* e\nu$	5.612 ± 0.049	5.424 ± 0.049	5.579 ± 0.048	5.611 ± 0.048
$D^{(*)}\pi e\nu$	< 0.052	< 0.025	< 0.050	< 0.075
$D^{**} e\nu$	2.285 ± 0.071	2.540 ± 0.075	2.331 ± 0.070	2.287 ± 0.070
$D'^{(*)} e\nu$	0.046 ± 0.011	0.023 ± 0.011	0.041 ± 0.011	0.045 ± 0.011
$D \to e$	0.982 ± 0.005	0.968 ± 0.005	0.980 ± 0.005	0.982 ± 0.005
$r_L/r_L^{(0)}$ On/Off lumi ratio	1.0002 ± 0.0007	1.0002 ± 0.0007	1.0002 ± 0.0007	1.0002 ± 0.0007
$\chi^2_{\mathrm{ON}} + \chi^2_{\mathrm{OFF}} + \chi^2_{\mathrm{constraints}}$	$27.4 + 69.7 + 0.1$	$31.9 + 70.9 + 0.2$	$27.8 + 69.9 + 0.1$	$26.8 + 69.7 + 0.1$
χ^2/ndof	$97.2/85$	$102.9/85$	$97.8/85$	$96.6/85$

- Fit repeated for 4 different predicted B → X_u e ν spectra
- Quoted values are corrected for final-state radiation
- Good χ^2/ndof
- Proceed to determine $|V_{ub}|$

$|V_{ub}|$ results, $p_{min} > 0.8$ GeV

- Values depend on model, choice of inputs

used in summary plot (arrows point to the DN, DGE, GGOU$_1$ and BLNP$_1$ rows)

