Pion screening mass at finite density

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Introduction

- Perturbation theory is valid only at large temperatures, owing to asymptotic freedom of QCD at high energy scales.
- At scales of our interest near the chiral phase transition, the perturbation theory fails as the coupling constant $\alpha_s > 1$.
- Thus, non-perturbative methods like lattice QCD need to be used.
- We will focus on finite temperature mesonic correlators, which are useful for calculating the mesonic energy eigenvalues using the spectral decomposition of the excitations.
- At finite temperatures, the temporal span is constrained as $N_\tau = 1/(aT)$ making the long distance behaviour for temporal correlator difficult to study. Instead, we calculate correlator propagating in spatial direction called screening correlator and extract screening mass from it.

Introduction

- Why are screening masses an important observable?

1. The inverse of a screening mass is the screening length, i.e., the spatial distance beyond which the effects of a test hadron are effectively screened. Thus, they provide important length scales of the system.
2. They give an idea about relevant degrees of freedom at high temperatures.
3. They are related to the pole masses via the same spectral function giving a handle on understanding it.
4. They can be used to locate the temperature where chiral and $U(1)_A$ symmetries are effectively restored.
5. They can be used to check the accuracy of the predications from the perturbation theory.

Introduction

- Correlator on lattice are calculated by introducing fermions on lattice. But discretizing fermions creates an issue called fermion doubling where you get unphysical contribution from additional fermions called doublers.
- To take care of this, we use what is called the staggered fermions.
- Staggered fermions resolve the problem of fermion doubling by effectively doubling the lattice spacing $a \to 2a$, reducing the Brillouin zone by half and thus eliminating the contributions of the doublers.
- Lattice is divided into hypercubes of unit length. Placing a single fermionic degree of freedom on each point in a hypercube gives us $2^4 = 16$ fermionic degrees of freedom. Using these 16 degrees of freedom, we get 4 flavors of Dirac fields which we call tastes of staggered fermions.

Introduction

- The partition function, after integrating the fermionic degrees of freedom, at finite temperature T and chemical potential $\boldsymbol{\mu}$:

$$\mathcal{Z}(T, \boldsymbol{\mu}) = \int \mathcal{D}U \, \Delta(T, \boldsymbol{\mu}) \, e^{-S_G(T)},$$

where U are the gauge links, S_G is the gauge action, and $\Delta(T, \boldsymbol{\mu})$ is the fermion determinant given by

$$\Delta(T, \boldsymbol{\mu}) = \prod_{f=u,d,s} \left[\det M_f(m_f, T, \mu_f) \right]^{1/4},$$

where $M_f(m_f, T, \mu_f)$ is the staggered fermion matrix for flavor f.

- The expectation value of operator $\mathcal{O}$ at finite $\boldsymbol{\mu}$ and T:

$$\langle \mathcal{O}(T, \boldsymbol{\mu}) \rangle = \frac{1}{\mathcal{Z}(T, \boldsymbol{\mu})} \int \mathcal{D}U \, e^{-S_G(T)} \, \mathcal{O}(T, \boldsymbol{\mu}) \, \Delta(T, \boldsymbol{\mu}).$$

Introduction

- The integration over the gauge links U is carried out using importance sampling.
- The current lattice QCD techniques available limit the simulation to only zero chemical potential as adding chemical potential makes the action complex making it difficult to use importance sampling. This issue is called the sign problem.
- Our analysis bypasses this difficulty in our analysis by expanding the screening correlator in a Taylor series of chemical potential and thus obtaining a first order correction to the screening mass.
- We will only consider isoscalar chemical potential $\mu_S \equiv \mu_u = \mu_d$.

Screening correlator

- Staggered meson operator $\mathcal{M}(x)$ is used to obtain the screening correlator.
- 4 tastes of staggered quarks give $4 \times 4 = 16$ meson tastes.
- Different taste mesons have different masses which become degenerate in the continuum limit.
- We consider the meson taste having a local operator for which the meson operator can be written as

$$\mathcal{M}_{\Gamma}(x) = \phi_{\Gamma}(x)\bar{\chi}_i(x)\chi_j(x)$$

  where $\bar{\chi}_i$ and χ_j are staggered quark fields with flavor indices i and j respectively, $\boldsymbol{x} = (x, y, z, \tau)$, and $\phi_{\Gamma}(x)$ is a phase factor that depends upon the spin and taste quantum numbers of the meson.
- We only consider the staggered light pseudoscalar meson, for which $(i, j) = (u, d)$ and $\phi(\boldsymbol{x}) = 1$ for all $\boldsymbol{x}$.

