



# Exclusive $|V_{cb}|$ at Belle

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9<sup>th</sup> International Workshop on the  
CKM Unitarity Triangle (CKM 2016)  
Nov 28-Dec 2, 2016, TIFR Mumbai, India

# Outline of this talk

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1. Belle  $B^0 \rightarrow D^{*-}l^+\nu$  with hadronic tag  
(CKM 2016 preliminary)

**BELLE-CONF-1612**



2. Belle  $B \rightarrow \overline{D}l^+\nu$  with hadronic tag

**Phys. Rev. D93, 032006 (2016)**

3. HFAG summer 2016 averages

# $B \rightarrow D^{(*)} \ell \nu$ decay width

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$$w = \frac{P_B \cdot P_{D^{(*)}}}{m_B m_{D^{(*)}}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

$$B \rightarrow D^* \ell \nu \quad \frac{d\Gamma}{dw} = \frac{G_F^2 m_{D^*}^3}{48\pi^3} (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \chi(w) \mathcal{F}^2(w) |V_{cb}|^2$$

$$B \rightarrow D \ell \nu \quad \frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} \mathcal{G}^2(w) |V_{cb}|^2$$

# Form factor parameterizations

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- Caprini, Lellouch, Neubert [Nucl.Phys. B530, 153(1998)]

$B \rightarrow D^* | \nu$

$$h_{A_1}(w) = h_{A_1}(1) [1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3],$$
$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2,$$

$B \rightarrow D | \nu$

$$\mathcal{G}(z) = \mathcal{G}(1)(1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3)$$

Parameters:  $F(1), \rho^2, R_1(1), R_2(1)$   
 $G(1), \rho^2$

- Boyd, Grinstein, Lebed [Phys. Rev. Lett. 74, 4603 (1995)]

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^N a_{i,n} z^n, \quad z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

Parameters: coefficients  $a_{i,n}$

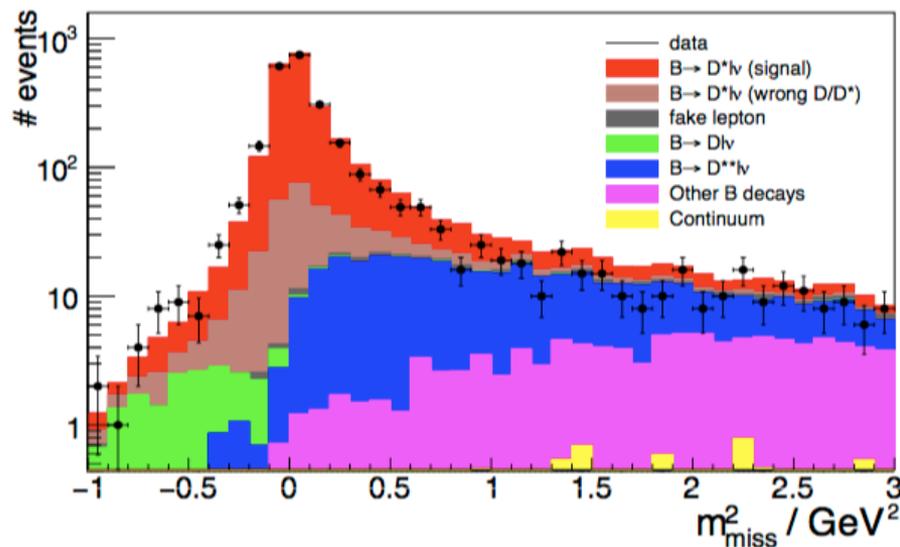
# Belle $B^0 \rightarrow D^{*-}l^+\nu$ with hadronic tag

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BELLE-CONF-1612 (preliminary)

# Event reconstruction and inclusive fit

- *Tag side*:  $B_{\text{tag}}$  reconstructed in over 1100 hadronic modes, 0.2% efficiency for neutral B mesons
- *Signal side*:  $B_{\text{sig}}$  assembled from an identified charged lepton (electron or muon) and a  $D^{*+}$  candidate
- $D^0 \rightarrow K^-\pi^+, K^-\pi^+\pi^0, K^-\pi^+\pi^-\pi^+$  (26.3%),  $D^+ \rightarrow K^-\pi^+\pi^+$  (9.4)%
- $D^{*+} \rightarrow D^0\pi^+, D^+\pi^0$  (98.4%)
- Signal is extracted from the missing mass distribution by an unbinned maximum likelihood fit



$\ell$	$\nu^{\text{sig}}$
$e + \mu$	$2374 \pm 53$
$e$	$1306 \pm 40$
$\mu$	$1066 \pm 34$

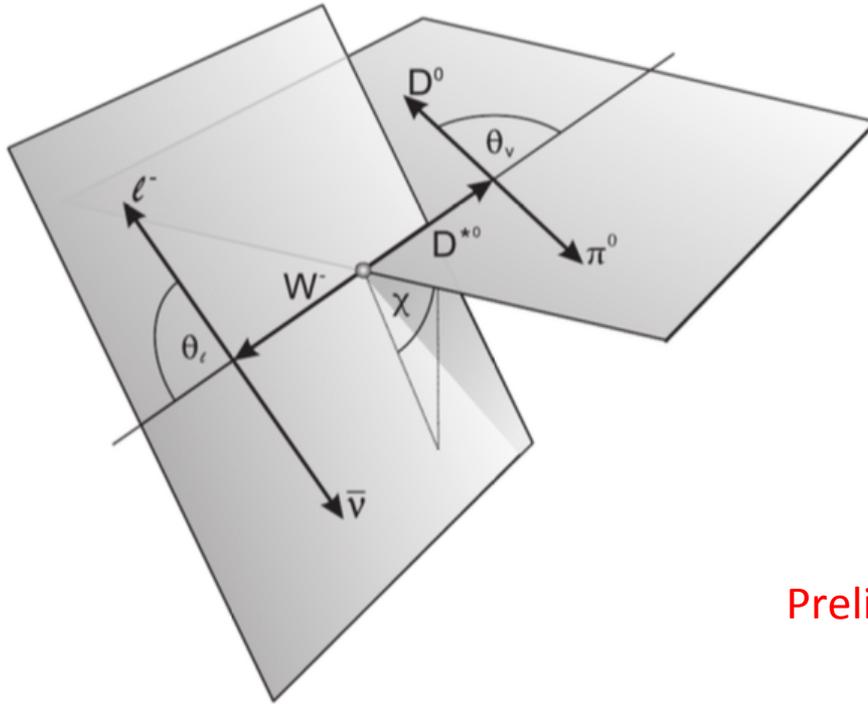
Preliminary!