$\Delta\mathcal{B}[10^{-3}]$	$\mathcal{B}[10^{-3}]$	$\lvert V_{ub}\rvert[10^{-3}]$	$\Delta\zeta(\Delta p)[\mathrm{ps}^{-1}]$
DN			
$1.397 \pm 0.078_{\rm exp}\ {}^{+0.214}_{-0.153}{}_{\rm SF}$	$1.494 \pm 0.084_{\rm exp}\ {}^{+0.239}_{-0.167}{}_{\rm SF}\ {}^{+0.030}_{-0.003}{}_{\rm theory}$	$3.794 \pm 0.107_{\rm exp}\ {}^{+0.292}_{-0.219}{}_{\rm SF}\ {}^{+0.078}_{-0.068}{}_{\rm theory}$	$61.43\ {}^{+0.20}_{-0.33}{}_{\rm SF}\ {}^{+2.28}_{-2.45}{}_{\rm theory}$
DGE			
$1.433 \pm 0.081_{\rm exp}$	$1.537 \pm 0.086_{\rm exp}\ {}^{+0.031}_{-0.003}{}_{\rm theory}$	$3.848 \pm 0.108_{\rm exp}\ {}^{+0.084}_{-0.070}{}_{\rm theory}$	$61.26\ {}^{+2.30}_{-2.58}{}_{\rm theory}$
$X_c\ell\nu$, m_c constraint fit of SF parameters, GGOU$_1$			
$1.554 \pm 0.082_{\rm exp}\ {}^{+0.095}_{-0.086}{}_{\rm SF}$	$1.665 \pm 0.087_{\rm exp}\ {}^{+0.103}_{-0.093}{}_{\rm SF}\ {}^{+0.002}_{-0.011}{}_{\rm theory}$	$3.959 \pm 0.104_{\rm exp}\ {}^{+0.164}_{-0.154}{}_{\rm SF}\ {}^{+0.042}_{-0.079}{}_{\rm theory}$	$62.76\ {}^{+1.59}_{-1.55}{}_{\rm SF}\ {}^{+2.58}_{-1.32}{}_{\rm theory}$
$X_c\ell\nu$, $X_s\gamma$ constraint fit of SF parameters, GGOU$_2$			
$1.551 \pm 0.081_{\rm exp}\ {}^{+0.117}_{-0.100}{}_{\rm SF}$	$1.661 \pm 0.086_{\rm exp}\ {}^{+0.127}_{-0.109}{}_{\rm SF}\ {}^{+0.008}_{-0.011}{}_{\rm theory}$	$3.936 \pm 0.102_{\rm exp}\ {}^{+0.202}_{-0.188}{}_{\rm SF}\ {}^{+0.168}_{-0.086}{}_{\rm theory}$	$63.38\ {}^{+2.15}_{-2.15}{}_{\rm SF}\ {}^{+2.87}_{-5.08}{}_{\rm theory}$
$X_c\ell\nu$, m_c constraint fit of SF parameters with $\mu_i = 2.0$ GeV, BLNP$_1$			
$2.268 \pm 0.125_{\rm exp}\ {}^{+0.191}_{-0.163}{}_{\rm SF}$	$2.418 \pm 0.134_{\rm exp}\ {}^{+0.205}_{-0.176}{}_{\rm SF}\ {}^{+0.003}_{-0.003}{}_{\rm theory}$	$4.563 \pm 0.126_{\rm exp}\ {}^{+0.230}_{-0.208}{}_{\rm SF}\ {}^{+0.162}_{-0.163}{}_{\rm theory}$	$68.93\ {}^{+1.88}_{-1.85}{}_{\rm SF}\ {}^{+5.20}_{-4.65}{}_{\rm theory}$
$X_c\ell\nu$, m_c constraint fit of SF parameters with $\mu_i = 1.5$ GeV, BLNP$_2$			
$2.924 \pm 0.112_{\rm exp}\ {}^{+0.152}_{-0.137}{}_{\rm SF}$	$2.160 \pm 0.120_{\rm exp}\ {}^{+0.164}_{-0.148}{}_{\rm SF}\ {}^{+0.002}_{-0.003}{}_{\rm theory}$	$4.406 \pm 0.122_{\rm exp}\ {}^{+0.203}_{-0.193}{}_{\rm SF}\ {}^{+0.176}_{-0.189}{}_{\rm theory}$	$66.00\ {}^{+1.88}_{-1.82}{}_{\rm SF}\ {}^{+6.06}_{-4.96}{}_{\rm theory}$
$X_c\ell\nu$, $X_s\gamma$ constraint fit of SF parameters with $\mu_i = 2.0$ GeV, BLNP$_3$			
$2.157 \pm 0.120_{\rm exp}\ {}^{+0.207}_{-0.176}{}_{\rm SF}$	$2.298 \pm 0.128_{\rm exp}\ {}^{+0.222}_{-0.189}{}_{\rm SF}\ {}^{+0.003}_{-0.003}{}_{\rm theory}$	$4.420 \pm 0.123_{\rm exp}\ {}^{+0.273}_{-0.251}{}_{\rm SF}\ {}^{+0.156}_{-0.158}{}_{\rm theory}$	$66.94\ {}^{+2.69}_{-2.62}{}_{\rm SF}\ {}^{+6.15}_{-5.02}{}_{\rm theory}$
$X_c\ell\nu$, $X_s\gamma$ constraint fit of SF parameters with $\mu_i = 1.5$ GeV, BLNP$_4$			
$1.931 \pm 0.108_{\rm exp}\ {}^{+0.172}_{-0.147}{}_{\rm SF}$	$2.059 \pm 0.115_{\rm exp}\ {}^{+0.185}_{-0.158}{}_{\rm SF}\ {}^{+0.002}_{-0.002}{}_{\rm theory}$	$4.273 \pm 0.119_{\rm exp}\ {}^{+0.263}_{-0.243}{}_{\rm SF}\ {}^{+0.170}_{-0.184}{}_{\rm theory}$	$69.88\ {}^{+2.70}_{-2.64}{}_{\rm SF}\ {}^{+5.26}_{-4.69}{}_{\rm theory}$

