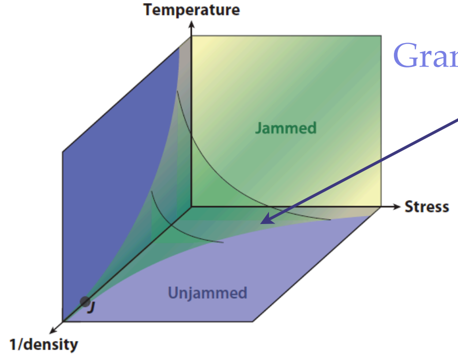
THE JAMMING TRANSITION IN GRANULAR MATERIALS

Liu & Nagel

Annu. Rev. Condens. Matter Phys. 2010. 1:347–69



Granular solids live on this plane

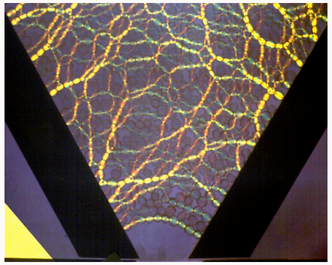
J-Point is a critical point

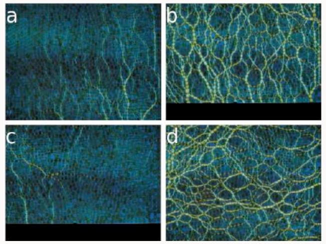
Mitch Mailman, Max Bi

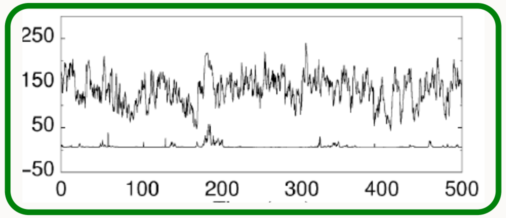
Bob Behringer & Jie Zhang, Duke University

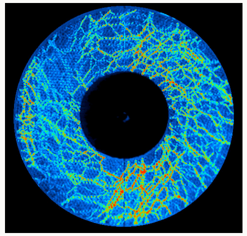
Anomalous Fluctuations

Dry grains: purely repulsive interactions







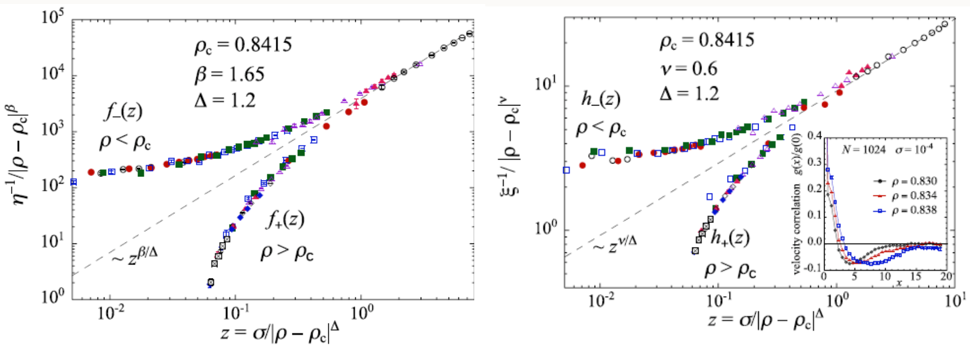


stress vs time

2-POINT CORRELATIONS NEAR J-POINT

- UNJAMMED SIDE: (Teitel & Olsson and Heussinger & Barrat)
  - PRL **102,** 218303 (2009): correlations in non-affine motions with exponent 0.8-1.0
  - PRL **99** 178001 (2007): correlations in transverse velocities under shear with exponent 0.6

