Infinite-randomness quantum critical points induced by dissipation

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- Motivation: superconducting nanowires and itinerant quantum magnets
- Strong-disorder renormalization group
- Infinite-randomness quantum critical points
- Stronger disorder effects for Ising symmetry: Smeared QPTs

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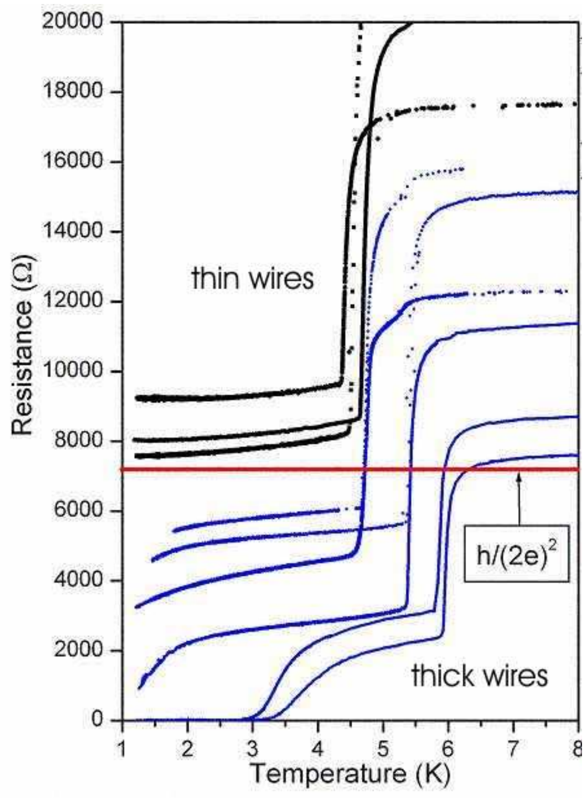
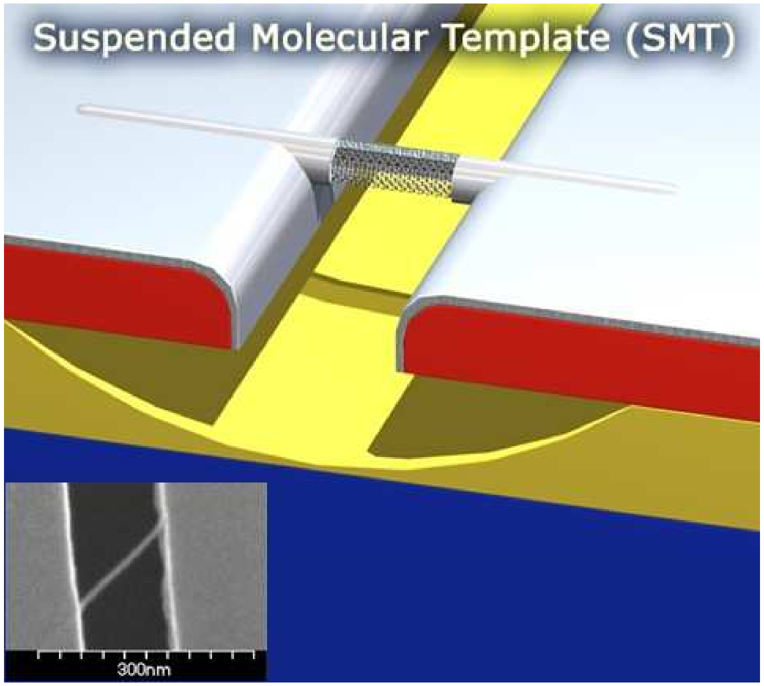
Experiment I: Superconductivity in ultrathin nanowires

- ultrathin MoGe wires (width $\sim$ 10 nm)
- produced by molecular templating using a single carbon nanotube

  [A. Bezryadin et al., Nature 404, 971 (2000)]

superconductor-metal QPT as function of wire thickness



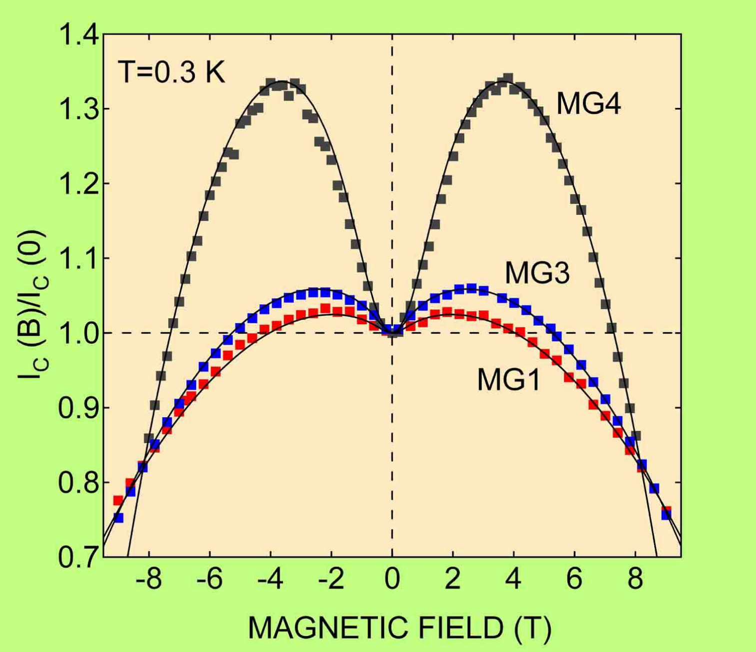


- thicker wires are superconducting at low temperatures
- thinner wires remain metallic

