Probing the Wigner distribution at the Electron-Ion Collider

+ an introduction to the physics of EIC

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Electron-Ion Collider (EIC)

A future (2030~) high-luminosity ($\sim 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$) polarized ep, eA collider at Brookhaven National Laboratory dedicated to the study of the nucleon and nucleus structure.

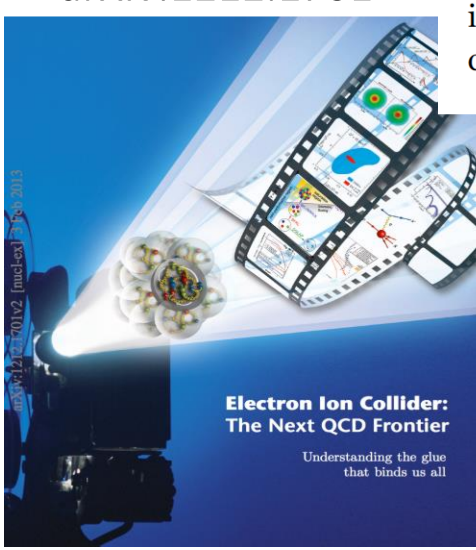
Brookhaven Picked as Site for Electron-Ion Collider

AIP aip.org/fyi/2020/brookhaven-picked-site-electron-ion-collider

January 17, 2020

The Department of Energy has selected Brookhaven National Laboratory as the site for its proposed Electron-Ion C
cost between $1.6 billion an

arXiv:1212.1701



Future DIS experiments worldwide

Planned DIS Colliders around the world

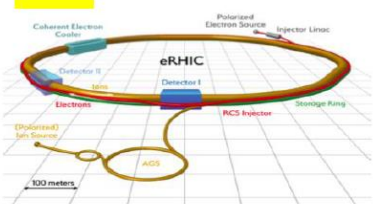
1812.08110

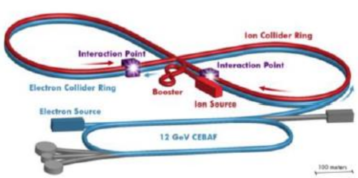
Facility	Years	E_{cm} (GeV)	Luminosity ($10^{33} cm^{-2} s^{-1}$)	Ions	Polarization
EIC in US	> 2028	$20 - 100 \rightarrow 140$	$2 - 30$	$p \rightarrow U$	e, p, d, ^{3}He, Li
EIC in China	> 2028	$16 - 34$	$1 \rightarrow 100$	$p \rightarrow Pb$	e, p, light nuclei
LHeC (HE-LHeC)	> 2030	200 - 1300 (1800)	10	depends on LHC	e possible
PEPIC	> 2025	$530 \rightarrow 1400$	$< 10^{-3}$	depends on LHC	e possible
VHEeP	> 2030	1000 - 9000	$10^{-5} - 10^{-4}$	depends on LHC	e possible
FCC-eh	> 2044	3500	15	depends on FCC-hh	e possible

EPPSU DIS Input

United States

EIC





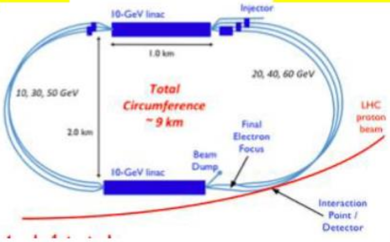
China

EicC

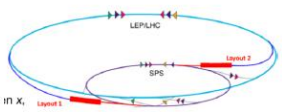
Europe (CERN)

LHeC

FCC-eh

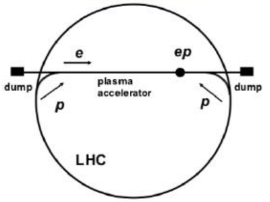


PEPIC



VHEeP

VLEeP



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Planned DIS Colliders around the world

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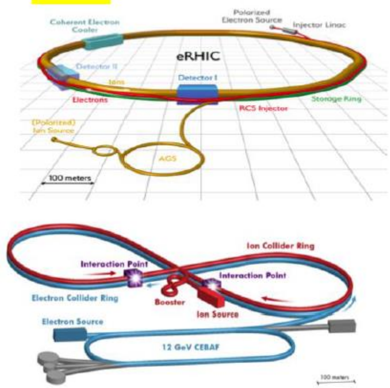
EIC in US	> 2028	20 - 100 → 140	2 - 30	p → U	e, p, d, ^{3}He, Li
EIC in China	[illegible]	[illegible]	[illegible]	[illegible]	e, p, light nuclei
LHeC (HE-LH	[illegible]	[illegible]	[illegible]	[illegible]	e possible
PEPIC	[illegible]	[illegible]	[illegible]	[illegible]	e possible
VHEeP	[illegible]	[illegible]	[illegible]	[illegible]	e possible
FCC-eh	[illegible]	[illegible]	[illegible]	[illegible]	e possible

EPPSU DIS Input

The era of precision EW, pQCD, and precision study of nucleon and nuclear structures in the next 20-30 years!

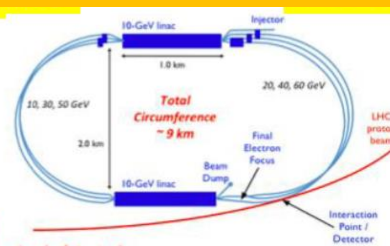
Unite

EIC



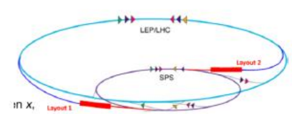
China

EicC



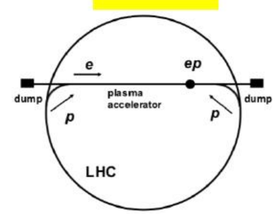
Europe (CERN)

PEPIC

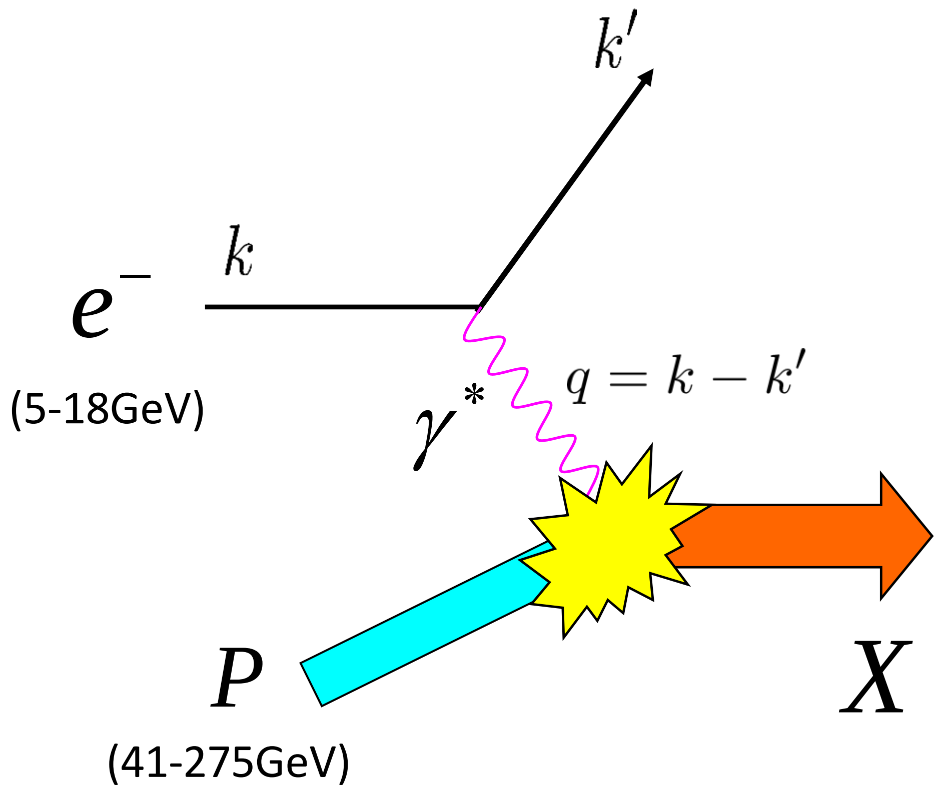


VHEeP

VLEeP



Experiment at EIC: Deep Inelastic Scattering (DIS)



Two most important kinematic variables

$$Q^2 = -q^2$$ photon virtuality (resolution)

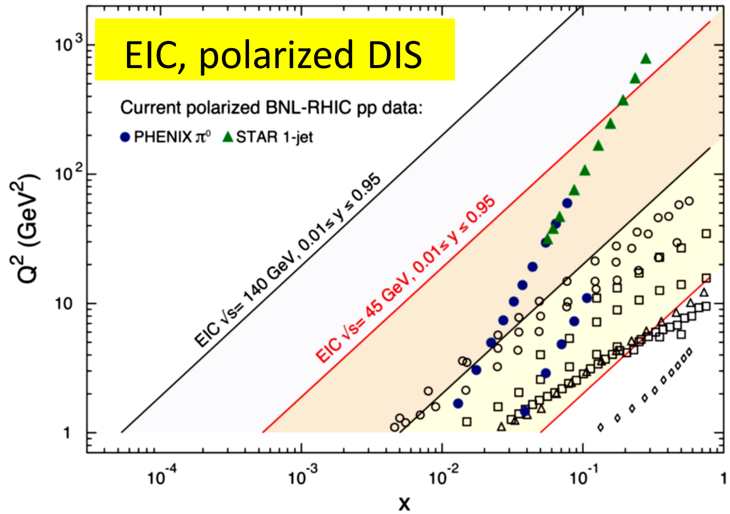
$$x = \frac{Q^2}{2P \cdot q}$$ Bjorken variable (inverse energy)

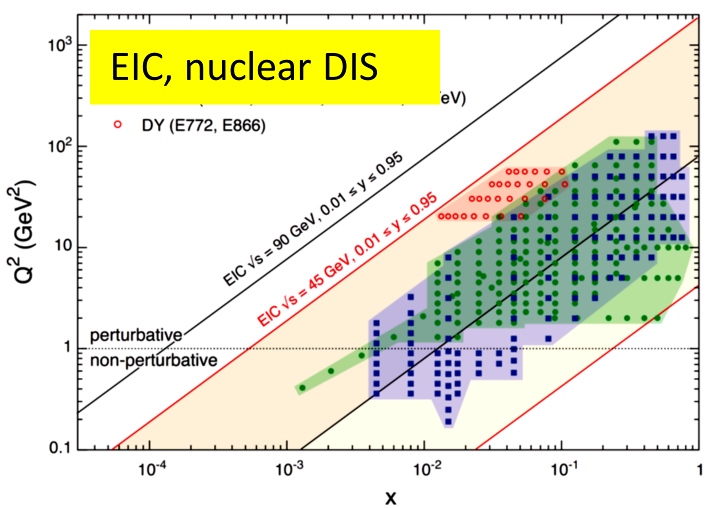
$$\approx \frac{E_{parton}}{E_{proton}}$$

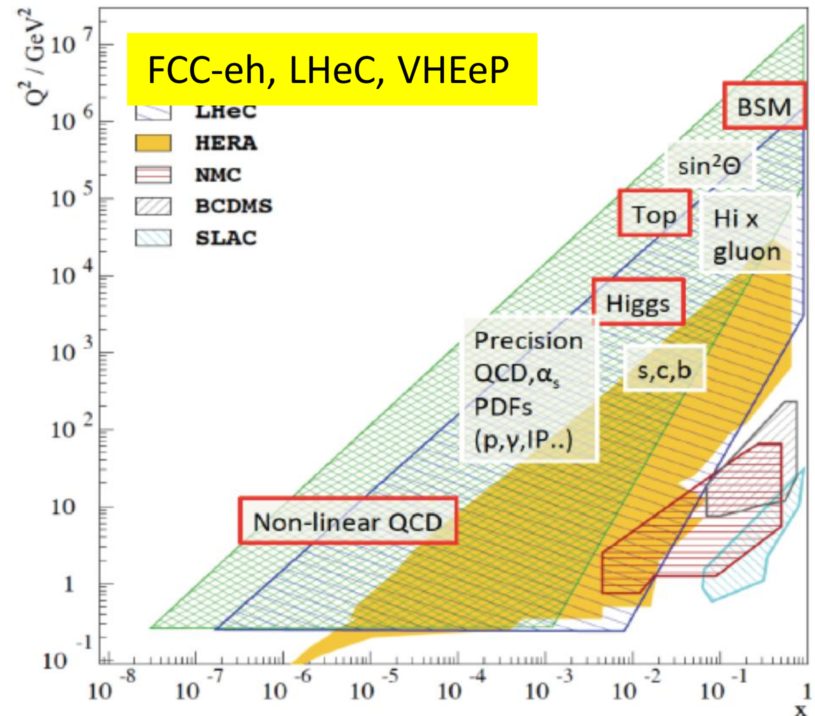
Proton, deuteron, helium, gold…any nucleus of your choice!

Electron, proton and light nuclei can be polarized.

