Superluminal neutrino: a quantum weak measurement effect?

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Symmetry in time: quantum vs classical

- Dynamical laws, both in quantum and classical theory, are time-symmetric.
- Measurements introduce time-asymmetry in quantum case.

Consider state of an isolated system at time t is known

t_0 t t_1

Classical:

its state both in past t_0 well as in future t_1 can be determined !
everything about the system can be determined at all times if the initial conditions and its dynamics is known at a given instant.

Quantum:

Even if the wavefunction is known at time t , and Hamiltonian is known at all times, the result of a measurement performed at a later time t_1 or previous time t_0 can not be exactly predicted

Both "past" and "future" equivalent (equally good or bad) as far as prediction of "present" is concerned !

So why not use both to predict the present (Y.Aharonov) !

Standard theory of measurements:

Ideal quantum measurement of an observable A

If the system before the measurement is in an eigenstate of A with an eigenvalue a_n , the outcome of the measurement is a_n and the quantum state of the system is not changed.

The interaction Hamiltonian between measuring device and system is

$$H_{int}(t) = g(t)\, P\, A$$

- P: momentum conjugate to the pointer variable Q of the measuring device
- g(t): normalized coupling function specifying the time of the measurement interaction

For ideal measurement,

- g(t) nonzero only for a short time δt
- free Hamiltonian negligible during this time

$$\frac{dQ}{dt} = \frac{i}{\hbar}\left[H_{int}, Q\right] = g(t)\, A \quad \Longrightarrow \quad Q_f - Q_i = A$$

Shift of the pointer variable during the interaction gives the outcome of the measurement

Conventional Measurement

- Any precise measurement of A requires Q to be precisely fixed before measurement.

Consider initial state of the pointer on measuring device a Gaussian: $e^{-Q^2/4\Delta^2}$

precise measurement requires $\Delta \rightarrow 0$

Effect of a measurement of A when system is in state $\Psi_0 = \sum_k c_k |a_k\rangle$

Measurement leads to coupled state

$$\Psi_t = e^{-i\int H\,dt} e^{-Q^2/4\Delta^2} \Psi_0 = \sum_k c_k\, e^{-(Q-a_k)^2/4\Delta^2} |a_k\rangle$$

- measurement gives an eigenvalue if $\Delta \rightarrow 0$

$$\langle \Psi_0 | \Psi_t \rangle = \sum_k |c_k|^2\, e^{-(Q-a_k)^2/4\Delta^2}$$

however

- $\Delta \rightarrow 0$ ➡ P with large uncertainty ➡ H_{int} large
- **Measurement necessarily disturbs the value of all the observables non-commuting with A**
- **Result of measurement of B after A will be different from B before A**

weak measurement

- For a good measurement, interaction between measuring device and system should be weak
  H → 0 → P → 0 → Δ → large >> spectrum length
  then state of pointer is a Gaussian centered around <A> with spread Δ

- One such measurement gives no information as Δ >> expectation value <A> ,
  M such measurements with reduce the uncertainty by $1/\sqrt{M}$

↓

- A sufficient large ensemble of systems all in a initial state Ψ will lead to measurement of <A> with any desired precision.

↓

- As measurements hardly disturb the ensemble, they characterize the ensemble during all intermediate times

↓

- Even non-commuting operators can be measured at the same time !
  (as a single measurement is imprecise)

weak values: role of pre/post selected ensembles

Y. Aharonov

- Measurement via an initial state ensemble only breaks time-symmetry.
- Time-symmetry can be preserved by selecting both past as well as future state of a state at time t

  → by defining a pre- and post-selected ensemble

- weak measurement of an operator A at time t between a pre-selected state Ψ_0 at time $t_0 < t$ and a post-selected state Ψ_1 at $t_1 > t$ yields values that need not be eigenvalues or even classically allowed !

  → these values are known as weak values !

weak value is the average of measurement results over a subset of data that correspond to a prescribed outcome of a projective measurement.

How weak values arise

- measure A at time t between a pre-selected state Ψ_0 at time t_0 and a post-selected state Ψ_1 at t_1

- initial wavefunction of the measuring device is $\Phi(Q)$ of width Δ.