$|V_{ub}|$ results, $p_{min} > 2.1$ GeV

- Values depend on model, choice of inputs

$\Delta\mathcal{B}[10^{-3}]$	$\mathcal{B}[10^{-3}]$	$\lvert V_{ub}\rvert[10^{-3}]$	$\Delta\zeta(\Delta p)[\text{ps}^{-1}]$
DN			
$0.330 \pm 0.018_{\text{exp}} {}^{+0.009}_{-0.009}{}_{\text{SF}}$	$1.471 \pm 0.081_{\text{exp}} {}^{+0.235}_{-0.164}{}_{\text{SF}} {}^{+0.124}_{-0.101}{}_{\text{theory}}$	$3.764 \pm 0.104_{\text{exp}} {}^{+0.290}_{-0.216}{}_{\text{SF}} {}^{+0.170}_{-0.148}{}_{\text{theory}}$	$14.75 {}^{+1.41}_{-1.70}{}_{\text{SF}} {}^{+1.23}_{-1.24}{}_{\text{theory}}$
DGE			
$0.331 \pm 0.018_{\text{exp}}$	$1.511 \pm 0.082_{\text{exp}} {}^{+0.090}_{-0.085}{}_{\text{theory}}$	$3.815 \pm 0.104_{\text{exp}} {}^{+0.182}_{-0.160}{}_{\text{theory}}$	$14.40 {}^{+1.29}_{-1.28}{}_{\text{theory}}$
$X_c\ell\nu$, m_c constraint fit of SF parameters, GGOU$_1$			
$0.342 \pm 0.018_{\text{exp}} {}^{+0.007}_{-0.006}{}_{\text{SF}}$	$1.634 \pm 0.087_{\text{exp}} {}^{+0.100}_{-0.090}{}_{\text{SF}} {}^{+0.109}_{-0.163}{}_{\text{theory}}$	$3.922 \pm 0.104_{\text{exp}} {}^{+0.160}_{-0.150}{}_{\text{SF}} {}^{+0.170}_{-0.251}{}_{\text{theory}}$	$14.06 {}^{+0.87}_{-0.82}{}_{\text{SF}} {}^{+1.99}_{-1.14}{}_{\text{theory}}$
$X_c\ell\nu$, $X_s\gamma$ constraint fit of SF parameters, GGOU$_2$			
$0.342 \pm 0.018_{\text{exp}} {}^{+0.008}_{-0.007}{}_{\text{SF}}$	$1.630 \pm 0.086_{\text{exp}} {}^{+0.122}_{-0.105}{}_{\text{SF}} {}^{+0.188}_{-0.189}{}_{\text{theory}}$	$3.899 \pm 0.103_{\text{exp}} {}^{+0.198}_{-0.185}{}_{\text{SF}} {}^{+0.381}_{-0.289}{}_{\text{theory}}$	$14.23 {}^{+1.12}_{-1.08}{}_{\text{SF}} {}^{+2.37}_{-2.42}{}_{\text{theory}}$
$X_c\ell\nu$, m_c constraint fit of SF parameters with $\mu_i = 2.0$ GeV, BLNP$_1$			
$0.397 \pm 0.022_{\text{exp}} {}^{+0.014}_{-0.012}{}_{\text{SF}}$	$2.359 \pm 0.130_{\text{exp}} {}^{+0.199}_{-0.170}{}_{\text{SF}} {}^{+0.173}_{-0.133}{}_{\text{theory}}$	$4.507 \pm 0.124_{\text{exp}} {}^{+0.226}_{-0.204}{}_{\text{SF}} {}^{+0.337}_{-0.275}{}_{\text{theory}}$	$12.36 {}^{+0.89}_{-0.83}{}_{\text{SF}} {}^{+1.66}_{-1.66}{}_{\text{theory}}$
$X_c\ell\nu$, m_c constraint fit of SF parameters with $\mu_i = 1.5$ GeV, BLNP$_2$			
$0.376 \pm 0.021_{\text{exp}} {}^{+0.011}_{-0.010}{}_{\text{SF}}$	$2.110 \pm 0.117_{\text{exp}} {}^{+0.158}_{-0.143}{}_{\text{SF}} {}^{+0.128}_{-0.087}{}_{\text{theory}}$	$4.356 \pm 0.120_{\text{exp}} {}^{+0.198}_{-0.190}{}_{\text{SF}} {}^{+0.317}_{-0.265}{}_{\text{theory}}$	$12.55 {}^{+0.92}_{-0.85}{}_{\text{SF}} {}^{+1.68}_{-1.64}{}_{\text{theory}}$
$X_c\ell\nu$, $X_s\gamma$ constraint fit of SF parameters with $\mu_i = 2.0$ GeV, BLNP$_3$			
$0.389 \pm 0.022_{\text{exp}} {}^{+0.015}_{-0.013}{}_{\text{SF}}$	$2.244 \pm 0.124_{\text{exp}} {}^{+0.215}_{-0.183}{}_{\text{SF}} {}^{+0.152}_{-0.117}{}_{\text{theory}}$	$4.367 \pm 0.121_{\text{exp}} {}^{+0.270}_{-0.248}{}_{\text{SF}} {}^{+0.313}_{-0.257}{}_{\text{theory}}$	$12.91 {}^{+1.25}_{-1.17}{}_{\text{SF}} {}^{+1.67}_{-1.67}{}_{\text{theory}}$
$X_c\ell\nu$, $X_s\gamma$ constraint fit of SF parameters with $\mu_i = 1.5$ GeV, BLNP$_4$			
$0.370 \pm 0.020_{\text{exp}} {}^{+0.012}_{-0.010}{}_{\text{SF}}$	$2.013 \pm 0.111_{\text{exp}} {}^{+0.179}_{-0.153}{}_{\text{SF}} {}^{+0.112}_{-0.075}{}_{\text{theory}}$	$4.225 \pm 0.116_{\text{exp}} {}^{+0.259}_{-0.239}{}_{\text{SF}} {}^{+0.296}_{-0.250}{}_{\text{theory}}$	$13.10 {}^{+1.30}_{-1.20}{}_{\text{SF}} {}^{+1.70}_{-1.66}{}_{\text{theory}}$

