Topology of electronic bands and Topological Order

R. Shankar

The Institute of Mathematical Sciences, Chennai

TIFR, 26^{th} April, 2011

Outline

IQHE and the Chern Invariant

Topological insulators and the Z_2 invariant

Topological order

Exactly solved models with topological order

A model with everything !

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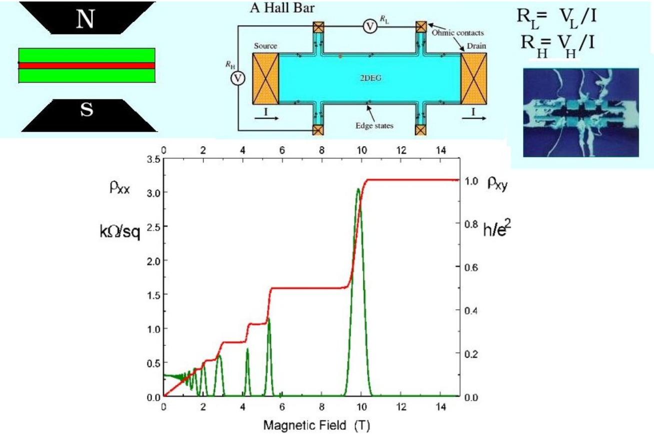
Quantum Hall Plateaus



$$R = \frac{25812.807557(18)}{n}\ \Omega$$

Why is R so accurately measurable ?

An answer

VOLUME 49, NUMBER 6 PHYSICAL REVIEW LETTERS 9 AUGUST 1982

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 30 April 1982)

The Hall conductance of a two-dimensional electron gas has been studied in a uniform magnetic field and a periodic substrate potential U. The Kubo formula is written in a form that makes apparent the quantization when the Fermi energy lies in a gap. Explicit expressions have been obtained for the Hall conductance for both large and small $U/\hbar\omega_c$.

PACS numbers: 72.15.Gd, 72.20.Mg, 73.90.+b

$$\sigma_{\mathrm{H}} = \frac{ie^2}{2\pi h}\sum\int d^2k\int d^2r\left(\frac{\partial u^*}{\partial k_1}\frac{\partial u}{\partial k_2} - \frac{\partial u^*}{\partial k_2}\frac{\partial u}{\partial k_1}\right)$$
$$= \frac{ie^2}{4\pi h}\sum\oint dk_j\int d^2r\left(u^*\frac{\partial u}{\partial k_j} - \frac{\partial u^*}{\partial k_j}u\right), \qquad (5)$$

VOLUME 71, NUMBER 22 PHYSICAL REVIEW LETTERS 29 NOVEMBER 1993

Chern Number and Edge States in the Integer Quantum Hall Effect

Yasuhiro Hatsugai
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and Institute for Solid State Physics, University of Tokyo, 7-22-1 Roppongi Minato-ku, Tokyo 106, Japan
(Received 12 July 1993)

Hall conductance of the filled jth band is

$$\sigma_{xy}^{j,\mathrm{bulk}} = -\frac{e^2}{h}[\ I(C_j) - I(C_{j-1})\ ] = \sigma_{xy}^{j,\mathrm{edge}}, \qquad (15)$$

Tight binding models

$$H = \sum_{i\alpha\ j\beta} c^{\dagger}_{i\alpha} h(R_i - R_j)_{\alpha\beta} c_{j\beta}$$

$$R_i = \sum_{a} i_a \vec{e}_a, \ \ \alpha, \beta = 1, \ldots, N_B$$

Fourier transform,

$$c_{i\alpha} = \int_k e^{i\vec{k}\cdot\vec{R}_i} c_{k\alpha}$$

Reciprocal lattice basis

$$\vec{G}_a \cdot \vec{e}_b = \delta_{ab}$$

$$\vec{k} \equiv \sum_{a} k_a \vec{G}_a, \qquad \vec{k} \cdot \vec{R}_i = \sum_{a} i_a k_a$$

Hence $k_a \sim k_a + 2\pi$, $\Rightarrow$ the Brillouin zone is always a topologically a torus.

Momentum space wave functions

$$H = \int_k \sum_{\alpha\beta} c^\dagger_\alpha(k)\, h(k)_{\alpha\beta}\, c_\beta(k)$$

The single particle hamiltonian is an $N_B \times N_B$ matrix at every k.

$$h(k)_{\alpha\beta} u^n_\beta(k) = \epsilon_n(k) u^n_\alpha(k)$$

$\epsilon^n(k),\ n = 1, ..., N_B$ form the energy bands.

$$c_n(k) \equiv \sum_{\alpha=1}^{N_B} (u^n(k))^*_\alpha c_\alpha(k)$$

$$|GS\rangle = \prod_{\epsilon_n(k)<\epsilon_F} c^\dagger_n(k)|0\rangle$$

The phase that launched a thousand scripts

- The wave functions, $u^n(k)$ and $e^{i\Omega^n(k)}u^n(k)$ represent the same physical state.
- The phase picked up by the wave-function when the state is Adiabatically transported defines a "Pancharathnam-Berry" connection:

$$
\begin{aligned}
u^n(k_1) &\rightarrow e^{i\int_{k_1}^{k_2} A_i^n(k)dk^i} u^n(k_2) \\
A_i^n(k) &= \frac{1}{2i}\left((u^n(k))^\dagger \partial_i u^n(k) - h.c\right)
\end{aligned}
$$

- $$F_{ij}^n(k) = \partial_i A_j^n(k) - \partial_j A_i^n(k)$$

  The Pancharathnam-Berry curvature field, $F_{ij}^n(k)$ is independent of the phase convention used to define the eigenstates.

