Karlsruhe Institute of Technology

Modeling of Hadronic Interactions

Lecture 1

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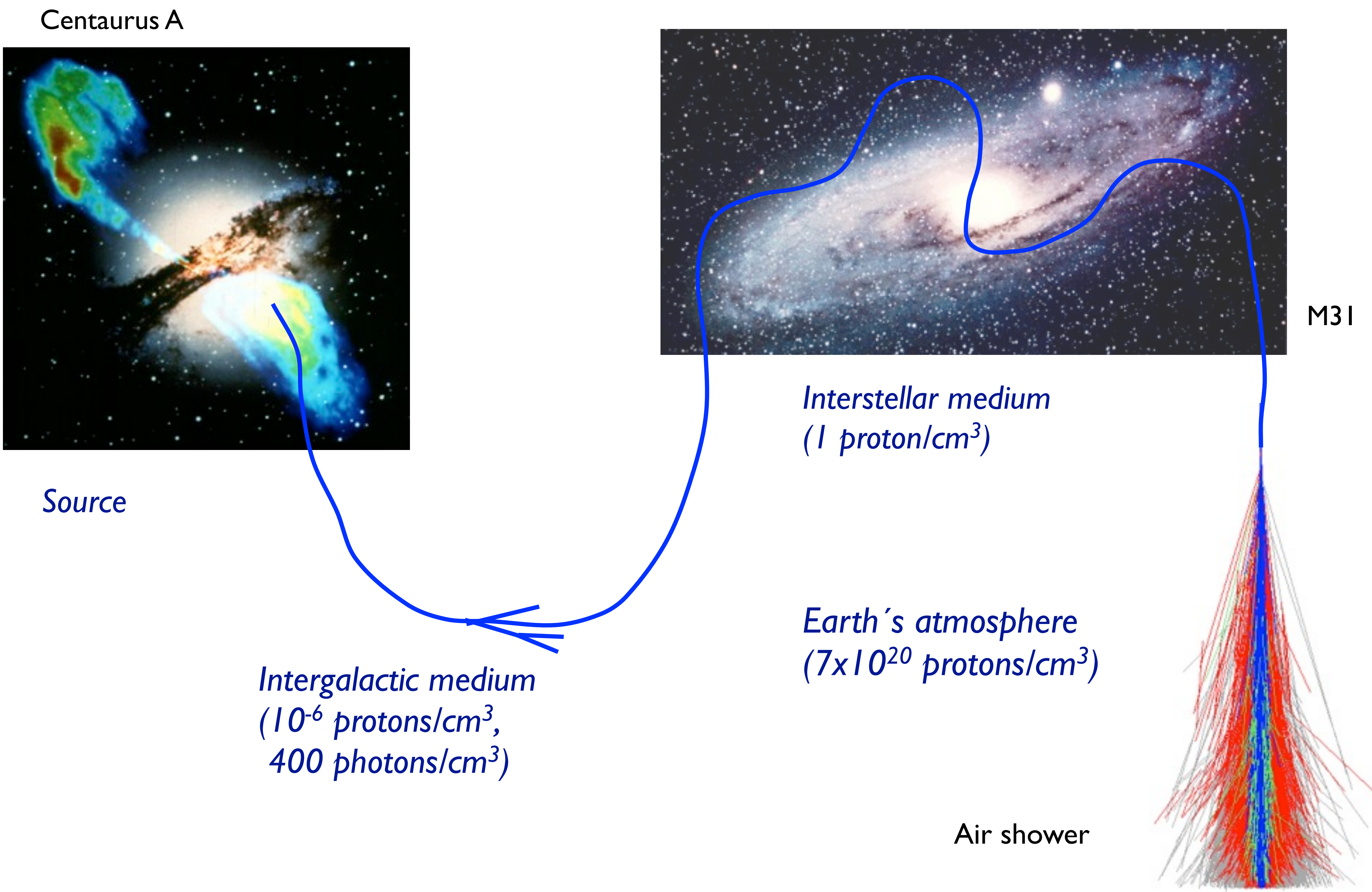
Examples of cosmic ray interactions



Outline

Lecture 1 – Low- and intermediate-energy interactions

- Particle production threshold: resonances
- Intermediate energies: two-string models
- Extension to nuclei and photons

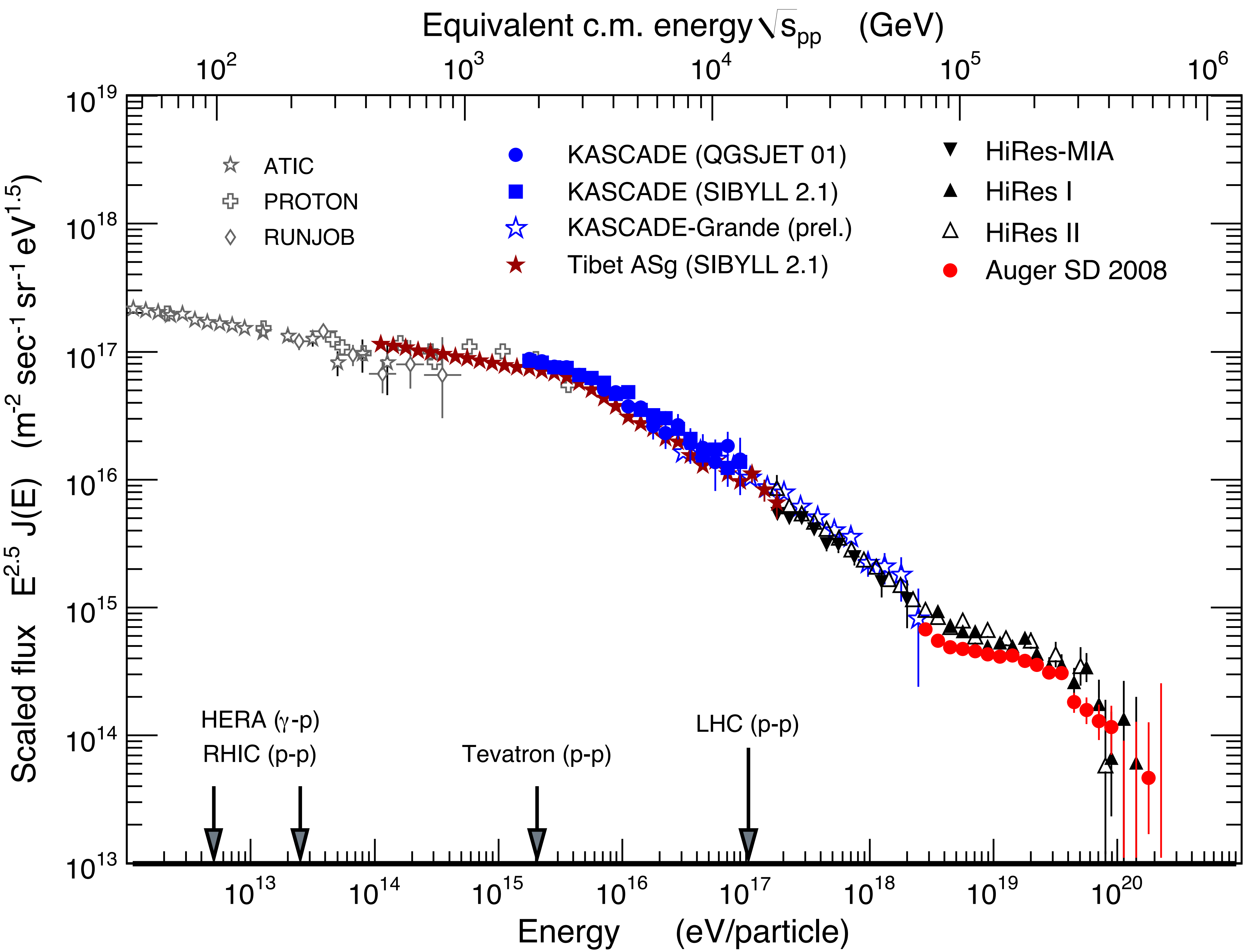
Lecture 2 – Interactions at very high energy

- Jets and minijets, multiple interactions
- Unitarization and saturation scenarios
- Comparison of models and uncertainties of extrapolations

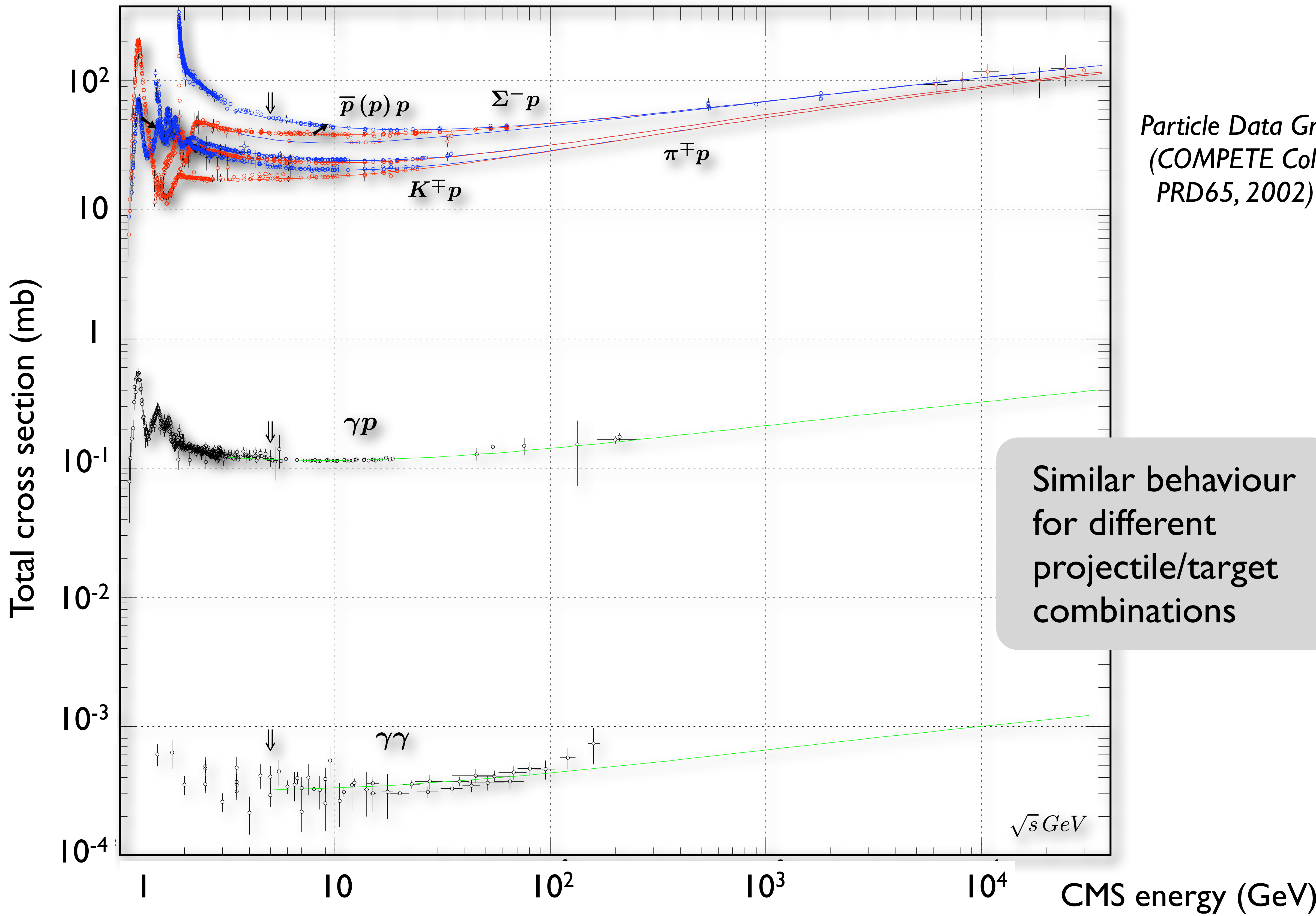
Lecture 3 – Air shower phenomenology and accelerator data

- Relation between hadronic interactions and air showers
- Accelerator experiments & discrimination potential of LHC
- Comparison of model predictions with accelerator data

Comparison of energies



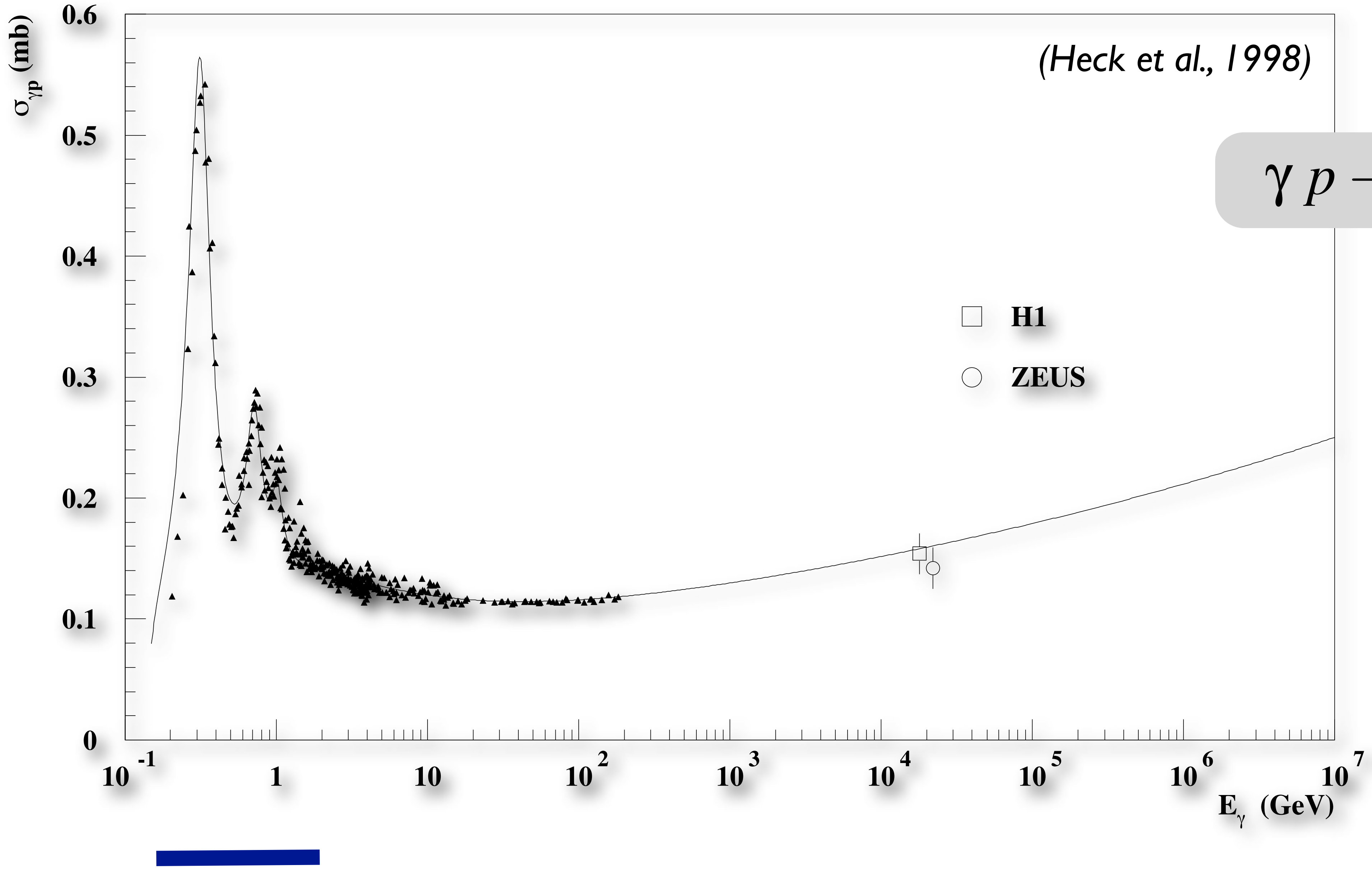
Compilation of total cross sections



Particle Data Group (COMPETE Collab. PRD65, 2002)

Similar behaviour for different projectile/target combinations

Simulation concepts: energy ranges



$$\gamma\, p \longrightarrow X$$

Resonances (fireball)

Scaling region (longitudinal phase space)

Minijet region (scaling violation)

???

Particle production close to the threshold:

Resonance models

Photoproduction of resonances

CMB: Energy threshold not sharp $E_{\gamma,\max} \approx 10^{-3}\mathrm{eV}$

$$E_{p,\Delta} = \frac{m_\Delta^2 - m_p^2}{2E_{\gamma,\max}(1-\cos\theta)} \approx 10^{20}\mathrm{eV}$$

In proton rest frame:

$$E_{\gamma,\mathrm{lab}} \approx 300\ \mathrm{MeV}$$

Decay branching ratio proton:neutron = 2:1

Mean proton energy loss 20%

Decay isotropic up to spin effects

Superposition of resonances

Baryon resonances and their physical parameters implemented in SOPHIA (see text). Superscripts $^+$ and ^{0} in the parameters refer to $p\gamma$ and $n\gamma$ excitations, respectively. The maximum cross section, $\sigma_{\max} = 4m_{\mathrm{N}}^2 M^2 \sigma_0/(M^2 - m_{\mathrm{N}}^2)^2$, is also given for reference

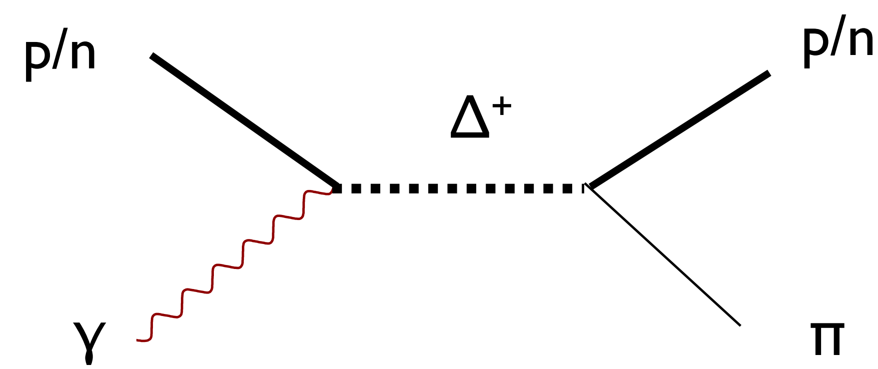
Resonance	M	Γ	$10^3 b_\gamma^+$	σ_0^+	$\sigma_{\max}^+$	$10^3 b_\gamma^0$	σ_0^0	$\sigma_{\max}^0$
$\Delta(1232)$	1.231	0.11	5.6	31.125	411.988	6.1	33.809	452.226
$N(1440)$	1.440	0.35	0.5	1.389	7.124	0.3	0.831	4.292
$N(1520)$	1.515	0.11	4.6	25.567	103.240	4.0	22.170	90.082
$N(1535)$	1.525	0.10	2.5	6.948	27.244	2.5	6.928	27.334
$N(1650)$	1.675	0.16	1.0	2.779	7.408	0.0	0.000	0.000
$N(1675)$	1.675	0.15	0.0	0.000	0.000	0.2	1.663	4.457
$N(1680)$	1.680	0.125	2.1	17.508	46.143	0.0	0.000	0.000
$\Delta(1700)$	1.690	0.29	2.0	11.116	28.644	2.0	11.085	28.714
$\Delta(1905)$	1.895	0.35	0.2	1.667	2.869	0.2	1.663	2.875
$\Delta(1950)$	1.950	0.30	1.0	11.116	17.433	1.0	11.085	17.462

Breit-Wigner resonance cross section

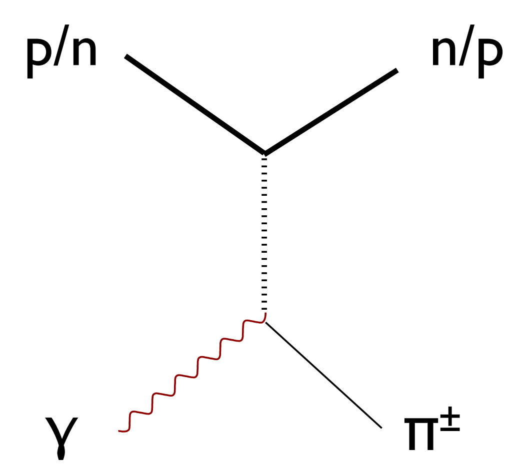
$$\sigma_{\mathrm{bw}}(s; M, \Gamma, J) = \frac{s}{(s - m_{\mathrm{N}}^2)^2} \frac{4\pi b_\gamma (2J+1) s \Gamma^2}{(s - M^2)^2 + s\Gamma^2}$$

Resonance and direct pion production

Resonance production
(s channel)

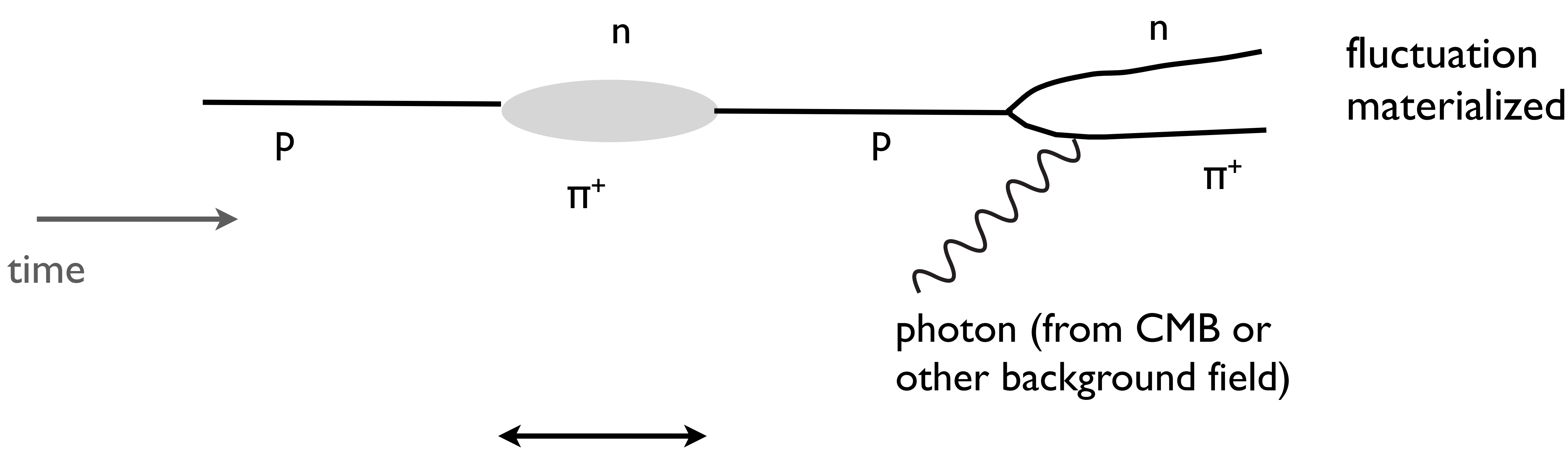


Direct pion production
(t channel)



Direct pion production

Possible interpretation: p fluctuates from time to time to n and π^+

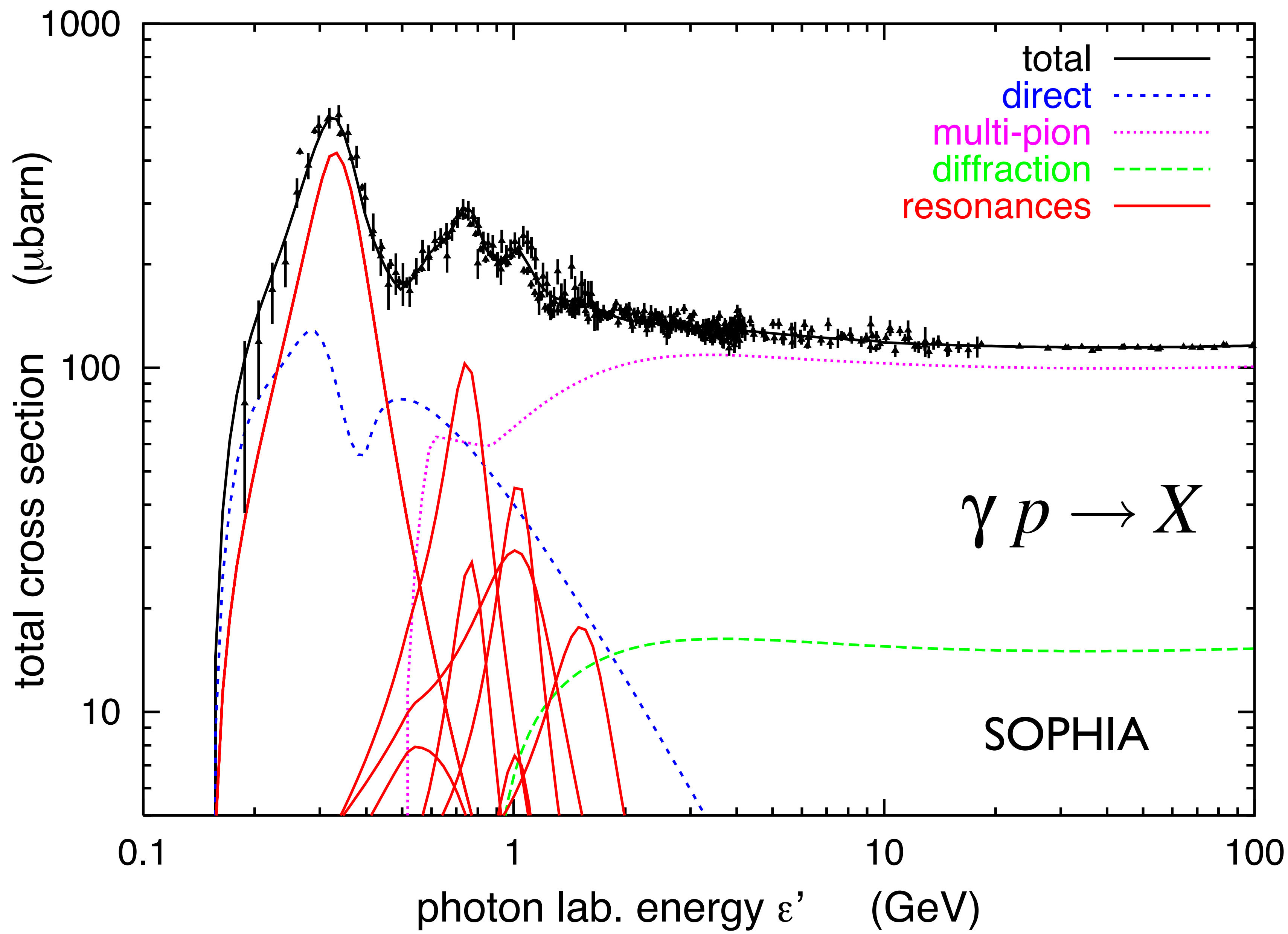


Heisenberg uncertainty relation $\Delta E \Delta t \approx 1$

Energy threshold very low: $E_{\mathrm{cm,min}} = m_\pi + m_p \approx 1.07\ \mathrm{GeV}$

(Δ^+ resonance: 1.232 GeV)

Putting all together: description of total cross section



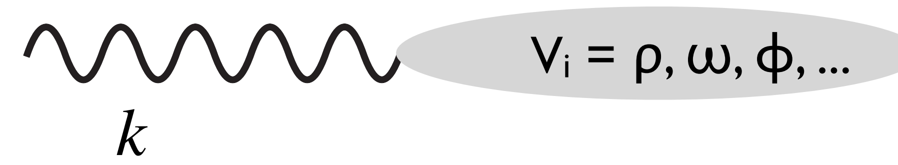
- PDG: 9 resonances, decay channels, angular distributions
- Regge parametrization at higher energy
- Direct contribution: fit to difference to data

Many measurements available, still approximations necessary

SOPHIA (Mücke et al. CPC124, 2000)