Description of fit

- Simultaneous binned χ^2 fit to on- and off-resonance samples
- Continuum component fitted to

$$a_0\left(\exp\left(a_1 p_i + a_2 p_i^2 + a_3 p_i^3\right) + \exp\left(a_4 p_i + a_5 p_i^2\right)\right)$$

- $B\bar{B}$ component fitted to sum of contributions

$$\sum_k b_k g_k(\mathbf{t}, p_i)$$

$g_k(\mathbf{t},p_i)$ – MC predictions for 6 $B \rightarrow X_c e\nu$ modes + $B \rightarrow X_u e\nu$ model

b_k – fitted corrections to default MC BFs

$\mathbf{t}$ – form-factor parameters (fixed)

- External constraint term added to χ^2 for $B \rightarrow De\nu$: $\dfrac{\left(b_{De\nu} - b_{De\nu}^{PDG}\right)^2}{\sigma_{PDG}^2}$
- 50 MeV bins except for 2.1-2.7 GeV (big bin);
this reduces dependence on theory in SF-dominated region

p_i = electron momentum in bin i

Free parameters a_j, b_k

Experimental systematic uncertainties

- relative uncertainties on partial BF

p^{min}(GeV/c)	0.8	1.5	2.1	2.3
Single Track efficiency	0.1	0.1	0.1	0.0
Charged track multiplicity	1.2	1.9	1.3	1.0
Particle identification	0.5	0.5	0.5	0.5
Hadron mis-ID background	0.7	0.7	0.8	0.5
Photon selection	0.4	0.3	0.4	0.2
Neural net event selection	$^{+3.0}_{-0.8}$	$^{+3.3}_{-1.2}$	$^{+3.6}_{-1.2}$	$^{+3.1}_{-2.1}$
non-$B\overline{B}$ background	0.5	0.5	0.5	0.8
$B \to X_u e\nu$ exclusive decays	0.3	0.2	0.3	0.3
$B \to D^{(*)} l\nu$ form factors	1.1	0.5	1.2	0.2
$B \to D^{**} e\nu$ form factors	0.6	0.4	0.6	0.0
$B \to D^{**} e\nu$ BF	0.4	1.1	0.5	0.1
$B \to D^{(\prime)} e\nu$ BF	0.2	0.9	0.2	0.0
Widths of $D^{(\prime)}$ states	0.2	0.5	0.2	0.0
J/ψ and $\psi(2S)$ background	0.1	0.2	0.1	0.1
τ background	0.2	0.7	0.3	0.1
B momentum	1.5	1.5	1.6	0.5
Bremsstrahlung	0.3	0.1	0.3	0.0
Final state radiation	0.6	0.6	0.5	0.6
Width of wide bin	0.4	0.4	0.3	0.0
$N_{B\overline{B}}$ normalization	1.1	1.1	1.1	1.1
Total exp. systematic uncertainty	$^{+4.2}_{-3.1}$	$^{+4.8}_{-3.7}$	$^{+4.7}_{-3.3}$	$^{+3.8}_{-3.0}$
Total exp. statistical uncertainty	3.8	5.0	3.5	2.8
Total exp. uncertainty	$^{+5.7}_{-4.9}$	$^{+7.0}_{-6.2}$	$^{+5.9}_{-4.8}$	$^{+4.7}_{-4.1}$