The Chern invariant

- For 2-d systems, the integral of F over the Brillouin zone is an integer invariant, the Chern number.

$$\nu^n = \frac{1}{8\pi} \int d^2k \; \epsilon_{ij} F^n_{ij}(k)$$

- $\frac{e^2}{h}\times$Chern Number=Quantized Hall Conductance

  Thouless, Kohmoto, Nightingale and den Nijs, (1982)

- Chern Number=Number of chiral edge channels

  Hatsugai, (1993)

The Chern number as a winding number

- At every k we have a N_B level system.
  eg. $N_B = 2$ we have a 2 level system.

$$h(k) = \bar{\epsilon}(k) + \frac{1}{2}\Delta\epsilon(k)\hat{n}(k) \cdot \vec{\tau}$$

- The physical states at every k form a CP_{N_B-1} manifold.
  eg at $N_B = 2$, we have the Bloch sphere ($CP_1 = S_2$).

$$u(k) = \begin{pmatrix} \cos\frac{\theta(k)}{2} \\ \sin\frac{\theta(k)}{2} e^{i\phi(k)} \end{pmatrix}$$

- The wavefunctions for every band define a map from the Brillouin zone to the space of physical states, CP_{N_B-1}.

The Chern number as a winding number

- When $d = 2$, we have maps from $T_2 \to CP_{N_B-1}$. These are topologically equivalent to maps from $S_2 \to CP_{N_B-1}$ since all loops in CP_{N_B-1} can be shrunk to points.
- The winding number of the map is exactly the Chern number.

$$\begin{aligned}\nu^{\pm} &= \pm\frac{1}{4\pi}\int d^2k\ \hat{n}\cdot\partial_1\hat{n}\times\partial_2\hat{n} \\ &= \pm\frac{1}{4\pi}\int d^2k\ F_{12}(k)\end{aligned}$$

Physical Effects: Anomalous velocity

Semiclassical equations of motion for wave packets:

$$\begin{aligned} \dot{x}_i &= \frac{\partial}{\partial k_i}\epsilon(k) + F_{ij}(k)\dot{k}_j \\ \dot{k}_i &= e\frac{\partial}{\partial x_i}V(x) + eB_{ij}(x)\dot{x}_j \end{aligned}$$

Ganesh Sundaram and Qian Niu, Phys. Rev. B23, 14915 (1999);

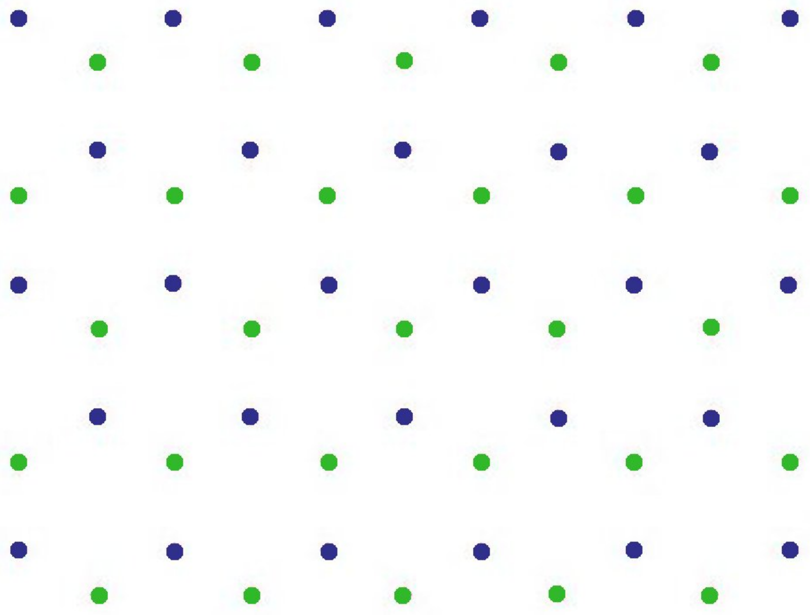
F. D. M. Haldane, PRL 93, 20662 (2004)

Leads to anomalous hall conductivity:

$$\sigma_H = \int_{\epsilon(k)<\epsilon_F} \frac{e^2}{2h}\epsilon_{ij}F_{ij}(k)$$

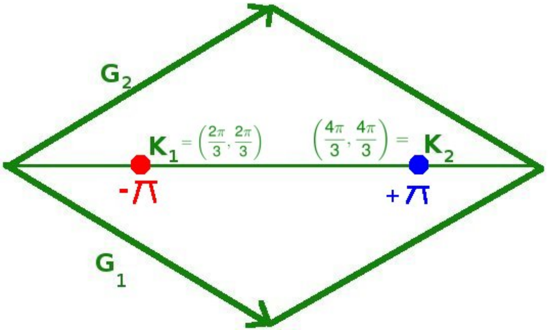
If the Fermi level is in a gap and all the bands are either filled or empty, then the hall conductivity is quantised.

Explicit models on the honeycomb lattice



Graphene

$$
\begin{aligned}
h(k) &= \alpha^x p_x(k) + \alpha^y p_y(k) + \beta M \\
&= \begin{pmatrix} M & p_x - i p_y \\ p_x + i p_y & -M \end{pmatrix} \\
p_x(k) &= t(1 + \cos k_1 + \cos k_2) \\
p_y(k) &= t(\sin k_1 - \sin k_2)
\end{aligned}
$$



Graphene

$k = K_1 + q,$

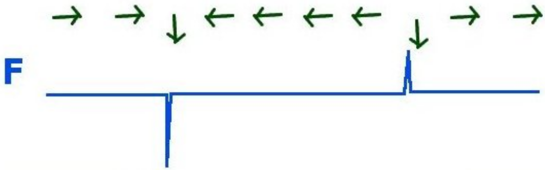
$$h(k) = \frac{t\sqrt{3}}{2}(\alpha_x q_x + \alpha_y q_y) + \beta M$$

