Hydrodynamics and fluctuations in relativistic heavy-ion collisions

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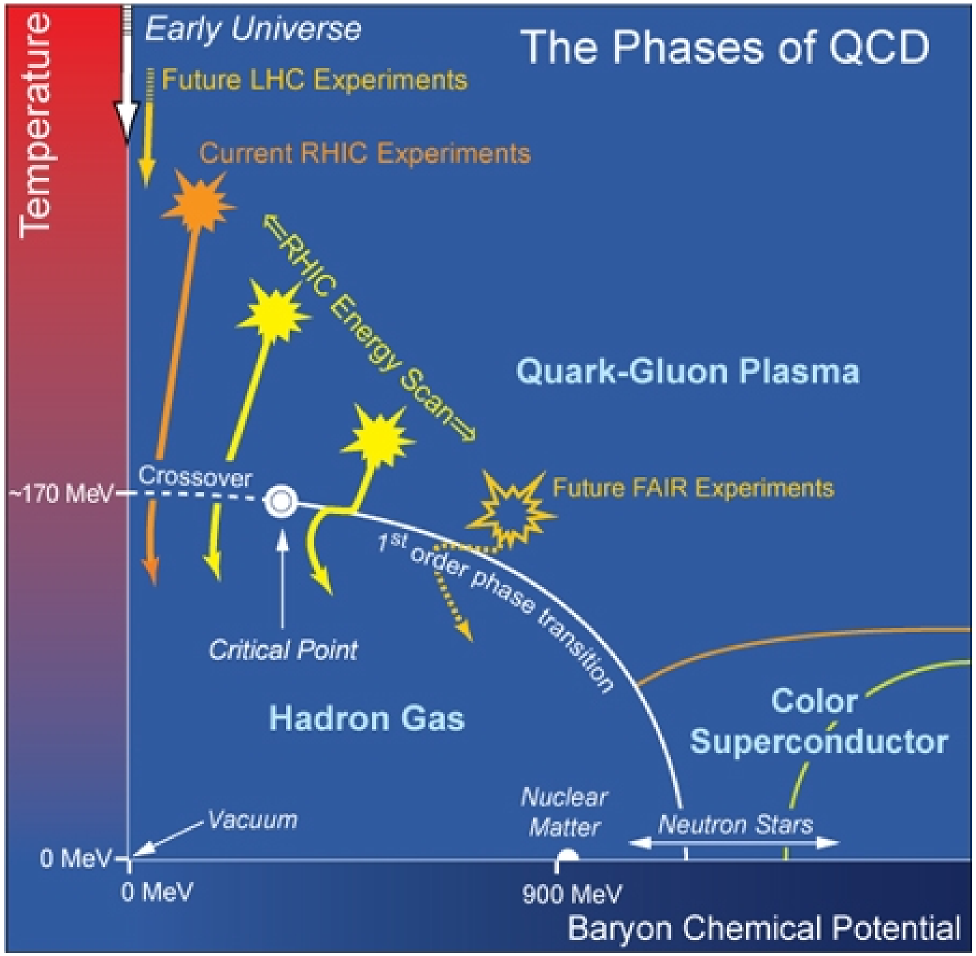
The relativistic heavy-ion programme

RHIC at Brookhaven :
Two beams of atomic Au nuclei, accelerated at energies up to 100 GeV per nucleon (since 2000)

LHC at CERN:
Two beams of atomic Pb nuclei will be accelerated at energies up to 2.7 TeV per nucleon (2010?)

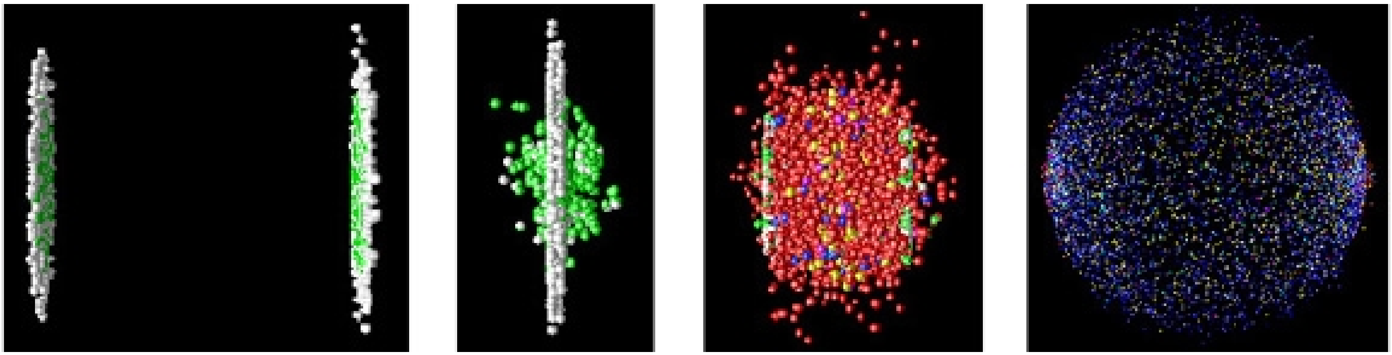
The phase diagram of strong interactions



Can we learn anything about hot gauge theories by smashing heavy ions together?