$$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell) = (4.95 \pm 0.11 \pm 0.21) \times 10^{-2}$$

$$R_{e\mu} = \frac{\mathcal{B}(B^0 \rightarrow D^{*-} e^+ \nu_e)}{\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)} = 1.04 \pm 0.05 \pm 0.01$$

Largest syst: tag calibration

# Differential fit result



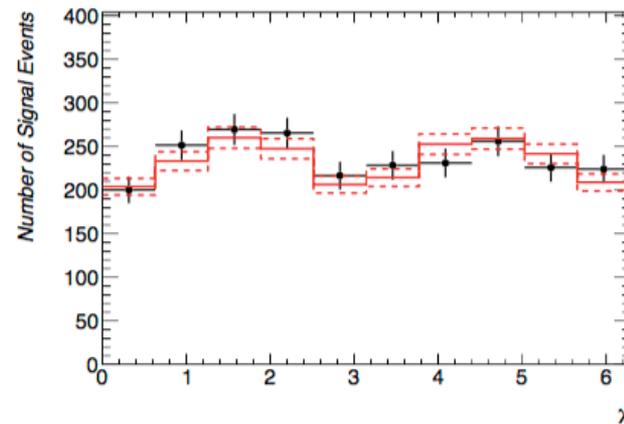
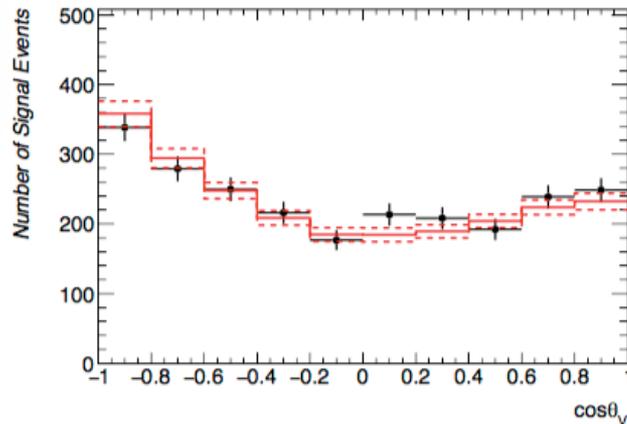
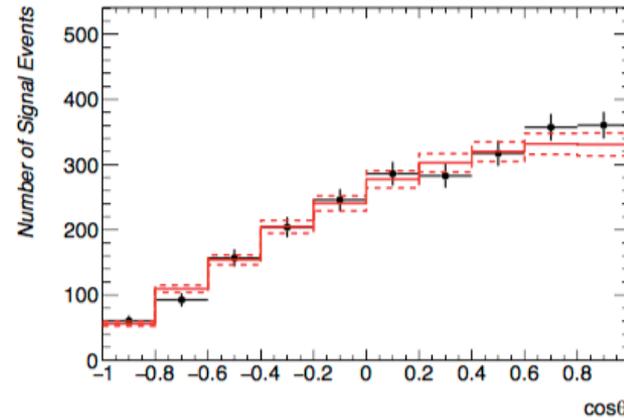
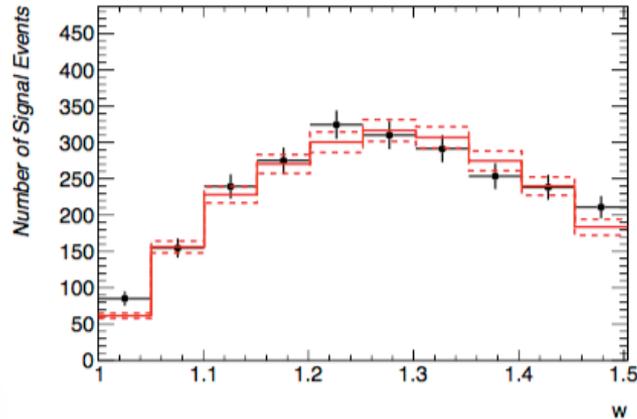
Preliminary!

- Yield is also extracted in  $4 \times 10$  bins of  $w$ ,  $\cos \theta_l$ ,  $\cos \theta_v$  and  $\chi$
- Overlapping samples  $\rightarrow$  statistical correlations between partial widths, determined with bootstrapping technique
- Unfolding of resolution effect with the SVD algorithm  
[Nucl.Instr.Meth. A372, 469 (1996)]

Variable	Bin	$\Delta\Gamma/\Delta x$ [ $10^{-15}\text{GeV}$ ]
$w$	1	$1.32 \pm 0.11$
	2	$2.08 \pm 0.15$
	3	$2.39 \pm 0.15$
	4	$2.57 \pm 0.16$
	5	$2.63 \pm 0.15$
	6	$2.46 \pm 0.14$
	7	$2.25 \pm 0.14$
	8	$2.08 \pm 0.13$
	9	$1.99 \pm 0.12$
	10	$1.83 \pm 0.13$
$\cos \theta_v$	1	$2.80 \pm 0.19$
	2	$2.30 \pm 0.14$
	3	$1.95 \pm 0.13$
	4	$1.70 \pm 0.11$
	5	$1.58 \pm 0.11$
	6	$1.65 \pm 0.11$
	7	$1.77 \pm 0.12$
	8	$2.00 \pm 0.14$
	9	$2.50 \pm 0.16$
	10	$3.19 \pm 0.25$
$\cos \theta_l$	1	$0.73 \pm 0.07$
	2	$1.18 \pm 0.09$
	3	$1.64 \pm 0.11$
	4	$2.04 \pm 0.13$
	5	$2.34 \pm 0.14$
	6	$2.50 \pm 0.15$
	7	$2.54 \pm 0.15$
	8	$2.68 \pm 0.16$
	9	$2.83 \pm 0.20$
	10	$2.82 \pm 0.24$
$\chi$	1	$1.86 \pm 0.16$
	2	$2.31 \pm 0.15$
	3	$2.59 \pm 0.16$
	4	$2.37 \pm 0.15$
	5	$1.95 \pm 0.13$
	6	$1.87 \pm 0.15$
	7	$2.11 \pm 0.15$
	8	$2.33 \pm 0.15$
	9	$2.15 \pm 0.15$
	10	$1.89 \pm 0.16$

