On the Validity of the Effective Potential and the Precision of Higgs Self Couplings

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Work done with Minho Son, Seung J Lee
Based on arXiv:1709.03232
The Shape of Things to come

\[ V_h = \frac{1}{2} m_h^2 h^2 + \frac{c_3}{6} \left( \frac{3 m_h^2}{\nu} \right) h^3 + \frac{d_4}{24} \left( \frac{3 m_h^2}{\nu^2} \right) h^4 \]

We want to reconstruct the global picture

unknown!! - \( c_3 = d_3 = 1 \) (SM) - measurement needed to test the validity of SM
Can Higgs potential be tested by self coupling measurements?

- Cubic higgs self coupling
- HH production via gluon fusion is the best channel

Negatively Interfering

\[ \sigma \sim \frac{m_h^2}{\hat{s}} \]

Threshold region = big backgrounds

\[ \sim 40 \text{ fb} @14 \text{ TeV} \]

\times BR

Very small signal rate
Nothing to discuss about Higgs self coupling using current data

Cubic coupling @ HL LHC using 3000/fb

✓ Very tough process

\( b\bar{b}\gamma\gamma (0.264\%) \)

Seems to be the best channel so far

We would see only ~ 10 events by the end of HL LHC

an exclusion at 95% C.L. of BSM models with \( \lambda_3/\lambda_{3,SM} \leq -1.3 \) and \( \lambda_3/\lambda_{3,SM} \geq 8.7 \)

Similarly for \( b\bar{b}\gamma\gamma, b\bar{b}\tau^+\tau^- \) by CMS
Nothing to discuss about Higgs self coupling using current data

Cubic coupling at HL LHC using 3000/fb

✓ Very tough process

$b\bar{b}\gamma\gamma$ (0.264%)

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Similarly for $b\bar{b}\gamma\gamma$, $b\bar{b}\tau^+\tau^-$ by CMS

-0.8 < $\lambda/\lambda_{SM}$ < 7.7 at 95% CL (excl.)
Cubic coupling @ 100 TeV using 30/ab

✓ $b\bar{b}\gamma\gamma$ becomes Golden channel at 100 TeV pp collider

40× enhanced xsec due to PDF

Barr, Dolan, Englert, Lima, M.Spannowsky 15'
Contino, Azatov, Panico, SON 15'
H. He, J. Ren, W. Yao 16'
Physics at 100 TeV
Contino, Panico, Papaefstathiou, Selvaggi, SON in progress

~ 3.4 % is possible with 30 ab$^{-1}$

Physics at 100 TeV, arXiv:1606.09408

✓ ILC via VBF at 1 TeV 8 ab$^{-1}$ ~10%

J. Tian, LC-REP-2013-003 M. Kurata,
HHabb, bbWW* combination
Quartic coupling @ 100 TeV

$h h h \rightarrow b \bar{b} b \bar{b} \gamma \gamma$, BR = 0.232%

$h h h \rightarrow b \bar{b} b \bar{b} \tau^+ \tau^-$, BR = 6.46%

If we observe cubic ~ $O(1)$ @ HL LHC or 100 TeV, then quartic from 100 TeV is very useful

What if we observe a large $\kappa_3$ at HL LHC?
Dynamics of EWPT

\[ V_{\text{EFF}}(\varphi, T) = D(T^2 - T_0^2)\varphi^2 - (ET + e)\rho^3 + \bar{\lambda}\varphi^4 + \cdots \]

- In SM, \( e = 0 \) while in BSM its nonzero
- \( E \) is generated at loop level (SM & BSM)
- \((ET+e)\) and \(\bar{\lambda}\) decide the nature of EWPT
- II order PT - both \( E \) and \( e \) vanish
- I order PT occurs with \( E > 0 \) and/or \( e > 0 \)
- I order PT has interesting consequences
- Eg: - baryogenesis (more about this later)
In SM, with $m_H = 125$ GeV, numerical simulations suggest a crossover