Do heavy-ion collisions have anything to do with temperature and thermodynamics?

What RHIC has taught us



During the expansion, the matter behaves collectively like a fluid.
This fluid has the smallest viscosity/entropy ratio ever seen: typically 2x the absolute lower bound postulated using gauge/gravity duality: $\eta/s=\hbar/4\pi k_B$.

Kovtun Son Starinets hep-th/0405231

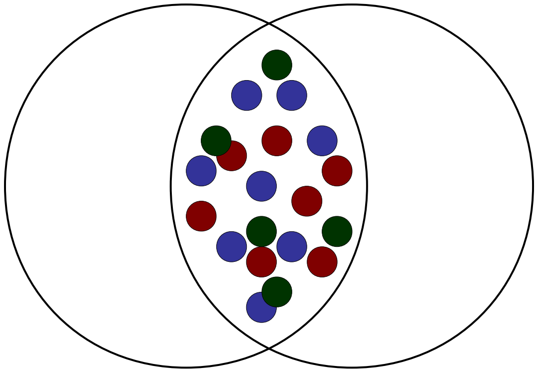
Outline

- A close look at the pattern of emitted particles: elliptic flow and « higher harmonics »
- A simple, universal prediction from hydrodynamics
- Comparing with experimental data
- Taking into account fluctuations
- Conclusion

Gombeaud, JYO, arXiv:0907.4664

A primer on nucleus-nucleus collisions

A typical Au-Au collision viewed in the transverse plane, perpendicular to beam axis



Collisions between partons/nucleons occur in the overlap area between the two nuclei: this is where matter is originally created

The non-overlapping parts don't interact (we call them « spectator » nucleons)

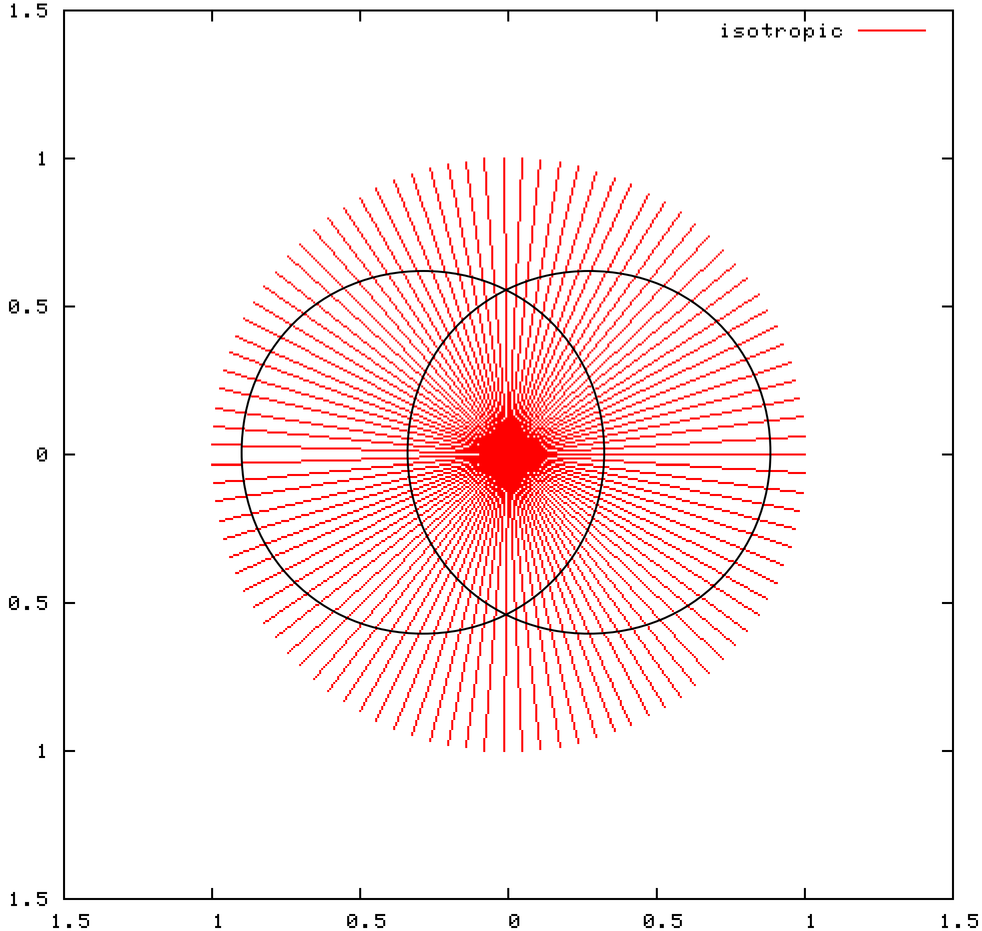
By measuring the number of spectators, or the number of participants, one estimates the centrality (impact parameter) of a given collision.