  After an impulsive measurement of A and projection onto a final state Ψ_1,
  the final state of the measuring device is

$$\langle \Psi_1 | \, e^{-i \int H\, dt/\hbar} \, | \Psi_0 \rangle \, \Phi(Q) = \langle \Psi_1 | \, e^{-i P A/\hbar} \, | \Psi_0 \rangle \, \Phi(Q)$$

For $\Delta \rightarrow$ large, |P| is small (weak measurement)

$$\begin{aligned} \langle \Psi_1 | \, e^{-i P A/\hbar} \, | \Psi_0 \rangle \, \Phi(Q) &= \langle \Psi_1 | \, 1 - i P A/\hbar \, | \Psi_0 \rangle \, \Phi(Q) \\ &\approx \langle \Psi_1 | \Psi_0 \rangle \, e^{-i P A_w/\hbar} \, \Phi(Q) \\ &\approx \langle \Psi_1 | \Psi_0 \rangle \, \Phi(Q - A_w) \end{aligned}$$

weak value

$$A_w = \frac{\langle \Psi_1 | \, A \, | \Psi_0 \rangle}{\langle \Psi_1 | \Psi_0 \rangle} = \text{a complex number} \neq \langle A \rangle$$

Example: spin ½ case

- **Pre-selected ensemble**

  ensemble of spin ½ particles, each one polarized up in the z-direction at time t_0 :

$$S_z|s_z\rangle = +\frac{1}{2}|s_z\rangle$$

- **Post-selected ensemble**

  measurement of spin in the x direction at t_1 for each member of ensemble, only those cases are selected for which the spin is up along x :

$$S_x|s_x\rangle = +\frac{1}{2}|s_x\rangle$$

→ this sub-ensemble is the post-selected ensemble.

- **weak measurement of spin in a direction ϕ at time t, $t_0 < t < t_1$:**

$$S_\phi = S_x \cos\phi + S_z \sin\phi$$

$$S_{\phi,\text{weak}} = \frac{\langle S_x|\ S_\phi\ |S_z\rangle}{\langle S_x|S_z\rangle} = \frac{1}{2}(\cos\phi + \sin\phi)$$

As both S_z , S_x=+1/2 at t (same as at t_0, t_1 since no external field to change spin)

At $\phi=\pi/4$ → $S_\phi = 1/\sqrt{2} \ > \ 1/2$ outside the range [+1/2,-1/2]

weak, super-weak & expectation value

Consider operator A with its eigenvalues a_n lying in the range $a_{min} \leq a_n \leq a_{max}$,
the initial state Φ of the detector is represented by a real wavepacket of width Δ,

- weak measurement at time t with an initial ensemble of size M at time t_0 gives expectation value <A> with error $\Delta/\sqrt{M}$

- the weak measurement, for both past and future ensemble chosen, replaces <A> by Re (A_w)
- Im(A_w) gives the shift of the device pointer for the operator conjugate to A

- Expectation value <A> never lies outside the range of a bound spectrum.
- Re(A_w) can lie outside the spectrum-range:
- It can be superweak value.

Neutrino Basics ?

spin $(1/2\hbar)$

Primary interaction : weak force.

Types : 3 known types or flavors

(named after their partner leptons)

electron neutrino ,

muon neutrino

tau

Sterile ?

Mass: implied by neutrino flavour oscillations

⇒ existence of a tiny neutrino magnetic moment of the order of 10−19 μB,

⇒ possibility of an electromagnetic interaction too among neutrinos

neutrino flavor eigenstates different from neutrino mass eigenstates (referred as 1, 2, 3, more ?)

Neutrinos have left-handed chirality. (C.S.Wu) ?

neutrino flavor oscillations:

neutrinos are able to oscillate between the flavor states while propagating through vacuum

$$\left| \text{flavour} \right\rangle = \sum_k U_{f\,k} \left| m_k \right\rangle$$

U : Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, lepton mixing matrix, or MNS matrix

MSW effect:

◆ even if all neutrinos are massless, oscillations may occur when neutrinos pass through matter. (L. Wolfenstein)

◆ modified neutrino oscillations in matter,
neutrinos in matter have a different effective mass than vacuum.

The electrons in matter change the energy levels of the propagation eigenstates of neutrinos due to charged current coherent forward scattering of the electron neutrinos (i.e., weak interactions). This scattering is analogous to the electromagnetic process leading to the refractive index of light in a medium.

