

# *Rare Kaon Decays*

*Rahul Sinha*

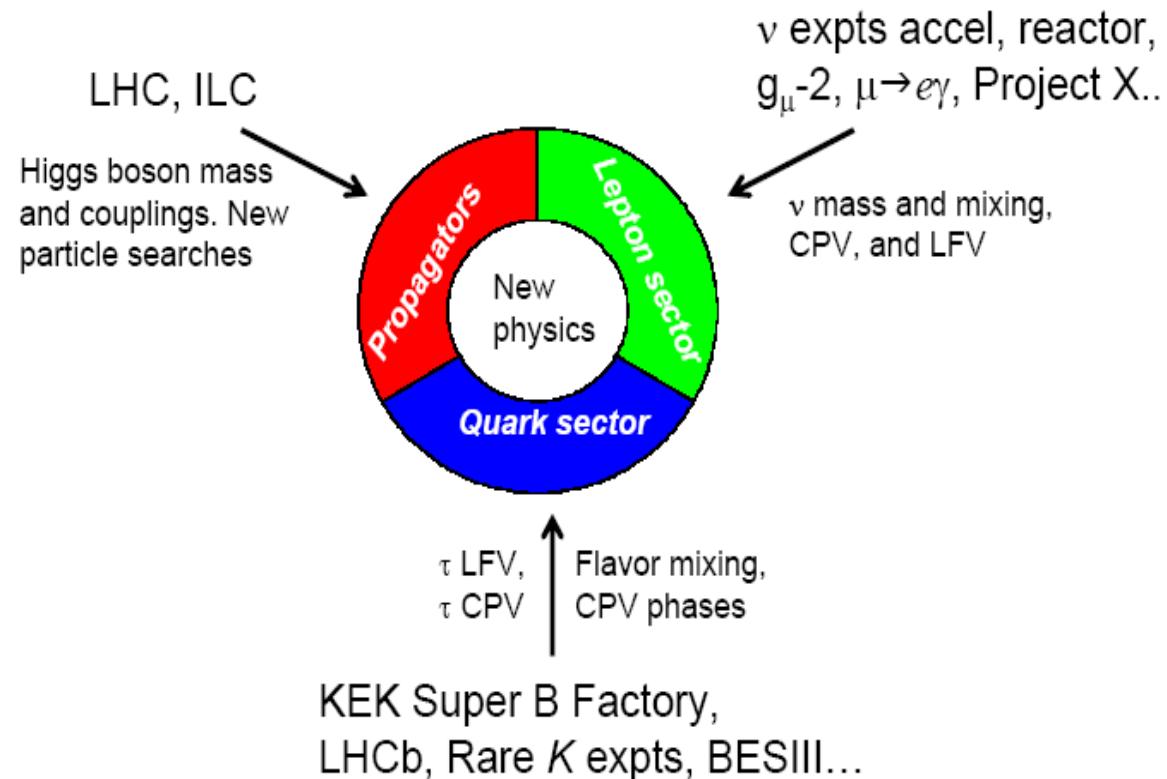
*The Institute of Mathematical Sciences,  
Chennai, INDIA*

## **Project X meeting**



**Jan13-14, 2011**





# *Why study Rare Kaon Decays ?*

- Search for explicit violation of Standard Model  
Lepton Flavor Violation
- Probe the flavor sector of the Standard Model  
FCNC
- Test fundamental symmetries  
CP, CPT
- Study the strong interactions at low energy  
Chiral Perturbation Theory, K structure
- Exploring lepton mass matrix  
Unique possibility of measuring double beta decay analogue for  $\mu$ .

