BTZ black-hole and Luttinger Liquids

B. Sathiapalan

Institute of Mathematical Sciences, Chennai

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Outline

(Collaborators:D. Maity, S. Sarkar, R. Shankar and Nilanjan Sircar)

Strongly Coupled Systems

- Many physical systems are described by field theories where the coupling constant flows (either in the IR or in the UV) to a fixed point where the correlation length (in units of the cutoff) diverges and the theory is scale invariant. These are "critical points".
- These are also conformally invariant usually (Polyakov, Zamolodchikov) though not always (Polchinski, Cardy).
- If the fixed point coupling constant is zero (as in QCD at high energies) then one can use perturbation theory to probe phenomena in the vicinity of the fixed point.

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Strongly Coupled Systems

- If the fixed point coupling is $O(1)$ then one cannot use perturbation theory.

Examples

- In classical statistical mechanics as you vary temperature one may encounter this critical point where there is a second order phase transition. One is dealing with thermal fluctuations.
- But even at zero temperature one may encounter a phase transitions as you vary a coupling constant - due to quantum fluctuations. For instances the phases of 3+1 Yang-Mills theory - confining/Higgs - have been studied (starting with 'tHooft etc) in the parameter g and θ_{QCD}.

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Examples

- This is the "quantum" critical point - of the condensed matter physicist.
- Mathematically these are not very different from what we are used to: Equilibrium Classical Statistical mechanics of a $d+1$ dimensional system and quantum mecahnics of a d dimensional system are described by the same field theory - one has merely to Wick rotate the time coordinate and make it a space coordinate. $T \leftrightarrow \hbar$.

$$\int \mathcal{D}\phi e^{\frac{i}{\hbar}\int_{-\infty}^{\infty} dt \int d^3x \mathcal{L}(\phi)} \leftrightarrow \int \mathcal{D}\phi e^{\frac{-1}{k_B T}\int_{\infty}^{\infty} dt \int d^3x \mathcal{L}(\phi)}$$

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Examples

- One can also take a d dimensional real time quantum field theory, Wick rotate and compactify time over a length $\beta\hbar$ and study equilibrium quantum statistical mechanics of the same d dimensional system.

$$\int \mathcal{D}\phi e^{\frac{i}{\hbar}\int_{-\infty}^{\infty} dt \int d^3x \mathcal{L}(\phi)} \leftrightarrow \int \mathcal{D}\phi e^{-\int_0^{\beta} dt \int d^3x \mathcal{L}(\phi)}$$

Examples

- When we have both thermal and quantum fluctuations and need to study transport and real time phenomena at finite temperature - then we are doing real time field theory at finite temperature.
- For eg. people have studied QCD at high temperature using perturbation theory - when the coupling is small. But if the coupling is not small, eg if we are in the vicinity of a quantum critical point, where $g_c \approx 1$ then we need other methods.

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AdS/CFT

- The AdS/CFT correpondence can be used fruitfully for these kind of problems.
- The correspondence is (in words):
  "String theory in a background that is asymptotically AdS is dual to a conformal field theory (CFT) on the boundary of AdS"
- A concrete and most studied example:
  "Type IIB superstrings in $AdS_5 \times S^5$ is dual to $N = 4$ supersymmetric (SU(N) Yang-Mills theory on the boundary viz.R^4 (Actually $R \times S^3$)."

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AdS/CFT

- More mathematically: $((x, r)$ are bulk coordinates and $r = \infty$ is the boundary, with boundary coordinates being $x)$

$$Z[\phi_B] = \int_{\phi(x,\infty)=\phi_B(x)} \mathcal{D}\phi_i(x, r) e^{iS_{String}[\phi_i(x,r)]}$$

$$= \int \mathcal{D}A(x) e^{iS[A(x)]+i\int_x \phi_{iB}(x)O^i(x)}$$

  A are boundary fields and O_i are operators of the boundary theory that are "dual" to the bulk field ϕ_i.