Lifetime of fluctuations

Consider photon with momentum k



Heisenberg uncertainty relation $\Delta E \, \Delta t \approx 1$

Length scale (duration) of hadronic interaction $\Delta t_{\text{int}} < 1\text{fm} \approx 5\text{GeV}^{-1}$

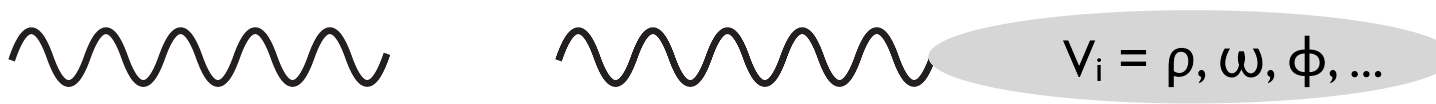
$$\Delta t \approx \frac{1}{\Delta E} = \frac{1}{\sqrt{k^2 + m_V^2} - k} = \frac{1}{k(\sqrt{1 + m_V^2/k^2} - 1)} \approx \frac{2k}{m_V^2}$$

Fluctuation long-lived for k > 3 GeV

$$\Delta t \approx \frac{2k}{m_V^2} > \Delta t_{\text{int}}$$

Multiparticle production: vector meson dominance

Photon is considered as superposition of ``bare´´ photon and hadronic fluctuation

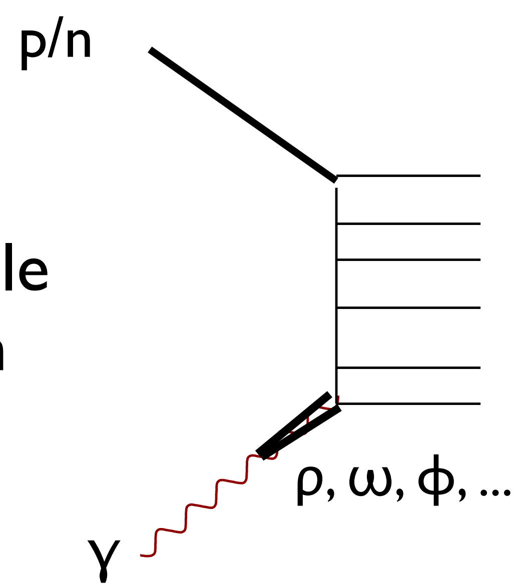


$$|\gamma\rangle = |\gamma_{\rm bare}\rangle + P_{\rm had} \sum_i |V_i\rangle$$

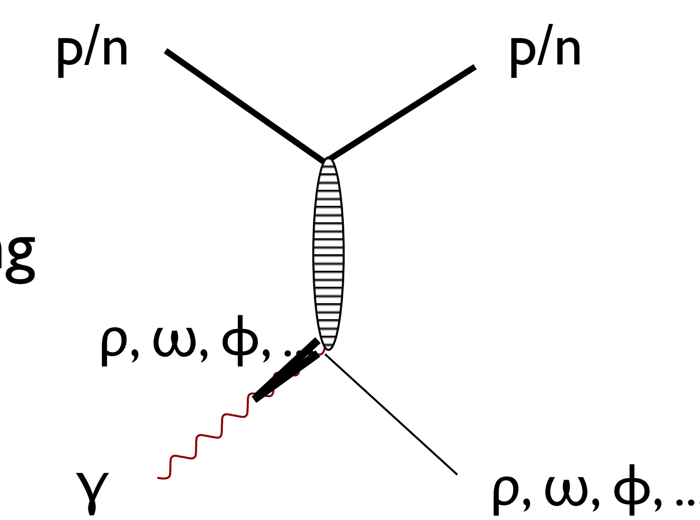
$$P_{\rm had} \approx \frac{1}{300} \dots \frac{1}{250}$$

Cross section for hadronic interaction ~1/300 smaller than for pi-p interactions

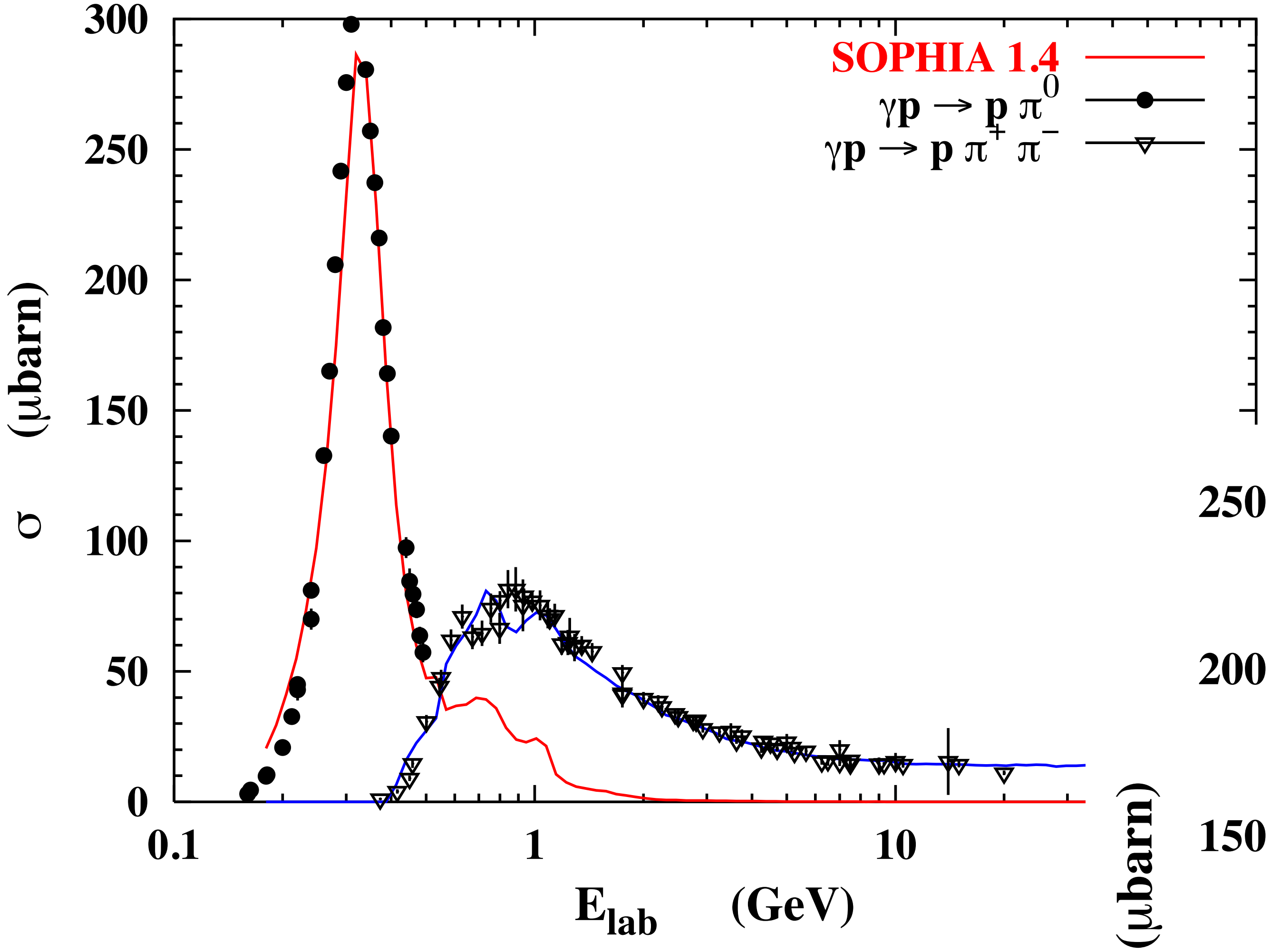
Multiparticle production

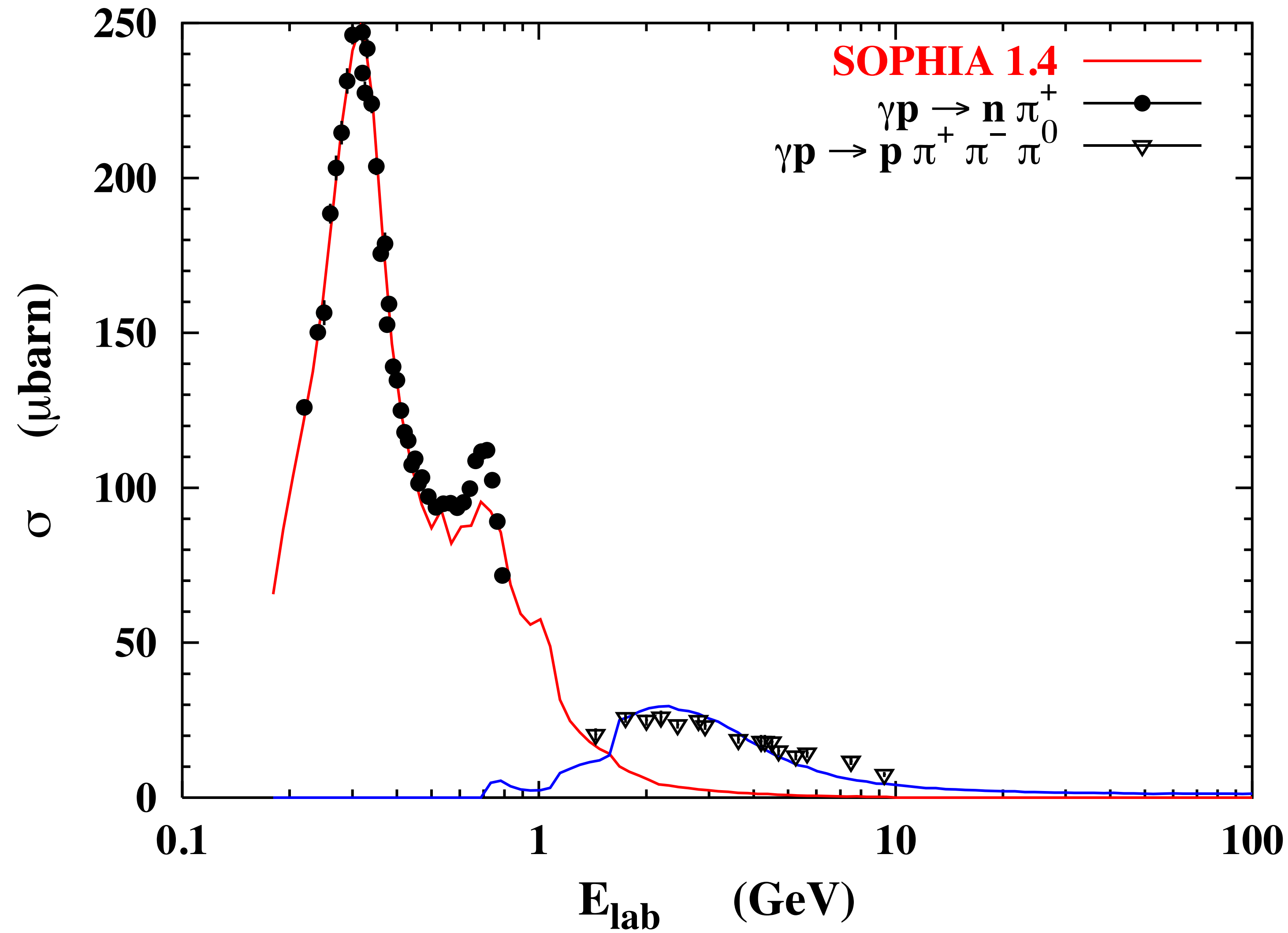


Elastic scattering



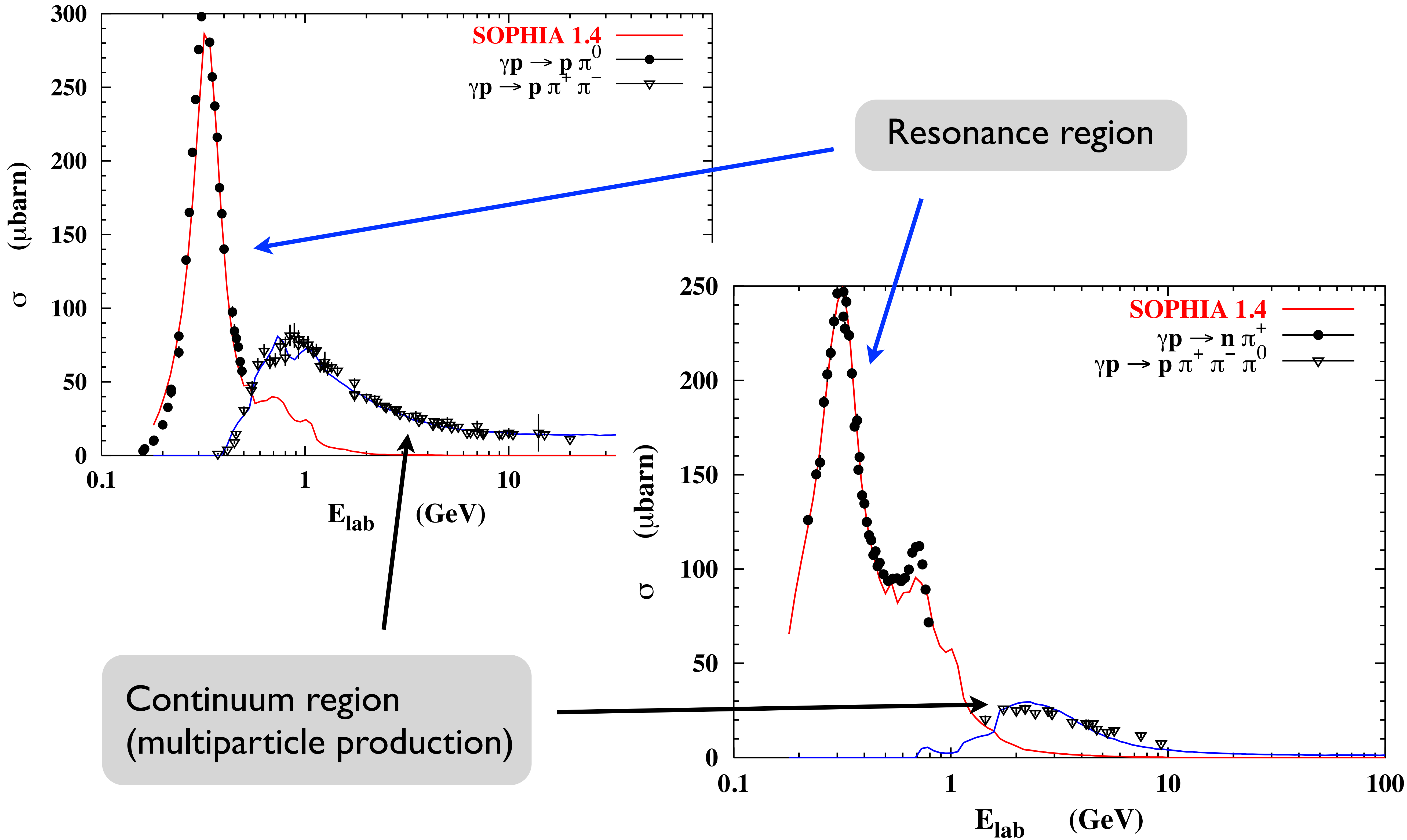
Comparison with measured partial cross sections





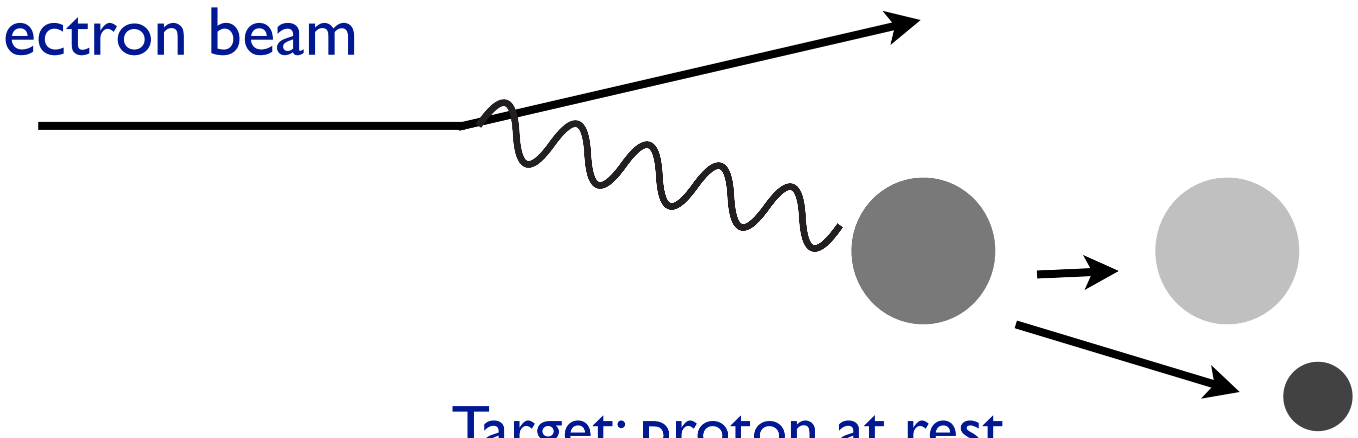
SOPHIA (Mücke et al. CPC124, 2000)

Comparison with measured partial cross sections

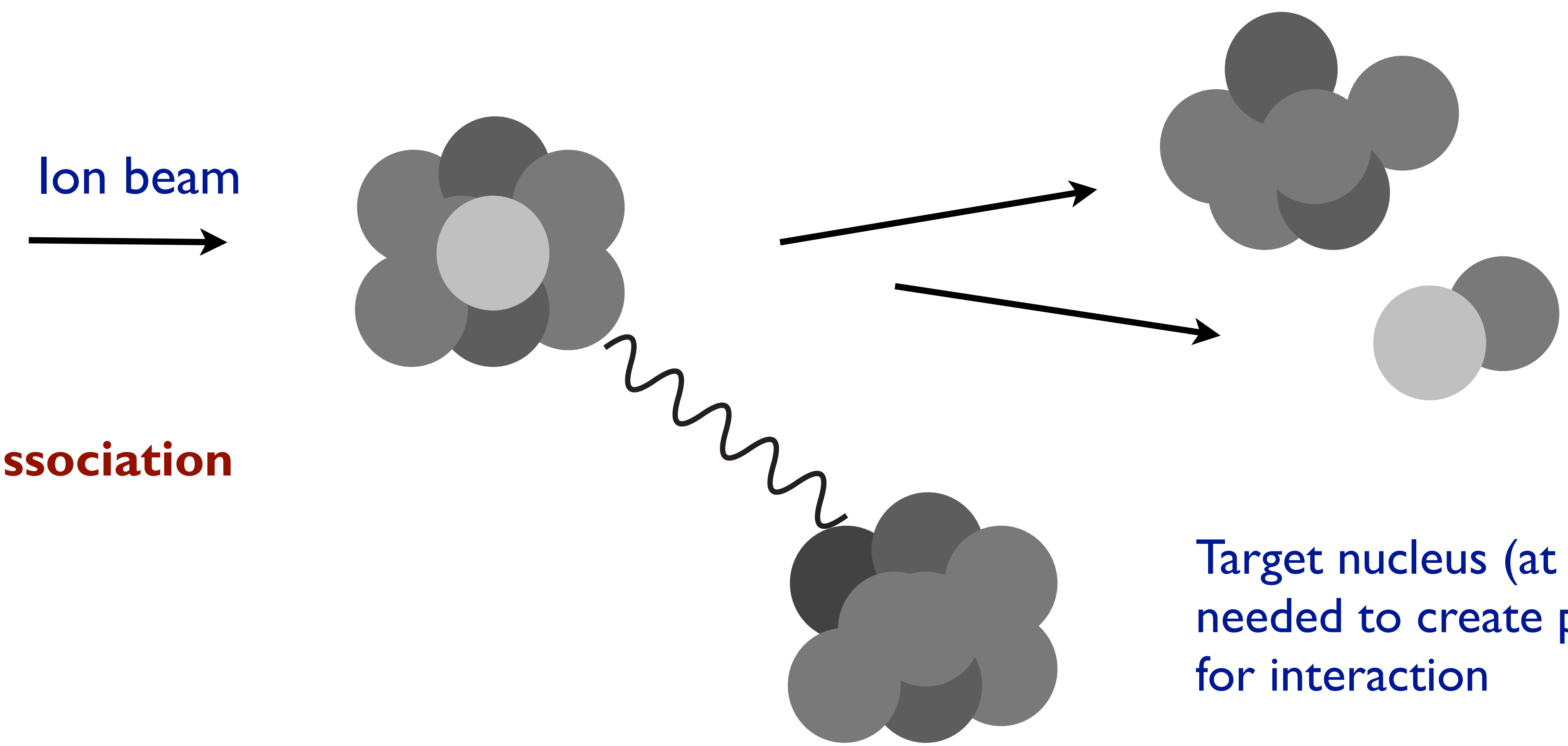


Measurement of nucleus disintegration

Photoproduction



Photodissociation

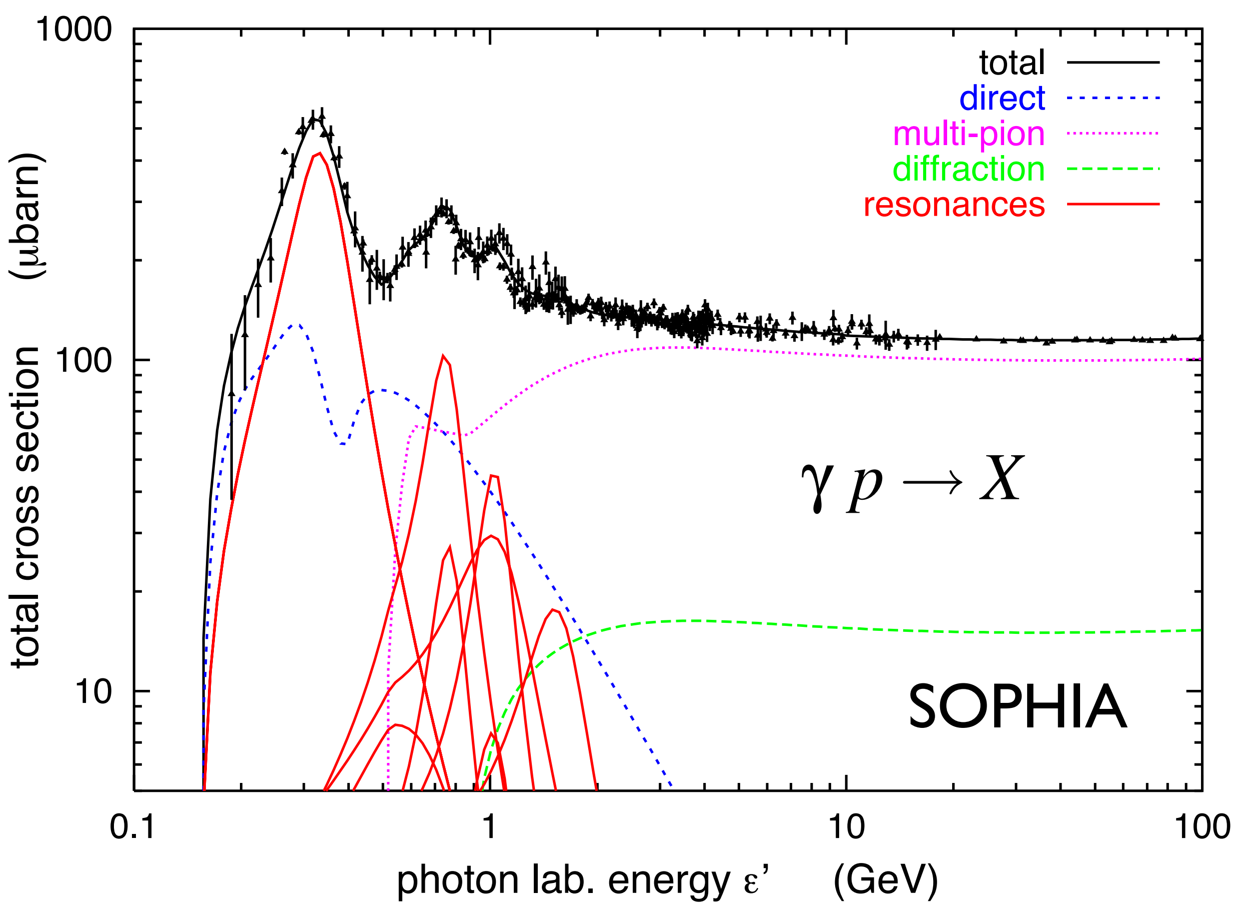


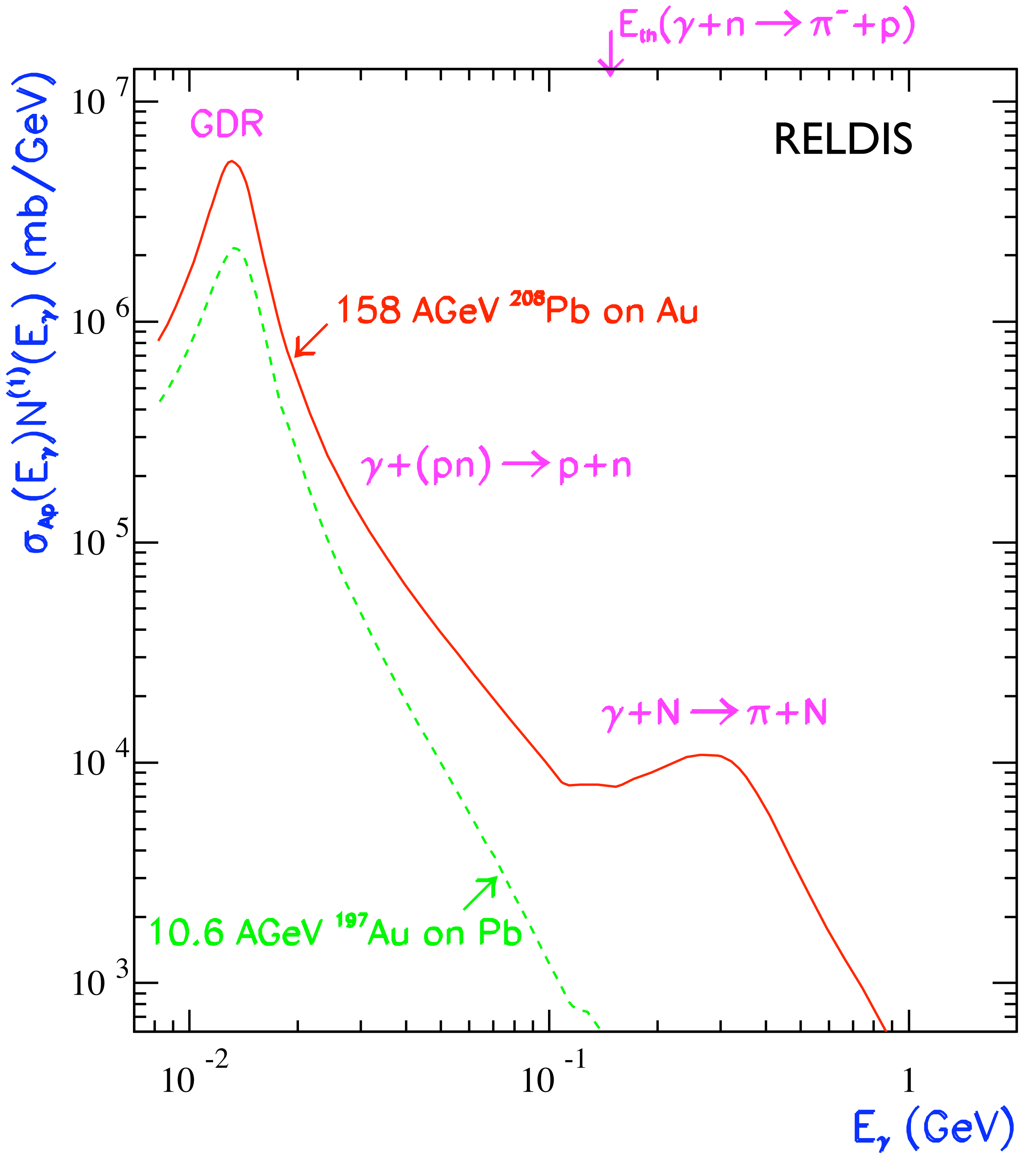
Effective em. dissociation cross section