Screening correlator

- The meson correlator $\mathcal{G}(\boldsymbol{x}, T, \boldsymbol{\mu})$ is the two-point function of the corresponding meson operator:

$$\begin{aligned}\mathcal{G}(\boldsymbol{x}, T, \boldsymbol{\mu}) &\equiv \langle \mathcal{M}(\boldsymbol{x})\overline{\mathcal{M}}(0)\rangle \\ &= \langle \mathrm{Tr}\big[P_u(\boldsymbol{x}, 0, \mu_u)P_d^{\dagger}(\boldsymbol{x}, 0, -\mu_d)\big]\rangle\end{aligned}$$

where the trace is over the color indices and $P_k(\boldsymbol{x}, \boldsymbol{y}, \mu_k)$ is the staggered quark propagator for the kth flavor.

- For isoscalar chemical potential, we define $G(\boldsymbol{x}, T, \mu_S)$:

$$G(\boldsymbol{x}, T, \mu_S) \equiv \mathrm{Tr}\big[P(\boldsymbol{x}, 0, \mu_S)P^{\dagger}(\boldsymbol{x}, 0, -\mu_S)\big]$$

Screening correlator

- The correlator now is given as:

$$\mathcal{G}(\boldsymbol{x}, T, \mu_S) = \frac{\int \mathcal{D}U \, e^{-S_G(T)} \, G(\boldsymbol{x}, T, \mu_S) \, \Delta(T, \mu_S)}{\int \mathcal{D}U \, e^{-S_G(T)} \, \Delta(T, \mu_S)}.$$

- Expanding $\mathcal{G}(\boldsymbol{x}, T, \mu_S)$ in a Taylor series in μ_S/T:

$$\mathcal{G}(\boldsymbol{x}, T, \mu_S) = \sum_{k=0}^{\infty} \frac{\mathcal{G}^{(k)}(\boldsymbol{x}, T)}{k!} \left(\frac{\mu_S}{T}\right)^k,$$

where the Taylor coefficients $\mathcal{G}^{(k)}(\boldsymbol{x}, T)$ are evaluated at $\mu_S = 0$. By differentiating w.r.t. $\hat{\mu}_S = \mu_S/T$, we find that the first three Taylor coefficients are given by

$$\begin{aligned} \mathcal{G}^{(0)}(\boldsymbol{x}, T) &= \langle G \rangle, \\ \mathcal{G}^{(1)}(\boldsymbol{x}, T) &= \langle G' \rangle, \text{ and} \\ \mathcal{G}^{(2)}(\boldsymbol{x}, T) &= \left\langle G'' + 2G' \frac{\Delta'}{\Delta} + G \frac{\Delta''}{\Delta} \right\rangle - \langle G \rangle \left\langle \frac{\Delta''}{\Delta} \right\rangle, \end{aligned} \tag{1}$$

Screening correlator

- The screening correlator $C(z, T, \mu_S)$ is obtained from $\mathcal{G}(\boldsymbol{x}, T, \mu_S)$ by summing over x, y and τ i.e.

$$C(z, T, \mu_S) = \frac{1}{N_\tau N_\sigma^2} \sum_{x,y,\tau} \mathcal{G}(\boldsymbol{x}, T, \mu_S).$$

- Its Taylor expansion follows simply:

$$C(z, T, \mu_S) = \sum_{k=0}^{\infty} \frac{C^{(k)}(z, T)}{k!} \left(\frac{\mu_S}{T}\right)^k,$$

$$C^{(k)}(z, T) = \frac{1}{N_\tau N_\sigma^2} \sum_{x,y,\tau} \mathcal{G}^{(k)}(\boldsymbol{x}, T).$$

Free theory

- Free theory correlator equation (JHEP 2007(03): 022) for $zT \gg 1$

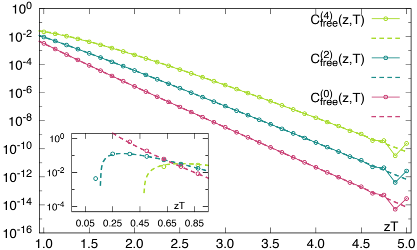
$$\frac{C_{\text{free}}(z,T,\mu_S)}{T^3} = \frac{3}{2}\frac{e^{-2\pi zT}}{zT}\left[\left(1+\frac{1}{2\pi zT}\right)\cos(2z\mu_S) + \frac{\mu_S}{\pi T}\sin(2z\mu_S)\right] + \mathcal{O}\left(e^{-4\pi zT}\right).$$

