QCD sum rules predictions for exclusive $b \to c$ transitions

Status 2016

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Motivation

$$\frac{\mathrm{d}\Gamma(\overline{B} \to D^* \ell \nu)}{\mathrm{d}\omega} \propto |V_{cb}|^2 |\mathcal{F}(\omega)|^2$$

$$|V_{cb}|\mathcal{F}(1) = 35.81 \pm 0.11\big|_{\text{stat}} \pm 0.44\big|_{\text{syst}}$$

[HFAG 2014, averg. of ALEPH, BaBar, Belle, CLEO, DELPHI, OPAL meas.]

$$\mathcal{F}(1) = 0.906 \pm 0.004\big|_{\text{stat}} \pm 0.012\big|_{\text{syst}}$$

[Fermilab/MILC PRD 89 (2014) 114504]

$$|V_{cb}| = (39.2 \pm 0.7) \cdot 10^{-3}$$

[PDG 2014 (w/ 2015 partial update)]

Continuum methods are important to

- provide complementary information
  - $\mathcal{F}(\omega_{\text{max}} \approx 1.5)$
  - shape of $\mathcal{F}(\omega)$
- cross check existing lattice results

[also for $B \to D\ell\nu$: $\mathcal{G}(\omega)$]

Outline

Review two continuum methods that have been successfully used to infer knowledge on $B \to D^{(*)}$ form factors

- Zero Recoil Sum Rules

  inclusive constraints on combination of form factors in a single phase-space point

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  inclusive constraints on combination of form factors in a single phase-space point

– Light-Cone Sum Rules with B-meson Light-Cone Distribution Amplitudes

  constraint on each form factor in small region around maximal hadronic recoil

– brief comment on Dispersive Bounds

  contraints on the shape of the form factors as functions of the momentum transfer

Zero Recoil Sum Rules: Basic Idea

Consider an artifical two-point function [at $p_{X_c} = M_{X_c} v$]

$$T_J(\varepsilon) \equiv \frac{1}{N_J} \int \mathrm{d}^4 x \, e^{i(v\cdot x)\varepsilon} \, \langle \overline{B}(M_B v) | \mathcal{T} \left\{ J^{\dagger,\mu}(x), J_\mu(0) \right\} | \overline{B}(M_B v) \rangle$$

$J^\mu = \bar{c}\gamma^\mu(\gamma_5) b$ $\varepsilon = M_{X_c} - M_D$: excitation energy above M_D

– can be obtained in two representations

OPE inclusive calculation: express in terms of local operators, and expand in $1/m_c$, $1/m_b$ and α_s

hadronic express in terms of spectral densities involving hadronic matrix elements of exclusive processes (form factors)

– sum rule: equate moments of $T(\varepsilon)$ in both represenations, and infer knowledge on the form factors

$$\oint_{|\varepsilon|=\varepsilon_M} \mathrm{d}\varepsilon \, T_A(\varepsilon) = |\mathcal{F}(1)|^2 + \ldots$$

– hadronic representation is sum of strictly positive quantities
$\Rightarrow$ upper bound on $B \to D^{(*)}$ form factors

Limitations

- triple expansion in α_s, $1/m_b$, $1/m_c$
- $\langle\overline{B}||\overline{B}\rangle$ matrix elements of operators comprise non-perturbative input
  - universal input: μ_π^2, μ_G^2, ρ_D^3, ρ_{LS}^3
  - can be extracted from $B \to X_c \ell\overline{\nu}$ data [see backups for definition]
- $1/m_c$ expansion might converge slowly or not at all

  $B \to D$ only $1/m_c^2$ corrections in BPS limit [Uraltsev Phys.Lett. B585 (2004) 253-262]

  $B \to D^*$ reverse setup ($D^{(*)} \to B^{(*)}$ sum rules) suggests that the terms in the $1/m_c$ expansion alternate in sign

  $\Rightarrow$ good convergence expected [Gambino/Mannel/Uraltsev JHEP 1210 (2012) 169]
- continuum background can be estimated in the OPE, but involves matrix elements of nonlocal operators ($\rho_{\pi\pi}, \ldots$) [Gambino/Mannel/Uraltsev JHEP 1210 (2012) 169]
  - ZRSR provides reliable upper bound on sum of form factor terms

