Bilayer graphene: Kinks, Superlattices, Transport(?)

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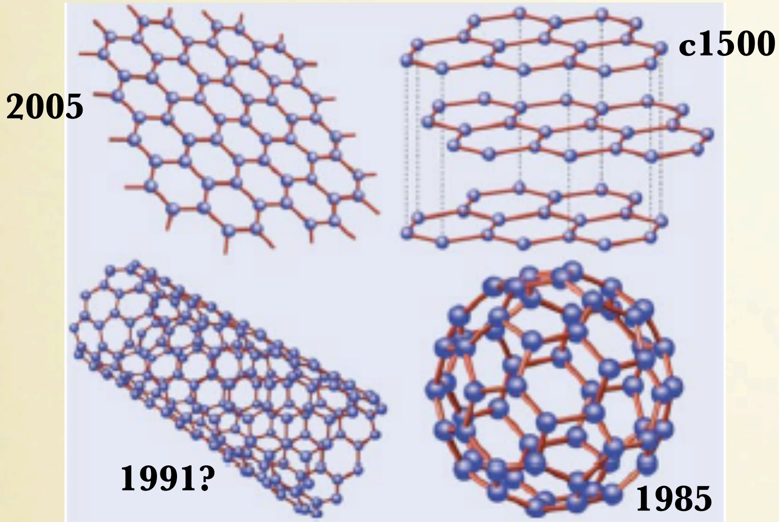
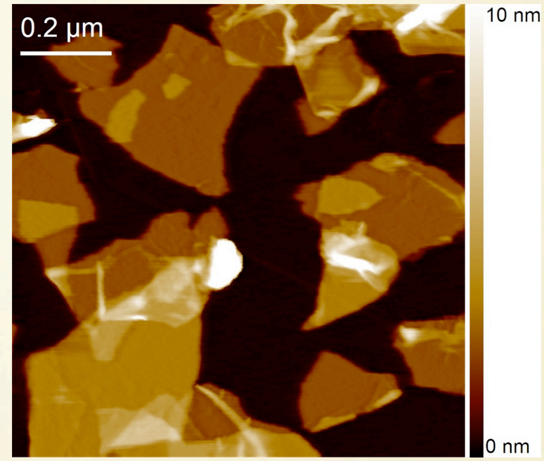
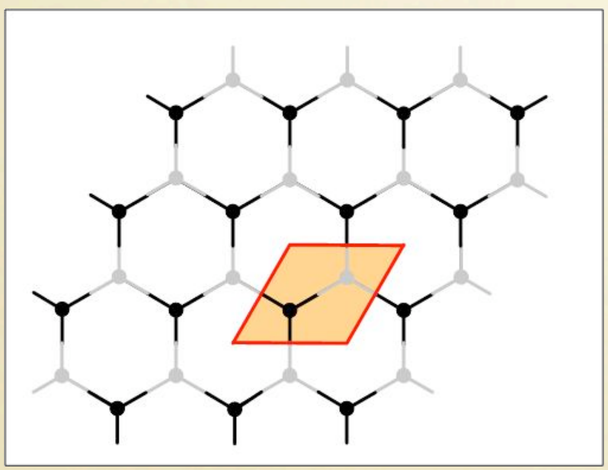
TIFR, 24 March 2011

Graphene



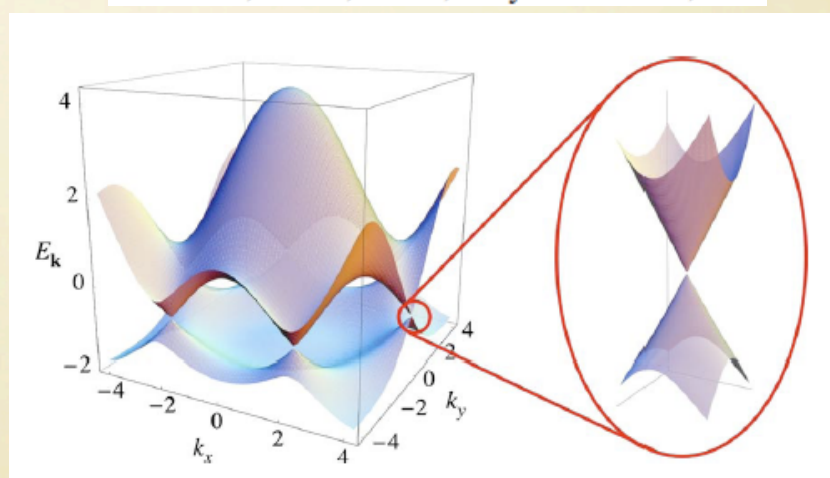


Novoselev, Geim,et al (2004)



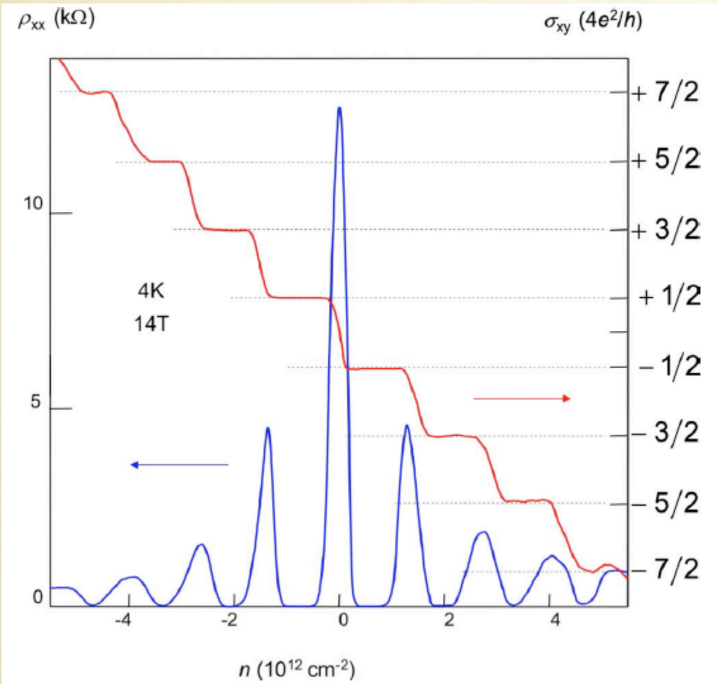
$$H_{\text{eff}} = v_F \vec{\sigma} \cdot \vec{k}$$