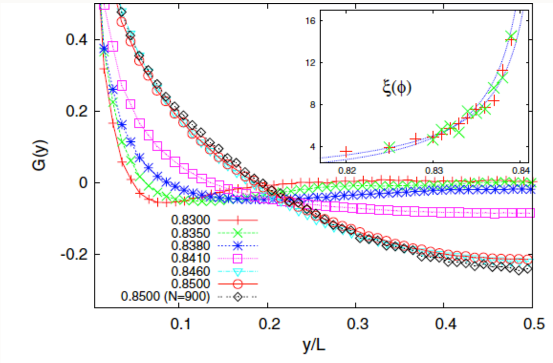
$$\xi \sim \delta\phi^{-\upsilon}$$



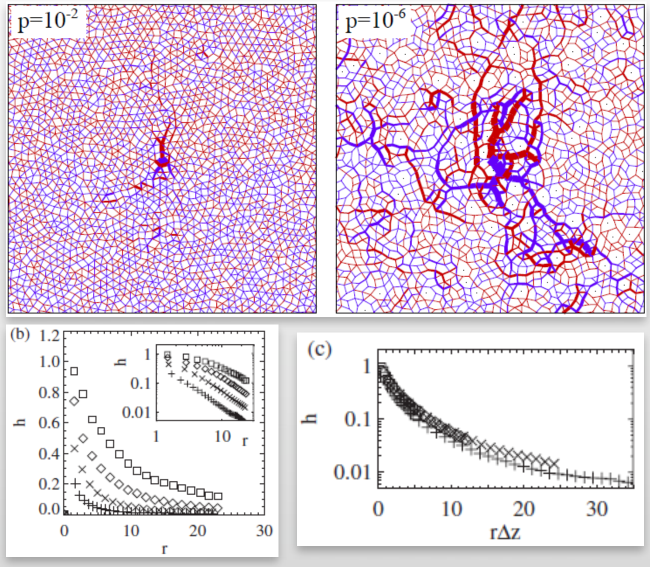
Emphasize no 2-point correlation function; Using "equal time tranverse velocity correlations," transverse to shear flow.

CORRELATIONS NEAR J-POINT

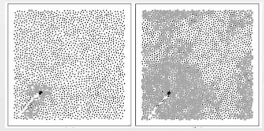
- JAMMED SIDE:
- Heussinger and Barrat, PRL **102,** 218303 (2009)
- Lois G. et al, PRE 80 (2009) 060303(R): stress fluctuation correlations are power laws.



MEASURING LENGTH SCALES FROM RESPONSE



A growing length scale is observed directly as a result of perturbations in simulation.



J.A. Drocco et al., PRL **95** 088001 (2005)

l^* is verified in scaling of force fluctuation measurements. Not a direct measurement.

Ellenbroek W. G. et al, PRL 97 (2006) 258001

h is a measure of the rms fluctuations of force magnitude due to a point inflation. ; after this, outline the talk. Say no correlation function.

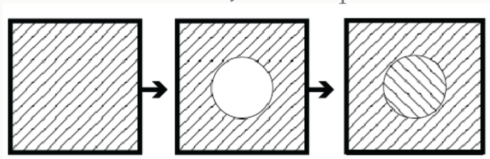
POINT-TO-SET CORRELATIONS

Probes boundary effects:

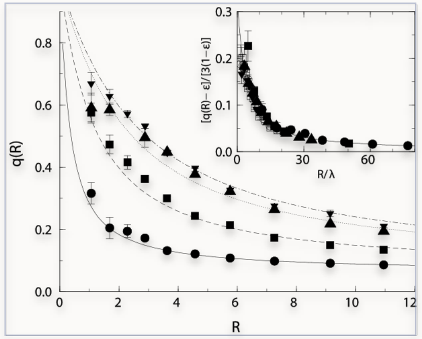
near a critical point, effects of boundaries should become long-ranged