$$F_{ij}(k) = \frac{1}{4\pi}\frac{M}{(q^2 + M^2)^{3/2}}$$

$k = K_2 + q,$

$$h(k) = \frac{t\sqrt{3}}{2}(\alpha_x q_x - \alpha_y q_y) + \beta M$$

$$F_{ij}(k) = -\frac{1}{4\pi}\frac{M}{(q^2 + M^2)^{3/2}}$$

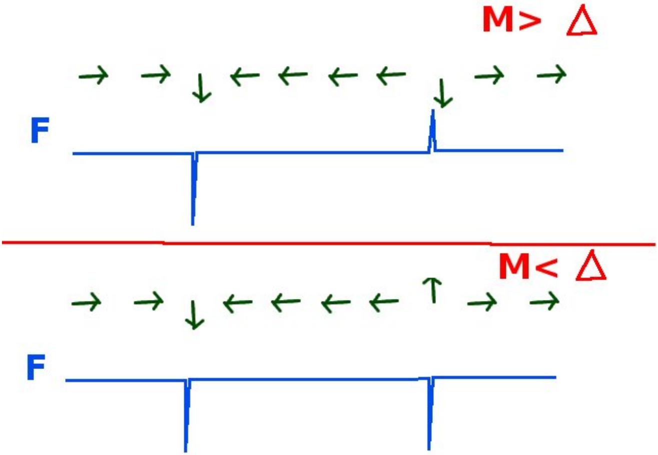


The Haldane model

$$\begin{aligned} h(k) &= \alpha^x p_x(k) + \alpha^y p_y(k) + \beta M(k) \\ p_x(k) &= t(1 + \cos k_1 + \cos k_2) \\ p_y(k) &= t(\sin k_1 - \sin k_2) \\ M(k) &= M + \Delta\,(\sin k_1 + \sin k_2 + \sin(-k_1 - k_2)) \end{aligned}$$

- $M >> \Delta,\ \nu = 0$, $M << \Delta,\ \nu = \pm 1$. Topological transition from $\nu = 0$ to $\nu = \pm 1$ phase at $M = 3\frac{\sqrt{3}}{2}$.
- Magnetic field not necessary for non-zero Chern number. Other time reversal symmetry breaking terms can also induce it.
- Bands can exchange units of Pancharathnam-Berry flux when they touch at Dirac points.

The Haldane model



Outline

Topological Insulators: Theory

Prediction of Insulating materials with metallic surfaces

PRL **95,** 146802 (2005) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2005

Z_2 Topological Order and the Quantum Spin Hall Effect

C. L. Kane and E. J. Mele

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **75**, 121306(R) (2007)

Topological invariants of time-reversal-invariant band structures

J. E. Moore[1,2] and L. Balents[3]

Three dimensional topological invariants for time reversal invariant Hamiltonians and the three dimensional quantum spin Hall effect

Rahul Roy

arXiv:cond-mat/0607531v3 [cond-mat.mes-hall] 21 Jul 2006

PRL **98,** 106803 (2007) PHYSICAL REVIEW LETTERS week ending 9 MARCH 2007

Topological Insulators in Three Dimensions

Liang Fu, C. L. Kane, and E. J. Mele

Topological Insulators: Real materials

ARTICLES | nature physics

PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1270

Topological insulators in Bi_2Se_3, Bi_2Te_3 and Sb_2Te_3 with a single Dirac cone on the surface

Haijun Zhang[1], Chao-Xing Liu[2], Xiao-Liang Qi[3], Xi Dai[1], Zhong Fang[1] and Shou-Cheng Zhang[3]*

Topological insulators are new states of quantum matter in which surface states residing in the bulk insulating gap of such systems are protected by time-reversal symmetry. The study of such states was originally inspired by the robustness to scattering of conducting edge states in quantum Hall systems. Recently, such analogies have resulted in the discovery of topologically protected states in two-dimensional and three-dimensional band insulators with large spin–orbit coupling. So far, the only known three-dimensional topological insulator is Bi_xSb_{1-x}, which is an alloy with complex surface states. Here, we present the results of first-principles electronic structure calculations of the layered, stoichiometric crystals Sb_2Te_3, Sb_2Se_3, Bi_2Te_3 and Bi_2Se_3. Our calculations predict that Sb_2Te_3, Bi_2Te_3 and Bi_2Se_3 are topological insulators, whereas Sb_2Se_3 is not. These topological insulators have robust and simple surface states consisting of a single Dirac cone at the Γ point. In addition, we predict that Bi_2Se_3 has a topologically non-trivial energy gap of 0.3 eV, which is larger than the energy scale of room temperature. We further present a simple and unified continuum model that captures the salient topological features of this class of materials.

LETTERS | nature physics

PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1274

Observation of a large-gap topological-insulator class with a single Dirac cone on the surface

Y. Xia[1,2], D. Qian[1,3], D. Hsieh[1,2], L. Wray[1], A. Pal[1], H. Lin[4], A. Bansil[4], D. Grauer[5], Y. S. Hor[5], R. J. Cava[5] and M. Z. Hasan[1,2,6]*

Recent experiments and theories have suggested that strong spin–orbit coupling effects in certain band insulators can give rise to a new phase of quantum matter, the so-called topological insulator, which can show macroscopic quantum-entanglement effects[1-7]. Such systems feature two-dimensional surface states whose electrodynamic properties are described not by the conventional Maxwell equations but rather by an attached axion field, originally proposed to describe interacting quarks[8-15]. It has been proposed that a topological insulator[2] with a single Dirac cone interfaced with a superconductor can form the most elementary unit for performing fault-tolerant quantum computation[14]. Here we present an angle-resolved photoemission spectroscopy study that reveals the first observation of such a topological state of matter featuring a single surface Dirac cone realized in the naturally occurring Bi_2Se_3 class of materials. Our results, work as a matrix material to observe a variety of topological quantum phenomena.

The topological-insulator character of BiSb[5,6] led us to investigate the alternative Bi-based compounds Bi_2X_3 (X = Se, Te). The undoped Bi_2Se_3 is a semiconductor that belongs to the class of thermoelectric materials Bi_2X_3 with a rhombohedral crystal structure (space group D^5_{3d} ($R\bar{3}m$); refs 17, 18). The unit cell contains five atoms, with quintuple layers ordered in the Se(1)–Bi–Se(2)–Bi–Se(1) sequence. Electrical measurements report that, although the bulk of the material is a moderately large-gap semiconductor, its charge transport properties can vary significantly depending on the sample preparation conditions[19], with a strong tendency to be n-type[20,21] owing to atomic vacancies or excess selenium. An intrinsic bandgap of approximately 0.35 eV is typically measured in experiments[22,23], whereas theoretical calculations estimate the gap to be in the range of 0.24–0.3 eV (refs 20, 24).

Time reversal in Quantum mechanics

$$\psi_2 = e^{-iH(t_2-t_1)}\psi_1$$

If there is an operator $\mathcal{T}$ such that,

$$\mathcal{T}\psi_1 = e^{-iH(t_2-t_1)}\mathcal{T}\psi_2$$

Then the system is time reversal invariant.