Wallace, P. R., 1947, Phys. Rev. **71**, 622



Castro-Neto et al, RMP (2009)

Anomalous integer quantum hall effect



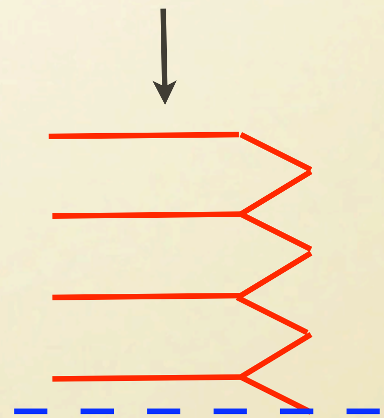
Novoselov/Geim, Philip Kim (2005)

Dirac fermions in an orbital B-field

$$v_F[\vec{\sigma}\cdot(-i\nabla + e\mathbf{A}/c)]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$$H^2 = v_F^2((\vec{p} + e\vec{A}/c)^2 + \vec{\sigma}\cdot\vec{B})$$

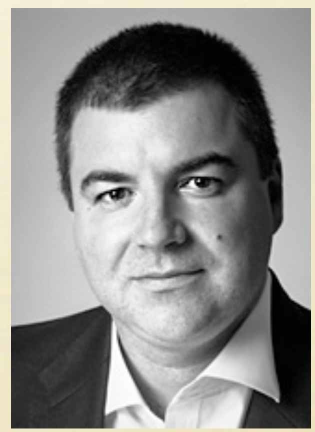
Non-relativistic spinful system with g=2



σ_{xy} counts number of Landau levels

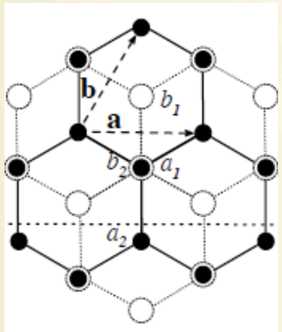
Dirac theory in the lab

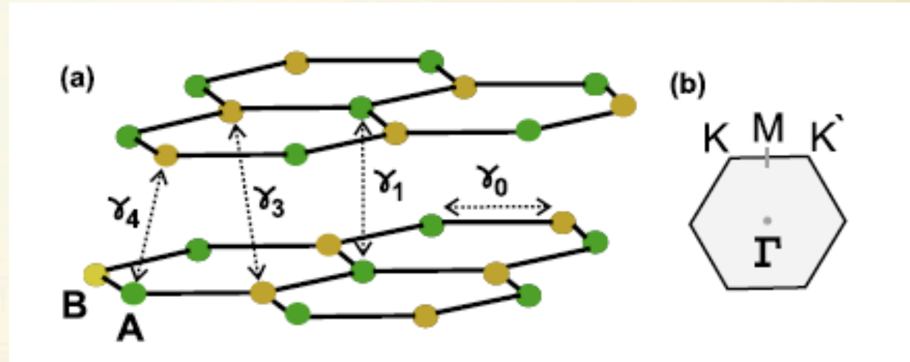
Weird quantum Hall effect
Klein paradox
Strange lensing effects of electron waves
.
.



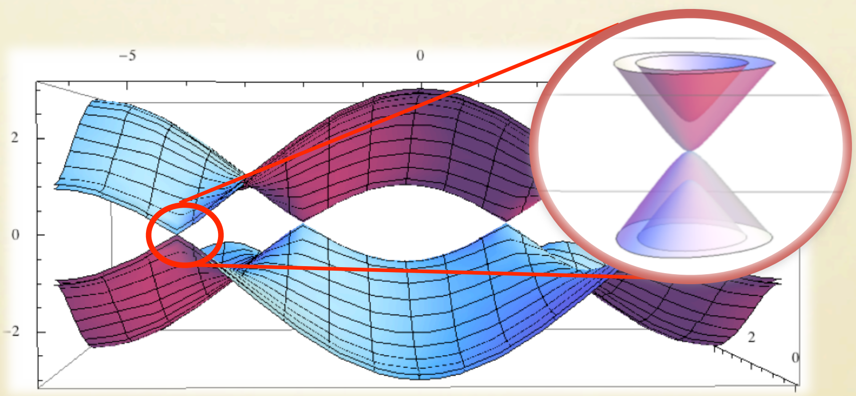
Bilayer Graphene

Bernal stacking

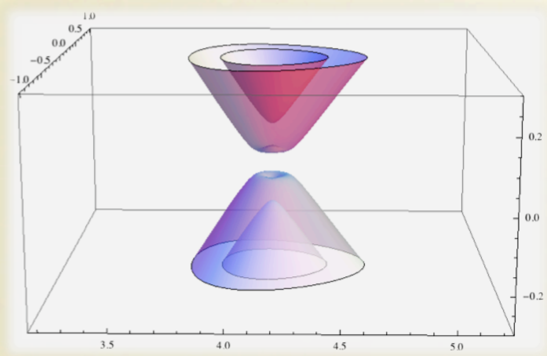




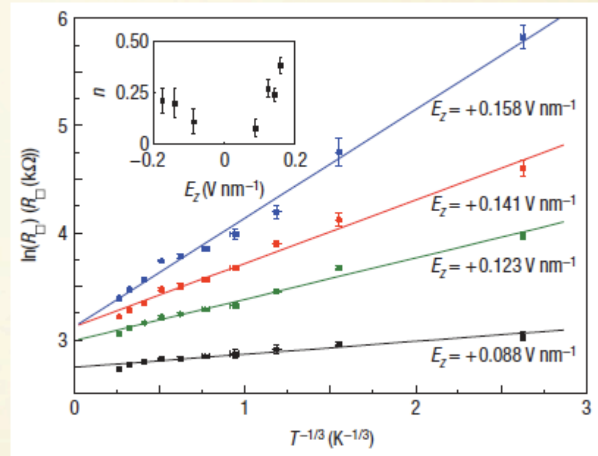
Castro-Neto et al, RMP (2009)



