Abstract

Nash proved in 1954 that any Riemannian manifold (M, g) of dimension n admits an isometric C^1-immersion into the Euclidean space $\mathbb{R}^q$, provided it admits a C^∞-immersion in $\mathbb{R}^q$ for some $q \geq n + 2$. In 1955 Kuiper improved this result by relaxing the dimension condition to $q \geq n + 1$. The technique of Kuiper was later adapted into the theory of Convex Integration by Gromov. We shall discuss some generalisations of Nash-Kuiper theorem for pairs of Riemannian metrics on the manifolds using the convex integration technique.

Main Result. Let h_1, h_2 be two Euclidean metrics on $\mathbb{R}^q$, such that for any $c \in (a, b)$, $c^2h_1 - h_2$ is an indefinite metric with signature (r_+, r_-), where $r_\pm \geq 2n$. If M is manifold of dimension n with a pair of Riemannian metrics (g_1, g_2) related by the inequalities $a^2g_1 < g_2 < b^2g_1$, then there exists an almost everywhere differentiable (Lipschitz) map $f : M \to \mathbb{R}^q$ satisfying $(df_x)^*h_i = g_i$ for $i = 1, 2$ for almost all $x \in M$.

We also discuss the existence of isometric C^1-immersions for such pairs. These are joint work with G. D'Ambra.