Main contribution:
giant dipole resonance

Dominant emission processes:

- single nucleon
- quasi-deuteron
- alpha particle

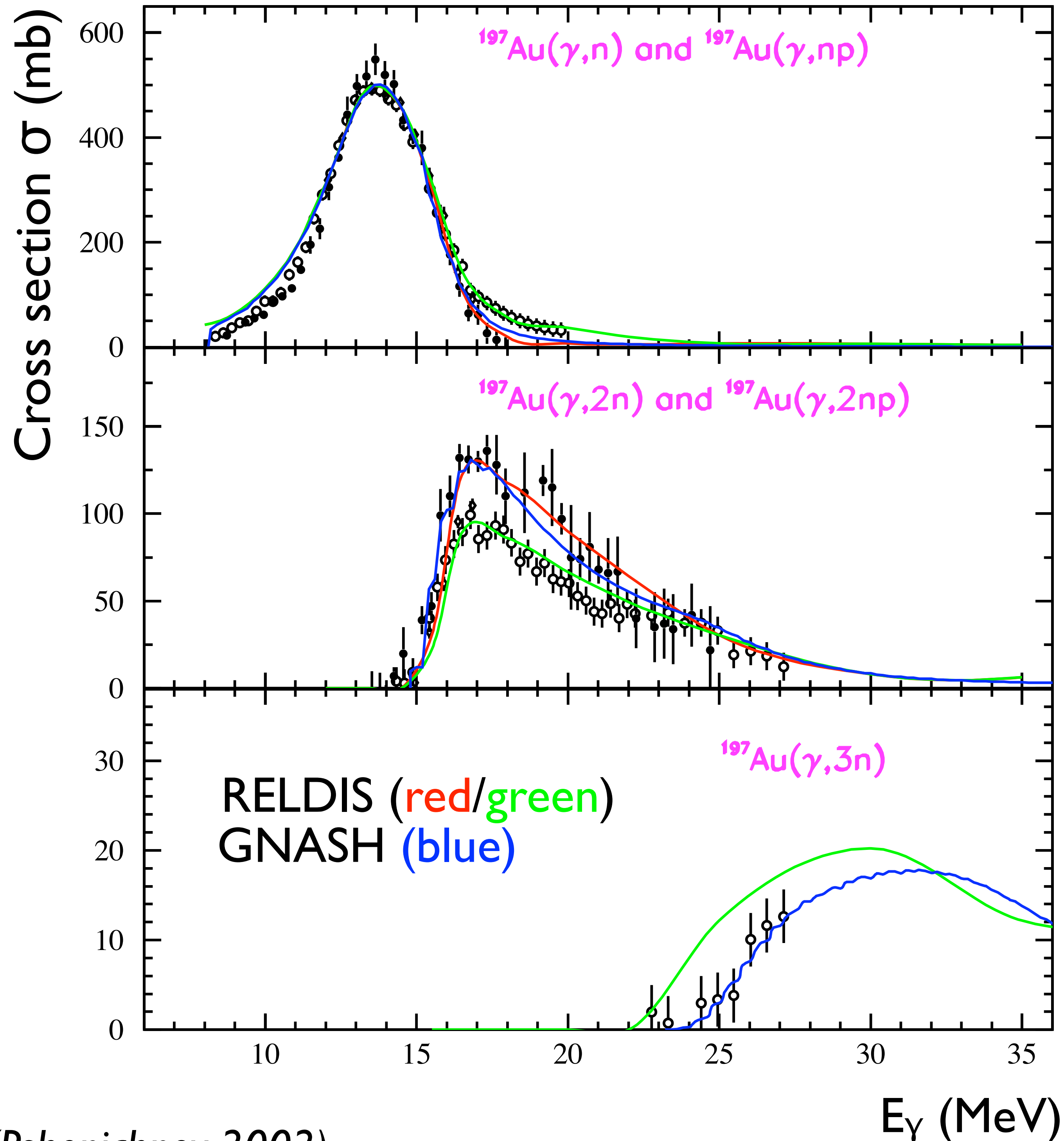




(Pshenichnov 2002)

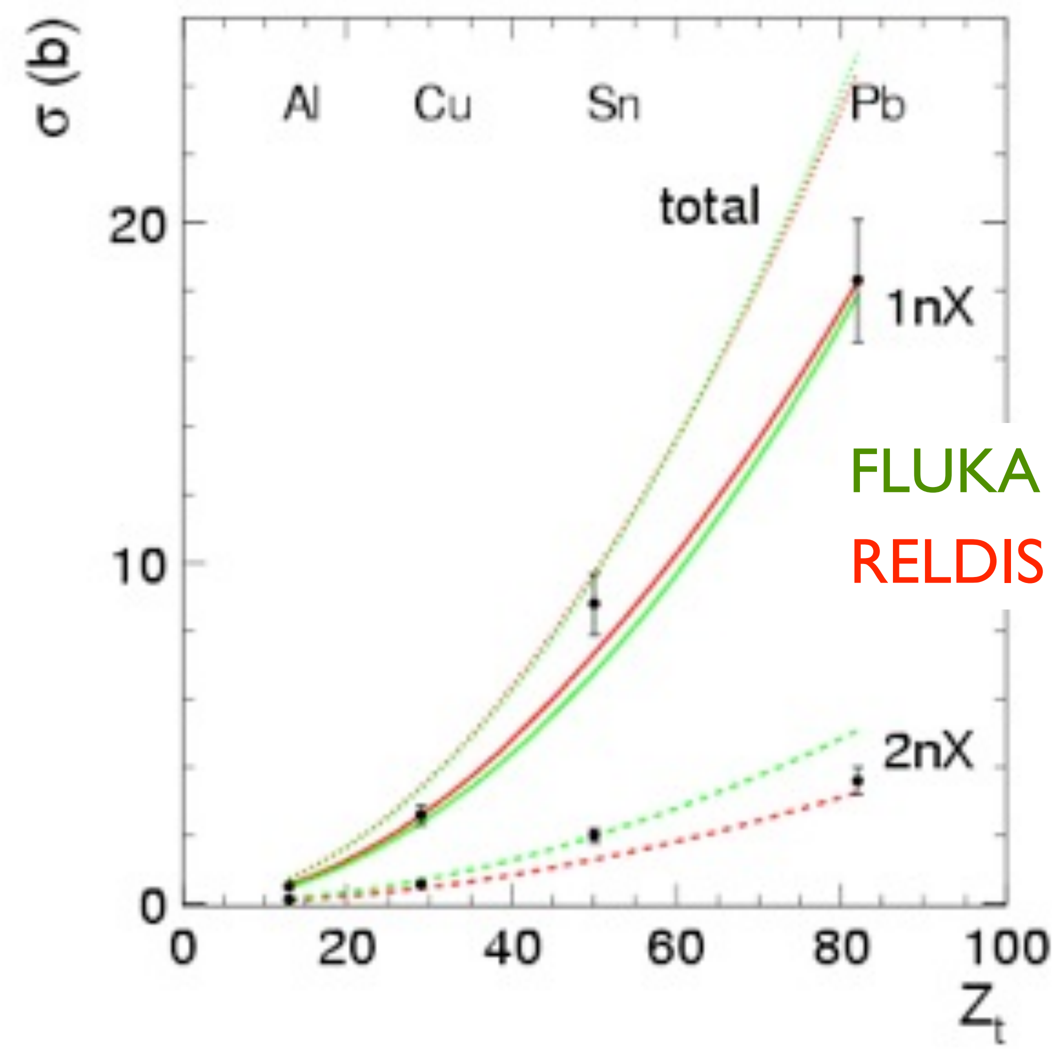
Example: photo-dissociation of nuclei

Saclay & Livermore data



(Pshenichnov 2002)

Projectile: 30 AGeV Pb,
different targets



(Smirnov, 2005)

Energy considerations

Energy of nucleus needed for formation of giant dipole resonance in CMB

Nucleus at rest

13 MeV

$$
\begin{aligned}
s &= (p_\gamma + p_A)^2 \\
&= p_\gamma^2 + p_A^2 + 2(p_\gamma \cdot p_A) \\
&= (Am_p)^2 + 2Am_pE_\gamma
\end{aligned}
$$

Iron: $E_A \sim 3\ 10^{20}$ eV
Helium: $E_A \sim 2\ 10^{19}$ eV

Nucleus with E_A in CMB field

10^{-3} eV

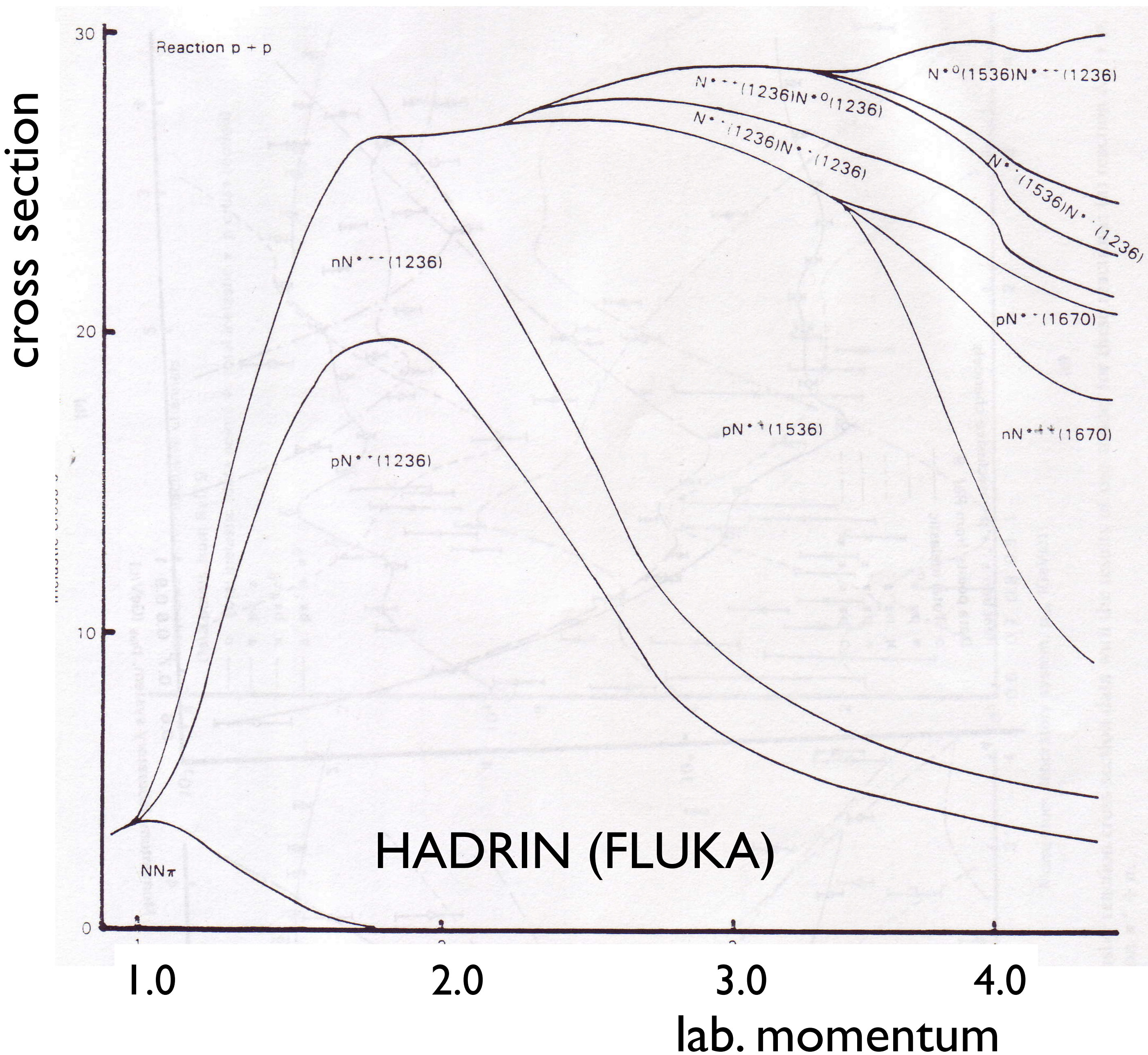
$$s = (Am_p)^2 + 2E_\gamma^{\text{CMB}} E_A (1 - \cos\theta)$$

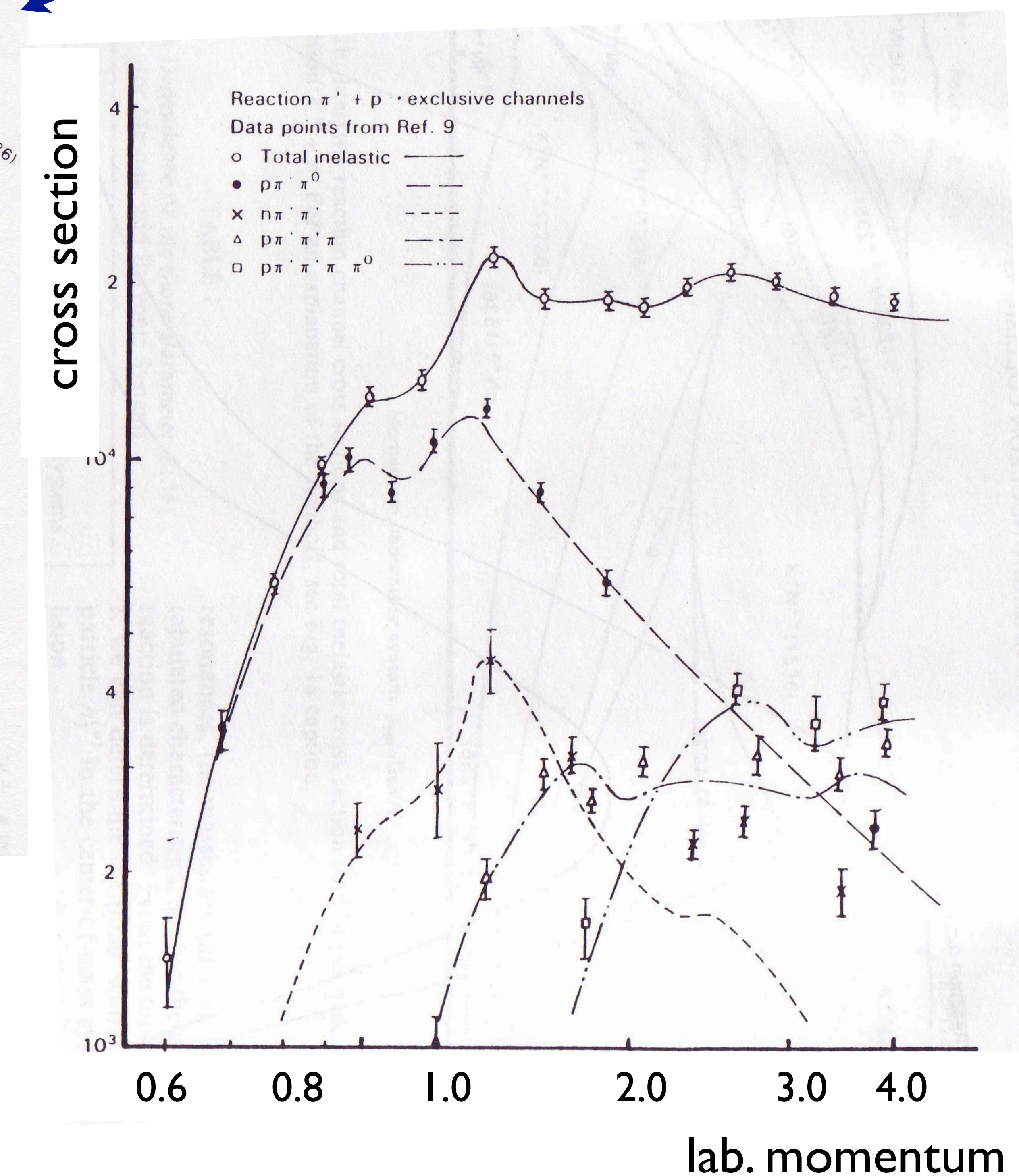
$$E_\gamma^{\text{CMB}} \geq A \frac{m_p E_\gamma}{(1-\cos\theta)E_A}$$

Light nuclei disintegrate very fast while traveling through CMB

Example: resonances in HADRIN

Large number of resonances





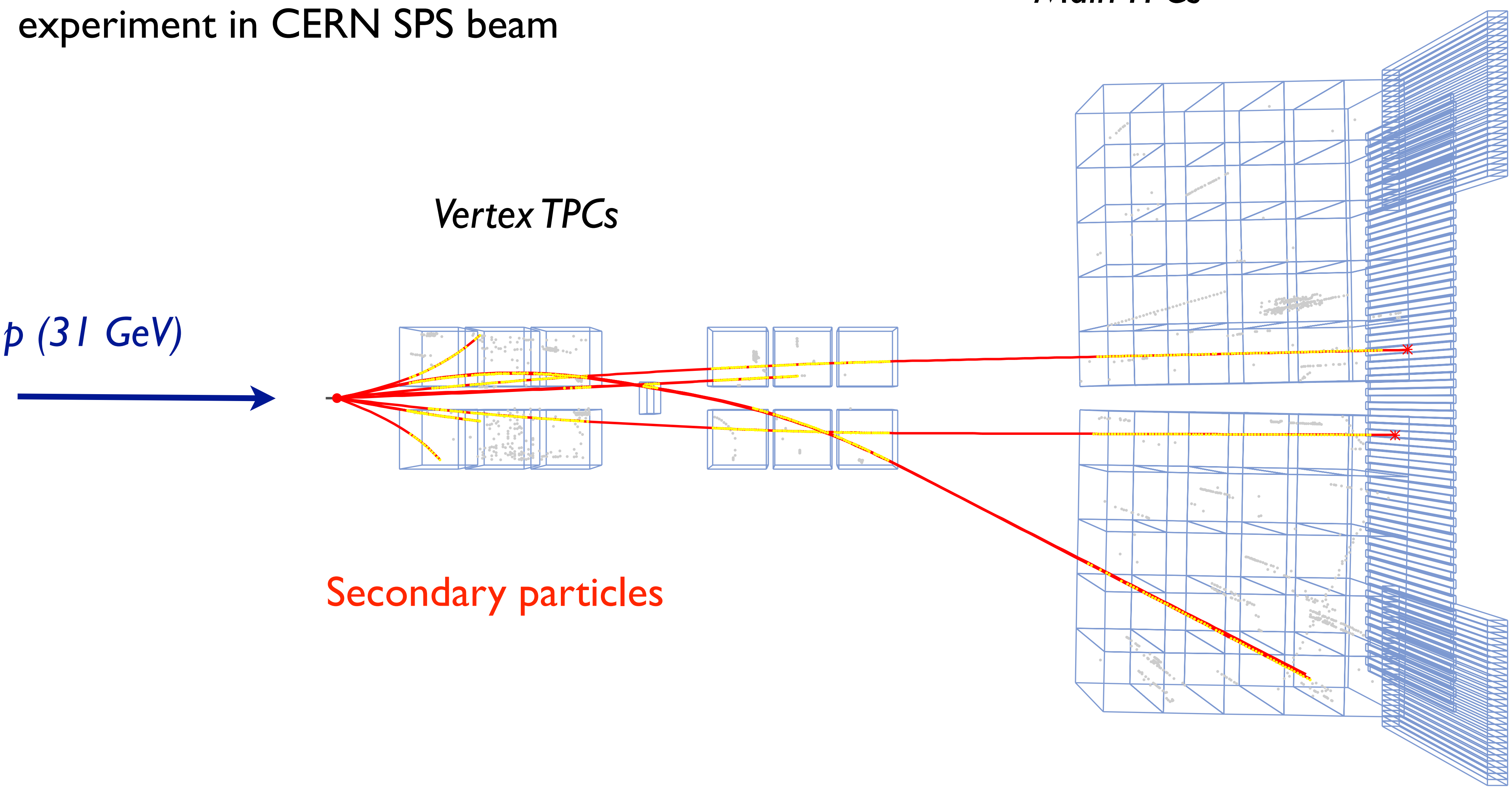
(Hänßgen, Ranft, CPC 39, 1984)

Particle production in intermediate energy range:

Two-string models

Example: p-C interaction at 30 GeV lab. momentum

NA61 experiment in CERN SPS beam

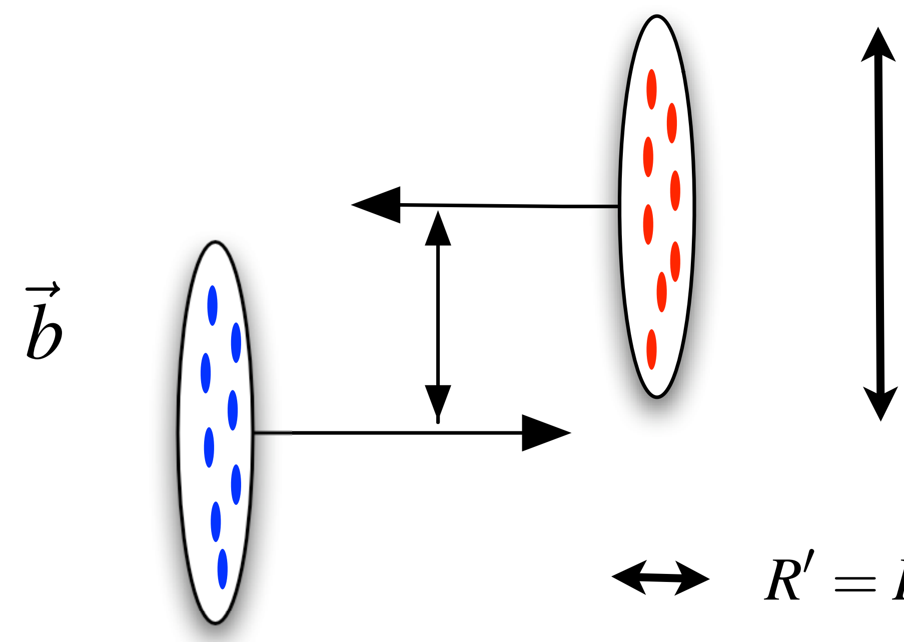


Typical particle multiplicities: 5 to 15 secondaries

Expectations from uncertainty relation

Assumptions:

- protons built up of partons
- partons liberated in collision process
- partons fragment into hadrons (pions, kaons,...) after interaction
- interaction viewed in c.m. system (other systems equally possible)



$$R \approx 1\text{fm} \approx 5\text{GeV}^{-1}$$

$$\leftrightarrow \quad R' = R/\Gamma = R\left(\frac{m_p}{E_p}\right)$$

Heisenberg uncertainty relation

$$\Delta x \, \Delta p_x \simeq 1$$

Longitudial momenta of secondaries

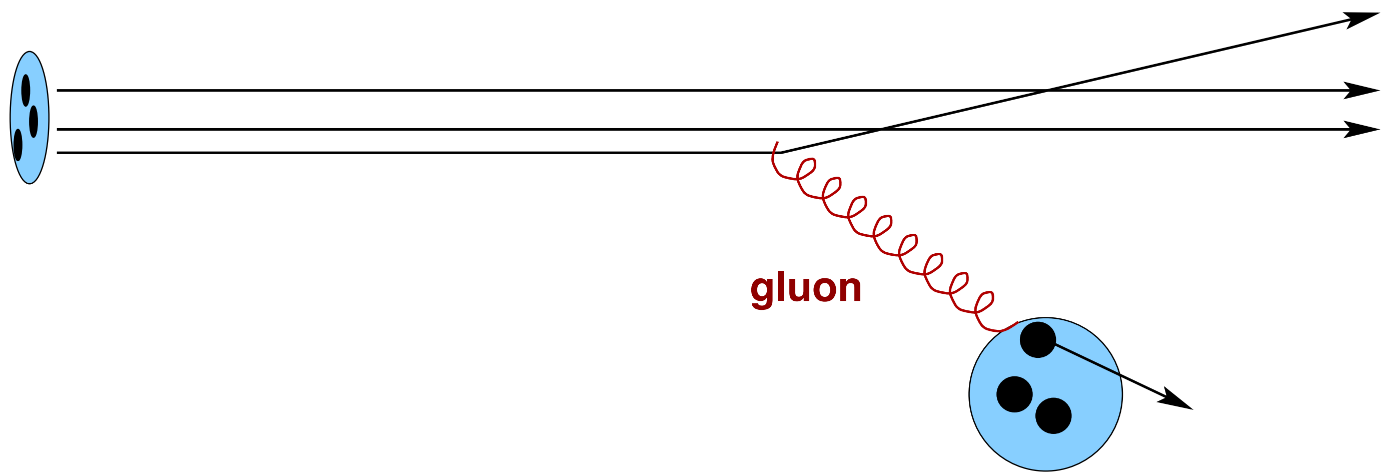
$$\langle p_\parallel \rangle \sim \Delta p_\parallel \approx \frac{1}{R'} \approx \frac{1}{5} E_p$$

Transverse momenta of secondaries

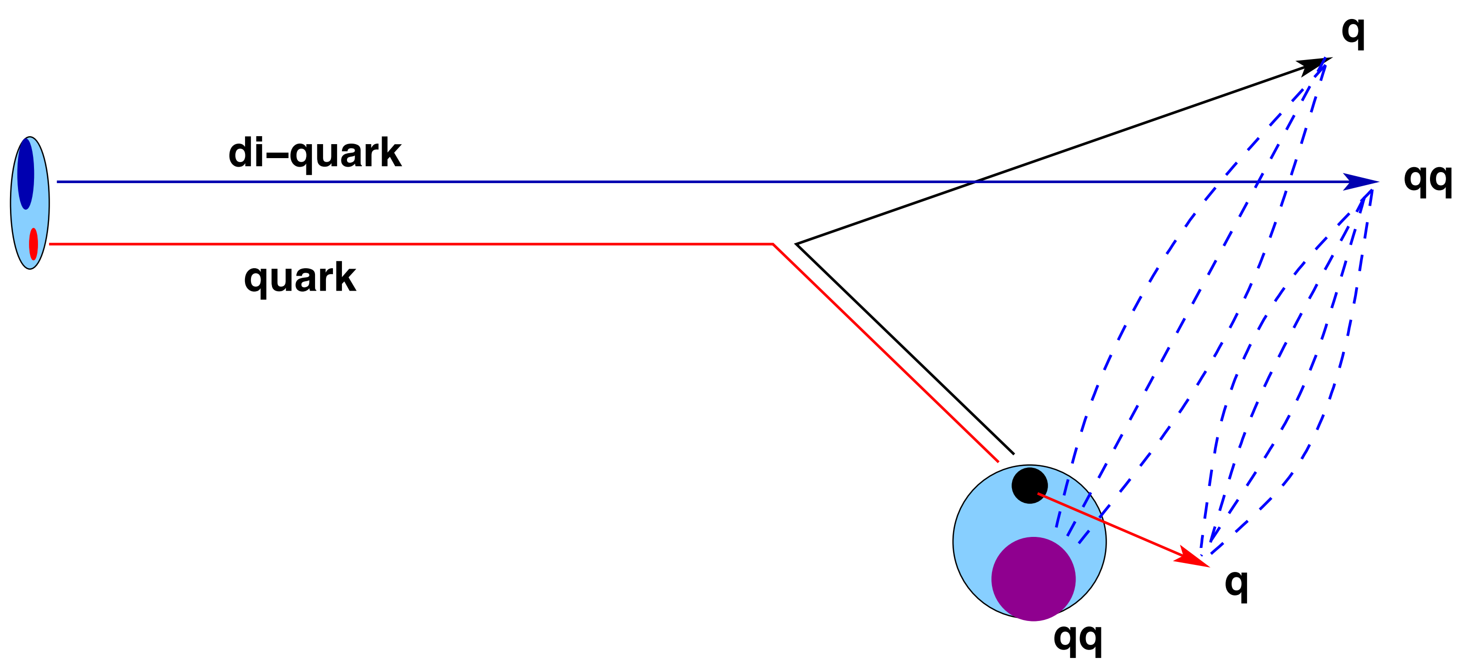
$$\langle p_\perp \rangle \sim \Delta p_\perp \sim \frac{1}{R} \approx 200\,\text{MeV}$$