- Field, $\Phi$ develops a vev when $T < T_c$
- $\Phi$ rolls smoothly from zero to nonzero value
- Such transitions are always in thermal equilibrium
- System remembers nothing about the unbroken phase
- Cosmologically uninteresting

In BSM, we can have first order phase transitions

- @Tc all 3 minima become equal
- Φ is still trapped in the origin but secondary minima is energetically favorable
- At Tn potential barrier is small
- Φ tunnels to true vacuum
- Out of equilibrium processes
- Relevant for cosmology
Phase transition model classes

I. Thermally Driven - new particles in early universe plasma
- barrier formation via thermal loop effects associated with bosonic zero modes
IA. barrier from light scalar via thermal cubic terms
IIB. from heavy particles with large coupling to Higgs field

II. Tree-Level Driven - BSM physics
IIA. Renormalizable Operators - effective $h^3$ operator from extra scalars which acquire nonzero vevs during EWPT eg:- SM+ real singlet
IIB. Non-Renormalizable Operators. - higher dimensional operators like $h^6$

III. Loop Driven
- Eq:- quartic correction of form $h^4 \ln h^2$ - which competes with unstable $-h^4$ term
Extensions to Higgs

Requires precision thermal field theory.

Chung et.al. 1209.1819
Baryogenesis

➔ Evidence from cosmology: \[ \frac{n_B}{s} = (8.59 \pm 0.11) \times 10^{-11} \] (Planck)

➔ Sakharov’s 3 conditions (1967), for baryogenesis
  ◆ Baryon number violation
  ◆ C and CP violation
  ◆ Out of equilibrium

➔ EW baryogenesis is one of the potential solutions

➔ Need new physics because in SM:
  ◆ EW phase transition is a crossover, instead of 1st order
  ◆ CP violation is too small
Criteria for baryon number violation

Baryon number can be violated by **non-perturbative EW processes**

Unstable solutions of $S_{\text{EFF}}$ i.e sphalerons interpolate b/w topologically distinct vacua

Sphaleron energy depends on **shape of potential** away from the minimum

**Litmus test for its global structure !!!**

For successful EWBG, EW sphaleron processes are out of equilibrium in broken phase

- washout avoidance condition

$$\frac{v(T_{\text{PT}})}{T_{\text{PT}}} \geq 1 \times \left( \frac{E_{\text{sph},0}}{9 \, \text{TeV}} \right)^{-1} \in [0.6, 1.4]$$

Strong first order PT

No unique value $\Rightarrow$ adds to uncertainty on Higgs self coupling
Testability

➔ LHC is running!

➔ What’s the sensitivity of HL-LHC, 100 TeV pp colliders, and future e+ e- colliders to the region of parameter space where SFOPT is allowed?

➔ Gravitational waves: Bubble collisions
Overview of effective potential at Finite T
The effective potential

Truncated Full Dressing (TFD):

- thermal mass $\Pi_i$ is still obtained in the high-T approximation

$$V_{eff} = V_{tree} + V_{CW}[m_i^2(h) + \Pi_i] + V_T[m_i^2(h) + \Pi_i]$$

$$V_T = \sum_{i=BF} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} \int_0^\infty dx x^2 \log \left[ 1 + \exp \left( -\sqrt{x^2 + \frac{(m_i^2(h) + \Pi_i)}{T^2}} \right) \right]$$

$$m_i^2(h) = m^2 + \text{coupling}\times h^2$$

For $v_c \gg T_c$ and $\gtrsim O(1)$ coupling,

integral needs to be exactly evaluated

✓ Validity of High-T approximation/Validity of perturbation

- not rigorously addressed in most literature in the context of BSM physics

Curtin, Meade, Ramani 16’ for a recent discussion

We do not intend to clarify this problem in this talk,
but just point out a few observations in the process of reproducing others.
In the High-T approximation