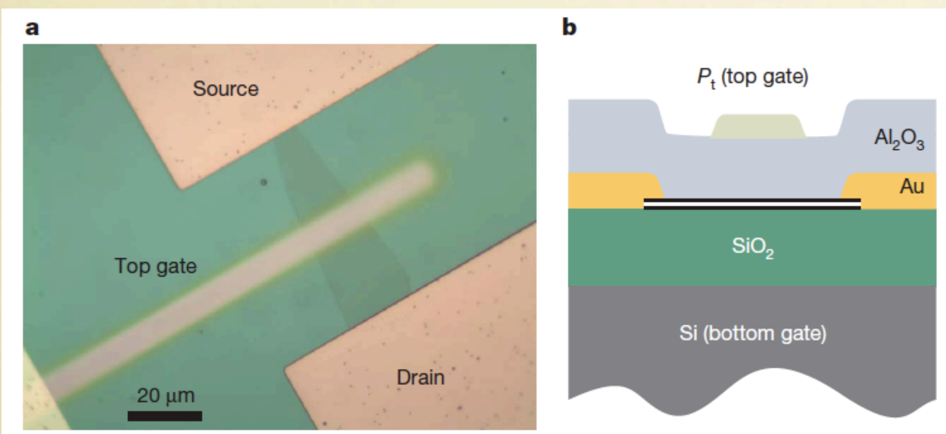
Gated bilayer graphene as a tunable gap semiconductor

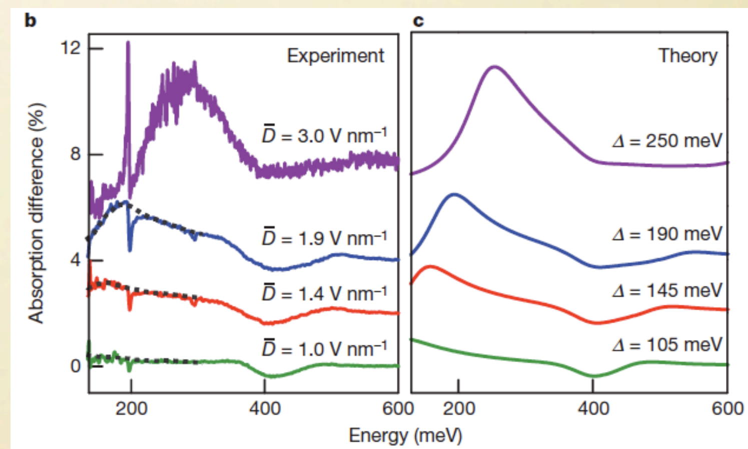


Perpendicular electric field opens a gap



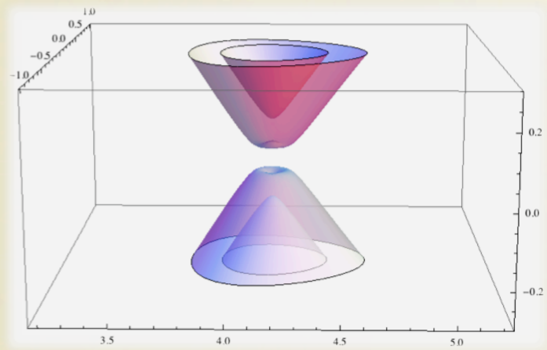
J. B. Oostinga, et al (Nature Mat., 2009)



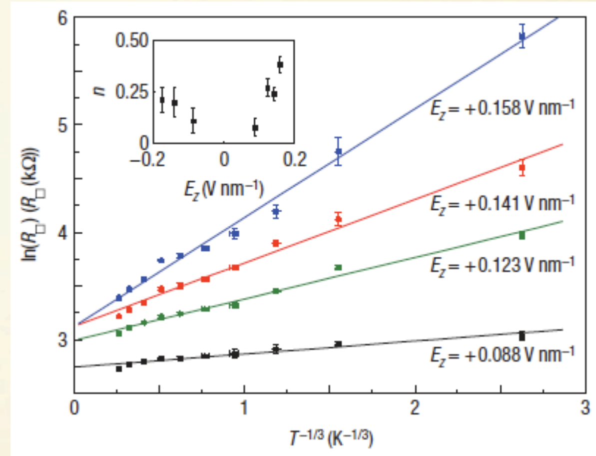


Y. Zhang et al (Nature, 2009)

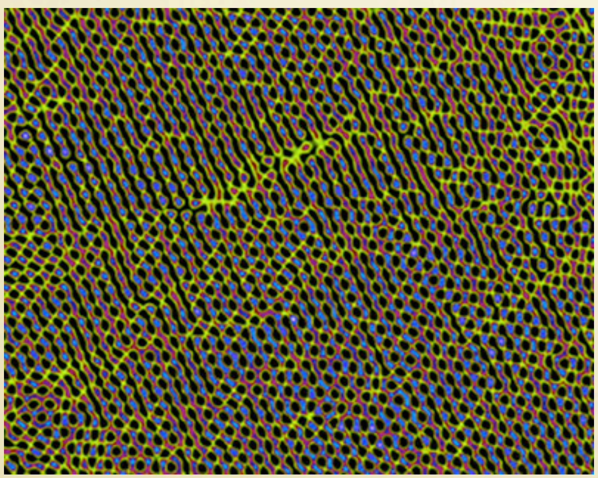
Possible disorder effects in BLG

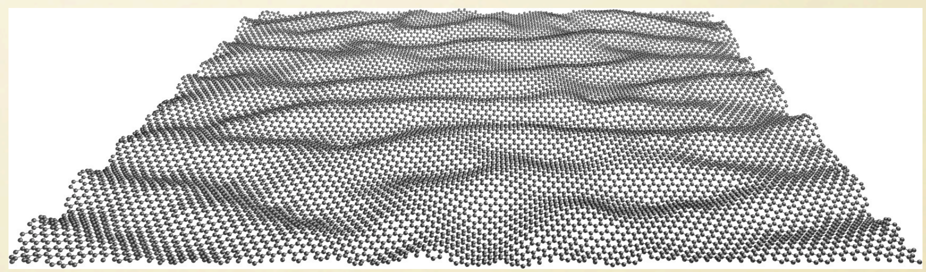


Perpendicular electric field opens a gap



J. B. Oostinga, et al (Nature Mat., 2009)





Schematic view and STM images of rippling of monolayer graphene (Eg: Ishigami et al, 2007)