- In the case of $AdS_5 \times S^5$: The bulk dilaton is dual to $TrF^2 + ...$ of the boundary theory. Also $g_{YM}^2 = g_s$ and $\lambda_{'tHooft} = g_{YM}^2 N = \frac{R^4}{(\alpha')^2}$.

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AdS/CFT-Finite Temperature

- So large λ means large R. But this is precisly where gravity is agood approximation to string theory. $N \to \infty$, $g_s \to 0$ with λ large is the best combinations: Tree level (classical) gravity in the bulk is equivalent all orders in λ (but leading in $\frac{1}{N}$) Yang-Mills!
- How do you put the system at finite temperature? Introduce a black hole in the bulk. The Hawking temperature of the black hole is the temperature of the boundary theory.
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Examples: Spectrum

- Glueball masses (Witten): Solve the dilaton wave equation with vanishing slope boundary condition at the horizon and look for normalizable solutions. This is an eigenvalue problem and gives glueball masses in (non supersymmetric) QCD! Can be compared with a (coarse) lattice calculation.

Real Time Quantities

- Modify boundary condition: Normalizable and has **ingoing** boundary conditions at the black hole horizon. Eigenvalues are complex. The imaginary part of the pole of the gluon propagator. Gives the "Lyapunov exponent" for classical Yang-Mills. Responsble for thermalization processes.

Real-Time Quantities

- Instead of eigenvalue problems, one can allow more general solutions and extract Green's functions $G(x_1, x_2) = \langle O(x_1) O(x_2) \rangle$ by calculating $\frac{\partial^2 Z}{\partial \phi_B(x_1) \partial \phi_B(x_2)}$.
- Fermions: In the case of fermions this computation reduces to $\frac{\psi_-}{\psi_+}$ where the chiralities are defined w.r.t γ^r - Dirac gamma matrix in the radial direction of AdS.

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Real-Time Quantities

- The best known example is the calculation of $\frac{\eta}{s}$:

$$\langle T_{xy} T_{xy} \rangle \approx -i\eta\omega$$

gives: $\eta = \frac{\pi}{8} N^2 T^3$ and using $s = \frac{\pi^2}{2} N^2 T^3$ we get $\frac{\eta}{s} = \frac{1}{4\pi}$

Non-Fermi Liquids

- Fermi Liquid Theory is expected to break down in many situations involving "strongly correlated electrons".
- Fermi liquid is essentially a fermi gas when interactions of the electrons with each other are included.
- What are the low energy excitations?
  Start with a filled fermi sphere and imagine adding one more fermion in a momentum eigenstate. One can imagine doing this with a Fermi gas first, and then quickly increasing the strength of the interaction parameter. The particle becomes a quasi particle with the same momentum.

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- But it is not an energy eigenstate and can decay. Due to phase space limitations the life time increases as $\frac{1}{(k-k_F)^2}$. So near k_F these are legitimate excitations. Thus one expects a quasi particle pole in the propagator with a width given by the above expectation.
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Non-Fermi Liquid

- Recently field theories in 2+1 dimensions were studied using AdS/CFT by H.Liu,McGreevy,Vegh. This is dual to a Reissner-Nordstrom (i.e.charged) black-hole in AdS_4. The presence of the gauge field component A_0 at the boundary provides a tunable chemical potential in the boundary theory and also breaks the Lorentz and conformal invariance.

Non-Fermi Liquid

- They found a sharp Fermi surface)with a quasiparticle peak) with a dispersion of the form $\omega = (k - k_F)^z$ with $z \approx 2$. A Landau Fermi liquid should have $z = 1$.
- They found a particle-hole asymmetry i.e. the peak looks different for $k > k_F$ and $k < k_F$ which again is not expected in a Fermi liquid.
- For $k < \frac{\mu}{\sqrt{6}}$ they find Green fns have a periodicity in $log\omega$ - suggestive of underlying discrete scale invariance.
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Non-Fermi Liquids: Luttinger Liquid

- In 1+1 dimensions fermions can be bosonized. These are collective excitations. Physically if an electron moves in one dimension it has to push all the electrons in front and make them move - they can't be "pushed aside" as in higher dimensions. So it is always "collective" motion. The boson can then be thought of as a longitudinal phonon (which is there in both solids and iquids).

