The black hole membrane paradigm redux

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Black Holes as Fluid Membranes

- A black hole primer
- Black holes & thermodynamics
- The old membrane paradigm
- Detour: relativistic fluid dynamics
- Blackfolds
- Charged branes and fluid/gravity correspondence
- Summary

A black hole primer

"The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."

Subrahmanyan Chandrasekhar

A black hole primer

- Classically black holes are specific solutions of Einstein's General Relativity.
- Physically they are characterized by regions of intense gravitational field which precludes any signal, including light, from escaping its grasp.
- The surface from above which the light-ray can escape out to the rest of the spacetime is called the *event horizon*.

Image courtesy: Wikimedia commons

Black holes in nature

- Black holes are the natural end-point of stellar evolution.
- As predicted beautifully by Chandrasekhar in 1934, a star that has run out of fuel succumbs to the inexorable gravitational pull and forms a black hole.
- All large galaxies are expected to have a supermassive black hole (mass ~ 10^8 M_{sun}) at their center.

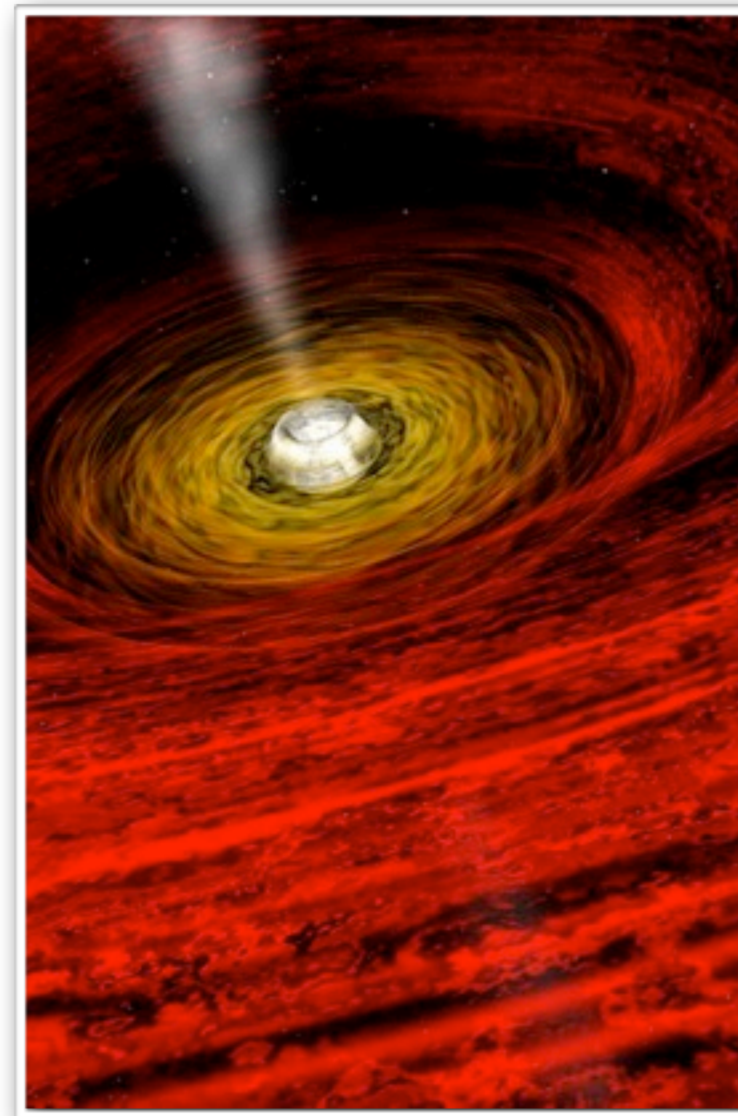


Image courtesy: Chandra X-Ray observatory

Detecting black holes

- Black holes are detected by the light emitted by the matter they accrete.
- Inspiralling black hole binaries are a powerful source of gravitational radiation, which one hopes to observer with gravity wave detectors in the near future.