Time reversal for Bloch hamiltonians,

$$\mathcal{T}u(k)_\alpha = (\sigma^y\ u^*(k))_\alpha$$

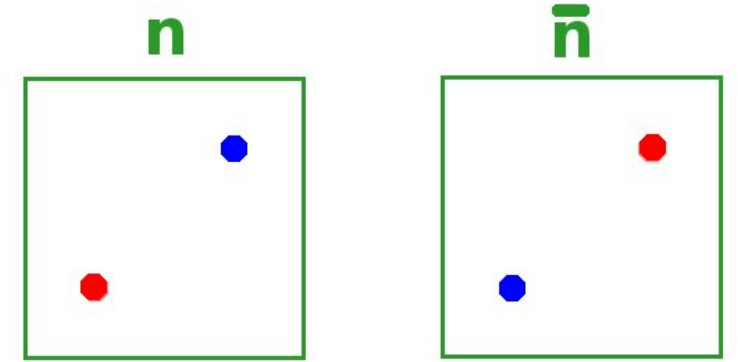
if

$$\sigma^y h^*(k)\sigma^y = h(-k)$$

Then system is time reversal invariant

Band Pairs

$$
\begin{aligned}
\mathcal{T} u_n(k) &= u_{\bar{n}}(-k) \\
\mathcal{T} u_{\bar{n}}(k) &= -u_n(-k) \\
\epsilon_n(k) &= \epsilon_{\bar{n}}(-k) \\
F_{ij}^{n}(k) &= -F_{ij}^{\bar{n}}(-k)
\end{aligned}
$$



The 2-d Z_2 invariant

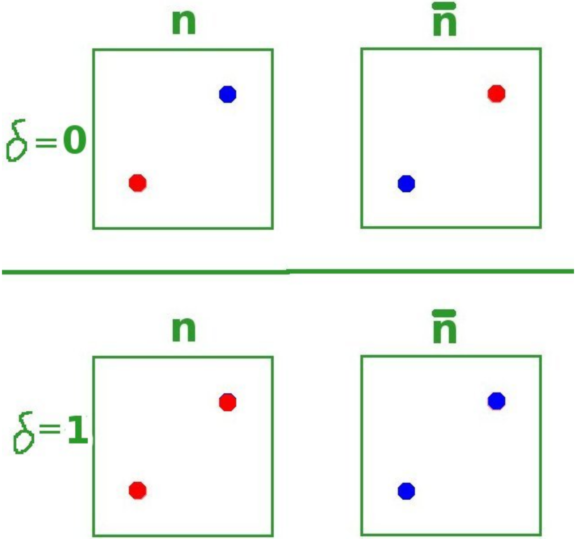
Consider a system with 2 occupied time reversed bands:

- The total Berry flux carried by them will always be zero.
- If the hamiltonian is smoothly perturbed, maintaining time reversal invariance, and they touch ($h(k)$ becomes degenerate), it will always happen at pairs of points $(k, -k)$.
- Time reversal invariance ensures that the flux exchanged by bands at these two points is always equal.
- The change of flux in each band is always even and hence the Chern index of each band modulo 2 is invariant under smooth time reversal symmetric perturbations.
- in general,

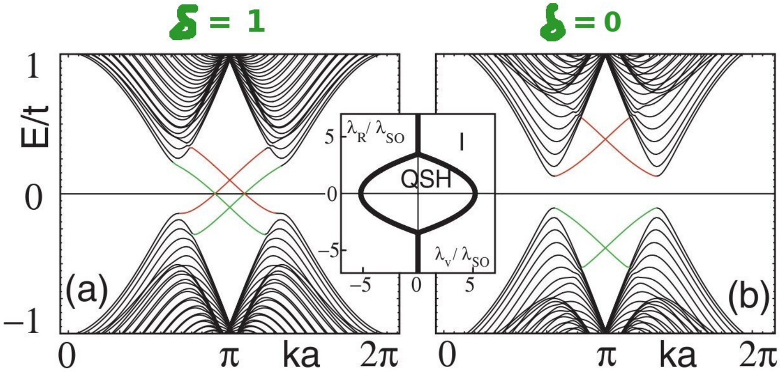
$$\delta = \left(\frac{1}{2} \sum_{n=1}^{N_B} |\nu_n| \right) \text{modulo } 2$$

is invariant.

The 2-d Z_2 invariant



Consequences at the edge



C. L. Kane and E. J. Mele PRL **95,** 146802 (2005)

- Even number of edge pairs (per edge) for $\delta = 0$
  Odd number of edge pairs (per edge) for $\delta = 1$
- No backscattering for a single pair due to time reversal symmetry.

The 3-d Z_2 invariants

Parameterise the 3-d torus by $-\pi \leq k_i \leq \pi, \; i = x, y, z$.

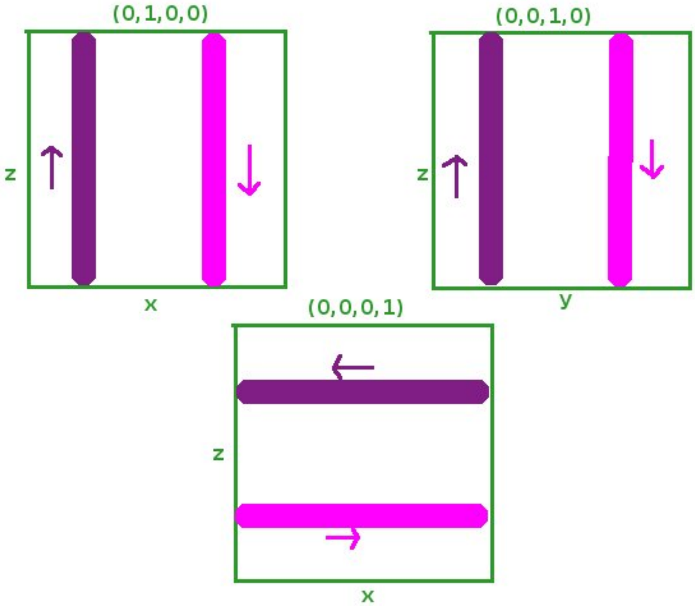
- There are many time reversal invariant planes. eg $k_i = 0, \pi$.
- The 2-d Z_2 invariants of these planes are all topological invariants.
- Of these, there are 4 independent invariants which can be chosen to be,

$$\delta_x \equiv \delta_{k_x=\pi}, \quad \delta_y \equiv \delta_{k_y=\pi}, \quad \delta_z \equiv \delta_{k_z=\pi}$$