Status 2016: $B \to D\mu\bar{\nu}$

ZRSR upper bounds/estimates of $\mathcal{G}(1)$:

- $O(\alpha_s)$ and partial $O(\alpha_s^2)$ terms (for the unit operator)
- up to $O(1/m^3)$ correction

U2004 $\mathcal{G}(1) < 1.04 \pm 0.02 \pm \delta_{\text{exp}}$ [Uraltsev Phys.Lett. B585 (2004) 253-262]

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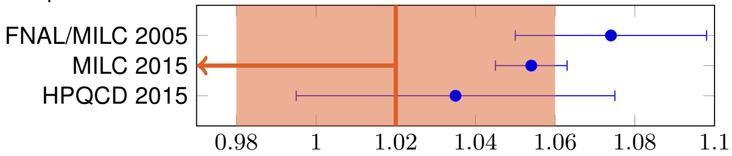
Comparison with lattice results



FNAL/MILC 2005 $G(1) = 1.074(18)_{\text{stat}}(16)_{\text{syst}}$

[Fermilab Lattice and MILC Collaborations Nucl. Phys. Proc. Suppl. 140, 461 (2005)]

MILC 2015 $G(1) = 1.054(4)_{\text{stat}}(8)_{\text{syst}}$ [MILC Collaboration Phys.Rev. D92 (2015) no.3, 034506]

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LvD2016 $\mathcal{G}(1) < 1.012 \pm 0.012$ preliminary! [Lancierini,DvD w.i.p.]

[based on inputs from Alberti,Gambino,Healy,Nandi Phys.Rev.Lett. 114 (2015) no.6, 061802]

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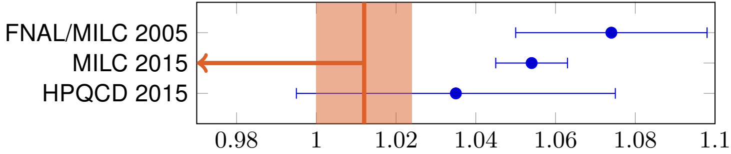

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- complete $O(\alpha_s^2)$ (for the unit operator)
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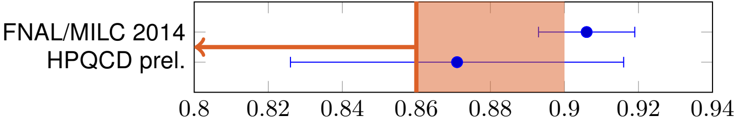

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[HPQCD Collaboration preliminary result shown at CKM2016]

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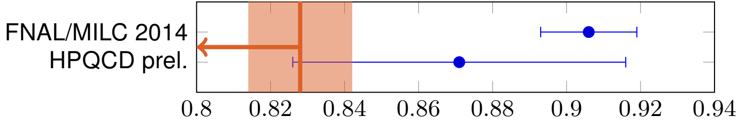

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What changed?

updated input values

2012

$$
\begin{aligned}
m_b^{\text{kin}}(1\,\text{GeV}) &= 4.6\,\text{GeV} \\
m_c^{\text{kin}}(1\,\text{GeV}) &= 1.2\,\text{GeV} \\
\mu_\pi^2(1\,\text{GeV}) &= 0.4\,\text{GeV}^2 \\
\mu_G^2(1\,\text{GeV}) &= 0.3\,\text{GeV}^2 \\
\rho_D^3(1\,\text{GeV}) &= 0.15\,\text{GeV}^3 \\
-\rho_{LS}^3(1\,\text{GeV}) &= 0.12\,\text{GeV}^3
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2016

$$m_b^{\text{kin}}(1\,\text{GeV}) = 4.561 \pm 0.021\,\text{GeV}$$
$$m_c^{\text{kin}}(1\,\text{GeV}) = 1.092 \pm 0.020\,\text{GeV}$$
$$\mu_\pi^2(1\,\text{GeV}) = 0.464 \pm 0.067\,\text{GeV}^2$$
$$\mu_G^2(1\,\text{GeV}) = 0.333 \pm 0.061\,\text{GeV}^2$$
$$\rho_D^3(1\,\text{GeV}) = 0.175 \pm 0.040\,\text{GeV}^3$$
$$-\rho_{LS}^3(1\,\text{GeV}) = 0.146 \pm 0.096\,\text{GeV}^3$$