Exploring *terra incognita*

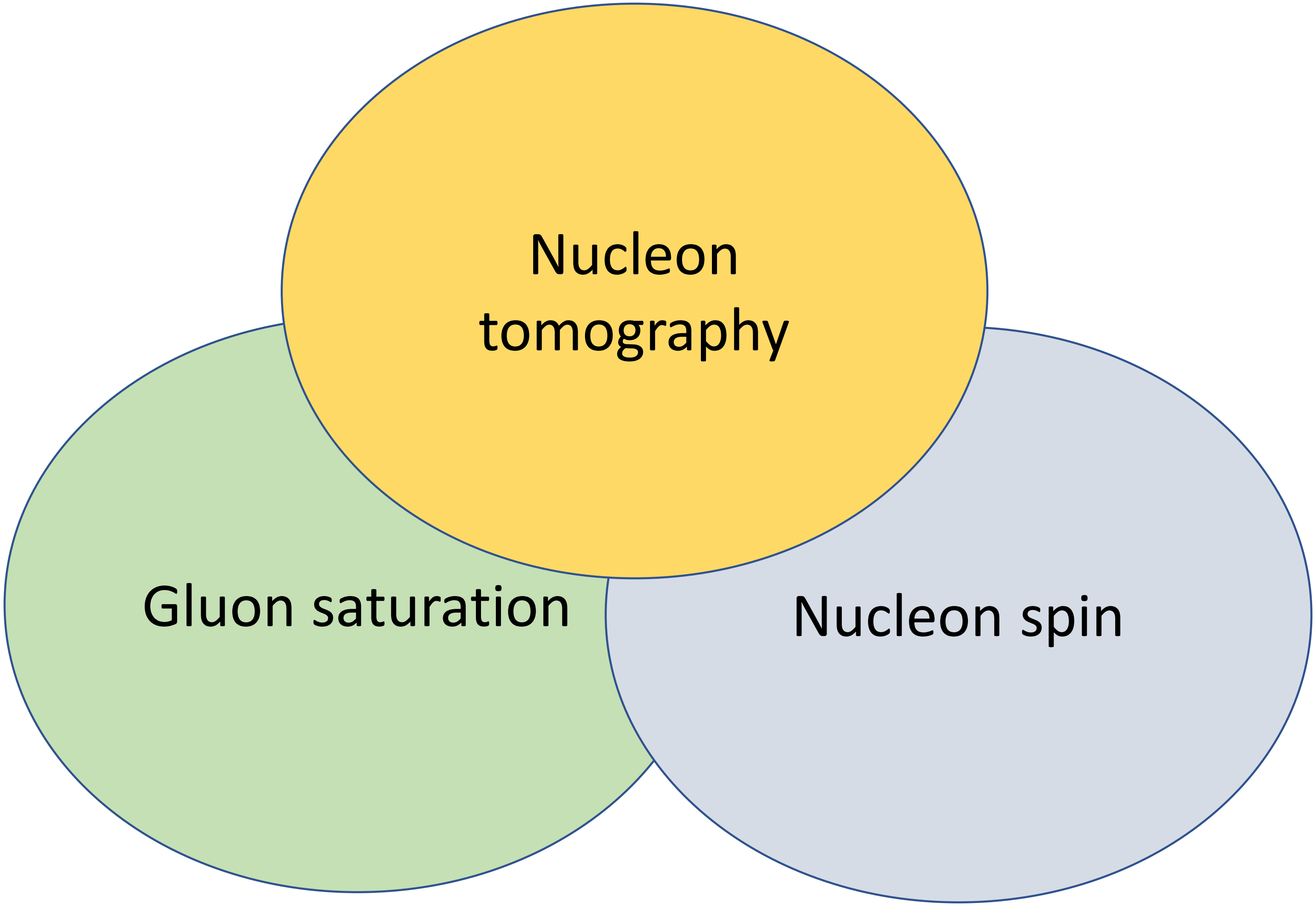




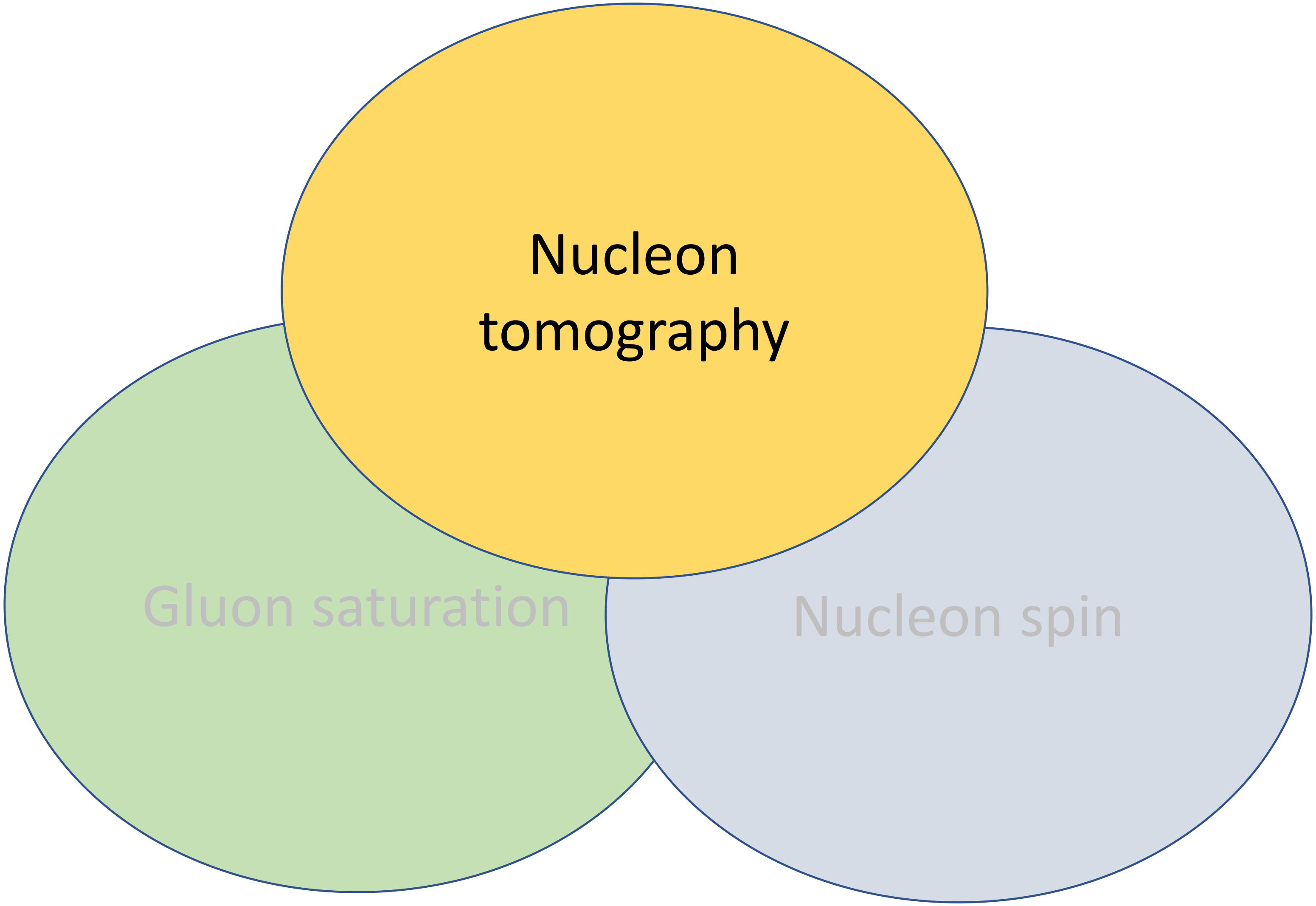


Unprecedented coverage in kinematics.
Tremendous physics opportunities.

Scientific goals of EIC



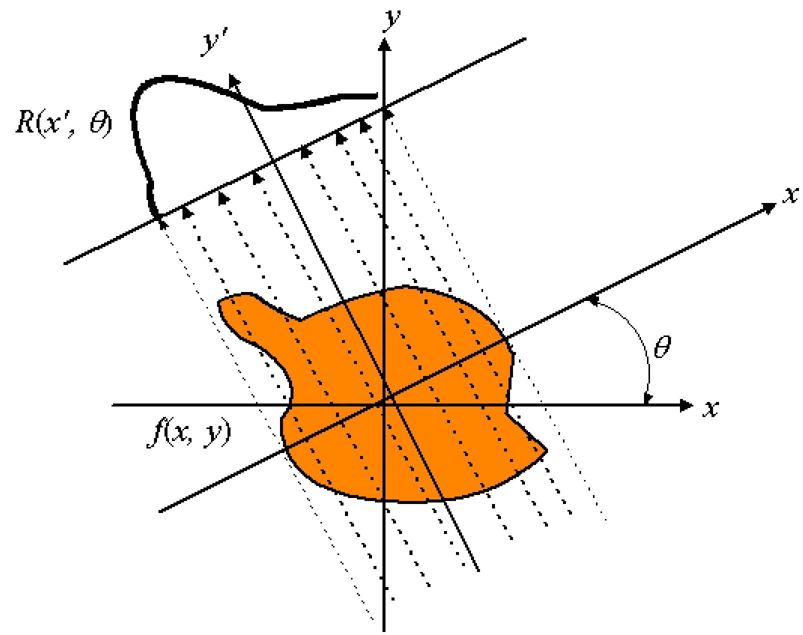
Scientific goals of EIC

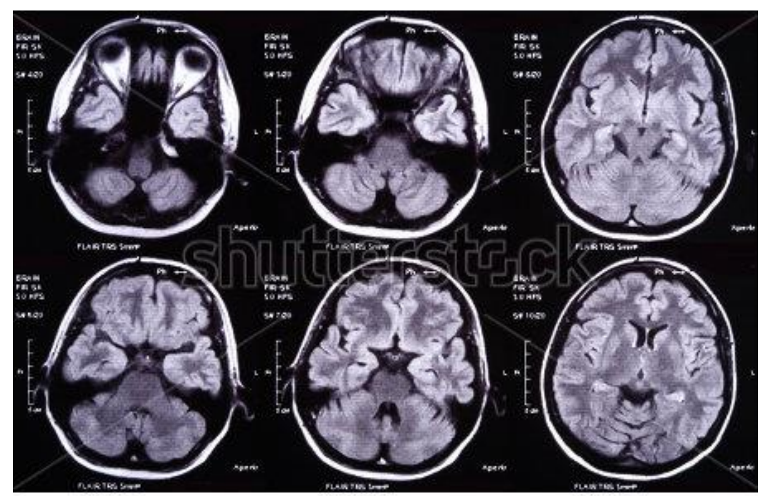


Tomography

CT = Computed Tomography

See inside an object without cutting



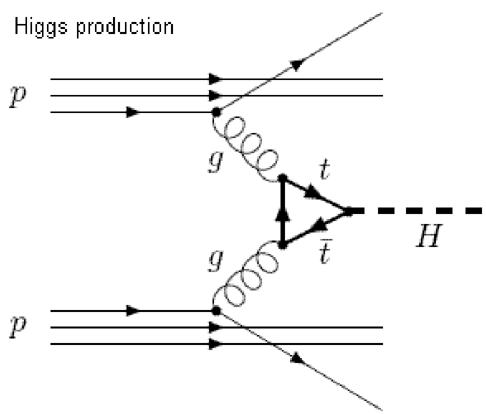


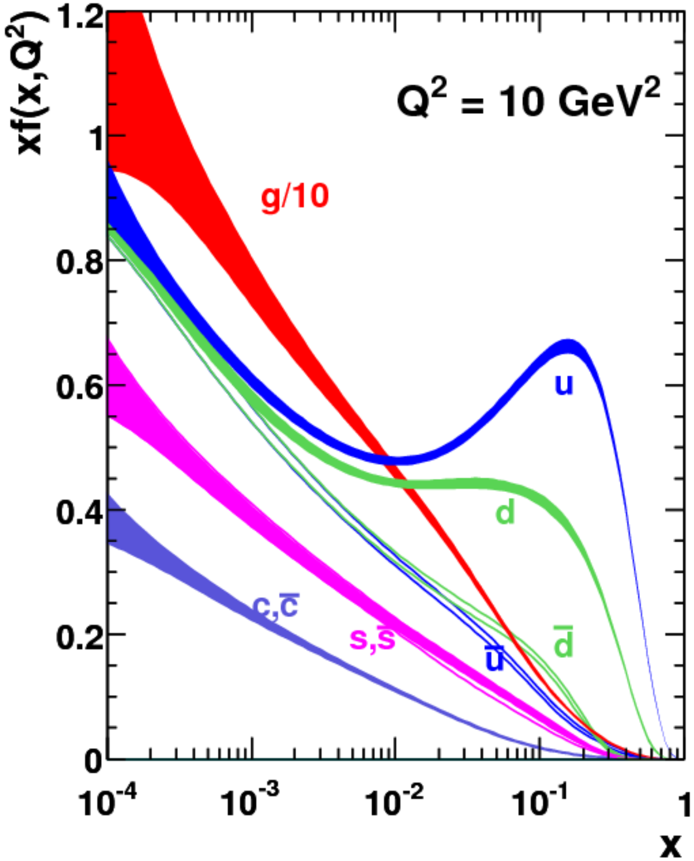
Parton distribution function

$$u(x) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P|\bar{u}(-z^-/2)\gamma^+ u(z^-/2)|P\rangle$$

Number distribution of up quarks
with momentum fraction x inside the proton

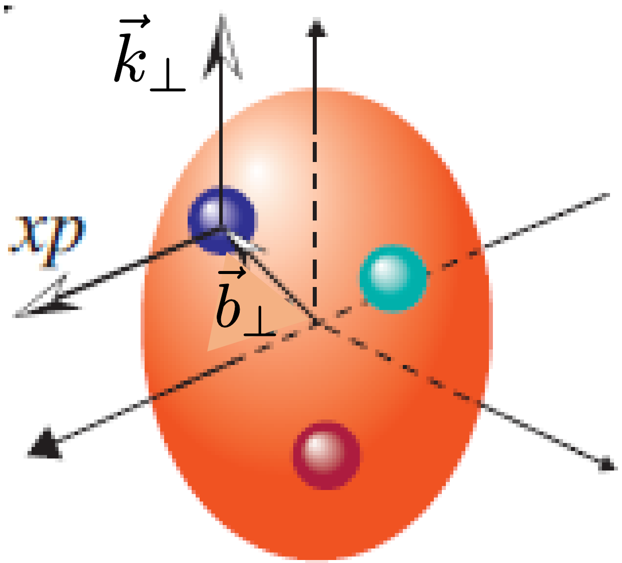
QCD factorization $\sigma = \sigma_0 \otimes g(x_1) \otimes g(x_2)$





Universality of PDF—the same function can be used for different processes.
Fundamental to the predictive power of pQCD

Multi-dimensional tomography



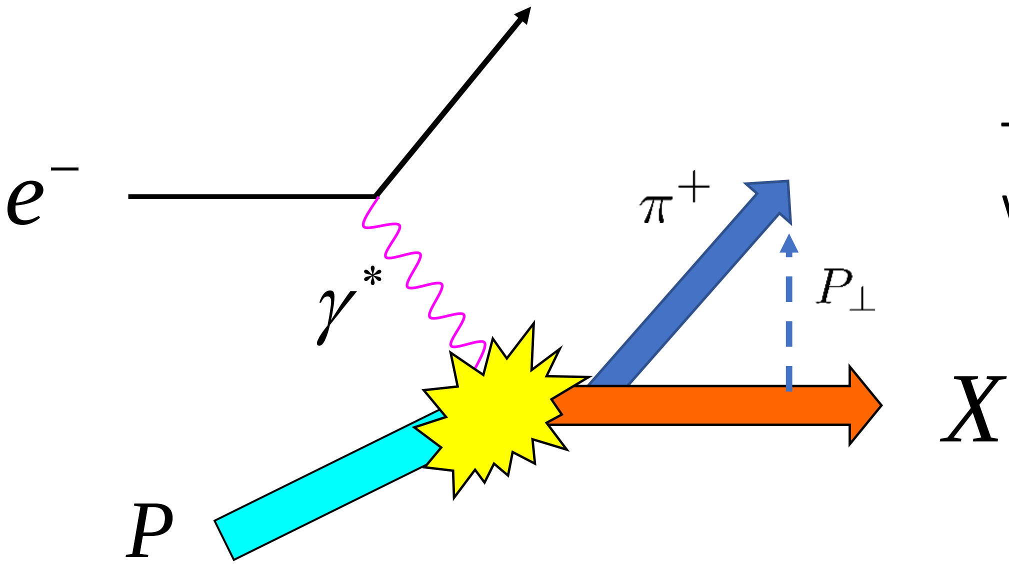
The nucleon is much more complicated!
Partons also have transverse momentum $\vec{k}_\perp$
and are spread in impact parameter space $\vec{b}_\perp$

$u(x)$ → $u(x, \vec{k}_\perp)$ Transverse momentum dependent distribution (TMD) 3D tomography

$u(x)$ → $u(x, \vec{b}_\perp)$ Generalized parton distribution (GPD) 3D tomography

$u(x)$ → $u(x, \vec{b}_\perp, \vec{k}_\perp)$ Wigner distribution 5D tomography

Semi-inclusive DIS



Tag one hadron species
with fixed transverse momentum $P_\perp$

When $P_\perp$ is small, TMD factorization

Collins, Soper, Sterman;
Ji, Ma, Yuan,…

$$\frac{d\sigma}{dP_\perp} = H(\mu) \int d^2 q_\perp d^2 k_\perp f(x, k_\perp, \mu, \zeta) D(z, q_\perp, \mu, Q^2/\zeta) \delta^{(2)}(z k_\perp + q_\perp - P_\perp) + \cdots$$

TMD PDF TMD FF

Open up a new class of observables where perturbative QCD is applicable!