Details of fit

- Linear correlation coefficients

	$D e\nu$	$D^* e\nu$	$D^{(*)}\pi e\nu$	$D^{**} e\nu$	$D'^{(*)} e\nu$	$X_u e\nu$	$D \to e$	a_0	a_1	a_2	a_3	a_4	a_5	$r_L/r_L^{(0)}$
$D e\nu$	1	-0.827	0.032	-0.398	-0.449	-0.305	-0.060	0.018	-0.048	0.058	-0.036	0.023	-0.032	0.001
$D^* e\nu$		1	-0.024	-0.158	0.784	-0.128	0.309	0.050	0.029	-0.146	0.126	0.038	0.125	0.008
$D^{(*)}\pi e\nu$			1	-0.031	0.004	0.027	0.012	-0.066	0.033	0.033	-0.048	-0.044	-0.052	-0.028
$D^{**} e\nu$				1	-0.601	0.598	-0.361	-0.062	0.030	0.055	-0.063	-0.055	-0.066	-0.012
$D'^{(*)} e\nu$					1	-0.236	0.206	0.069	-0.051	-0.034	0.053	0.070	0.063	0.001
$X_u e\nu$						1	-0.252	-0.461	0.310	0.252	-0.369	-0.425	-0.363	-0.107
$D \to e$							1	-0.108	0.204	-0.189	0.104	-0.102	0.037	-0.116
a_0								1	-0.827	-0.196	0.670	0.980	0.671	0.139
a_1									1	-0.315	-0.190	-0.870	-0.209	-0.103
a_2										1	-0.801	-0.122	-0.818	0.012
a_3											1	0.610	0.947	0.035
a_4												1	0.627	0.027
a_5													1	-0.006
$r_L/r_L^{(0)}$														1

Latest results from HFAG, courtesy C. Bozzi

Measurement	Accepted region	$\Delta\mathcal{B}[10^{-4}]$	Notes
CLEO [521]	$E_e > 2.1\,\mathrm{GeV}$	$3.3 \pm 0.2 \pm 0.7$	
BABAR [520]	$E_e > 2.0\,\mathrm{GeV},\ s_{\mathrm{h}}^{\mathrm{max}} < 3.5\,\mathrm{GeV}^2$	$4.0 \pm 0.2 \pm 0.3$	
BABAR [518]	$E_e > 2.0\,\mathrm{GeV}$	$5.7 \pm 0.4 \pm 0.5$	
Belle [522]	$E_e > 1.9\,\mathrm{GeV}$	$8.5 \pm 0.4 \pm 1.5$	
BABAR [511]	$M_X < 1.7\,\mathrm{GeV}/c^2, q^2 > 8\,\mathrm{GeV}^2/c^2$	$6.9 \pm 0.6 \pm 0.4$	
Belle [523]	$M_X < 1.7\,\mathrm{GeV}/c^2, q^2 > 8\,\mathrm{GeV}^2/c^2$	$7.4 \pm 0.9 \pm 1.3$	
Belle [524]	$M_X < 1.7\,\mathrm{GeV}/c^2, q^2 > 8\,\mathrm{GeV}^2/c^2$	$8.5 \pm 0.9 \pm 1.0$	used only in BLL average
BABAR [511]	$P_+ < 0.66\,\mathrm{GeV}$	$9.9 \pm 0.9 \pm 0.8$	
BABAR [511]	$M_X < 1.7\,\mathrm{GeV}/c^2$	$11.6 \pm 1.0 \pm 0.8$	
BABAR [511]	$M_X < 1.55\,\mathrm{GeV}/c^2$	$10.9 \pm 0.8 \pm 0.6$	
Belle [510]	$p_\ell^* > 1\mathrm{GeV}/c$	$19.6 \pm 1.7 \pm 1.6$	
BABAR [511]	(M_X, q^2) fit, $p_\ell^* > 1\mathrm{GeV}/c$	$18.2 \pm 1.3 \pm 1.5$	
BABAR [511]	$p_\ell^* > 1.3\mathrm{GeV}/c$	$15.5 \pm 1.3 \pm 1.4$	