[Alberti,Gambino,Healey,Nandi Phys.Rev.Lett. 114 (2015) no.6, 061802]

old values exhibit $> 3\sigma$ tension, $\approx 2\sigma$ deviation, $< 1\sigma$ agreement

Light-Cone Sum Rules: Basic Idea

– construct an artificial correlator

$$F_{\alpha\,\mu}(p,q) = i\int \mathrm{d}^4x\, e^{ip\cdot x}\langle 0|\mathcal{T}\{\bar{d}\Gamma_\alpha c(x), \bar{c}\gamma_\mu(1-\gamma_5)b(0)|\overline{B}\rangle$$

$$= \frac{\langle 0|\bar{d}\Gamma_\alpha|D^{(*)}\rangle\,\langle D^{(*)}|\bar{c}\gamma_\mu(1-\gamma_5)b|\overline{B}\rangle}{M^2_{D^{(*)}}-p^2} + \text{multi-body contributions}$$

p: momentum of $D^{(*)}$ q: momentum of leptons

– for q^2 "sufficiently faw away from zero recoil" the integral is dominated by light-like distances x^2

– apply Operator Product Expansion on the light cone
  – input: universal non-perturbative non-local matrix elements $\langle 0|\bar{d}(x)\Gamma h_v(0)|\overline{B}\rangle$
  – parametrized as B-meson light-cone distribution amplitudes
  – note: defined in HQET, subject to power corrections

– relate to $B \to D^{(*)}$ form factors and $D^{(*)}$ decay constants

Complementary Information

- complementary to ZRSR
  - each form factor can be obtained individually
- complementary to Lattice and ZRSR
  - by contruction the LCSRs apply at/close to maximum hadronic recoil
  - can be used to anchor parametrization of the FFs for arbitrary momentum transfer
  - so far not used in experimental analyses

Status ~~2016~~ 2008

$\overline{B} \to D\mu\overline{\nu}$:

$$\mathcal{G}(\omega_{\text{max}}) = 0.61 \pm 0.11\big|_{\text{SR}} \pm 0.10\big|_{f_B} \pm 0.07\big|_{f_D}$$

$\overline{B} \to D^*\mu\overline{\nu}$:

$$h_{A_1}(\omega_{\text{max}}) = 0.65 \pm 0.12\big|_{\text{SR}} \pm 0.11\big|_{f_B} \pm 0.07\big|_{f_{D^*}}$$

$$R_1(\omega_{\text{max}}) = 1.32 \pm 0.04\big|_{\text{SR}} \qquad \text{[CLN: } R_1 = 1.22\text{]}$$

$$R_2(\omega_{\text{max}}) = 0.91 \pm 0.17\big|_{\text{SR}} \qquad \text{[CLN: } R_1 = 0.84\text{]}$$

[Faller,Khodjamirian,Klein,Mannel Eur.Phys.J. C60 (2009) 603-615]

Uncertainty budgets:

- f_B due to normalization of B-meson LCDA
- $f_{D^{(*)}}$ due to decay constant in dispersion relation
- SR due to Sum Rule parameters (λ_B, M^2, s_0, ...)