# Rare decays: $K_L$ decays

$\Delta S = 1$

<i>Mode</i>	<i>Expt. value</i>	
• $\star K_L \rightarrow \pi^0 \nu \bar{\nu}$	$< 6.7 \times 10^{-8}$	$= (2.7 \pm 0.4) \times 10^{-11}$
• $K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$	$< 4.7 \times 10^{-5}$	
• $\star K_L \rightarrow \pi^+ \pi^- e^+ e^-$	$< (3.11 \pm 0.19) \times 10^{-7}$	
• $K_L \rightarrow \pi^0 \pi^0 e^+ e^-$	$< 6.6 \times 10^{-9}$	
• $K_L \rightarrow \mu^+ \mu^-$	$(6.84 \pm 0.11) \times 10^{-9}$	
• $K_L \rightarrow e^+ e^-$	$(9^{+6}_{-4}) \times 10^{-12}$	
• $K_L \rightarrow \pi^0 \mu^+ \mu^-$	$< 3.8 \times 10^{-10}$	
• $K_L \rightarrow \pi^0 e^+ e^-$	$< 2.8 \times 10^{-10}$	
• $K_L \rightarrow e^\pm \mu^\mp$	$< 4.7 \times 10^{-12}$	
• $K_L \rightarrow e^\pm e^\pm \mu^\mp \mu^\mp$	$< 4.12 \times 10^{-11}$	
• $K_L \rightarrow \pi^0 e^\pm \mu^\mp$	$< 7.6 \times 10^{-11}$	
• $K_L \rightarrow \pi^0 \pi^0 e^\pm \mu^\mp$	$< 1.7 \times 10^{-10}$	

↑  
CP  
↓

↑  
LF  
↓

# *K<sup>+</sup> decays*

$$\Delta S = 1$$

• ☼ $K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$(1.7 \pm 1.1) \times 10^{-10}$	$= (8.5 \pm 0.07) \times 10^{-11}$
• $K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu}$	$< 4.3 \times 10^{-5}$	
• $K^+ \rightarrow \pi^+ e^+ e^-$	$(3.00 \pm 0.09) \times 10^{-7}$	
• $K^+ \rightarrow \pi^+ \mu^+ \mu^-$	$(8.1 \pm 1.4) \times 10^{-8}$	
• $K^+ \rightarrow \mu^- \nu e^+ e^+$	$2.0 \times 10^{-8}$	
• $K^+ \rightarrow \pi^+ \mu^+ e^-$	$< 1.3 \times 10^{-11}$	Lepton Family Number
• $K^+ \rightarrow \pi^+ e^+ \mu^-$	$< 5.2 \times 10^{-10}$	
• $K^+ \rightarrow \pi^- \mu^+ e^+$	$< 5.0 \times 10^{-10}$	
• $K^+ \rightarrow \pi^- e^+ e^+$	$< 6.4 \times 10^{-10}$	Lepton number
• $K^+ \rightarrow \pi^- \mu^+ \mu^+$	$< 3.0 \times 10^{-9}$	
• $K^+ \rightarrow \pi^+ \gamma$	$< 2.3 \times 10^{-9}$	Angular momentum

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

## Theoretically clean mode

Buras:  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K^0 \rightarrow \pi^0 \nu \bar{\nu}$ , ratio  $x_d/x_s$  of  $B_d^0 - \overline{B_d^0}$  to  $B_s^0 - \overline{B_s^0}$  mixing and class of asymmetries in neutral  $B$  decays cleanest observables, being essentially free from hadronic uncertainties.

## Hadronic matrix element of the operator

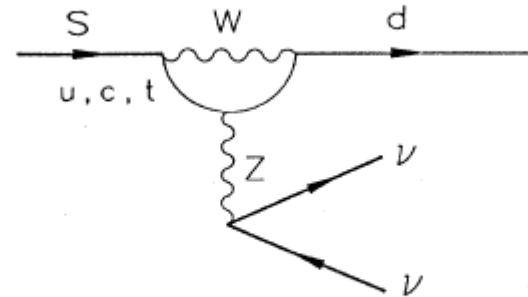
$$\bar{s}\gamma_\mu(1-\gamma_5)d \quad \bar{\nu}\gamma_\mu(1-\nu_5)\nu$$