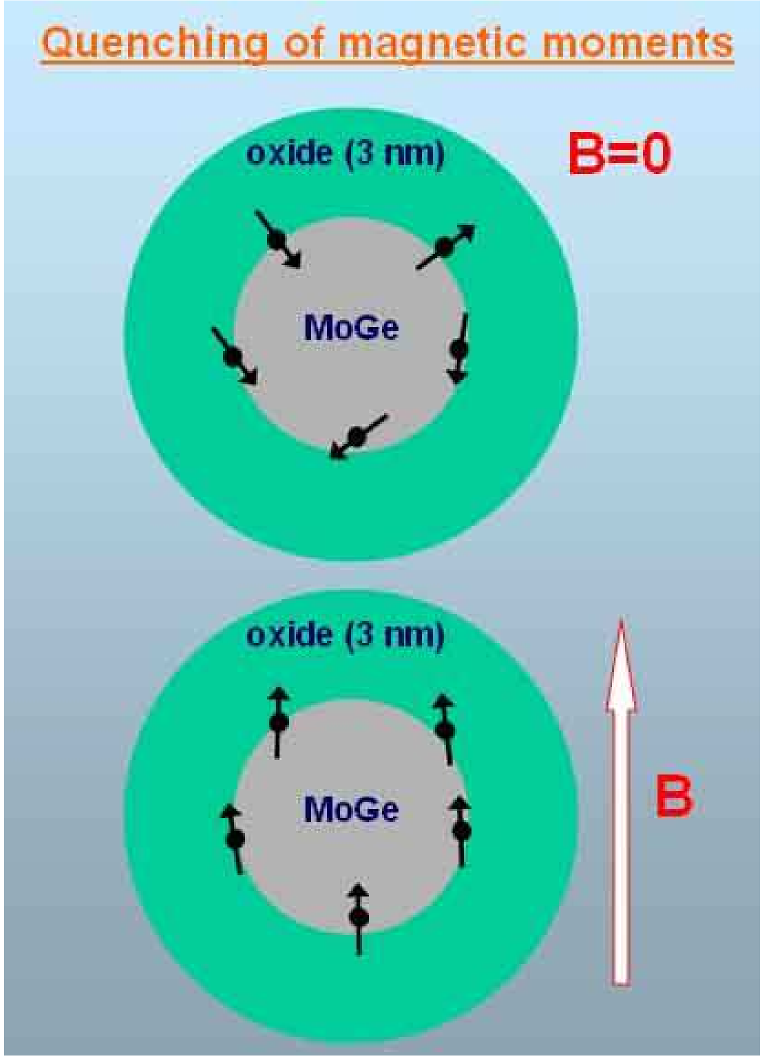
superconductor-metal QPT as function of wire thickness

Pairbreaking mechanism

- pair breaking by surface magnetic impurities
- random impurity positions
  $\Rightarrow$ quenched **disorder**
- gapless excitations in metal phase
  $\Rightarrow$ Ohmic **dissipation**



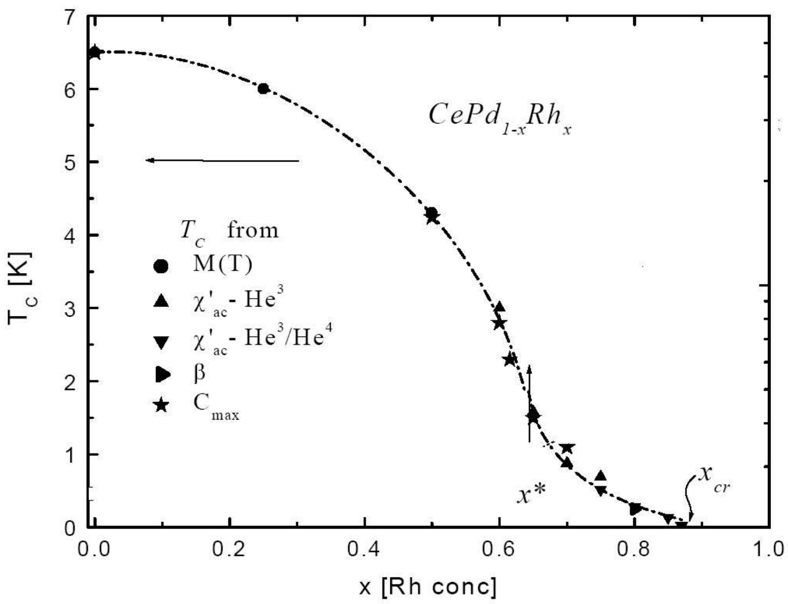
weak field enhances superconductivity

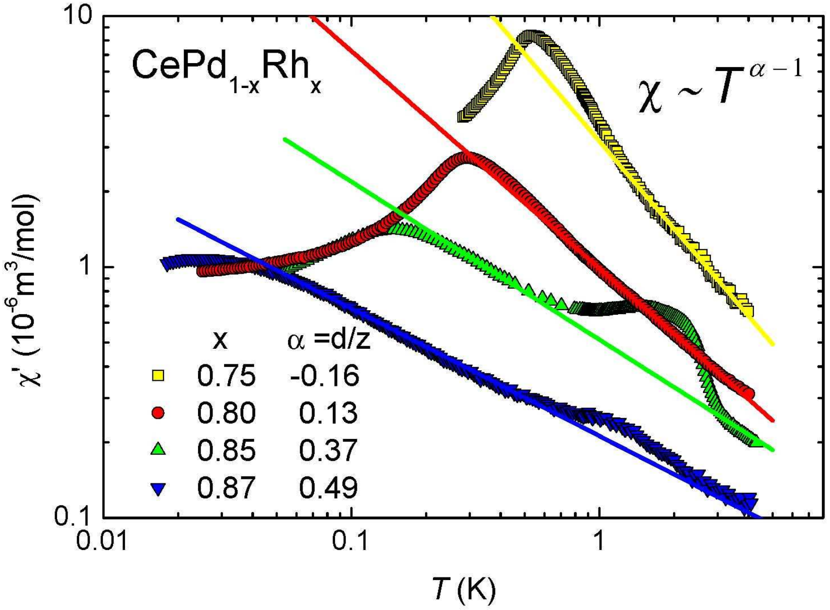


magnetic field aligns the impurities and reduces magnetic scattering

Experiment II: Itinerant quantum magnets

- quantum phase transitions between paramagnetic metal and ferromagnetic or antiferromagnetic metal
- transition often controlled by chemical composition ⇒ **disorder** appears naturally
- magnetic modes damped due to coupling to fermions ⇒ Ohmic **dissipation**
- typical example: ferromagnetic transition in CePd$_{1-x}$Rh$_x$





(Sereni et al., Phys. Rev. B **75** (2007) 024432 + Westerkamp, private communication)

What is the fate of a quantum phase transition under the combined influence of disorder and dissipation?