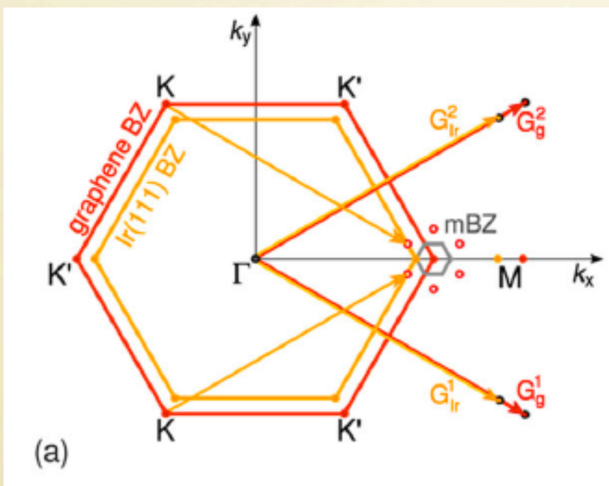
Similar effects expected in BLG

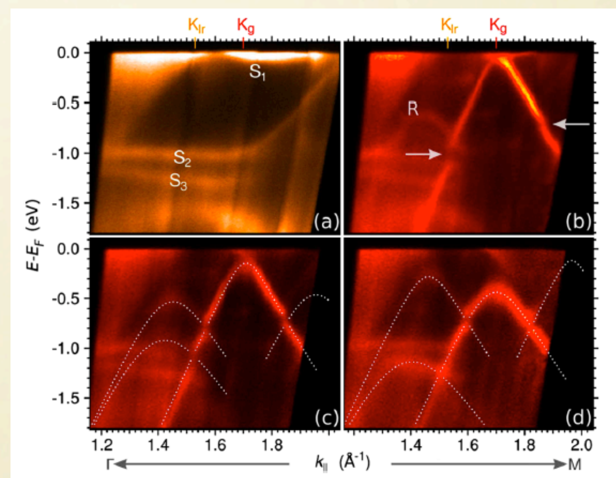
1. Chemical potential modulations - will lead to electron/hole puddles at long wavelength
2. Bias modulations - will lead to inhomogeneous gaps

Superlattices in BLG

1. Chemical potential and bias modulations - anything useful from periodic modulations?

2. Superlattices for `band structure engineering' - for example, monolayer graphene on Ir(111)

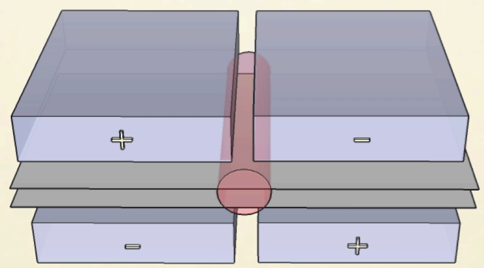




<u>Spatially varying electric field - a single kink problem</u>

. If interactions induce an interlayer CDW - what happens at CDW domain walls?

. If we can use multiple gates to change E_{perp}?



. Answer - get fermion modes bound to the interface

. Closely related to the problem of Peierls domain wall bound states in polyacetylene

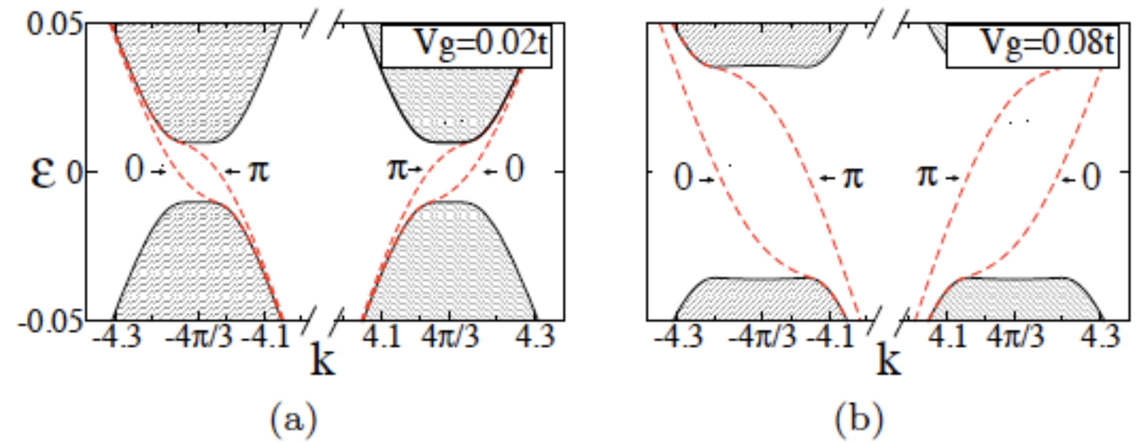
Single particle dispersion

Ivar Martin et al (PRL 2008)

$$\hat{H} = \begin{pmatrix} V_1 & v_F\pi^\dagger & 0 & 0 \\ v_F\pi & V_1 & t_\perp & 0 \\ 0 & t_\perp & V_2 & v_F\pi^\dagger \\ 0 & 0 & v_F\pi & V_2 \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_{A1} \\ \psi_{B1} \\ \psi_{A2} \\ \psi_{B2} \end{pmatrix}$$

$$\tilde{H} = \begin{pmatrix} -\frac{V}{2}\left(1 - \frac{c^2p^2}{t_\perp^2}\right) & -\frac{c^2\pi^{\dagger 2}}{t_\perp} \\ -\frac{c^2\pi^2}{t_\perp} & \frac{V}{2}\left(1 - \frac{c^2p^2}{t_\perp^2}\right) \end{pmatrix}$$

$$\pi = -i\partial_x + \partial_y$$

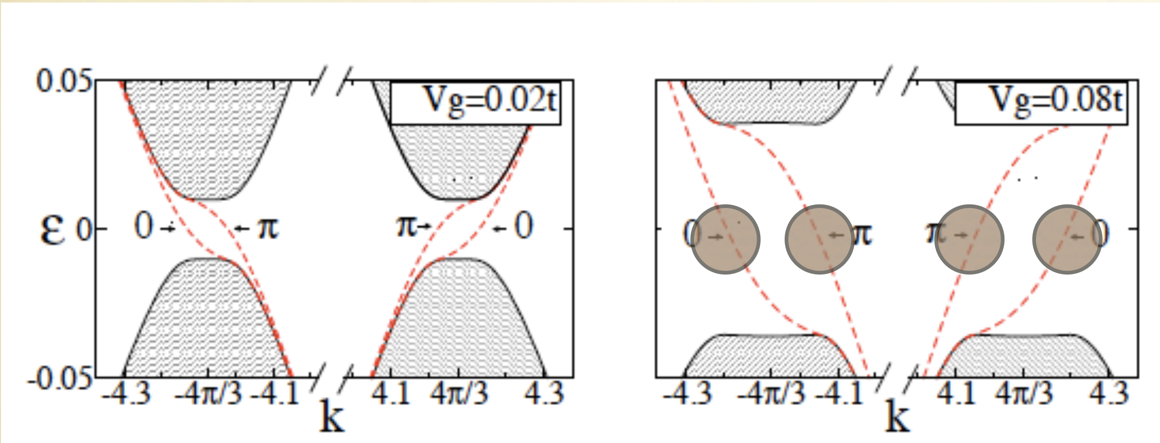


$0/\pi$ are eigenstates of “generalized parity”