- By differentiating w.r.t. $\hat{\mu_S}$, we obtain the first few Taylor coefficients as (with $\hat{z} \equiv zT$)

$$\frac{C^{(0)}_{\text{free}}(z,T)}{T^3} = \frac{3e^{-2\pi\hat{z}}}{2\hat{z}}\left(1+\frac{1}{2\pi\hat{z}}\right), \qquad \frac{C^{(2)}_{\text{free}}(z,T)}{T^3} = 6\hat{z}e^{-2\pi\hat{z}}\left(\frac{1}{2\pi\hat{z}}-1\right),$$

$$\frac{C^{(4)}_{\text{free}}(z,T)}{T^3} = 24\hat{z}^3 e^{-2\pi\hat{z}}\left(1-\frac{3}{2\pi\hat{z}}\right), \qquad C^{(1)}_{\text{free}}(z,T) = C^{(3)}_{\text{free}}(z,T) = 0.$$

Free theory



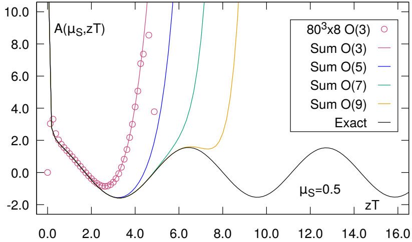


Figure: (left) Free theory Correlator with its derivatives for $80^3 \times 8$ lattice along with theoretical curve. (right) Summed $A(\mu_S, zT) = \hat{z}C(\mu_S, zT)e^{2\pi\hat{z}}$ for $\mu = 0.5$ along with the exact expression.

Free theory

- We rewrite the correlator in the familiar form $C = A\,e^{-Mz}$, now having complex amplitude and complex screening mass

$$
\begin{aligned}
\frac{C_{\rm free}(z,T,\mu_S)}{T^3} &= \frac{3}{2}\frac{e^{-2\pi zT}}{zT}\left[\left(1+\frac{1}{2\pi zT}\right)\cos(2z\mu_S)+\frac{\mu_S}{\pi T}\sin(2z\mu_S)\right] \\
&= \mathrm{Re}\Big[A(\mu_S)e^{-zM(\mu_S)}\Big], \\
&= e^{-zM_R(\mu_S)}\Big[A_R(\mu_S)\cos(zM_I(\mu_S)+A_I(\mu_S)\sin(zM_I(\mu_S)\Big], \\
M(\mu_S) &= 2\pi T+2i\mu_S \equiv M_R(\mu_S)+iM_I(\mu_S), \\
A(\mu_S) &= \frac{3}{2zT}\left(1+\frac{1}{2\pi zT}\right)\left(1-i\frac{\mu_S}{\pi T}\right) \equiv A_R(\mu_S)-iA_I(\mu_S).
\end{aligned}
$$

- Note, M_R and A_R are even functions of μ_S, while M_I and A_I are odd functions of μ_S.

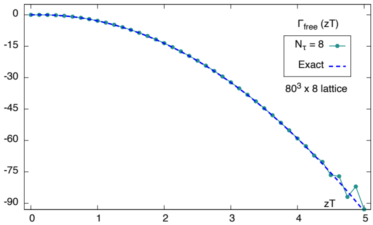
Free theory

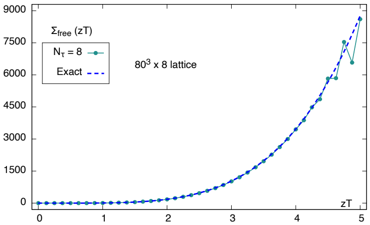
The exponential factor of the $C^{(n)}$ cancel out if we define the ratios

$$\Gamma(\hat{z}) \equiv \frac{C^{(2)}(z,T)}{C^{(0)}(z,T)} \quad \text{and} \quad \Sigma(\hat{z}) \equiv \frac{C^{(4)}(z,T)}{C^{(0)}(z,T)},$$