Random-First-Order Theory: Entropic droplets

Lennard-Jones Liquids

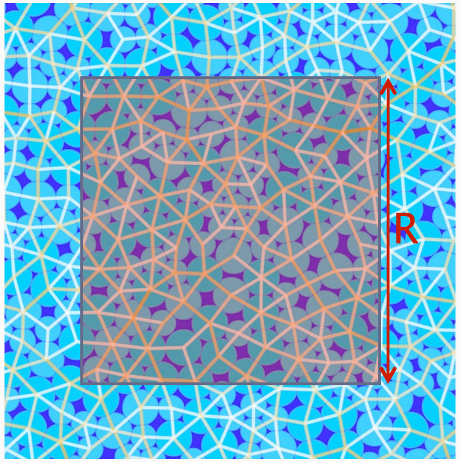


A. Cavagna et al., PRL 98 (2007) 187801,
A. Montanari and G. Semerjian, J. Stat. Phys. 125 (2006)
G. Biroli et al Nature Physics, 4, 771, (2008)



Text: Probes boundary effects near any critical point.

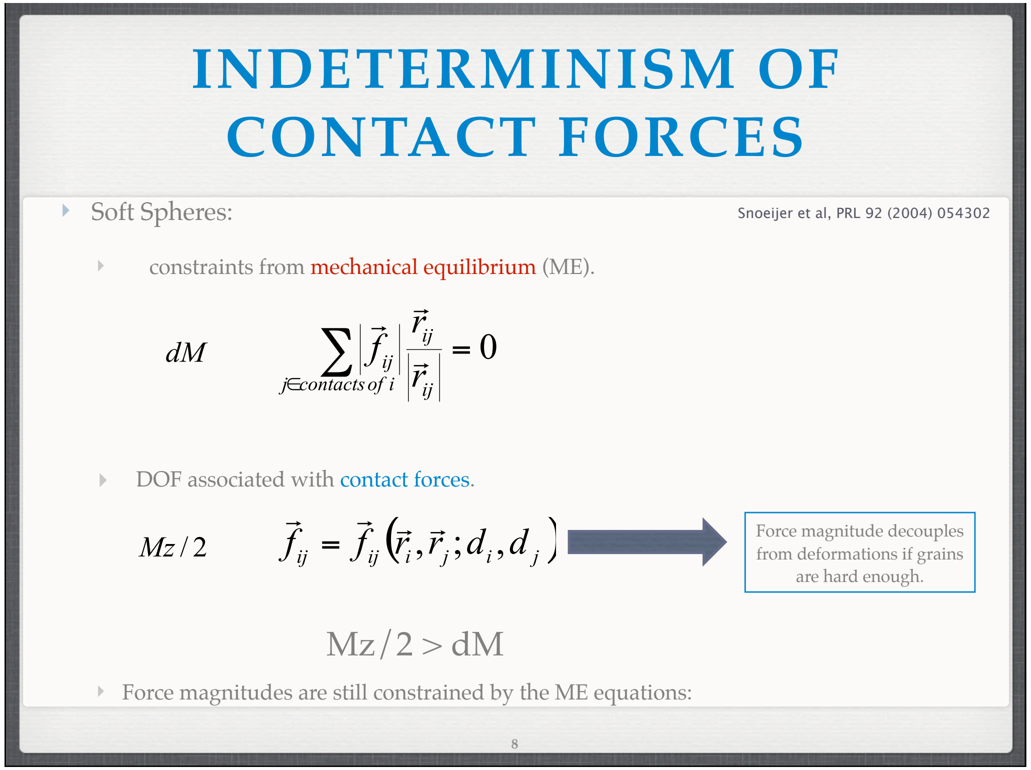
FORMULATION OF PTS FOR GRANULAR SYSTEMS



- Keep grain configuration fixed outside of B(R).
- Find another configuration in ME by only reconfiguring the interior.
- Generate geometries using quench protocol

Force network image originated at:http://jamming.research.yale.edu/multimedia/soft/gallery.htmlR

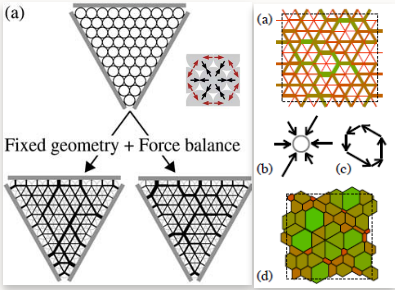
Just say: this is very difficult in practice. No thermalization to aid equilibration w/ boundary. Instead, there is a simple model that helps out (FNE).