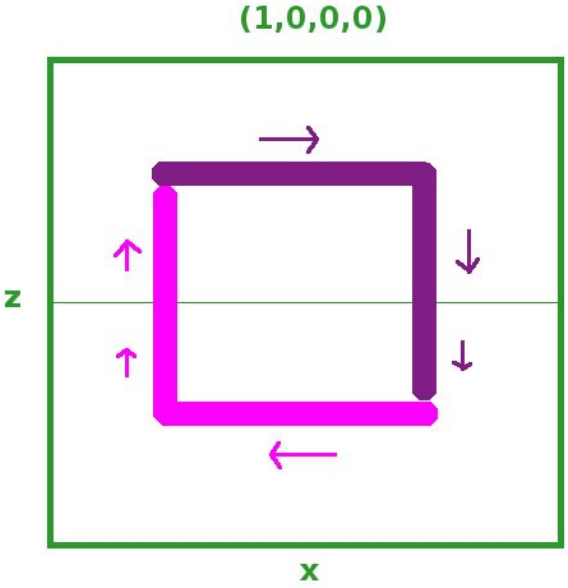
$$\delta_0 \equiv \delta_{k_z=0}\delta_{k_z=\pi}$$

- If $\delta_0 = -1$, then the system is called a “Strong topological insulator".

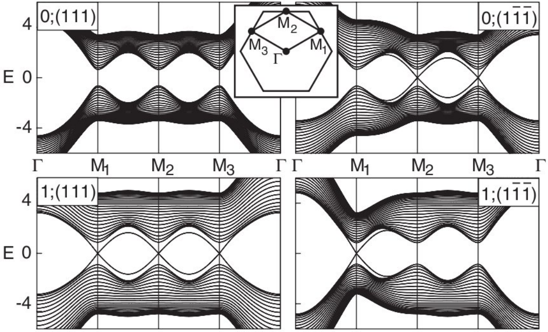
$(\delta_0, \ \delta_x, \ \delta_y, \ \delta_z)$



$(\delta_0,\ \delta_x,\ \delta_y,\ \delta_z)$



Consequences at the surface



Liang Fu, C. L. Kane, and E. J. Mele PRL **98,** 106803 (2007)

- Even number of surface Dirac cones for $\delta = 0$
  Odd number of surface Dirac cones for $\delta = 1$
- Gaplessness for odd number of Dirac points protected by time reversal symmetry.

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IQHE and the Chern Invariant

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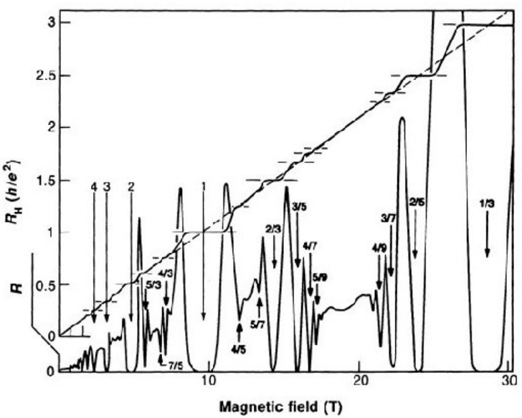
Exactly solved models with topological order

A model with everything !

FQHE: Partially filled Landau Bands

D. C. Tsui, H. L. Stormer, and A. C. Gossard, *Two-dimensional magnetotransport in the Extreme Quantum Limit*, Phys. Rev. B 48 (1982) 1559



$$\nu = \frac{p}{2mp+1}$$ Jainendra Jain

Topological order in FQHE

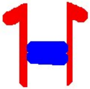
- For $\nu = 1/3$: unique ground state on the sphere but 3-fold degenerate ground state on the torus. In general FQHE states have a genus dependent degeneracy
- States characterised by gapped, fractionally charged quasi-particles obeying anyonic statistics.
- Gapless chiral excitations at the edge.
- The longwavelength physics described by topological field theories (Chern-Simons theories).

Strongly correlated systems

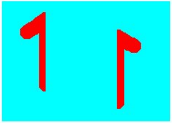
PHYSICAL REVIEW B VOLUME 37, NUMBER 1 1 JANUARY 1988

Gauge theory of high-temperature superconductors and strongly correlated Fermi systems

G. Baskaran and P. W. Anderson
Joseph Henry Laboratories, Department of Physics, Jadwin Hall, Princeton University, P.O. Box 708, Princeton, New Jersey 08544
(Received 6 July 1987)



Confined
Magnetic ordered



Deconfined
Spin Liquid

PHYSICAL REVIEW B VOLUME 62, NUMBER 12 15 SEPTEMBER 2000-II

Z_2 gauge theory of electron fractionalization in strongly correlated systems

T. Senthil and Matthew P. A. Fisher
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106–4030
(Received 25 October 1999)

Topological entanglement entropy

PRL **96,** 110404 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

Topological Entanglement Entropy

Alexei Kitaev[1,2] and John Preskill[1]

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[2]*Microsoft Research, One Microsoft Way, Redmond, Washington 98052, USA*
(Received 13 October 2005; published 24 March 2006)

We formulate a universal characterization of the many-particle quantum entanglement in the ground state of a topologically ordered two-dimensional medium with a mass gap. We consider a disk in the plane, with a smooth boundary of length L, large compared to the correlation length. In the ground state, by tracing out all degrees of freedom in the exterior of the disk, we obtain a marginal density operator ρ for the degrees of freedom in the interior. The von Neumann entropy of ρ, a measure of the entanglement of the interior and exterior variables, has the form $S(\rho) = \alpha L - \gamma + \cdots$, where the ellipsis represents terms that vanish in the limit $L \to \infty$. We show that $-\gamma$ is a universal constant characterizing a global feature of the entanglement in the ground state. Using topological quantum field theory methods, we derive a formula for γ in terms of properties of the superselection sectors of the medium.