can be measured in the leading decay  $K^+ \rightarrow \pi^0 e^+ \nu$

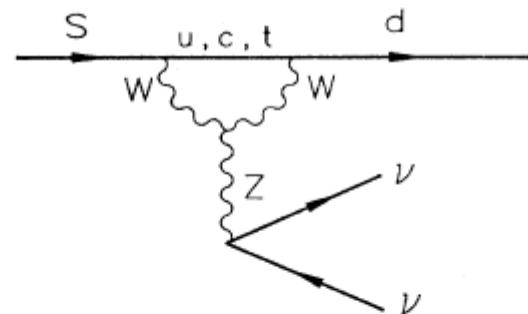
$$B_{SD}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{\kappa_+ \alpha^2 B(K_{e3})}{2\pi^2 \sin^4 \theta_W |V_{us}|^2} \sum_t |X_t \lambda_t + X_c \lambda_c|^2 = \\ 8.9 \times 10^{-11} A^4 [(\rho_0 - \bar{\rho})^2 + \bar{\eta}^2]$$



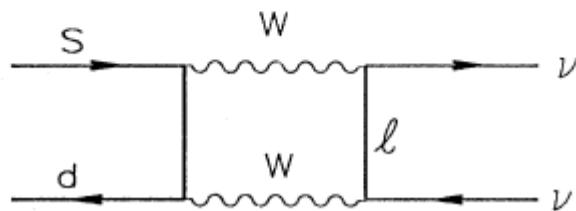
(a)



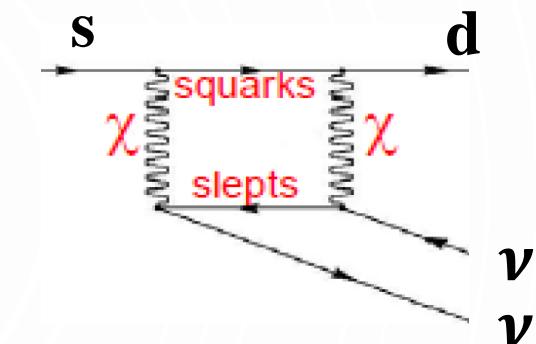
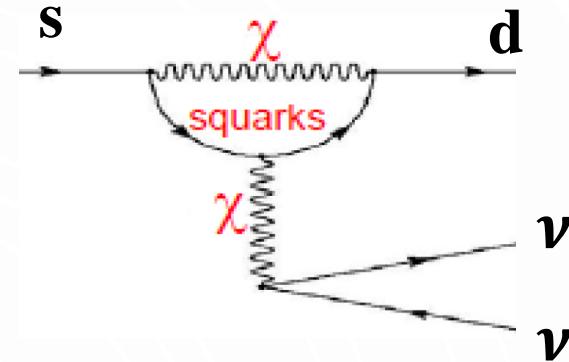
(b)



(c)



**SM contribution**



## SUSY Contribution

**Project X sensitive to 1000  
SM events.**

**BSM rates 10x SM rates.**

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

Buras:  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , ratio  $x_d/x_s$  of  $B_d^0 - \overline{B_d^0}$  to  $B_s^0 - \overline{B_s^0}$  mixing and class of asymmetries in neutral  $B$  decays cleanest observables, being essentially free from hadronic uncertainties.

- Purely CP-Violating (Littenberg, 1989)
- Totally dominated from t-quark
- Computed to NLO in QCD ( Buchalla, Buras, 1999)
- No long distance contribution SM  $\sim 3 \times 10^{-11}$

Backgrounds:

$$K_L \rightarrow 2\pi^0, \pi^0 e^+ e^-, \pi^0 \gamma\gamma$$

Difficult mode to measure



*Rates of  $K^0 \rightarrow \pi^0 \nu \bar{\nu}$  and  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  sensitive to other NP models also*

## The Randall-Sundrum (RS) idea

**Island Universes in Warped Space-Time**

According to string theory, our universe might consist of a three-dimensional "brane," embedded in higher dimensions. In the model developed by Lisa Randall and Roman Sundrum, gravity is much weaker on our brane than on another brane, separated from us by a fifth dimension. (Time is the unseen fourth dimension.)