QCD-inspired interpretation: color flow (i)

Partonic view:



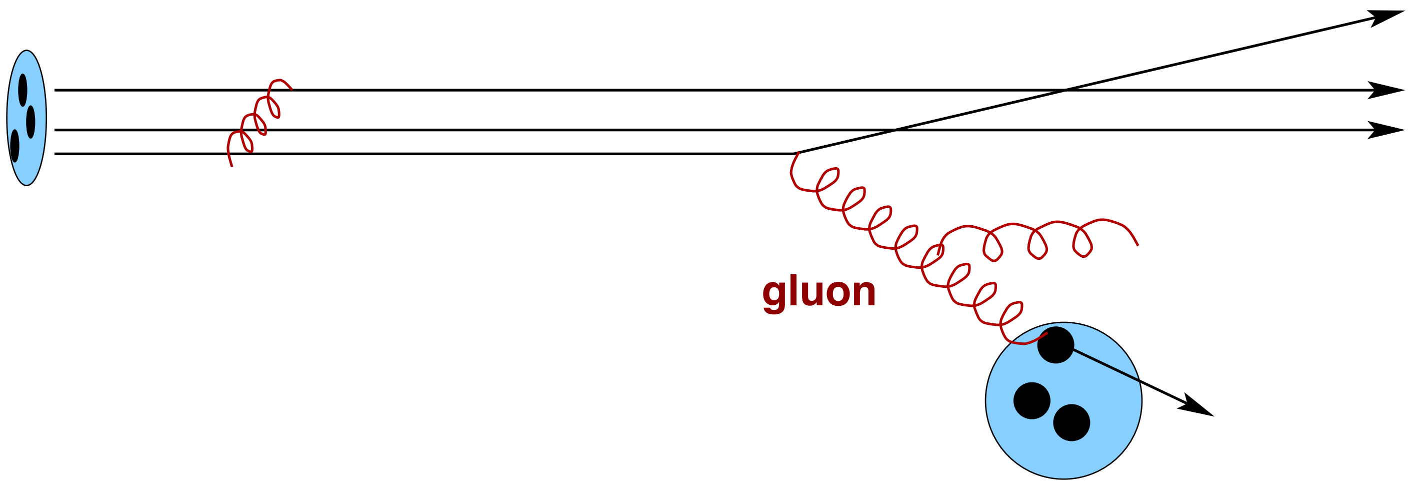
Color flow:



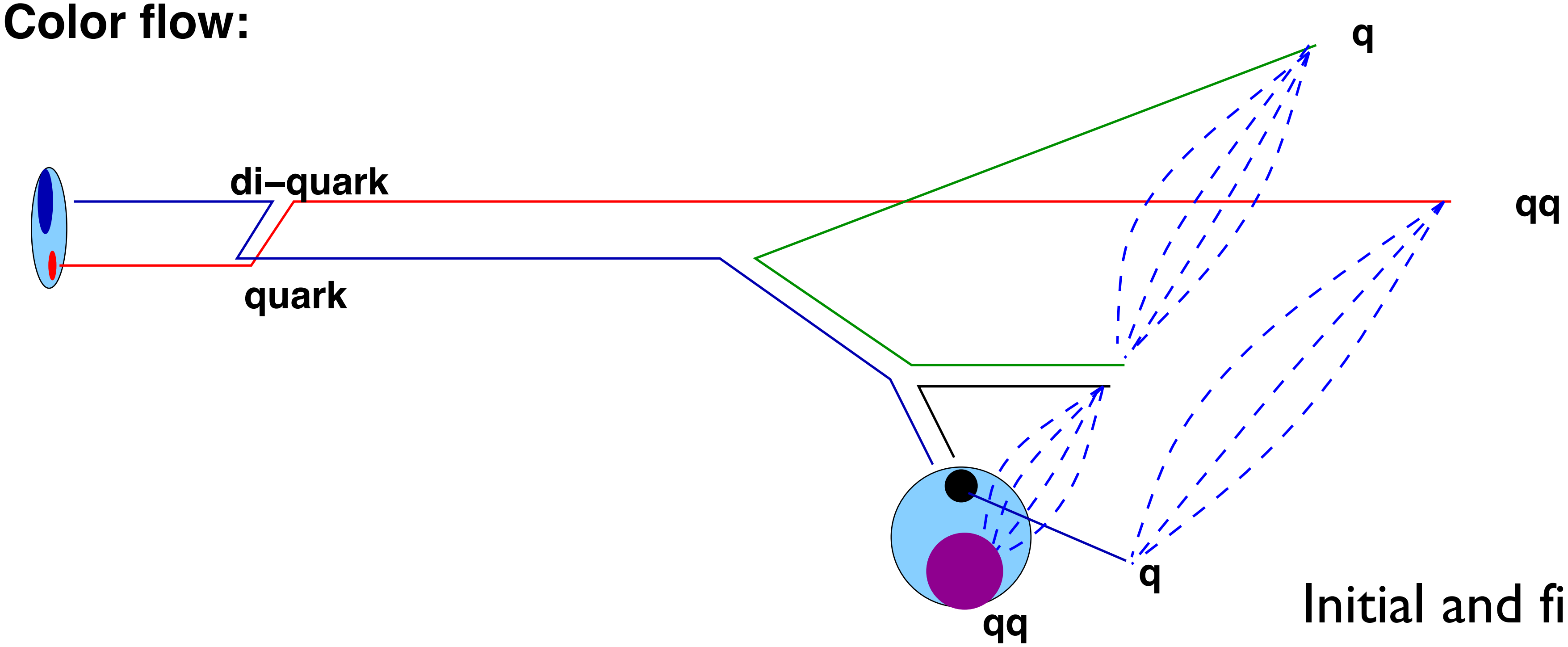
One-gluon exchange:
two color fields (strings)

QCD-inspired interpretation: color flow (ii)

Partonic view:



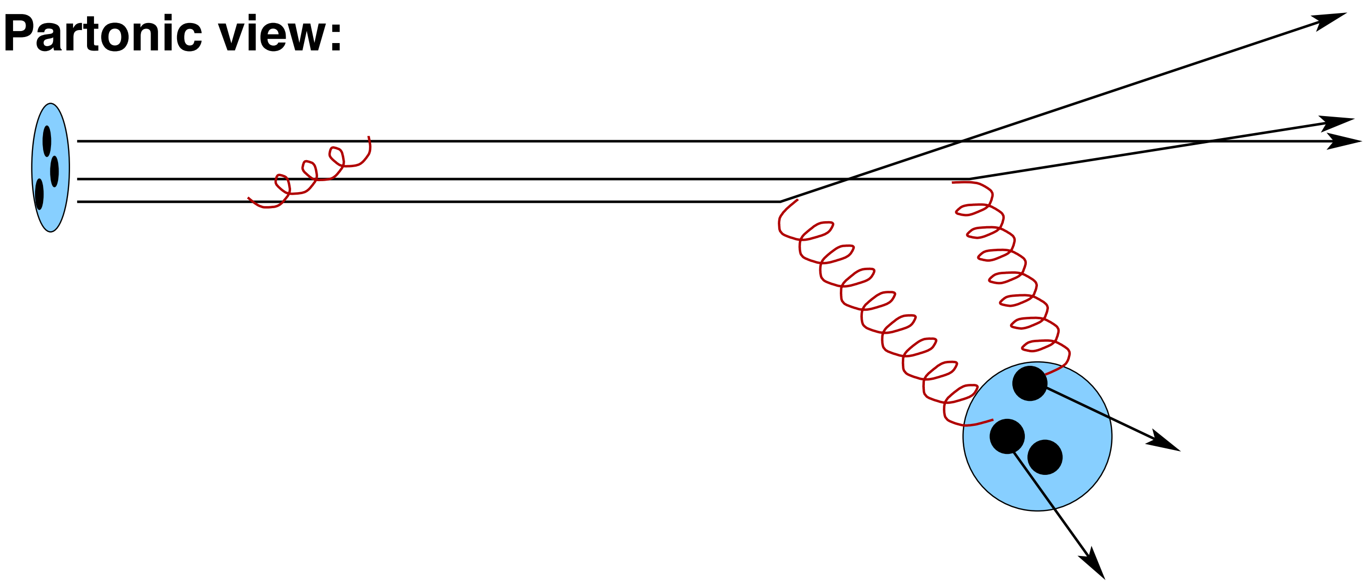
Color flow:



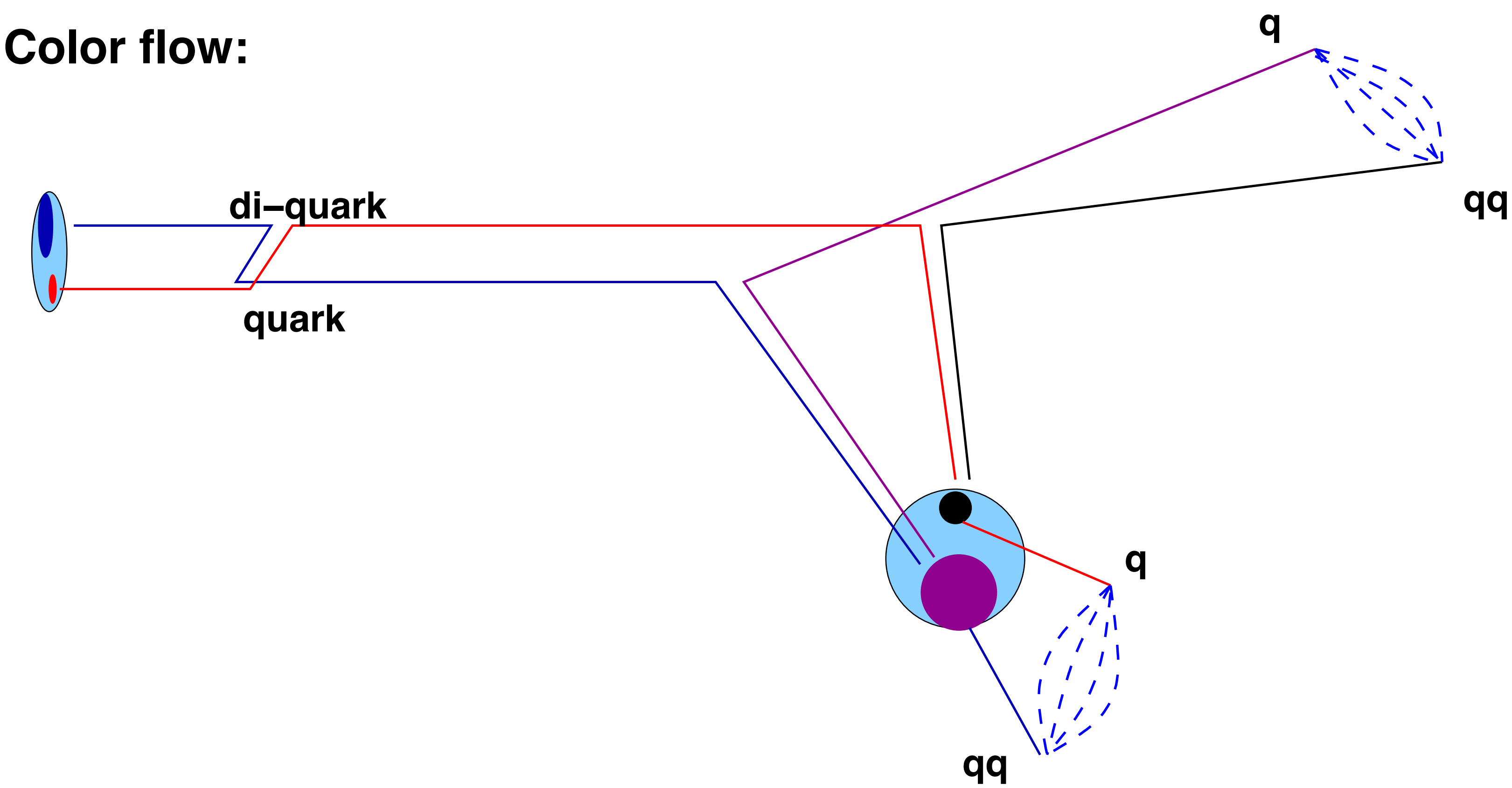
Initial and final state radiation does not change topology

QCD-inspired interpretation: color flow (iii)

Partonic view:



Color flow:



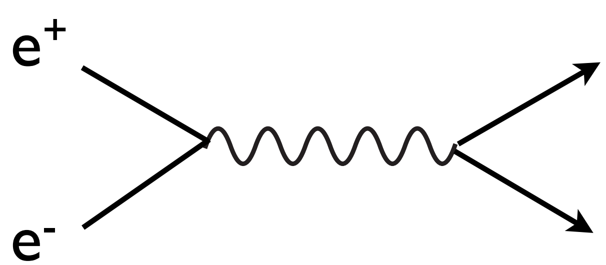
Two-gluon exchange: diffraction dissociation

DPMJET III, EPOS: detailed color flow simulation for each event

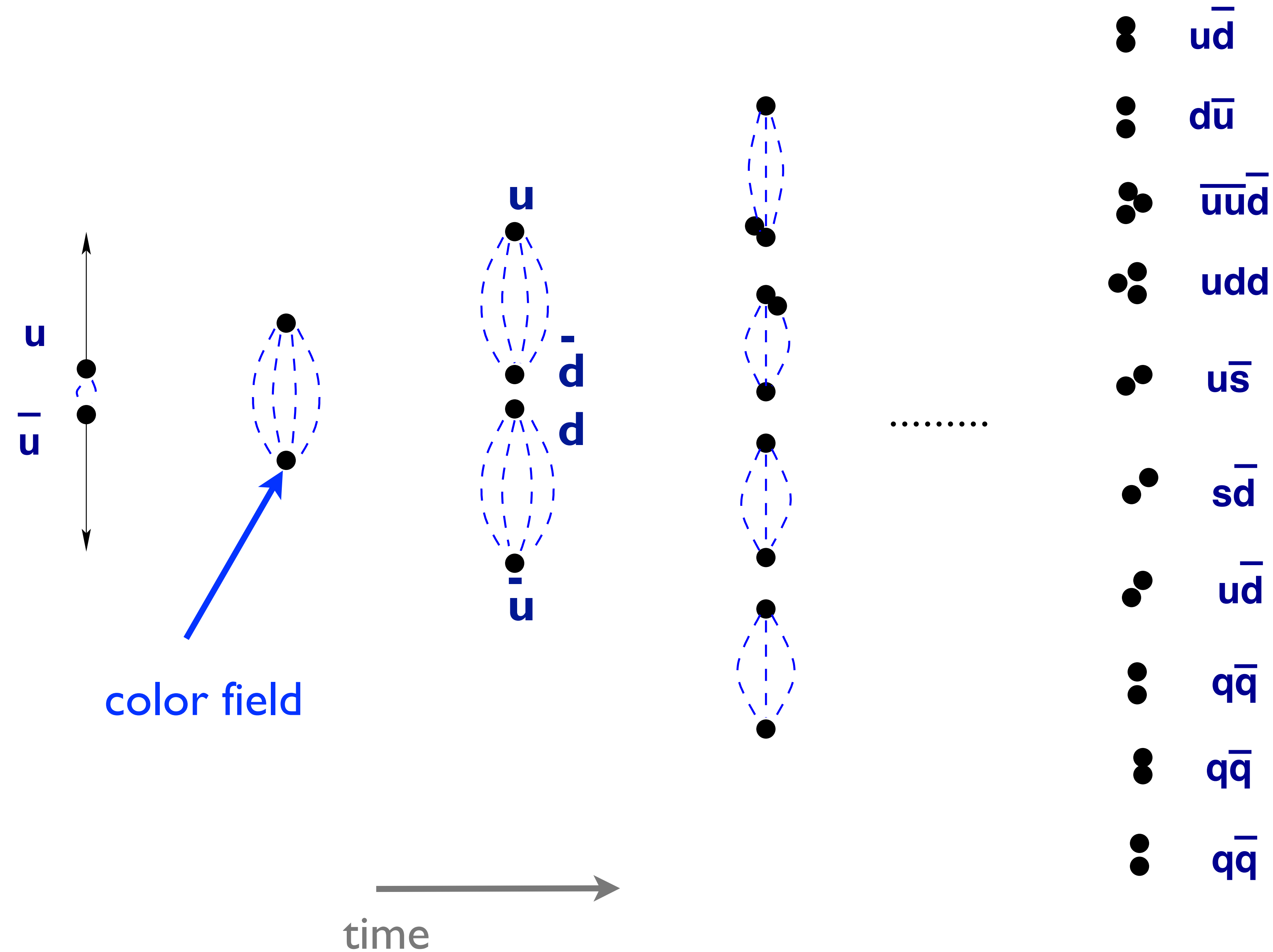
DPMJET II, SIBYLL, QGSJET 01: pomeron always only two-string configuration

Simplest case: e^+e^- annihilation into quarks

Annihilation at high energy



Quarks together are color-neutral system

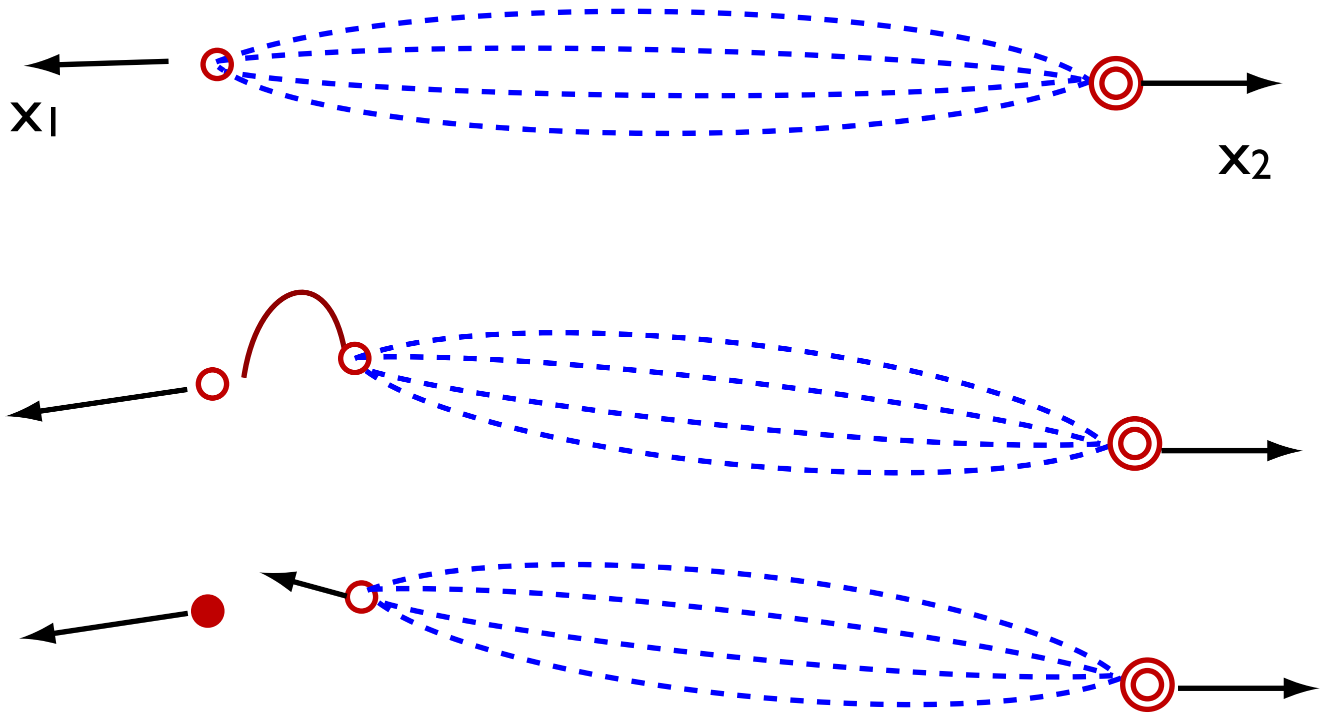


Chain of hadrons

String fragmentation

Fragmentation function (SIBYLL)

String characterized by momentum fractions of partons at ends



$$E_{\rm str} = \frac{1}{2}\sqrt{s}(x_1 + x_2)$$

$$p_{\rm str} = \frac{1}{2}\sqrt{s}(x_1 - x_2)$$

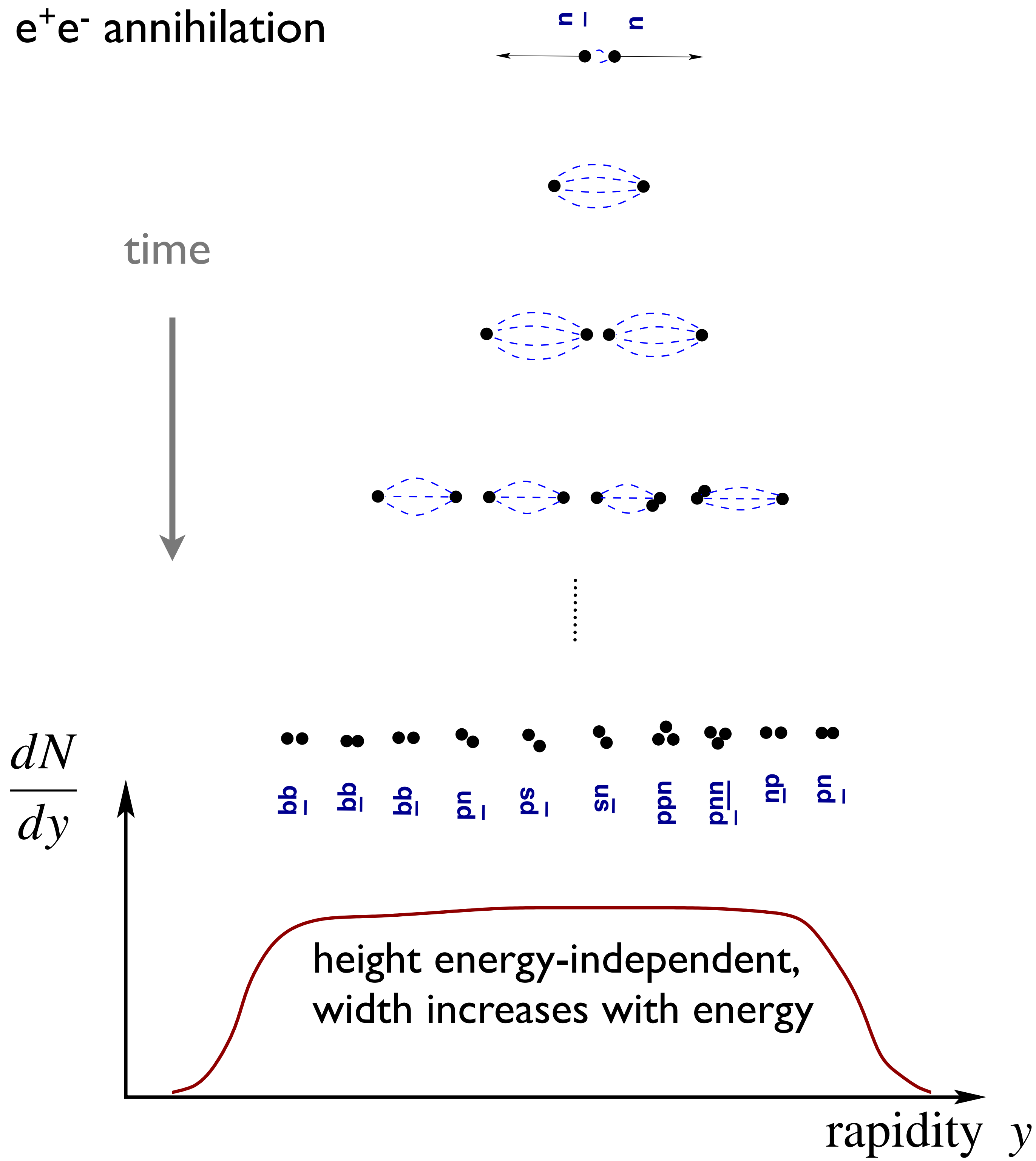
$$m_\perp^2 = p_\perp^2 + m^2$$

Momentum fraction z of new meson relative to quark at string end

$$f(z) = \frac{(1-z)^a}{z} \exp\left\{-\frac{bm_\perp^2}{z}\right\}$$

String fragmentation and rapidity

Example:
q-qbar pair produced
in e^+e^- annihilation



Rapidity

$$y = \frac{1}{2} \ln \frac{E + p_{\|}}{E - P_{\|}}$$

Rapidity of massless particles

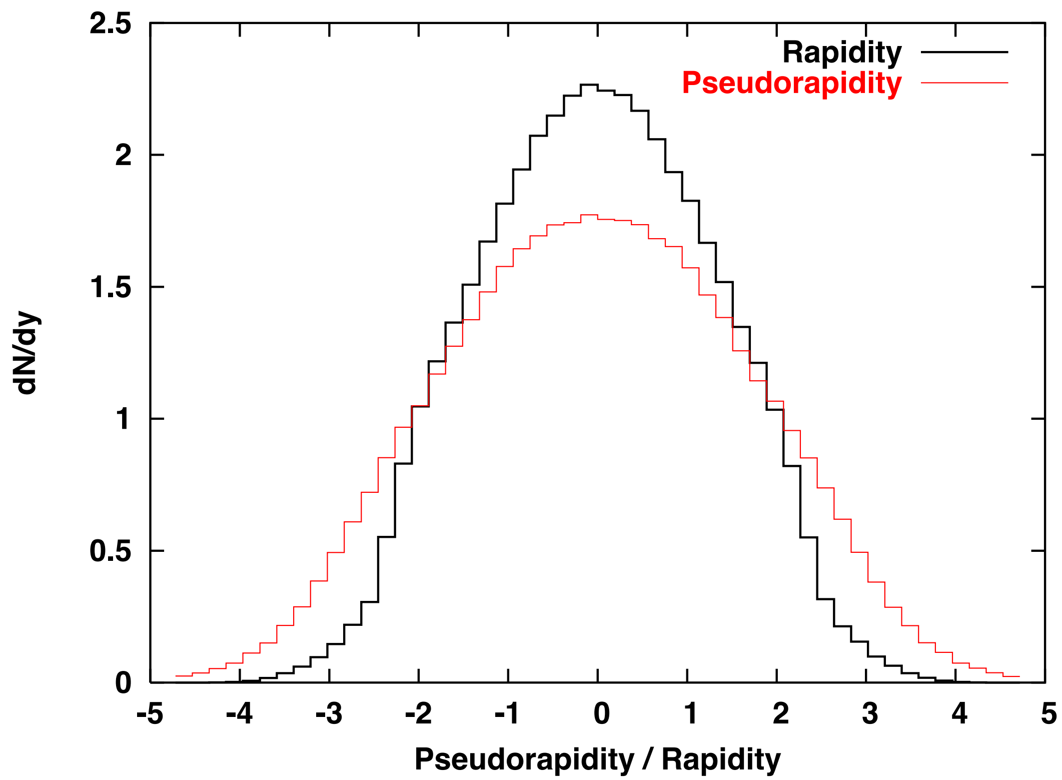
$$y = \frac{1}{2} \ln \frac{1 + \cos\theta}{1 - \cos\theta} = -\ln \tan \frac{\theta}{2}$$