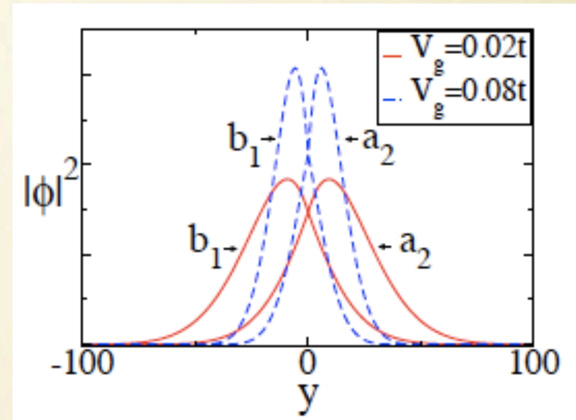
Somewhat like anomalous quantum Hall effect at each valley

Interaction effects

M. Killi et al, PRL (2010)



(a) (b)



. Isolate states in the vicinity of the Fermi points

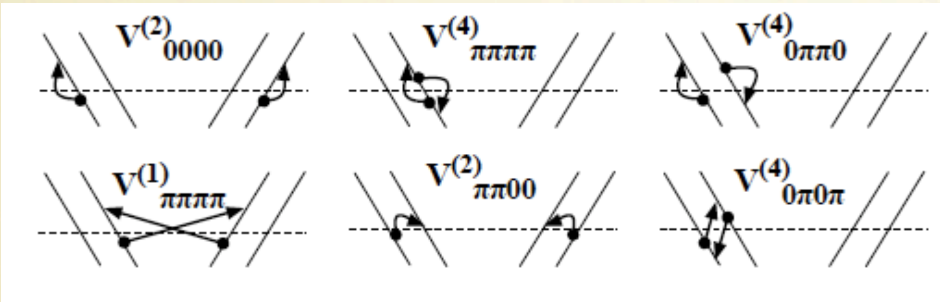
. Coulomb forward scattering dominates due to wave function spread (eg: fat nanotubes - Balents,Kane,Fisher)

. Bosonize including forward scattering terms

Interaction effects

M. Killi et al, PRL (2010)

Various scattering processes



→ Keep

→ Drop

Check

$$V^{(1)}_{0000}/V^{(2)}_{0000} \sim 10^{-3}$$

$$V^{(4)}_{0\pi 0\pi}/V^{(2)}_{0000} \sim 10^{-2}$$

Bosonization

$$\partial_x \hat{\phi}_{\alpha\sigma} = -\pi\left(\hat{\rho}_{R\alpha\sigma} + \hat{\rho}_{L\alpha\sigma}\right)$$

$$\partial_x \hat{\theta}_{\alpha\sigma} = \pi\left(\hat{\rho}_{R\alpha\sigma} - \hat{\rho}_{L\alpha\sigma}\right)$$

$$H_1 = \frac{1}{2\pi}\int dx (\partial_x \Phi)^T \hat{u}\cdot\hat{K}^{-1}(\partial_x\Phi) + (\partial_x\Theta)^T \hat{u}\cdot\hat{K}(\partial_x\Theta)$$

$$\hat{u}\cdot\hat{K}^{-1} = V_F \mathbb{1} + \frac{V_F}{2\pi}\begin{pmatrix} g_A & g_B & g_A & g_B \\ g_B & g_A & g_B & g_A \\ g_A & g_B & g_A & g_B \\ g_B & g_A & g_B & g_A \end{pmatrix}$$

$$\hat{u}\cdot\hat{K} = V_F \mathbb{1}.$$

Interaction effects

M. Killi et al, PRL (2010)

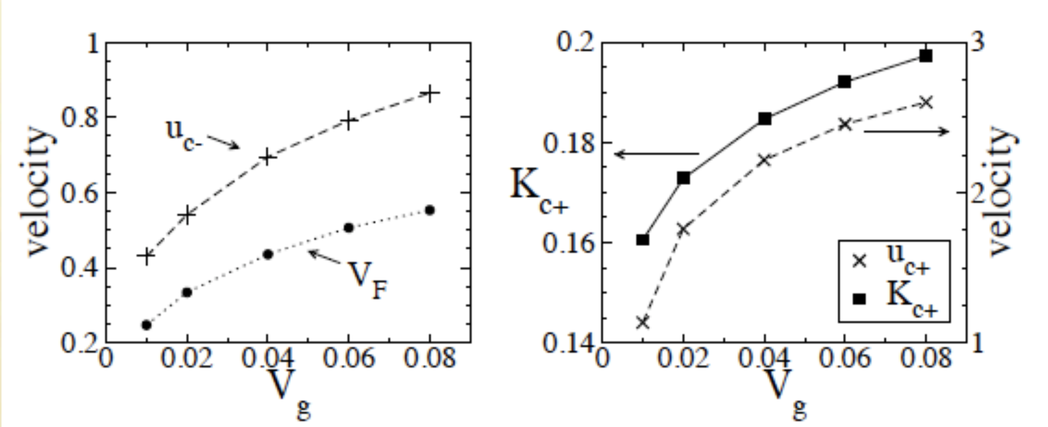
Spin modes unaffected

$$K_{s\pm} = 1$$
$$u_{s\pm} = V_F$$

Charge modes "renormalized"

$$u_{c\pm} = V_F\,(1 + y_{c\pm})^{\frac{1}{2}}$$
$$K_{c\pm} = (1 + y_{c\pm})^{-\frac{1}{2}}\,,$$
$$y_{c\pm} = 2(V_A \pm V_B)/\pi V_F$$

Luttinger parameter and velocity are tunable via electric field



$$K_{c-} \approx 0.63$$

. Bare Fermi velocity changes with field: "kinetic"
. Confinement length transverse to 'wire' also changes: "interaction"