Mention frictionless, disordered, soft spheres for jammed side. Talk only about the soft sphere, overcompressed inequality. Write the scale associated with the decoupling of forces and coordinates.

Can say, if asked, the relevant parameter is <f>/<rij><drij/dFij> is small but not zero.

FORCE NETWORK ENSEMBLE



- Overcompressed packings are underdetermined (hyperstatic) w.r.t. contact force variables:
- Matrix equation for ME

$$z \geq 2d$$

$$A\vec{f} = P$$

$$P_1 \ldots P_{dM} = 0$$

$$P_{dM+1} = P_0$$

- For a single (underdetermined) packing geometry, multiple force configurations satisfy ME. Small deformations lead to large changes in force magnitude.

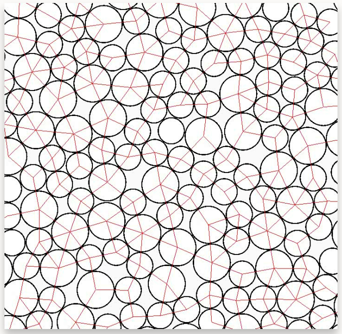
- Large fluctuation in contact forces. Reproduced qualitatively in FNE.

Tighe B. et al, Soft Matter, 2908-2917 (2010), McNamara S and Herrmann H, PRE 70 (2004) 061303

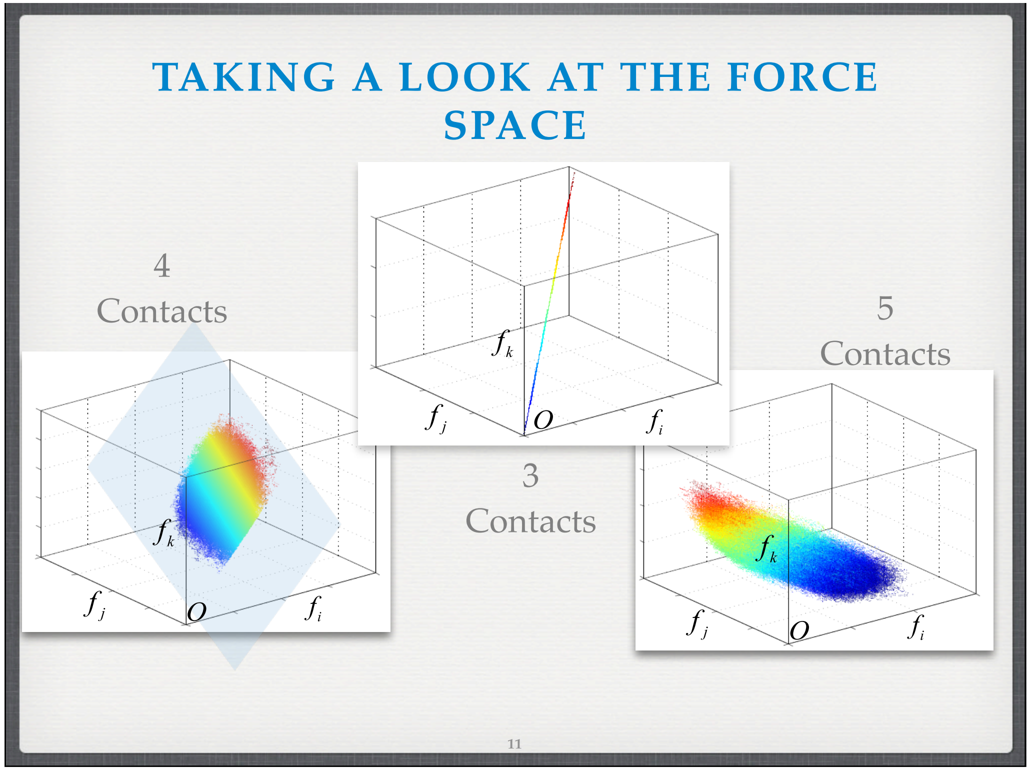
FORCE NETWORK ENSEMBLE FOR AMORPHOUS GEOMETRIES