Luttinger Liquid

- Mathematically this shows up in the equivalence of the Thirring model with the Sine Gordon model. Equivalently:

$$\psi_L \approx e^{i\phi_L} \qquad \psi_R \approx e^{-i\phi_R}$$

- $$\bar{\psi}\gamma^\mu\partial_\mu\psi \approx (\partial_\mu\phi)^2 = (\partial_t\phi)^2 - v^2(\partial_x\phi)^2$$

  v is the velocity of the excitation. This has a Lorentz invariant form with v being the velocity of "light".

Luttinger Liquid

- Mass term: $\psi_L^\dagger \psi_R \approx e^{-i(\phi_L+\phi_R)}$
- Currents: $\bar{\psi}\gamma^\mu\psi \approx \epsilon^{\mu\nu}\partial_\mu\phi$ So a four Fermi -Thirring Interaction: $(\bar{\psi}\gamma^\mu\psi)^2 \approx (\partial_\mu\phi)^2$ - just renormalizes the kinetic term and hence also the dimension of the operator $e^{i\phi_L}$.
- Its correlation changes from $\frac{1}{k_L}$ to k_L^α for some α. Thus the quasi particle pole is lost in the presence of interactions - non-Fermi liquid.
- Non Lorentz invariant ρ^2 ("charge-charge") energy term adds $(\partial_x\phi)^2$. This changes the velocity of the excitation.

Luttinger Liquids

- Bosonization thus solves one strongly coupled problem-massless Thirring model. But one can have mass perturbations or other releveant perturbations when scale invariance is broken. Then the theory goes into a massive phase - studied first by Kosterlitz -Thouless.
- Since it is 1+1 dimensional theory there are many analytical approaches.

AdS_3

- One can apply AdS_3/CFT to these problems.

$$S_{EinsteinMaxwell} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R + \frac{2}{l^2} - 4\pi G F_{\mu\nu}F^{\mu\nu}\right)$$

$$+i(\bar{\Psi}\Gamma^M\mathcal{D}_M\Psi - m\bar{\Psi}\Psi)$$

AdS_3

- Example: The equations for a Fermion in pure AdS_3 are $(z = 1/r)$:
$$z\partial_z \psi_+ = iz(\omega - k)\psi_-$$
$$z\partial_z \psi_- = iz(\omega + k)\psi_+$$
with solutions $\psi_+ = e^{i\sqrt{\omega^2-k^2}z}$ and $\psi_- = \sqrt{\frac{\omega+k}{\omega-k}}\psi_+$. For fermions the ratio $\frac{\psi_-}{\psi_+}$ gives the Green's function and this is $\sqrt{\frac{\omega+k}{\omega-k}}$. This is the Green function of a dimension one chiral operator in 1+1 CFT.

BTZ Black-hole

- The (charged, non rotating) BTZ black hole has an asymptotic AdS_3 structure. The metric is

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\theta^2$$

where, $f(r) = \frac{1}{l^2} - \frac{8GM}{r^2} - \frac{8\pi G Q^2}{r^2} ln(\frac{r}{l})$ and $F_{tr} = \frac{Q}{r}$.