Signature:

$$dI/dV \sim V^{\alpha}$$
$$\alpha_{\text{edge}} = \tfrac{1}{4}(K_{c+}^{-1} + K_{c-}^{-1} - 2)$$

$$\alpha_{\text{edge}} : 1.1 \rightarrow 1.4$$

Summary of single kink physics

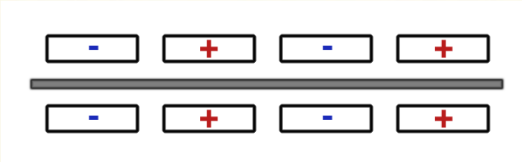
1. A tunable 2-band Luttinger liquid using bilayer graphene

. Bilayer graphene is a tunable gap semiconductor
. ‘Kink’ in the bias leads to a LL localized at the interface
. LL is spin-charge-band separated with 3 mode velocities
. LL has tunable Luttinger parameter in total charge channel

M. Killi, T. C. Wei, I. Affleck, A. Paramekanti (PRL 2010)

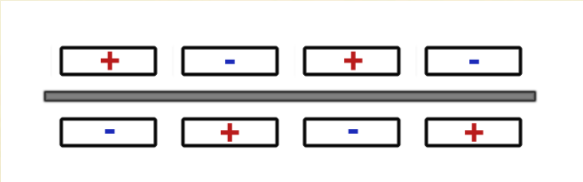
Superlattices

1. Chemical Potential Modulation



$$
\begin{aligned}
0 < y \leq w \quad &: \quad V_1(x,y) = V_2(x,y) = 2U(1 - w/\lambda) \\
w < y \leq \lambda \quad &: \quad V_1(x,y) = V_2(x,y) = -2Uw/\lambda
\end{aligned}
$$

- Interlayer Bias Modulation



$$
\begin{aligned}
0 < y \leq w \quad &: \quad V_1(x,y) = -V_2(x,y) = 2U(1 - w/\lambda) \\
w < y \leq \lambda \quad &: \quad V_1(x,y) = -V_2(x,y) = -2Uw/\lambda
\end{aligned}
$$

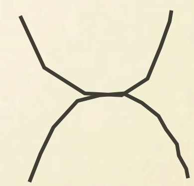
Superlattices

Full Hamiltonian at low energy

$$\hat{H} = -\frac{v_F^2}{t_\perp}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} + \begin{pmatrix} V_1(\mathbf{x}) & 0 \\ 0 & V_2(\mathbf{x}) \end{pmatrix}$$

Kinetic energy part

$$H_{kin} = \sum_{\mathbf{p}} \left(\varepsilon_e(\mathbf{p})\beta^\dagger_{\mathbf{p}}\beta_{\mathbf{p}} + \varepsilon_h(\mathbf{p})\alpha^\dagger_{\mathbf{p}}\alpha_{\mathbf{p}}\right)$$



Superlattice potential scattering

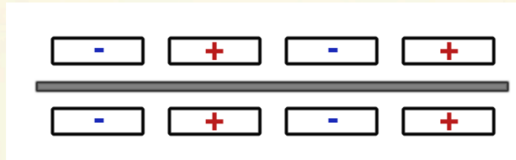
$$\sum\nolimits_{\mathbf{p},\mathbf{G}} \Psi^\dagger(\mathbf{p}) W_{\mathbf{p},\mathbf{G}} \Psi(\mathbf{p}-\mathbf{G})$$

Scattering depends crucially on angles

$$W_{\mathbf{p},\mathbf{G}} = \frac{1}{2}\begin{pmatrix} V_1(\mathbf{G}) + V_2(\mathbf{G})e^{2i\theta} & V_1(\mathbf{G}) - V_2(\mathbf{G})e^{2i\theta} \\ V_1(\mathbf{G}) - V_2(\mathbf{G})e^{2i\theta} & V_1(\mathbf{G}) + V_2(\mathbf{G})e^{2i\theta} \end{pmatrix}$$

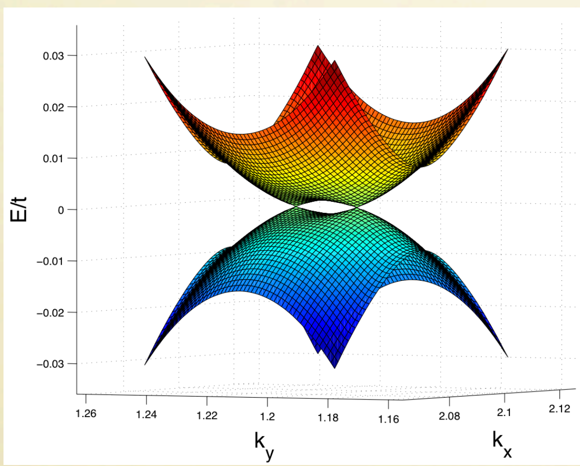
1D chemical potential superlattices

Scattering depends crucially on angles

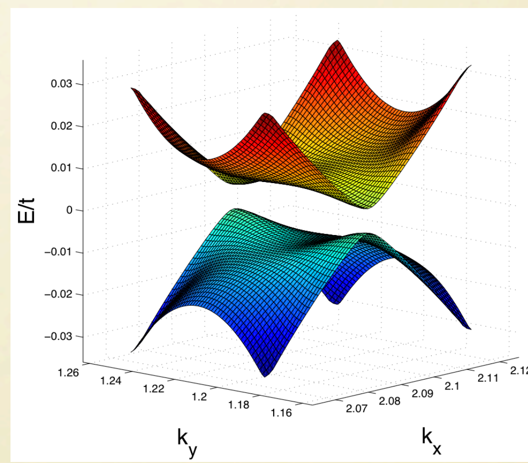


$$W_{\mathbf{p},\mathbf{G}}=\frac{1}{2}\begin{pmatrix} V_1(\mathbf{G})+V_2(\mathbf{G})e^{2i\theta} & V_1(\mathbf{G})-V_2(\mathbf{G})e^{2i\theta} \\ V_1(\mathbf{G})-V_2(\mathbf{G})e^{2i\theta} & V_1(\mathbf{G})+V_2(\mathbf{G})e^{2i\theta} \end{pmatrix}$$

Small U



Large U



Form 2 ***anisotropic*** Dirac cones at small potentials

Increase potential Dirac points ***move*** along k_y=0 towards the MZB

Upon reaching the MZB a ***gap opens***

1D chemical potential superlattices

Perturbative Results

$$W_{\mathbf{p},\mathbf{G}} = \frac{1}{2}\begin{pmatrix} V_1(\mathbf{G})+V_2(\mathbf{G})e^{2i\theta} & \cancel{V_1(\mathbf{G})-V_2(\mathbf{G})e^{2i\theta}} \\ \cancel{V_1(\mathbf{G})-V_2(\mathbf{G})e^{2i\theta}} & V_1(\mathbf{G})+V_2(\mathbf{G})e^{2i\theta} \end{pmatrix}$$