\[ V_{\text{eff}} = V_{\text{tree}} + V_{CW} + V_T + V_{\text{ring}} \]

\[ V_{CW} = \sum_{i=t,\psi,\zeta,\eta,\gamma,\ldots} (-1)^{F_i} \frac{g_i}{64\pi^2} \left[ m_i^4(h) \left( \log \frac{m_i^2(h)}{m_i^2(\nu)} - \frac{3}{2} \right) + 2m_i^2(h)m_i^2(\nu) \right] \]

\[ V_T = \sum_{i=B,F} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} J_{B/F} \left( \frac{m_i^2(h)}{T^2} \right) \]

\[ V_{\text{ring}} = \sum_{i=\text{bosons}} \frac{\bar{g}_i T}{12\pi} \left[ m_i^3(h) - \left( m_i^2(h) + \Pi_i(T) \right)^{\frac{3}{2}} \right] \]

\[ x^2 = \frac{m^2}{T^2} \ll 1 \]

\[ J_{B/F}(x^2) = \int_0^\infty dy \, y^2 \log[1 + \exp(-\sqrt{y^2 + x^2})] \]

\[ J_B(x^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12} x^2 - \frac{\pi}{6} x^3 - \frac{1}{32} x^4 \log \left( \frac{x^2}{c_b} \right) \]

\[ J_F(x^2) \approx -\frac{7\pi^4}{360} - \frac{\pi^2}{24} x^2 - \frac{1}{32} x^4 \log \left( \frac{x^2}{c_f} \right) \]
The effective potential

Truncated Full Dressing (TFD): We call this Prescription A

\[ V_{eff} = V_{tree} + V_{CW}[m_i^2(h) + \Pi_i] + V_T[m_i^2(h) + \Pi_i] \]

\[ V_T(m_i^2(\phi), T) = \sum_i (-1)^{F_i} \frac{g_iT^4}{2\pi^2} J_{B/F} \left( \frac{m_i^2(\phi)}{T^2} \right) \]

\[ J_B(\alpha^2) \equiv J_B(\alpha^2; n) = -\sum_{k=1}^{n} \frac{1}{k^2} \alpha^2 K_2(\alpha k), \]

\[ J_F(\alpha^2) \equiv J_F(\alpha^2; n) = -\sum_{k=1}^{n} \frac{(-1)^k}{k^2} \alpha^2 K_2(\alpha k), \]

✓ Validity of High-T approximation/Validity of perturbation

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Benchmark Scenarios
Most commonly considered frameworks

\[ V_{\text{eff}} = \sum_{i=t,W,Z,h,G,\text{BSM}} V_i \]

Higgs portal:
new scalar \( S \)

\[ Z_2 \text{-symmetry, e.g. } \langle S \rangle = 0 \text{ vs } \langle S \rangle \neq 0 \]

\[ N_S: \text{multiplicity} \quad \text{E.g. can help with weak coupling} \]

Effective Field Theory:
higher-dimensional operators

\[ \mathcal{O}_H = (\partial |H|^2)^2 \text{ vs } \mathcal{O}_6 = |H|^6 \]

Weakly coupled theory

Strongly coupled theory

PGB vs non-pGB
Higgs Portal

\[ \mathcal{O}_H \ll \mathcal{O}_6 \text{ possible} \]

Azatov, Contino, Panico, Son 15'
Higgs portal
\[ V_{\text{tree}} = -\frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{2} \lambda_{HS} h^2 S^2 + \frac{1}{2} m_0^2 S^2 + \frac{1}{4} \lambda_S S^4 \]

Based on naïve criterion, existence of degenerate vacua, with \( \nu_c/T_c > 1 \) 

* Note cutoff \( \lambda_{HS} > 5 \) by hand

1. One-step strong 1\textsuperscript{st} phase transition (dotted RED)
   \[ V(0, 0) \rightarrow V(\nu, 0), \langle S \rangle = 0 \]