DOI: 10.1103/PhysRevLett.96.110404 PACS numbers: 03.65.Ud, 03.67.Mn, 71.10.Pm, 73.43.Nq

PRL **96,** 110405 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

Detecting Topological Order in a Ground State Wave Function

Michael Levin and Xiao-Gang Wen

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(Received 25 October 2005; published 24 March 2006)

A large class of topological orders can be understood and classified using the string-net condensation picture. These topological orders can be characterized by a set of data $(N, d_i, F^{ijk}_{lmn}, \delta_{ijk})$. We describe a way to detect this kind of topological order using only the ground state wave function. The method involves computing a quantity called the "topological entropy" which directly measures the total quantum dimension $D = \sum_i d_i^2$.

DOI: 10.1103/PhysRevLett.96.110405 PACS numbers: 11.15.−q, 03.65.Ud, 11.25.−w, 71.10.−w

Topological order

Topological order realised in quantum Hall liquids and quantum spin liquids.
Characterised by:

- Genus dependent degeneracy of all states
- Emergent gauge fields
- Quasi-particles with fractional quantum numbers and statistics
- Related to “pattern of quantum entanglement" ?

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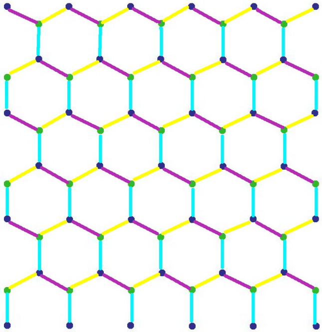
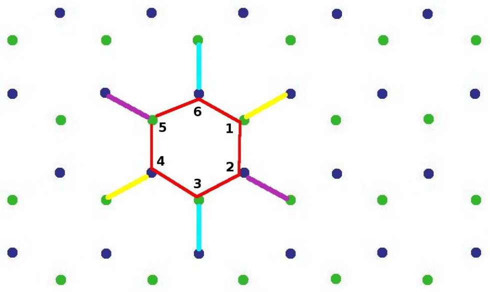
Kitaev's honeycomb model

Alexei Kitaev,"Anyons in an exactly solved model and beyond", cond-mat/0506438, Annals of Physics.



$$H = J_x \sum_{<ij>} \sigma_i^x \sigma_j^x + J_y \sum_{<ij>} \sigma_i^y \sigma_j^y + J_z \sum_{<ij>} \sigma_i^z \sigma_j^z$$

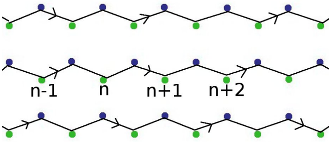
Conserved quantities



$$W_p \equiv \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$
$$[W_p, H] = 0 = [W_p, W_{p'}]$$
$$W_p^2 = 1$$

Conserved quantities have the properties of magnetic fluxes of a Z_2 gauge theory

Jordan-Wigner transformation for $S = \frac{1}{2}$



The disorder variables:

$$\mu_{i_l} = \prod_{m<l} \left(\sigma^z_{i_m}\right)$$

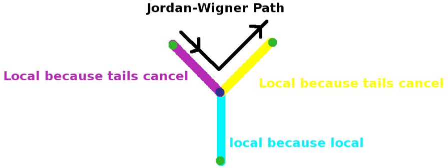
Jordan-Wigner fermions:

$$\xi_{i_n} = \sigma^x_{i_n}\mu_{i_n} \qquad \eta_{i_n} = \sigma^y_{i_n}\mu_{i_n}$$

$$\{\xi_{i_n}, \xi_{i_m}\} = 2\delta_{nm} = \{\eta_{i_n}, \eta_{i_m}\}$$

$$\{\xi_{i_n}, \eta_{i_m}\} = 0$$

Majoranisation



$$H = J_x \sum_{\langle ij \rangle} i\xi_i\xi_j + J_y \sum_{\langle ij \rangle} i\xi_i\xi_j + J_z \sum_{\langle ij \rangle} i\xi_i u_{ij}\xi_j$$

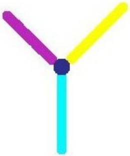
$$u_{ij} = i\eta_i\eta_j$$

Hamiltionian of Majorana fermions, ξ_i, interacting with **static** Z_2 gauge fields, u_{ij}, in the gauge,

$$u_{\langle ij \rangle} = u_{\langle ij \rangle} = 1$$

Generalisations

Spin-1/2 Kitaev Model generalises to any lattice constructed out of:



Generalises to any lattice using Clifford Algebras

PHYSICAL REVIEW D **68**, 065003 (2003)

Quantum order from string-net condensations and the origin of light and massless fermions

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Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 24 February 2003; published 3 September 2003)

$$\chi_{P_{xy}\mathbf{m}} = (-)^{m_x m_y}\chi_{\mathbf{m}}. \qquad (125)$$

VIII. QED AND QCD FROM A BOSONIC MODEL ON A CUBIC LATTICE

Topological order in the Kitaev Model

- Fermionic “spin-1/2" excitations.
- Non-abelian anyonic excitations in a particular phase: Unpaired Majorana fermions bound to a vortex (flux defect).
- 4-fold degeneracy on a torus corresponding to Z_2 fluxes passing through the “holes". Saptarshi Mandal PhD thesis

Our work

- Spin liquid: Very short ranged spin-spin correlations.

  G. Baskaran, Saptarshi Mandal and R. Shankar, PRL **98**, 247201 (2007)

  True at all S. Huge classical spin degeneracy: Very much like frustrated quantum antiferromagnets:

  G. Baskaran, Diptiman Sen and R. Shankar, Phys. Rev. **B 78** 115116 (2008); D. Dhar, Kabir Ramola and Samarth Chandra Phys. Rev. **B**
- First order transitions to magnetically ordered phases (confined phases) when an Ising interaction is added. Criterion that perturbation does not induce long range correlations:

  Saptarshi Mandal, Subhro Bhattacharjee, Krishnendu Sengupta, R. Shankar and G. Baskaran
- Simple chains where Majorana Fermions can be manipulated:

  Abhinav Saket and S. R. Hassan and R. Shankar, Phys. Rev. **B 82**, 174409 (2010)
- Creating unpaired Majorana fermions by coupling to an impurity spin:

  Kusum Dhochak and Vikram Tripathi and R. Shankar, Phys. Rev. Lett. **105**, 117201 (2010)

Outline

IQHE and the Chern Invariant

Topological insulators and the Z_2 invariant

Topological order

Exactly solved models with topological order

A model with everything !