GRAVITY BRANE (where gravity is concentrated)

Fifth dimension  
Space is warped by energy throughout five-dimensional space-time. As a result, gravity is much weaker on our brane.

Gravitons, which transmit gravity, are closed strings, which are not confined to either brane.

Warped space-time  
Because space-time is warped, things are exponentially bigger and lighter closer to our brane.

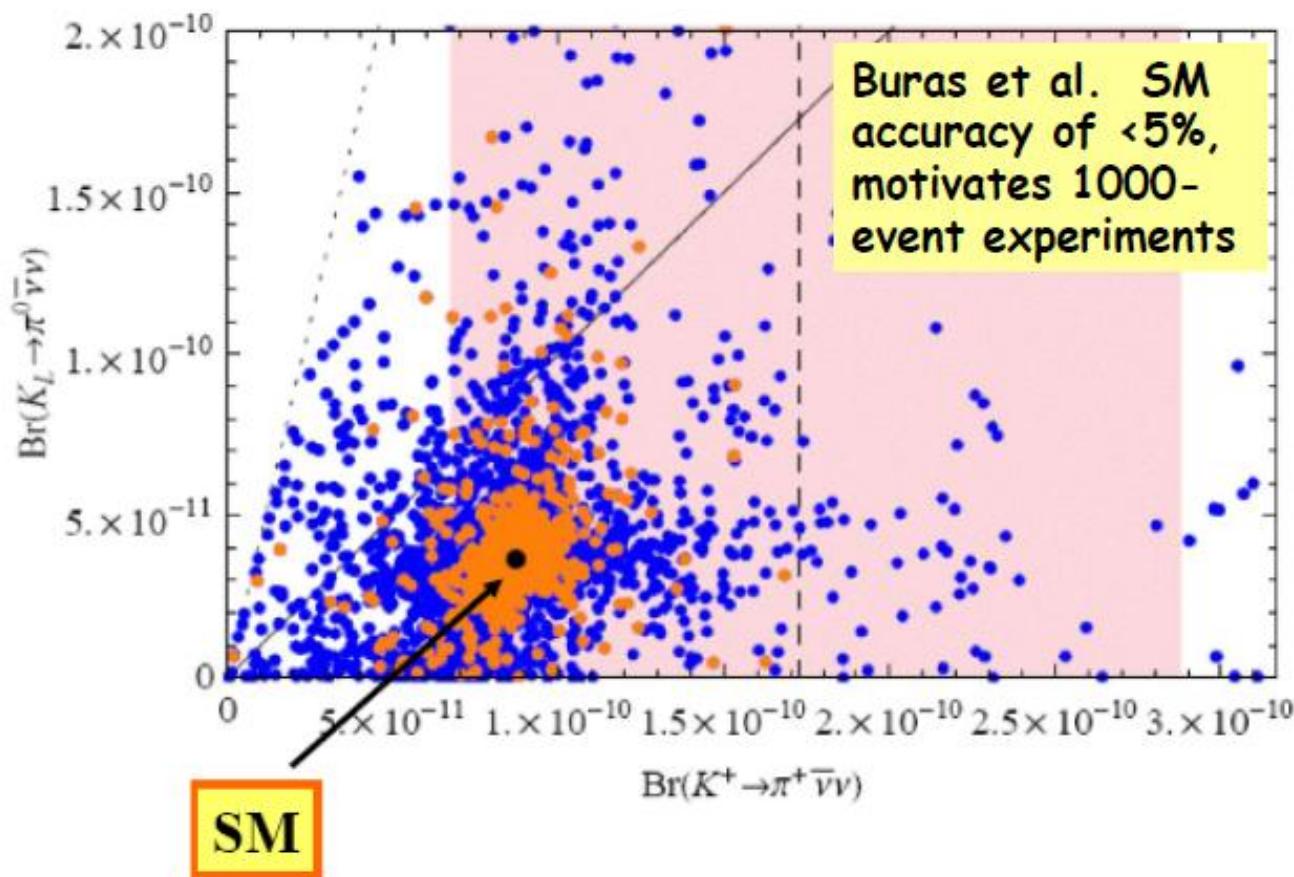
The ends of open strings, whose oscillations are particles and forces other than gravity, are stuck to our brane.