- Motivation: superconducting nanowires and itinerant quantum magnets
- **Strong-disorder renormalization group**
- Infinite-randomness quantum critical point
- Stronger disorder effects for Ising symmetry: Smeared QPTs

Dissipative $O(N)$ order parameter field theory

N-component $(N > 1)$ order parameter field $\varphi(\mathbf{x},\tau)$ in d dimensions
derived by standard methods (Hubbard-Stratonovich transformation etc.)

$$S = T \sum_{\mathbf{q},\omega_n} \left(r + \xi_0^2 \mathbf{q}^2 + \gamma\,|\omega_n|\right) |\varphi(\mathbf{q},\omega_n)|^2 + \frac{u}{2N} \int \mathrm{d}^d x \mathrm{d}\tau \; \varphi^4(\mathbf{x},\tau)$$

Disorder: $\left\{ \begin{array}{c} \text{distance } r \text{ from criticality} \\ \text{bare correlation length } \xi_0 \\ \text{Ohmic dissipation constant } \gamma \end{array} \right\}$ random functions of position

- Superconductor-metal quantum phase transition in nanowires $(d = 1, N = 2)$
  $\varphi(\mathbf{x},\tau)$ represents local Cooper pair operator (Sachdev, Werner, Troyer 2004)
- Hertz' theory of itinerant quantum Heisenberg antiferromagnets $(d = 3, N = 3)$
  $\varphi(\mathbf{x},\tau)$ represents staggered magnetization (Hertz 1976)

Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- asymptotically exact if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

discretized order-parameter field theory for “rotor” variables $\phi_i(\tau)$

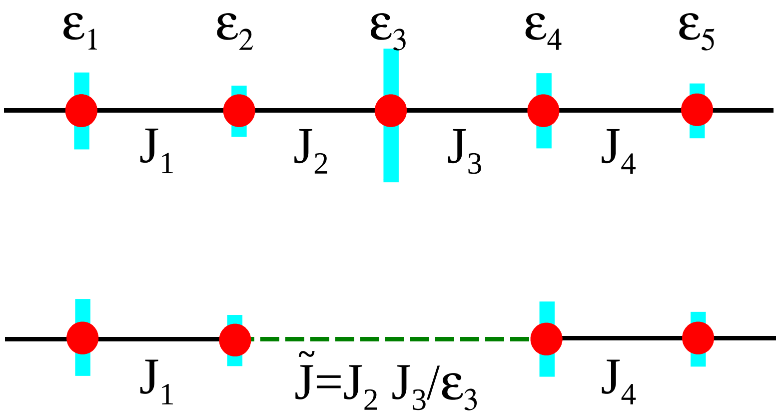
$$S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|) \, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \, \phi_i(-\omega_n) \, \phi_{i+1}(\omega_n)$$

the competing local energies are:

- interactions (bonds) J_i favoring the ordered phase
- local “gaps” ϵ_i favoring the disordered phase

$\Rightarrow$ in each RG step, integrate out largest among all J_i and ϵ_i

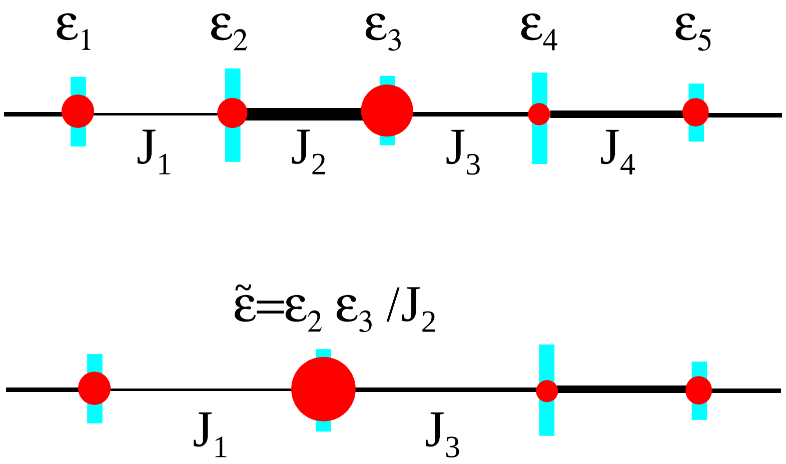
Recursion relations in one dimension



if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$:

- site 3 is removed from the system
- coupling to neighbors is treated in 2nd order perturbation theory

new renormalized bond $\tilde{J} = J_2 J_3/\epsilon_3$



if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$:

- rotors of sites 2 and 3 are parallel
- can be replaced by single rotor with moment $\tilde{\mu} = \mu_2 + \mu_3$

renormalized gap $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$

Renormalization-group flow equations

- strong disorder RG step is iterated, gradually reducing maximum energy Ω
- competition between cluster aggregation and decimation
- leads to larger and larger clusters connected by weaker and weaker bonds

⇒ **flow equations** for the full probability distributions $P(J)$ and $R(\epsilon)$

$$\begin{aligned} -\frac{\partial P}{\partial \Omega} &= [P(\Omega) - R(\Omega)]\, P + R(\Omega) \int dJ_1 dJ_2\, P(J_1)P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right) \\ -\frac{\partial R}{\partial \Omega} &= [R(\Omega) - P(\Omega)]\, R + P(\Omega) \int d\epsilon_1 d\epsilon_2\, R(\epsilon_1)R(\epsilon_2)\, \delta\left(\epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega}\right) \end{aligned}$$

Flow equations are identical to those of the **random transverse-field Ising chain**

Note symmetry between J and ϵ!