TMD is becoming precision physics

Define Fourier transform $$\int d^2k_\perp e^{ik_\perp r_\perp} f(k_\perp \ldots) = f(r_\perp \ldots)$$

RG equation

$$\frac{\partial}{\partial \ln \mu} f(x, r_\perp, \mu, \zeta) = \gamma_F f(x, r_\perp, \mu, \zeta)$$

Known to three loops
Moch, Vermeseren, Vogt (2005)

Collins-Soper equation

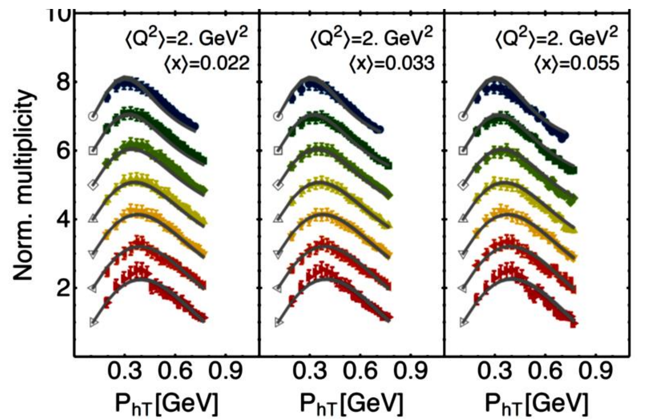
$$\frac{\partial}{\partial \ln \zeta} f(x, r_\perp, \mu, \zeta) = -\mathcal{D}(r_\perp) f(x, r_\perp, \mu, \zeta)$$

Recently computed to three loops
Li, Zhu (2017); Vladimirov (2017)

Computable from lattice QCD at large $r_\perp$
Ebert, Stewart, Zhao (2018)

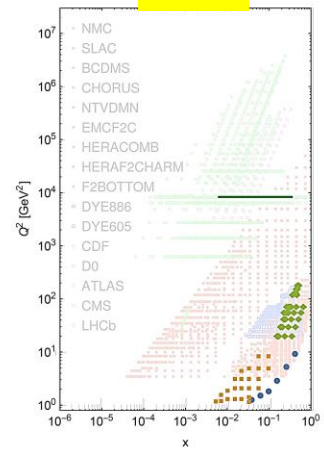
TMD global analysis

	Framework	W+Y	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	W	✘	✘	✔	✔	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✘	✘	✔	✔	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✘	✘	✔	✔	>100 (?)
Pavia 2013 arXiv:1309.3507	LO	W	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	LO	W	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x,Q²) bin	1 (x,Q²) bin	✔	✔	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✘	✔	✔	✔	200 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✘	✘	✔	✔	309
BSV 2019 arXiv:1902.08474	NNLO-NNLL	W	✘	✘	✔	✔	457

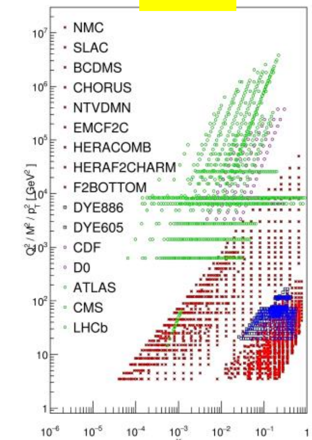


Still in its infancy. Fully blossoms in the EIC era!

TMD



PDF



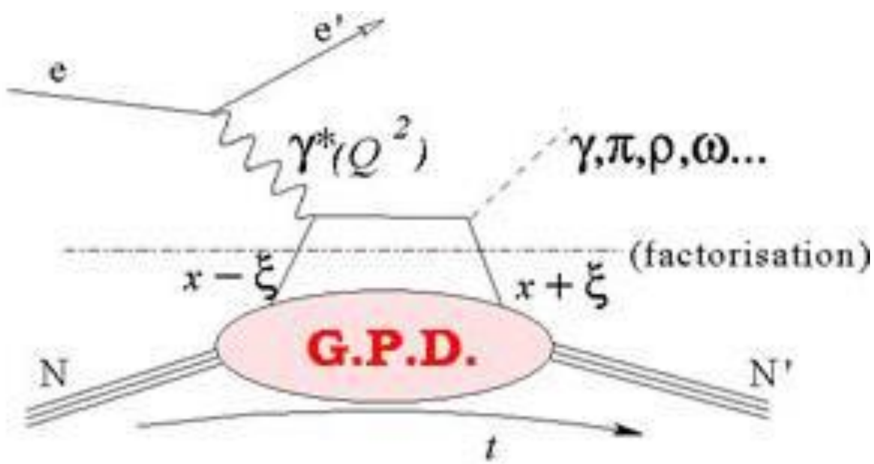
Generalized parton distributions (GPD)

$$P^+ \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \langle P'S'|\bar{\psi}(0)\gamma^\mu\psi(y^-)|PS\rangle$$

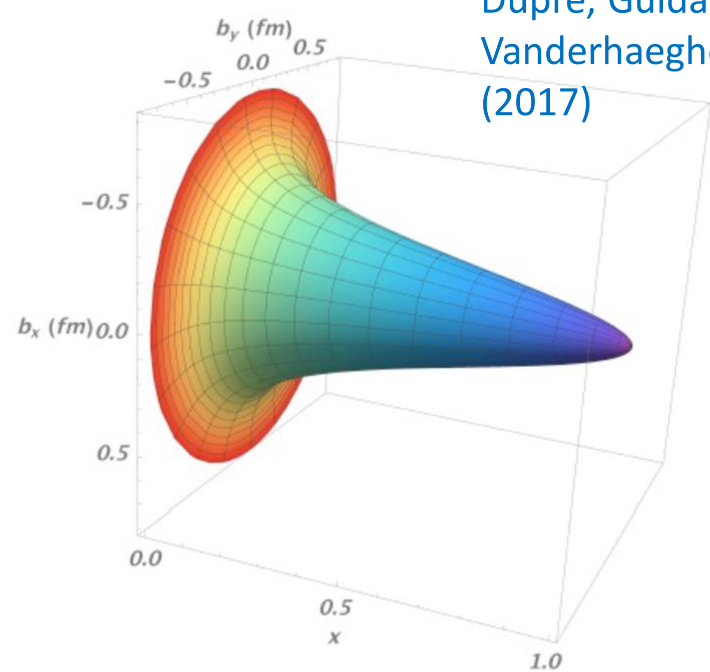
$$= H_q(x,\Delta)\bar{u}(P'S')\gamma^\mu u(PS) + E_q(x,\Delta)\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}u(PS) \qquad \Delta = P' - P$$

Fourier transform → Distribution of partons in impact parameter space $b_\perp$

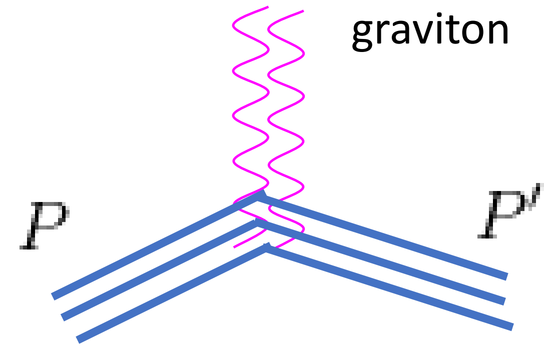
Measurable in
Deeply Virtual Compton Scattering (DVCS)



Dupre, Guidal, Vanderhaeghen (2017)



Nucleon gravitational form factors



$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}Mg^{\mu\nu}\right]u(P)$$

All the form factors are interesting and measurable!

$A_{q,g}$ Momentum fraction

$B_{q,g}$ Ji sum rule

$$\frac{1}{2} = J_q + J_g$$

$$J_q = \frac{1}{2}\int dx(H_q(x) + E_q(x))$$

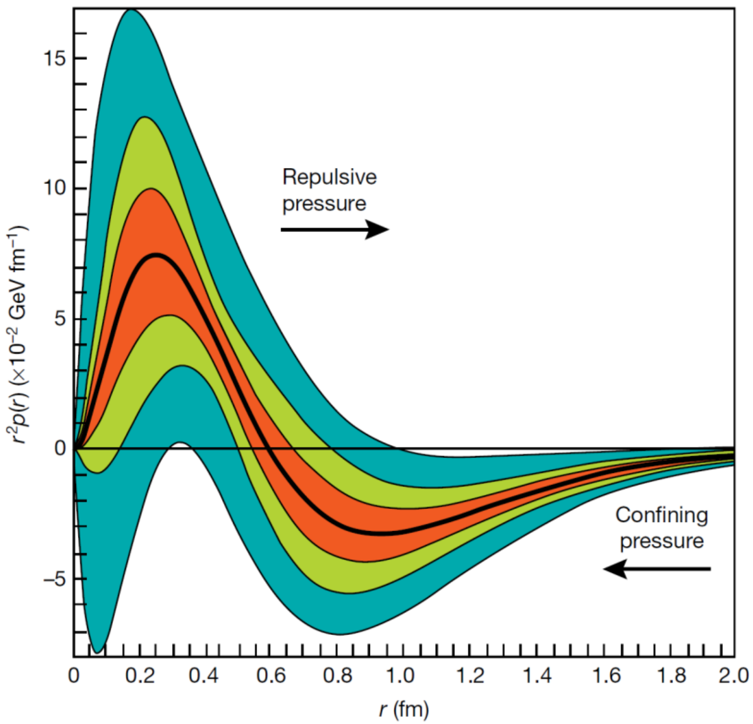
$D_{q,g}$ `Pressure’ and `shear’ inside proton

$\bar{C}_{q,g}$ Mass, pressure

D-term: the last global unknown

$$\langle P'|T^{ij}|P\rangle \sim (\Delta^i\Delta^k - \delta^{ik}\Delta^2)D(t)$$

Burkert, Elouadrhiri, Girod (Nature, 2018)



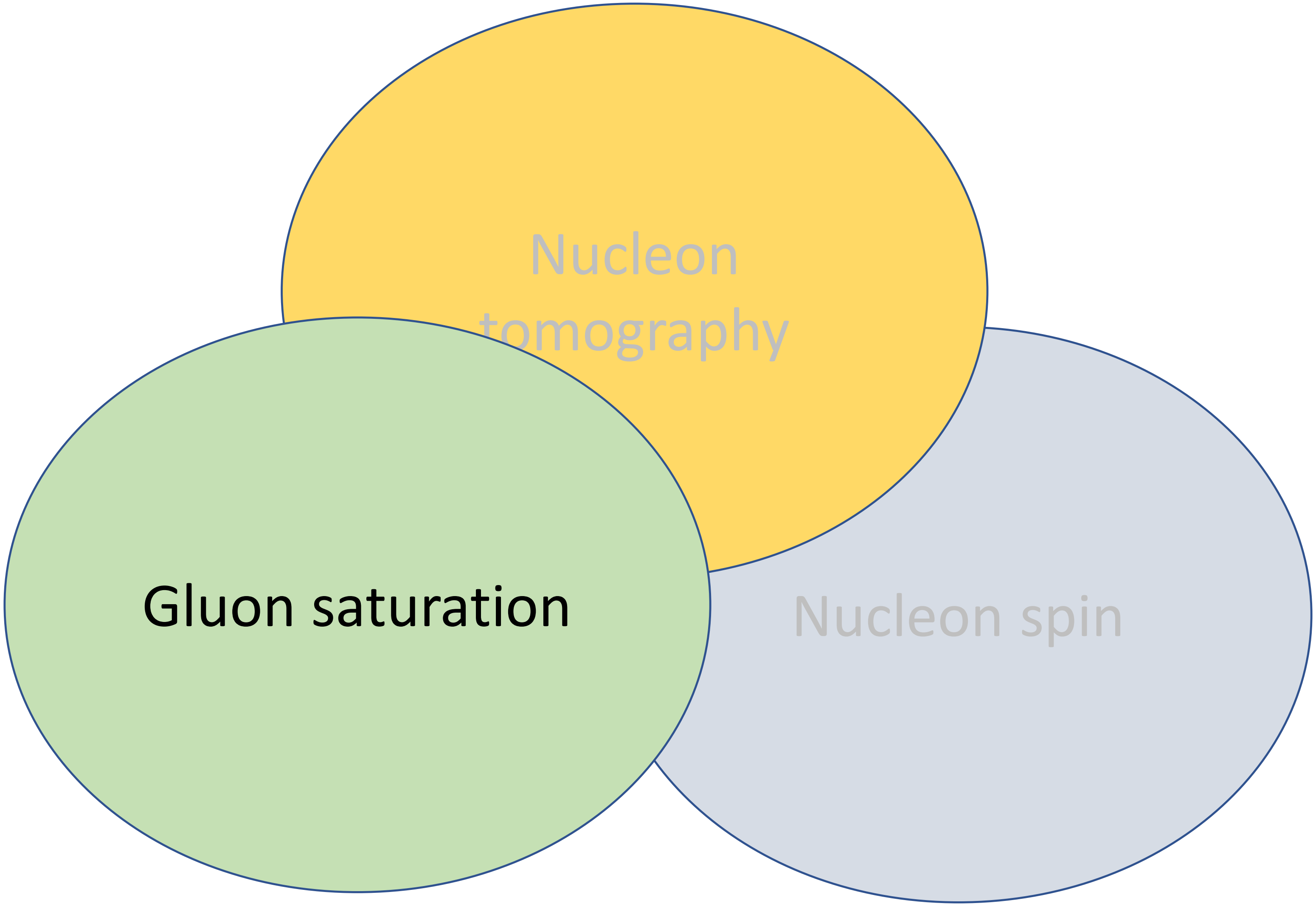
$D(t=0)$ is a conserved charge of the nucleon, just like mass and spin!

Related to the radial pressure distribution inside a nucleon   Polyakov, Schweitzer,...

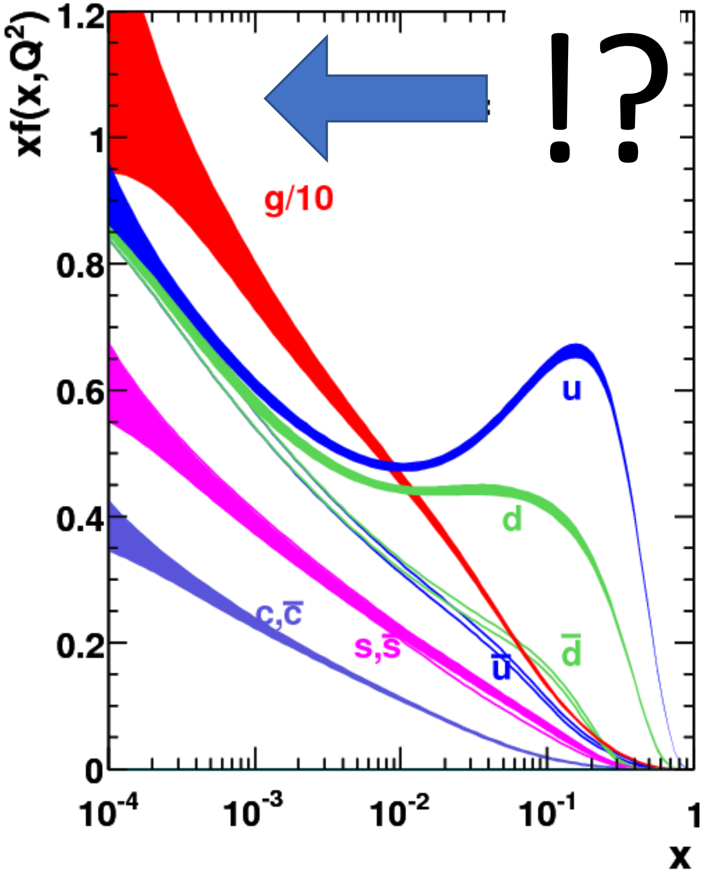
$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$

First extraction at Jlab, large model dependence.
Need significant lever-arm in Q^2 to disentangle various moments of GPDs → EIC

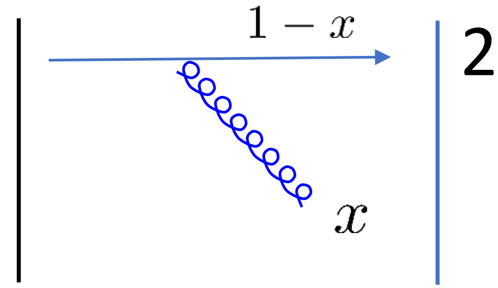
Scientific goals of EIC



QCD at small-x



Probability to emit a soft gluon diverges



$\propto \frac{1}{x}$

A myriad of small-x gluons
in a high energy hadron/nucleus!