Proposals for engineered realisations

VOLUME 91, NUMBER 9 PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2003

Controlling Spin Exchange Interactions of Ultracold Atoms in Optical Lattices

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[2]*Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA*
(Received 25 October 2002; published 26 August 2003)

Quantum emulation of a spin system with topologically protected ground states using superconducting quantum circuits

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[4]*Center for Theoretical Physics, Physics Department, University of Michigan, Ann Arbor, MI 48109-1040, USA*
(Dated: December 3, 2009)

Phys. Rev. B 81, 014505 (2010)

epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS
October 2008
EPL, **84** (2008) 20001
www.epljournal.org
doi: 10.1209/0295-5075/84/20001

Reproducing spin lattice models in strongly coupled atom-cavity systems

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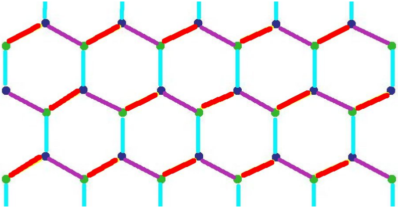
[2] *Science Department, Technical University of Crete - Chania, Crete, Greece, 73100, EU*
[3] *Centre for Quantum Technologies, National University of Singapore - 2 Science Drive 3, Singapore 117543*

The Demler et. al. Model



$$H = \sum_{\langle ij \rangle} t\, C_i^\dagger C_j + h.c + \sum_{\langle ij \rangle_x} t' \left(C_i^\dagger \sigma^x C_j + h.c \right)$$

$$+ + \sum_{\langle ij \rangle_y} t' \left(C_i^\dagger \sigma^y C_j + h.c \right) + \sum_{\langle ij \rangle_z} t' \left(C_i^\dagger \sigma^z C_j + h.c \right)$$

$$+ U \sum_i n_{i\uparrow} n_{i\downarrow}$$

S.R. Hassan, S. Goyal and R. Shankar (IMSc.)

David Senechal and A -M. Tremblay (Sherbrooke)

$U = 0$

$$h(k) = \begin{pmatrix} 0 & \Sigma(k) \\ \Sigma^\dagger(k) & 0 \end{pmatrix}$$

$$\Sigma(k) = t\left(1 + e^{ik_2} + e^{-ik_1}\right) + t'\left(\sigma^z + \sigma^x e^{ik_2} + \sigma^y e^{-ik_1}\right) = \Sigma^\dagger(-k)$$

At $t' = 0$,

$$h(k) = \sigma^y h^*(-k)\sigma^y$$

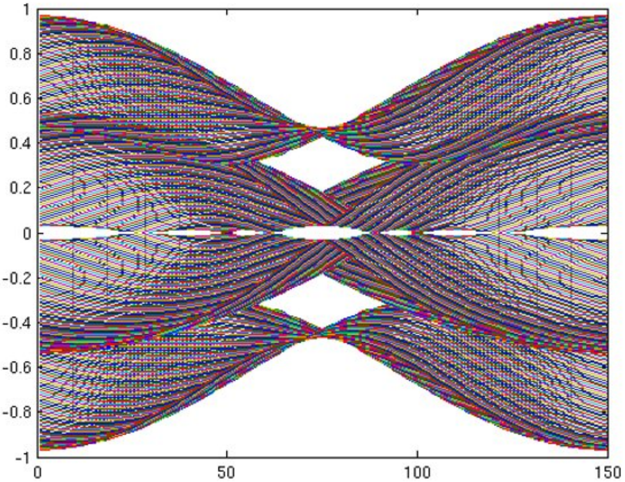
At $t = 0$,

$$h(k) = \beta\sigma^y h^*(-k)\sigma^y\beta$$

$$\beta \equiv \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

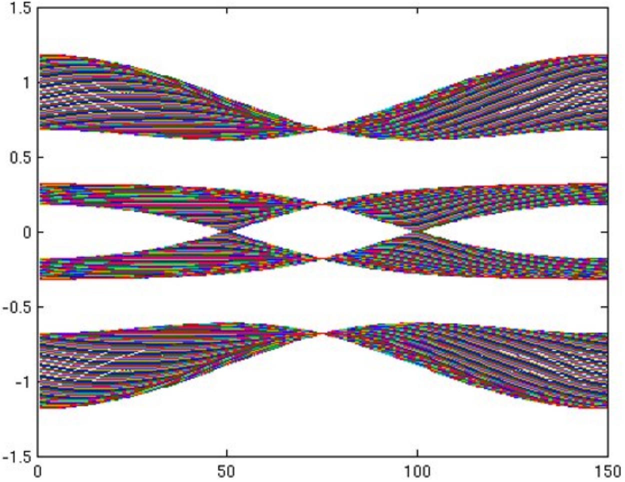
At $t, t' \neq 0$, time reversal symmetry is broken.