- Setting $z = 1/r$, and after some rescalings, $f(z) = 1 - z^2 + \frac{Q^2}{2} z^2 ln(z)$ and the temperature of the black hole is $T = -\frac{f'(1)}{4\pi} = \frac{(1 - \frac{Q^2}{4})}{2\pi}$

BTZ Black-hole

- Equation and boundary condition for Fermion Green's function:

$$zf(z)\partial_z G(z) + G(z)^2 z(\omega + \mu\, ln(z) - n\sqrt{f(z)}) + z(\omega + \mu\, ln(z) + n\sqrt{f(z)}) = 0 \qquad (1)$$

The ingoing boundary condition at the horizon implies

$$G(1) = i \qquad (2)$$

BTZ Black-hole

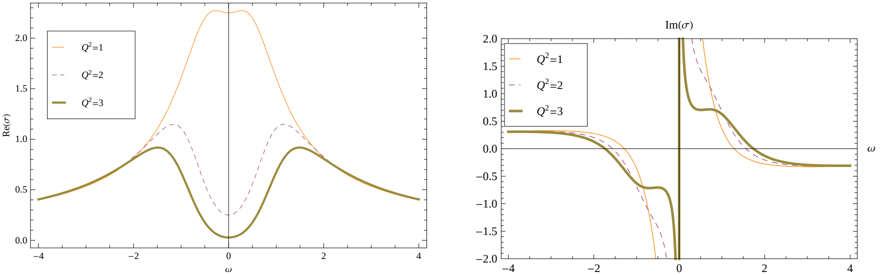

Figure: Conductivity

BTZ Black-hole

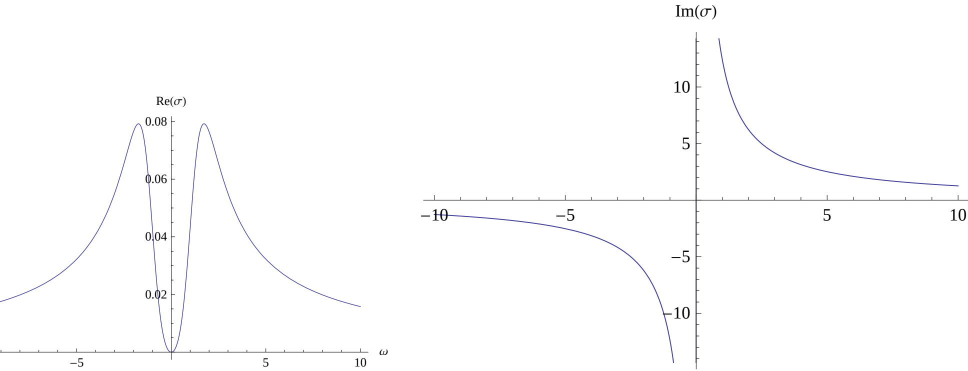

Figure: Conductivity at zero temperature

BTZ Black-hole

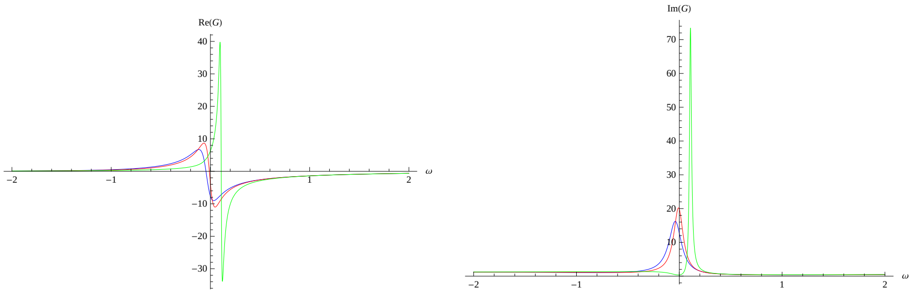

Figure: Real and Imaginary part of Fermion Greens' function for various values of Q: .5 (Blue), 1 (Red), 1.9 (Green), with $k = -5$ and $\mu = 1$

Gapless Phase

- There are quasiparticle peaks. Not necessarily poles.
- Can check that the scaling dimension in ω is what you expect from the asymptotic analysis.
- One important observation is that $ImG \neq 0$ as $\omega \approx 0$. So we are still in the gapless (Luttinger Liquid) phase.