Fixed points

If bare distributions do not overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: no clusters formed – disordered phase

$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: all sites connected – ordered phase

If bare distributions do overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: rare clusters – disordered Griffiths phase

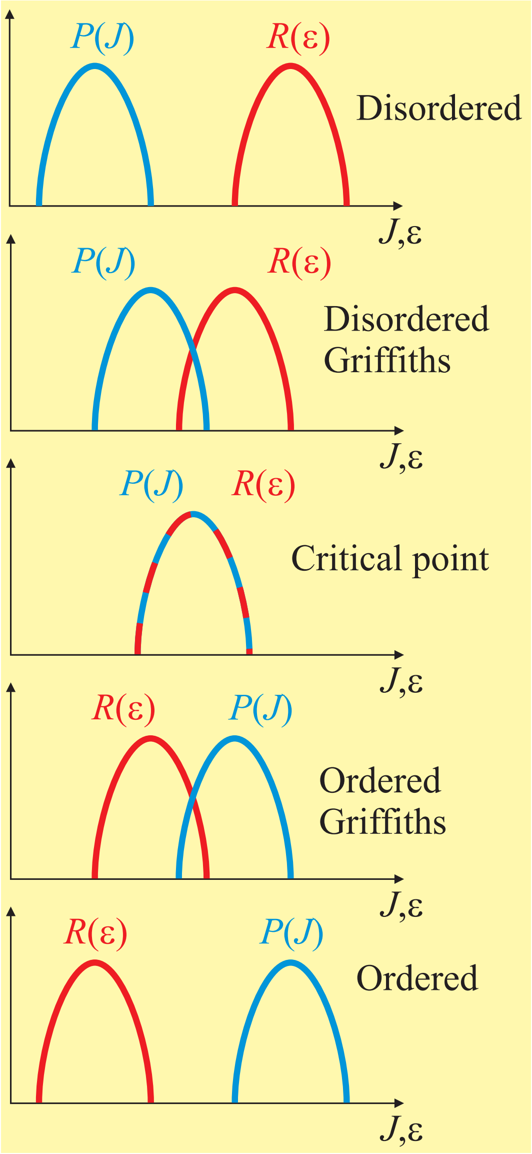
$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: rare "holes" – ordered Griffiths phase

$\langle \ln \epsilon \rangle = \langle \ln J \rangle$: cluster aggregation and decimation balance at all energies – **critical point**

$$\boxed{\mathcal{P}(\zeta) = \frac{1}{\Gamma} e^{-\zeta/\Gamma}, \quad \mathcal{R}(\beta) = \frac{1}{\Gamma} e^{-\beta/\Gamma}}$$

log. variables $\zeta = \ln(\Omega/J)$, $\beta = \ln(\Omega/\epsilon)$, $\Gamma = \ln(\Omega_0/\Omega)$

Distributions become infinitely broad at critical point



initial (bare) distributions

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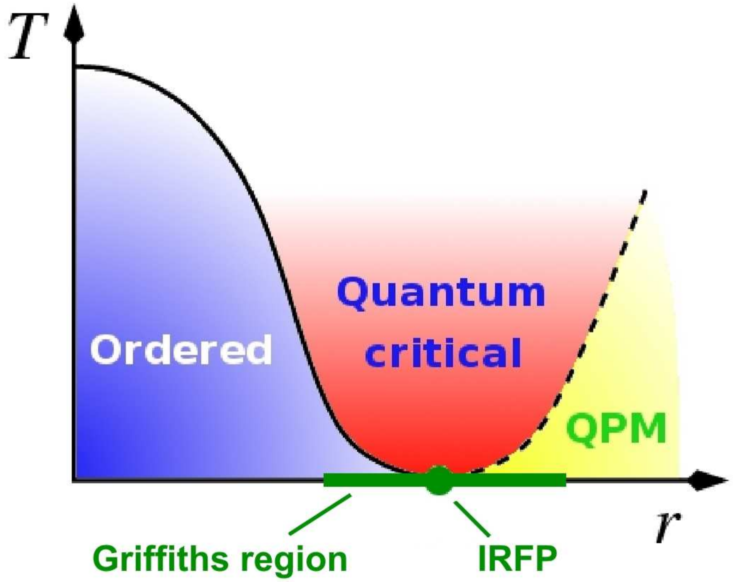
Critical behavior

- at critical FP, disorder scales to ∞
  $\Rightarrow$ **infinite-randomness critical point**
- activated dynamical scaling $\ln(1/\Omega) \sim L^{\psi}$ with tunneling exponent $\psi = 1/2$
- moments of surviving clusters grow like $\mu \sim \ln^{\phi}(1/\Omega)$ with $\phi = (1+\sqrt{5})/2$
- average correlation length diverges as $\xi \sim |r|^{-\nu}$ with $\nu = 2$

dissipative $O(N)$ order parameter is in universality class of **dissipationless** random transverse-field Ising model.



finite-temperature phase boundary and crossover line take unusual form

$$T_c \sim \exp(-\text{const}\, |r|^{-\nu\psi})$$

Quantum Griffiths regions:

- power-law dynamical scaling with nonuniversal exponent

Quantum-critical thermodynamics

to calculate thermodynamic properties at temperature T:
run RG down to energy scale $\Omega = T$ and consider remaining clusters as free