What are the directions of created particles?

Random parton-parton collisions occurring on scales << nuclear radius.

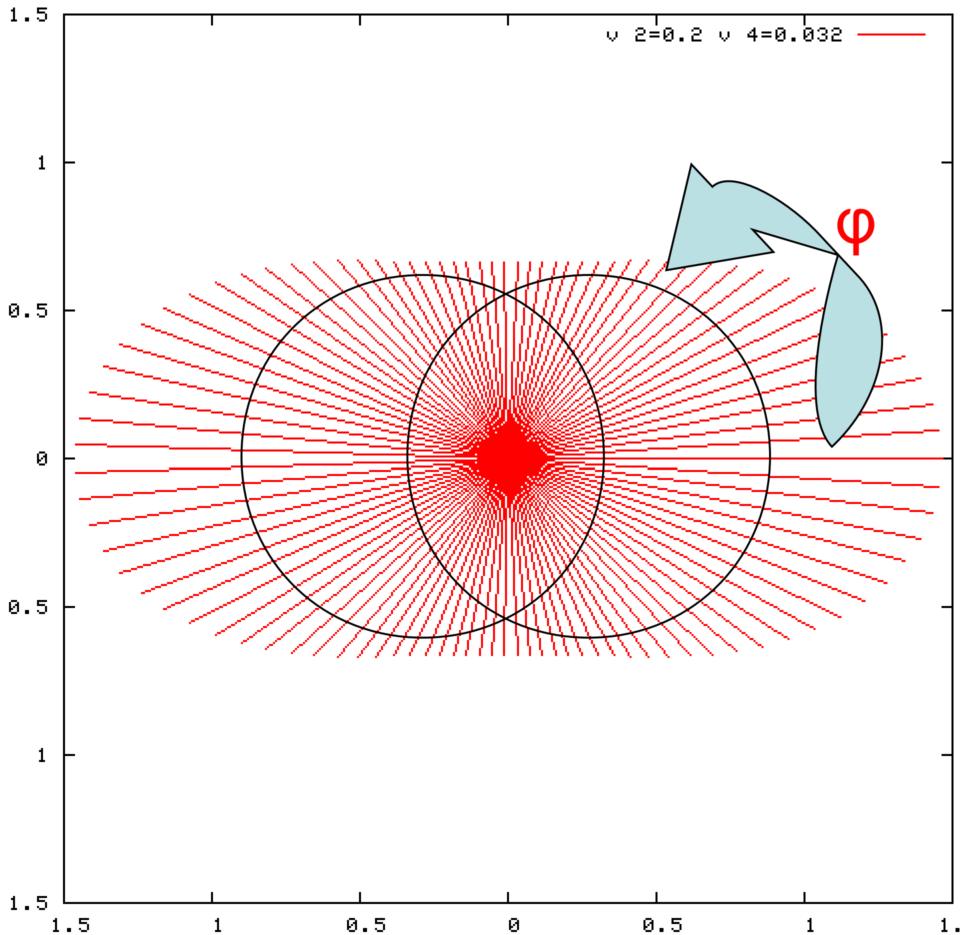
No preferred direction in the production process.

Isotropic azimuthal distribution



What we see is :

Bar length = number of particles in the direction = Azimuthal (φ) distribution plotted in polar coordinates



We call this elliptic flow. We think it is created by pressure gradients in the overlap area

(for pions with transverse momentum ~ 2 GeV/c)

Anisotropic flow

Fourier series expansion of the azimuthal distribution:

Using the $\varphi \to -\varphi$ and $\varphi \to \varphi+\pi$ symmetries of overlap area:

$$dN/d\varphi = 1 + 2v_2\cos(2\varphi) + 2v_4\cos(4\varphi) + \ldots$$

$v_2 = \langle\cos(2\varphi)\rangle$ (<…> means average value) is elliptic flow

$v_4 = \langle\cos(4\varphi)\rangle$ is a (much smaller) « higher harmonic »