Along **p || G,** θ=0 and particle/hole states ***decouple***

Level repulsion pushes ***conduction*** band ***down*** and ***valence*** band ***up***

Location of DP

$$p_y^* \sim \pm\frac{2\sqrt{2}m^*U\lambda}{\pi^2}$$

Velocity Anisotropy

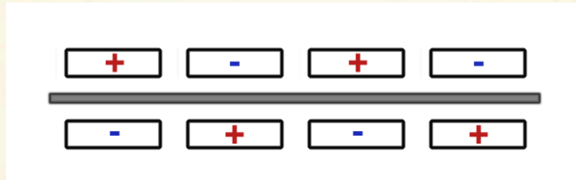
$$v_x = 4\sqrt{2}\lambda U/\pi^2$$

Critical Pot.

$$U_c \sim \frac{\pi^3}{2\sqrt{2}m^*\lambda^2} \approx 0.03t$$

1D electric field superlattices

Scattering depends crucially on angles

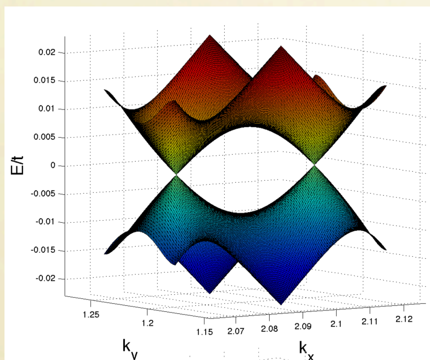


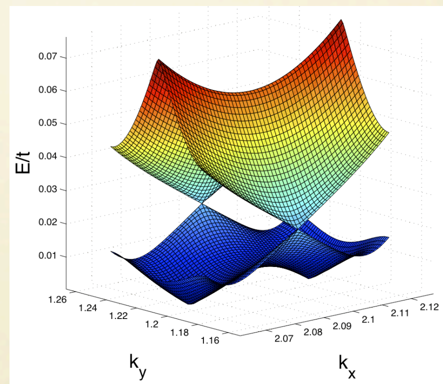
$$W_{\mathbf{p},\mathbf{G}}=\frac{1}{2}\begin{pmatrix} V_1(\mathbf{G})+V_2(\mathbf{G})e^{2i\theta} & V_1(\mathbf{G})-V_2(\mathbf{G})e^{2i\theta} \\ V_1(\mathbf{G})-V_2(\mathbf{G})e^{2i\theta} & V_1(\mathbf{G})+V_2(\mathbf{G})e^{2i\theta} \end{pmatrix}$$

Valence and Conduction Band

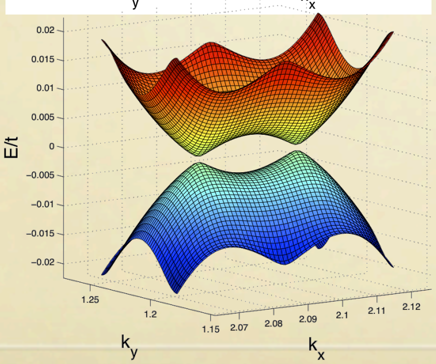
Lowest Conduction Bands

w=λ/2





w≠λ/2



When w=λ/2, 4 ***anisotropic*** Dirac cones

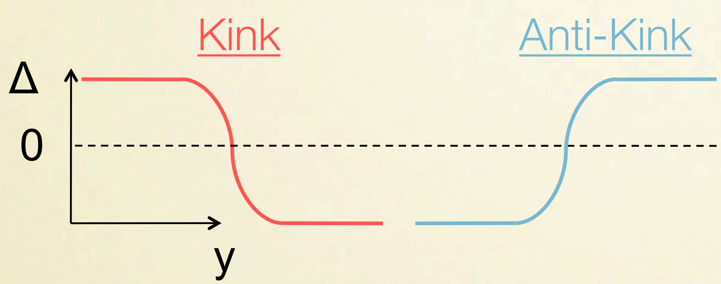
-2 at zero energy (k_y=0)

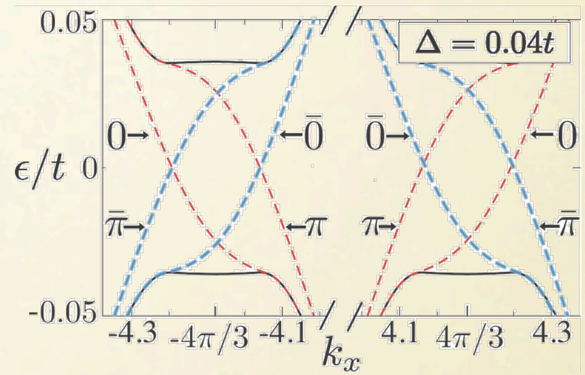
-2 at finite energy (k_y=π/λ or MZB)

When w≠λ/2, a ***gap opens*** at all DP

1D electric field superlattices

Effective Theory

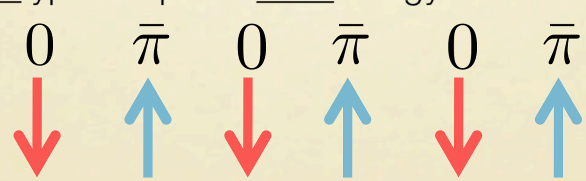




For a given kink, each unidirectional mode of a given K-point couples to the opposite moving modes of the two neighbouring anti-kinks with the same valley index.