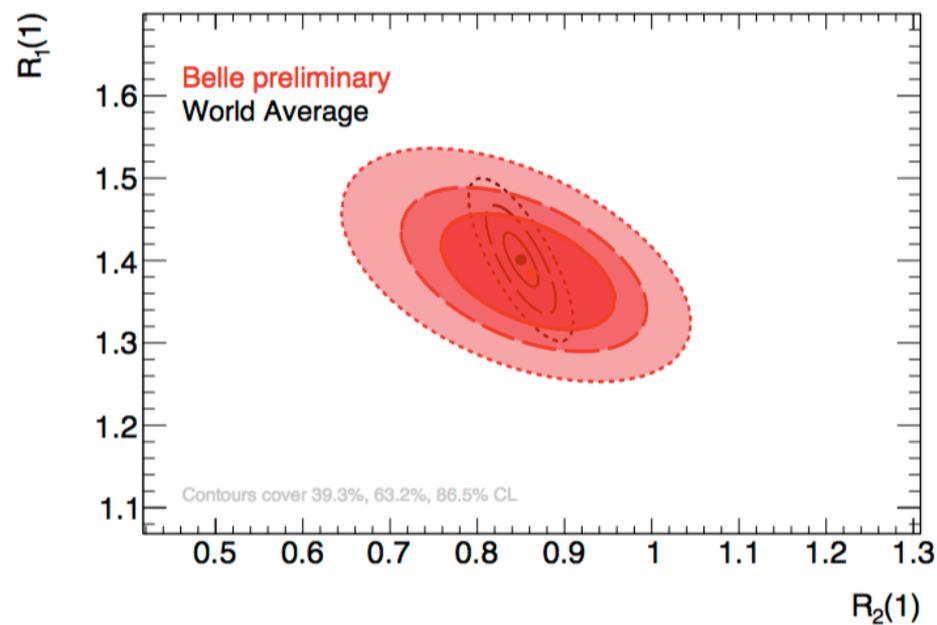
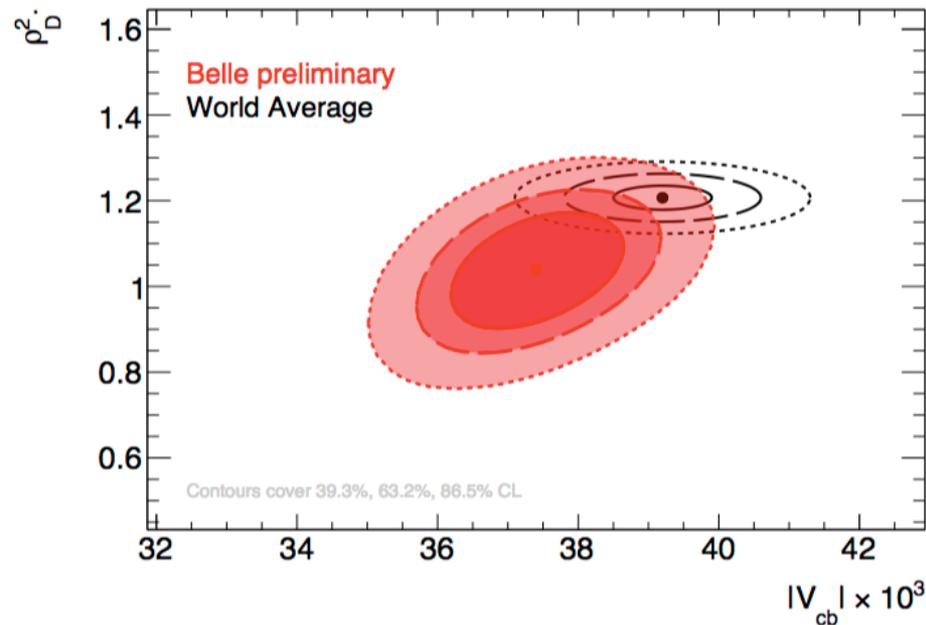
# CLN fit to the differential widths

$$\chi^2 = \left( \boldsymbol{\nu}_{\text{sig}} - \boldsymbol{\nu}_{\text{sig}}^{\text{pred}} \right) C^{-1} \left( \boldsymbol{\nu}_{\text{sig}} - \boldsymbol{\nu}_{\text{sig}}^{\text{pred}} \right) + \left( h_{A1}(1) - h_{A1}^{\text{la}}(1) \right)^2 / \left( \sigma_{h_{A1}(1)}^{\text{la}} \right)^2$$



Points with error bars: Belle data,  
red histogram: fit result, dashed histogram:  $\Delta\chi^2=1$  contour

# CLN fit to the differential widths (2)



Parameter	This result
$ V_{cb}  \times 10^3$	$37.4 \pm 1.2$
$\rho_D^2$	$1.04 \pm 0.13$
$R_1(1)$	$1.38 \pm 0.07$
$R_2(1)$	$0.86 \pm 0.10$

$$C = \begin{pmatrix} 1 & 0.42 & -0.20 & -0.16 \\ 0.42 & 1 & 0.21 & -0.87 \\ -0.20 & 0.21 & 1 & -0.47 \\ -0.16 & -0.87 & -0.47 & 1 \end{pmatrix}$$

$\chi^2 = 39.8$  for 40-4 degrees of freedom

Preliminary!

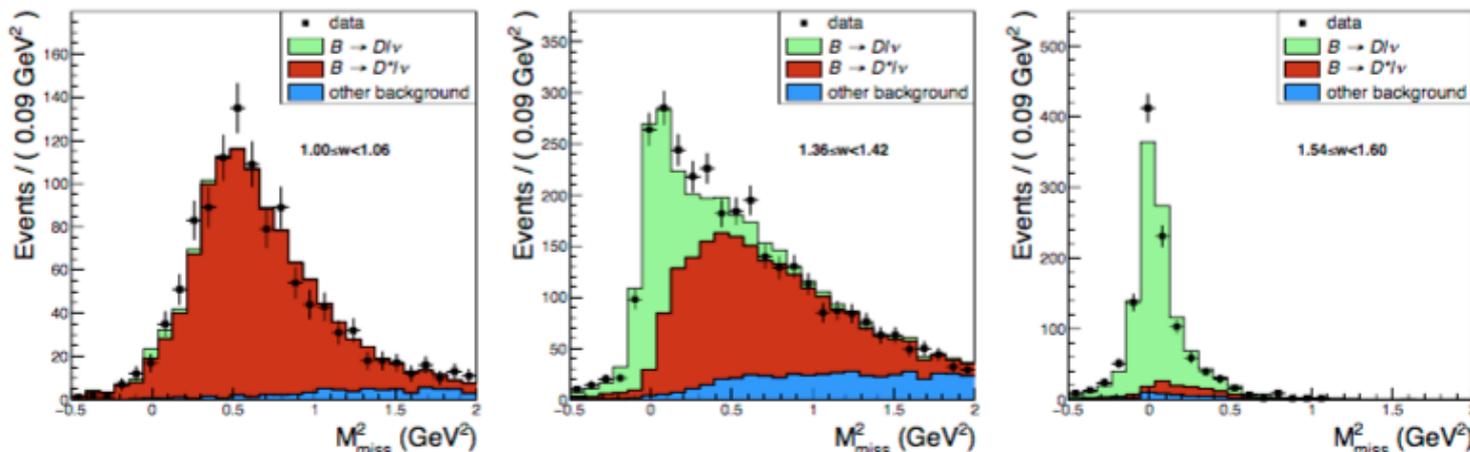
# Belle $B \rightarrow \bar{D}l\nu$ with hadronic tag