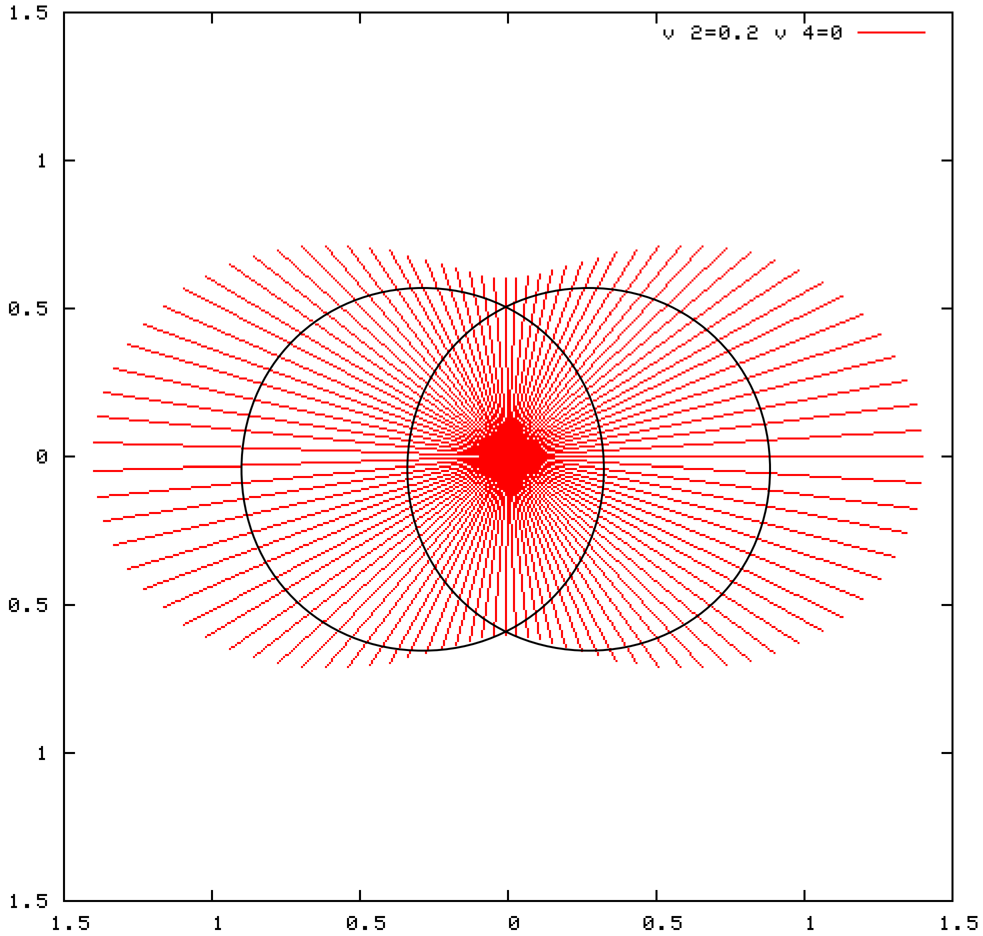
higher harmonics v_6, etc are 0 within experimental errors.

This talk really is about v_4

Azimuthal distribution without v_4

A small effect: Average value 0.3%, maximum value 3%

Should we care?



The beauty is in the details!

A primer on hydrodynamics

- Ideal gas (weakly-coupled particles) in global thermal equilibrium. The phase-space distribution is (Boltzmann)

$$dN/d^3p\,d^3x = \exp(-E/T)$$

Isotropic!

- A fluid moving with velocity v is in (local) thermal equilibrium in its rest frame:

$$dN/d^3p\,d^3x = \exp(-(E-\mathbf{p}.\mathbf{v})/T)$$

Not isotropic: Momenta parallel to v preferred

- At RHIC, the fluid velocity depends on φ: typically $v(\varphi)=v_0+2\varepsilon \cos(2\varphi)$

The simplicity of v_4

- Within the approximation that particle momentum **p** and fluid velocity **v** are parallel (good for large momenta)