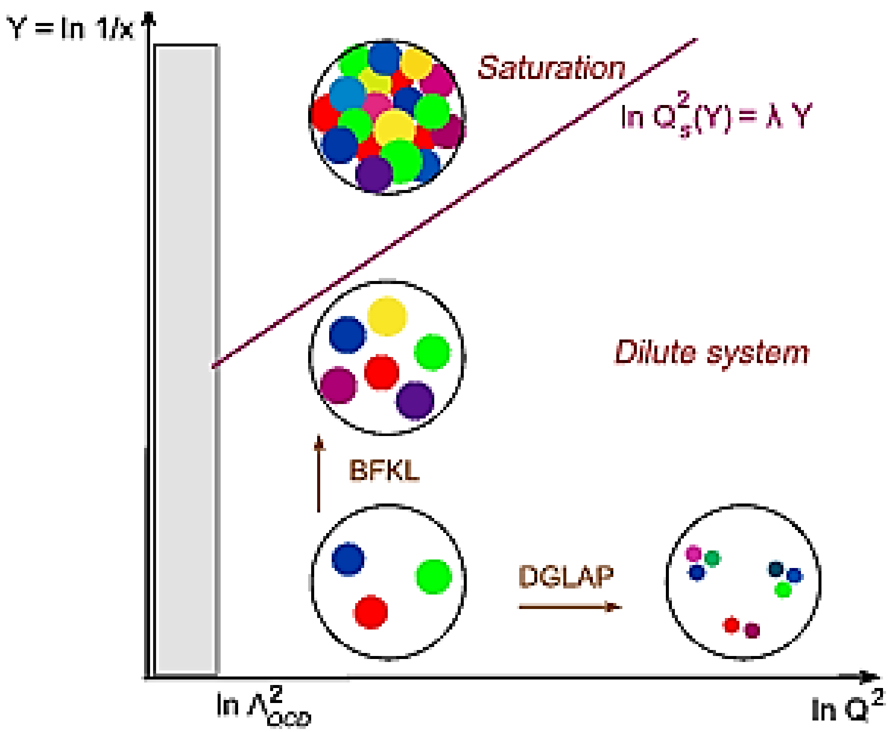
as predicted by BFKL
(Balitsky-Fadin-Kuraev-Lipatov)

$$\sum_n \frac{1}{n!}(\alpha_s \ln 1/x)^n \sim \left(\frac{1}{x}\right)^{\alpha_s}$$

Gluon saturation

The gluon number eventually saturates, forming the universal QCD matter at high energy called the Color Glass Condensate.

Gribov, Levin, Ryskin (1980); Mueller, Qiu (1986); McLerran, Venugopalan (1993)

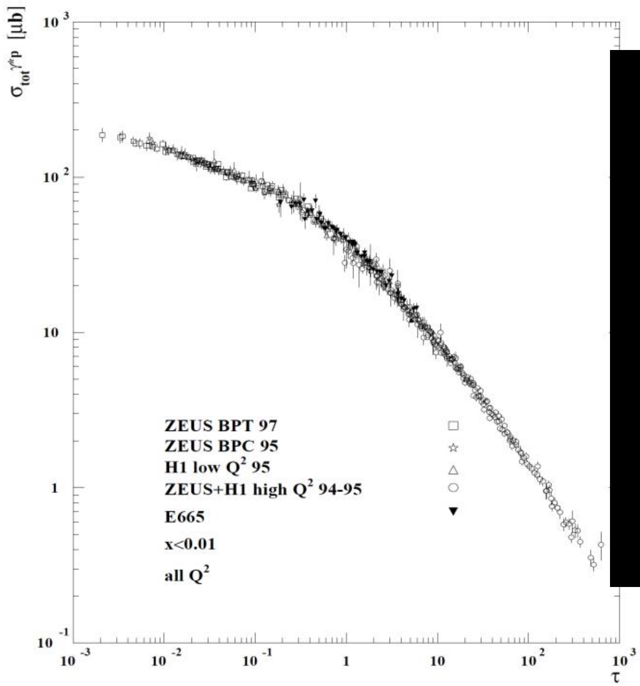


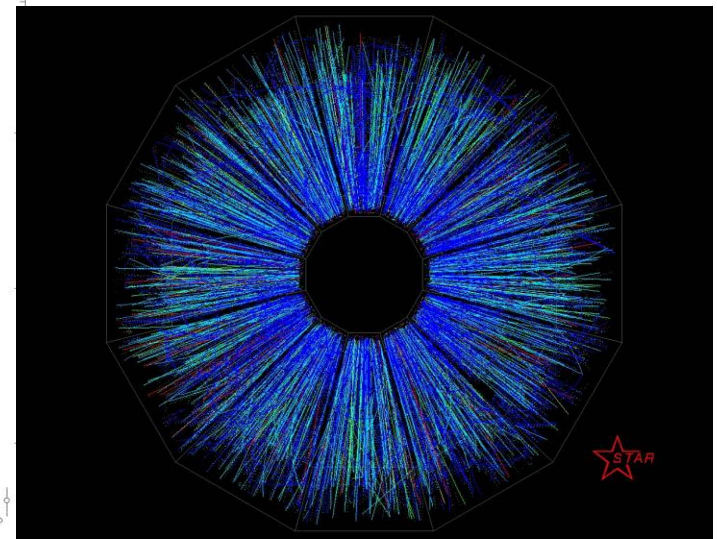
CGC characterized by one hard scale, the saturation momentum

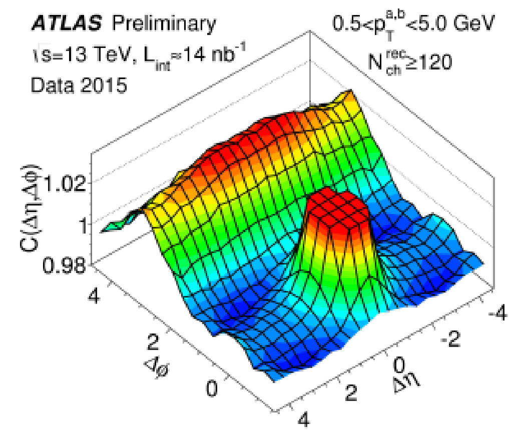
$$Q = Q_s(x) \gg \Lambda_{QCD}$$

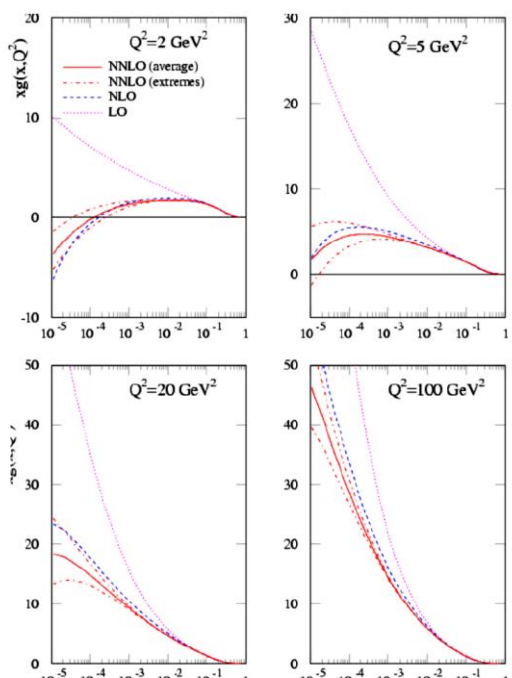
Particle production around the scale $P_\perp \sim Q_s$ calculable including the saturation effect.

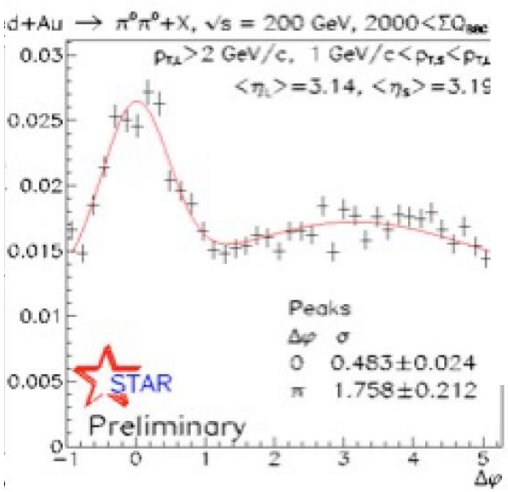
Has saturation been observed at HERA, RHIC, LHC?

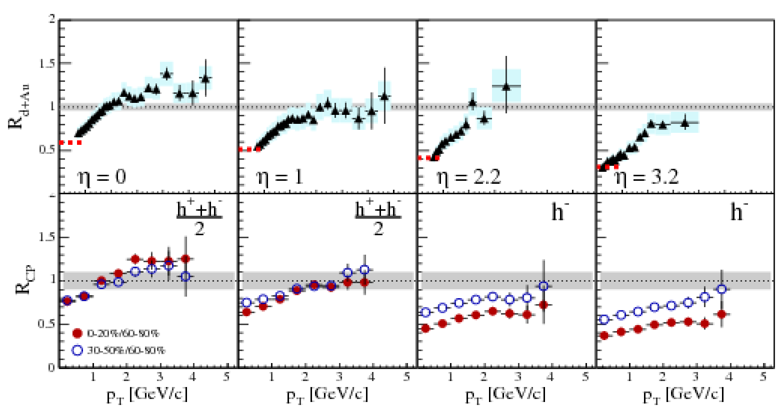








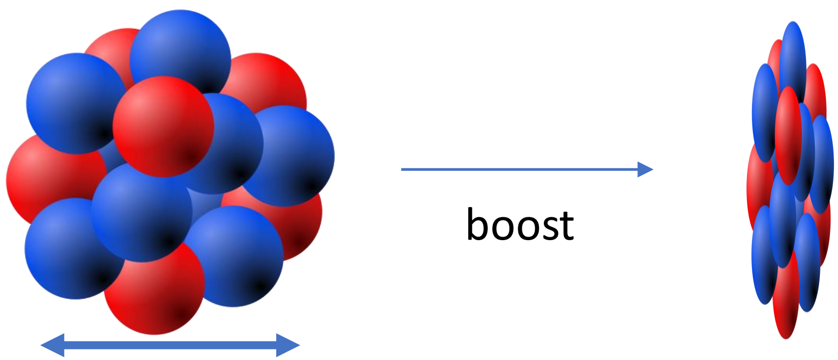




eA collision at EIC : ideal place to study saturation

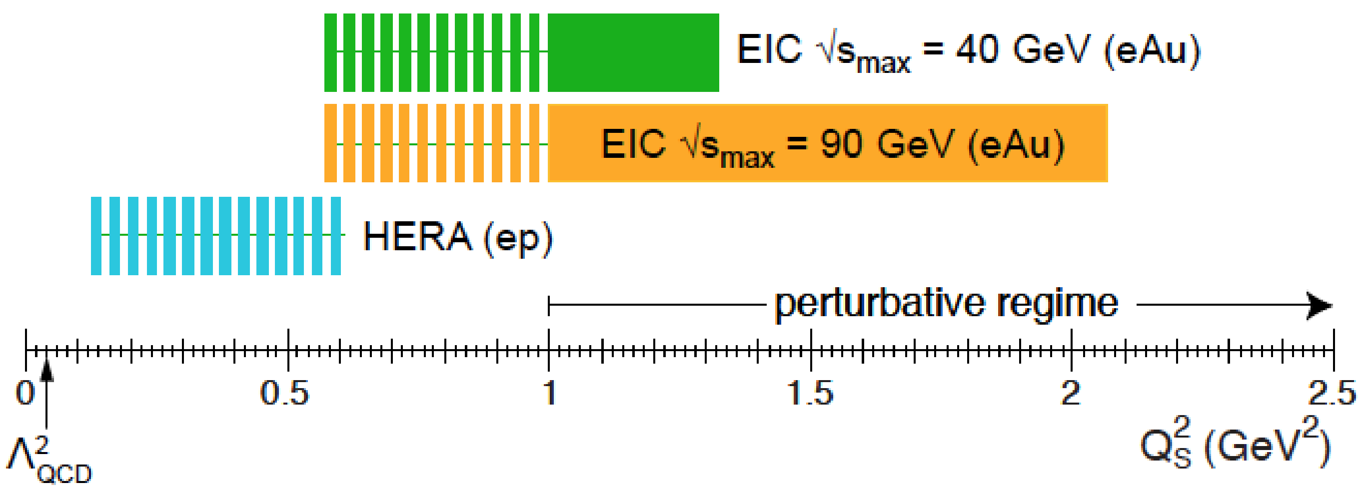
No initial state interactions (advantage over LHC, RHIC)

Nuclear enhancement of the saturation momentum (advantage over HERA)



$$R_A \sim A^{1/3}$$

$$Q_s^2 \propto A^{1/3}$$



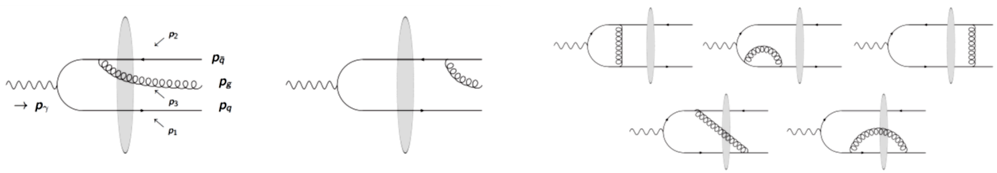
Can saturation become precision physics?

No all-order proof of factorization.
`Leading order' already contains infinitely many diagrams with infinitely many twists.