- Null vectors generalize wheel move: $A\hat{g} = 0$
- Random walk with reflecting BC:

$$\vec{f} = \vec{f}(P_0) + \sum_{i=1}^{\delta z} c_i \hat{g}_i \qquad \vec{f} \geq 0$$



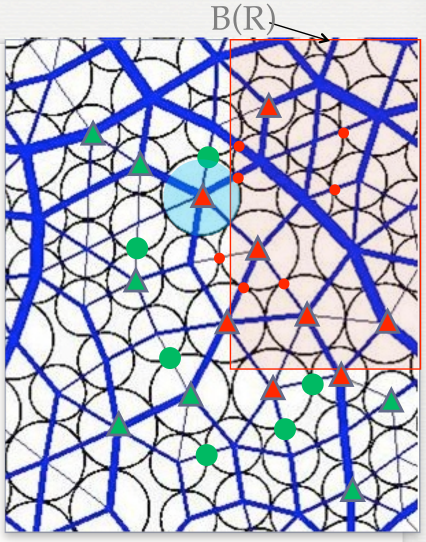
Random walk is in a continuous space. ci's are chosen from a uniform distribution [-c;c]; The c's are chosen to be of order <f> and were adjusted early on for efficiency.



Put in C= f*f and explain that the PTS is the overlap of two of the points here.

BOUNDARY CONDITIONS AND THE CONSTRUCTION OF PTS

- Begin with static packing and $\vec{f}_0$
- PTS is the overlap with an equivalent force network
- Freeze force variables outside of boundary B(R): $C(R) = \hat{f}_0 \cdot \hat{f}(R)$

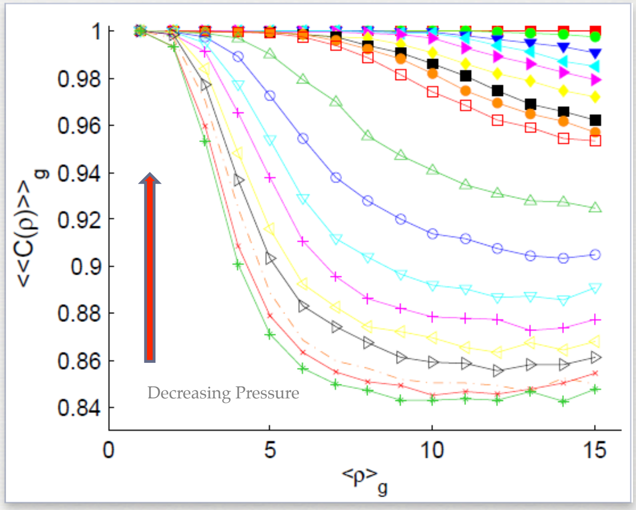


▲ Grains contribute 2 ME constraints.
▲ No additional constraints.
• Fluctuating DOF.
● Frozen DOF.

Put circles/triangles next to equations to illustrate where they come from. Get rid of geometry matrix. Put it in random walk slide.

RESULTS FOR PTS

- 900 grains, 2D, bidisperse.
- Packing fraction from 10^{-3} to 10^{-1}.
- 40 packing geometries.



BULK-SURFACE ARGUMENT

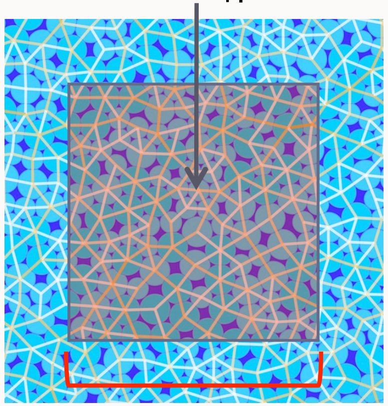
- BULK-SURFACE ARGUMENT
  - Nullity is the excess contacts