$$dN/d\varphi=\exp(2\varepsilon\, p \cos(2\varphi)/T)$$

- Expanding to order ε, the $\cos(2\varphi)$ term is

$$v_2=\varepsilon\, p/T$$

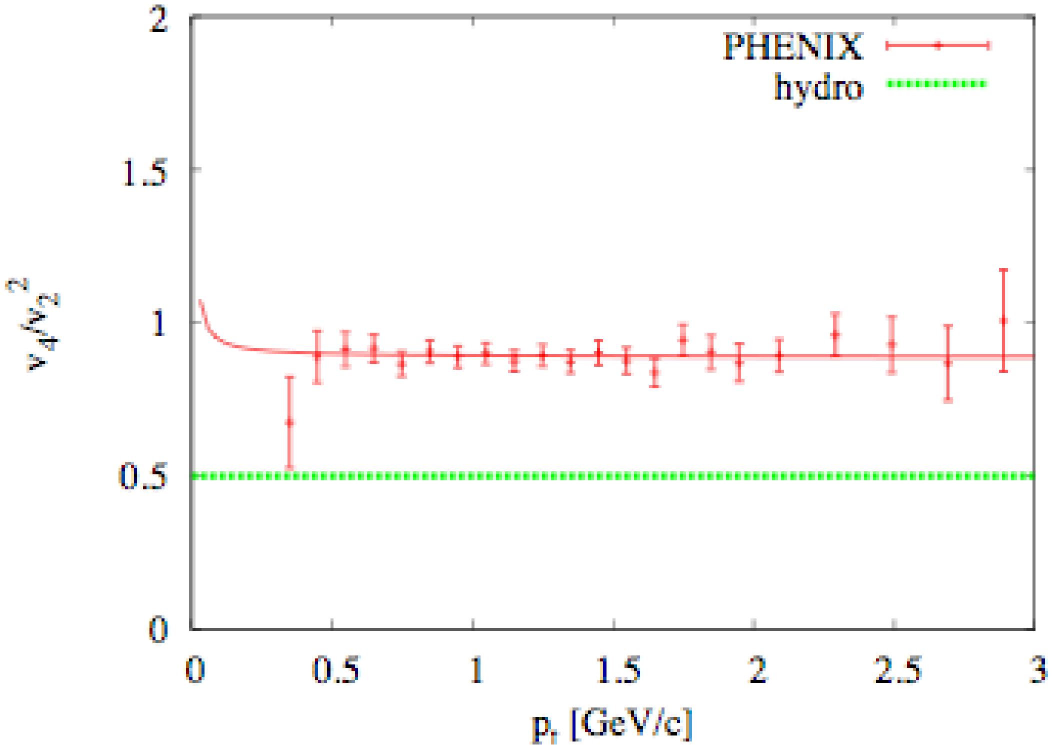
- Expanding to order ε^2, the $\cos(4\varphi)$ term is

$$v_4=\tfrac{1}{2}\,(v_2)^2$$

Hydrodynamics has a universal prediction for $v_4/(v_2)^2$!

Should be independent of equation of state, initial conditions, centrality, particle momentum and rapidity, particle type