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Phys. Rev. D93, 032006 (2016)

# Event reconstruction

- Identical event reconstruction method
- 10  $D^+$  and 13  $D^0$  modes are used on the signal side, covering 28.9% and 40.1% of the width
- Signal extraction from  $M_{\text{miss}}^2$  in 10 bins of  $w$
- 16,992 +/- 192 signal events  
(5150 +/- 95 neutral, 11,843 +/- 167 charged B events)



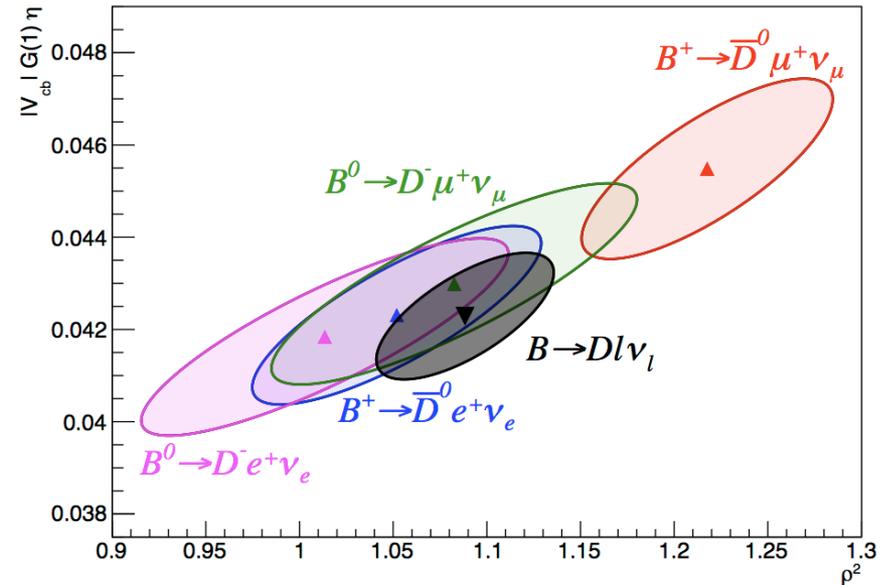
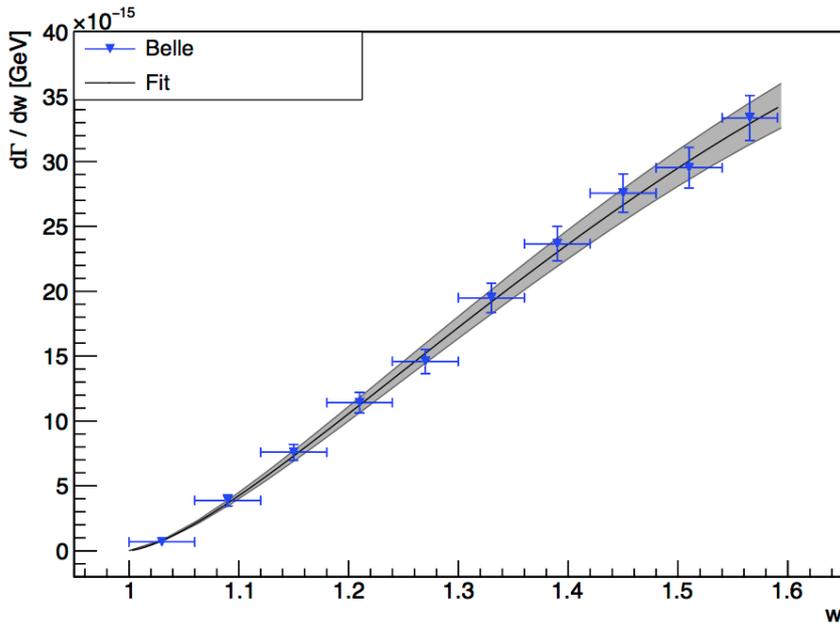
# Differential width and branching fraction

$i$	$w_{i,\min}$	$w_{i,\max}$	$\Delta\Gamma_i/\Delta w [10^{-15}\text{GeV}]$	$\rho_{ij,\text{syst}}$									
				0	1	2	3	4	5	6	7	8	9
0	1.00	1.06	$0.68 \pm 0.21 \pm 0.05$	1.000	0.682	0.677	0.663	0.654	0.656	0.664	0.648	0.608	0.560
1	1.06	1.12	$3.88 \pm 0.38 \pm 0.18$		1.000	0.976	0.974	0.969	0.972	0.972	0.961	0.933	0.900
2	1.12	1.18	$7.59 \pm 0.50 \pm 0.35$			1.000	0.991	0.987	0.990	0.989	0.980	0.959	0.929
3	1.18	1.24	$11.42 \pm 0.58 \pm 0.54$				1.000	0.993	0.993	0.990	0.980	0.961	0.934
4	1.24	1.30	$14.59 \pm 0.64 \pm 0.69$					1.000	0.996	0.992	0.985	0.972	0.952
5	1.30	1.36	$19.49 \pm 0.69 \pm 0.91$						1.000	0.996	0.991	0.979	0.956
6	1.36	1.42	$23.66 \pm 0.76 \pm 1.10$							1.000	0.995	0.981	0.952
7	1.42	1.48	$27.56 \pm 0.79 \pm 1.27$								1.000	0.992	0.968
8	1.48	1.54	$29.52 \pm 0.80 \pm 1.34$									1.000	0.985
9	1.54	$w_{\max}$	$33.37 \pm 0.86 \pm 1.50$										1.000

Sample	Signal yield	$\mathcal{B}$ [%]
$B^0 \rightarrow D^- e^+ \nu_e$	$2848 \pm 72 \pm 17$	$2.44 \pm 0.06 \pm 0.12$
$B^0 \rightarrow D^- \mu^+ \nu_\mu$	$2302 \pm 63 \pm 13$	$2.39 \pm 0.06 \pm 0.11$
$B^+ \rightarrow \bar{D}^0 e^+ \nu_e$	$6456 \pm 126 \pm 66$	$2.57 \pm 0.05 \pm 0.13$
$B^+ \rightarrow \bar{D}^0 \mu^+ \nu_\mu$	$5386 \pm 110 \pm 51$	$2.58 \pm 0.05 \pm 0.13$
$B^0 \rightarrow D^- \ell^+ \nu_\ell$	$5150 \pm 95 \pm 29$	$2.39 \pm 0.04 \pm 0.11$
$B^+ \rightarrow \bar{D}^0 \ell^+ \nu_\ell$	$11843 \pm 167 \pm 120$	$2.54 \pm 0.04 \pm 0.13$
$B \rightarrow D \ell \nu_\ell$	$16992 \pm 192 \pm 142$	$2.31 \pm 0.03 \pm 0.11$