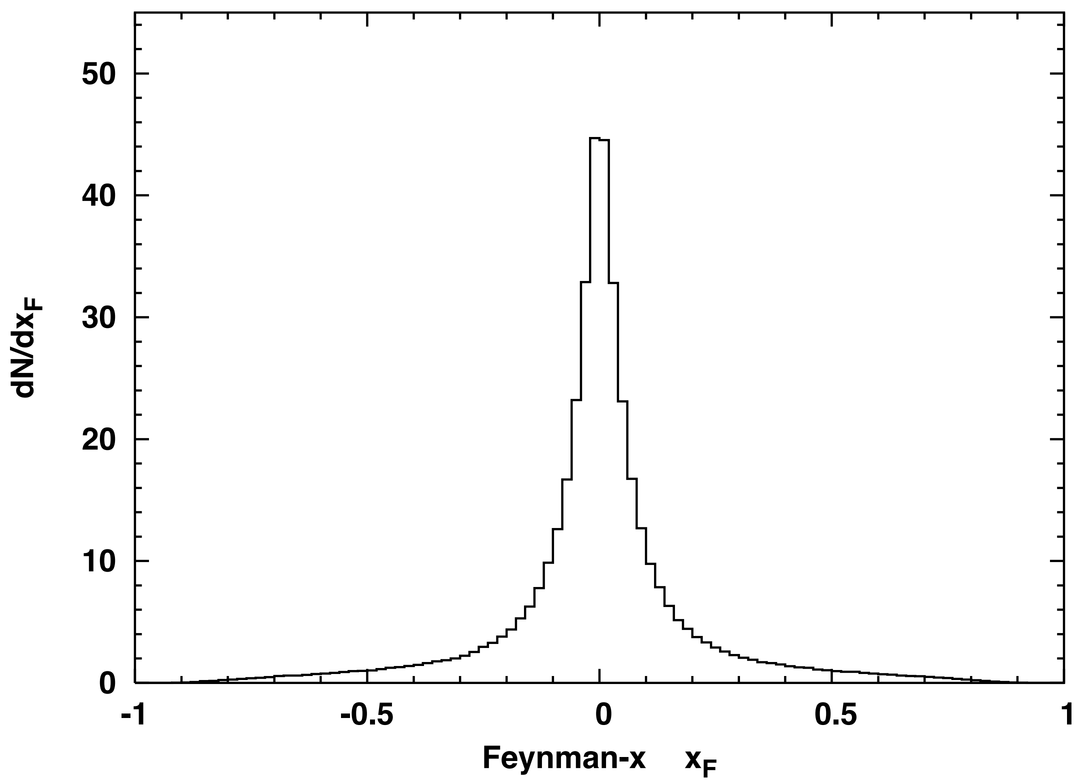
Pseudorapidity (all particles)

$$\eta = -\ln \tan \frac{\theta}{2}$$

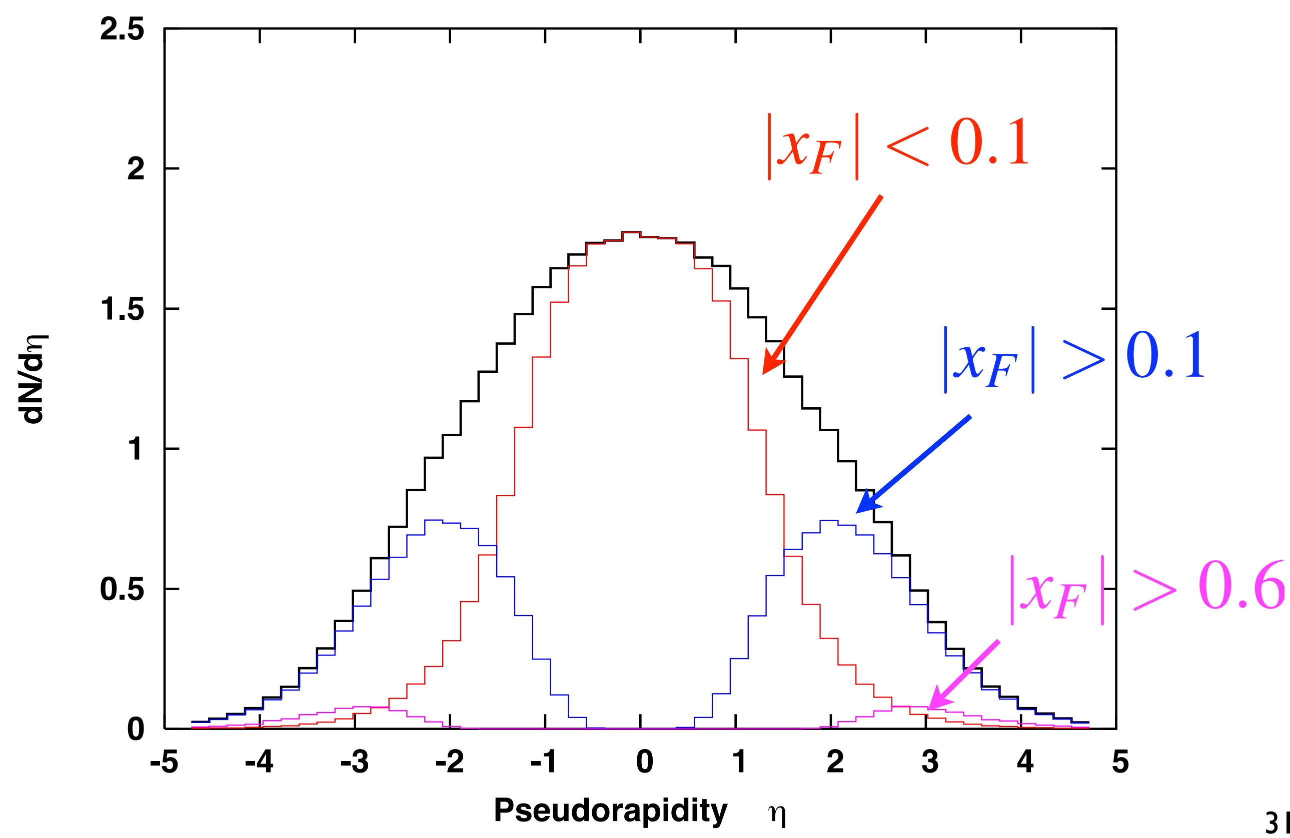
Rapidity and pseudorapidity

Example: 100 GeV p-p collisions,
charged secondaries





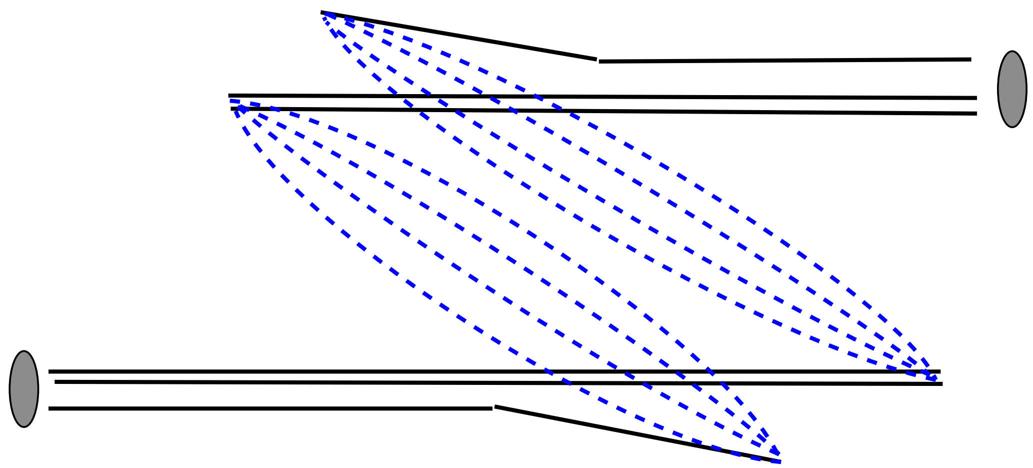
Rapidity and pseudorapidity very similar

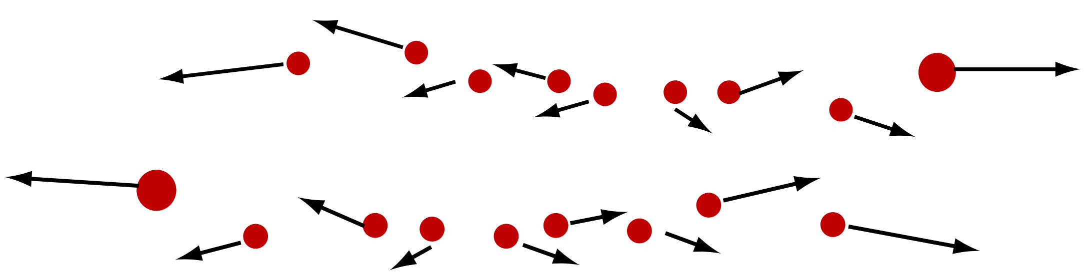


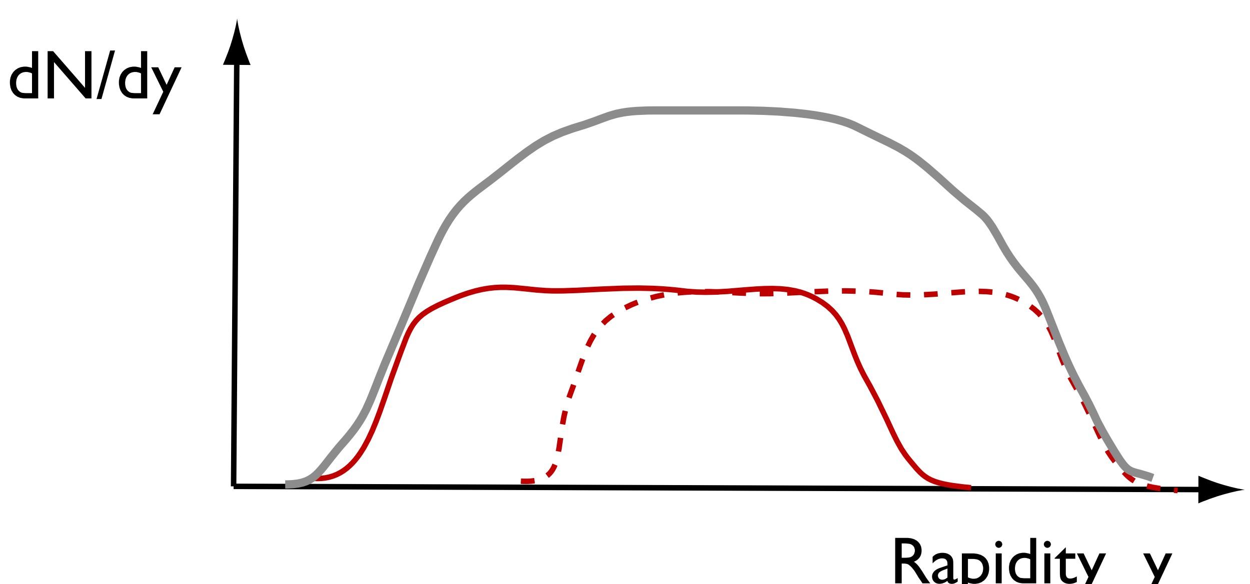
Feynman-x

$$x_F = \frac{p_\parallel}{p_{\max}} \approx \frac{2p_\parallel}{\sqrt{s}}$$

Predictions of two-string models







(Capella et al., Physics Reports 1994)

Two-string models:

- Feynman-scaling
- long-range correlations
- leading particle effect
- delayed threshold for baryon pair production

Feynman scaling

$$2E\,\frac{dN}{d^3p} = \frac{dN}{dy\,d^2p_\perp} \longrightarrow f(x_F, p_\perp)$$

Distribution independent of energy

$$\frac{dN}{dx} \approx \tilde{f}(x) \qquad\qquad x = E/E_{\text{prim}}$$

Momentum fractions: soft string ends

Asymmetric momentum sharing of valence quarks: most energy given to di-quark

Quark in nucleon
(example: SIBYLL)

$$f_{\mathrm{q|nuc}}(x) \sim \frac{(1-x)^3}{(x^2+\mu^2)^{\frac{1}{4}}}$$

Many other parametrizations work well in describing data (example: DPMJET)

$$f_{\mathrm{q|nuc}}(x) \sim \frac{(1-x)^{\frac{3}{2}}}{\sqrt{x}}$$

$$f_{\mathrm{q|mes}}(x) \sim \frac{1}{\sqrt{x(1-x)}}$$

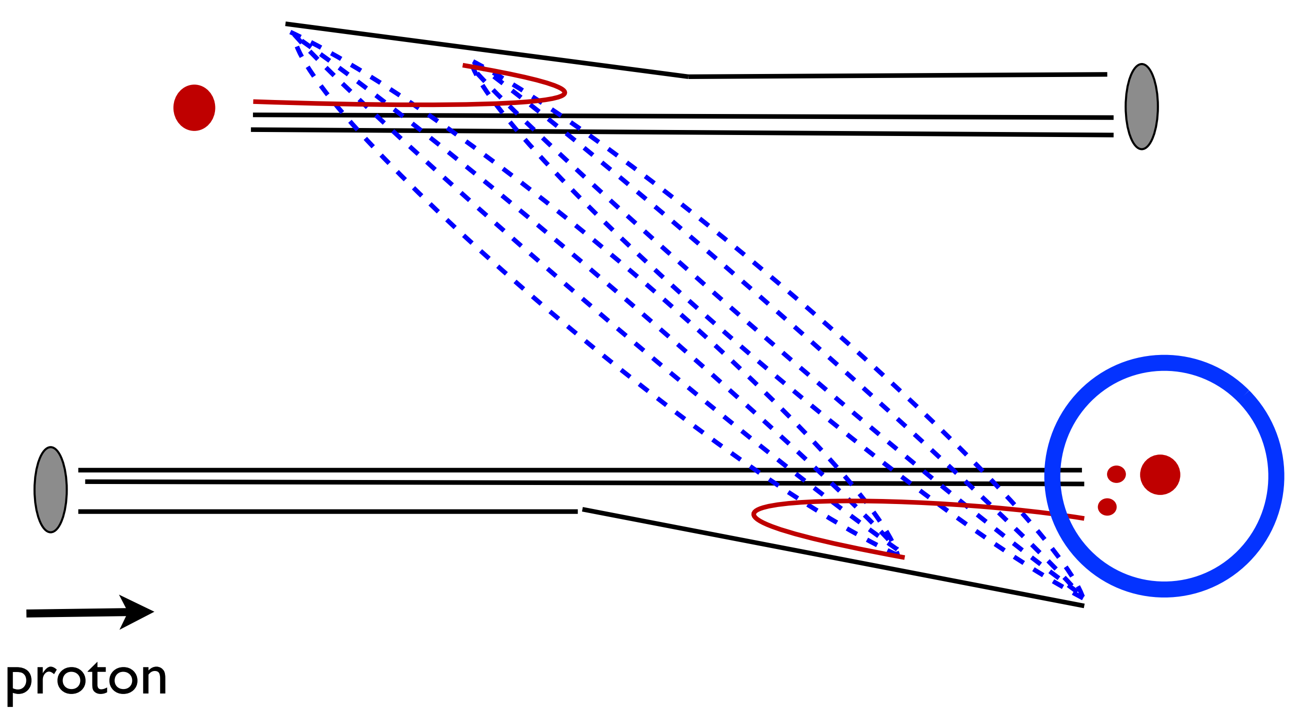
Sea quark momentum fractions

$$f_{\mathrm{q_{sea}}}(x) \sim \frac{1}{x}$$

or

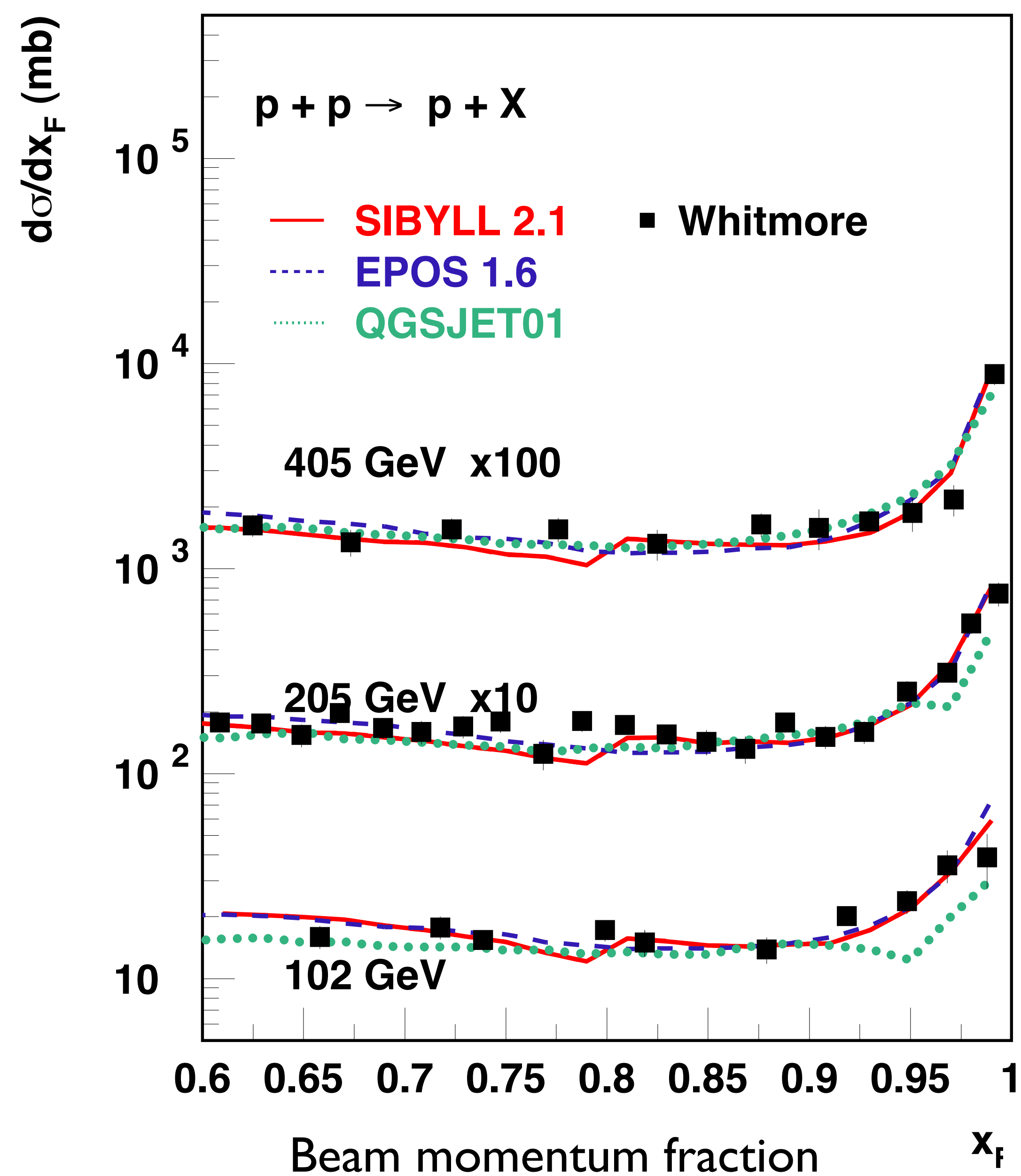
$$f_{\mathrm{q_{sea}}}(x) \sim \frac{1}{\sqrt{x}}$$

Particle production spectra (i)

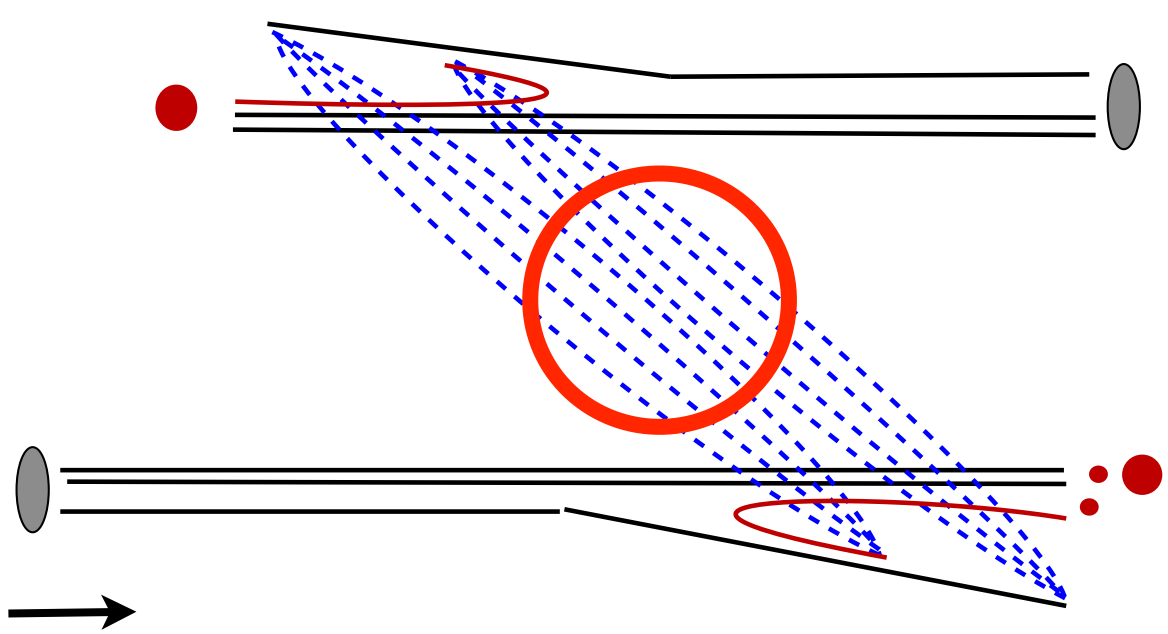


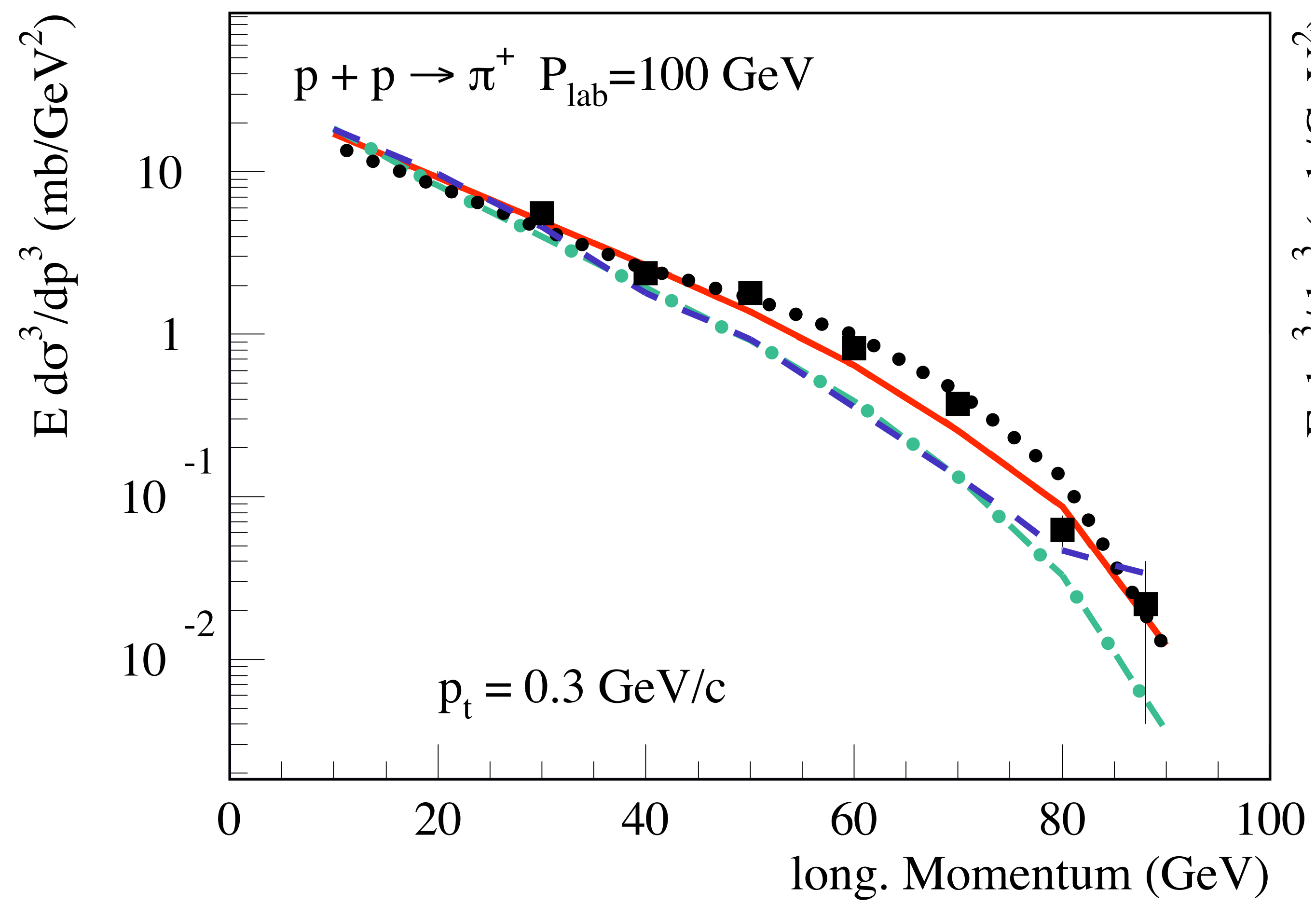
Fluctuations: Generation of sea quark anti-quark pair and leading/excited hadron

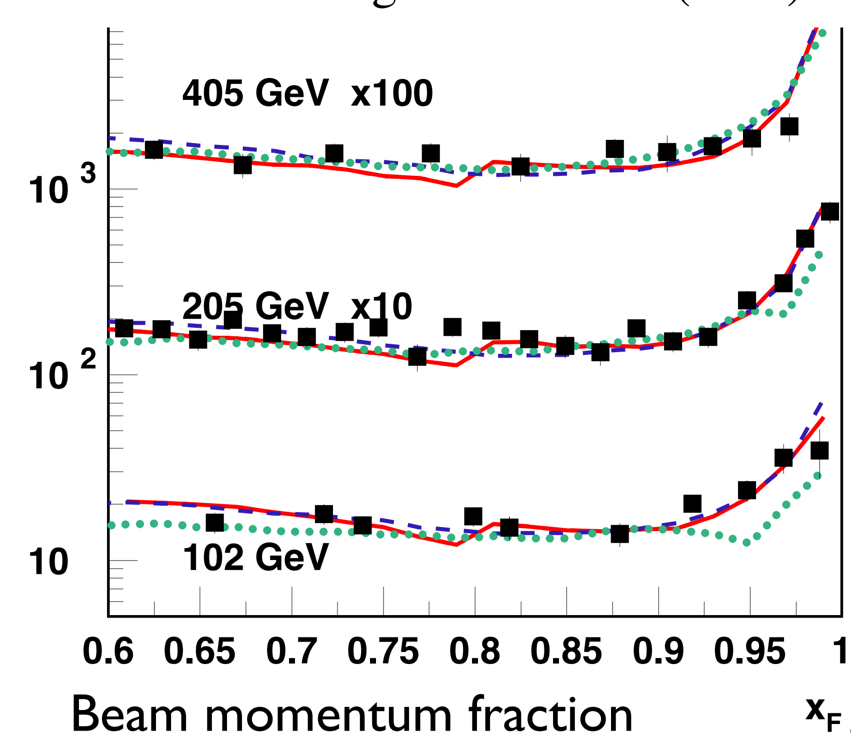
Leading particle effect



Particle production spectra (ii)