PHENIX results for v_4



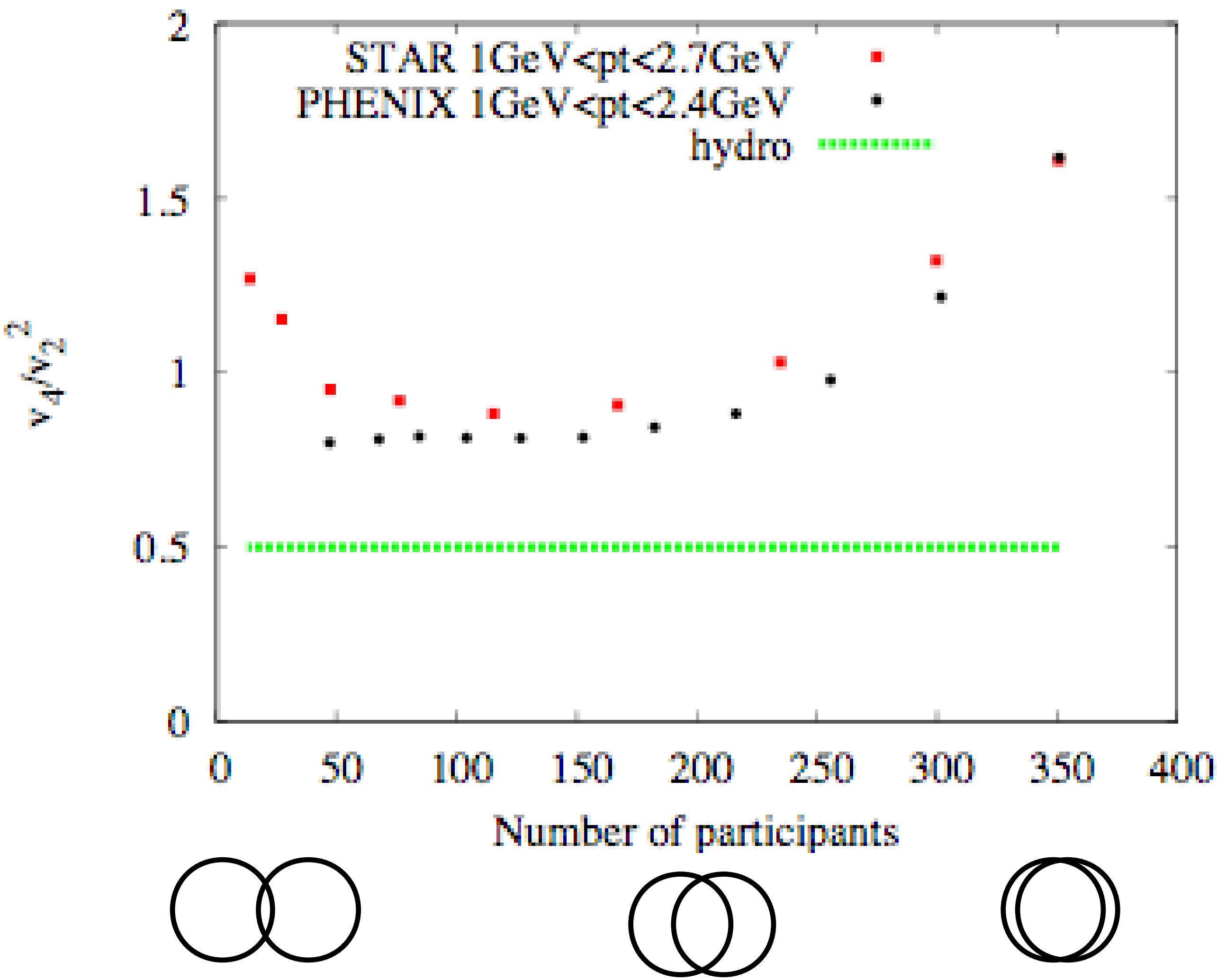
PHENIX data for charged pions

Au-Au collisions at 100+100 GeV

20-60% most central

The ratio is independent of p_T, as predicted by hydro.
But… the value is significantly larger than 0.5