NLL Balitsky-Kovchegov (BK) Balitsky, Chirilli (2008)
NNLL BK Caron-Huot, Herranen (2016)

Factorization should be checked order by order. Currently NLO for a few processes.
Chirilli, Xiao, Yuan; Beuf; Roy, Venugopalan…

e.g., NLO exclusive diffractive dijet, vector meson production at EIC

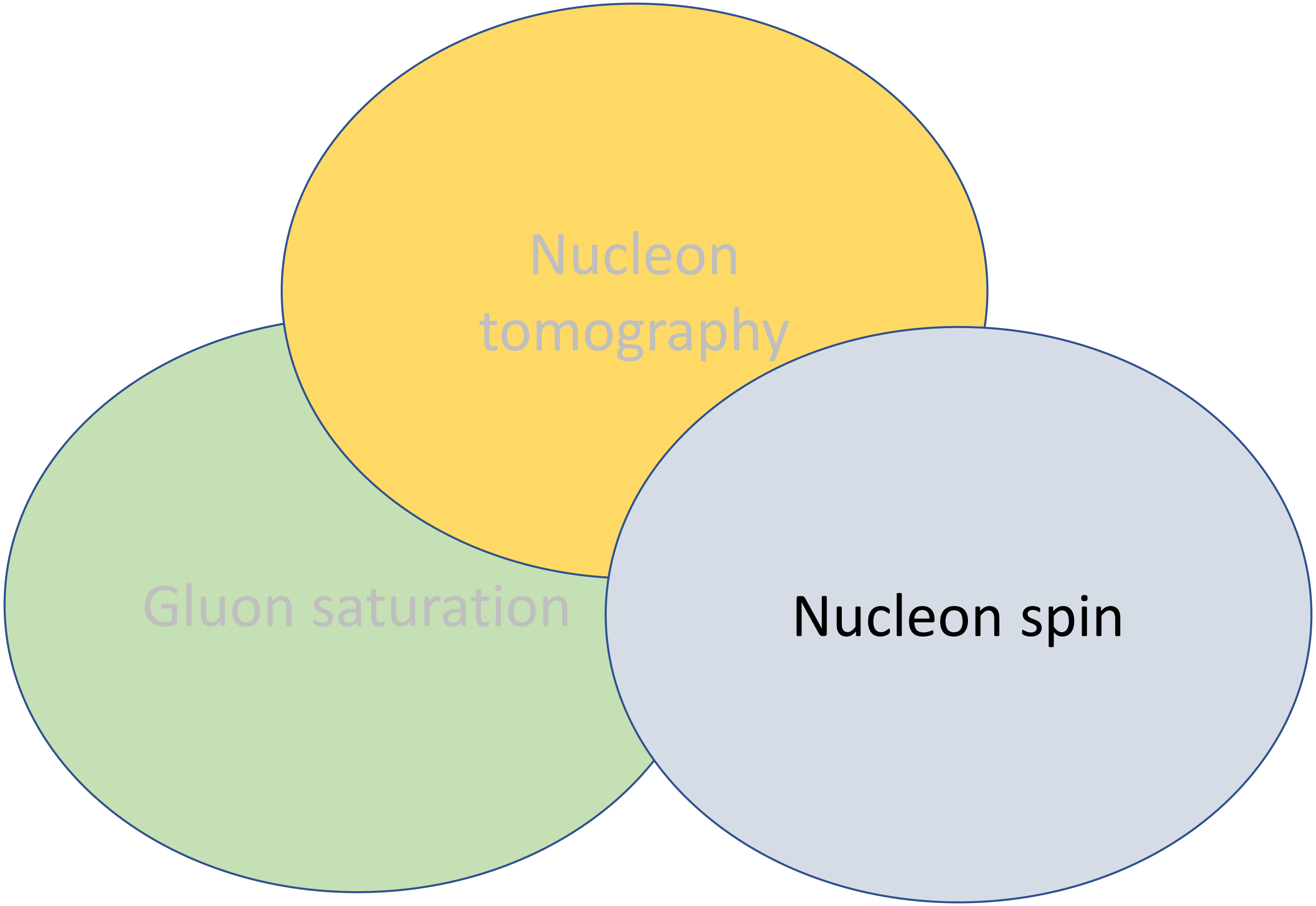


Boussarie, Grabovsky, Szymanowski, Wallon (2016)

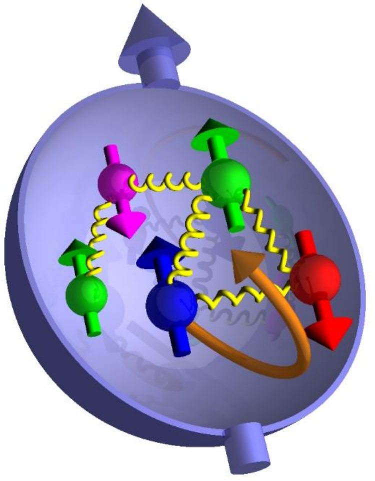
Need also `collinear improvement' Iancu, Mueller, Soyez, Triantafyllopoulos

NLO global analysis of the dipole S-matrix? cf. Albacete, Armesto, Milhano, Salgado (2009)

Scientific goals of EIC



Proton spin decomposition



The proton has spin ½.
The proton is not an elementary particle.

➡ Jaffe-Manohar sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity ($\frac{1}{2}\Delta\Sigma$)

Gluons' helicity (ΔG)

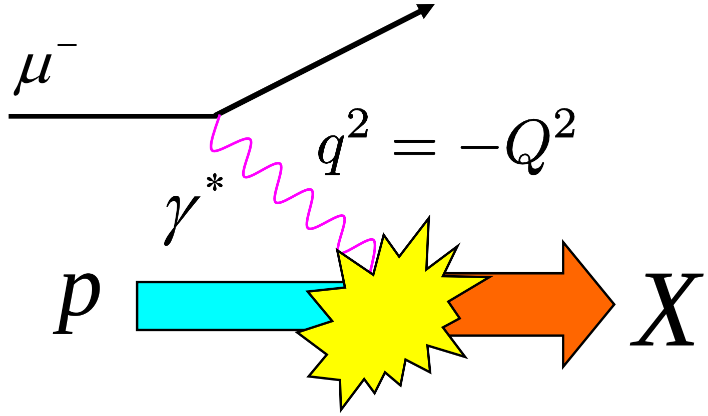
Orbital angular Momentum (OAM) (L^q, L^g)

$\Delta\Sigma = 1$ in the quark model

$\Delta\Sigma$ from polarized DIS

Longitudinal double spin asymmetry in polarized DIS

$$A_{LL} = \frac{\mu^\uparrow p^\downarrow - \mu^\uparrow p^\uparrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\downarrow} \sim \left(1+\frac{\sigma_L}{\sigma_T}\right)\frac{2xg_1}{F_2}$$



$$\int_0^1 dx g_1(x,Q^2) = \frac{1}{2}\sum_f e_f^2 \int_0^1 dx(\Delta q_f(x,Q^2) + \Delta\bar{q}_f(x,Q^2)) + \cdots$$

$$= \frac{1}{9}(\Delta u + \Delta d + \Delta s) \quad \leftarrow \Delta\Sigma$$

$$+\frac{1}{12}(\Delta u - \Delta d)$$

$$+\frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)$$

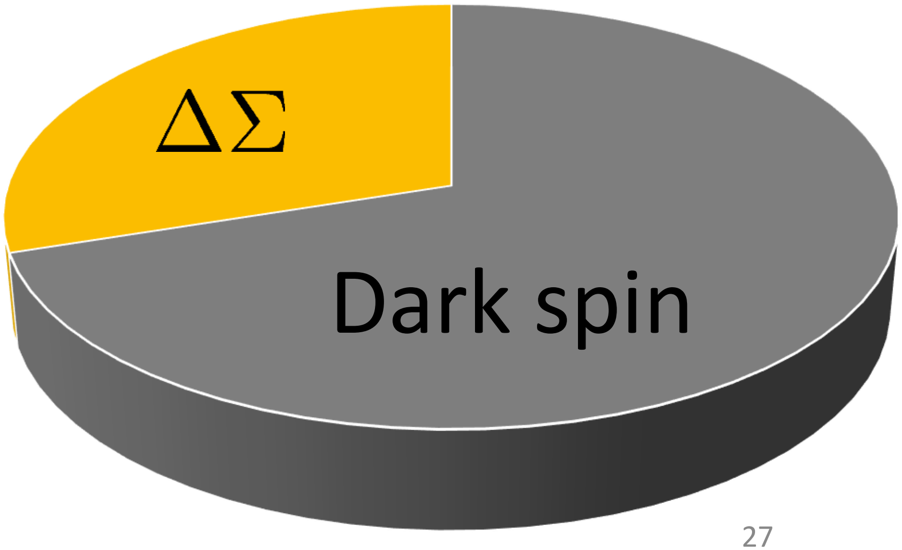
`Spin crisis'

In 1987, EMC (European Muon Collaboration) announced a very small value of the quark helicity contribution

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \ !?$$

Recent value from NLO QCD global analysis

$$\Delta\Sigma = 0.25 \sim 0.3$$



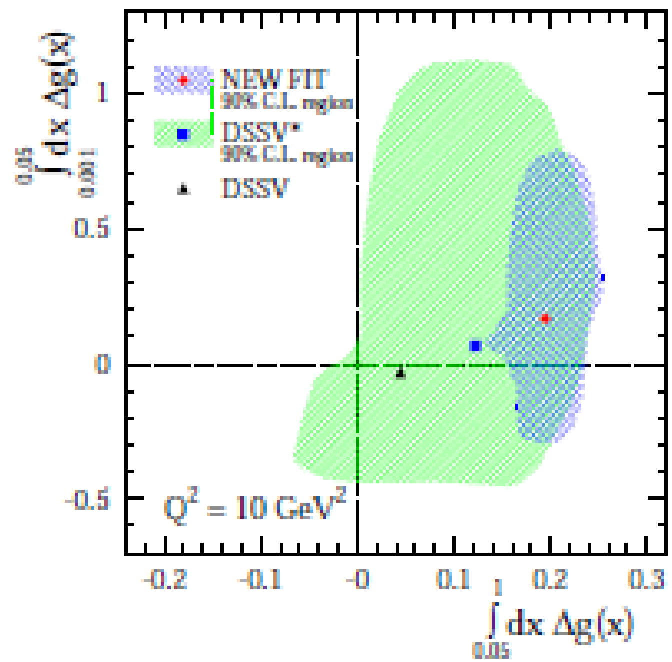
Evidence of nonzero ΔG

Global analysis based on RHIC 200 GeV data.

$$\int_{0.05}^{1} dx\Delta g(x,Q^2=10\text{GeV}^2) = 0.20^{+.06}_{-.07} \qquad \text{DSSV++}$$

$$\int_{0.05}^{0.2} dx\Delta g(x,Q^2=10\text{GeV}^2) = 0.17\text{+-}0.06 \qquad \text{NNPDFpol1.1}$$

$$\int_{0.001}^{0.8} dx\Delta g(x,Q^2 = 1\text{ GeV}^2) = 0.5\text{+-}0.4 \qquad \text{JAM15}$$

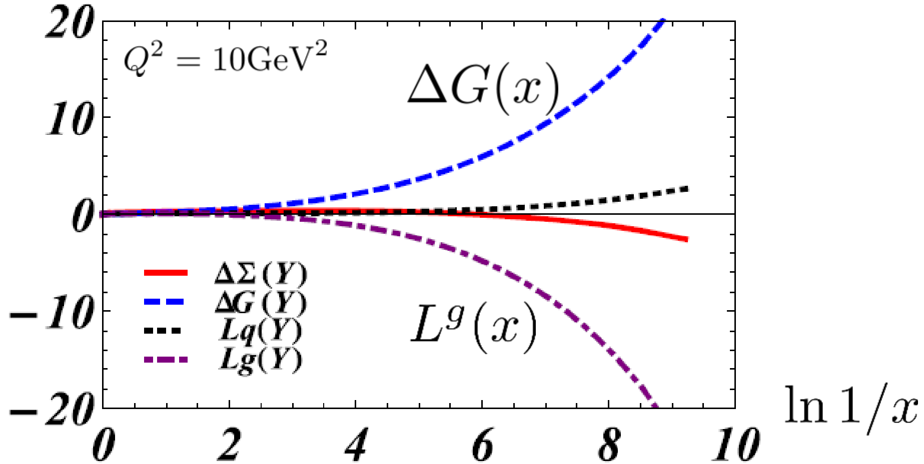


Huge uncertainty from the small-x region

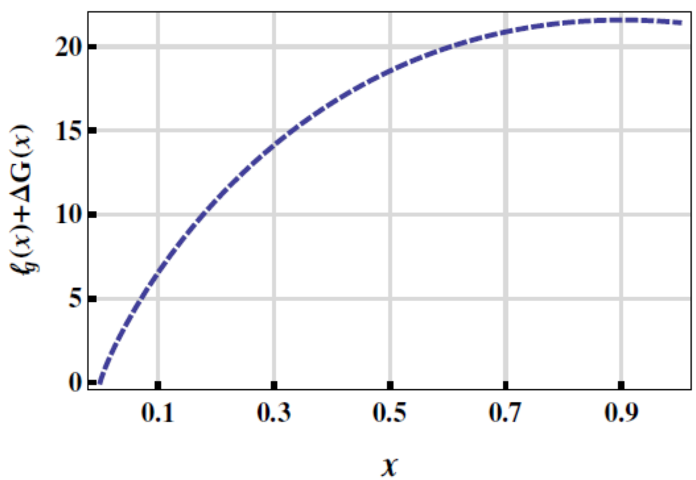
EIC will pin down the value of ΔG ....
Does this finally solve the spin puzzle? No!

Don't forget Orbital Angular Momentum. It's there!

Significant cancellation between helicity and OAM at small-x



YH, Yang (2018)



More, Mukherjee, Nair (2018)

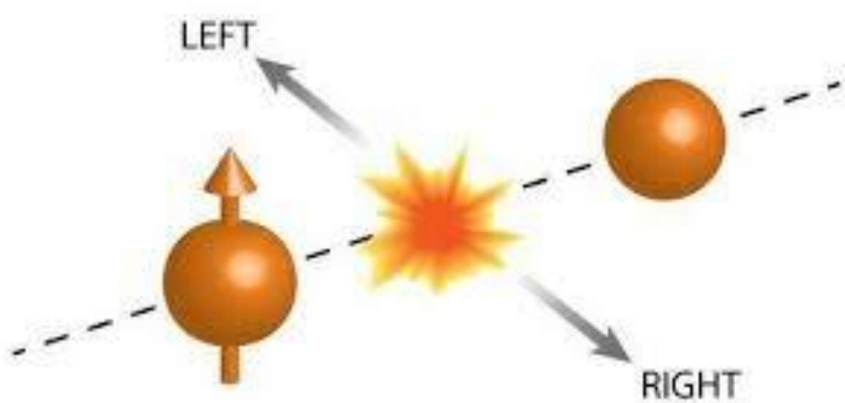
All-loop resummation of small-x double logarithms $(\alpha_s \ln^2 1/x)^n$

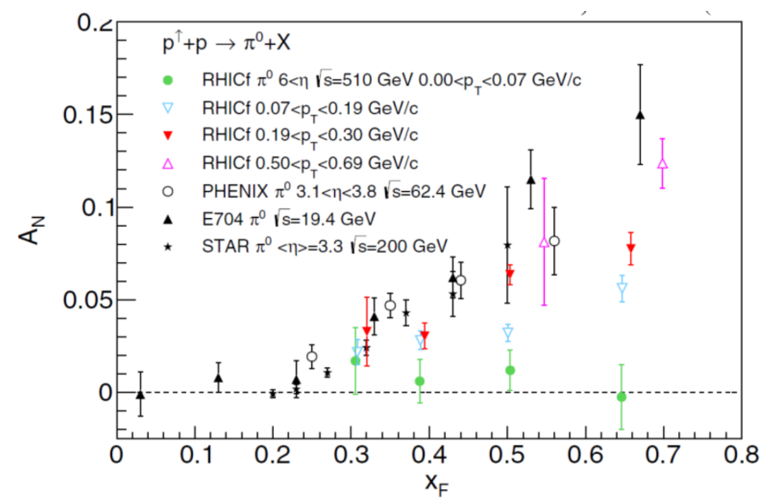
$$L_g(x) \approx -2\Delta G(x)$$

Boussarie, YH, Yuan (2019)

Single spin asymmetry

Polarize spin perpendicularly to the direction of motion and hit an unpolarized target.