There are other possibilities in which neutrino can oscillate even if they are massless. If Lorentz invariance is not an exact symmetry, neutrinos can experience Lorentz-violating oscillations

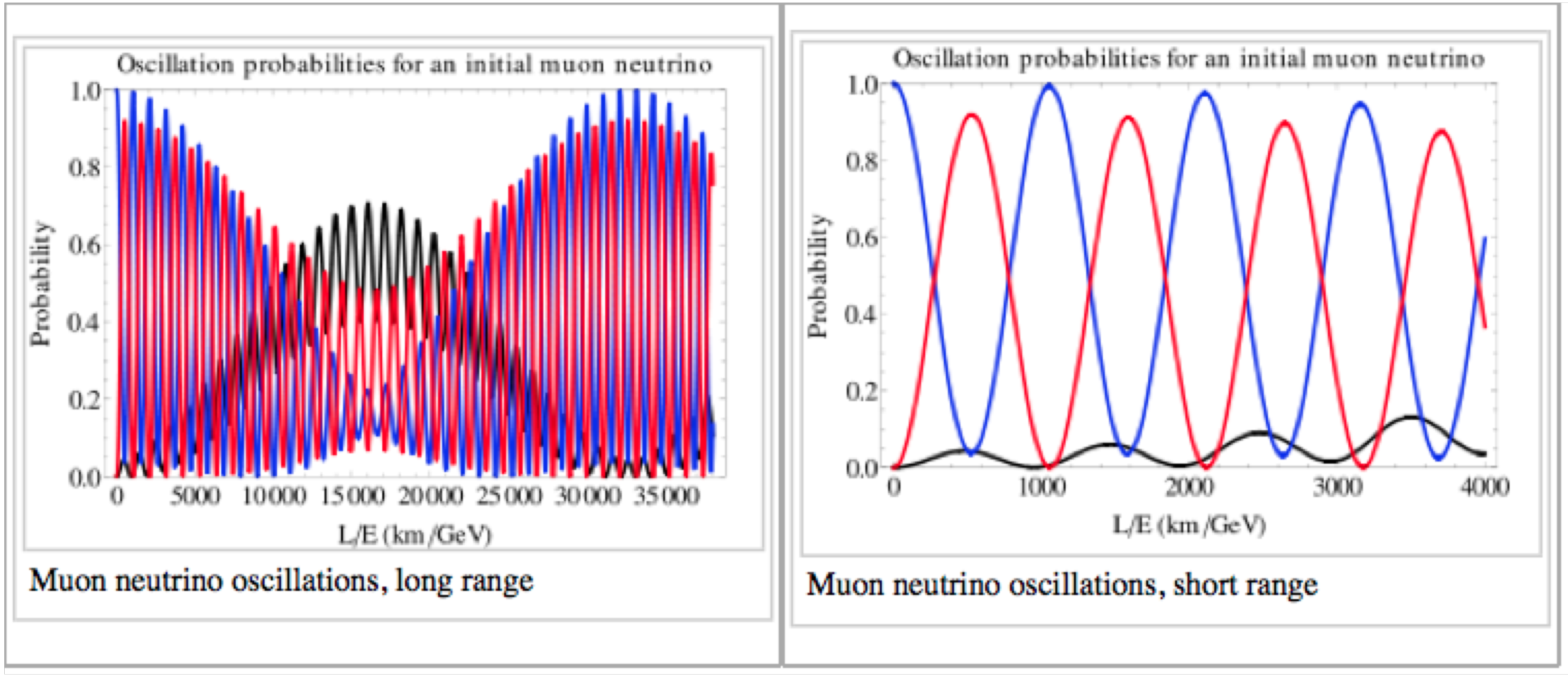
m_j being mass eigenstates, their propagation can be described by plane wave solutions

$$\left|m_j(t)\right\rangle = e^{i\left(p_j.r - E_j t\right)} \left|m_j(0)\right\rangle$$

$$E_j = c\sqrt{p_j^2 + c^2 m_j^2} \approx E + \frac{m_j^2}{2E}$$

Neutrino masses << 1 ev and energy E = c p ~ 1 gev

Eigenstates with different masses propagate at different speeds. The heavier ones lag behind while the lighter ones pull ahead. The mass eigenstates being combinations of flavor eigenstates, this difference in speed causes interference between the corresponding flavor components of each mass eigenstate. Constructive interference causes it to be possible to observe a neutrino created with a given flavor to change its flavor during its propagation.