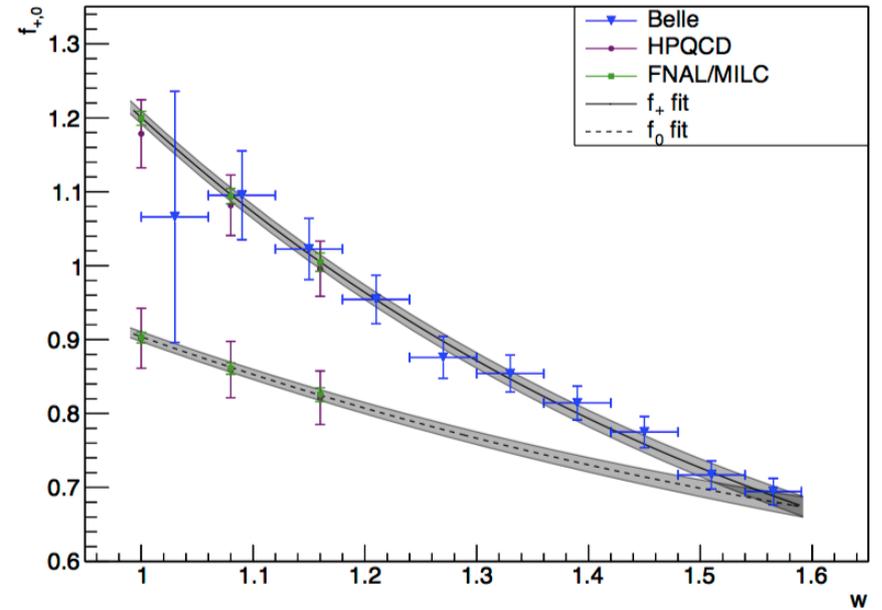
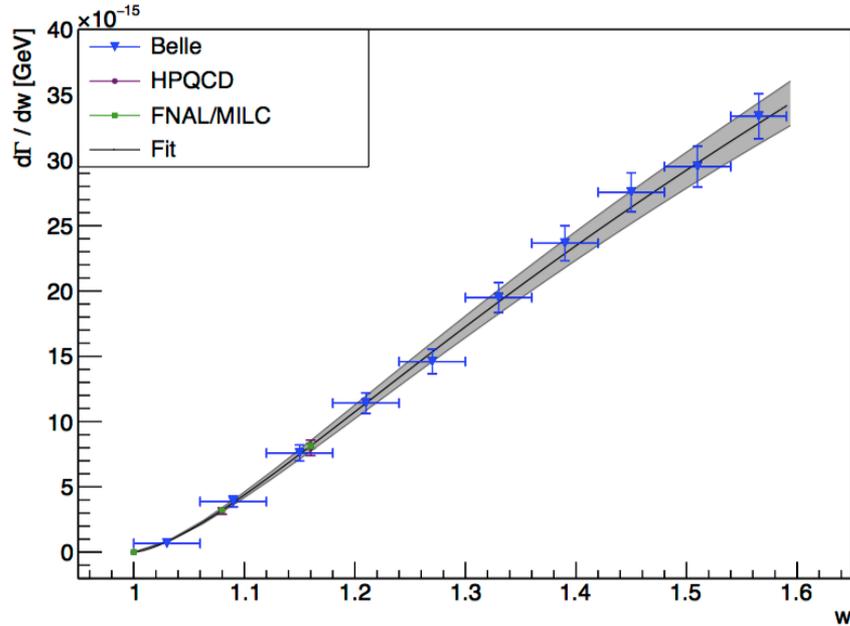
$$R_D^{\mu/e} = \mathcal{B}(B \rightarrow D\mu\nu)/\mathcal{B}(B \rightarrow De\nu) = 0.995 \pm 0.022(\text{stat}) \pm 0.039(\text{syst})$$

# CLN fit to the differential widths



	$B^+ \rightarrow \bar{D}^0 e^+ \nu_e$	$B^+ \rightarrow \bar{D}^0 \mu^+ \nu_\mu$	$B^0 \rightarrow D^- e^+ \nu_e$	$B^0 \rightarrow D^- \mu^+ \nu_\mu$	$B \rightarrow D l \nu_l$
$\eta_{EW} \mathcal{G}(1)  V_{cb}  [10^{-3}]$	$42.31 \pm 1.94$	$45.48 \pm 1.96$	$41.84 \pm 2.14$	$42.99 \pm 2.18$	$42.29 \pm 1.37$
$\rho^2$	$1.05 \pm 0.08$	$1.22 \pm 0.07$	$1.01 \pm 0.10$	$1.08 \pm 0.10$	$1.09 \pm 0.05$
Correlation	0.81	0.77	0.85	0.84	0.69
$\eta_{EW}  V_{cb}  [10^{-3}]$	$40.14 \pm 1.86$	$43.15 \pm 1.89$	$39.69 \pm 2.05$	$40.78 \pm 2.09$	$40.12 \pm 1.34$
$\chi^2/n_{df}$	2.19/8	2.71/8	9.65/8	4.36/8	4.57/8
Prob.	0.97	0.95	0.29	0.82	0.80