Static order parameter susceptibility:

each surviving cluster contributes μ^2/T

$$\chi(r,T) = \frac{1}{T} n(\Omega = T)\mu^2(\Omega = T) = \frac{1}{T}\left[\ln(1/T)\right]^{2\phi - d/\psi}\,\Theta_\chi\left(r^{\nu\psi}\ln(1/T)\right)$$

Specific heat:

each surviving cluster contributes T to the total energy

$$C(r,T) = \frac{\partial}{\partial T}[T n(\Omega = T)] = \left[\ln(1/T)\right]^{-d/\psi}\,\Theta_C\left(r^{\nu\psi}\ln(1/T)\right)$$

Dynamics and transport

to calculate dynamic OP susceptibilities at external frequency ω (and $T = 0$):
run RG down to energy scale $\Omega = \gamma_{\mathrm{eff}}\omega = \gamma\mu(\Omega)\omega$

single-cluster contributions:

$$\chi_j(\omega + \mathrm{i}\delta) = \frac{\mu_j^2}{\epsilon - \mathrm{i}\mu_j\gamma\omega}, \quad \chi_j^{\mathrm{loc}}(\omega + \mathrm{i}\delta) = \frac{\mu_j}{\epsilon - \mathrm{i}\mu_j\gamma\omega}$$

Dynamic susceptibilities at $T = 0$:

$$\begin{aligned} \mathrm{Im}\chi(r,\omega) &\sim \frac{1}{\omega}\,[\ln(1/\omega)]^{\phi - d/\psi}\; X\left(r^{\nu\psi}\ln(1/\omega)\right) \\ \mathrm{Im}\chi^{\mathrm{loc}}(r,\omega) &\sim \frac{1}{\omega}\,[\ln(1/\omega)]^{-d/\psi}\; X^{\mathrm{loc}}\left(r^{\nu\psi}\ln(1/\omega)\right) \end{aligned}$$

Transport properties: optical conductivity, dc conductance – work in progress

Quantum Griffiths singularities

in disordered Griffiths phase:

thermodynamics is characterized by **nonuniversal power laws**

local susceptibility $\chi^{\mathrm{loc}}(r,T) \sim T^{d/z'-1}$

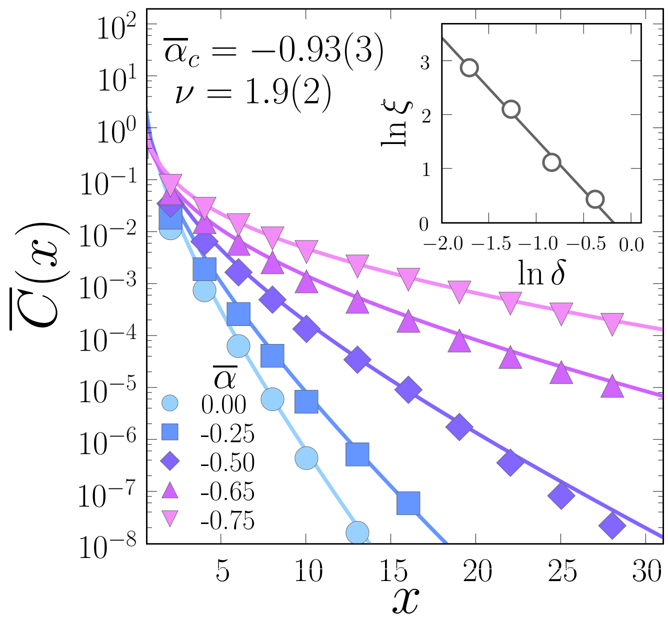
specific heat $C(r,T) \sim T^{d/z'-1}$

magnetization in field $m(r,H) \sim H^{d/z'}$

dynamical exponent

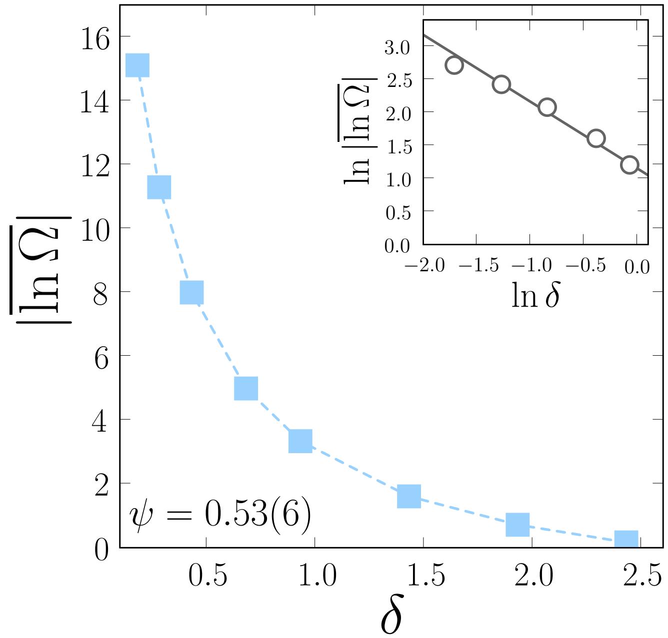
$z' \sim r^{-z\nu}$ **diverges** at infinite-randomness critical point

