Exact quantum entropy of black holes

1. From Bekenstein-Hawking to AdS_2 functional integrals

Sameer Murthy

Tata Institute of Fundamental Research, Mumbai

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Outline

1. Introduction and idea of exact black hole entropy
2. Gravitational origin of finite size effects
3. Holography and quantum entropy
4. Gravitational quantum entropy

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Black Hole thermodynamics

- Black holes are solutions to Einstein's equations characterized by an event horizon.
- Classically, signals from behind the horizon cannot reach an asymptotic observer.
- Quantum mechanically, they radiate like a hot body.
- One can assign thermodynamic quantities like temperature and entropy to a black hole based purely on its gravitational properties

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Black Hole Entropy

- Area law Bekenstein ['72]; Hawking ['75]

$$S_{BH} = \frac{A}{4G} \,. \tag{1}$$

- In a consistent quantum theory of gravity, there must be a statistical interpretation of black hole entropy in terms of underlying microstates.
- This is a *universal* requirement for a theory of quantum gravity, that must hold for all black holes in all phases of the theory.
- For a theory under construction such as string theory, a useful strategy in such a situation is to focus on such universal properties, and try to verify/falsify the theory in controllable examples in a controllable phase.

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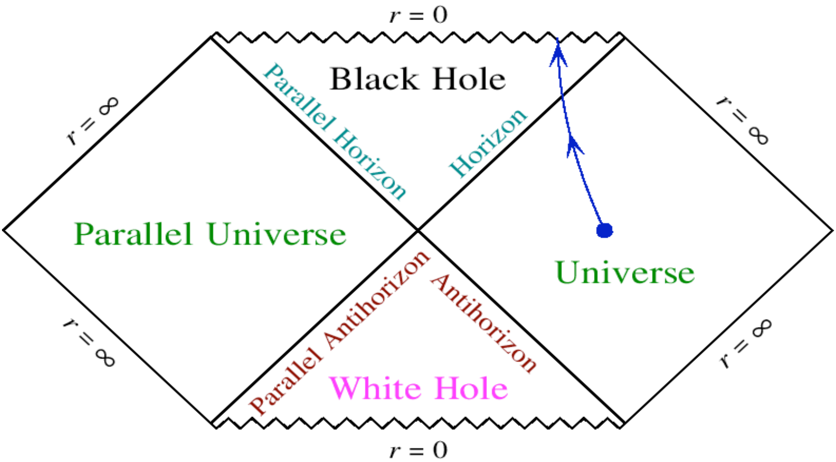
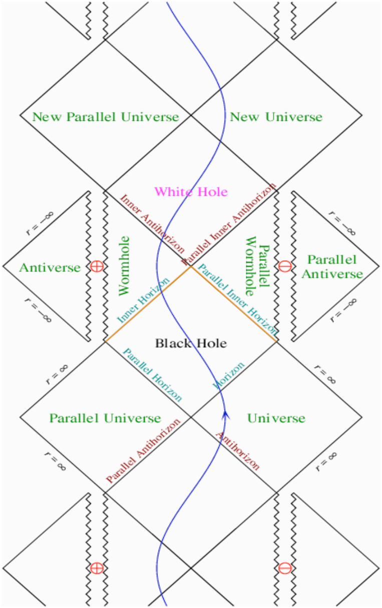
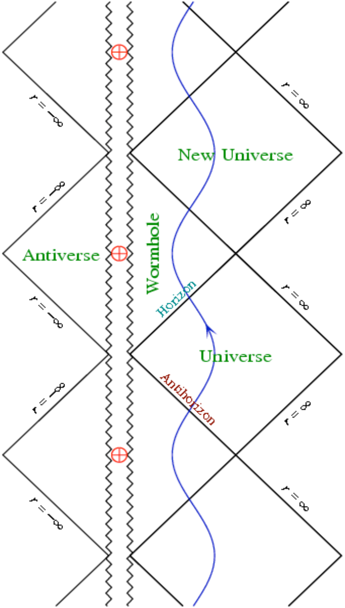


Schwarzschild black hole

http://casa.colorado.edu/∼ajsh/astr2030_06



Reissner-Nordstrom black hole



Extremal RN black hole

Supersymmetric phases of string theory

Supersymmetric black holes

- Focus on black hole entropy in the zero temperature limit $\Rightarrow$ supersymmetric, extremal black holes.
- These black holes are stable objects of the theory.
- The number of supersymmetric states do not change when the coupling is varied.