# BGL fit to differential widths and lattice data



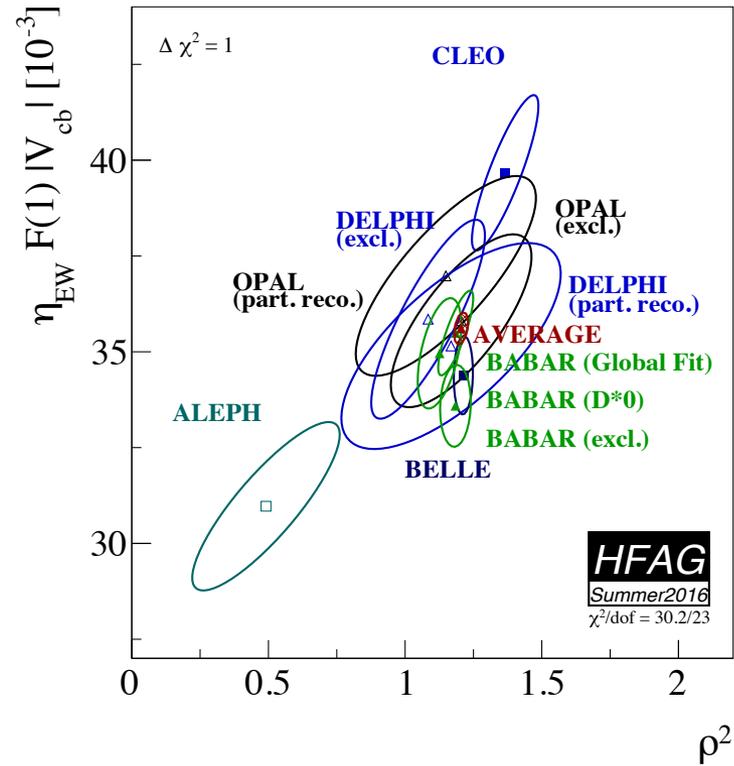
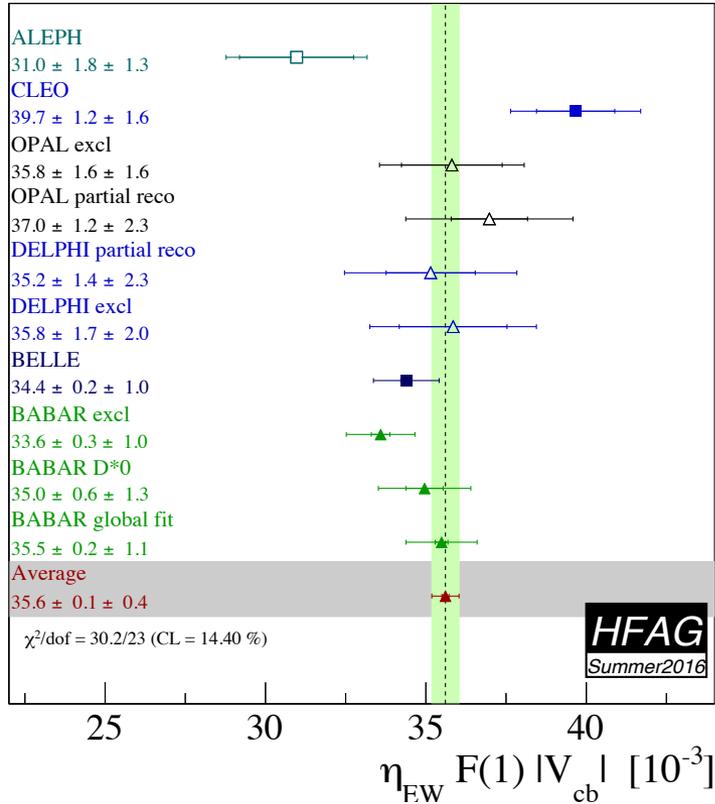
	$N = 2$	$N = 3$	$N = 4$
$a_{+,0}$	$0.0127 \pm 0.0001$	$0.0126 \pm 0.0001$	$0.0126 \pm 0.0001$
$a_{+,1}$	$-0.091 \pm 0.002$	$-0.094 \pm 0.003$	$-0.094 \pm 0.003$
$a_{+,2}$	$0.34 \pm 0.03$	$0.34 \pm 0.04$	$0.34 \pm 0.04$
$a_{+,3}$	–	$-0.1 \pm 0.6$	$-0.1 \pm 0.6$
$a_{+,4}$	–	–	$0.0 \pm 1.0$
$a_{0,0}$	$0.0115 \pm 0.0001$	$0.0115 \pm 0.0001$	$0.0115 \pm 0.0001$
$a_{0,1}$	$-0.058 \pm 0.002$	$-0.057 \pm 0.002$	$-0.057 \pm 0.002$
$a_{0,2}$	$0.22 \pm 0.02$	$0.12 \pm 0.04$	$0.12 \pm 0.04$
$a_{0,3}$	–	$0.4 \pm 0.7$	$0.4 \pm 0.7$
$a_{0,4}$	–	–	$0.0 \pm 1.0$
$\eta_{EW} V_{cb} $	$40.01 \pm 1.08$	$41.10 \pm 1.14$	$41.10 \pm 1.14$
$\chi^2/n_{df}$	24.7/16	11.4/16	11.3/16
Prob.	0.075	0.787	0.787

Lattice data	$\eta_{EW} V_{cb} [10^{-3}]$	$\chi^2/n_{df}$	Prob.
FNAL/MILC [15]	$40.96 \pm 1.23$	6.01/10	0.81
HPQCD [32]	$41.14 \pm 1.88$	4.83/10	0.90
FNAL/MILC & HPQCD [15, 32]	$41.10 \pm 1.14$	11.35/16	0.79

# Summer 2016 HFAG results for $|V_{cb}|$ exclusive

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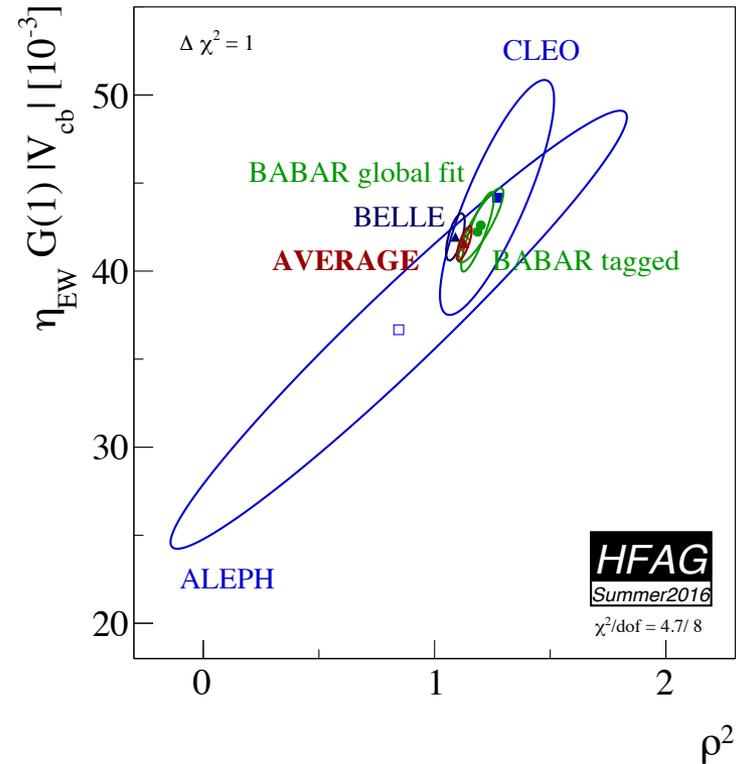
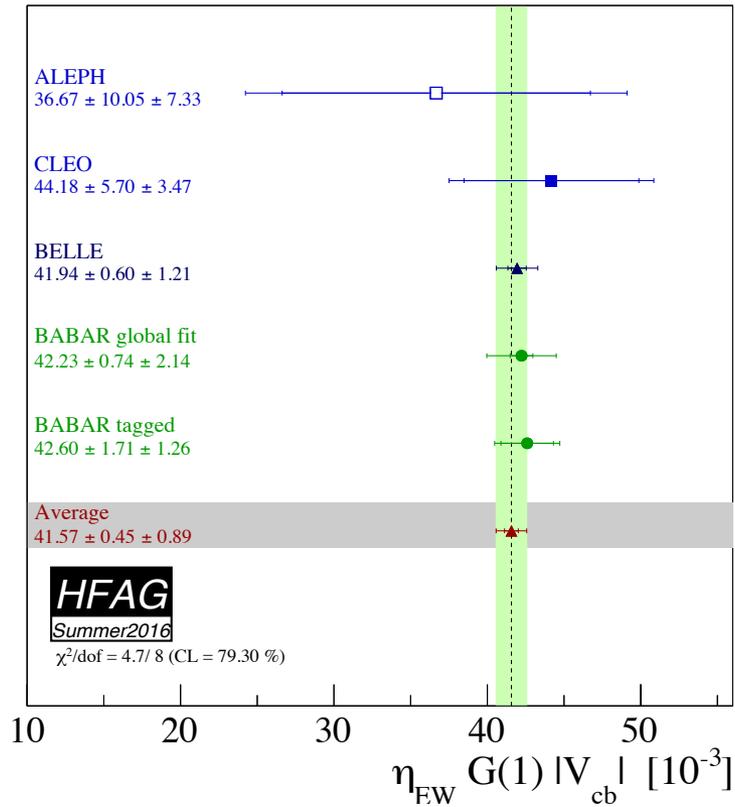
# $\eta_{EW} F(1) |V_{cb}|$ ( $B \rightarrow D^* | \nu$ )