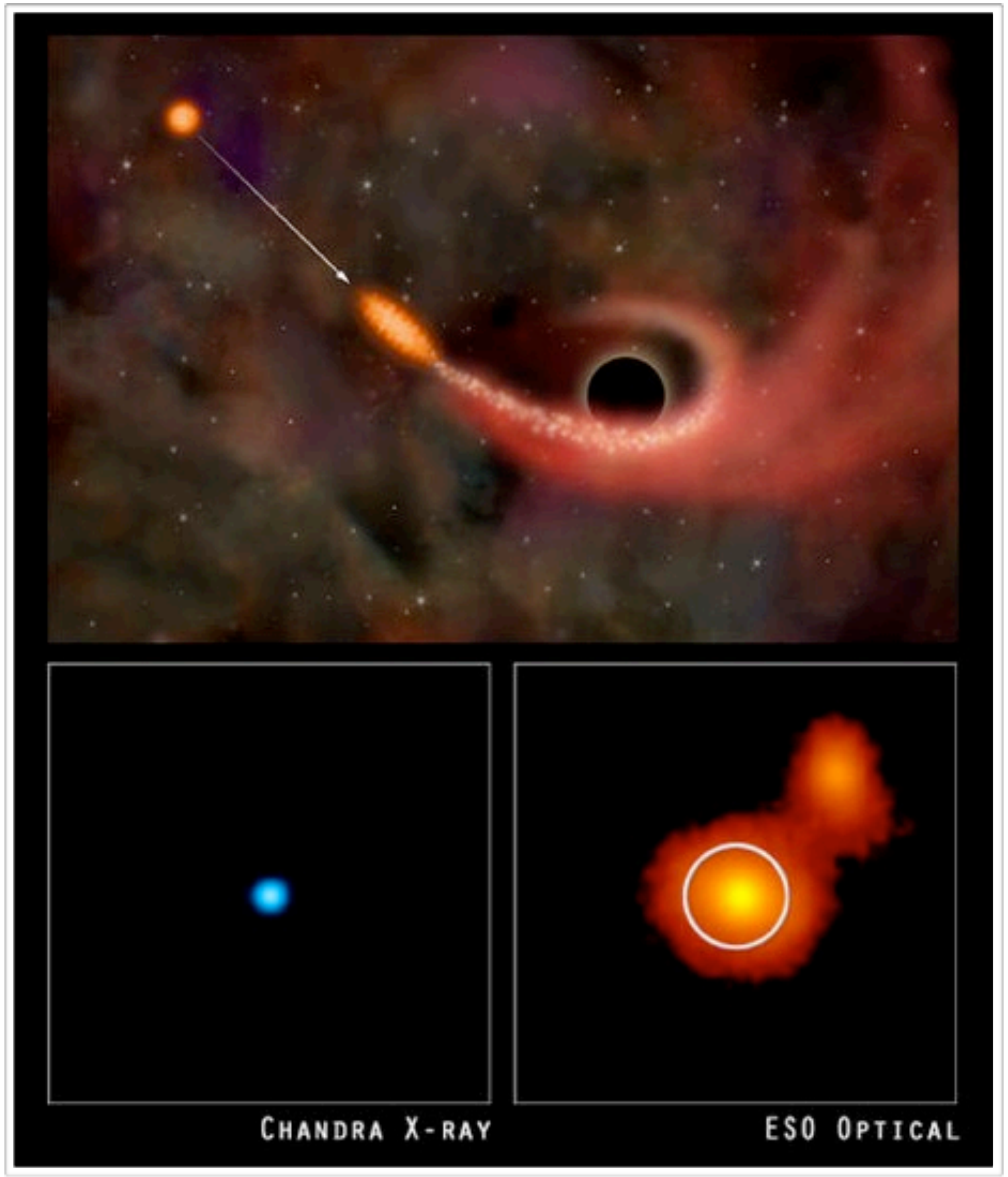


Image courtesy:Wikimedia commons

The black hole event horizon

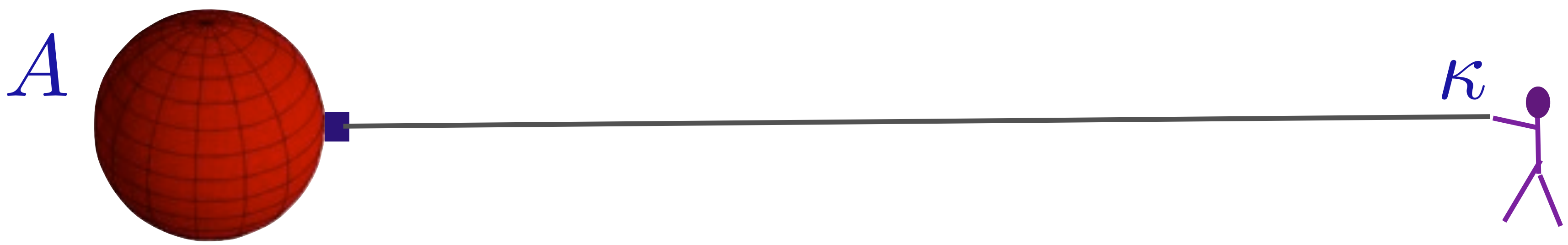
- The event horizon is a co-dimension one surface generated by null geodesics in the spacetime.

- Of some interest to us will be the area A_H of spatial sections of this surface, which we will measure in natural units using the Planck scale

$$\ell_P = \sqrt{\frac{\hbar G_N}{c^3}}$$

- We will call this dimensionless area the black hole entropy $S_{BH} \sim \frac{A_H}{\ell_P^2}$

- Another quantity of interest is the *surface gravity* ϰ of the event horizon, which roughly speaking measures the local acceleration due to gravity on the horizon.



Black hole thermodynamics

Thermodynamic Laws

- T is constant in thermal equilibrium.
- Energy conservation

$$dE = T\,dS + \text{work terms}$$

- Entropy is non-decreasing in any physical process (*Boltzmann*)

$$\delta S \geq 0$$

Laws of Black hole mechanics

- The surface gravity κ is constant over the horizon of a stationary bh.
- Total spacetime energy conservation

$$dM = \frac{1}{8\pi}\,\kappa\,dA_H + \text{work terms}$$

- Area of the event horizon is non-decreasing in any physical process (*Hawking*)

$$\delta A_H \geq 0$$

Black hole entropy

- Does the analogy between laws of thermodynamics and those of black hole mechanics have a fundamental significance?

- Black holes actually carry entropy S_{BH}, which should somehow be associated with the number of states a black hole can be in. (Bekenstein)

- Evidence: the discovery by Hawking in 1974 that once we turn on quantum effects black holes radiate. They do so like a black body at a temperature

$$T_{BH} = \frac{\hbar\,\kappa}{2\pi}$$

- There are many interesting questions associated with black hole evaporation, such as the black hole information paradox, but for this talk, we will focus on the fact that black holes are indeed thermodynamic objects with an entropy S_{BH}.

Black hole entropy: A puzzle

- If we grant that black holes carry entropy S_{BH}, we then have to ask what are the states that this entropy corresponds to?

- It is natural to posit that the entropy is related to the internal structure of the black hole.

- However, this poses a puzzle: the black hole entropy being proportional to the area of the horizon is not extensive!

- A corollary of black holes carrying entropy is that it seems to imply that gravity is effectively like thermodynamics: a statistical description valid on macroscopic length scales. This is bizarre (but with a kernel of truth).

The membrane paradigm: motivations

- Surface dynamics associated with the event horizon should capture the dynamics of the internal states of the black hole.