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AdS_2 Near Horizon Geometry

- The ($T = 0$) small ω behaviour of the Green function can be obtained exactly: The dominant contribution to this comes from the near horizon region of the extremal black-hole which is AdS_2 (this is universal) and can be solved exactly. (Faulkner et al)
- Metric becomes:

$$ds^2 = \frac{1}{z^2}[-(f(z))^2 dt^2 + \frac{dz^2}{f(z)^2} + dx^2]$$

with $f(z) \approx 2(1-z)^2$.
- Do a scaling: $1 - z = \frac{\omega}{2\zeta}$ with $z \to 1, \quad \omega \to 0$ and ζ finite.

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Real-Time Quantities

- Result of this analysis: $ImG(\omega = 0) = 0$ for $qQ < \frac{k}{\sqrt{2}}$. $ImG \approx (\omega)^{2\nu_k}$. Beyond that $ImG(\omega = 0) \neq 0$ and ν_k becomes imaginary. So there is **periodicity** in $ln\ \omega$!

Log periodicity

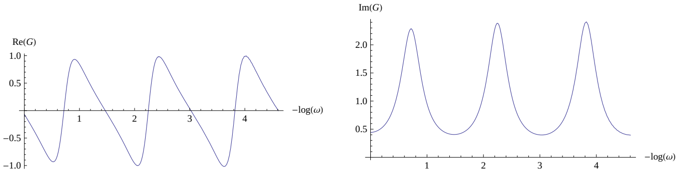

Figure: Log Periodicity of Real and Imaginary part of Fermion Greens' function w.r.t *Log*(ω) at zero temperature for $\mu = 4$, $k = .5$

Luttinger Liquid

- Can one understand such (k-independent) non analyticity in ω from the boundary 1+1 dimensional theory? In a typical Luttinger liquid one expects $(\omega - k)^\alpha$ as the singularity.
- Assume velocities are not the same - since there is no Lorentz Invariance this is possible. We will set $v_S = 0$.

$$S = \int dxdt[i\bar{\psi}(\gamma^0\partial_0 + \gamma^1 v_F\partial_1)\psi + \frac{1}{2}(\partial_0\phi\partial^0\phi - v_S^2\partial_1\phi\partial^1\phi)]$$

$$+g\int dxdt\ i\bar{\psi}\psi cos\ \beta\phi$$

Luttinger Liquid

Consider the self energy correction with $v_S = 0$:

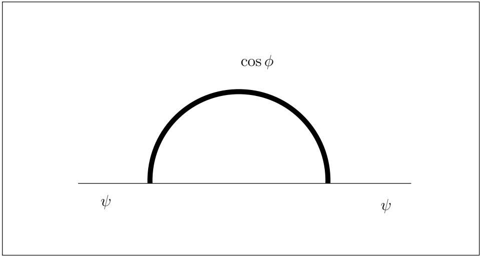

Figure: 1 loop correction to ψ propagator

Luttinger Liquid

- The graph evaluates to

$$\Sigma(p) \approx \int \frac{d\omega dk}{(2\pi)^2} \frac{(\omega + v_F k)}{(\omega^2 - v_F^2 k^2)((p^0 - \omega)^2)^{\frac{\beta^2}{4\pi} - 1}}$$

$$\approx \frac{[ln\ p^0 - C - \psi(p^0)]}{(p^0)^{\frac{\beta^2}{4\pi} - 3}}$$

.

- As expected we get a p independent non analyticity. $v_S \approx 0$ is some non propagating localized object -"impurity".

Summary

- There is a class of problems in condensed matter physics where AdS/CFT techniques can be applied.
- The 1+1 theory considered here shows some interesting non-Fermi liquid behaviour.
- We have given a proposal for the modified Luttinger Liquid behaviour at the boundary.
- Any connection with experiments in wires or other one dimensional systems needs to be explored.