Source: wikipedia

Probability of a flavour α to be observed as flavour β

$$P_{\alpha \to \beta} = \left| \langle n_\beta | n_\alpha \rangle \right|^2 = \left| \sum_k U^*_{\alpha k} \, U_{\beta k} \, e^{-i m_k^2 (L/2E)} \right|$$

When only two neutrinos participate significantly in mixing

For channel μ ➔ τ , sufficient to consider 2 X 2 mixing matrix U as mixing angle between 1 and 3 is small and as two of the mass states are very close as compared to third one (in atmospheric mixing)

Probability of a flavour α to be observed as flavour β

$$P_{\alpha \rightarrow \beta} = \delta_{\alpha\beta} - \sin^2 2\theta \sin^2\left(\frac{t\,\Delta E}{2\hbar}\right)$$

The maximum probability of conversion : governed by mixing angle θ

Frequency of oscillation: governed by mass-difference

OPERA experiment designed for detection of neutrino oscillations
In channel $\nu_\mu \rightarrow \nu_\tau$

Only two neutrinos participate significantly : $|\mu\rangle$ and $|\tau\rangle$

⬇

Sufficient to consider a 2 X 2 mixing matrix with mass-eigenstates $|+\rangle$ and $|-\rangle$ of energies E_+ and E_-

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Flavour μ $$|\mu\rangle = \cos\theta\,|+\rangle + \sin\theta\,|-\rangle$$

Initial state of neutrino beam

$$|\Psi_0(x)\rangle = \left(\cos\theta\,|+\rangle + \sin\theta\,|-\rangle\right) \exp\left(\frac{ipx}{\hbar}\right) f(x)$$

f(x) envelope of the initial wave packet two mass states move with different speeds and different energies

$$v_{\pm} = v \pm \frac{1}{2}\Delta v \qquad E_{\pm} = E \pm \frac{1}{2}\Delta E$$

wave arriving at the detector after travelling for time t

$$|\Psi(x,t)\rangle = \left(\exp\left(\frac{i\,t\,E_+}{\hbar}\right) f\left(x - t\,v_+\right)\cos\theta\,|+\rangle + \exp\left(\frac{i\,t\,E_-}{\hbar}\right) f(x - t\,v_-)\sin\theta\,|-\rangle \right) \exp\left(\frac{i\,p\,x}{\hbar}\right)$$

Detector postselects the muon flavour

Final spatial wavepacket after postselection

$$F(x,t) = \langle \mu | \Psi(x,t) \rangle = N \left(\cos\theta\,\langle + | + \sin\theta\,\langle - | \right) |\Psi(x,t)\rangle$$

ξ the deviation from the center of wavepacket
(assuming it moves with mean velocity v)

$$x = v t + \xi$$

Final spatial wavepacket after postselection

$$F(\xi, t) = N \exp\left(i \frac{p x - t E}{\hbar}\right) \left[\cos^2\theta \exp\left(-i \frac{t \Delta E}{2\hbar}\right) f\left(\xi - \frac{1}{2} t \Delta v\right) + \sin^2\theta \exp\left(i \frac{t \Delta E}{2\hbar}\right) f\left(\xi + \frac{1}{2} t \Delta v\right)\right]$$

Shift in the measured final position of the wavepacket

$$\overline{\xi} = \int_{-\infty}^{\infty} d\xi \; \xi \left| F(\xi, t) \right|^2$$

For initial envelope f(x) as a Gaussian of width w centered at x=0

$$\overline{\xi} = \frac{1}{2}\, t\, \Delta v \;\; \frac{\cos 2\theta}{1 - \sin^2 2\theta \left(1 - \cos\left(\frac{t\, \Delta E}{\hbar}\right) \exp\left(- \left(\frac{t\, \Delta v}{2\, w} \right)^2 \right) \right)}$$