Modes of the ***same*** type at ***finite*** energy

Modes of the ***opposite*** type couple at ***zero*** energy



1D electric field superlattices

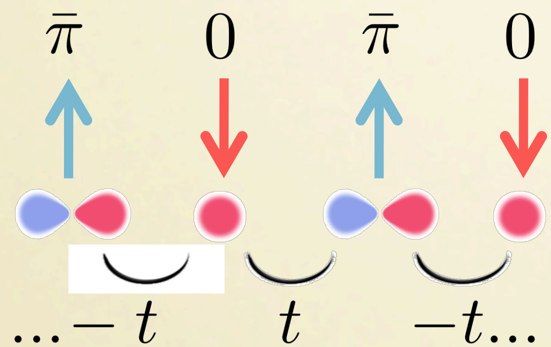
Perturbative Results

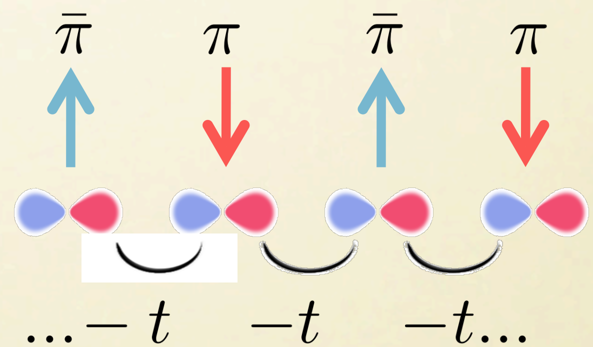
What is the appropriate form of the coupling between neighbouring wires?

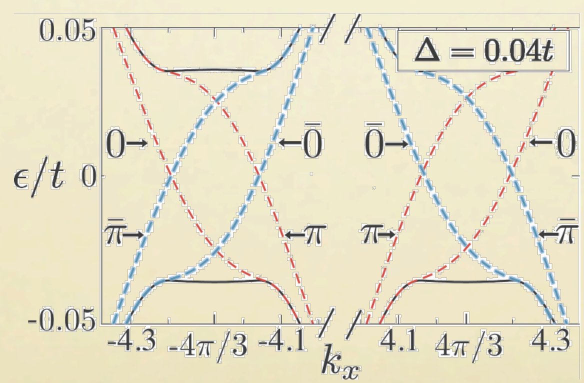
Wavefunctions satisfy a generalized ‘parity’ operator:

P : x → -x and Layer 1 ←→ Layer 2

$$\begin{pmatrix} w(x) \\ w(-x) \end{pmatrix}, \begin{pmatrix} v(x) \\ -v(-x) \end{pmatrix}$$







1D electric field superlattices

Perturbative Results

What is the appropriate form of the coupling between neighbouring wires?

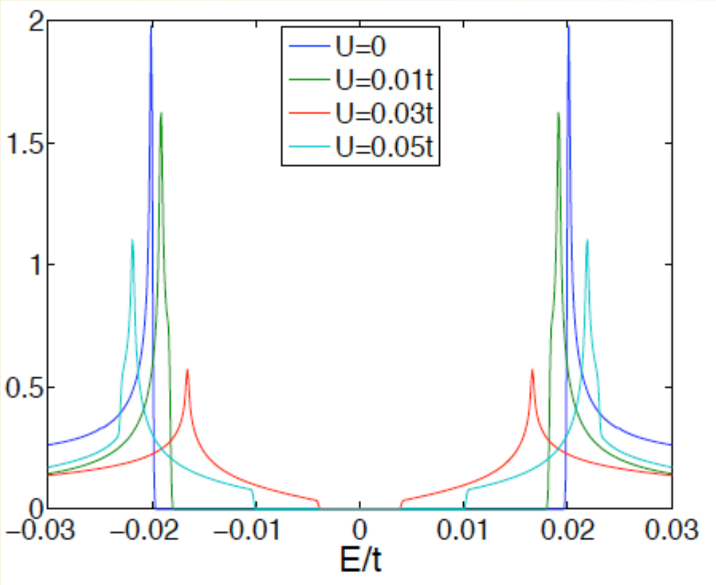
Wavefunctions satisfy a generalized ‘parity’ operator:

P : x → -x and Layer 1 ←→ Layer 2 $\begin{pmatrix} w(x) \\ w(-x) \end{pmatrix}, \begin{pmatrix} v(x) \\ -v(-x) \end{pmatrix}$

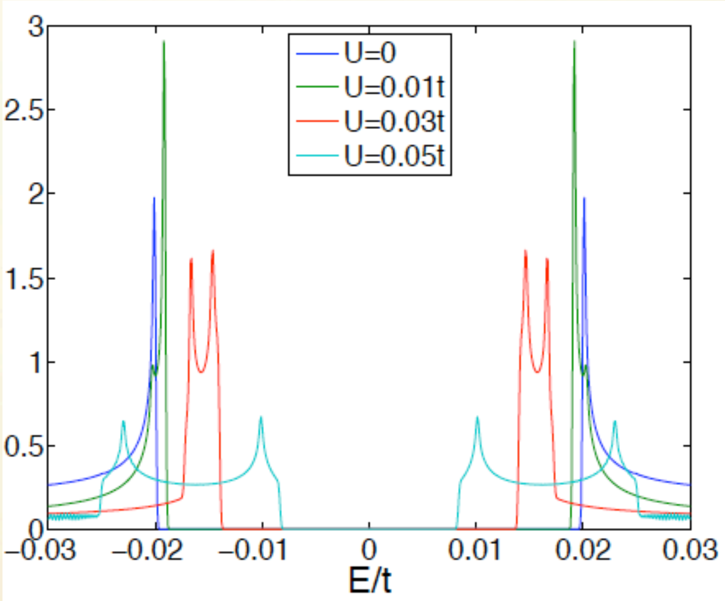
$$H(p_x) = v_0 \sum_n \left((-1)^n (p_x - p_x^*) c^\dagger_{p_x n} c_{p_x n}\right) - \sum_n (g(-1)^n + \delta)\left(c^\dagger_{p_x n} c_{p_x n+1} + h.c.\right)$$

$$E = \pm\sqrt{\xi^2(p_x) + 4\delta^2\cos^2(p_y) + 4g^2\sin^2(p_y)}$$

DOS with uniform bias + modulations



chemical potential modulations



bias modulations

Modulation induced subgap modes

Summary

1. A tunable 2-band Luttinger liquid using bilayer graphene

. Bilayer graphene is a tunable gap semiconductor
. ‘Kink’ in the bias leads to a LL localized at the interface
. LL is spin-charge-band separated with 3 mode velocities
. LL has tunable Luttinger parameter in total charge channel

M. Killi, T. C. Wei, I. Affleck, A. Paramekanti (PRL 2010)

2. Superlattices lead to new Dirac points with tunable velocity

. Superlattices lead to new Dirac points with tunable velocity
. Electric field superlattices map onto coupled chains of topological modes
. Such modulations might lead to subgap modes and contribute to transport

M. Killi, Si Wu, A. Paramekanti (in preparation)