The number of particles produced on the left hand side and right hand side are different.

40 years old puzzle in QCD, several mechanisms known

EIC will nail down the origin of SSA

Conclusion

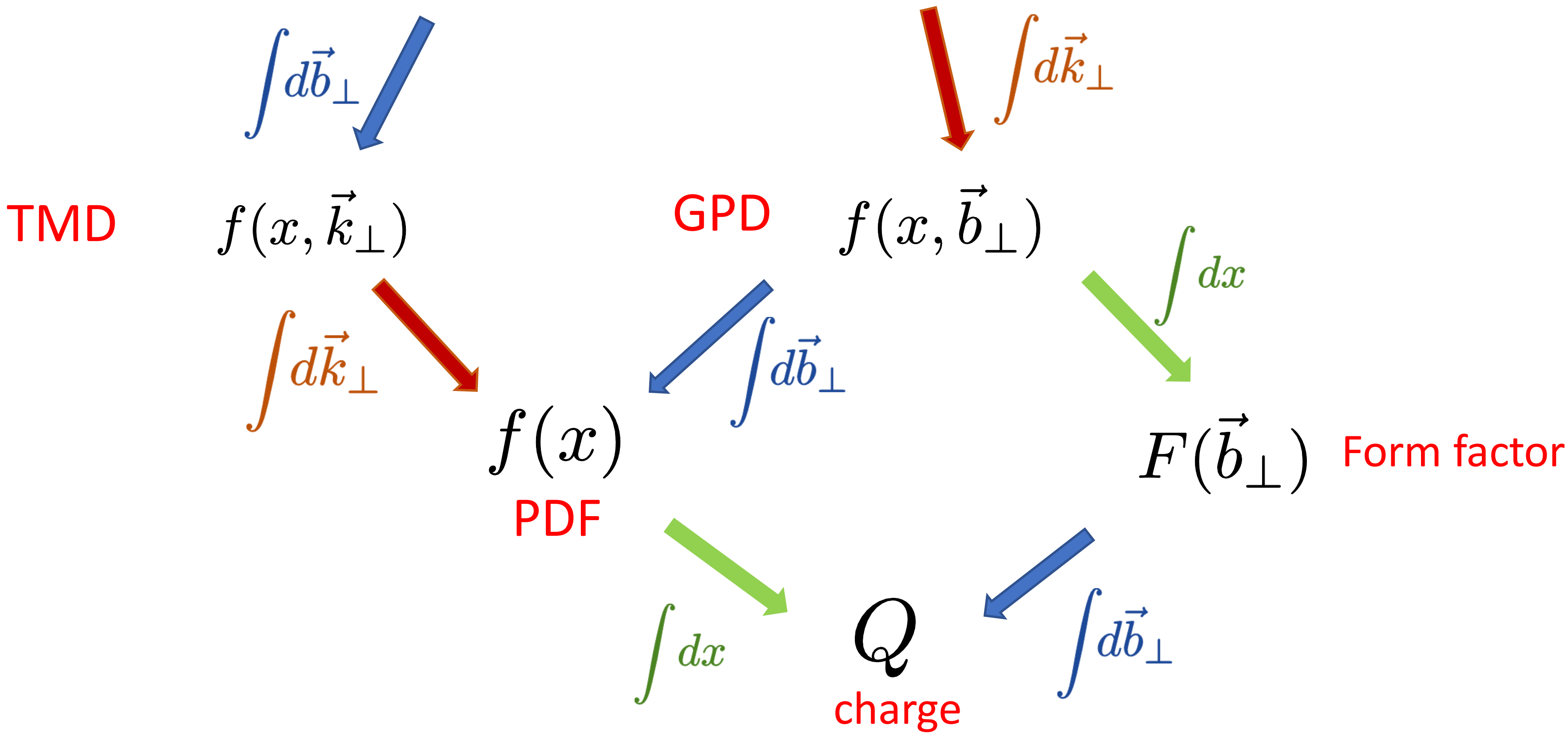
- In 10-15 years from now, DIS experiments will be running in the US, China and maybe also in Europe.
- Tremendous physics opportunities for theory, experiments, and lattice QCD. Exciting times ahead.

Probing the Wigner/GTMD distribution at the EIC

5D tomography:
Wigner distribution— the “mother distribution”

Belitsky, Ji, Yuan (2003)

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2)\gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$



5D tomography: GTMD and Husimi

Wigner distribution

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$

Fourier transform

$$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$$

Gaussian smearing in k, b

Generalized TMD (GTMD)

$$G(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

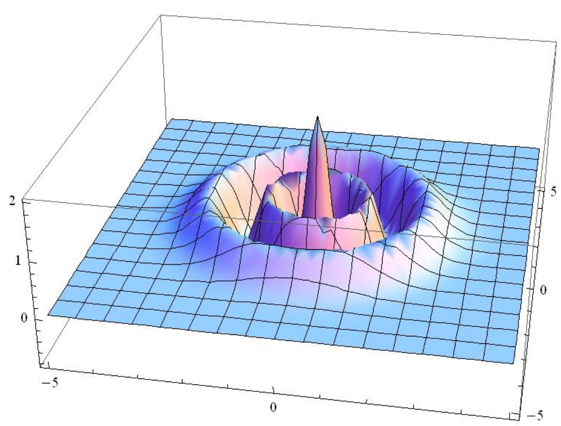
Meissner, Metz, Schlegel (2009)

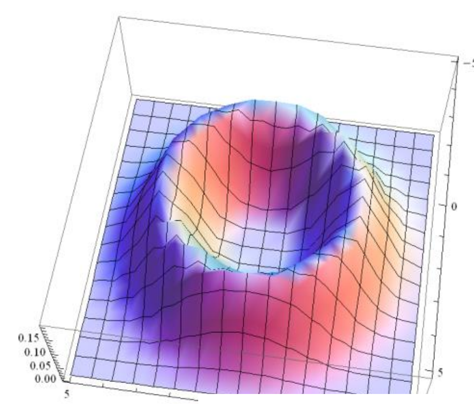
Husimi distribution

$$H(x, \vec{k}_\perp, \vec{b}_\perp)$$

Hagiwara, YH (2015)

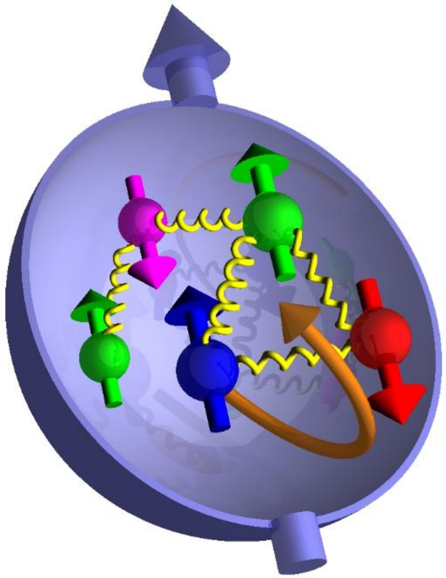
Wigner and Husimi of the 4^{th} excited state of a 1D harmonic oscillator.





Positive definite!

Wigner distribution and orbital angular momentum



Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Canonical Orbital angular momentum (OAM)

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

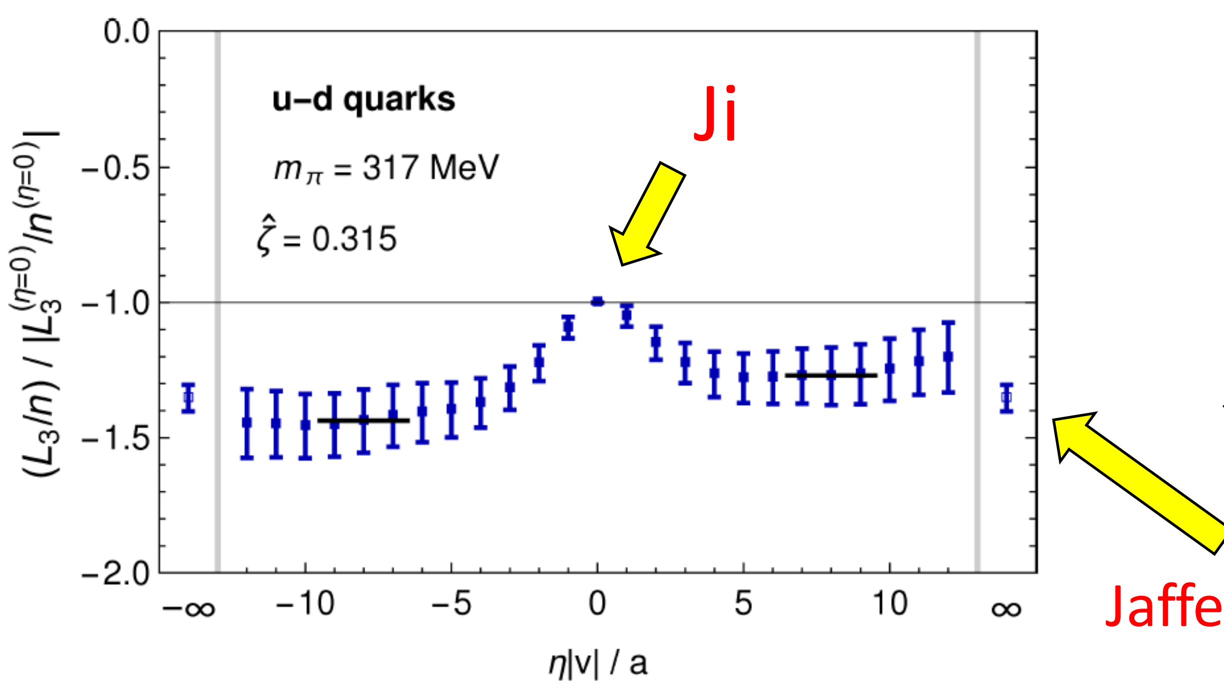
Lorce, Pasquini, (2011);
YH (2011)
Ji, Xiong, Yuan (2012)

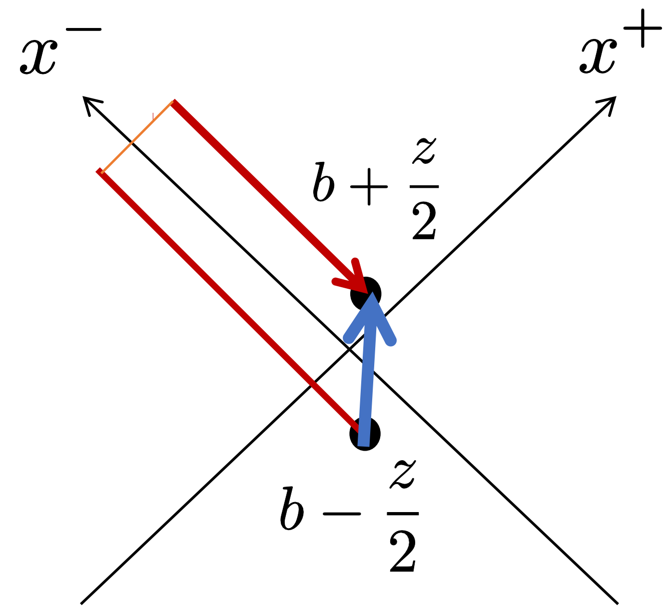
Wilson line and OAMs

Jaffe-Manohar OAM from staple Wilson line YH (2011)

Ji's OAM from straight Wilson line Ji, Xiong, Yuan (2012)

First lattice calculation Engelhardt (2017)





Jaffe-Manohar

`Entropy' of partons

Hagiwara, YH, Xiao, Yuan (2018)

Phase space distribution naturally defines an entropy.
Use the QCD Husimi distribution

$$S(x) \equiv -\int d^2b_\perp d^2k_\perp xH(x,b_\perp,k_\perp)\ln xH(x,b_\perp,k_\perp)$$

$$S(x) \sim \frac{N_c}{\alpha_s}Q_s^2(x)S_\perp \propto A\left(\frac{1}{x}\right)^{\#\alpha_s}$$

$x \to 0$

cf. Kutak (2011)
Kovner-Lublinsky (2015)

Measure of `complexity' of the multiparton system.
Saturation of entropy due to the Pomeron loop effect?
Connection to the `jet entropy' in the final state? Neill, Waalewijn (2018)

Gluon Wigner/GTMD at small-x

$$xW(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\Delta/2 | F^{+i}(-z/2) F^{+}_{i}(z/2) | P+\Delta/2\rangle$$

At small-x, approximate $e^{ixP^+z^-} \approx 1$ YH, Xiao, Yuan (2016)

$$xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s} \int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp} \left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S_x(\vec{b}_\perp,\vec{r}_\perp)$$

\`\`Dipole S-matrix” $S_x(\vec{b}_\perp,\vec{r}_\perp) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$

$\cos 2\phi$ correlation expected

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = W_0(x,k_\perp,b_\perp) + 2\cos 2(\phi_k - \phi_b) W_1(x,k_\perp,b_\perp) + \cdots$$

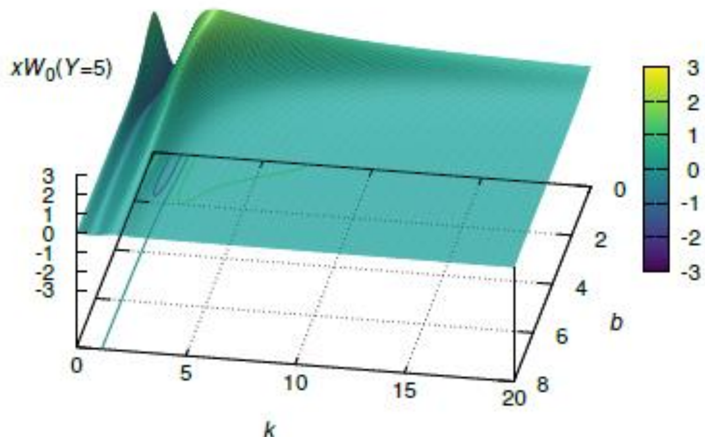
\`Elliptic Wigner’ distribution

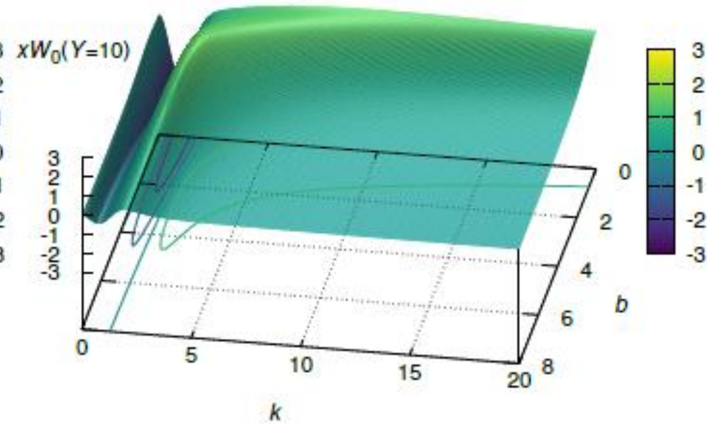
Gluon Wigner from Balitsky-Kovchegov equation

Hagiwara, YH, Ueda (2016)

Peak at the saturation momentum

$W_0(k_\perp, b_\perp)$

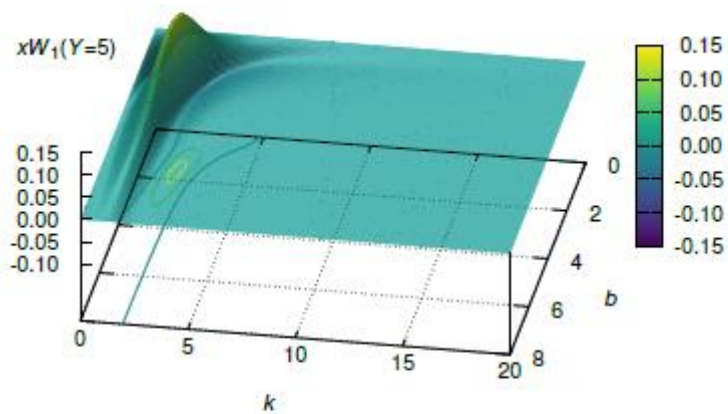


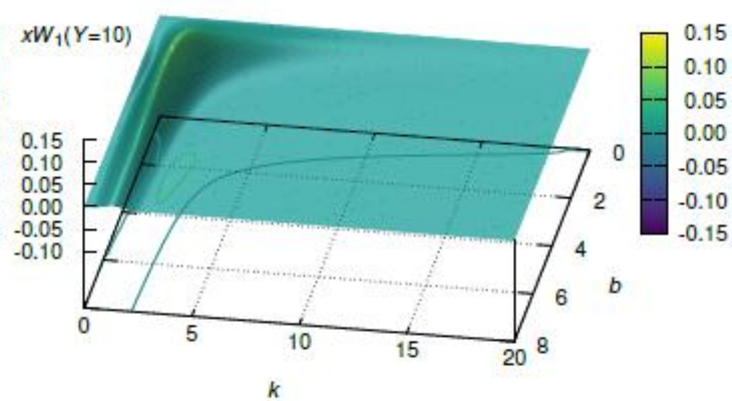


$$Y = \ln \frac{1}{x} = 5$$

$$Y = 10$$

$W_1(k_\perp, b_\perp)$





Elliptic part small in magnitude (a few percent effect). No geometric scaling.