$$\begin{aligned}\Gamma_{\text{free}}(\hat{z}) &= -4\hat{z}^2\left(1-\frac{1}{2\pi\hat{z}}\right)\Big/\left(1+\frac{1}{2\pi\hat{z}}\right),\\ &= -4\hat{z}^2+\frac{4\hat{z}}{\pi}-\frac{2}{\pi^2}+\mathcal{O}\left(\hat{z}^{-1}\right),\\ &\equiv \alpha_2\hat{z}^2+\alpha_1\hat{z}+\alpha_0,\end{aligned}$$

$$\begin{aligned}\Sigma_{\text{free}}(\hat{z}) &= 16\hat{z}^4\left(1-\frac{3}{2\pi\hat{z}}\right)\Big/\left(1+\frac{1}{2\pi\hat{z}}\right)\\ &= 16\hat{z}^4-\frac{32\hat{z}^3}{\pi}+\frac{16\hat{z}^2}{\pi^2}+\mathcal{O}\left(\hat{z}\right),\\ &\equiv \beta_4\hat{z}^4+\beta_3\hat{z}^3+\beta_2\hat{z}^2.\end{aligned}$$

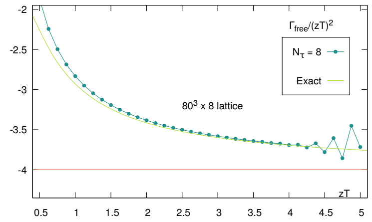


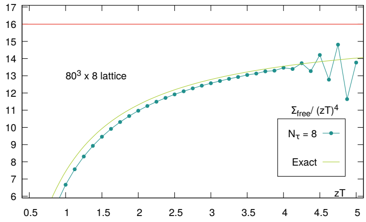


Free theory

- To compare the approach of Γ and Σ for the large $\hat{z}$ limit, we look at $\Gamma/\hat{z}^2$ and $\Sigma/\hat{z}^4$

$$\frac{\Gamma_{\text{free}}}{\hat{z}^2} = -4 + \frac{4}{\pi\hat{z}} - \frac{2}{\pi^2\hat{z}^2}\,, \qquad \frac{\Sigma_{\text{free}}}{\hat{z}^4} = 16 - \frac{32}{\pi\hat{z}} + \frac{16}{\pi^2\hat{z}^2}\,.$$





- The curves approach the value of the constant term asymptotically as the contribution from other terms decrease at large $\hat{z}$.
- $\Gamma/\hat{z}^2$ approach the negative constant value of -4 from above while $\Sigma/\hat{z}^4$ approach a positive constant value of 16 from below.

Finite temperature

- Like free theory, we postulate that the finite-temperature screening correlator can be written with complex screening mass and screening amplitude as

$$\frac{C(z,T,\mu_S)}{T^3} = e^{-zM_R(\mu_S)}\Big[A_R(\mu_S)\cos\left(zM_I(\mu_S)\right) + A_I(\mu_S)\sin\left(zM_I(\mu_S)\right)\Big].$$

- M_R and A_R are even functions of μ_S, while M_I and A_I are odd functions of μ_S. This follows from the hermitian conjugate of $G(\boldsymbol{x},T,\mu_S) = \mathrm{Tr}\big[P(\boldsymbol{x},0,\mu_S)P^\dagger(\boldsymbol{x},0,-\mu_S)\big]$

$$\begin{aligned} G(\boldsymbol{x},T,\mu_S)^* &= G(\boldsymbol{x},T,-\mu_S) \\ \implies C(z,T,\mu_S)^* &= C(z,T,-\mu_S) \end{aligned}$$

- Hence, odd (even) derivatives of M_R and A_R (of M_I and A_I) vanish.
- Next, like in free theory we obtain Γ and Σ by taking derivative of the correlator expression.