[see backups for details]

Briefly: Dispersive Bounds

– dispersively relate hadronic matrix elements to vacuum-to-vacuum matrix elements

$$\Pi^{\mu\nu} \equiv i \int \mathrm{d}^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\{J^{\dagger,\mu}(x), J^{\nu}(0)\}|0\rangle$$

q: momentum of $B\,D$ pair

  – relates $\overline{B} \to D^{(*)}$, $\overline{B}^* \to D^{(*)}$, and further exclusive matrix elements with each other
  – use of analytic structure of $\Pi^{\mu\nu}$ in plane of complex-valued momentumm transfer

– can be used to infer knowledge of the shape of the $\overline{B} \to D^{(*)}$ form factors as functions of momentum transfer
  – inspired CLN parametrization

– crucial input: normalization of form factors at one kinematical point
  – last results date from 1998, in dire need of update

[Caprini,Lellouch,Neubert Nucl.Phys. B530 (1998) 153-181]

Summary

- few new developments

- zero recoil sum rules still at odds with (some) lattice inputs

  $B \to D$ MILC 2015 at $\sim 3\sigma$ tension, HPQCD 2014 compatible
  $B \to D^*$ FNAL/MILC 2015 at $\sim 5\sigma$ tension, HPQCD prel. compatible

- light-cone sum rules provide information complementary to lattice results
  - so far, not used in fits to $\overline{B} \to D^{(*)}\mu\overline{\nu}$ spectra as functions of recoil ω

- dispersive bounds used to guide CLN parametrization
  - input parameters from 1998
  - in desparate need of an update

Appendix

Definition of the Hadronic Matrix Elements

at order $1/m^2$:

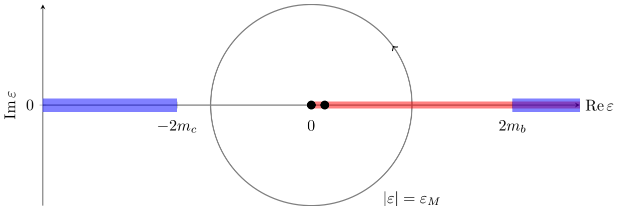
$$\mu_\pi^2 = -\frac{1}{2M_B}\langle\overline{B}|\bar{h}_v(iD_\perp)^2 h_v|\overline{B}\rangle\,, \quad \mu_G^2 = -\frac{i}{4M_B}\langle\overline{B}|\bar{h}_v\sigma^{\mu\nu}[iD_{\perp\mu}, iD_{\perp\nu}]h_v|\overline{B}\rangle$$

at order $1/m^3$:

$$\rho_D^3 = +\frac{1}{4M_B}\langle\overline{B}|\left[iD_{\perp\mu},[i(v\cdot D),iD_\perp^\mu]\right]|\overline{B}\rangle\,,$$
$$\rho_{LS}^3 = -\frac{i}{4M_B}\langle\overline{B}|\sigma^{\mu\nu}\left[iD_{\perp\mu},[i(v\cdot D),iD_{\perp\nu}]\right]|\overline{B}\rangle$$

[see e.g. Mannel,Turczyk,Uraltsev JHEP 1011 (2010) 109]

Zero-Recoil Sum Rule



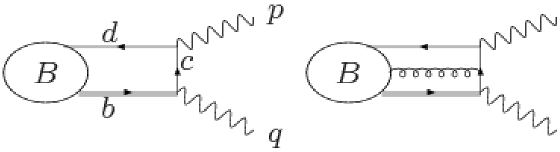
OPE Representation

- pole(s) at $\varepsilon = 0$
- parasitic branch cut from $-2m_c \to -\infty$
- parasitic branch cut from $+2m_b \to +\infty$
- separation scale $\mu \approx \varepsilon_M = 0.75\,\text{GeV}$ chosen for large distance from parasitic branch cuts, while still large enough to separate hard from soft modes in the OPE

Hadronic Representation

- pole for D at $\varepsilon = 0$
- pole for D^* at $\varepsilon = M_{D^*} - M_D - i\ldots$
- branch cuts $(D + n \times \pi, \ldots)$ from $n \times M_\pi \to +\infty$

Light-Cone Sum Rules with B-meson LCDAs



Input parameters

- $\lambda_B = 460 \pm 110$ MeV: inverse moment of the two-particle LCDA
- $M^2 = 3 \dots 6$ GeV^2: Borel parameter window
- $s_0^{D^{(*)}} = 6.0(8.0)$ GeV^2: hadronic threshold

[Faller,Khodjamirian,Klein,Mannel Eur.Phys.J. C60 (2009) 603-615]