$$\eta_{EW} F(1) |V_{cb}| = (35.61 \pm 0.11 \pm 0.41) \times 10^{-3}$$

$$[2014: \eta_{EW} F(1) |V_{cb}| = (35.81 \pm 0.11 \pm 0.44) \times 10^{-3}]$$

# $\eta_{EW} G(1) |V_{cb}| (B \rightarrow D|v)$



$$\eta_{EW} G(1) |V_{cb}| = (41.57 \pm 0.45 \pm 0.89) \times 10^{-3}$$

$$[2014: \eta_{EW} G(1) |V_{cb}| = (42.65 \pm 0.72 \pm 1.35) \times 10^{-3}]$$

# $|V_{cb}|$ exclusive

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- $B \rightarrow D^* \ell \nu$

- $\eta_{EW} F(1) |V_{cb}| = (35.61 \pm 0.11_{\text{stat}} \pm 0.41_{\text{syst}}) \times 10^{-3}$

- $\eta_{EW} F(1) = (0.920 \pm 0.014)$  [FNAL/MILC, PRD89, 114504]

- $|V_{cb}| = (38.71 \pm 0.47_{\text{exp}} \pm 0.59_{\text{th}}) \times 10^{-3}$

- $B \rightarrow D \ell \nu$

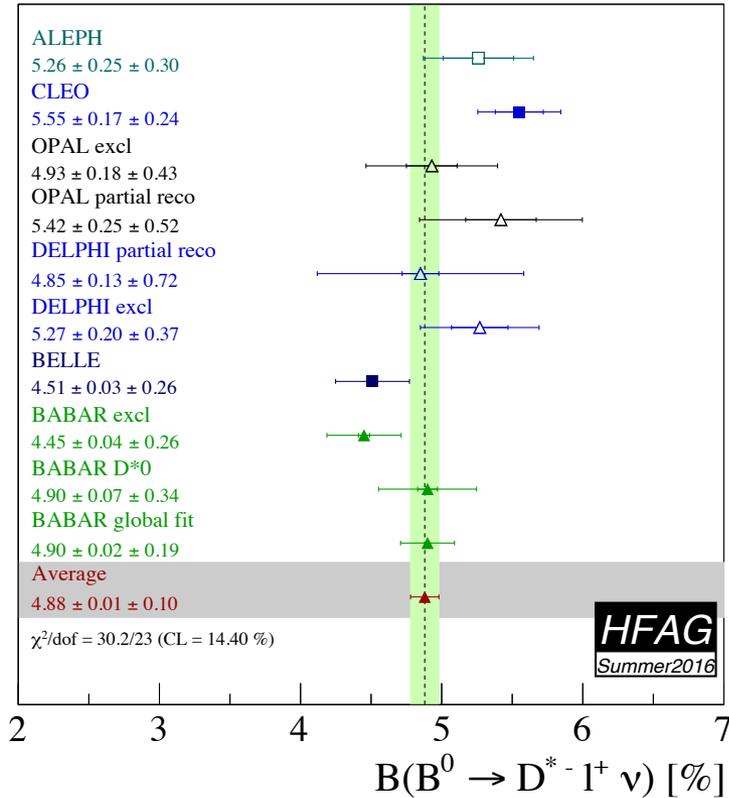
- $\eta_{EW} G(1) |V_{cb}| = (41.57 \pm 0.45 \pm 0.89) \times 10^{-3}$

- $G(1) = 1.0541 \pm 0.0083$  [FNAL/MILC, PRD92, 034506]

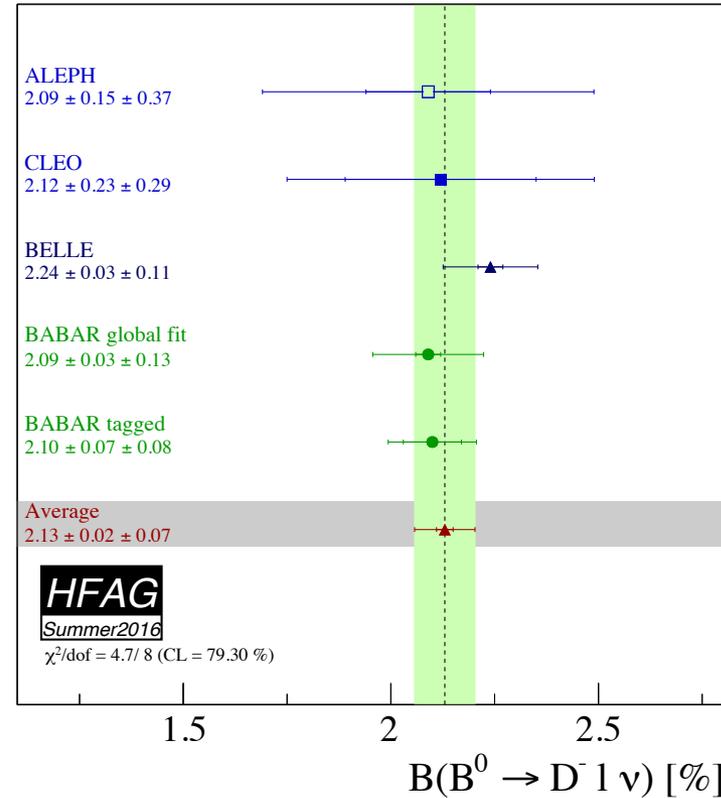
- $\eta_{EW} = 1.0066 \pm 0.0016$  [NPB 196, 83]