Finite temperature

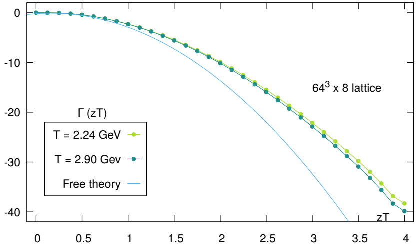
- Expression for Γ and Σ are

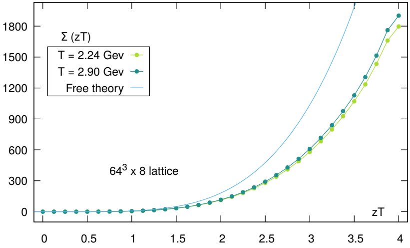
$$\begin{aligned}\Gamma(z) &= -z^2\left(M_I'\right)^2 + z\left[2\frac{A_I'}{A_R}M_I' - M_R''\right] + \frac{A_R''}{A_R},\\ &\equiv \alpha_2\hat{z}^2 + \alpha_1\hat{z} + \alpha_0,\\ \Sigma(z) &= z^4\left(M_I'\right)^4 + z^3\left[6M_R''M_I'^2 - 4\frac{A_I'}{A_R}M_I'^3\right] + \mathcal{O}(z^2),\\ &\equiv \beta_4\hat{z}^4 + \beta_3\hat{z}^3 + \beta_2\hat{z}^2 + \mathcal{O}(\hat{z})\end{aligned}$$

- Like free theory, Γ is quadratic in $\hat{z}$ and Σ are quartic in $\hat{z}$.
- The lowest order corrections M_I' and M_R'' to the screening mass can be obtained from the coefficients of these polynomials as

$$\hat{M}_I' = (-\alpha_2)^{1/2} = \beta_4^{1/4} \quad \text{and} \quad \hat{M}_R'' = \frac{1}{4}\left(2\alpha_1 - \frac{\beta_3}{\alpha_2}\right).$$

Finite temperature



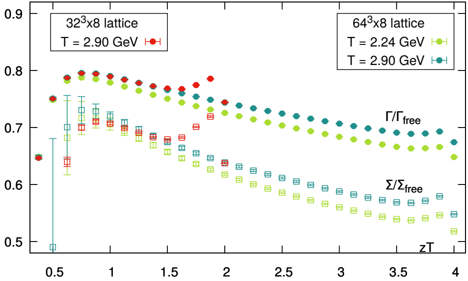


N_σ	β	T[GeV]	m_l	m_s	configurations
32	9.670	2.90	0.0001399	0.002798	12700
64	9.670	2.90	0.0001399	0.002798	6000
64	9.360	2.24	0.00018455	0.003691	6000

Table: The list of HISQ configurations used for the finite temperature. All the configurations used here have $N_\tau = 8$ with strange quark mass tuned to physical value.

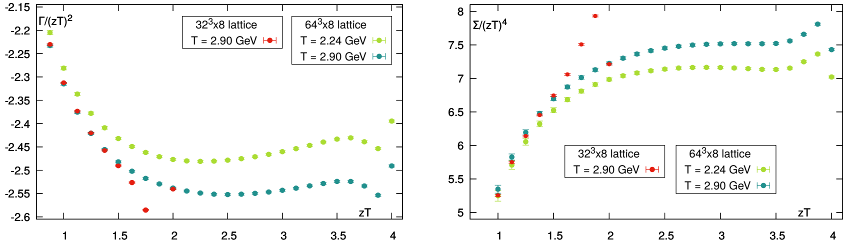
Finite temperature

$$\frac{\Gamma}{\Gamma_{\text{free}}} = \frac{\alpha_2 \hat{z}^2 + \alpha_1 \hat{z} + \alpha_0}{-4\hat{z}^2 + 4\hat{z}/\pi - 2/\pi^2}, \qquad \frac{\Sigma}{\Sigma_{\text{free}}} = \frac{\beta_4 \hat{z}^4 + \beta_3 \hat{z}^3 + \beta_2 \hat{z}^2}{16\hat{z}^4 - 32\hat{z}^3/\pi + 16\hat{z}^2/\pi^2}.$$



- For large $\hat{z}$, both curves seem to slowly approach the plateauing values corresponding to the ratio of the highest polynomial coefficients.
- The data points curve upwards near $\hat{z} = N_\sigma/(2N_\tau)$ suggesting significant boundary effects (except the $\hat{z} = N_\sigma/(2N_\tau)$ point itself).

Finite temperature



- The boundary effects are clearly visible with $\hat{z} = N_\sigma/(2N_\tau)$ point unaffected.
- Like free theory curves, these are expected to approach the value of the constant term asymptotically at large $\hat{z}$.
- Unlike the free theory $\Gamma/\hat{z}^2$ $(\Sigma/\hat{z}^4)$ for the finite temperature curve encounters minima (maxima) at $\hat{z} = \hat{z}_\Gamma$ $(\hat{z} = \hat{z}_\Sigma)$ before approaching the asymptotic value from below (above).
- This difference in the large $\hat{z}$ approaching behaviour enforces the signs of α_1 (β_3) and α_0 (β_2) to be different from the free theory value.
- We also note that these minima and maxima shift to larger $\hat{z}$ with increasing temperature and eventually vanishing for free theory.