- What is a good description of such surface dynamics? One should be able to derive it from Einstein's equations.

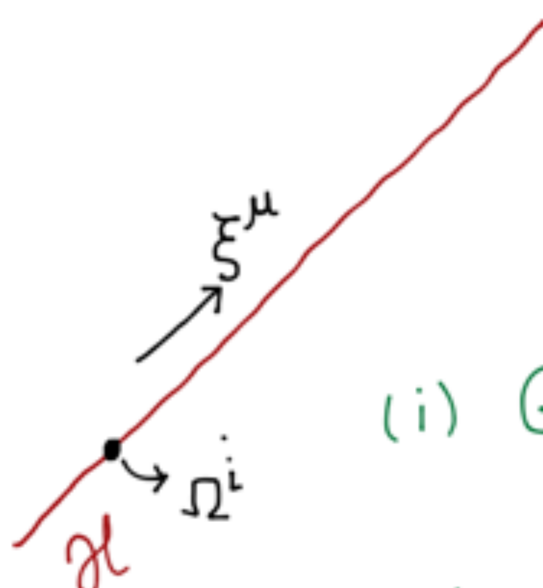
- The membrane paradigm for black holes was invented to understand some of these aspects and demystify the characteristics of the black hole and to describe the associated physics as one would for "ordinary bodies".

- *Claim*: the internal dynamics of a black hole can be modeled effectively as a membrane with electromechanical properties. The dynamics of Einstein's equations allows determination of the response of the black hole to external disturbances.

Damour (1978); Price, Thorne (1986)

The membrane paradigm: derivation

- Dynamical equations: project Einstein's equations along the event horizon which is a null hypersurface, using the the generator of the event horizon, (which is a null vector in the spacetime).



Projecting Einstein's eqns:

(i) $G_{AB}\xi^A\xi^B \quad \propto \quad R_{AB}\xi^A\xi^B$

(ii) $G_{AB}\xi^A e^B \quad \propto \quad R_{AB}\xi^A e^B$

The membrane paradigm

- Dynamical equations: project Einstein's equations along the event horizon which is a null hypersurface, using the the generator of the event horizon, (which is a null vector in the spacetime).

- Surface dynamics controlled by the extrinsic curvature induced on the horizon, essentially the gradient of the horizon generator, which provides a measure of gravitational energy momentum.

- The diffeomorphism symmetry inherent in Einstein's equations implies the conservation for the "gravitational energy-momentum" obtained via such a projection as an identity.

- The equations associated with the membrane paradigm are these conservation equations; these can be written in a form that is tantalizingly similar to fluid dynamical equations albeit of a peculiar kind.

Kinematics: variables for the paradigm

- From the horizon generator we can determine the extrinsic curvature of the horizon; introduce a basis on spatial sections with vectors e_μ

$$\nabla_\mu \xi = -K_\mu^\nu \, e_\nu$$

- Components of this extrinsic curvature are decomposed based on their transformations of the spatial rotation group.

$$\text{expansion:} \qquad \theta = -K_a^a \ ,$$

$$\text{shear:} \qquad \sigma_{ab} = -\gamma_{ac} \, K^c{}_b + \frac{1}{2} \gamma_{ab} \, \theta$$

$$\text{vorticity vector:} \qquad \Omega_a = -K_a^\xi$$

- The surface gravity can also be recovered from the extrinsic curvature by looking at the component along the generator:

$$\text{surface gravity:} \qquad \kappa = -K_\xi^\xi \ ,$$

Membrane dynamics

- The equations on a given spatial section of the horizon are interpreted as fluid dynamical equations and those along the generator as a gravitational analog of the Clausius equation (relating entropy production to heat).

$$D_t p_a = -\nabla_a \left(\frac{\kappa}{8\pi}\right) + \frac{1}{16\pi}\,\nabla_b \sigma_a{}^b - \frac{1}{16\pi}\,\nabla_a \theta - \xi^\mu \, T^{\text{matter}}_{\mu a}$$

Equation bears similarity to a hydrodynamic equation, with

$$P = \frac{\kappa}{8\pi}\,, \qquad \eta = \frac{1}{16\pi}\,, \qquad \zeta = -\frac{1}{16\pi}$$

$$\eta/s = \frac{1}{4\pi}$$

The equation along the generators is the famous Raychaudhuri equation:

$$D_t s - \frac{1}{\kappa}\, D_t^2 s = \frac{1}{T}\left(2\,\frac{1}{16\pi}\,\sigma_{ab}\,\sigma^{ab} - \frac{1}{16\pi}\,\theta^2 + \text{horizon-momenta}^2\right)$$

Damour (1978); Price, Thorne (1986)

Inadequacies of the membrane paradigm

- The equations for the most part are essentially kinematical: conservation of the horizon stress tensor is guaranteed a-priori.
- The derivation is predicated on the equation of motion of gravity being solved and one subsequently focuses on the projection of the full dynamics.
- Role of the horizon momentum density unclear.
- The negative value of the bulk viscosity, usually attributed to the teleological nature of the event horizon, indicates a serious pathology of the horizon fluid.
- Analysis of linearized fluctuations about the black hole does not show features of hydrodynamic behaviour.
- More critically, hydrodynamics is a low energy effective theory. At no stage in the derivation of the membrane paradigm is one focussing on the low energy excitations of the black hole.

Relativistic hydrodynamics

- Hydrodynamics is an IR effective field theory, valid when systems attain local but not global thermal equilibrium.
- We require that deviations away from equilibrium are long-wavelength in nature, i.e., we allow fluctuations that occur at scales larger than the typical mean free path of the theory. $\ell_m \ll L \qquad t_m \ll t$
- This allows for a gradient expansion: higher derivative operators are suppressed by powers of our expansion parameter ℓ_m/L .
- The dynamical content of fluid dynamics is just conservation. The energy momentum tensor and charge currents if any should be covariantly conserved.

$$\nabla_\mu T^{\mu\nu} = 0 \ , \qquad \nabla_\mu J^\mu = 0$$

- Conservation alone does not make for a good dynamical system since there are more dof than equations, but things simplify in the long-wavelength limit.