More data : centrality dependence



Au-Au collision

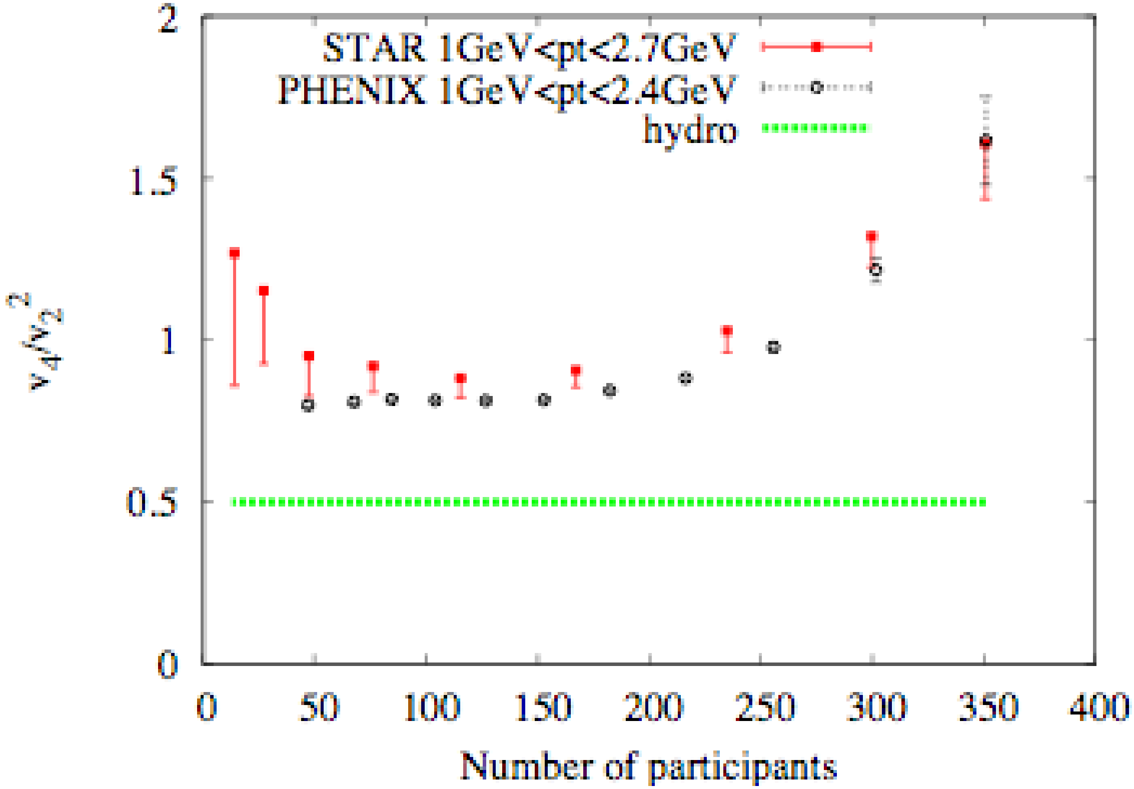
100+100 GeV

per nucleon

Data > hydro

Small discrepancy between STAR and PHENIX data

Estimating experimental errors



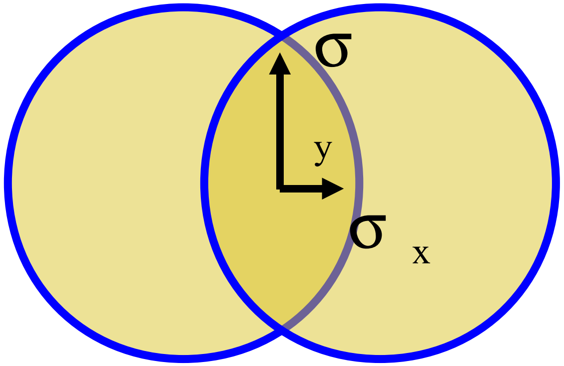
v_2 and v_4 are not measured directly but inferred from azimuthal correlations (more later on this).
There are many sources of correlations (jets, resonance decays,…): this is the « nonflow » error which we can estimate (order of magnitude only)

Difference between STAR and PHENIX data compatible with non-flow error

How do we understand the discrepancy with hydrodynamics??

Eccentricity scaling

We understand elliptic flow as the consequence of the almond shape of the overlap area

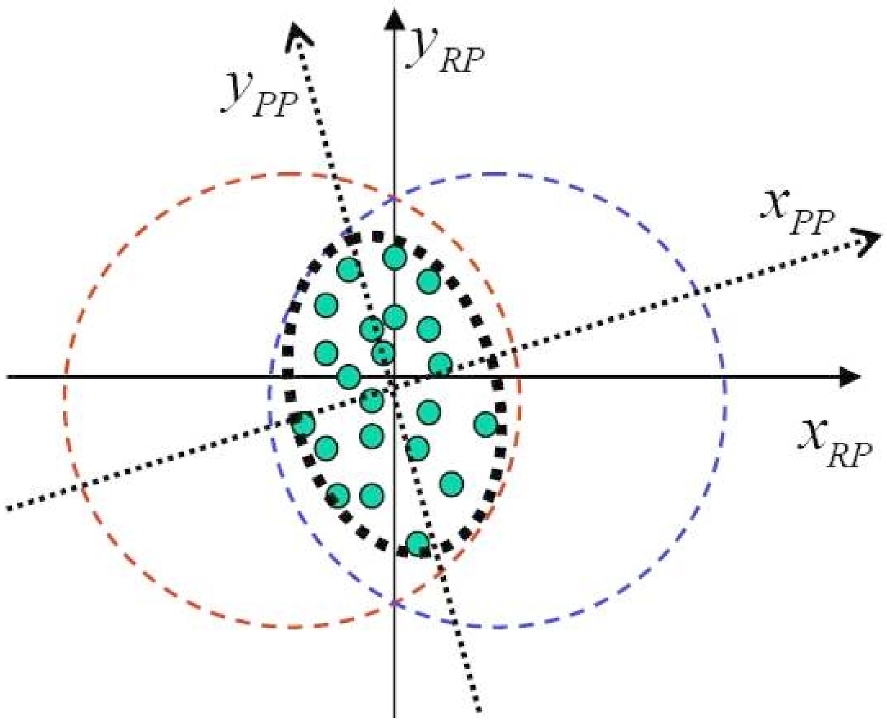


It is therefore natural to expect that v_2 scales like the eccentricity ε of the initial density profile, defined as :

$$\varepsilon = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$

(this is confirmed by numerical hydro calculations)

Eccentricity fluctuations



Depending on where the participant nucleons are located within the nucleus at the time of the collision, the actual shape of the overlap area may vary: the orientation and eccentricity of the ellipse defined by participants fluctuates.