Strategy: microscopic

- Specify an ensemble of states with charges Q_i in string theory. These charges are carried by the fundamental objects of string theory *i.e.* strings, branes, monopoles, ...
- g_s controls the gravitational force. When $g_s Q_i \ll 1$, these objects exert a weak gravitational force, and one can describe the fluctuations as a weakly coupled field theory.
- Enumerate these microstates $d(Q_i)$ and compute the statistical entropy $S_{stat} = \log(d)$.

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Strategy: macroscopic

- For $g_s Q_i \gg 1$, the backreaction is strong, and the correct description is as solutions to the low energy effective theory *i.e.* general relativity coupled to matter fields.
- Black hole solution to the effective action carrying charges $\{Q_i\}$. Weak curvatures at the horizon for $Q_i \gg 1$. Measure its thermodynamic entropy S_{BH}.

In a large class of examples, for a BPS black hole with charge vector Q_i, the Bekenestein-Hawking entropy agrees with the logarithm of the degeneracy of the corresponding quantum microstates in the thermodynamic limit. Strominger & Vafa [1996]

$$\log(d(Q_i)) = \frac{A(Q_i)}{4} + O(1/Q_i)\,. \qquad (2)$$

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Early successful example Strominger & Vafa [1996]

- Five dimensional BPS Black hole carrying four types of charges $(Q1, Q5, p, \ell)$ in type IIB string theory on $K3 \times S^1$

$$S_{BH} = \frac{A}{4G} = 2\pi\sqrt{Q_1 Q_5 p - \ell^2} \,, \qquad (3)$$

- $d(Q_1, Q_5, p, \ell)$ given by the number of excitations of a certain chiral two dimensional SCFT with central charge $6Q_1Q_5$, $L_0 = p$ and $J_0 = \ell$:

$$d(Q_1, Q_5, p, \ell) = \exp(2\pi\sqrt{Q_1 Q_5 p - \ell^2}) + \cdots \,. \qquad (4)$$

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p	$Q_1 Q_5$	ℓ	$d(Q_1 Q_5, p, \ell)$	$\log(d)$	S_{BH}
1	1	0	5424	8.59	6.28
2	2	0	2540544	14.74	12.57
3	3	0	1254480000	20.95	18.85
3	3	1	991591800	20.71	18.59
3	3	2	483665920	20.00	17.77
3	3	-1	991591800	20.71	18.59
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$$\log(d) \xrightarrow{Q_i \to \infty} 2\pi \sqrt{Q_1 Q_5 p - \ell^2} = S_{BH} \, .$$

Questions

This beautiful approximate agreement raises important questions:

- What exact formula is (2) an approximation to?
- Do black holes know about *finite size effects*?
- Can we systematically compute these corrections to the gravitational entropy formula, in a $1/Q$ expansion, and perhaps even *exactly* for arbitrary finite values of the charges?

$$\log(d(Q)) = \frac{A(Q)}{4} + O(1/Q)\,, \tag{5}$$
$$\stackrel{?}{=} \log(W(Q))\,.$$

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Aim

- In these two talks, I shall report on the progress made in the last few years towards answering these questions.
- In particular, for a class of examples in superstring theory, under some reasonable assumptions, one can now compute $W(Q)$ exactly.
- Main objective: Compute $W(Q)$ and compare with $d(Q)$.