Relativistic hydrodynamics

- In the long-wavelength limit the dynamical dof are reduced, to local charge densities, local temperature and a (normalized) velocity field which indicates direction of flow of energy flux.

$$T^{\mu\nu}(x) = [P(x) + \rho(x)]\; u^\mu\, u^\nu + P(x)\, g^{\mu\nu} + \Pi^{\mu\nu}(x)$$

$$J_I^\mu = q_I\, u^\mu + J_{I,\text{diss}}^\mu$$

- The definition of the velocity field can be fixed by a choice of fluid frame; typically one chooses the velocity to be the timelike eigenvector of the energy -momentum tensor (defining thus the Landau frame).

- Further specification of the fluid requires constitutive relations which require the operators which characterize the dissipative tensors.

- In addition, a fluid also has an entropy current, which satisfies the 2$^{\text{nd}}$ law.

$$\mathcal{J}_S^\mu = s\, u^\mu + \mathcal{J}_{S,\text{diss}}^\mu \;, \qquad \nabla_\mu \mathcal{J}_S^\mu \geq 0$$

Relativistic hydrodynamics

- The dissipative parts of stress-tensor and charge currents can be expanded out in a basis of on-shell inequivalent operators built from the dynamical variables and their derivatives.
- From the velocity field we can for instance define:

$$\begin{aligned} \theta &= \nabla_\mu u^\mu = P^{\mu\nu}\,\nabla_\mu u_\nu \\ a^\mu &= u^\nu\,\nabla_\nu u^\mu \equiv \mathscr{D} u^\mu \\ \sigma^{\mu\nu} &= \nabla^{(\mu} u^{\nu)} + u^{(\mu}\,a^{\nu)} - \frac{1}{d-1}\,\theta\,P^{\mu\nu} = P^{\mu\alpha}\,P^{\nu\beta}\,\nabla_{(\alpha} u_{\beta)} - \frac{1}{d-1}\,\theta\,P^{\mu\nu} \\ \omega^{\nu\mu} &= \nabla^{[\mu} u^{\nu]} + u^{[\mu}\,a^{\nu]} = P^{\mu\alpha}\,P^{\nu\beta}\,\nabla_{[\alpha} u_{\beta]}\ . \end{aligned}$$

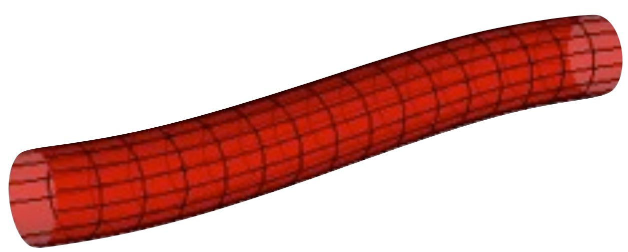
- At first order, upon using the conservation of ideal fluid to eliminate themal gradients, we have

$$\Pi^{\mu\nu}_{(1)} = -2\,\eta\,\sigma^{\mu\nu} - \zeta\,\theta\,P^{\mu\nu}$$

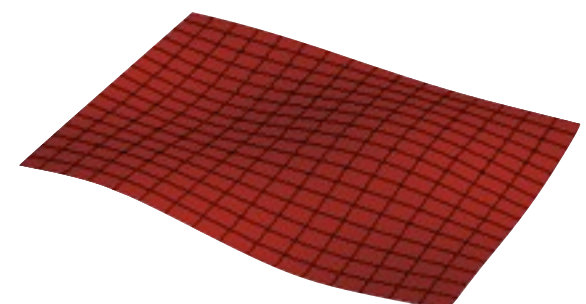
- Second law requires that $\eta \geq 0\ , \qquad \zeta \geq 0$

From black holes to black branes

- Part of the issue is the black holes in 3+1 dimensions are spherical (Hawking's topology theorem) and inherently have only one scale.
- In higher dimensions, one can have extended horizons, with multiple scales.