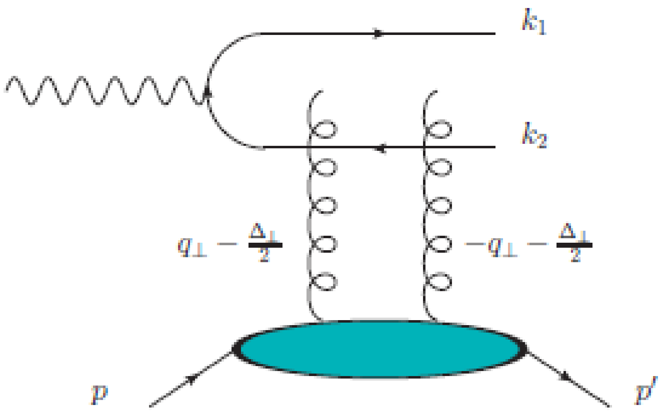
Can we probe Wigner/GTMD in experiments?

Distributions with more variables → more exclusive processes

Must be diffractive (proton remains intact) → $\Delta_\perp$-dependence.

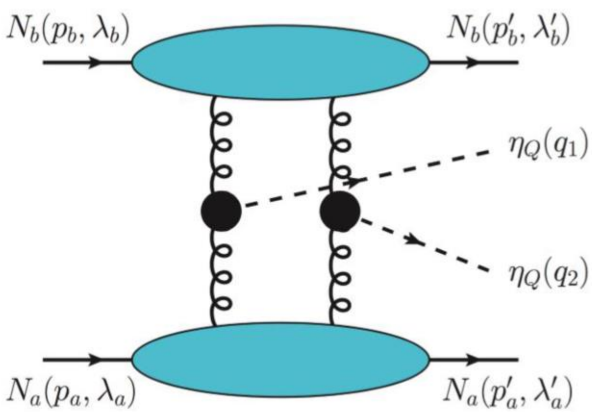
In addition to the recoiling proton, measure two particles (jets) → $k_\perp$-dependence

Diffractive dijet in DIS



YH, Xiao, Yuan (2016)

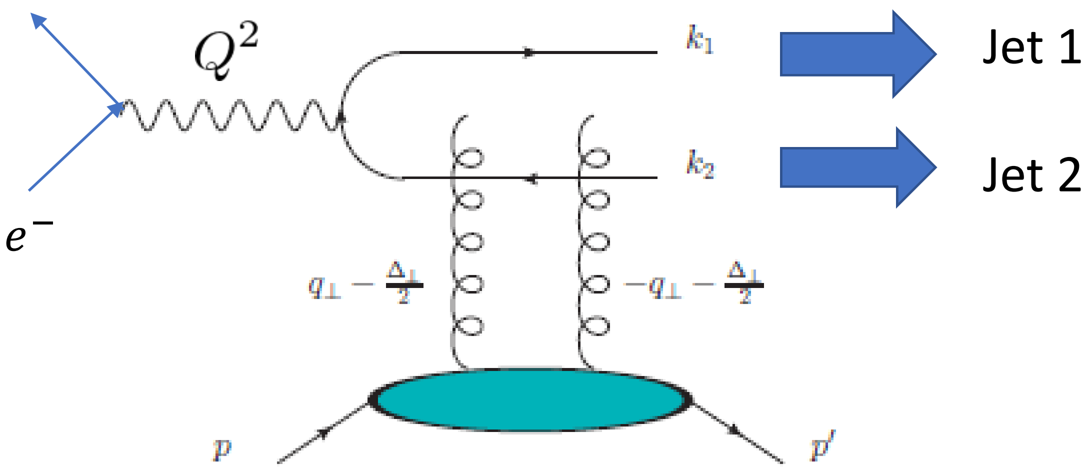
Double quarkonium production in pp



Bhattacharya, Metz, Ojha, Tsai, Zhou (2018)

Probing Wigner (GTMD) in diffractive dijet production

YH, Xiao, Yuan (2016), see also, Altinoluk, Armesto, Beuf, Rezaeian (2015)



$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$
$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

GTMD

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{\Delta}_\perp d^2\vec{P}_\perp} \propto z(1-z)[z^2 + (1-z)^2] \int d^2q_\perp d^2q'_\perp S(q_\perp, \Delta_\perp) S(q'_\perp, \Delta_\perp)$$
$$\times \left[\frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{q}_\perp}{(P_\perp - q_\perp)^2 + \epsilon^2}\right] \cdot \left[\frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{q}'_\perp}{(P_\perp - q'_\perp)^2 + \epsilon^2}\right]$$

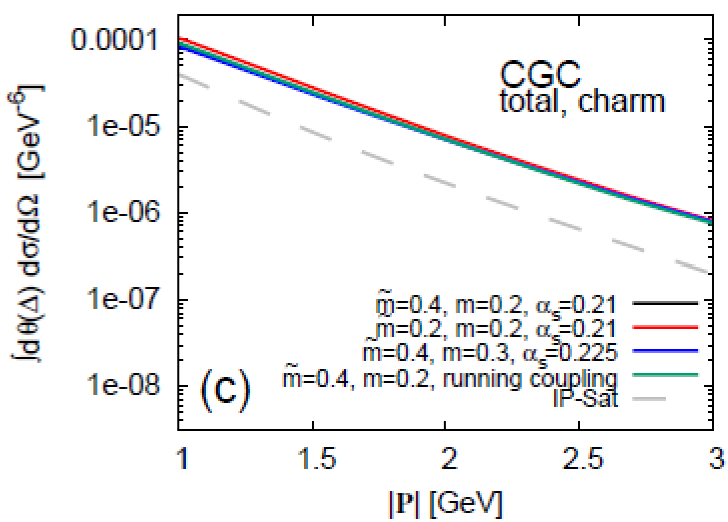
$$\sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta) d\tilde{\sigma}$$

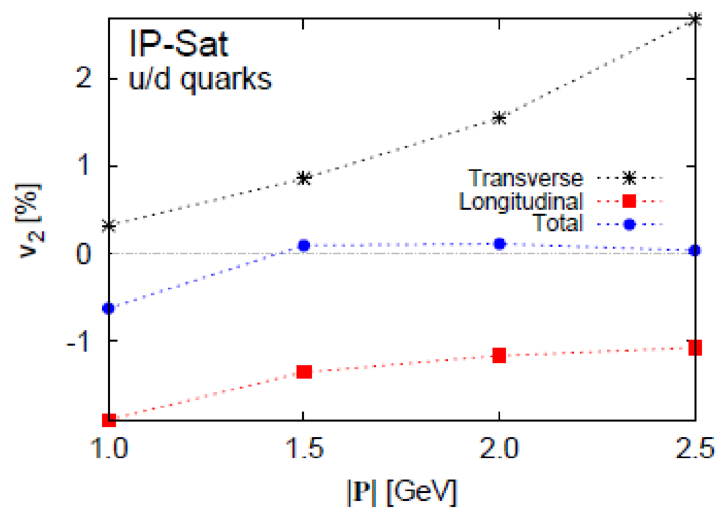
$$\epsilon^2 = z(1-z)Q^2$$

Diffractive dijet at work

Mantysaari, Mueller, Schenke (2019)

First realistic calculation from b-dependent JIMWLK



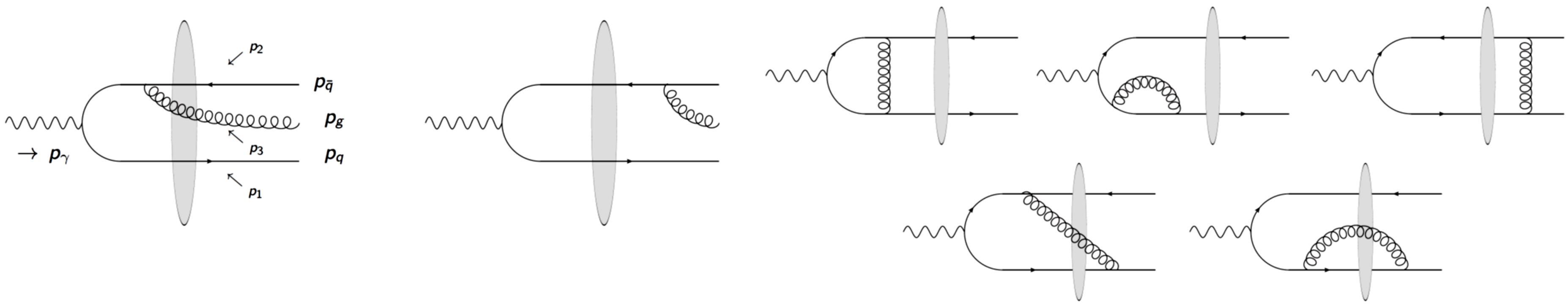


$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{\Delta}_\perp d^2\vec{P}_\perp} \sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta)d\tilde{\sigma} + \cos(\phi_P - \phi_\Delta)d\sigma'$$

surprise!

Factorization at NLO

Boussarie, Grabovsky, Szymanowski, Wallon (2016)



$$\Phi_L^{(0)} = \frac{2x\bar{x}p_V^+ Q}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2},$$

$$\Phi_T^{(0)} = -\frac{(x-\bar{x})p_V^+(\bar{x}\vec{p}_{1\perp} - x\vec{p}_{2\perp}) \cdot \vec{\varepsilon}_{\gamma_T}}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2}$$

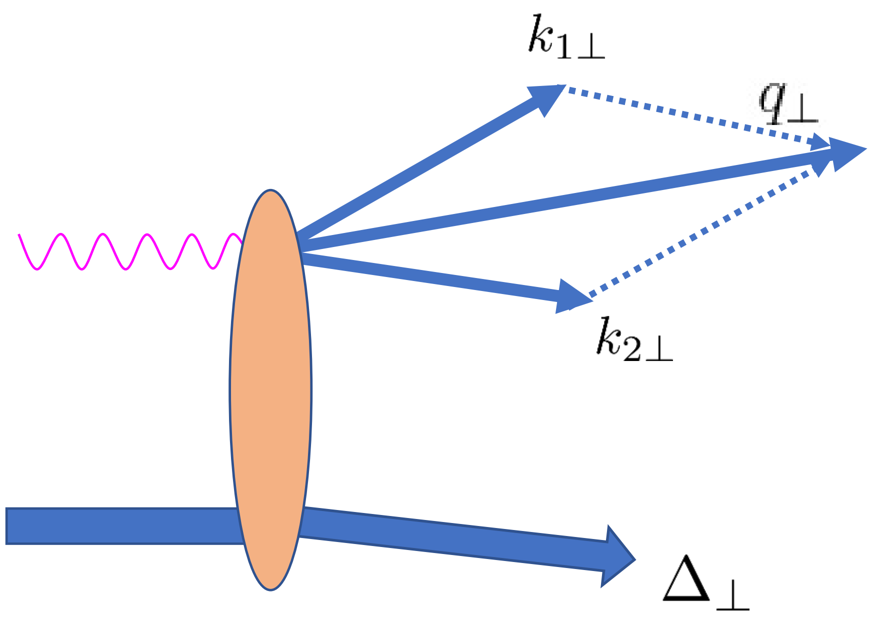
No end point singularity, even for a transverse photon and even in the photoproduction limit and even at NLO.
With null transverse momenta in the t channel, one could encounter $x \in \{0, 1\}$ end point singularities as $\frac{1}{x\bar{x}Q^2}$ thus breaking collinear factorization.