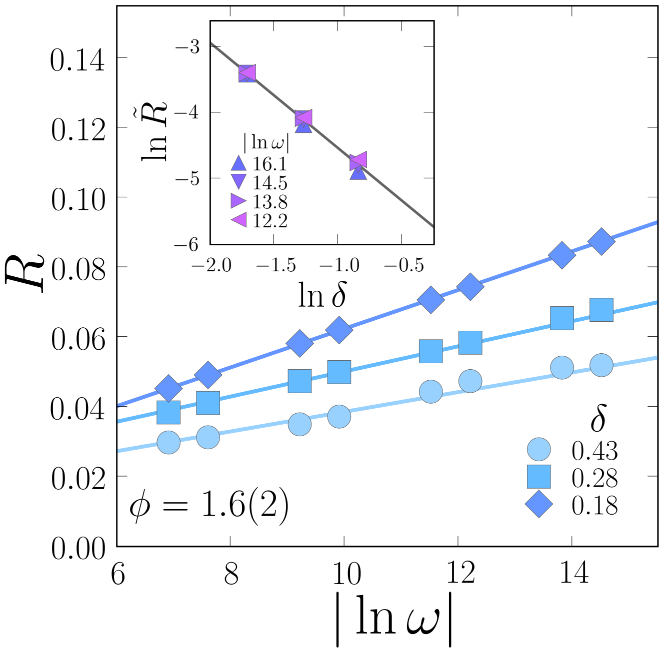
Numerical confirmation



- A. Del Maestro et al. (2008) solved disordered large-N problem numerically exactly
- calculated equal time correlation function C, energy gap Ω, and ratio R of local and order parameter dynamic susceptibilities

	ν	ψ	ϕ
SDRG	2	1/2	$(\sqrt{5}+1)/2$
Numerics	1.9(2)	0.53(6)	1.6(2)





Order parameter symmetry

- our explicit calculations are for an infinite number of OP components, $N = \infty$

Are the results valid for the physical cases $N = 2$ (superconductor-metal transition) and $N = 3$ (Hertz' antiferromagnetic transition)?

Analysis:

- infinite-randomness FP is due to **multiplicative** structure of recursion relations
- bond renormalization $\tilde{J} = J_2 J_3 / \epsilon_3$ follows from 2nd order perturbation theory, does not depend on N
- multiplicative structure of gap renormalization $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$ corresponds to **exponential** dependence of the gap on the cluster size
- applies to all **continuous symmetry** cases $N > 1$ (Mermin-Wagner)
- Ising OPs are different with even stronger disorder effects

Infinite-randomness critical point for all continuous symmetry cases $N > 1$

Generalizations: $d > 1$, nonohmic damping

Higher dimensions $d > 1$

- infinite randomness scaling scenario also appears in 2D and probably in 3D
- renormalization group must be implemented numerically because lattice connectivity changes
- critical exponent values are different, only known numerically

Nonohmic damping

- if damping term is nonohmic, $\gamma|\omega_n|^{2/z_0}$, recursion relations change

$$\tilde{\epsilon_2}^{-x} = \alpha\left[\epsilon_2^{-x} + \epsilon_3^{-x}\right] + \mathcal{O}\left(J_2^{-x}\right) \qquad \text{with } x = (2 - z_0)/z_0$$

- **subohmic** case, $z_0 > 2$: quantum phase transition **destroyed by smearing**
- **superohmic** case, $z_0 < 2$: transition survives, likely with **conventional scaling**

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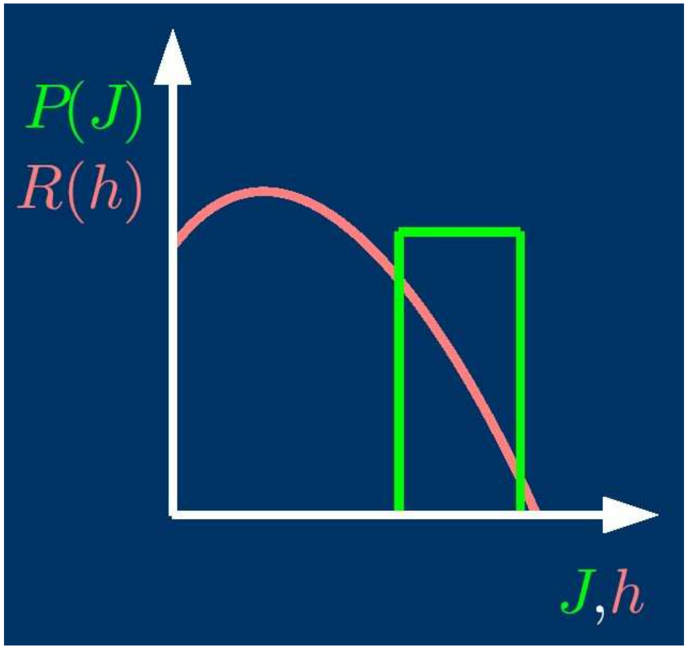
Dissipative random transverse-field Ising chain

$$H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x + \sum_{i,n} \sigma_i^z \lambda_{i,n}(a_{i,n}^\dagger + a_{i,n}) + \sum_{i,n} \nu_{i,n} a_{i,n}^\dagger a_{i,n}$$

J_i: exchange interaction between z-components of spin σ_i

h_i: transverse magnetic field, acting on x-component of spin σ_i

$a_{i,n}^\dagger, a_{i,n}$: harmonic oscillator bath coupling to z-component of spin σ_i



Bath spectral function

$$\mathcal{E}(\omega) = \pi \sum_n \lambda_{i,n}^2 \delta(\omega - \nu_{i,n}) = 2\pi\alpha\omega e^{-\omega/\omega_c}$$

α: dimensionless dissipation strength

ω_c: oscillator energy cutoff

Linear low freq. spectrum: Ohmic dissipation

Strong-disorder renormalization group

Integrate out local high energy modes: $\Omega = \max(J_i, h_i, \omega_c/p)$

To reduce maximum energy from Ω to $\Omega - d\Omega$:

1. Integrate out all oscillators with frequencies $\nu \in [p(\Omega - d\Omega), p\Omega]$

$$\tilde{h}_i = h_i \exp\left(-\alpha_i \int_{p(\Omega-\mathrm{d}\Omega)}^{p\Omega} \frac{\mathrm{d}\omega}{\omega}\right) = h_i \left(1 - \alpha\mu_i \frac{\mathrm{d}\Omega}{\Omega}\right)$$