Relative shift of mass wavepackets

effect of measurement

$t\,\Delta v \ll w$ Independent of the width or shape of the wavepacket

effective velocity shift

$$\Delta v_{eff} = \frac{\overline{\xi}}{t} = \frac{1}{2}\, \Delta v \;\; \frac{\cos 2\theta}{1 - \sin^2 2\theta \sin^2\left(\frac{t\, \Delta E}{2\hbar}\right)}$$

difference of group velocities

possible amplification factor

connection with weak measurement:

preselected state, arriving at detector

$$|\mathrm{pre}\rangle = \cos\theta \exp\left(-\mathrm{i}\frac{tE_+}{\hbar}\right)|+\rangle + \sin\theta \exp\left(-\mathrm{i}\frac{tE_-}{\hbar}\right)|-\rangle,$$

postselected state $|\mathrm{post}\rangle = |\mu\rangle = \cos\theta|+\rangle + \sin\theta|-\rangle.$

measured observable: velocity difference operator

$$\Delta\hat{v} = \tfrac{1}{2}\Delta v\sigma_z = \tfrac{1}{2}\Delta v\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

weak value

$$\Delta v_{\mathrm{weak}} = \frac{\langle \mathrm{post}|\Delta\hat{v}|\mathrm{pre}\rangle}{\langle \mathrm{post}|\mathrm{pre}\rangle} = \tfrac{1}{2}\Delta v \frac{\cos 2\theta \cos\frac{t\Delta E}{2\hbar} - \mathrm{i}\sin\frac{t\Delta E}{2\hbar}}{\cos\frac{t\Delta E}{2\hbar} - \mathrm{i}\cos 2\theta \sin\frac{t\Delta E}{2\hbar}}$$

and

$$\mathrm{Re}\,\Delta v_{\mathrm{weak}} = \tfrac{1}{2}\Delta v \frac{\cos 2\theta}{1 - \sin^2 2\theta \sin^2\left(\frac{t\Delta E}{2\hbar}\right)} = \Delta v_{\mathrm{eff}}$$

For neutrinos with momentum p, the energies of two mass states are

$$E_{\pm} = c\sqrt{p^2 + c^2 m_{\pm}^2} \approx c\,p + \frac{c^3 m_{\pm}^2}{2p}$$

$$v_{\pm} = \frac{\partial E_{\pm}}{\partial p} \approx c - \frac{c^3 m_{\pm}^2}{2\,p^2}$$

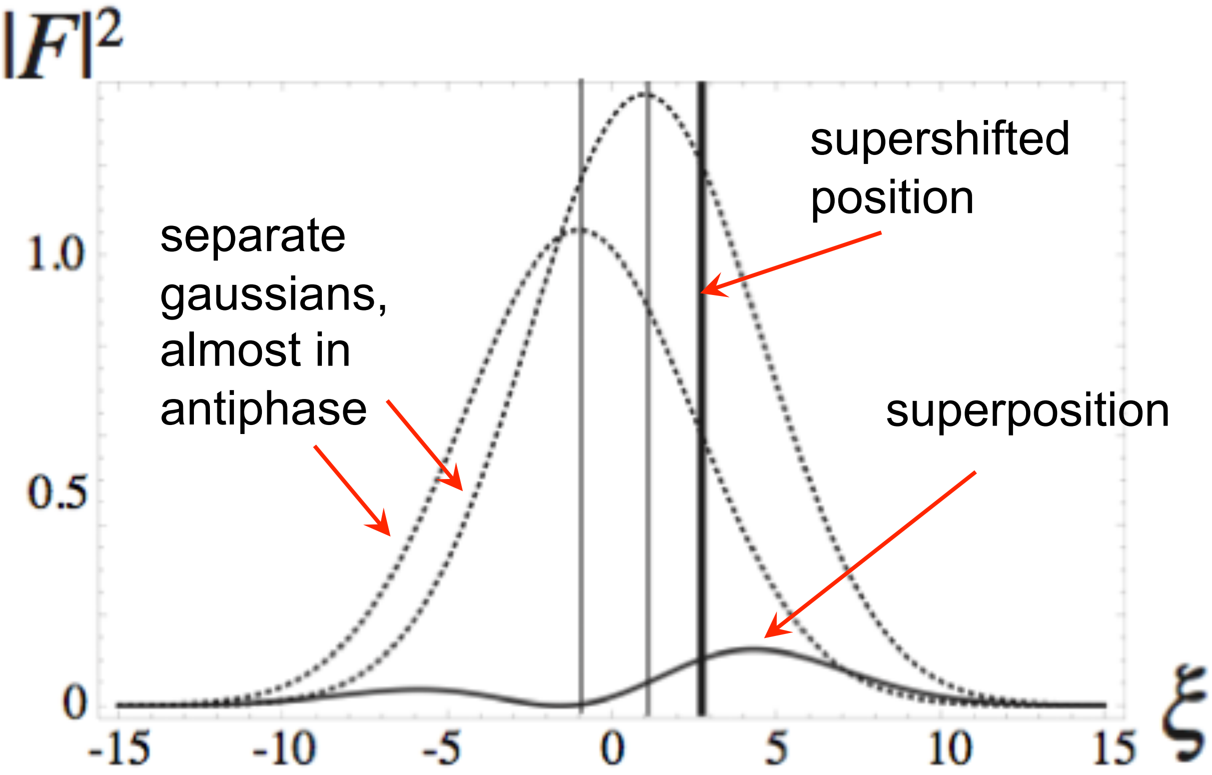
$$\Delta E = \frac{c^3}{2p}\left(m_{+}^2 - m_{-}^2\right)$$