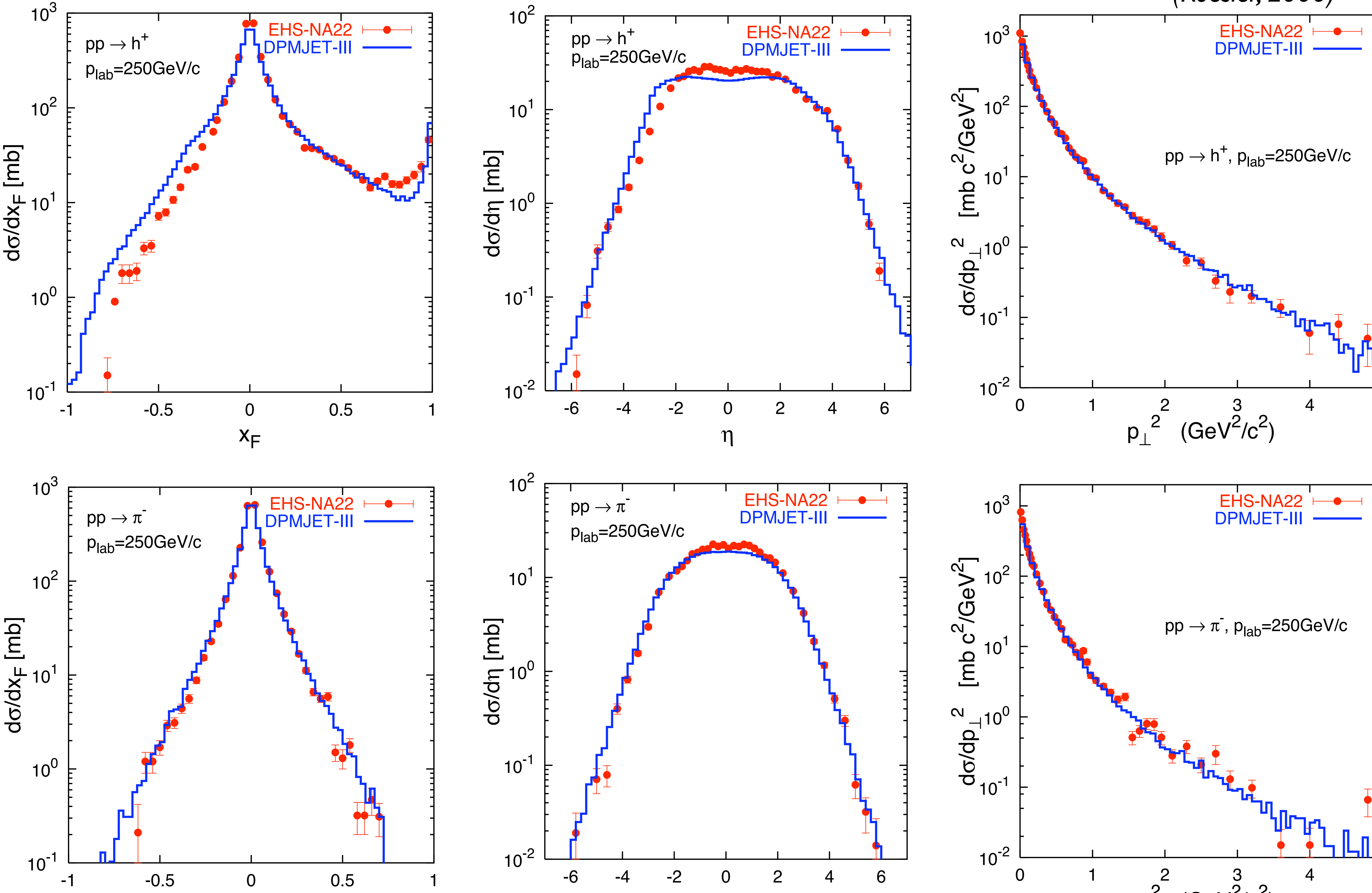




Fluctuations: Generation of sea quark anti-quark pair and leading/excited hadron

NA22 European Hybrid Spectrometer data

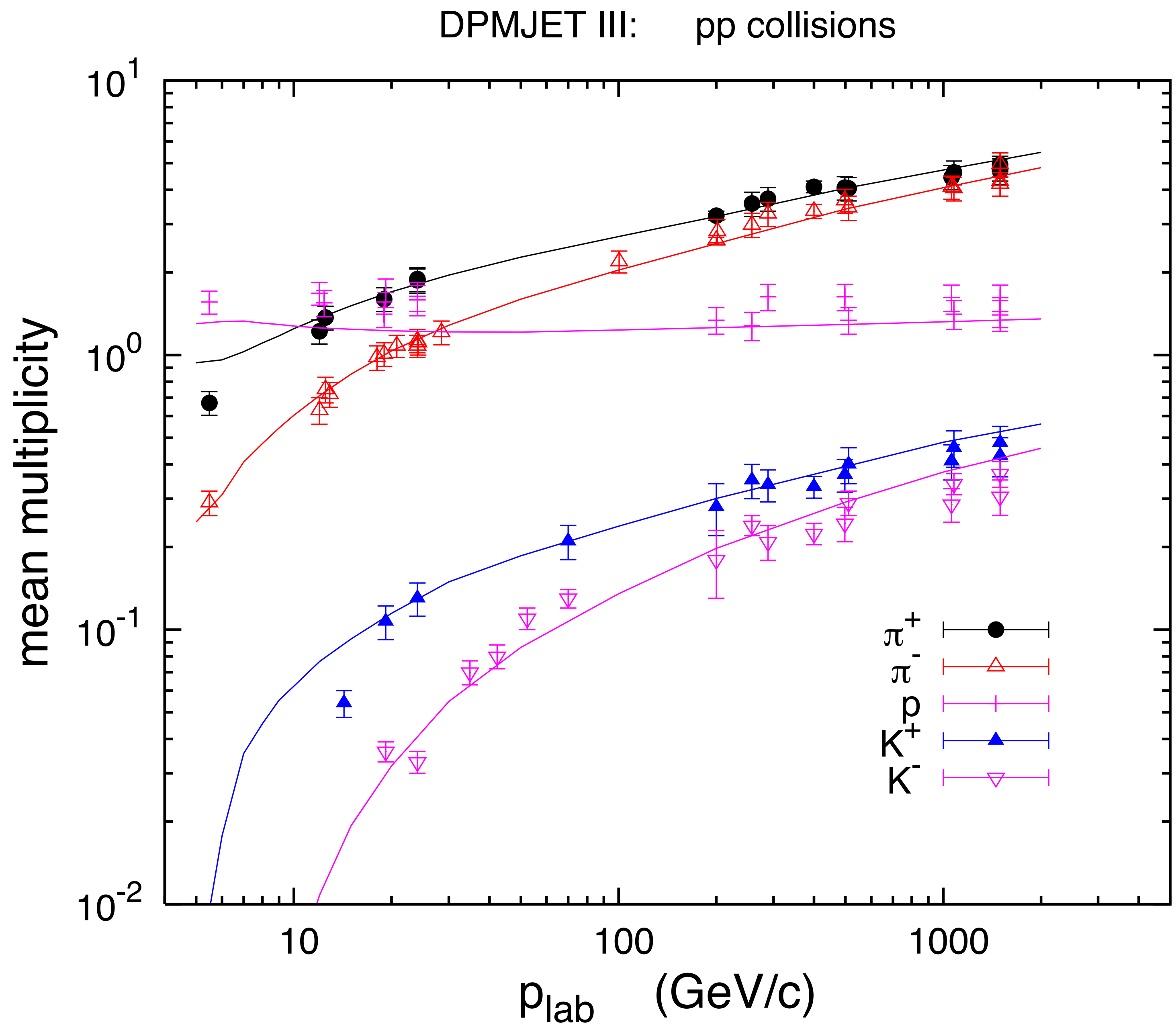
(Roesler, 2006)



Secondary particle multiplicities

proton - proton, $E_{lab} = 200GeV$

	Exp.	DPMJET-III
charged	7.69 ± 0.06	7.64
neg.	2.85 ± 0.03	2.82
p	1.34 ± 0.15	1.26
n	0.61 ± 0.30	0.66
π^+	3.22 ± 0.12	3.20
π^-	2.62 ± 0.06	2.55
K^+	0.28 ± 0.06	0.30
K^-	0.18 ± 0.05	0.20
Λ	0.096 ± 0.01	0.10
$\bar{\Lambda}$	0.0136 ± 0.004	0.0105



Secondary particle multiplicities

Power-law increase of number of secondary particles

$$n_{\mathrm{ch}} \sim s^{0.1}$$

Leading particles

proton - proton, $E_{lab} = 200\mathrm{GeV}$

	Exp.	DPMJET-III
charged	7.69 ± 0.06	7.64
neg.	2.85 ± 0.03	2.82
p	1.34 ± 0.15	1.26
n	0.61 ± 0.30	0.66
π^+	3.22 ± 0.12	3.20
π^-	2.62 ± 0.06	2.55
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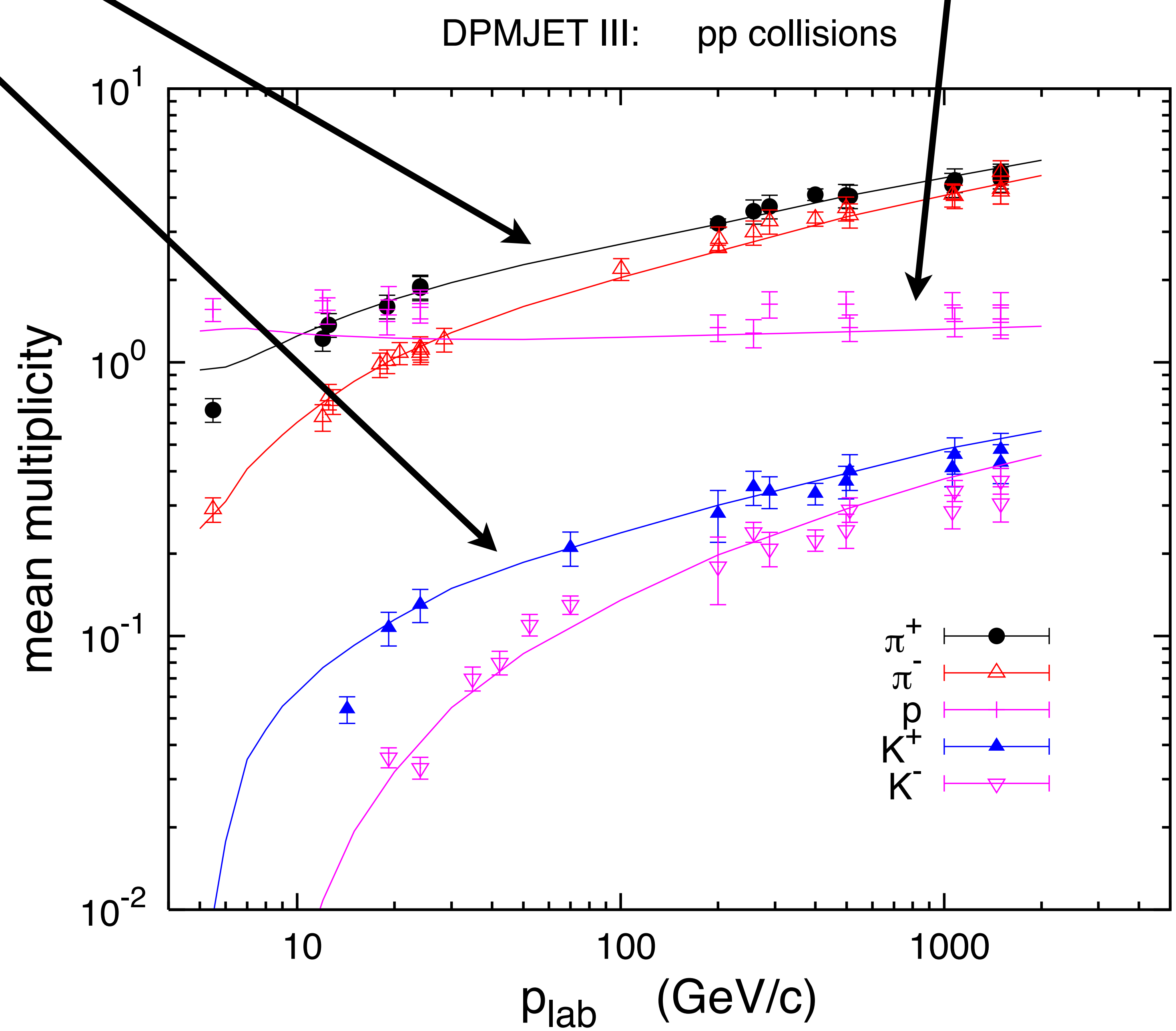
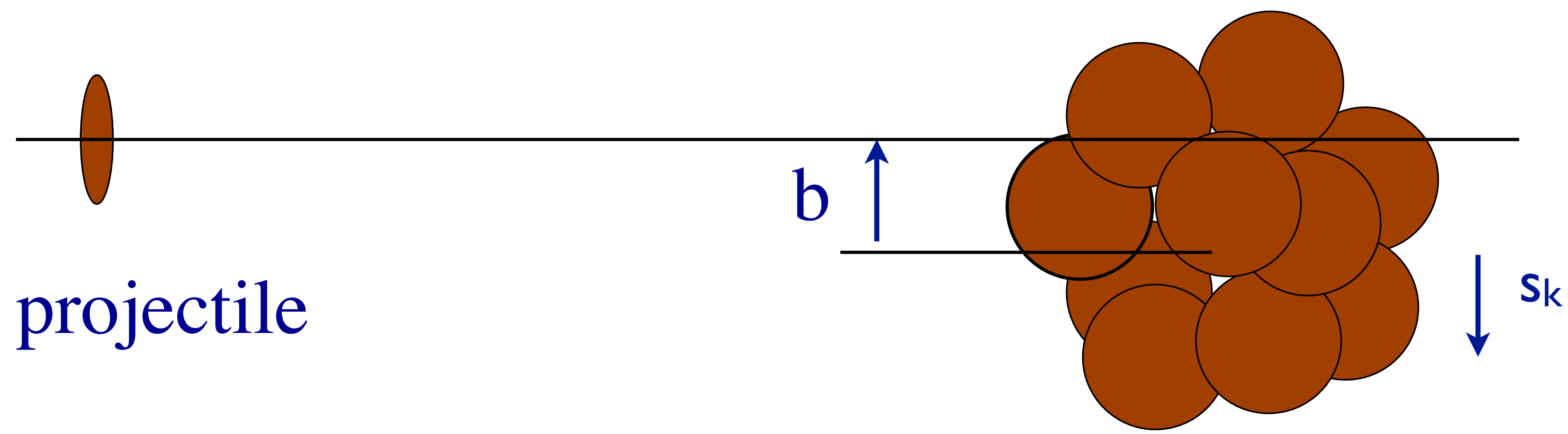


Interaction of hadrons with nuclei



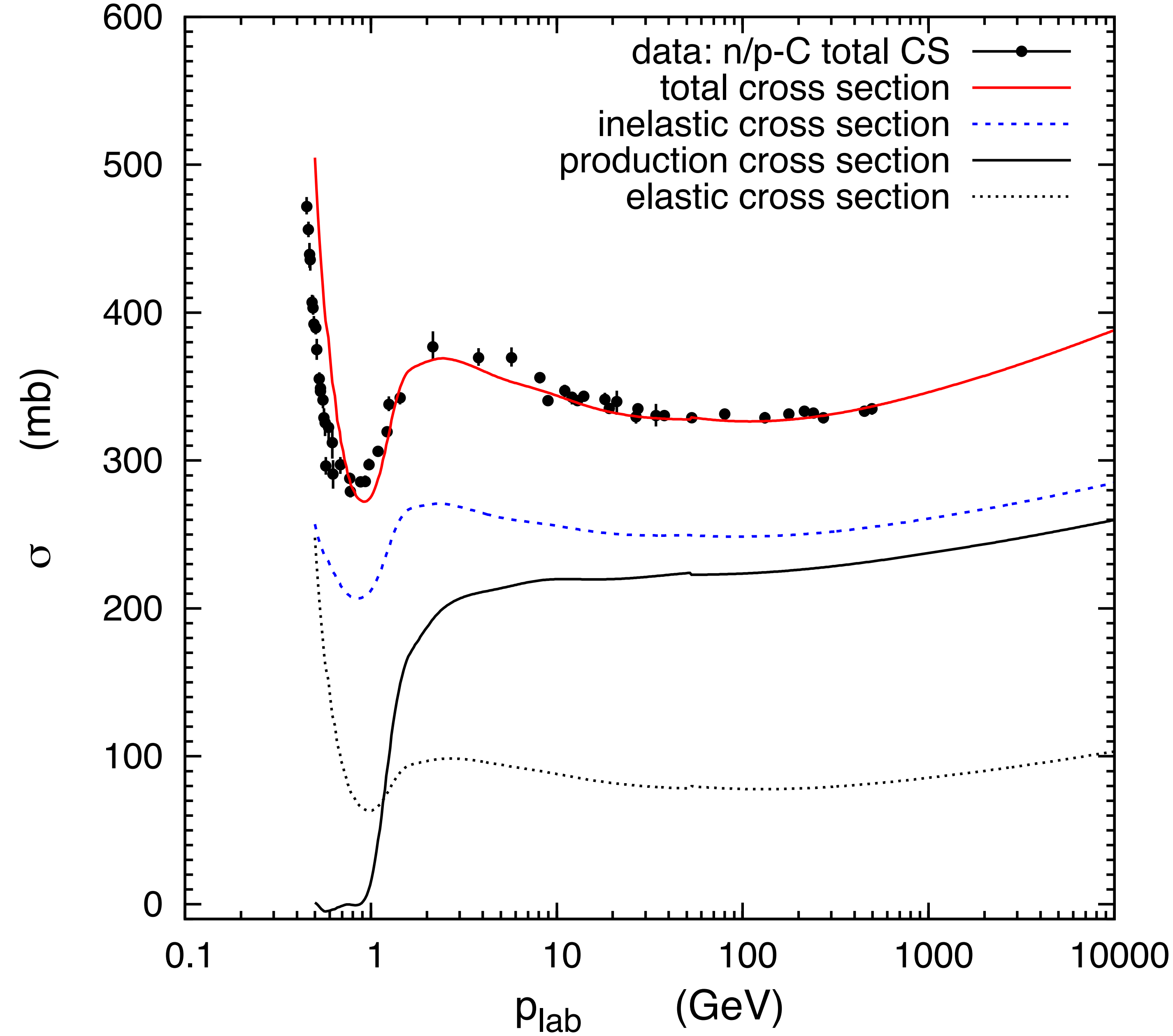
Standard Glauber approximation:

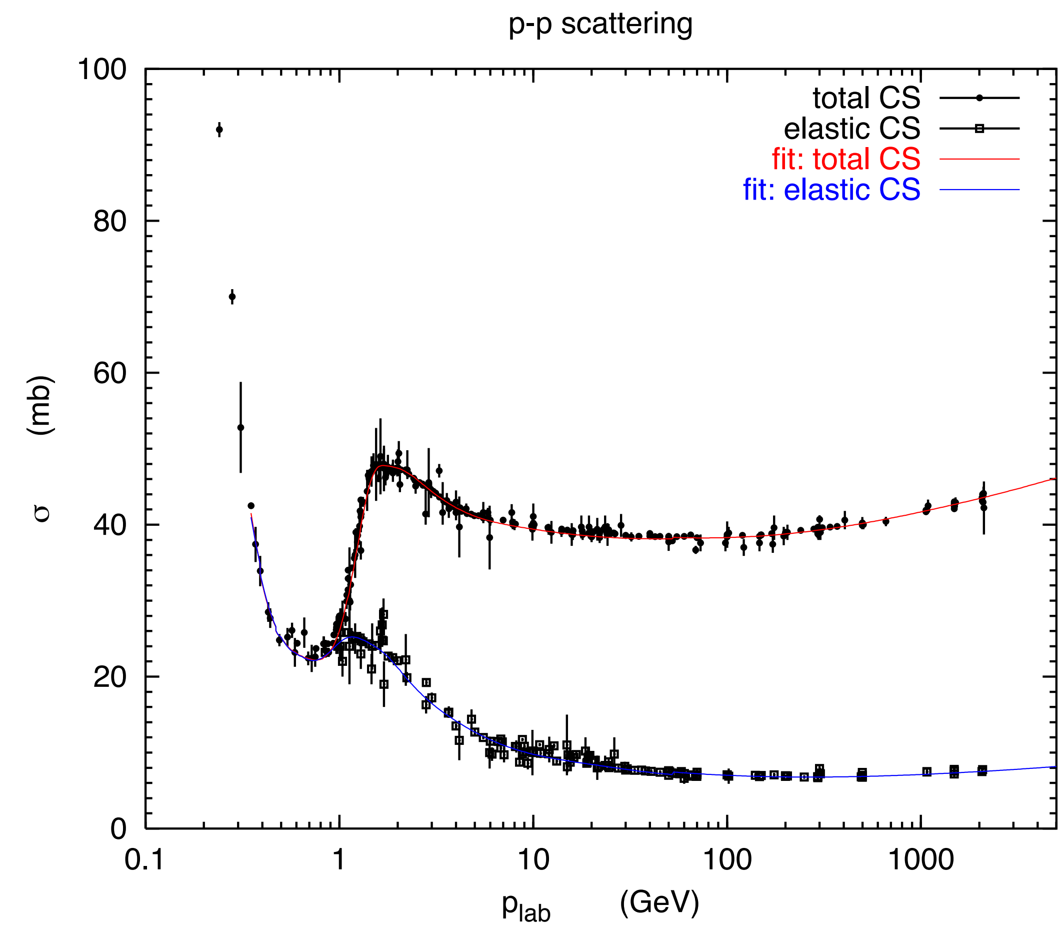
$$\sigma_{\mathrm{inel}} = \int d^2\vec{b} \left[1 - \prod_{k=1}^{A} \left(1 - \sigma_{\mathrm{tot}}^{NN} T_N(\vec{b} - \vec{s}_k) \right) \right] \approx \int d^2\vec{b} \left[1 - \exp\left\{ -\sigma_{\mathrm{tot}}^{NN} T_A(\vec{b}) \right\} \right]$$

$$\sigma_{\mathrm{prod}} \approx \int d^2\vec{b} \left[1 - \exp\left\{ -\sigma_{\mathrm{ine}}^{NN} T_A(\vec{b}) \right\} \right]$$

Coherent superposition of elementary nucleon-nucleon interactions

Example: proton-carbon cross section



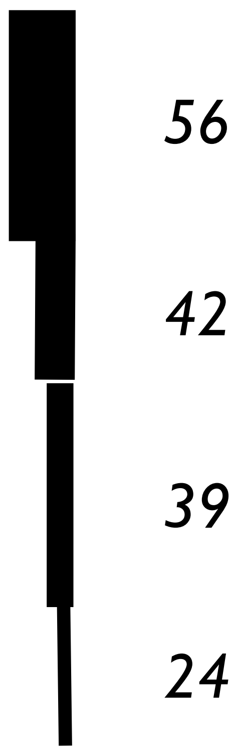


$$\sigma_{\text{prod}}^{\text{pC}} = \frac{A\sigma_{\text{ine}}^{\text{pp}}}{\langle n_{\text{part}} \rangle}$$

Number of participating target nucleons (1.8 at 100 GeV)

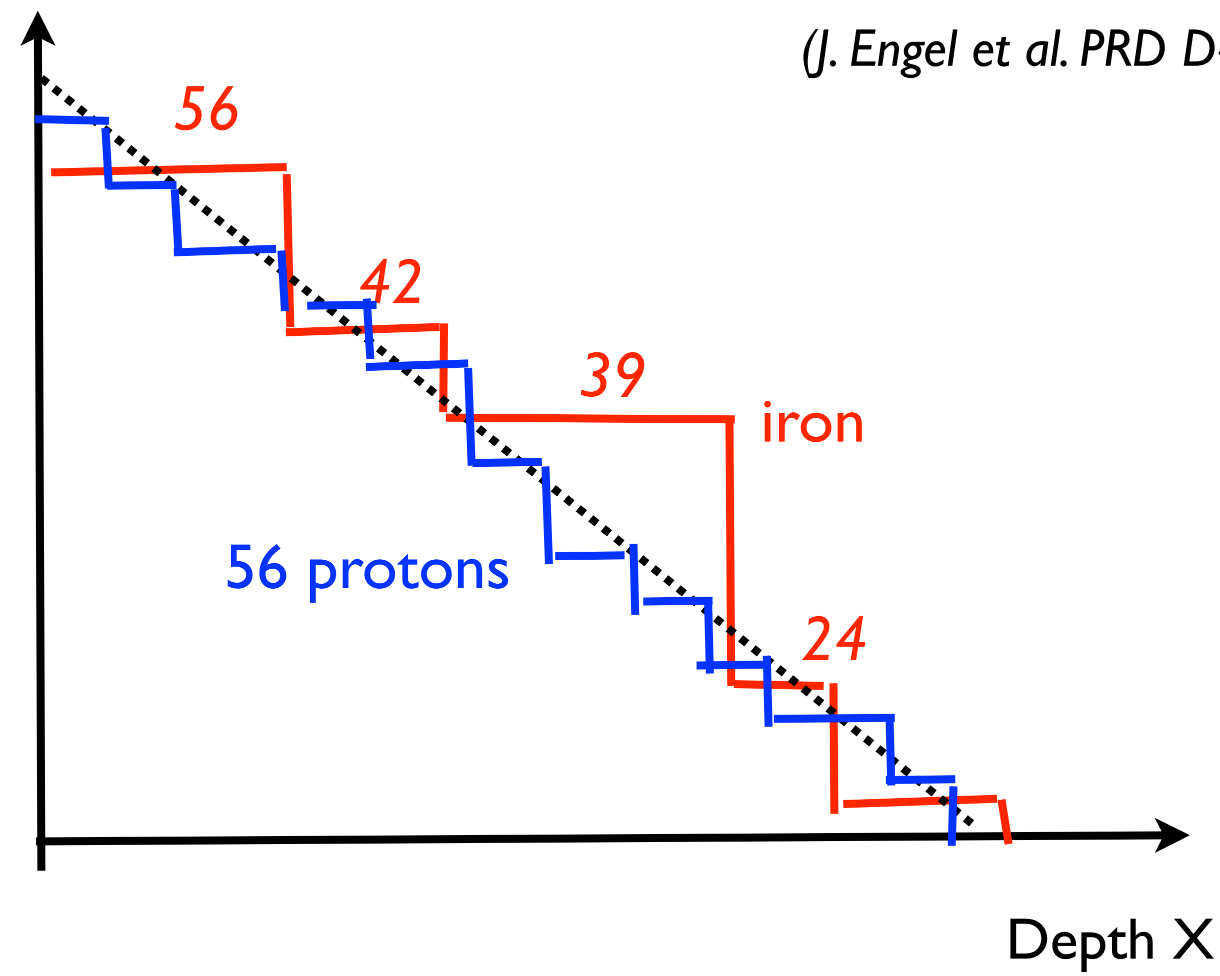
Superposition model: correct prediction of mean Xmax

iron nucleus



Number of nucleons without interaction

(J. Engel et al. PRD D46, 1992)