2. Decimate all transverse fields $h_i \in [\Omega - d\Omega, \Omega]$

$$\tilde{J} = J_{i-1} J_i / h_i$$

3. Decimate all interaction energies $J_i \in [\Omega - d\Omega, \Omega]$

$$\tilde{h} = h_i h_{i+1} / J_i, \quad \tilde{\mu} = \mu_i + \mu_{i+1}$$

Extra downward renormalization of the transverse fields due to dissipation

Renormalization-group flow equations

Flow equations for the probability distributions $P(J)$ and $R(h,\mu)$

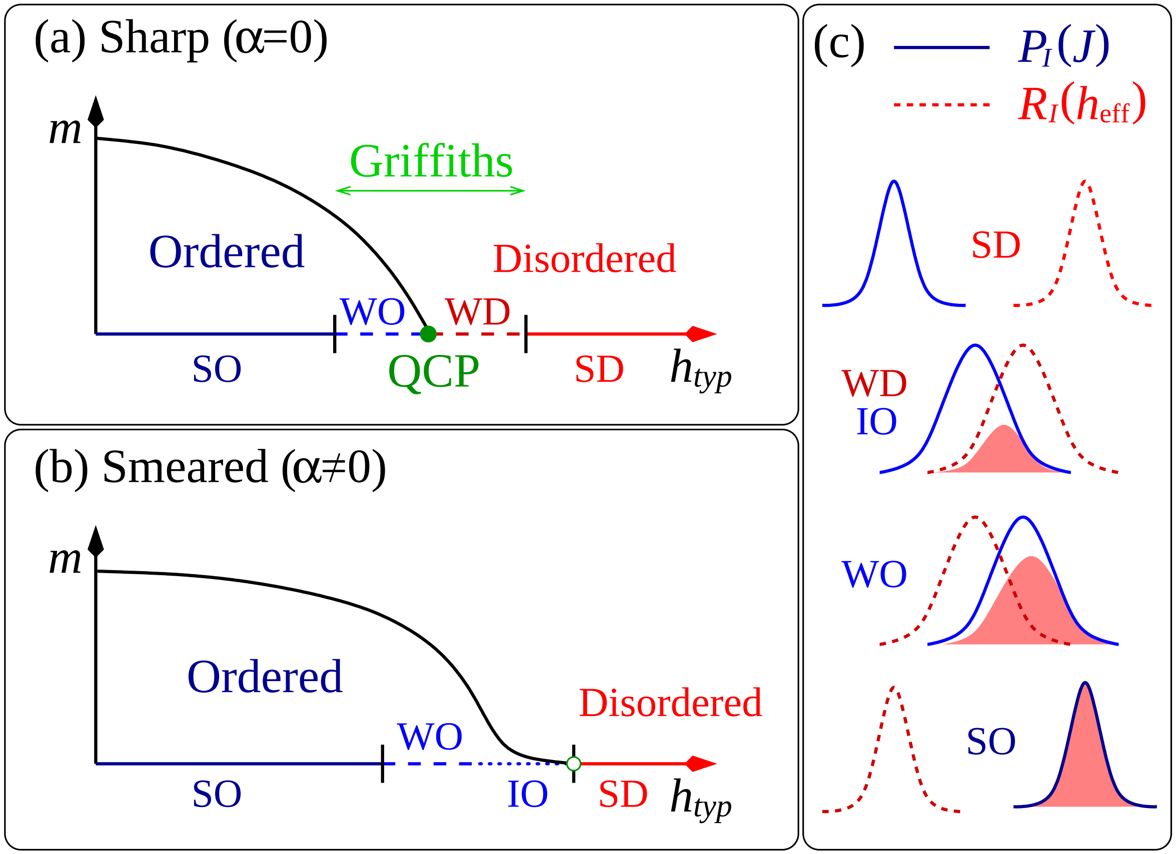
$$
\begin{aligned}
-\frac{\partial P}{\partial \Omega} &= \left[P(\Omega) - (1-\alpha\bar{\mu})R_h(\Omega)\right] P + (1-\alpha\bar{\mu})R(\Omega)\int dJ_1 dJ_2\, P(J_1)P(J_2)\,\delta\left[J - \frac{J_1 J_2}{\Omega}\right] \\
-\frac{\partial R}{\partial \Omega} &= \left[(1-\alpha\bar{\mu})R_h(\Omega) - P(\Omega)\right] R + \frac{\alpha\mu}{\Omega}\left[R + h\frac{\partial R}{\partial h}\right] + \\
&\quad + P(\Omega)\int dh_1 dh_2 d\mu_1 d\mu_2\, R(h_1,\mu_1)R(h_2,\mu_2)\,\delta\left[h - \frac{h_1 h_2}{\Omega}\right]\delta[\mu - \mu_1 - \mu_2]
\end{aligned}
$$

$(1-\alpha\bar{\mu})$: probability for decimating field vanishes for $\mu > 1/\alpha$
$\Rightarrow$ important finite "volume" scale $1/\alpha$

- clusters act as Ohmic spin-boson problem with effective damping constant $\alpha\mu$
- if $\alpha\mu > 1$, they undergo **localization transition** (Caldeira, Leggett, Weiss)

Large clusters freeze independently $\Rightarrow$ quantum phase transition is **smeared**