	BLNP	DGE	GGOU	ADFR	BLL
Input parameters					
scheme	SF	$\overline{MS}$	kinetic	$\overline{MS}$	$1S$
Ref.	[529, 530]	Ref. [531]	see Sec. 5.2.2	Ref. [531]	Ref. [532]
m_b (GeV)	4.582 ± 0.026	4.18 ± 0.043	4.554 ± 0.018	4.188 ± 0.043	4.704 ± 0.029
μ_π^2 (GeV^2)	$0.145\ ^{+0.091}_{-0.097}$	-	0.414 ± 0.078	-	-
Ref.	$\lvert V_{ub}\rvert$ values				
E_e [521]	$4.22 \pm 0.49^{+0.29}_{-0.34}$	$3.86 \pm 0.45^{+0.25}_{-0.27}$	$4.23 \pm 0.49^{+0.22}_{-0.31}$	$3.42 \pm 0.40^{+0.17}_{-0.17}$	-
M_X, q^2 [523]	$4.51 \pm 0.47^{+0.27}_{-0.29}$	$4.43 \pm 0.47^{+0.21}_{-0.19}$	$4.52 \pm 0.48^{+0.25}_{-0.28}$	$3.93 \pm 0.41^{+0.18}_{-0.17}$	$4.68 \pm 0.49^{+0.30}_{-0.30}$
E_e [522]	$4.93 \pm 0.46^{+0.26}_{-0.29}$	$4.82 \pm 0.45^{+0.23}_{-0.23}$	$4.95 \pm 0.46^{+0.16}_{-0.21}$	$4.48 \pm 0.42^{+0.20}_{-0.20}$	-
E_e [518]	$4.52 \pm 0.26^{+0.26}_{-0.30}$	$4.30 \pm 0.24^{+0.25}_{-0.23}$	$4.52 \pm 0.26^{+0.17}_{-0.24}$	$3.93 \pm 0.22^{+0.20}_{-0.20}$	-
$E_e, s_{\mathrm{h}}^{\mathrm{max}}$ [520]	$4.48 \pm 0.22^{+0.31}_{-0.36}$	$4.14 \pm 0.20^{+0.28}_{-0.27}$	-	$3.62 \pm 0.18^{+0.18}_{-0.17}$	
p_ℓ^* [510]	$4.50 \pm 0.27^{+0.20}_{-0.22}$	$4.62 \pm 0.28^{+0.13}_{-0.13}$	$4.62 \pm 0.28^{+0.09}_{-0.10}$	$4.50 \pm 0.30^{+0.20}_{-0.20}$	-
M_X [511]	$4.24 \pm 0.19^{+0.25}_{-0.25}$	$4.47 \pm 0.20^{+0.19}_{-0.24}$	$4.30 \pm 0.20^{+0.20}_{-0.21}$	$3.83 \pm 0.18^{+0.20}_{-0.19}$	-
M_X [511]	$4.03 \pm 0.22^{+0.22}_{-0.22}$	$4.22 \pm 0.23^{+0.21}_{-0.27}$	$4.10 \pm 0.23^{+0.16}_{-0.17}$	$3.75 \pm 0.21^{+0.18}_{-0.18}$	-
M_X, q^2 [511]	$4.32 \pm 0.23^{+0.26}_{-0.28}$	$4.24 \pm 0.22^{+0.18}_{-0.21}$	$4.33 \pm 0.23^{+0.24}_{-0.27}$	$3.75 \pm 0.20^{+0.17}_{-0.17}$	$4.50 \pm 0.24^{+0.29}_{-0.29}$
P_+ [511]	$4.09 \pm 0.25^{+0.25}_{-0.25}$	$4.17 \pm 0.25^{+0.28}_{-0.37}$	$4.25 \pm 0.26^{+0.26}_{-0.27}$	$3.57 \pm 0.22^{+0.19}_{-0.18}$	-
p_ℓ^*, (M_X, q^2) fit [511]	$4.33 \pm 0.24^{+0.19}_{-0.21}$	$4.45 \pm 0.24^{+0.12}_{-0.13}$	$4.44 \pm 0.24^{+0.09}_{-0.10}$	$4.33 \pm 0.24^{+0.19}_{-0.19}$	-
p_ℓ^* [511]	$4.34 \pm 0.27^{+0.20}_{-0.21}$	$4.43 \pm 0.27^{+0.13}_{-0.13}$	$4.43 \pm 0.27^{+0.09}_{-0.11}$	$4.28 \pm 0.27^{+0.19}_{-0.19}$	-
M_X, q^2 [524]	-	-	-	-	$5.01 \pm 0.39^{+0.32}_{-0.32}$
Average	$4.45 \pm 0.15^{+0.21}_{-0.22}$	$4.52 \pm 0.16^{+0.15}_{-0.16}$	$4.52 \pm 0.15^{+0.11}_{-0.14}$	$4.04 \pm 0.13^{+0.18}_{-0.12}$	$4.62 \pm 0.20^{+0.29}_{-0.29}$