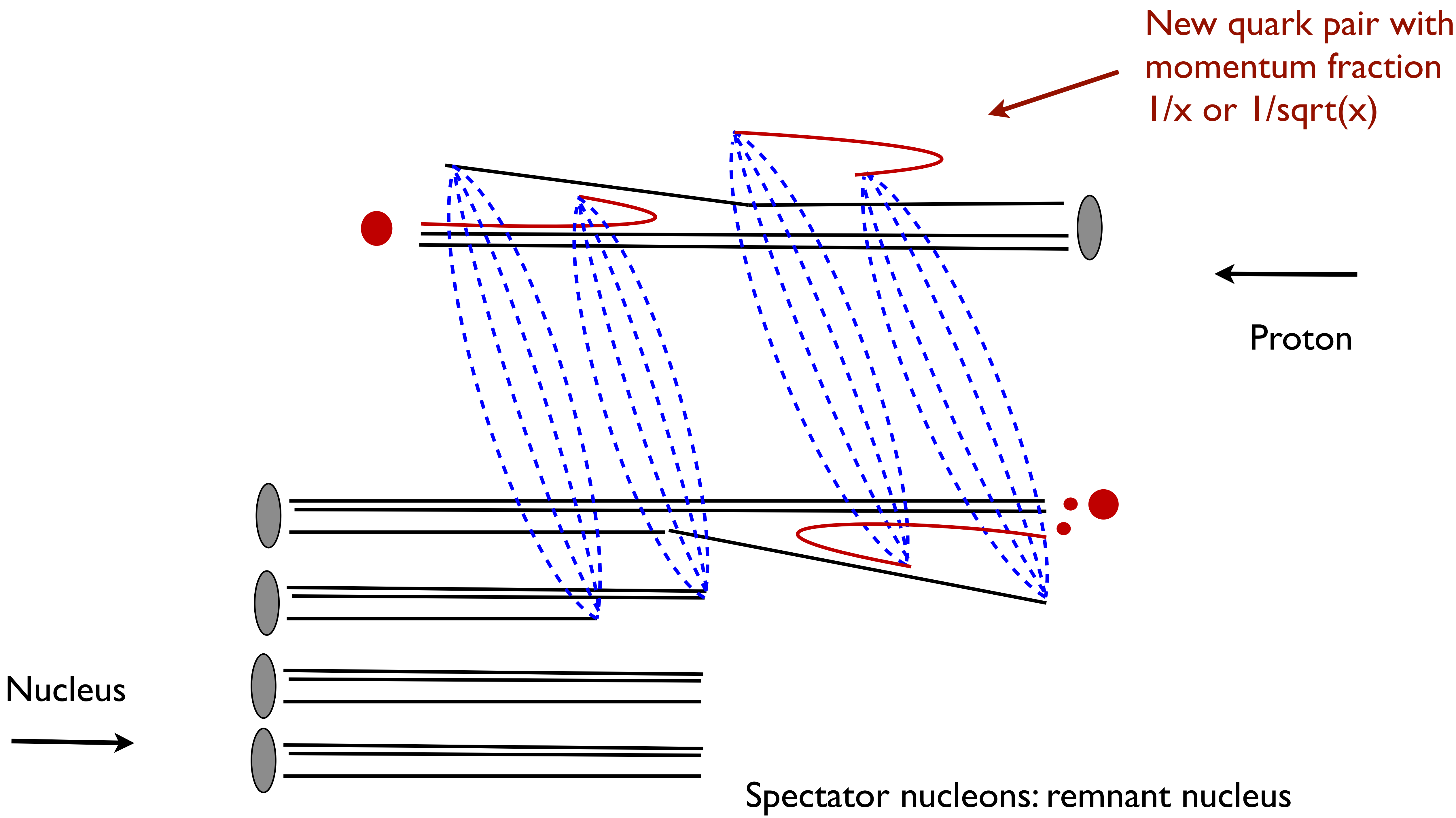


Glauber approximation (unitarity)

$$n_{\text{part}} = \frac{\sigma_{\text{Fe-air}}}{\sigma_{\text{p-air}}}$$

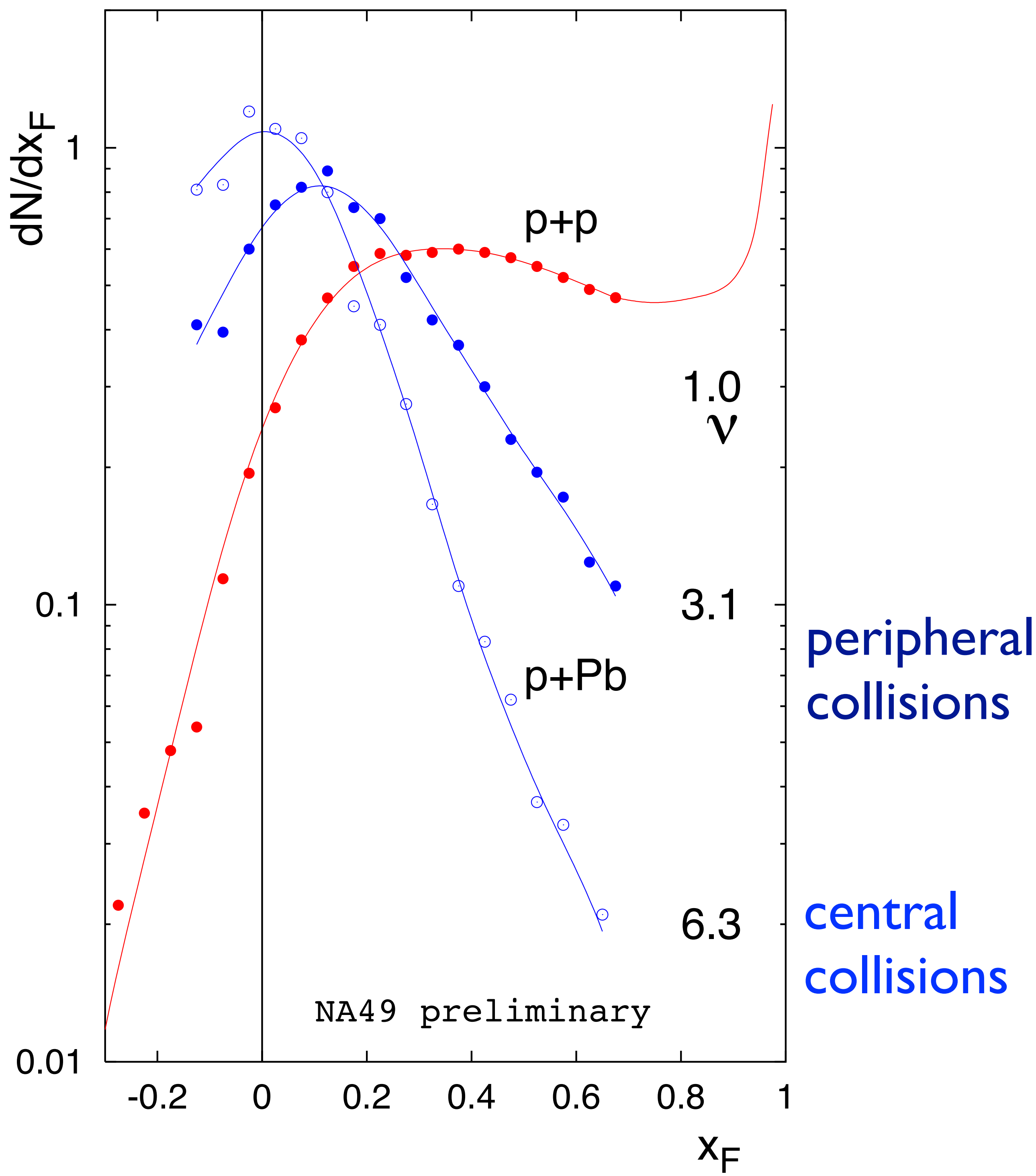
Superposition and semi-superposition models applicable to inclusive (averaged) observables

String configuration for nucleus as target

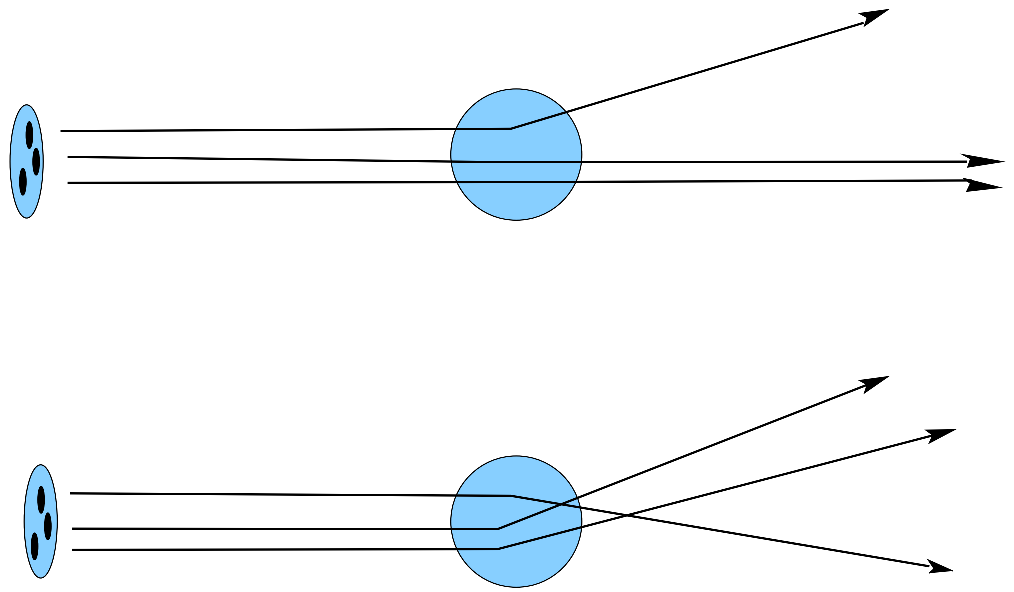


Leading particle effect and nuclei

Projectile component of net proton spectrum



E_{lab} = 149 GeV



Saturation:

- no leading particle effect,
- secondaries of highest energy are mesons

Comparison of low/intermediate energy models

DPMJET II & III *(Ranft / Roesler, RE, Ranft, Bopp)*	• microscopic (universal) model • resonances for low energy hadron projectiles (HADRIN, NUCRIN) • two- and multi-string model
FLUKA *(Ferrari, Sala, Ranft, Roesler)*	• microscopic (universal) model • resonances (PEANUT), photodissociation • two-string model, DPMJET at high energy
GHEISHA *(Fesefeld)*	• parametrization of data (GEANT 3) • wide range of projectiles/targets • limited to E_{lab} < 500 GeV
UrQMD (Bleicher et al.)	• combination of microscopic model with data parametrization (no Glauber calc.) • optimized for interactions of nuclei
SOPHIA *(Mücke, RE, et al.)*	• dedicated photon-nucleon model • resonances, two-strings, E_{lab} < 500 GeV
RELDIS *(Pshenichnov)*	• dedicated photodissociation model for nuclei, wide range of nuclei

Basic features of multiparticle production

Particle production threshold (low energy)

- Resonances, nearly isotropic decay
- Energy loss ~20% in $p\gamma$ interactions
- Photodissociation of nuclei

Multiparticle production (intermediate energy)

- Leading particle effect
  - ~50% of energy carried by leading nucleon
  - incoming proton: 66% proton, 33% neutron
- Secondary particles
  - power-law increase of multiplicity
  - quark counting: ~33% π^0, 66% $\pi^\pm$
  - transverse momentum energy-independent
  - baryons are pair-produced, delayed threshold
  - scaling of secondary particle distributions
- Diffraction (rapidity gaps)
  - elastic scattering & low-mass diffraction dissociation
  - large multiplicity fluctuations

Appendix: Glauber approximation

1.E.5
8.A.1

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HIGH-ENERGY SCATTERING OF PROTONS BY NUCLEI

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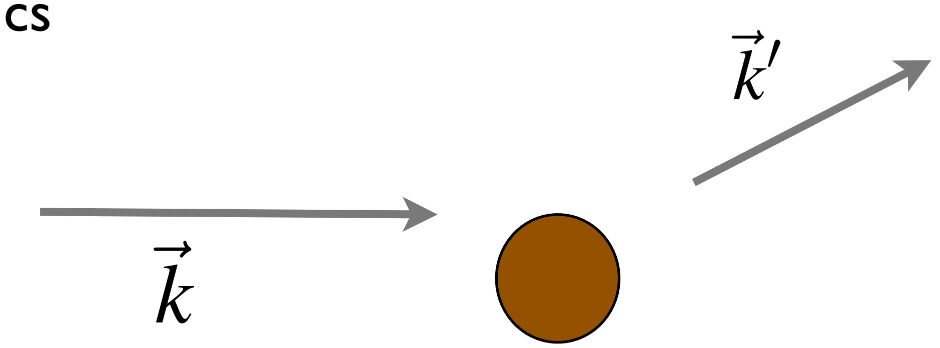
Received 19 February 1970

Basic building blocks are p-p and p-n scattering

Abstract: The theory of high-energy hadron-nucleus collisions is discussed by means of the multiple-diffraction theory. Effects of the Coulomb field are accounted for in elastic scattering by light and heavy nuclei. Inelastic scattering is treated by means of the shadowed single collision approximation at small momentum transfer and the corresponding multiple collision expansion at large momentum transfers. The theory is compared with the measurements of Bellettini et al. on proton-nucleus scattering at 20 GeV/c by finding density distributions for the nuclei which provide least-squares fits to the data. The nucleon densities found are closely comparable in dimensions to the known charge densities. The predicted sums of the angular distributions of elastic and inelastic scattering reproduce the experimental angular distributions fairly closely.

Amplitude for proton-proton scattering

Kinematics



Momentum transfer

$$\vec{q} = \vec{k}' - \vec{k}$$

$$t = (k' - k)^2$$

High-energy approximation

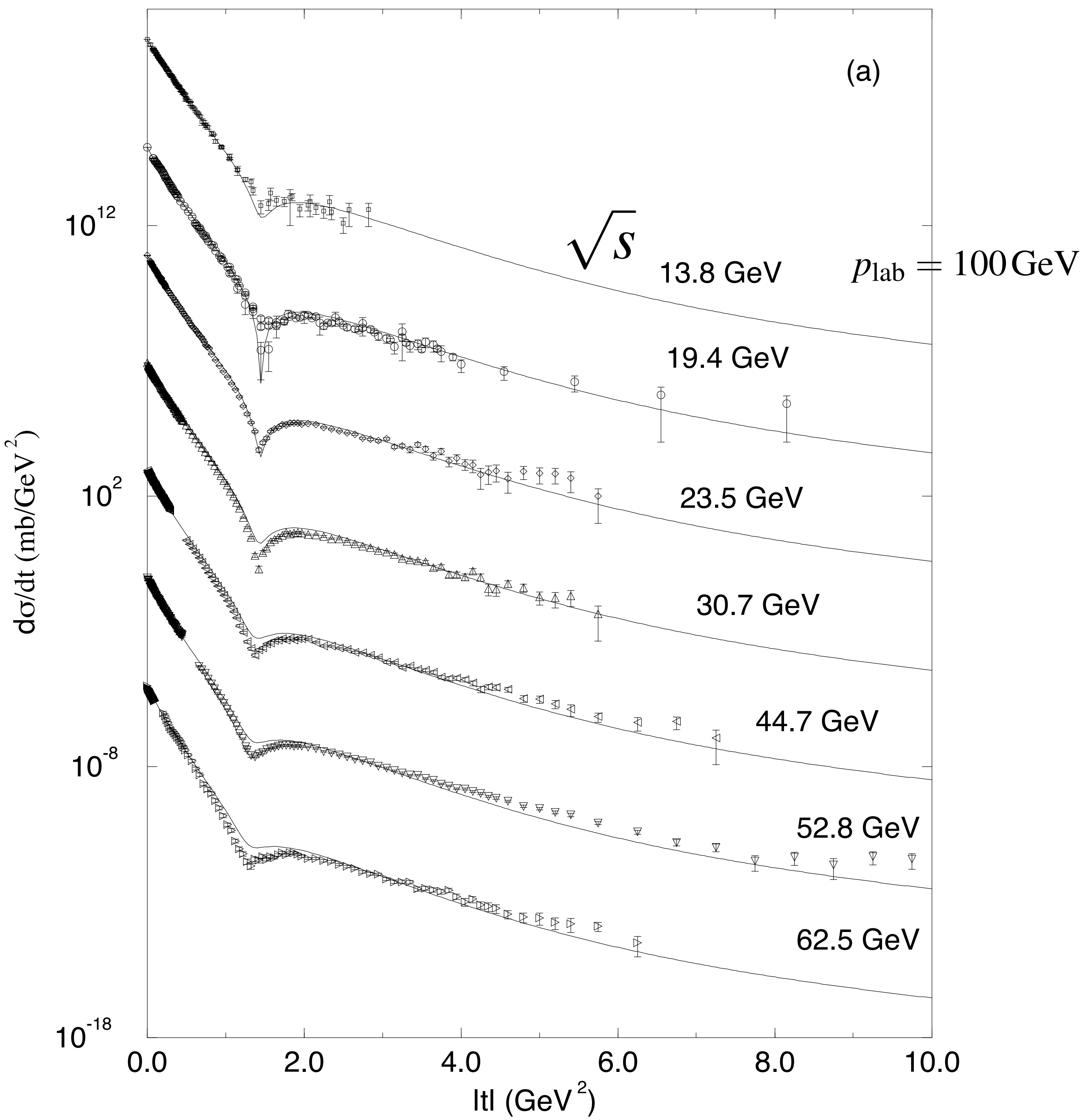
$$t = -4\, k_{\rm cms}^2 \sin^2(\theta/2) \approx -\vec{q}_\perp^{\,2}$$

Amplitude conventions (elastic scattering)

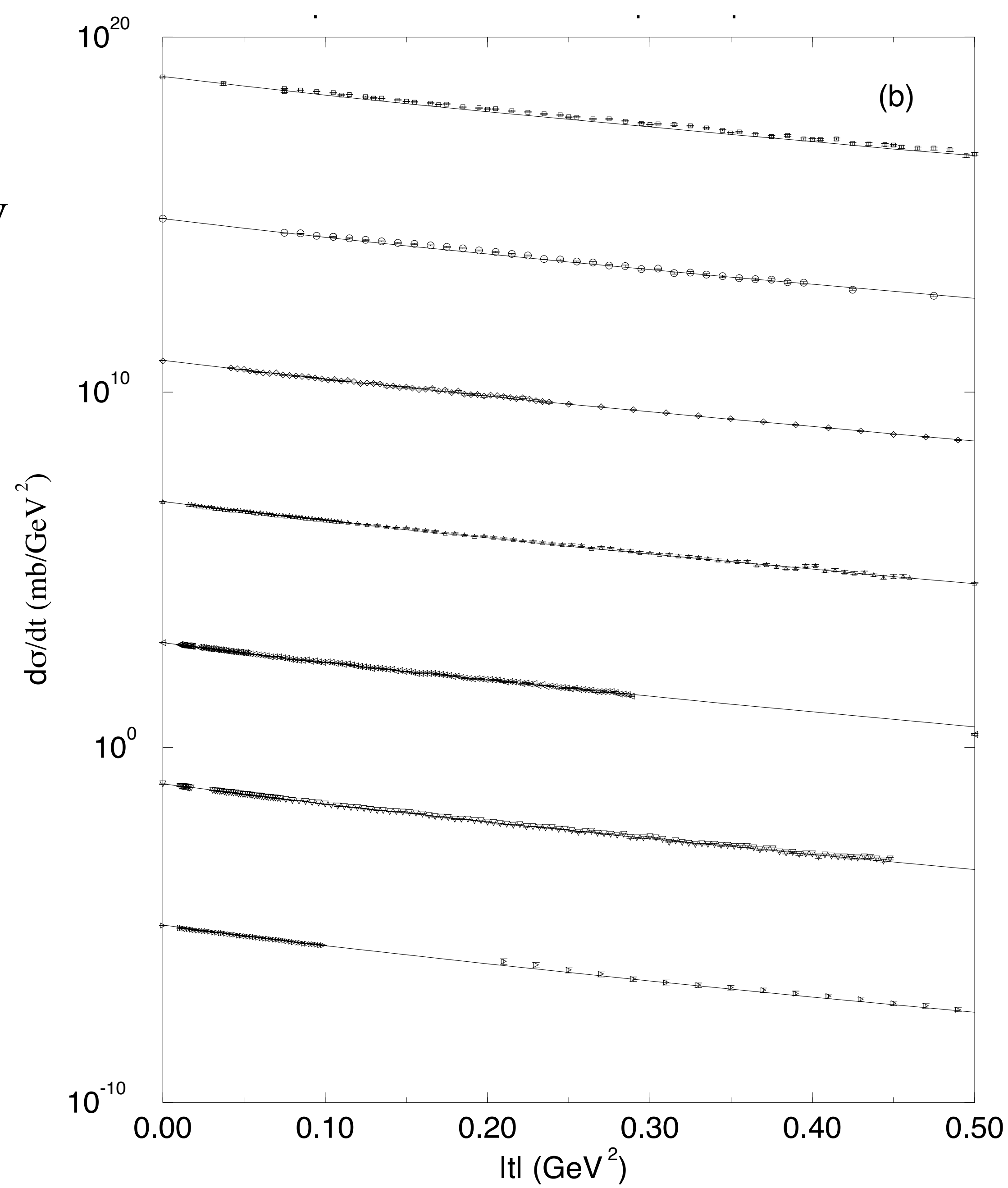
$$\frac{d\sigma_{\rm ela}}{d\Omega} = |f(\vec{q})|^2$$

$$\frac{d\sigma_{\rm ela}}{dt} = \frac{\pi}{k^2}\, |f(\vec{q})|^2$$

Elastic proton-proton scattering: data



Zoom into low-*t* region



Main contribution to cross section can be approximated by exponential in *t*

Approximation for p-p scattering amplitude

Ansatz: exponential in $t = -\vec{q}_\perp^2$

$$f_{pp}(q^2) = f_{pp}(0)\; e^{-\frac{1}{2}\beta^2\vec{q}^2}$$

slope parameter

normalization factor taken from optical theorem

Optical theorem (applies to all scattering processes)

$$\sigma_{\text{tot}} = \frac{4\pi}{k}\,\Im m(f(q^2 \to 0))$$

$$f_{pp}(0) = (i+\alpha)\frac{k\,\sigma_{\text{tot}}}{4\pi}$$

$$f_{pp}(q^2) = (i+\alpha)\frac{k\,\sigma_{\text{tot}}}{4\pi}\; e^{-\frac{1}{2}\beta^2\vec{q}^2}$$

Alpha parameter

$$\alpha = \frac{\Re e f(q^2 \to 0)}{\Im m f(q^2 \to 0)}$$

Relation between parameters

Valid in approximation of exponential *t* behaviour

$$\frac{d\sigma_{\rm ela}}{dt} = \frac{1}{16\pi}(1+\alpha^2)\sigma_{\rm tot}^2 e^{-\beta^2|t|}$$

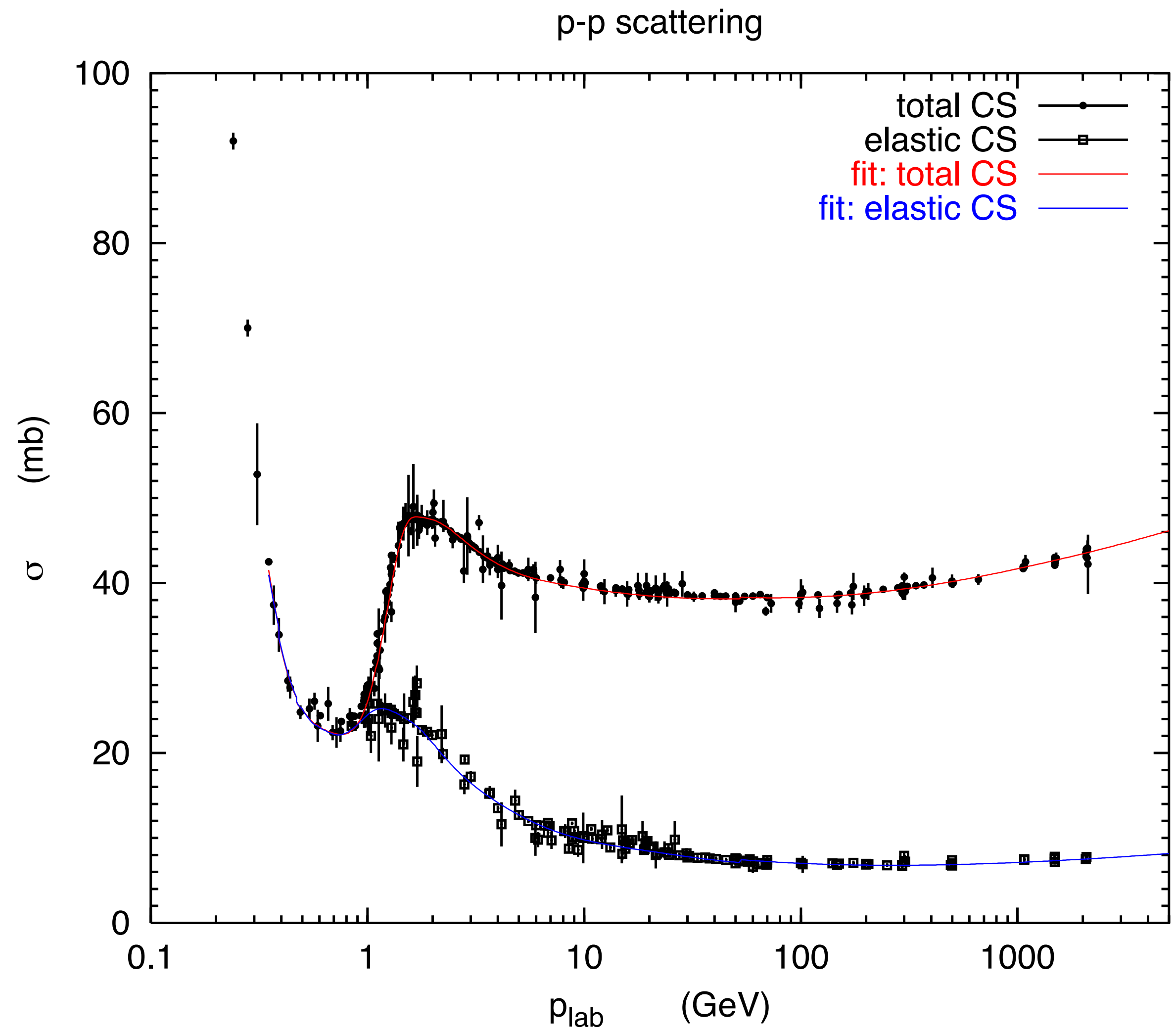
slope in dσ/dt plot

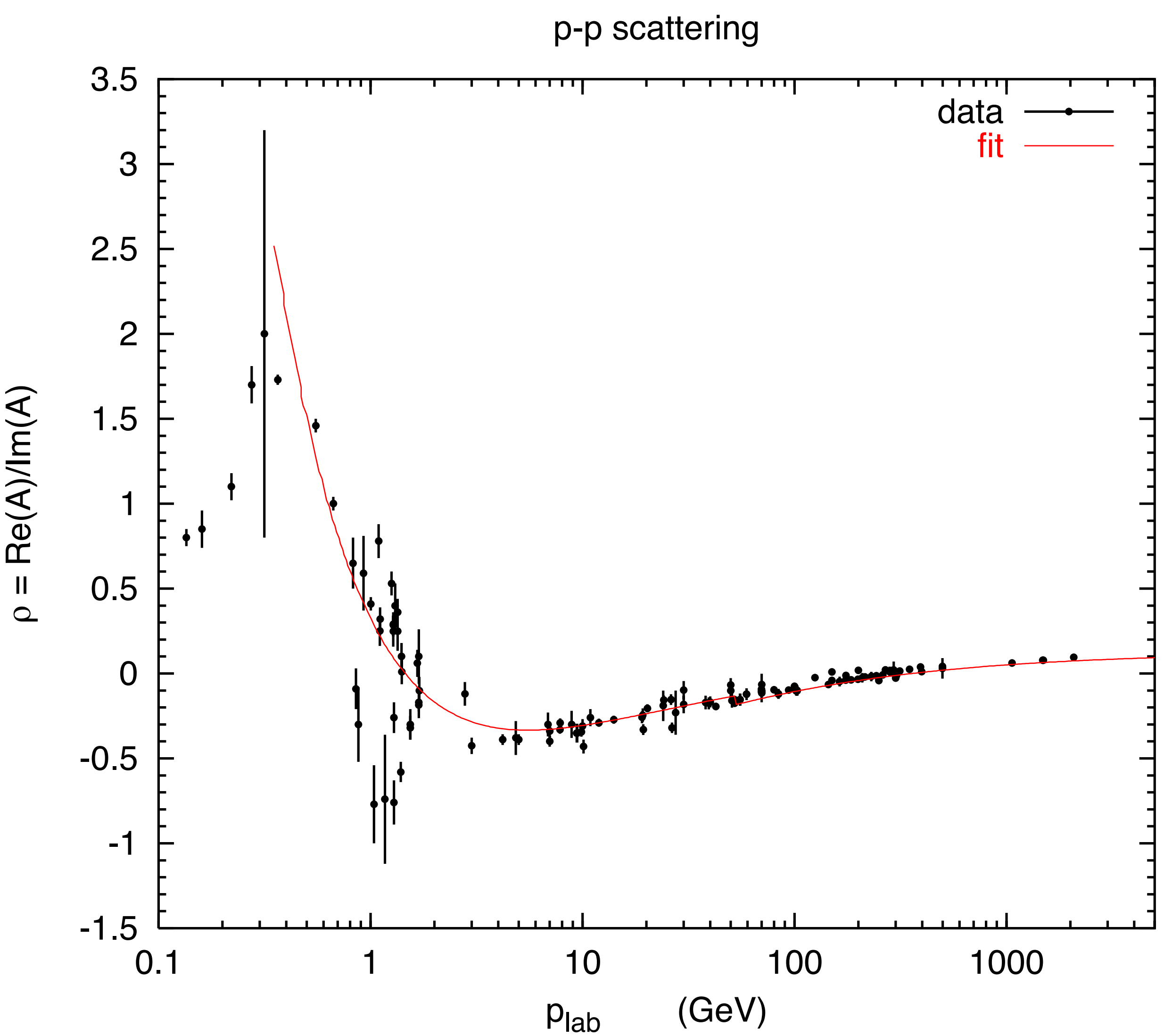
$$\sigma_{\rm ela} = \frac{\sigma_{\rm tot}^2}{16\pi\beta^2}(1+\alpha^2)$$