$$\delta n(\rho) = \alpha\phi\rho^2\delta z - \beta\sqrt{\phi}\rho$$

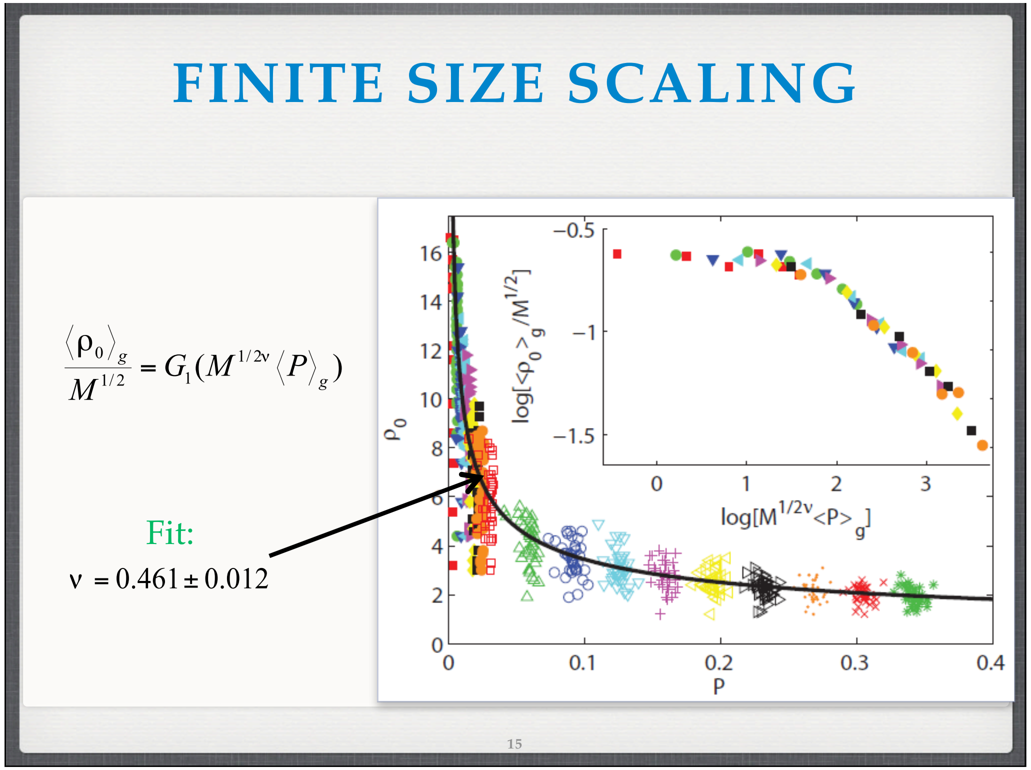
  - Identify root:

$$\rho_0 \propto \frac{1}{\sqrt{\phi}\delta z} \propto P^{-1/2}$$

$\sim M \propto \phi\rho^2$



$\sim \sqrt{M} \propto \sqrt{\phi}\rho$

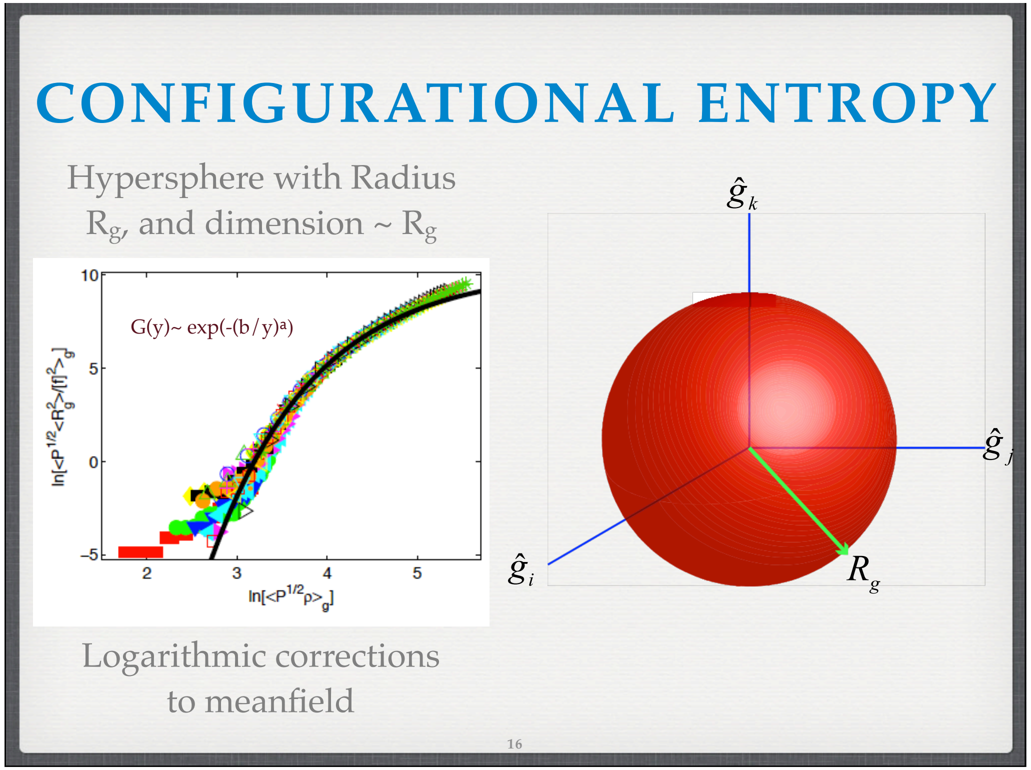


Emphasize

Discuss the complication with mixing ensembles.

Put a .5 fit on the main plot, make a .5 fss plot to see the difference.

WRITE FSS show a single, divering, dominant, scale.



This is an approximation. Free energy landscape is complex, but we're assuming 2 states, a and b. What we should have is a sum over all p(b) times the corresponding overlap of the reference state and b. Instead, what is done in RFOT is to put corrections to the free energy in the partition function associated with an effective surface tension resulting from pinning state b to the frozen boundary conditions. So p(b) then depends on how "different" b is from the reference state. For us, all states are equally good. The two-state picture actually makes more sense, and one expects a quick decay of C, since you're either in state a or you're not. What isn't captured is that some states are "closer" to each other in the sphere than others. C = 1/V * (1-q0) + q0. Come up with a better name than configurational entropy.

Write relation of solution space structure to PTS.

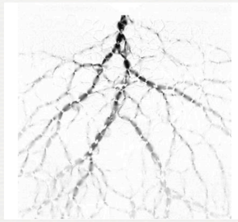
Write V is the volume of the solution space.

Re-title in terms of connection to C.

Define connected correlation function as well here.

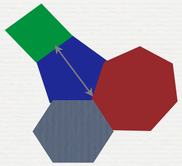
IMPLICATIONS OF GROWING LENGTH-SCALE

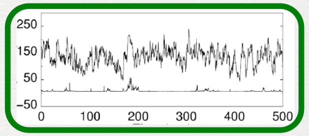
- PTS correlation length measures how far boundary forces propagate: diverges at zero pressure



- In granular packings, forces propagate (unlike elastic media)