Dijet cross section with soft gluon resummation

To leading order, dijet total transverse momentum
is equal to the proton recoil momentum

$$q_\perp = k_{1\perp} + k_{2\perp} = -\Delta_\perp$$

$$\frac{d\sigma}{d^2q_\perp} = \frac{d\sigma}{d^2\Delta_\perp}$$



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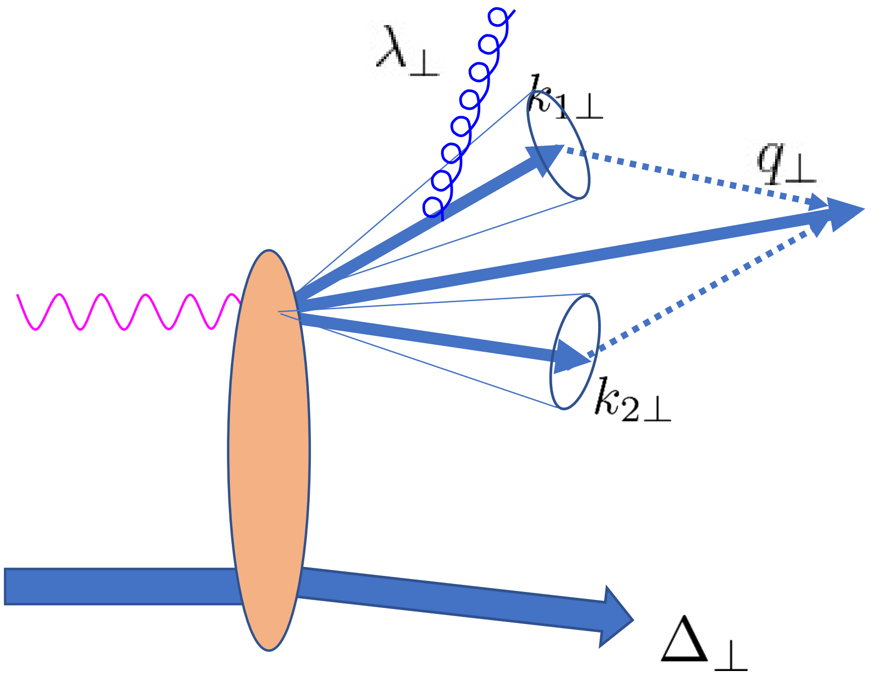


With soft radiation, this becomes

$$q_\perp = -\Delta_\perp + \lambda_\perp$$

Interesting signals can be smeared. Resum Sudakov and nonglobal logs.

$$\frac{d\sigma}{d^2 q_\perp} \sim \int d^2\Delta_\perp \frac{d\sigma}{d^2\Delta_\perp} S(|q_\perp + \Delta_\perp|)$$

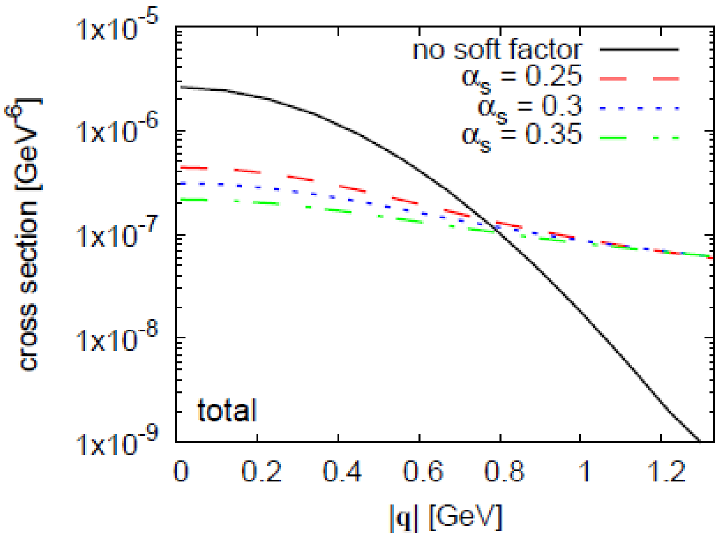
← soft factor

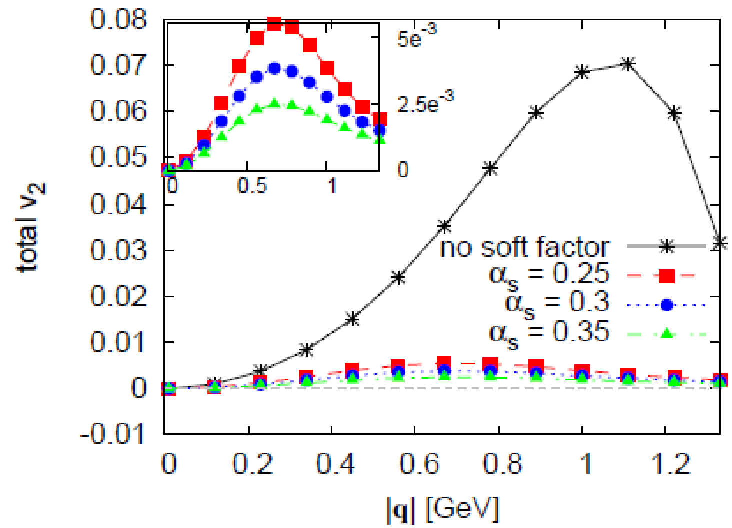
Predictions for EIC

YH, Mueller, Ueda, Yuan 1907.09491

Top EIC energy $W = 140\ \mathrm{GeV}^2, \quad Q^2 = 25\ \mathrm{GeV}^2$

Same code as in Mantysaari, Mueller, Schenke (2019)





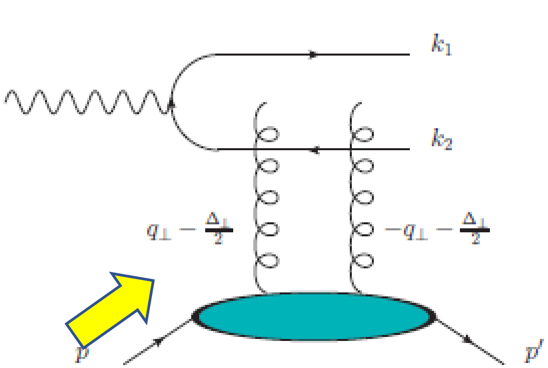
Suppression of back-to-back $q_\perp \approx 0$ cross section.
Elliptic modulation completely washed out.

$q_\perp$ cannot be a proxy for $\Delta_\perp$. Need to measure $\Delta_\perp$ directly

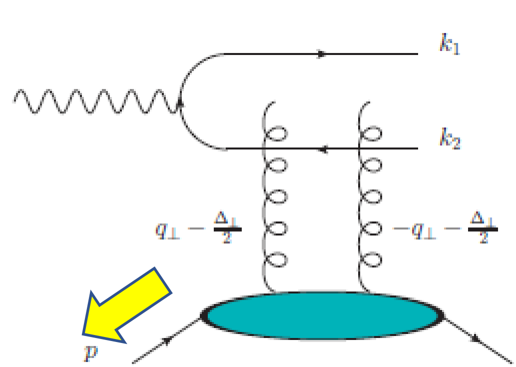
Towards measuring the orbital angular momentum

Longitudinal single spin asymmetry in dijet production

$d\Delta\sigma \sim$ |



$|^2$ $-$ |



$|^2$

Large-x gluon	Ji, Yuan, Zhao
Small-x gluon	YH, Nakagawa, Xiao, Yuan, Zhao
quark	Bhattacharya, Metz, Ojha, Tsai, Zhou

Sensitive to the OAM distribution

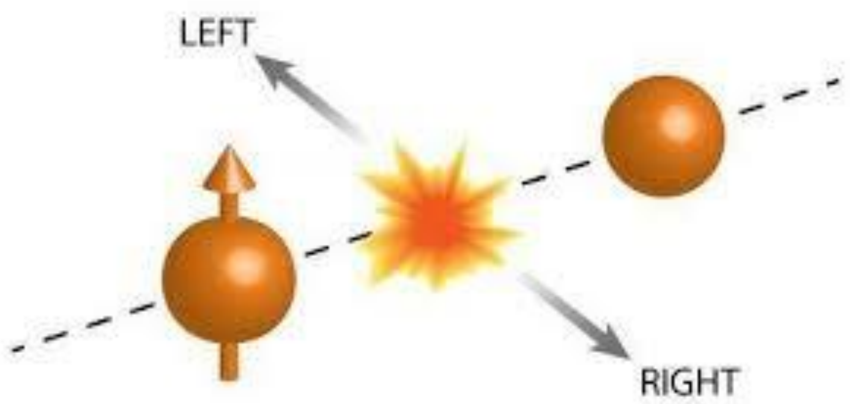
$$W(x,\vec{k}_\perp,\vec{b}_\perp) = W_0(x,k_\perp,b_\perp) + S^+ \sin(\phi_k - \phi_b) W_{OAM}(x,k_\perp,b_\perp) + \cdots$$

Sivers function

Spin-momentum correlation in a transversely polarized proton

$$f(x,k_\perp) = f_0(x,k_\perp) + (\vec{S}_\perp \times \vec{k}_\perp)\cdot \hat{p} f_{1T}^{\perp}(x,k_\perp)$$

One of the origins of observed large single spin asymmetry (SSA)



Gluon Sivers can also be studied at EIC → SSA of open charm, J/ψ, dijet,…

Zheng, Aschenauer, Lee, Xiao, Yin (2018);
Rajesh, Kishore, Mukherjee (2018)

Sivers function as a GTMD

$$f(x,k_\perp) = f_0(x,k_\perp) + \underline{(\vec{S}_\perp \times \vec{k}_\perp) \cdot \hat{p}}\, f_{1T}^{\perp}(x,k_\perp)$$

Use the nucleon spinor

$$k_\perp^i \bar{u}(PS_\perp)\sigma^{+i}u(PS_\perp) f_{1T}^{\perp}(x,k_\perp)$$

Nonforward generalization (GTMD)

$$k_\perp^i \bar{u}(P'S')\sigma^{+i}u(PS) F_{12}(x,k_\perp,\Delta_\perp)$$

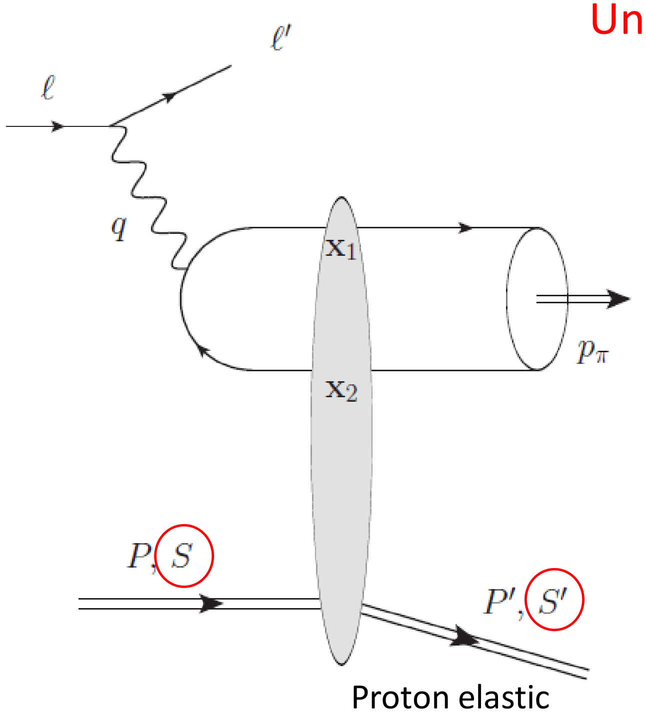
Reduces to Sivers in the forward limit

This spinor product is also nonvanishing if longitudinally polarized $S^\mu = S_L^\mu$ and if helicity flips $S^\mu = -S'^\mu$

$$\bar{u}(P,-S_L)\sigma^{+i}u(PS_L) = (\pm i, -1)$$

Exclusive π^0 production at EIC

Boussarie, YH, Szymanowski, Wallon (2020)



Unpolarized cross section at small-x, $t \approx 0$

$$\frac{d\sigma}{dt} = \sum_{SS'=\pm} \frac{d\sigma_{SS'}}{dt} \approx \frac{d\sigma_{+-}}{dt} + \frac{d\sigma_{-+}}{dt}$$

$$\frac{d\sigma}{dx_B dQ^2 d|t|} = \frac{\pi^5 \alpha_{\rm em}^2 \alpha_s^2 f_\pi^2}{2^3 x_B N_c^2 M^2 Q^6} (1 - y + \frac{y^2}{2})$$
$$\times \left[\int_0^1 dz \frac{\phi_\pi(z)}{z\bar{z}} \int d\boldsymbol{k}^2 \frac{\boldsymbol{k}^2}{\boldsymbol{k}^2 + z\bar{z}Q^2} x f_{1T}^{\perp g}(x, \boldsymbol{k}^2) \right]^2$$

Leading contribution coming from gluon Sivers!

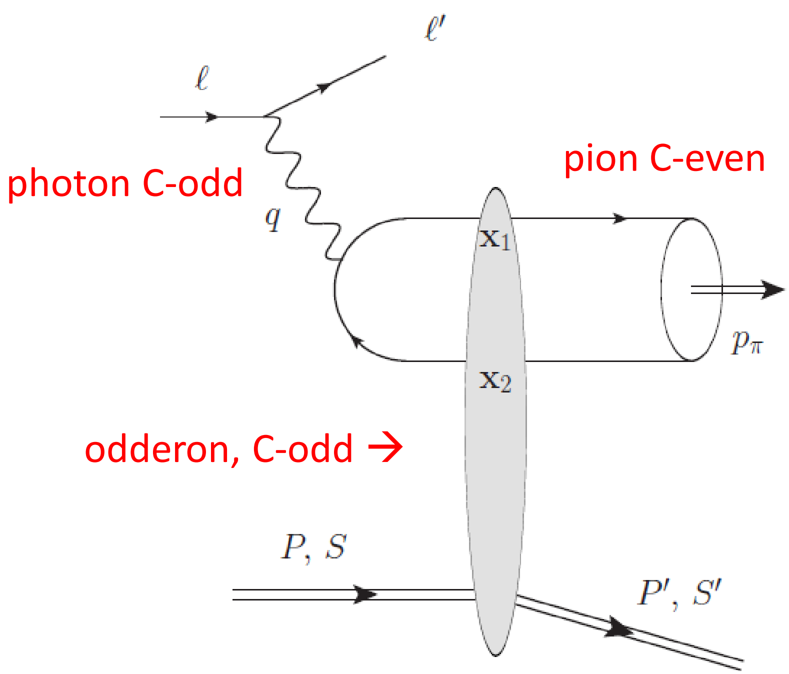
Connection to Odderon

Odderon: Predicted in the 70s as a C-odd counterpart of Pomeron

Experimentally elusive for decades. Finally found at the LHC? (TOTEM collaboration)

LHC Scientists Discover First Evidence of Particle Proposed Nearly 50 Years Ago

The odderon was first proposed in 1973, but actual evidence of its existence eluded scientists until now.



Spin-dependent odderon
=gluon Sivers at small-x

Zhou; 1308.5912

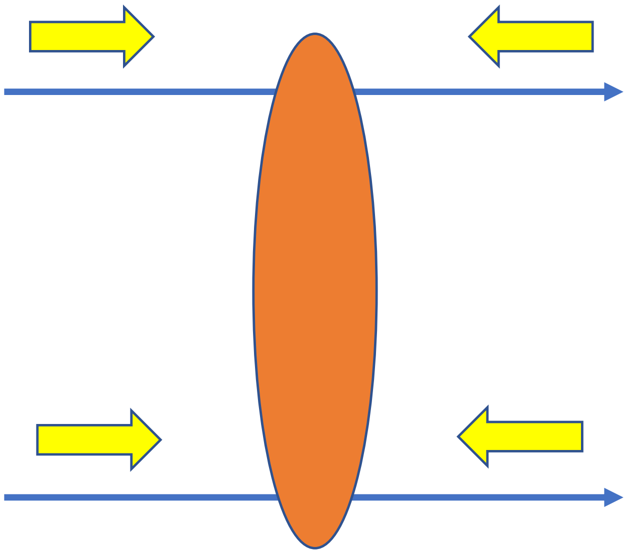
New connection between EIC and LHC

Gluon Sivers=Odderon in pp at the LHC

Hagiwara, YH, Pasechnik, Zhou; 2003.03680

Elastic pp scattering, unpolarized

$$\frac{d\sigma}{dt} = \sum_{S_1 S_2 S_3 S_4 = \pm} \frac{d\sigma_{S_1 S_2 \to S_3 S_4}}{dt}$$



Double helicity-flip from gluon Sivers=Odderon

$$\left.\frac{d\sigma}{dt}\right|_{t=0} = \frac{\sigma_{\rm tot}^2}{16\pi}(1+\rho^2+2|r_2|^2)$$

rho-parameter (spin non-flip)

$$\rho(s,t) = \frac{{\rm Re}T(s,t)}{{\rm Im}T(s,t)}$$

Conclusions

- Let’s get 5 dimensional. Even richer physics than TMD and GPD combined. Not discussed in the white paper.
- Wigner/GTMD measurable in ep, pp, pA, including the elliptic part and spin-dependent part (connection to OAM). Many interesting applications especially at small-x.
- Need more foundational works. Proper definition, factorization, evolution…