black string

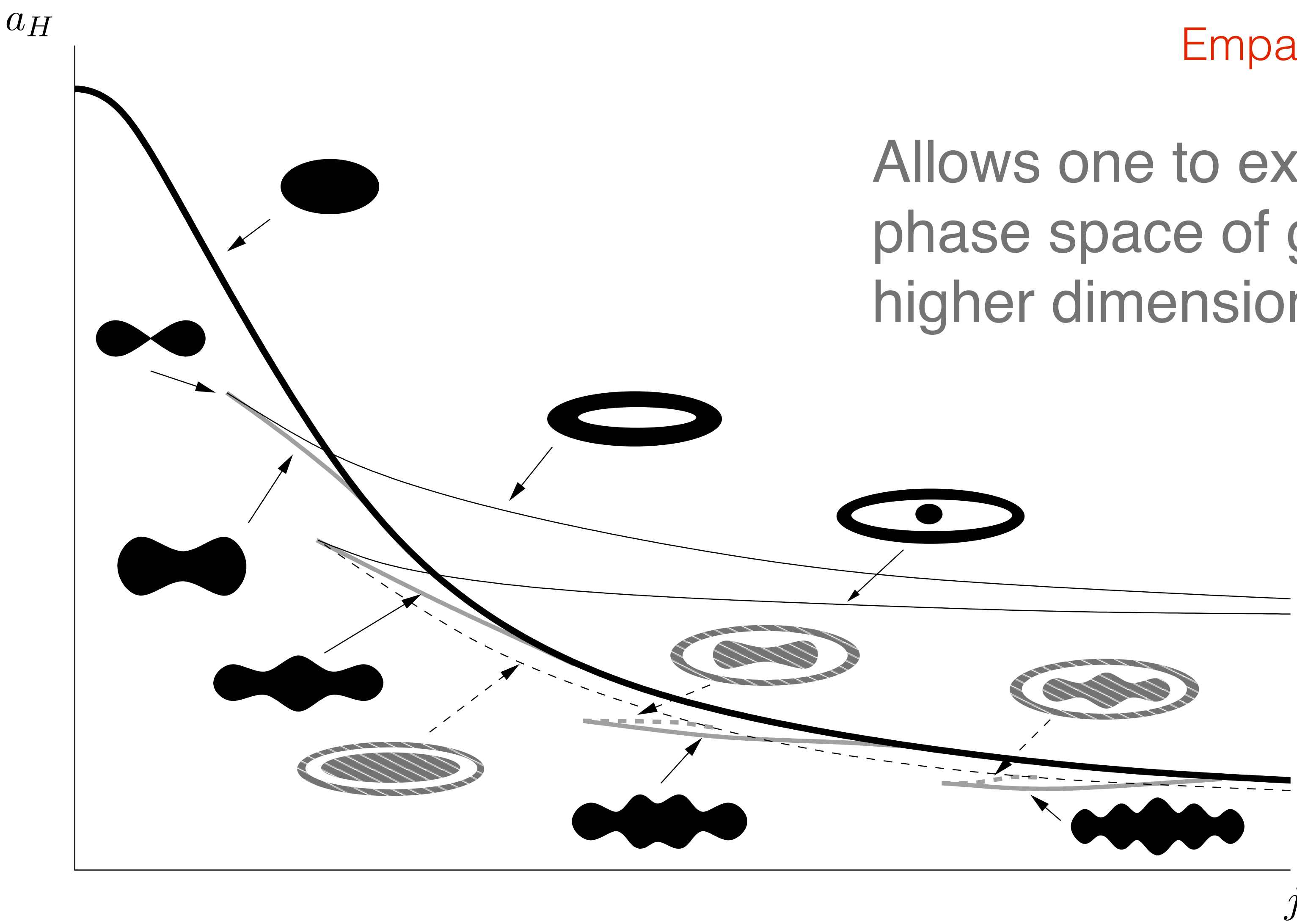


black brane

- Separation of scales allows us to investigate the behaviour of infra-red physics of black holes systematically.
- Historically, this was done in the context of black holes in a universe with a negative cosmological constant first: *fluid/gravity correspondence.*
- More generally the *blackfold* approach allows one to investigate this physics and moreover allows construction of approximate black hole solutions in higher dimensions.

Blackfolds

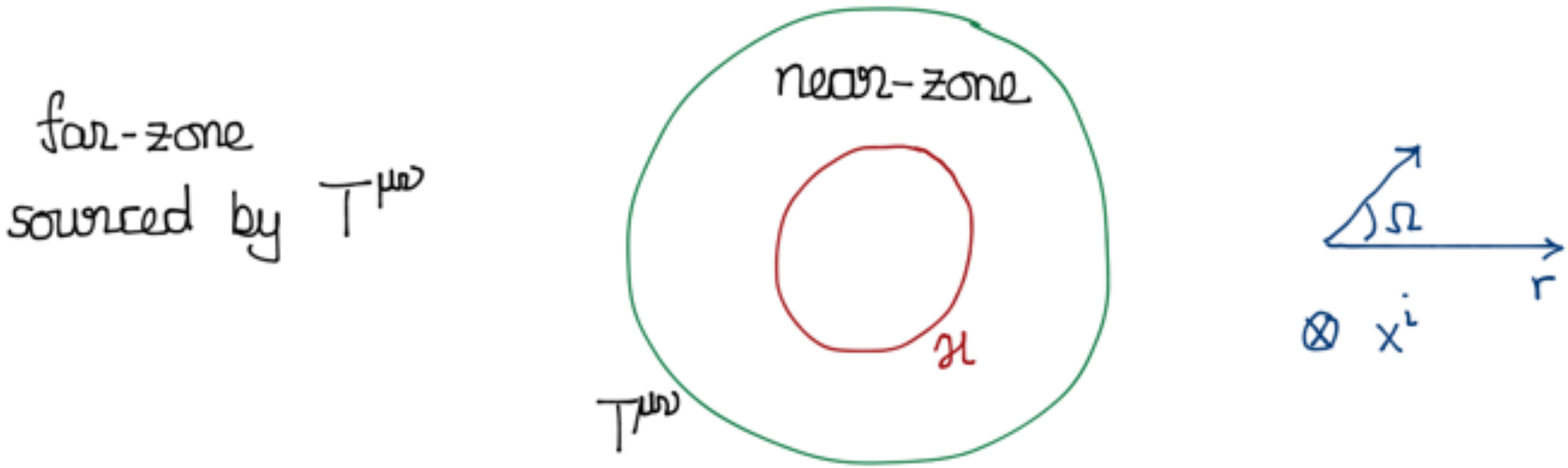
- A world-volume effective field theory for the dynamics of black branes.
- Gives approximate black hole solutions to Einstein's equations when the horizons in question admit two widely separated scales.



Emparan, Harmark, Nairchos, Obers (2009)

Allows one to explore the vast classical phase space of gravitational solutions in higher dimensions.