Smeared quantum phase transition

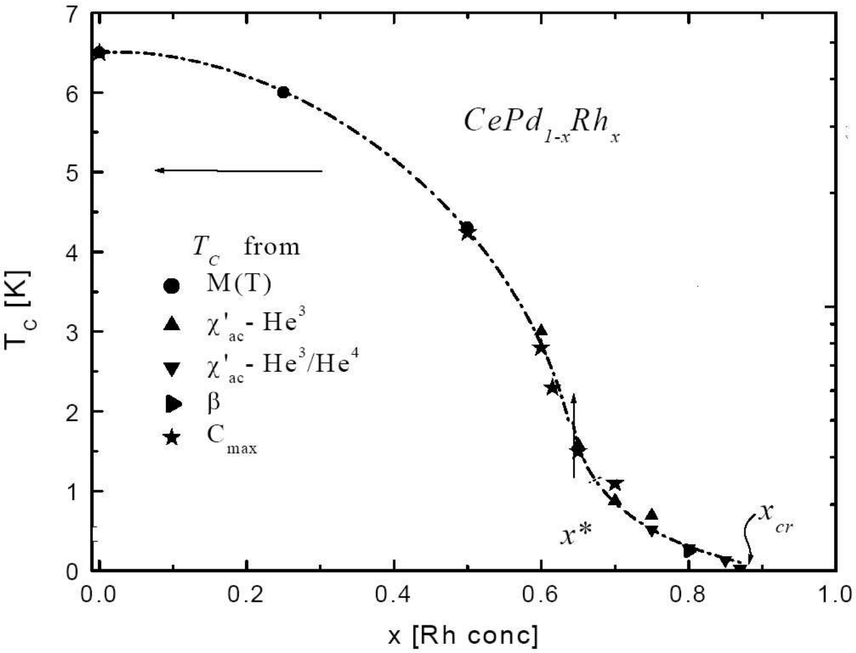


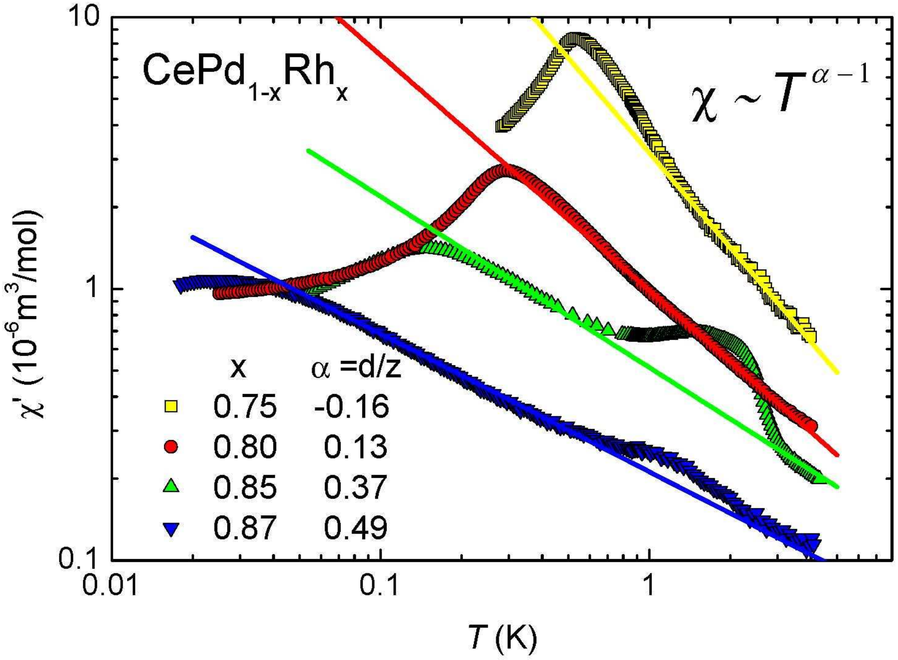
- quantum critical point and disordered Griffiths phase destroyed
- replaced by **inhomogeneously ordered** region in the tail of the ordered phase

Low temperature thermodynamics: dominated by large frozen clusters

Example: uniform susceptibility $\chi \sim T^{-1-1/z}$

Infinite-randomness physics in CePd$_{1-x}$Rh$_x$??





- ferromagnetic phase shows pronounced tail, evidence for glassy behavior in tail, possibly due to RKKY interactions
- above tail: nonuniversal power-laws characteristic of quantum Griffiths effects

(Sereni et al., Phys. Rev. B **75** (2007) 024432 + Westerkamp, private communication)

Classification of weakly disordered phase transitions according to importance of rare regions

T. Vojta, J. Phys. A **39**, R143–R205 (2006)

Dimensionality of rare regions	**Griffiths effects**	**Dirty critical point**	**Examples** (classical PT, QPT, non-eq. PT)
$d_{RR} < d_c^-$	weak exponential	conv. finite disorder	class. magnet with point defects dilute bilayer Heisenberg model
$d_{RR} = d_c^-$	strong power-law	infinite randomness	Ising model with linear defects random quantum Ising model disordered directed percolation (DP)
$d_{RR} > d_c^-$	RR become static	smeared transition	Ising model with planar defects itinerant quantum Ising magnet DP with extended defects

Conclusions

- We have performed a strong-disorder renormalization group study of the QPT in **disordered dissipative systems** with continuous symmetry order parameters
- 1D: analytical solution gives **infinite-randomness** critical point in the universality class of the random transverse-field Ising model
- 2D: numerical solution displays analogous scenario, exponent values different
  3D: preliminary numerical results point in same direction
- unconventional transport properties, work in progress
- discrete OP symmetry: **destruction** of the sharp quantum phase transition by **smearing**

For details see: J. A. Hoyos, C. Kotabage, T. Vojta, Phys. Rev. Lett. **99**, 230601 (2007)
J. A. Hoyos and T. Vojta, Phys. Rev. Lett. **100**, 240601 (2008)
T. Vojta, J. A. Hoyos, C. Kotabage, Phys. Rev. B **79**, 024401 (2009)

Interplay between disorder and dissipation leads to exotic quantum critical behavior.