Assuming that v_2 scales like the eccentricity, eccentricity fluctuations translate into v_2 (elliptic flow) fluctuations

Why fluctuations change $v_4/(v_2)^2$

The reference directions x and y are not known experimentally:
thus $v_2=\langle\cos(2\varphi)\rangle$ and $v_4=\langle\cos(4\varphi)\rangle$ are not measured directly

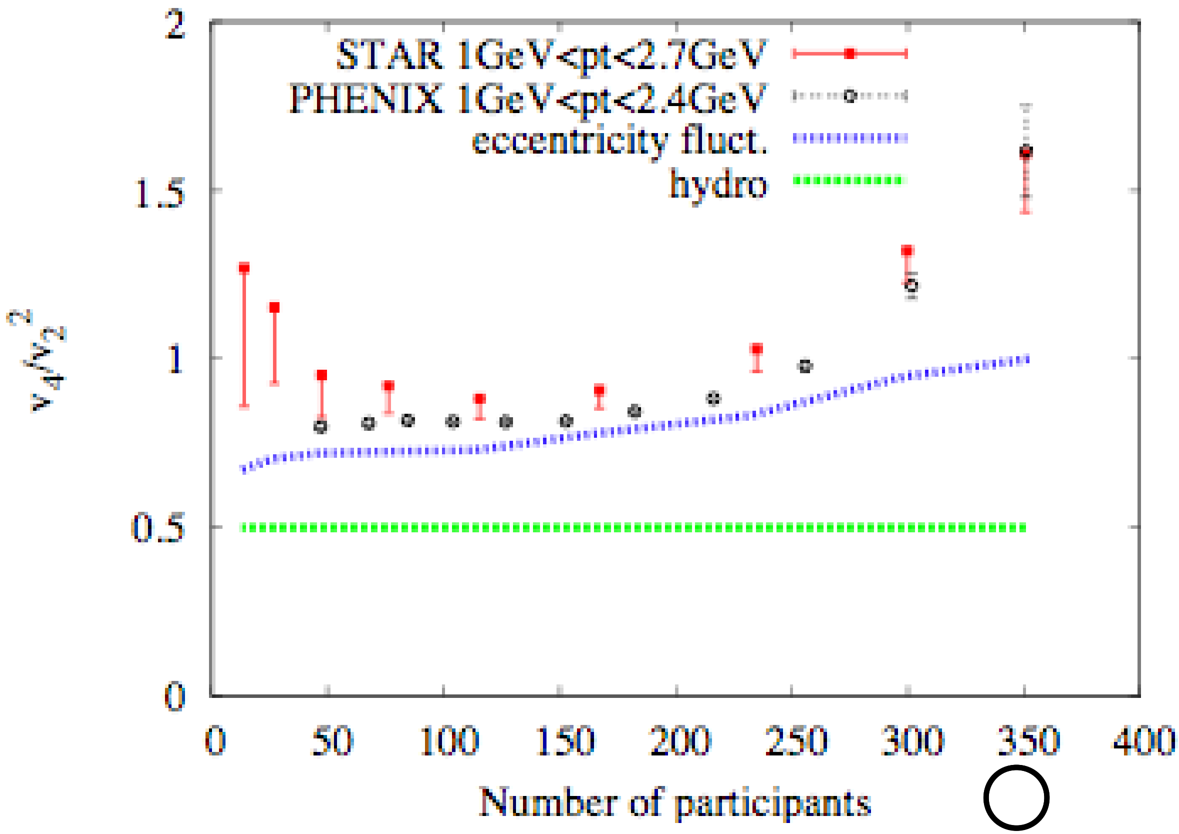
v_2 from 2-particle correlations: $\langle\cos(2\varphi_1-2\varphi_2)\rangle=\langle(v_2)^2\rangle$

v_4 from 3-particle correlations: $\langle\cos(4\varphi_1-2\varphi_2-2\varphi_3)\rangle=\langle v_4\,(v_2)^2\rangle$

If v_2 and v_4 fluctuate, the measured $v_4/(v_2)^2$ is really
$\langle v_4\,(v_2)^2\rangle/\langle(v_2)^2\rangle^2$. Inserting the prediction from hydrodynamics,

$$[v_4/(v_2)^2]_{exp}=\tfrac{1}{2}\,\langle(v_2)^4\rangle/\langle(v_2)^2\rangle^2$$

Data versus eccentricity fluctuations



Eccentricity fluctuations can be modelled using a Monte-Carlo program provided by the PHOBOS collaboration: Throw the dice for the positions of nucleons, with probability given by the nuclear density: No free parameter!

Fluctuations explain most of the discrepancy between data and hydro, except for central collisions which suggest $\langle (v_2)^4 \rangle / \langle (v_2)^2 \rangle^2 = 3$

By symmetry, $v_2=0$ for central collisions, *except for fluctuations!*

Conclusions

- The fourth harmonic, v_4, of the azimuthal distribution gives a further, independent indication that the matter produced at RHIC expands like a relativistic fluid
- We are colliding nuclei=complex quantum systems. We clearly see in the data large fluctuations which originate from the wavefunction of the colliding nuclei
- The (by now standard) model of eccentricity fluctuations fails for central collisions. We need a better understanding of fluctuations.

Is v_2 linear in momentum?

Hydro by Huovinen et al.
hydro tuned to fit central
spectra data.