- $|V_{cb}| = (39.18 \pm 0.94_{\text{exp}} \pm 0.31_{\text{th}}) \times 10^{-3}$

# Branching fractions



$$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}) = (4.88 \pm 0.10)\%$$



$$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}) = (2.13 \pm 0.07)\%$$

# Summary

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- Belle  $B^0 \rightarrow D^{*+}l\nu$  with hadronic tag [[BELLE-CONF-1612](#)]
  - About 2400 reconstructed decays
  - $\text{Br}(B^0 \rightarrow D^{*+}l\nu) = (4.95 \pm 0.11_{\text{stat}} \pm 0.21_{\text{syst}})\%$
  - CLN fit:  $|V_{cb}| = (37.4 \pm 1.2) \times 10^{-3}$
- Belle  $B \rightarrow Dl\nu$  with hadronic tag [[PRD93, 032006 \(2016\)](#)]
  - About 17,000 signal events (charged and neutral)
  - $\text{Br}(B^0 \rightarrow Dl\nu) = (2.31 \pm 0.03_{\text{stat}} \pm 0.11_{\text{syst}})\%$
  - CLN fit:  $|V_{cb}| = (39.86 \pm 1.33) \times 10^{-3}$
  - BGL fit:  $|V_{cb}| = (40.83 \pm 1.13) \times 10^{-3}$
- HFAG summer 2016 averages
  - $B \rightarrow D^{*+}l\nu$ :  $|V_{cb}| = (38.71 \pm 0.47_{\text{exp}} \pm 0.59_{\text{th}}) \times 10^{-3}$
  - $B \rightarrow Dl\nu$ :  $|V_{cb}| = (39.18 \pm 0.94_{\text{exp}} \pm 0.31_{\text{th}}) \times 10^{-3}$

# BACKUP

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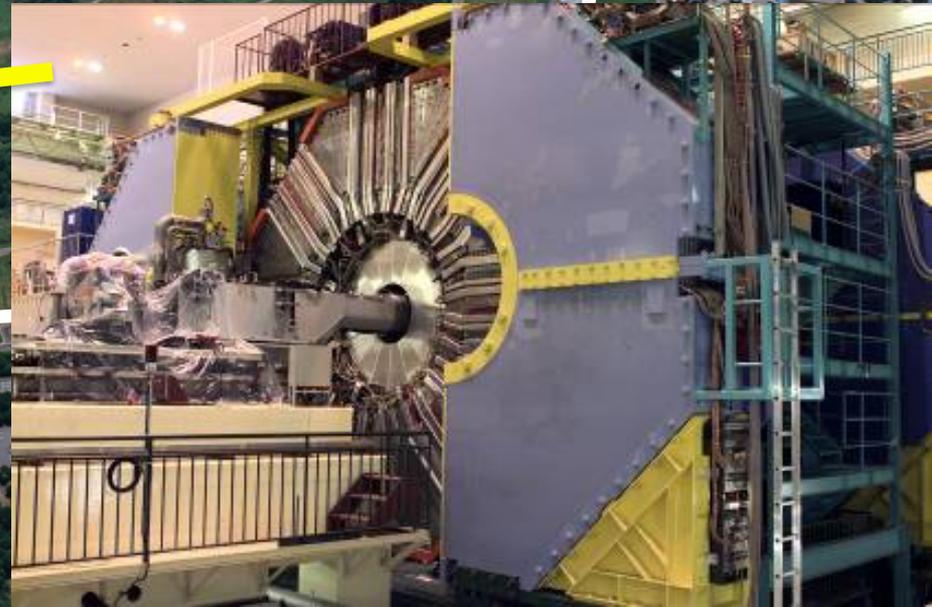
# 1999 – 2010: B factory at KEK (Japan)

KEKB double  
ring  $e^+e^-$  collider

Linac

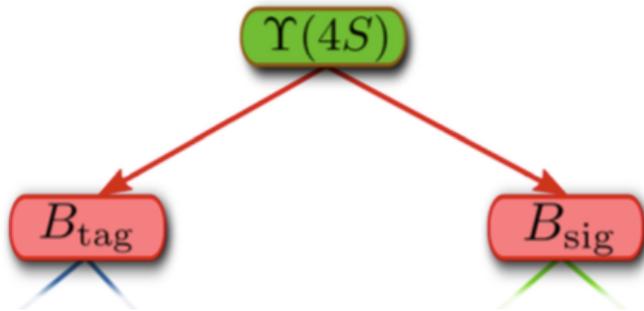
$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$

Belle detector



- World largest B meson sample  
~771 million  $B\bar{B}$  events
- Over 450 Belle physics publications

# Tagging techniques for $\Upsilon(4S)$ events



- Tagging provides:
  - Background suppression
  - Information on  $B_{\text{sig}}$  (4-momentum)

PURITY



EFFICIENCY



## Untagged

- No requirement on  $B_{\text{tag}}$
- High efficiency, low purity

## Semileptonic tag

- $B_{\text{tag}} \rightarrow D^* l \nu$
- Efficiency  $\sim O(0.2\%)$

## Hadronic tag

- $B_{\text{tag}} \rightarrow \text{hadrons}$
- Efficiency  $\sim O(0.1\%)$

# Systematic uncertainties on $\text{BR}(B^0 \rightarrow D^{*-}l^+\nu)$

Error Source	$\Delta\mathcal{B}$ [%]
Tagging Calibration	3.6
$N_{B\bar{B}}$	1.4
$f_{+0}$	1.1
PDF shapes	0.9
$\mathcal{B}(D \rightarrow K\pi(\pi)(\pi))$	0.4
$\mathcal{B}(D^* \rightarrow D\pi)$	0.2
$\mathcal{B}(\bar{B} \rightarrow D^{**}l\bar{\nu}_l)$	0.2
$e$ PID	0.2
$\mu$ PID	0.1
$\pi_{\text{slow}}$ Eff.	0.1
$\mathcal{B}(\bar{B} \rightarrow Dl\bar{\nu}_l)$	< 0.1
$B \rightarrow D^{(*,**)}l\bar{\nu}_l$ FFs	< 0.1
Lepton Fakes	< 0.1
$K$ PID	< 0.1
Total	4.2