Based on

- Atish Dabholkar, João Gomes, S.M., *"Quantum Black Holes, Localization, and the Topological String,"* arXiv:1012.0265.
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Finite size effects and testing quantum gravity

- The leading Bekenstein-Hawking entropy is too universal since it follows from the Einstein-Hilbert action which is the leading low energy effective action in all phases.
- Finite size effects depend on the phase (compactification) under consideration which governs the higher derivative terms in the effective action. They thus provide a sensitive probe of short distance degrees of freedom.
- One can hope to learn more about the microscopic degrees of freedom of quantum gravity, effective actions in string theory, nonperturbative functional integral of quantum gravity, exact holography.
- Analogous to how one might study the specific heat of metals to deduce whether electrons or phonons are the relevant degrees of freedom in different phases.

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An extension of the Bekenstein-Hawking formula

- For black holes in a semi-classical theory of gravity described by a local Lagrangian density $\mathcal{L}$, the black hole entropy is given by

  Wald[1994]; Iyer & Wald [1994]; Jacobson, Kang, & Myers [1993]

$$S_{\rm Wald} = -2\pi \int_H d^{d-2}x \sqrt{h} \, \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \, \epsilon^{\mu\nu} \epsilon^{\rho\sigma} \; . \tag{6}$$

- When

$$\mathcal{S}_{\rm grav} = \mathcal{S}_{EH} = \frac{1}{8\pi G} \int d^4x \, \sqrt{g} \, R \, , \tag{7}$$

$$S_{\rm Wald} = \frac{1}{4G} A_{BH} = S_{BH} \, . \tag{8}$$

- The Wald entropy has been computed for BPS black holes in $\mathcal{N} = 2$ supergravity, and then compared successfully with microscopic predictions of string theory. Cardoso, de Wit, Kappelli, Mohaupt [2000]

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BPS black hole (Q_1, Q_5, p, ℓ)

A. Castro, S. M.; N. Banerjee [2008]

- Recall that the microstate degeneracies are those of a particular SCFT with $c = 6Q_1Q_5$, $L_0 = p$, and $J_0 = \ell$.
- The SCFT admits powerful (modular) symmetries which makes it possible to estimate the degeneracy of states beyond the leading Cardy approximation.
- One has

$$S_{\rm micro} = S^0_{\rm micro} + S^1_{\rm micro} + \cdots$$

with

$$S^0_{\rm micro}(Q_1, Q_5, n) = 2\pi\sqrt{Q_1 Q_5 p - \ell^2}$$

and $S^1_{\rm micro}$ given by a transcendental function, whose beginning values are as follows:

p	$Q_1 Q_5$	ℓ	$d(Q_1 Q_5, p, \ell)$	$\log(d)$	$S^0_{\rm micro}$	$S^0_{\rm micro} + S^1_{\rm micro}$
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$$S^0_{\rm micro} + S^1_{\rm micro} = 2\pi\sqrt{Q_1 Q_5 p - \ell^2}\,\left(1 + \frac{3}{2p}\right) + O(1/p^2)$$

Example: Type IIB string theory on $K3 \times S^1$

- Low energy effective action of Gravity+Gauge+Scalars ($I = 1, \ldots 23$)

$$\mathcal{S}_{\rm grav} = \frac{1}{8\pi G} \int d^5x \sqrt{g} \Big(-R - G_{IJ}\, \partial_a M^I\, \partial^a M^J - \frac{1}{2} G_{IJ}\, F^I_{ab}\, F^{Jab} + c_{IJK}\, A^I_a\, F^J_{bc}\, F^K_{de}\, \epsilon^{abcde} \Big) . \quad (9)$$

- Four derivative corrections governed by mixed gauge-gravitational Chern-Simons term

$$\delta \mathcal{S}_{\rm grav} = c_{2I}\, \epsilon_{abcde} A^{Ia}\, R^{bcfg}\, R^{de}{}_{fg} \; , \qquad (10)$$

  and its supersymmetric completion. Hanaki, Ohashi, Tachikawa [2006]

- Parameters G_{IJ}, c_{IJK}, c_{2I} are given by the geometric data of $K3$.