Only 3 parameters out of the 4 need to be known, for example $\sigma_{\rm tot}, \ \sigma_{\rm ela}, \ \alpha$

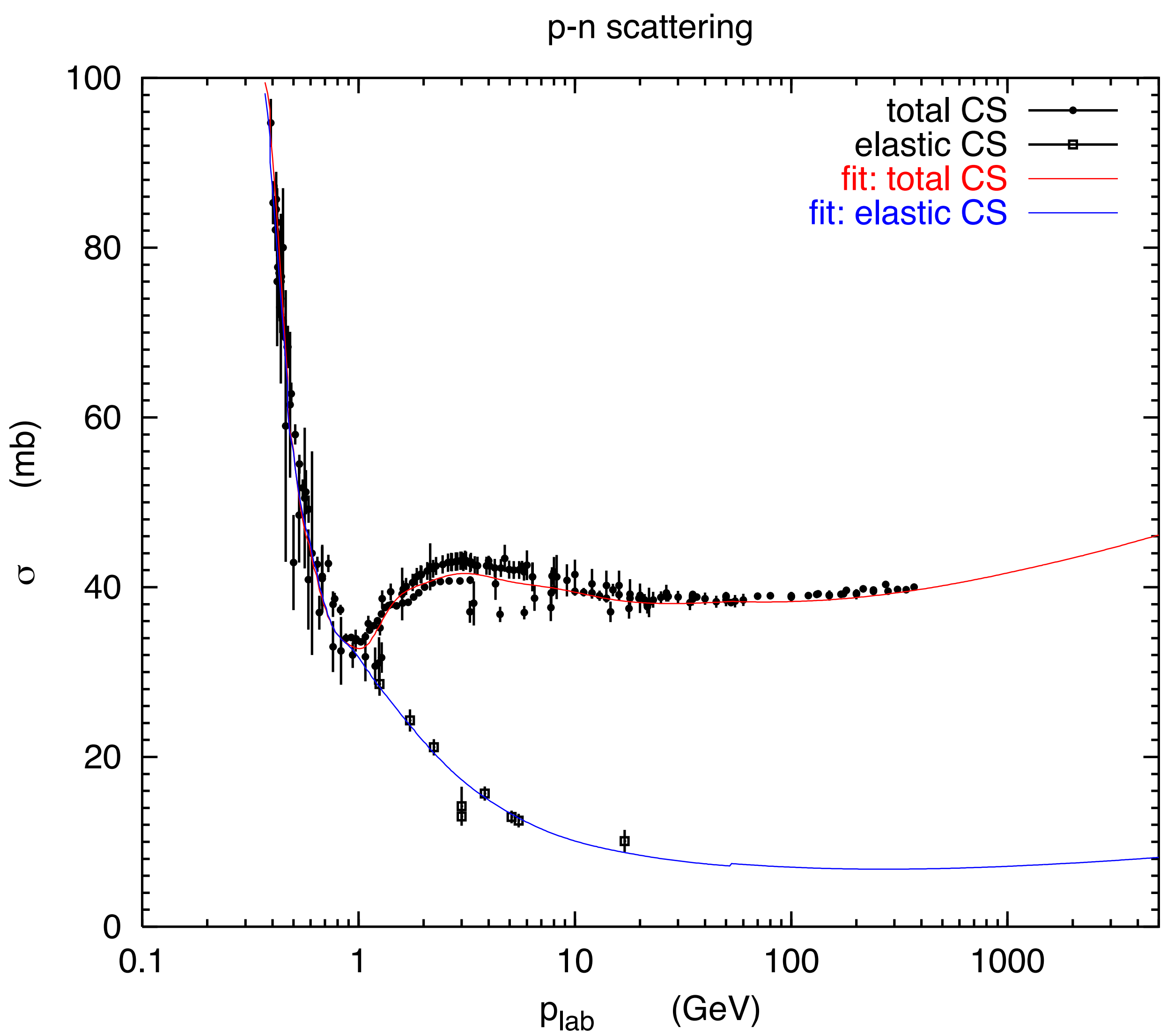
Relations often used to measure total cross section

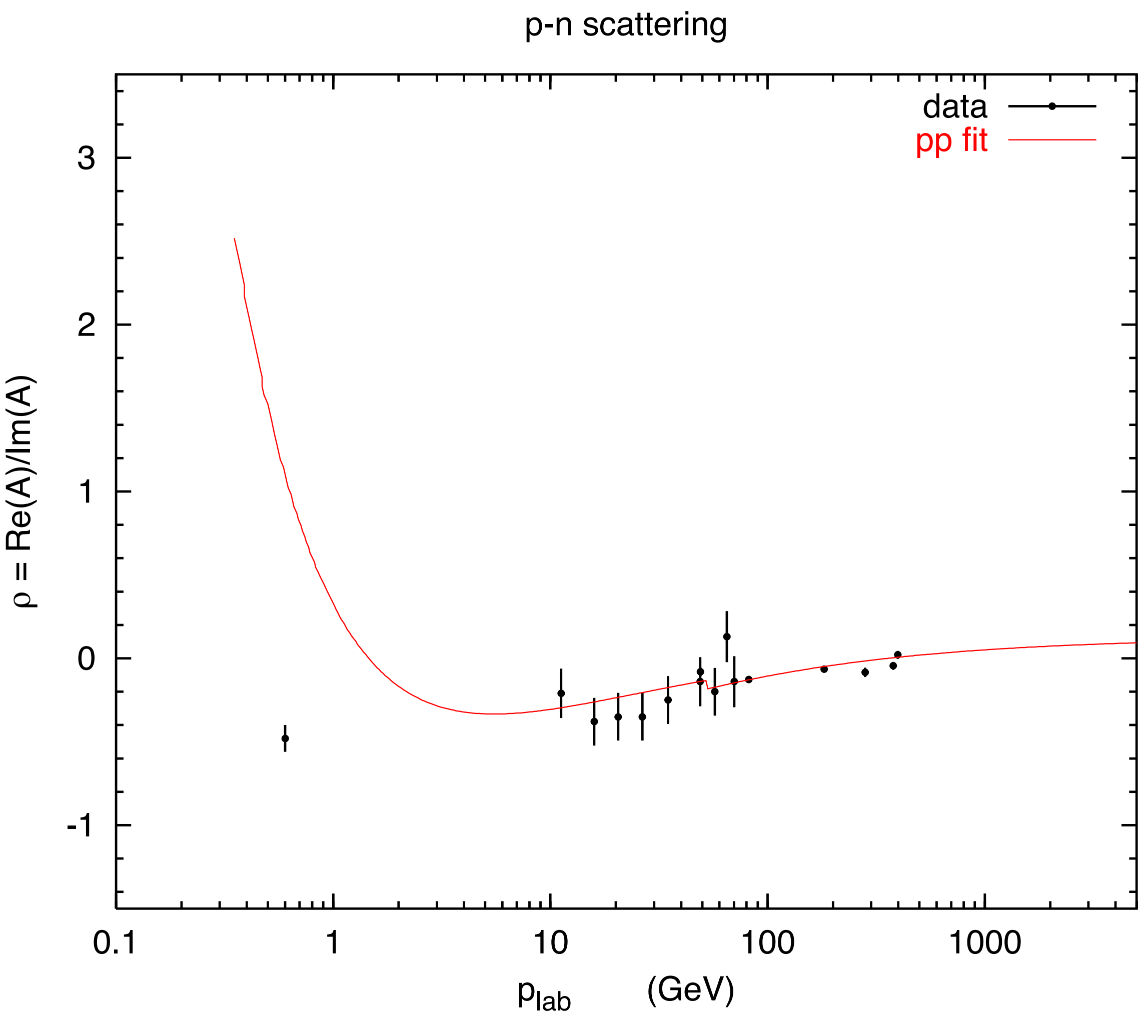
Parametrization of proton-proton data



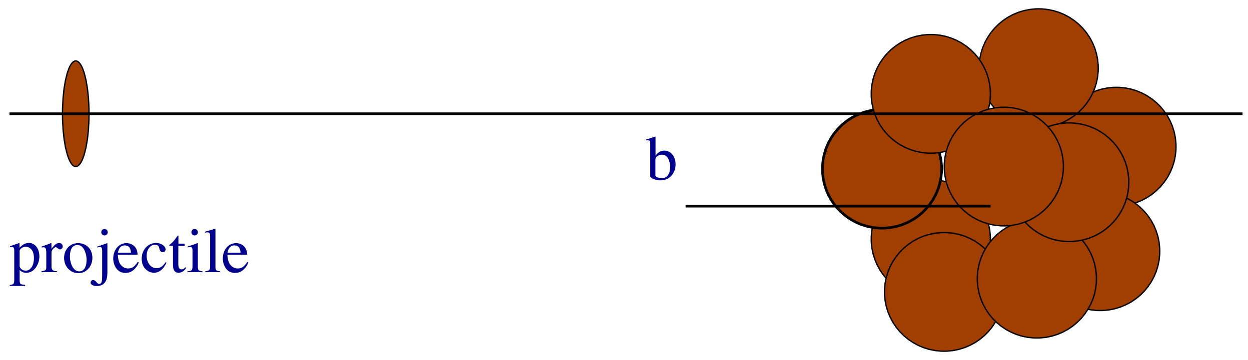


Parametrization of proton-neutron data





Glauber model in a nutshell



Transformation of amplitude in impact parameter (vector transverse to collision axis)

$$f(s, \boldsymbol{q}^2) = \frac{ik}{2\pi} \int e^{i\boldsymbol{q}\cdot\boldsymbol{b}} \Gamma(\boldsymbol{b}) d^2 b,$$

$$\Gamma(\boldsymbol{b}) = 1 - e^{i\chi(\boldsymbol{b})}$$

Phase shift function
(complex valued)

Scattering off two nucleons

$$\Gamma_{\text{Glauber}}(\boldsymbol{b}) = 1 - e^{i(\chi_1(\boldsymbol{b}) + \chi_2(\boldsymbol{b}))} = 1 - (1 - \Gamma_1)(1 - \Gamma_2)$$

scattering on nucleus corresponds to only adding phase shifts

Glauber expression for scattering amplitude

$$f_{fi}^{hA}(s, \boldsymbol{q}^2) = \frac{ik}{2\pi} \int e^{i\boldsymbol{q}\cdot\boldsymbol{b}} \psi_f^\star(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A) \Gamma_{hA}(\boldsymbol{b}, \boldsymbol{s}_1 \ldots \boldsymbol{s}_A) \psi_i(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A) d^2b \prod_{j=1}^{A} d^3 r_j$$

wave function of
nucleus in final state

wave function of
nucleus in initial state

$$\Gamma_{hA}(\boldsymbol{b}, \boldsymbol{s}_1 \ldots \boldsymbol{s}_A) = 1 - \exp\left\{ i \sum_{j=1}^{A} \chi_j(\boldsymbol{b} - \boldsymbol{s}_j) \right\} = 1 - \prod_{j=1}^{A} \left[1 - \Gamma_{hN}(\boldsymbol{b} - \boldsymbol{s}_j)\right]$$

Total and elastic cross sections

$$\sigma_{hA}^{\mathrm{tot}} = \frac{4\pi}{|\boldsymbol{k}|} \Im m \left\{ f_{ii}^{hA}(s, \boldsymbol{q}^2 \to 0) \right\}$$

$$\sigma_{hA}^{\mathrm{ela}} = \int \frac{1}{|\boldsymbol{k}|^2} \left| f_{ii}^{hA}(s, \boldsymbol{q}^2) \right|^2 d^2 q$$

Wave function of nuclei

Neglecting correlations

$$\psi_i^\star(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A)\psi_i(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A) = \prod_{j=1}^{A} \rho_j(\boldsymbol{r}_j)$$

Single nucleon density

Normalization

$$\int \rho_j(\boldsymbol{r}_j) d^3 r_j = 1$$

Light nuclei up to A = 18: potential of harmonic oscillator

$$\rho_s(\boldsymbol{r}) = \frac{1}{\pi^{3/2} a_0^3} e^{-r^2/a_0^2} \quad \text{and} \quad \rho_p(\boldsymbol{r}) = \frac{2r^2}{3\pi^{3/2} a_0^5} e^{-r^2/a_0^2}$$

Heavier nuclei: Woods-Saxon potential

$$\rho(\boldsymbol{r}) = \frac{\rho_0}{1 + \exp\left(\frac{|\boldsymbol{r}| - r_0}{a_0}\right)} \quad \text{with} \quad \rho_0 = \frac{3}{4\pi r_0^3} \frac{1}{1 + (a_0 \pi / r_0)^2}$$

Quasi-elastic scattering

Elastic scattering

$$\sigma_{hA}^{\text{ela}} = \int \frac{\pi}{k^2} \left| f_{ii}^{hA}(q^2) \right|^2 d^2q$$

final state = initial state

Elastic+quasi-elastic scattering

$$\sigma_{hA}^{\text{ela}} + \sigma_{hA}^{\text{qel}} = \sum_f \int \frac{\pi}{k^2} \left| f_{fi}^{hA}(q^2) \right|^2 d^2q$$

sum over all final states

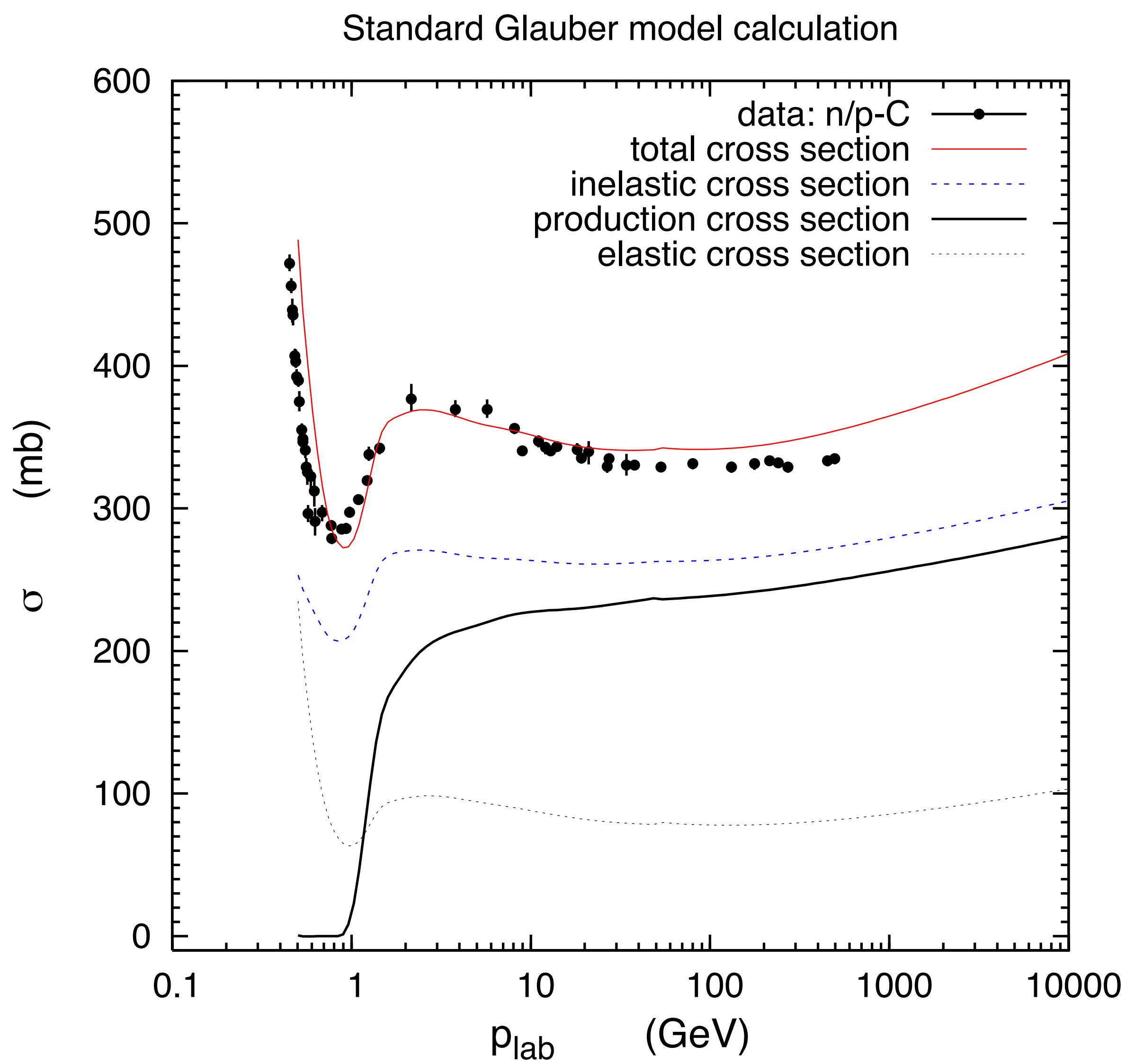
Completeness relation for wave function in final state if summed over all possible configurations

$$\sum_f \psi_f^\star(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A)\psi_f(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A) \prod_{j=1}^{A} d^3r_j = 1$$

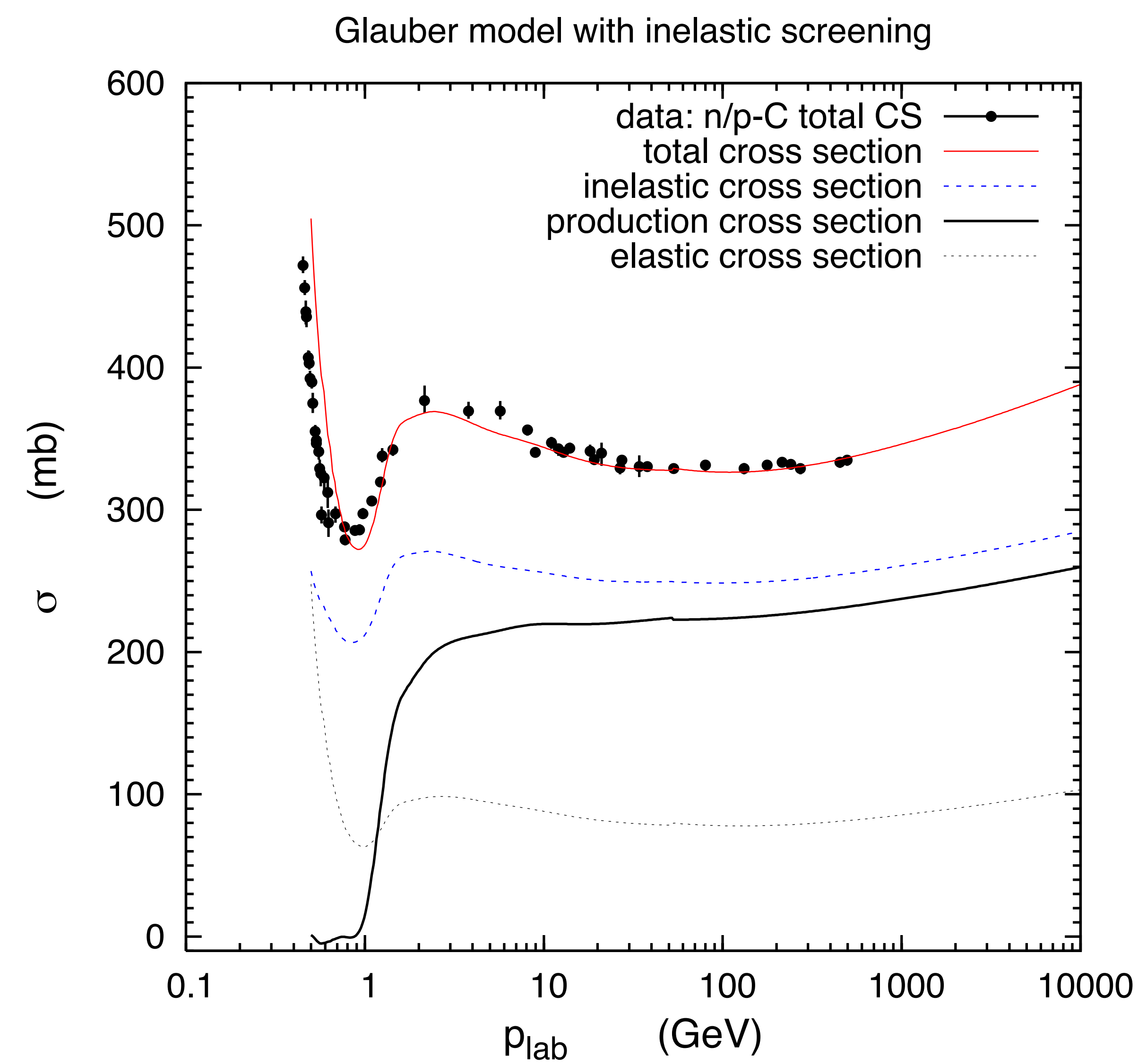
No need to know all quasi-elastic final states in detail

$$\sigma_{hA}^{\text{ela}} + \sigma_{hA}^{\text{qel}} = \int \left| 1 - \prod_{j=1}^{A} [1 - \Gamma_{hN}(\boldsymbol{b} - \boldsymbol{s}_j)] \right|^2 \left(\prod_{j=1}^{A} \rho_j(\boldsymbol{r}_j) d^3r_j \right) d^2b$$

Predictions from Glauber model

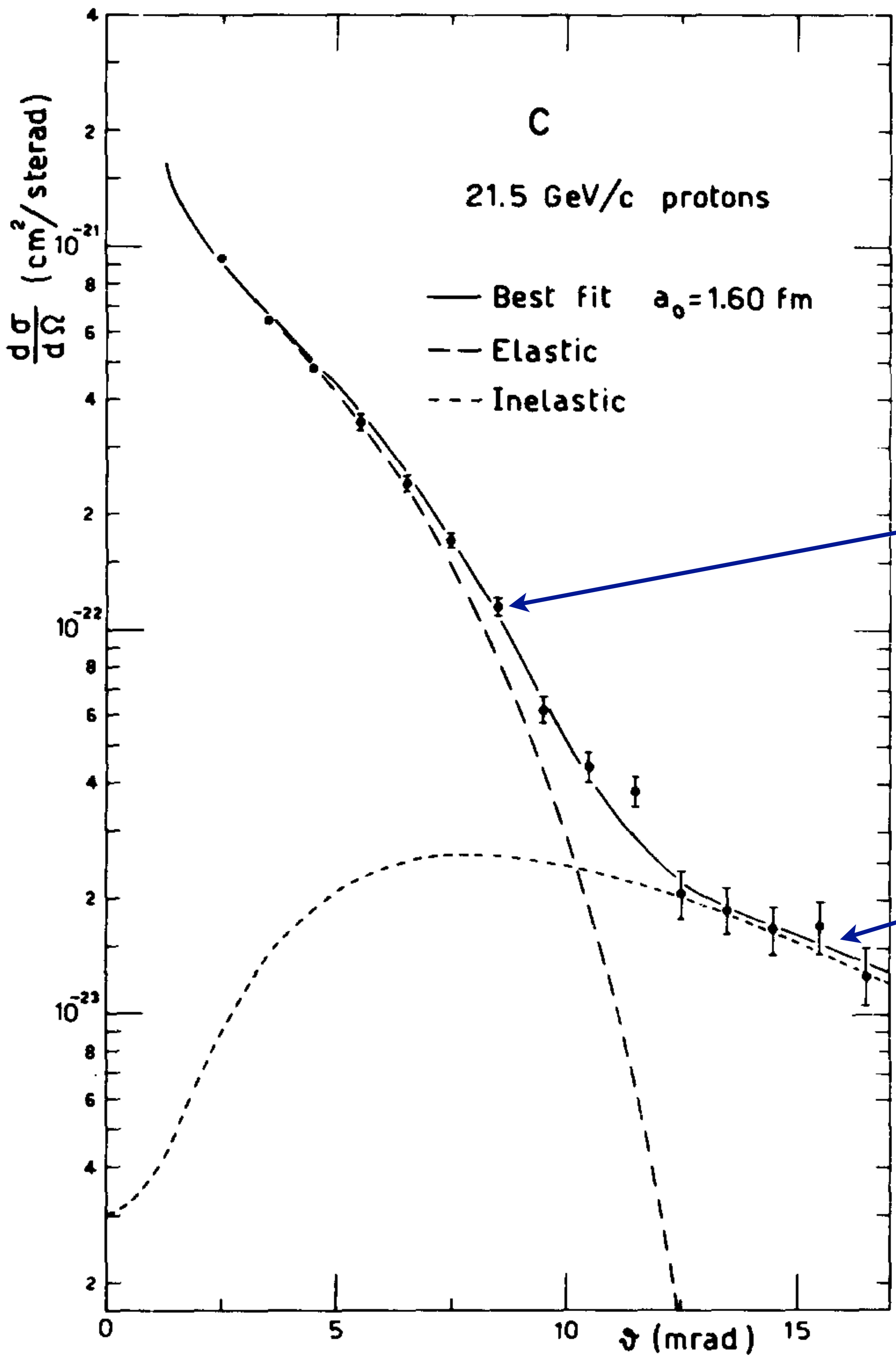


Calculation as published in 1970



Improved calculation with inelastic screening effects

Angular dependence: coherent and incoherent parts



(Glauber & Matthiae, NPB 1970)

Coherent (elastic) scattering

Incoherent (quasi-elastic) scattering

Glauber approximation works very well