- Related to isostaticity: marginally stable solid

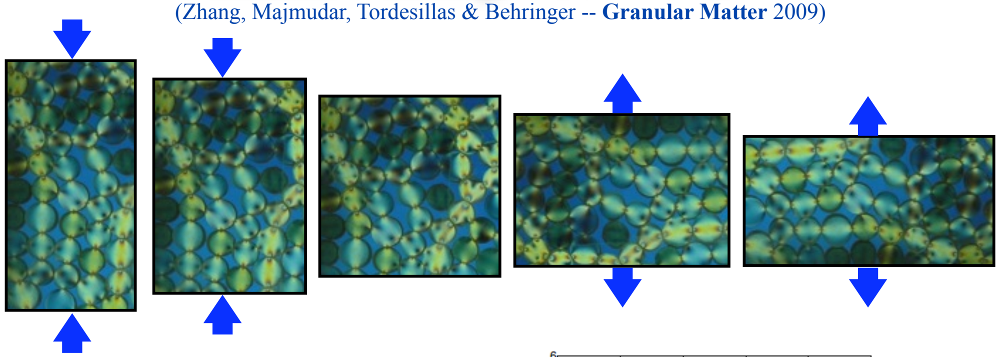


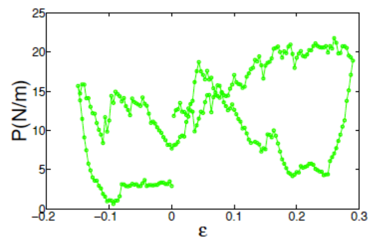


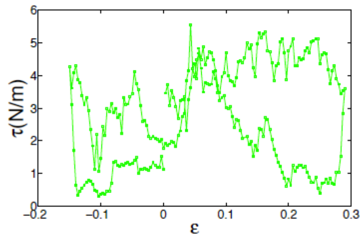
- Mosaic of “force tiles”. Tiles have to have a length ~ PTS correlation length

Jammed Packings below Point J

(Zhang, Majmudar, Tordesillas & Behringer -- **Granular Matter** 2009)



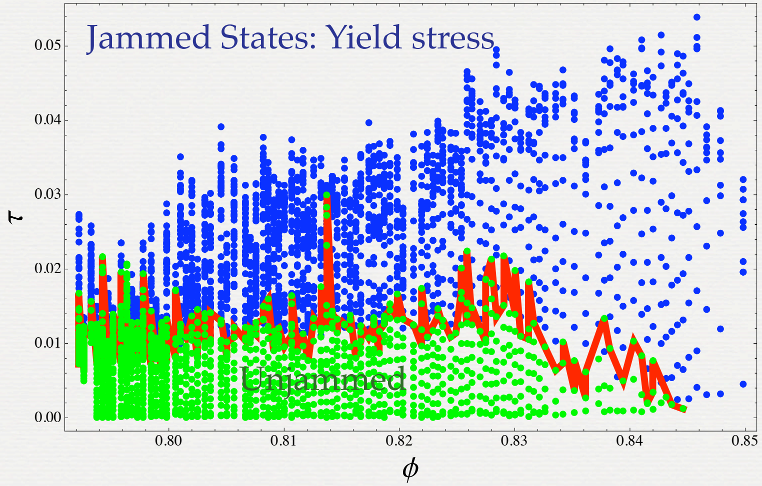




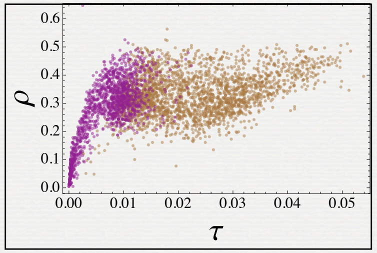
Quasistatic cyclic strain creating jammed packings

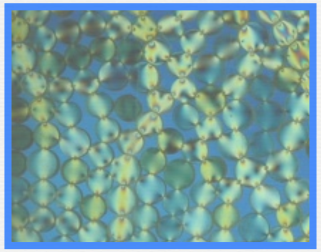
SHEARED, JAMMED STATES

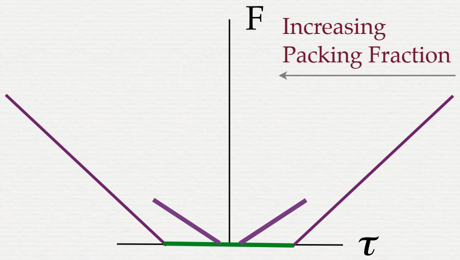
ZHANG & BEHRINGER STATES CREATED BY QUASISTATICALLY SHEARING PHOTOELASTIC DISKS



Anisotropy of Fabric







Define a free energy F: derivative wrt shear is fabric anisotropy

With increasing packing fraction:
- slope decreases
- excluded region shrinks
- At J-point, both go to zero

PTS correlation length increases