BPS black hole (Q_1, Q_5, p, ℓ) A. Castro, S. M.[2008]

- Using this higher derivative Lagrangian and the Wald formula, one gets a formula for the corrected entropy of the black hole

$$S_{\rm Wald} = 2\pi \sqrt{Q_1 Q_5 p - \ell^2}\,\Big(1 + \frac{3}{2p}\Big) + O(1/p^2) \qquad (11)$$

  in the regime $Q_1 Q_5 \to \infty$, p finite.

- This agrees with the analytic expansion of $S^0_{\rm micro} + S^1_{\rm micro}$ in the same regime.

- The macroscopic and microscopic entropies also agree in other ranges of large but finite charges.

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Good amount of progress in this area

This kind of agreement has been found for all 1/4-BPS black holes in a class of $\mathcal{N} = 4$ string theories in four and five dimensions.

- New exact microscopic degeneracy formulas have been found.
- Estimation methods for these formulas have been developed, mainly using methods of number theory and automorphic form theory.
- Efficient methods to compute the Wald entropy for the effective Lagrangians of the different string theories have been developed.

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Quantum corrections

- Wald entropy can incorporate the corrections to Bekenstein-Hawking entropy from all higher-derivative *local* terms in the effective action.
- But one should really use the 1PI quantum effective actions which include in general nonanalytic and nonlocal terms.
- These terms are in many cases essential for duality invariance.

We need

- A manifestly duality covariant formalism that generalizes Wald entropy to be able to discuss the finite size effects systematically.
- An IR regulator consistent with the symmetries of the theory.
- Such a generalization has been proposed in the recent work of Sen [2008]. We will now review this definition of exact quantum entorpy.

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Crucial ingredient: AdS_2 space

- Three dimensional Lorentzian space labelled by (x, y, z)

$$ds^2 = dx^2 - dy^2 - dz^2 \,. \qquad (12)$$

- Two dimensional space embedded in this three-dimensional space

$$x^2 - y^2 - z^2 = -a^2 \,.$$

  Hyperbolic space with radius a, $SO(2,1)$ isometry.

- $(x = a \sinh\eta \cosh t \,,\ y = a\cosh\eta \,,\ z = a\sinh\eta\sinh t)$ gives

$$ds^2 = a^2(d\eta^2 - \sinh^2\eta \, dt^2) \,.$$

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Appearance of AdS_2

- Intuition that the degrees of freedom of a black hole live on (or inside) the horizon.
- All known supersymmetric black holes develop a universal AdS_2 factor in the near horizon geometry.
- Time translation symmetry gets enhanced to $SO(2,1)$.
- Assume this is always the case, partially proven.

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1. Introduction and idea of exact black hole entropy
2. Gravitational origin of finite size effects
3. Holography and quantum entropy
4. Gravitational quantum entropy

AdS/CFT and extremal black holes

- AdS/CFT correspondence relates a quantum gravitational theory in AdS_{d+1} to a conformal field theory CFT_d.
- Obtained by focussing on the geometry of the near horizon region of an extremal black brane solution. The dual CFT is obtained as the IR limit of the fluctuations of the brane configuration.
- For $d = 1$, there should be a CFT_1 dual to the AdS_2 gravitational theory, living on the boundary of AdS_2.
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Particularities of AdS_2/CFT_1

- Generically, the brane fluctuations has a collection of degenerate ground states separated from excited states by a mass gap.
- Contrast with higher dimensions where there can be massless modes with long wavelength fluctuations.
- This is consistent with the fact that Lorentzian AdS_2 only supports zero energy excitations. Any finite energy excitation destroys the boundary conditions of AdS_2.
- The partition function is the trace over the CFT, which is simply the *number of ground states*. (Microstate degeneracy).
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Two special features of AdS_2

1. Choice of Ensemble

- In two dimensions Coulomb potential grows at the boundary instead of falling. Hence this growing mode must be held fixed and the constant mode can fluctuate.

$$A_r = 0\,, \quad A_\theta(r) \stackrel{r\to\infty}{\longrightarrow} er + c\,. \tag{13}$$

- By Gauss law, fixing e means that we are working in the fixed charge sector. Hence, the natural ensemble from the perspective of the AdS_2 boundary conditions is the *microcanonical* ensemble.