Blackfolds: qualitative picture

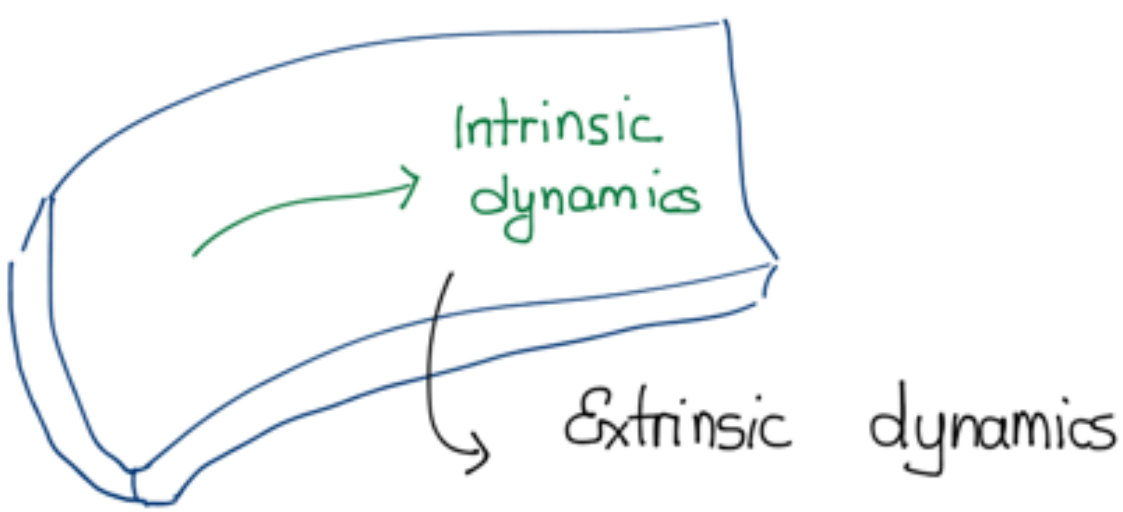


❖ Dynamics of black branes naturally splits into two

  ⋆ Intrinsic dynamics: dynamics along the world-volume, essentially given by the conservation of the brane stress tensor.

  ⋆ Extrinsic dynamics: which describes how the brane bends in response to the ambient curvature.

Blackfolds: low energy dynamics



- ⋆ Intrinsic: conservation of the induced stress tensor along the horizon directions.

$$\nabla_\mu T^{\mu\nu} = 0$$

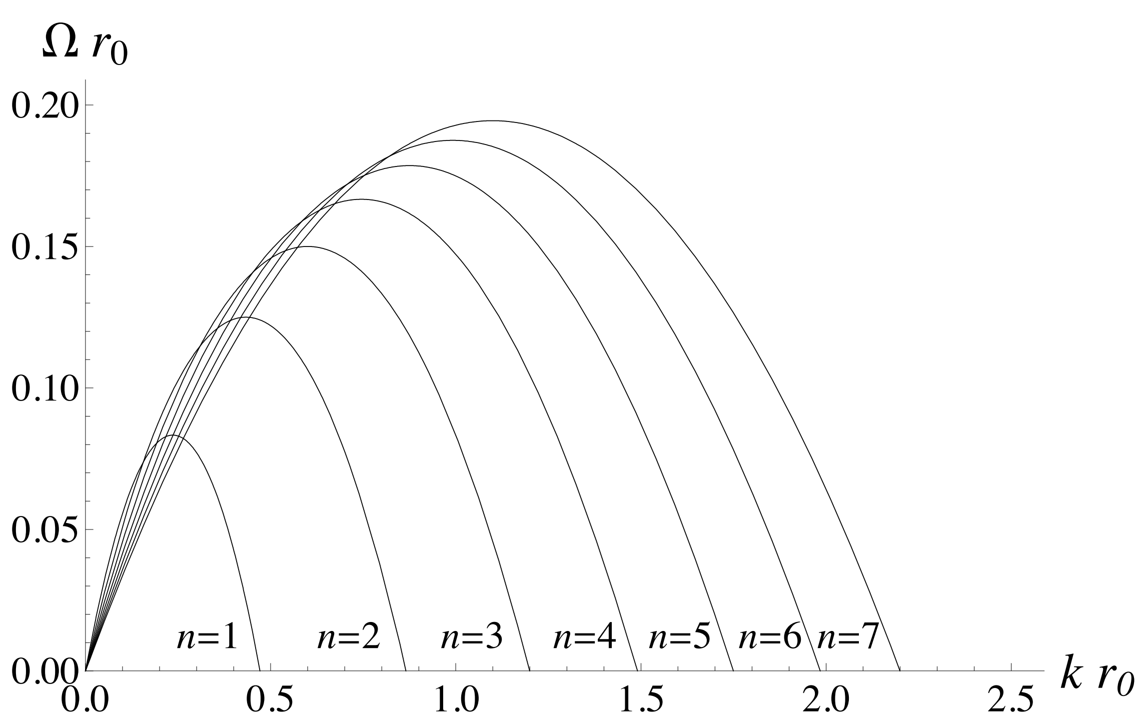
- ⋆ Extrinsic: minimization of the stress induced by the extrinsic curvature of the brane.