2. Two-step strong 1\textsuperscript{st} phase transition (dotted GREEN)
   \[ V(0, 0) \rightarrow V(0, \nu_s) \rightarrow V(\nu, 0) \]
   
   \[ V(0, \nu_s) > V(\nu, 0) \rightarrow \]
   
   \[ \lambda_s > \lambda_s^{\text{min}} = \lambda \frac{m_0^4}{m^4} = \frac{2(m_s^2 - \nu^2 \lambda_{HS})^2}{m_h^2 \nu^2} \]
At low temperatures, or at the interior of a bubble in the cooling universe, there exists a “true vacuum” (broken phase)

System at $T = T_c$

In 4D, True vacuum bubble forms

Expands until false vacuum bubble disappears.

At high temperatures, or far away from the bubble, “false vacuum”
**Bubble nucleation**

As the Universe continues to cool, bubbles of the broken-minimum phase are nucleated.

Nucleation probability per unit time per unit volume at temperature $T$ is given by

$$P \sim T^4 \exp\left(-\frac{S_3}{T}\right)$$

$S_3$ is the bubble action

Nucleation temperature $T_N$ is the temperature at which the nucleation probability per Hubble volume becomes of order one; for electroweak phase transition, this corresponds to $S_3 / T_N \approx 100$.

**To avoid washout** $v(T_N) / T_N > 1$
Two step PT

Most two-step region (III) is gone:
- fail to nucleate bubbles

We will focus on the one-step PT

Tiny two-step region:
- fine-tuned to live here
Precision of the Higgs self couplings

✓ Depending on the criteria on $\frac{v_c}{T_c}$, the target precision can fluctuate by $O(1)$ amount

✓ ~ 10% deviation of the Higgs
✓ Achievable @100 TeV pp collider and ILC
Validity of High-T approximation

High-T approx. fails
(The issue is more pronounced in two step PT)

** \( \frac{m_3(v) + \Pi}{T_c} \) will be even bigger \( \frac{\lambda_3}{\lambda_3^{SM}} \)

\[
x^2 = \frac{m^2}{T^2} \ll 1: \text{High T approximation}
\]

\[
J_B(x^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12} x^2 - \frac{\pi}{6} x^3 - \frac{1}{32} x^4 \log \left( \frac{x^2}{c_B} \right)
\]

\[
J_F(x^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24} x^2 - \frac{1}{32} x^4 \log \left( \frac{x^2}{c_f} \right)
\]

\[
x^2 = \frac{m^2}{T^2} \gg 1: \text{Low T approximation}
\]

\[
J_B(x^2; n) = -\sum_{k=1}^{n} \frac{1}{k^2} x^2 K_2(x \cdot k)
\]

\[
J_F(x^2; n) = -\sum_{k=1}^{n} \frac{(-1)^k}{k^2} x^2 K_2(x \cdot k)
\]
The results look similar in these plots
Consistent TFD

Pseudo Full Dressing

Prescription A

Prescription B

More focused

$v_c < v = 246$ GeV is satisfied better
Decoupling of higher singlet masses for one-step PT

Singlet decouples at $\mu_S \sim 550$ GeV

Failure of High T appx

Higher singlet masses do not decouple!!

$m_s/T_c >> 1$
Decoupling of higher singlet masses for one-step PT

Failure of High T appx

⇒ Higher masses still continue to contribute in high T limit
⇒ Severely affect the precision of cubic coupling when \( \nu_c/T_c < 1 \) region
Cubic vs. Quartic

\[ V(h) = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6} \left( \frac{3 m_h^2}{v} \right) h^3 + d_4 \frac{1}{24} \left( \frac{3 m_h^2}{v^2} \right) h^4 \]

Roughly

\[ \frac{\lambda_4}{\lambda_4^{SM}} \sim 2 \frac{\lambda_3}{\lambda_3^{SM}} \]
EFT - higher dimension operators
1st order phase transition in EFT approach