Eigenvalues: $\Delta t = 0.5$

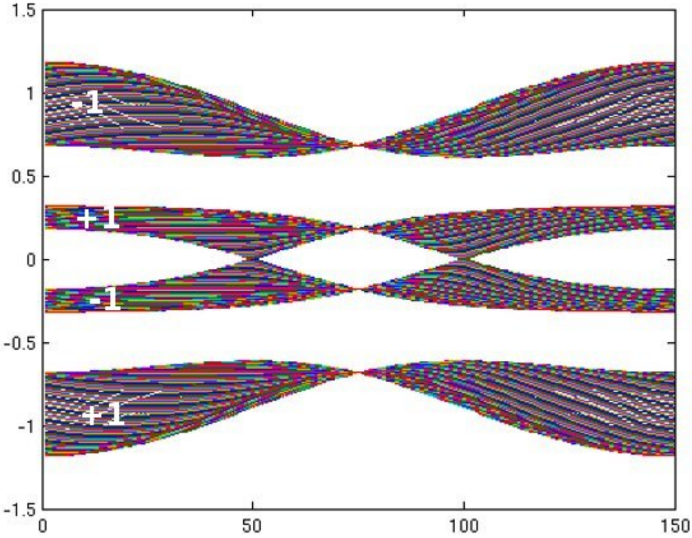


Eigenvalues: $\Delta t = 1.0$

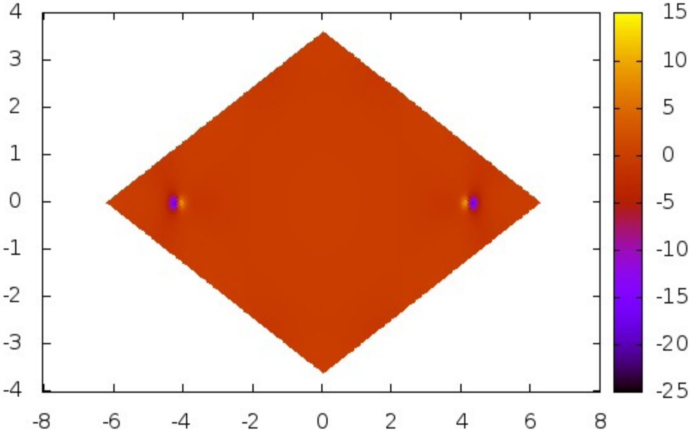
The bands stop overlapping at $\Delta t = \sqrt{6} - \sqrt{3} = 0.717$



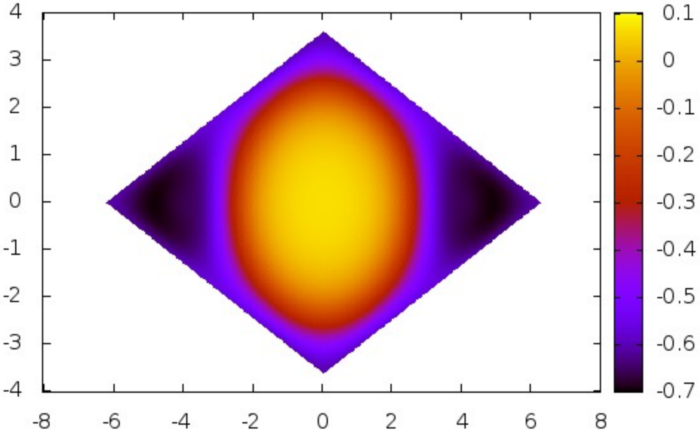
Chern Numbers



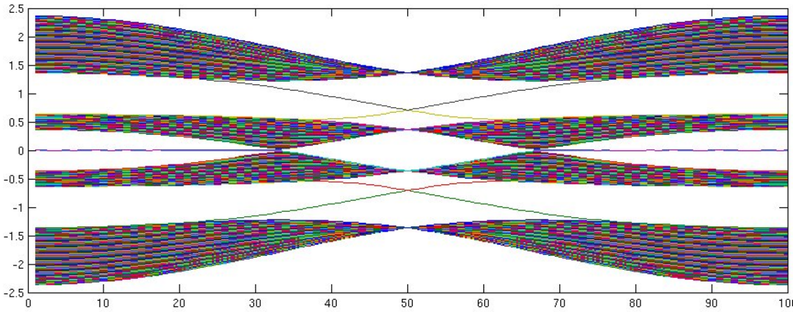
Pancharatnam-Berry field: $\Delta t = 0.1$



Pancharatnam-Berry field: $\Delta t = 1$



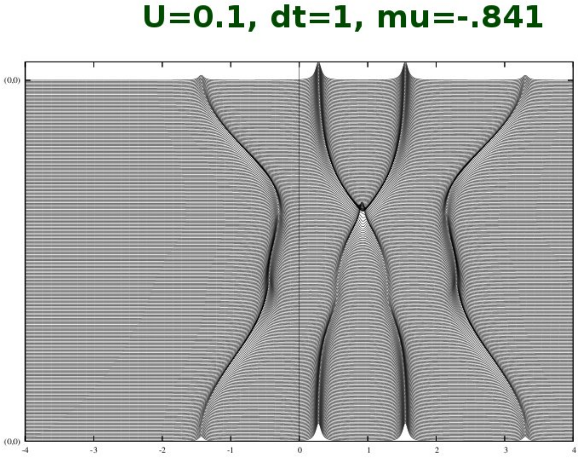
Chiral edge states



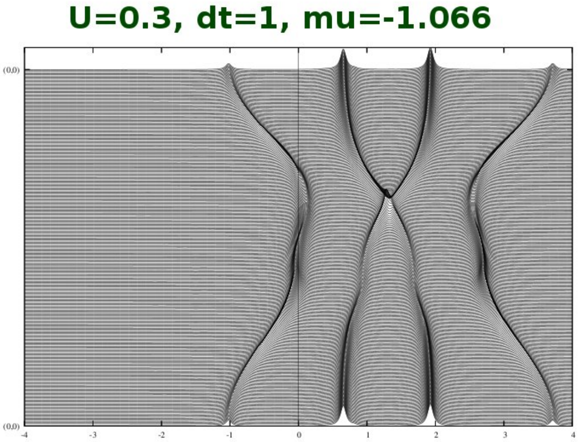
$U > 0$: Variational Cluster Perturbation Theory

- Choose a cluster hamiltonian H' with variational parameters approximating the effect of the environment. (We choose a 6 site cluster and the hopping parameters as the variational parameters).
- Solve the cluster hamiltonian exactly and the compute the self energy functional, $\Omega_t[G]$.
- Find the saddle point of Ω_t to determine G.

The Greens Function



The Greens Function



Conclusions

- The model proposed by Duan, Demler and Lukin to realise the Kitaev hamiltonian in cold atom systems has time reversal non-invariant spin-orbit couplings except at $t' = 0$ (graphene) and $t = 0$
- For certain parameter regimes, this model has four non-overlapping bands that carry non-zero Chern numbers. This implies a non-zero angular momentum of the ground state at quarter filling. Thus we will have a rotating condensate in a static optical lattice.
- The gap at quarter filling presists in a region in the $U - \Delta t$ space. There seem to be two phases, A quantum Hall state and a Chiral Metal.
- At $t = 0$, half filling we have a topological insulator.
- FQHE states at large U, partially filled bands ?? (Next problem)

THANK YOU !


12:05:07
12:20:21
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