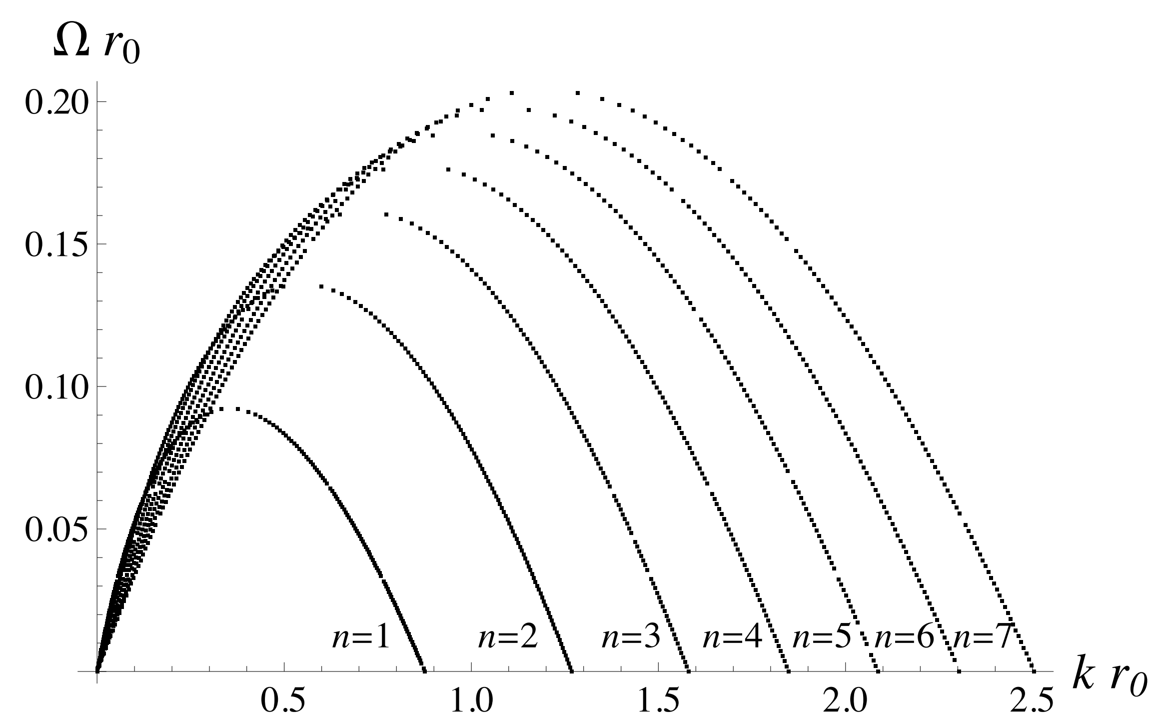
$$K_{\mu\nu}{}^{\rho} \, T^{\mu\nu} = 0$$

- ❖ In the long wavelength limit (momenta along the world-volume)
  - ⋆ Intrinsic: Black branes behave like fluids under strains along the horizon.
  - ⋆ Extrinsic: Dynamics is that of elastic solids for strains normal to the horizon.

Predictions of blackfolds

- Black branes are known to be unstable to long-wavelength fluctuations.
- This feature should be visible in the effective field theory approach and indeed the intrinsic dynamics of the branes i.e., fluid dynamics carries a clear signature of this instability.

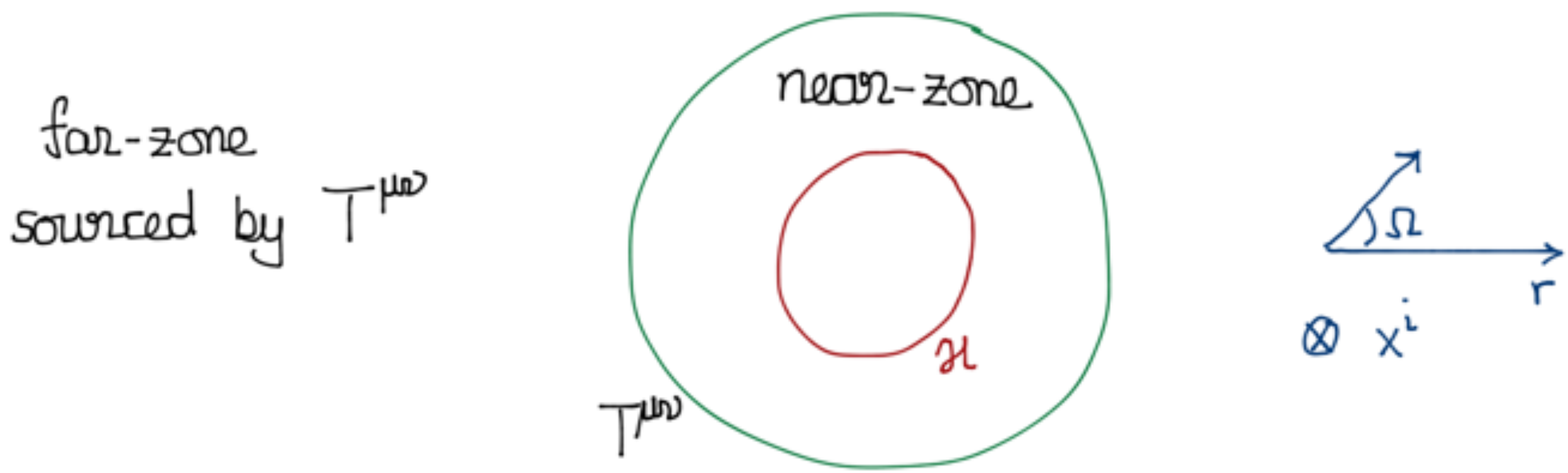




Camps, Emparan, Haddad (2010).

Branes in a box

- Focus solely on the near-zone physics, by isolating the black hole from the ambient geometry: this should bear close resemblance to the logic of the membrane paradigm.
- Natural to use Dirichlet boundary conditions on the hypersurface separating the near and far zones.



Branes in a box

- Putting a black hole/brane in a box has consquences on its dynamics.
- Given the new boundary conditions the black hole has to adapt itself and in particular its horizon needs to adjust appropriately. Indeed one can have multiple black hole solutions in the box.
- One important consequence is the change in the nature of the black hole thermodynamics:
  - Small black holes: Don't see the box and have thermodynamics of an asymptotically flat space solution, including the negative specific heat.
  - Large black holes: Are sensitive to the box and associated boundary conditions. They can come to equilibrium and have positive specific heat.