E.g. dim-6 operator

\[ V_{EFT} = m^2 |H|^2 + \lambda |H|^4 + \frac{c_6}{v^2 \frac{m_h^2}{2v^2}} |H|^6 \]

\[ V_{EFT} = \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{8} \frac{c_6}{v^2} \frac{m_h^2}{2v^2} h^6 \]

\[ 0 = \left. \frac{dV_{EFT}}{dh} \right|_{h=v} = m^2 + \lambda h^2 + \frac{3c_6 m_h^2}{8v^4} h^4 \]

\[ m_h^2 = \left. \frac{d^2V_{EFT}}{dh^2} \right|_{h=v} = m^2 + 3\lambda h^2 + \frac{15}{8} c_6 m_h^2 \]

\[ \lambda_3 = \left. \frac{d^3V_{EFT}}{dh^3} \right|_{h=v} = \frac{3m_h^2}{v} (1 + c_6) \]

\[ \lambda_4 = \left. \frac{d^4V_{EFT}}{dh^4} \right|_{h=v} = \frac{3m_h^2}{v^2} (1 + 6 c_6) \]

\[ \frac{\lambda_3}{\lambda_3^{SM}} = 1 + c_6 \]

\[ \frac{\lambda_4}{\lambda_4^{SM}} = 1 + 6 c_6 \]
1st order phase transition in EFT approach

E.g. dim-6 operator

\[ V_{\text{EFT}} = \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{8} \frac{c_6}{v^2} \frac{m_h^2}{2v^2} h^6 \]

\( (m^2, \lambda, c_6) \) 3 diff. local curvatures

: 1st order PT becomes in principle possible

When keeping only \( T^2 \)-term as thermal effect, \( m^2 (T) = m^2 + a T^2 \),
the analytic solution is possible

\[ \left. \frac{dV_{\text{eff}}}{dh} \right|_{h=v_c, T=T_c} = 0 \quad \text{Should be extreme point} \]

\[ V(v_c, T_c) = V(0, T_c) \quad \text{Degeneracy of the vacua} \]

\[ v_c^2 = -\frac{4m^2(T_c)}{\lambda} = -\frac{2\lambda v^4}{c_6 m_h^2} \]

\[ c_6 = \frac{2}{3} \frac{1}{1 - \frac{2}{3} \frac{v_c^2}{v^2}} \]

\[ \frac{2}{3} < c_6 < 2 \]

\[ \lambda = \frac{m_h^2}{2v^2} \left( 1 - \frac{3}{2} c_6 \right) \]
Re-summed higher-dim operators

\[ V_{\text{tree}} = -\mu^2 |H|^2 + \lambda |H|^4 + \sum_{n=1}^{\infty} \frac{c_{4+2n} m_h^2}{v^{2n} 2v^2} |H|^{4+2n} \]

\[ c_{4+2n} = c (v/f)^{2n} \text{ with } c \sim \mathcal{O}(1) \]

\[ V_{\text{tree}} = -\frac{\mu^2}{2} h^2 + \frac{\lambda}{4} h^4 + \frac{1}{8} \frac{c m_h^2}{f^2} \frac{h^6}{2v^2} h^6 \frac{1}{1 - \frac{h^2}{2f^2}} \]

\[ \lambda_3 = \left. \frac{d^3 V_{\text{tree}}(h)}{dh^3} \right|_{h=v} = \frac{3m_h^2}{v} \left[ 1 + 16c \frac{\xi}{(2 - \xi)^4} \right] \]

\[ \lambda_4 = \left. \frac{d^4 V_{\text{tree}}(h)}{dh^4} \right|_{h=v} = \frac{3m_h^2}{v^2} \left[ 1 + 32c \frac{(6 + \xi)\xi}{(2 - \xi)^5} \right] \]

NDA scaling for all coeff

Deviation in quartic is Bigger than cubic by

\[ 2(6 + \xi)/(2 - \xi) \]
We wish to add more interesting BSM scenarios

→ HL-LHC has sensitivity 0f $\lambda_3/\lambda_{3,SM} - 1$ at 68%CL has two intervals, $[-1.0, 1.8] \cup [3.5, 5.1]$

→ Interval around SM can test $O(1)$ deviations and exclude EFT cases
Lessons

➔ Break-down of the high-temperature approximation is more pronounced in the two-step SFOEPT.