(Wikipedia)

BRANE (our universe)

## $K_L \rightarrow \pi^0 \bar{\nu}\nu$ vs. $K^+ \rightarrow \pi^+ \bar{\nu}\nu$ (RS)

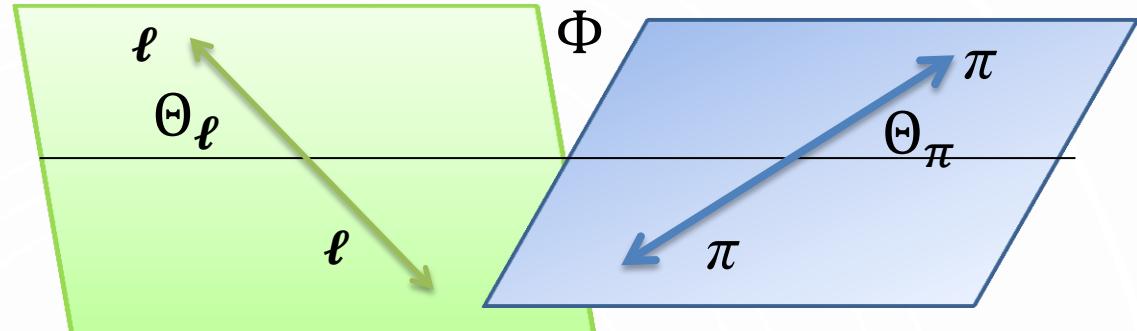
(Up to Factor 3 and 2 Enhancements)



*Effect of Warped Extra Dimension Models on Branching Fractions*



$$K_L \rightarrow \pi\pi\ell^+\ell^-$$



$$d\Gamma = \frac{G_F^2}{2^{12}\pi^6 M_K^5} \sin^2\Theta_C X \sigma_\pi \left[ 1 - \frac{4m_l^2}{s_l} \right]^2 I(s_\pi, s_l, \Theta_\pi, \Theta_l, \Phi) ds_\pi ds_l d\cos\Theta_\pi d\cos\Theta_l d\Phi ,$$

$$\begin{aligned} I &= I_1 \cos 2\Theta_l + I_3 \sin^2\Theta_l \cos 2\Phi \\ &\quad + I_4 \sin 2\Theta_l \cos \Phi + I_5 \sin \Theta_l \cos \Phi \\ &\quad + I_6 \cos \Theta_l + I_7 \sin \Theta_l \sin \Phi \\ &\quad + I_8 \sin 2\Theta_l \sin \Phi + I_9 \sin^2\Theta_l \sin 2\Phi , \end{aligned}$$