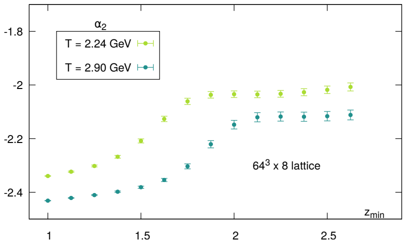
Evaluating polynomial coefficients:

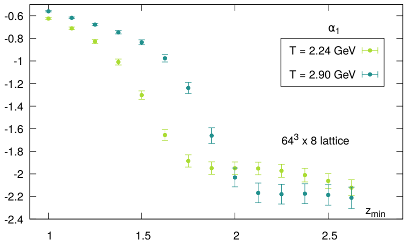
- Instead of keeping all the three coefficients as fit parameters, the lowest order coefficients $\alpha_0(\beta_2)$ were estimated in terms of $\alpha_1(\beta_3)$ using the location of the extremum points $\hat{z}_\Gamma(\hat{z}_\Sigma)$ discussed earlier.

$$\alpha_0 = -\frac{\alpha_1 \hat{z}_\Gamma}{2}\,, \qquad \beta_2 = -\frac{\beta_3 \hat{z}_\Sigma}{2}\,.$$

- The polynomial fits obtained for various windows between $\hat{z}_{\mathrm{min}}$ and $\hat{z}_{\mathrm{max}}$ show very little variation for changing $\hat{z}_{\mathrm{max}}$. Thus, we fix the upper fit window to $\hat{z}_{\mathrm{max}} = 3.25$ and obtain a plateau for the fit coefficients by varying the $\hat{z}_{\mathrm{min}}$.

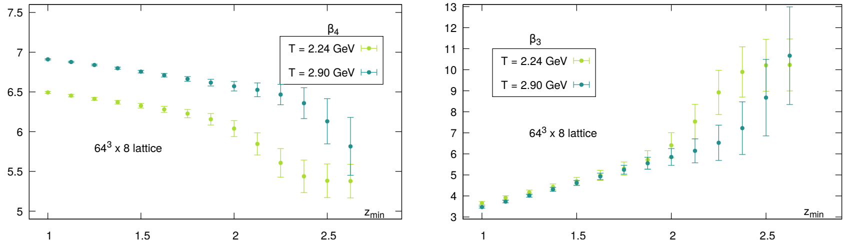
Finite temperature





Temp T	$\hat{z}_\Gamma$	α_2	α_1
2.24 GeV	2.269(23)	-2.034(13)	-1.955(57)
2.90 GeV	2.500(16)	-2.117(18)	-2.175(87)
Free theory		-4	10.2

Finite temperature



Temp T	$\hat{z}_\Sigma$	β_4	β_3
2.24 GeV	2.860(50)	5.383(218)	10.091(1255)
2.90 GeV	3.125(25)	5.815(365)	10.667(2321)
Free theory		16	$-32/\pi \approx -10.186$

Finite temperature

Temp T	M''_R	M'_I
2.24 GeV	0.263(169)	1.426(5)
2.90 GeV	0.172(328)	1.455(6)
Free theory	0	2

- The α's reach a plateau value at a smaller $\hat{z}$ and with smaller error bars when compared with the β's.
- The plateau values are reached at a smaller $\hat{z}$ for lower temperatures.
- α_2 and β_4 have the same sign for finite temperature and for free theory.
- The second highest coefficients α_1 and β_3 have different sign for free theory and finite temperatures.
- $\hat{M}''_R$ and $\hat{M}'_I$ also approach the free theory value with increasing temperature.
- $\hat{M}''_R$ seem to have a positive value at finite temperatures.

Conclusion and future work

- We verified the free theory expression for screening correlator derived analytically at finite isoscalar chemical potential by looking at its derivatives on lattice.
- We derived a new procedure for calculating the screening masses at small finite chemical potential using symmetry arguments.
- We calculated $\hat{M}_R''$ and $\hat{M}_I'$ at two temperatures.
- To get further accurate values, we need to go to larger lattices and measure at lower temperature where the noise is larger.
- We need to understand the behaviour at non-zero isovector chemical potential.