➔ Criteria for SFOEPT?
  ◆ $\frac{v_c}{T_c} > 1$ (1.4) requires the measurement of the coupling at $\sim 15\%$ (35\%) precision achievable at ILC (via VBF process at higher c.o.m energy) and 100 TeV pp collider,
  ◆ more stronger criteria, $\frac{v_c}{T_c} > 0.6$ requires $\sim 5\%$ precision of $\lambda_3$ which is likely plausible only at 100 TeV pp collider

➔ Various BSM scenarios can appear in different islands in $(\lambda_3, \lambda_4)$ space
  ◆ $\lambda_4$ could be important to distinguish different scenarios, and it has chance @ 100 TeV in case that a large deviation of $\lambda_3$, is observed

➔ What if we observe any hint of Strong 1st order Phase Transition?
  ◆ Likely strongly coupled dynamics not far away from EW scale?

➔ Very strong EWPT could generate a stochastic background of gravitational waves - signal is potentially within reach of future space-based gravitational wave interferometers, such as eLISA

Grojean et al' 99, Beniwal et al '17, Huang et. al. '17
The Road Ahead

Future is bright!!
Backup slides
FIG. 1: Free energy for the fermions (left) and bosons (right) as a function of $m/T$ in the high-$T$ approximation (black-dashed), low-$T$ approximation in Eq. (9) (red-dashed) with $n = 1$, and in the exact form (black-solid). The dotted-blue line is the low-$T$ approximation with the approximated $K_2$ as in [42].
For the one-step phase transition the quartic coupling $\lambda_S$ does not play much role directly in the phenomenology apart from ensuring the stability of the potential at a large field.

Fix $\lambda_S = 0$ and scan over $\mu_S = [10, 1310]$ GeV (in steps of 10 GeV) and $\lambda_{HS} = [0, 5]$ (in steps of 0.05).

For two-step cascade, the $\lambda_S$ needs to stay above the minimum $\lambda_{\text{min} S}$ so that $(v, 0)$ remains the global minimum.

Scan intervals $m_S = [65, 700]$ GeV (in steps of 5 GeV) and $\lambda_{HS} = [0, 5]$ (in steps of 0.05) for a few choices of $\lambda_S$, parameterized as $\lambda_S = \lambda_{\text{min} S} + \delta S$.

Assume: singlet mass is heavier than roughly $m_h/2$ to avoid the Higgs decays to the singlet scalar.

Impose arbitrary hard cutoffs $\lambda_{HS} < 5$ and $\lambda_S < 5$ (smaller than $4\pi$ which is the typical unitarity bound), to avoid the strongly coupled regime.
Probing triple-Higgs coupling with double Higgs production

- Consistency of check of EWSB
- Reconstructing the Higgs potential
- Sensitivity through yields and kinematics
- Large enhancement through BSM possible
- Exhaustive program at the (HL-)LHC

\[ \mathcal{L} = -\frac{1}{2} m_h^2 h^2 - \lambda_3 \frac{m_h^2}{2v} h^3 - \lambda_4 \frac{m_h^2}{8v^2} h^4 \]