$$\Delta v = -\frac{c^3}{2p^2}\left(m_{+}^2 - m_{-}^2\right) = -\frac{\Delta E}{p}$$

Possibility of superluminal velocity measurement

$\sin^2\left(\frac{t\,\Delta E}{2\hbar}\right) \approx 1$ and $\sin^2(2\theta) \approx 1$ => Near orthogonal pre, post states

Distorted wavepacket for the case $\frac{t\,\Delta E}{2\hbar} \approx \frac{\pi}{2}$ and $\theta \approx \frac{0.99\,\pi}{4}$



Opera experiment

wide-range variation of energies of the neutrinos with average

$$c\,p = 28.1 \text{ GeV}$$

$$c^4\left(m_+^2 - m_-^2\right) \approx 2.43 \times 10^{-3} \text{ eV}^2$$

$$\frac{\Delta v}{c} = -\frac{c^4}{2c^2p^2}\left(m_+^2 - m_-^2\right) \approx -1.5 \times 10^{-24}$$

$$\Delta v_{eff} \approx 10^{-24}\ c \bullet \frac{\cos 2\theta}{1 - \sin^2 2\theta \quad \sin^2\left(\frac{t\,\Delta E}{2\hbar}\right)}$$

$$\Delta v_{opera} \approx 10^{-5}\ c$$

so need amplification factor ~10^{19}

Opera experiment

$$c\,p = 28.1\ \text{GeV}$$

$$c^4\left(m_+^2 - m_-^2\right) \approx 2.43 \times 10^{-3}\ \text{eV}^2$$

Path length travelled by neutrinos from CERN to Gran Sasso = d =730 KM

$$\frac{t\,\Delta E}{2\,\hbar} = \frac{d}{2\,c\,\hbar}\,\frac{c^4\left(m_+^2 - m_-^2\right)}{2c\,p} \approx \frac{7.3\times 10^5}{6\times 10^8}\,\frac{2.43 \times 10^{-3}}{56.2\times 10^9} \approx 0.08$$

$$\rightarrow \quad \sin^2\left(\frac{t\,\Delta E}{2\hbar}\right) \approx 0.03$$

MINOS experiment gives mixing angle $\sin^2 2\theta > 0.9$

$$\text{amplification factor} \quad \frac{\cos 2\theta}{1-\sin^2 2\theta \quad \sin^2\left(\frac{t\,\Delta E}{2\hbar}\right)} < 1$$

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FAST TRACK COMMUNICATION

Can apparent superluminal neutrino speeds be explained as a quantum weak measurement?

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Abstract
Probably not.

Points to ponder which could influence the result ?

- individual masses of neutrinos not exactly known,

- consideration of more than two neutrinos
  (constructive interference of many mass-states may lead to superoscillations, or equlvalently the superweak values of velocity difference operator)

- more drastic atmospheric effects on the propagation of mass eigenstates

•If the neutrino mass proves to be of Majorana type (making the neutrino its own antiparticle), it is possible that the MNS matrix has more than one phase.

• Neutrinos may have another source of mass through the Majorana mass term. This type of mass applies for electrically-neutral particles since otherwise it would allow particles to turn into anti-particles, which would violate conservation of electric charge.

• The apparently innocent addition of right-handed neutrinos has the effect of adding new mass scales, completely unrelated to the mass scale of the Standard Model. Consideration of heavy right-handed neutrinos may explain the excess speed ?