$$F_1 = Xf + \sigma_\pi s \cos\Theta_\pi g ,$$

$$F_2 = \sigma_\pi (s_\pi s_l)^{1/2} g ,$$

$$F_3 = \sigma_\pi X (s_\pi s_l)^{1/2} \frac{h}{M_K^2} ,$$

$$\begin{aligned} I_1 &= \frac{1}{4} [\{ |F_1|^2 + \frac{3}{2}(|F_2|^2 + |F_3|^2) \sin^2\Theta_\pi \} \\ &\quad + (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3})] , \\ I_2 &= -\frac{1}{4} [\{ |F_1|^2 - \frac{1}{2}(|F_2|^2 + |F_3|^2) \sin^2\Theta_\pi \} \\ &\quad + (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3})] , \\ I_3 &= -\frac{1}{4} [\{ |F_2|^2 - |F_3|^2 \} + (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3})] , \\ I_4 &= \frac{1}{2} \text{Re}(F_1^* F_2) \sin\Theta_\pi + (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3}) , \\ I_5 &= -\{ \text{Re}(F_1^* F_3) \sin\Theta_\pi - (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3}) \} , \\ I_6 &= -\{ \text{Re}(F_2^* F_3) \sin^2\Theta_\pi - (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3}) \} , \\ I_7 &= -\{ \text{Im}(F_1^* F_2) \sin\Theta_\pi - (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3}) \} , \\ I_8 &= \frac{1}{2} \text{Im}(F_1^* F_3) \sin\Theta_\pi + (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3}) , \\ I_9 &= -\frac{1}{2} [\text{Im}(F_2^* F_3) \sin^2\Theta_\pi + (F_{1,2,3} \rightarrow \tilde{F}_{1,2,3})] . \end{aligned}$$



$$\begin{aligned}\mathcal{A} &= \frac{\int_0^{\pi/2} \frac{d\Gamma}{d\Phi} d\Phi - \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\Phi} d\Phi}{\int_0^{\pi/2} \frac{d\Gamma}{d\Phi} d\Phi + \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\Phi} d\Phi} \\ &= 15\% \sin[\Phi_{+-} + \delta_0(m_K^2) - \bar{\delta}_1] \\ &\approx 14\%.\end{aligned}$$

*Strong phase*

Heiliger & Sehgal Phys. Rev. D48, 4146 (1993).

*Weak phase*

$$K_L \rightarrow \pi^+(p_+) \pi^-(p_-) \ell^+(k_+) \ell^-(k_-)$$

Under CP:

$\mathbf{p}_{\pm} \xrightarrow{CP} -\mathbf{p}_{\mp}$ $\mathbf{k}_{\pm} \xrightarrow{CP} -\mathbf{k}_{\mp}$	$\cos \Theta_{\pi} \rightarrow -\cos \Theta_{\pi}$ $\cos \Theta_{\ell} \rightarrow -\cos \Theta_{\ell}$ $\cos \Phi \rightarrow \cos \Phi$	$\sin \Theta_{\pi} \rightarrow \sin \Theta_{\pi}$ $\sin \Theta_{\ell} \rightarrow \sin \Theta_{\ell}$ $\sin \Phi \rightarrow -\sin \Phi$
--	---	--

## Signal of T-reversal violation

Several papers supporting and several other disputing signal is genuine T-violation



*CPT is introduced through the Hamiltonian*

$$\mathcal{H} = E \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} - iD\mathcal{I}. \text{ CPT restored if } \theta = \frac{\pi}{2}$$

*A complete calculation without CPT in mixing is underway.*

*CPT violation should be studied in K since large numbers of K mesons will be produced.*

# Conclusion

- *After more than 60 years K meson continues to be produced in lab and is still a valuable source for understanding new physics.*
- *Will continue to be a studied at least until a  $5\sigma$  signal is observed in  $K_L \rightarrow \pi^0\nu\bar{\nu}$ .*

$$K_1 = (K^0 + \bar{K}^0)/\sqrt{2},$$

$$K_2 = (K^0 - \bar{K}^0)/\sqrt{2},$$

$$K_S = (K_1 + \tilde{\epsilon} K_2)/\sqrt{1+|\tilde{\epsilon}|^2},$$

$$K_L = (K_2 - \tilde{\epsilon} K_1)/\sqrt{1+|\tilde{\epsilon}|^2}.$$