EFT Lagrangian
Dihiggs production

<table>
<thead>
<tr>
<th>Process</th>
<th>8 TeV</th>
<th>14 TeV</th>
<th>100 TeV</th>
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<tbody>
<tr>
<td>gF</td>
<td>0.38</td>
<td>1</td>
<td>14.7</td>
</tr>
<tr>
<td>VBF</td>
<td>0.38</td>
<td>1</td>
<td>18.6</td>
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<td>9.7</td>
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<td>12.5</td>
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<td>ttH</td>
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<td>61</td>
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<tr>
<td>bbH</td>
<td>0.34</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>gF to HH</td>
<td>0.24</td>
<td>1</td>
<td>42</td>
</tr>
<tr>
<td>Decay Channel</td>
<td>Branching Ratio</td>
<td>Total Yield (3000 fb⁻¹)</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------------</td>
<td>-------------------------</td>
<td></td>
</tr>
<tr>
<td>$b\bar{b} + b\bar{b}$</td>
<td>33%</td>
<td>40,000</td>
<td></td>
</tr>
<tr>
<td>$b\bar{b} + W^+W^-$</td>
<td>25%</td>
<td>31,000</td>
<td></td>
</tr>
<tr>
<td>$b\bar{b} + \tau^+\tau^-$</td>
<td>7.3%</td>
<td>8,900</td>
<td></td>
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<tr>
<td>$ZZ + b\bar{b}$</td>
<td>3.1%</td>
<td>3,800</td>
<td></td>
</tr>
<tr>
<td>$W^+W^- + \tau^+\tau^-$</td>
<td>2.7%</td>
<td>3,300</td>
<td></td>
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<tr>
<td>$ZZ + W^+W^-$</td>
<td>1.1%</td>
<td>1,300</td>
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<tr>
<td>$\gamma\gamma + b\bar{b}$</td>
<td>0.26%</td>
<td>320</td>
<td></td>
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<tr>
<td>$\gamma\gamma + \gamma\gamma$</td>
<td>0.0010%</td>
<td>1.2</td>
<td></td>
</tr>
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</table>

Table 1: Branching ratios for different HH final states, and their corresponding approximate expected yields in 3000 fb⁻¹ of data before any event selection is applied, assuming a total production cross section of 40.8 fb and $m_H = 125$ GeV.
Fig. 65: Estimated precision on the measurement of the Higgs trilinear self-coupling. The left panel shows as a function of the cut on the invariant mass of the photon pair $\Delta m_{\gamma\gamma}$ for the three detector benchmark scenarios, “Low” (dot-dashed red), “Medium” (dashed green) and “High” (solid black). In the right panel the result is shown as a function of the cut on the maximal rapidity of the reconstructed objects $\eta_{\text{max}}$ assuming the “Medium” detector benchmark (the solid red and dashed green curves correspond to a variation of the photon and $b$-jets acceptances respectively). All the results have been obtained for an integrated luminosity of 30 $\text{ab}^{-1}$.
Double Higgs Production:

\[ e^+ e^- \rightarrow ZHH \]

\[ e^+ e^- \rightarrow vvHH \]

Require at least \( \sim 500 \) GeV for the direct measurement of the triple-Higgs coupling via double Higgs production.
Baryogenesis ... *in a nutshell*

First order electroweak phase transition:
\[ \langle \phi \rangle = 0 \quad \langle \phi \rangle \neq 0 \]
via bubble nucleation

**CP-violation:**
Generation of particle/antiparticle asymmetry:
\[ \langle \phi \rangle = 0 \quad \langle \phi \rangle \neq 0 \]

**B-violation:**
Generation of baryon asymmetry:
\[ \langle \phi \rangle = 0 \quad \langle \phi \rangle \neq 0 \]

B+L-violating EW Sphalerons convert baryons back to antileptons.

Sphaleron proc. must be quenched!
Strong $1^{st}$ order phase transition
$\leftrightarrow \lambda_{HS} \sim O(1)$ for $N_S = 1$

The perturbation breaks down? It looks like

$\lambda_4 \sim 0.13$, and we assumed $\lambda_S \sim 0$, ignored overall $m_h^2/m_{\tilde{s}}^2$ factor

**“ring-improved” version of $V_T$**

Curitn, Meade, Ramani’16