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2. Index = Degeneracy

- AdS_2 has $SU(1,1)$ symmetry. If there are at least four unbroken supersymmetries, the closure of the algebra implies $SU(1,1|2)$ superalgebra.

- Hence the horizon has an $SU(2)$ spherical symmetry.
  e.g.: Supersymmetric black holes in four dimensions do not rotate.
  Microcanonical ensemble $\Rightarrow J = 0$.

- If J is a generator then microstates associated with the horizon are invariant.

$$Tr[(-1)^J] = Tr[1] \,. \qquad (14)$$

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AdS_2/CFT_1 and exact quantum entropy

- Consider a black hole with charge vector (q, p). The quantum entropy is defined by a Euclidean functional integral over all field configurations which asymptote to AdS_2.

- For a theory with some vector fields A^i and scalar fields ϕ^a, we have the fall-off conditions

$$\begin{aligned} ds_0^2 &= v_* \left[\left(r^2 + \mathcal{O}(1)\right) d\theta^2 + \frac{dr^2}{r^2 + \mathcal{O}(1)} \right], \\ \phi^a &= u_*^a + \mathcal{O}(1/r), \qquad A^i = -i\, e_*^i (r - \mathcal{O}(1)) d\theta . \end{aligned}$$

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Definition of quantum entropy

$$W(q,p) = \left\langle \exp\left[-\, i\, q_i \int_0^{2\pi} A^i \, d\theta \right] \right\rangle_{\mathrm{AdS}_2}^{\mathrm{finite}} \, .$$

The constants v_*, e_*^i, u_*^a which set the boundary conditions of the functional integral are determined purely in terms of the charges by the *attractor mechanism*.

The quantum entropy is thus purely a function of the charges (q,p).

The action in the functional integral suffers from an infrared divergence due to infinite volume of the AdS_2. To obtain a well-defined functional integral one must regulate and renormalize. Holographic renormalization.

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Renormalized functional integral

- Put a cutoff at a large $r = r_0$. Locality of the effective Lagrangian + the attractor mechanism implies that the bulk effective action has the form
$$S_{\rm bulk} = C_0 r_0 + C_1 + \mathcal{O}(r_0^{-1}) \,,$$
with C_0, C_1 independent of r_0.

- The linear divergence can then be removed by a boundary counter-term corresponding to a boundary cosmological constant. There could be more general boundary terms in principle, but the they are not needed in the supersymmetric examples.

- With this prescription, in the semi-classical limit one obtains
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Concrete objectives of the quantum entropy program

Our main goal is to put these formal definitions to use in concrete examples:

- Compute $W(q, p)$ for arbitrary finite charges by evaluating the functional integral of string field theory on the AdS_2 background.
- Compute $d(q, p)$ from bound state dynamics of branes and check if it equals $W(q, p)$ computed above.

First example of *exact* holography.

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Outline

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2. Gravitational origin of finite size effects
3. Holography and quantum entropy
4. Gravitational quantum entropy

- Evaluating the formal expression for W(q,p) by doing the string field theory functional integral is of course highly nontrivial.
- It may seem too ambitious to try to evalute the functional integral of full string field theory on the black hole background.
- It turns out that using localization techniques one can go surprisingly far and reduce the functional integral to an ordinary integral.
- With enough supersymmetry, it seems possible to in fact evaluate both $d(q, p)$ and $W(q, p)$ exactly.

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- Explain the technique of localization to evaluate supersymmetric functional integrals exactly.
- Apply this to the gravity functional integral over AdS_2 to get an exact expression for $W(Q_i)$.
- Compare the macroscopic answer $W(Q_i)$ with the microscopic $d(Q_i)$ in a simple example.
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