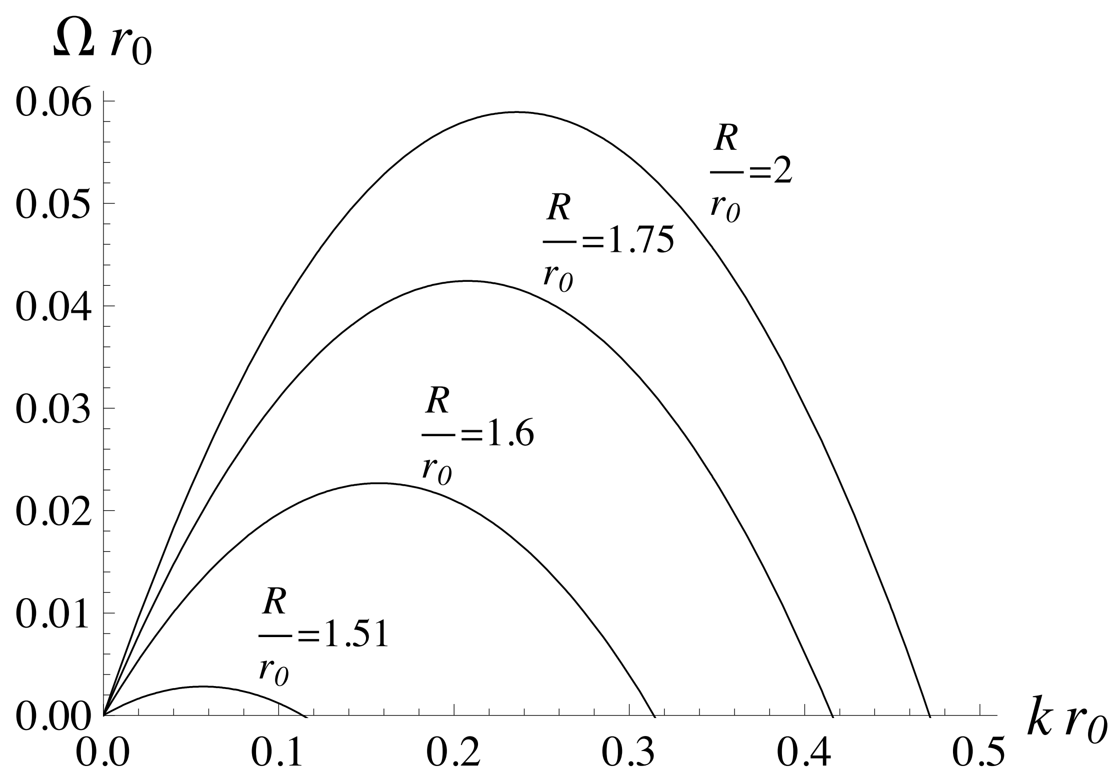
Natural way to implement a covariant gravitational box: put the black hole in a universe with a negative cosmological constant.

Branes in a box

- Einstein's equations can be solved in the near-zone in a certain long-wavelength approximation (focus on large black branes).
- Solution is parameterized by variables that characterize the low energy intrinsic dynamics of the hypersurface on which we impose the boundary conditions.
- This dynamics is the aforementioned fluid dynamics: one finds the hypersurface is described by a relativistic fluid with explicit constitutive relations determining its transport properties.
- The collective modes transverse to the brane world-volume lead to the elastic solid dynamics.
- To leading order the longitudinal and transverse modes are decoupled; the coupling between the dof occurs at higher orders.

Branes in a box: Inferences

- The fluid dynamical behaviour being sensitive to the boundary conditions displays this for e.g., via a modification of the spectrum of small amplitude fluctuations.



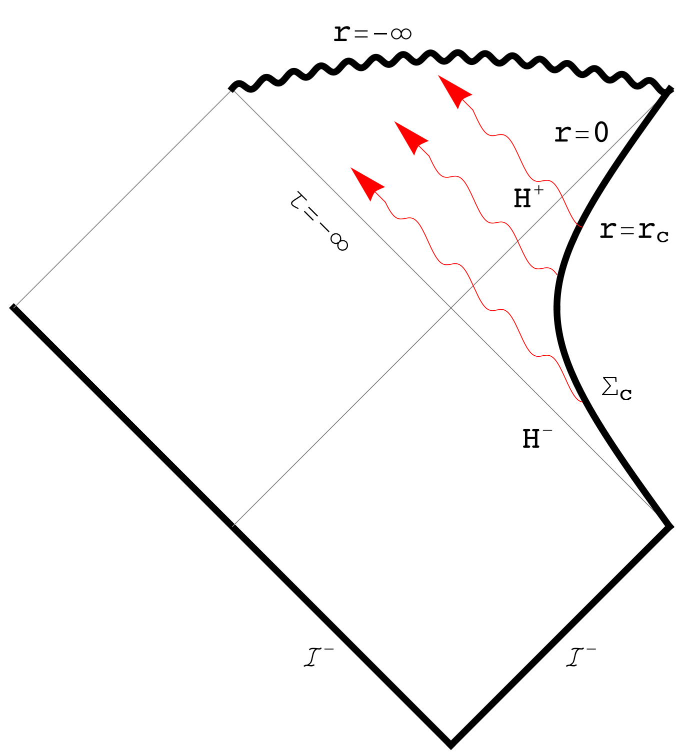
- One sees the disappearance of the GL mode and the speed of sound monotonically increases as we move the hypersurface close to the horizon.

- Near-horizon scaling behaviour: *incompressible Navier-Stokes dynamics.*

Towards the membrane paradigm

- The membrane paradigm should arise when we enclose the black hole/brane with a box that hugs the horizon (cf., stretched horizon).

- The near horizon geometry for branes becomes quite simple: it is simply a Rindler spacetime, the geometry seen by an accelerating observer in Minkowski spacetime.



$$\partial_i v^i = 0 \ , \qquad \partial_i p + \partial_t v_i + v^j \, \partial_j v_i - \nu \, \partial^2 v_i = 0$$

- The low energy physics of the Rindler region is universal to all branes and is simply the *incompressible Navier-Stokes dynamics.*

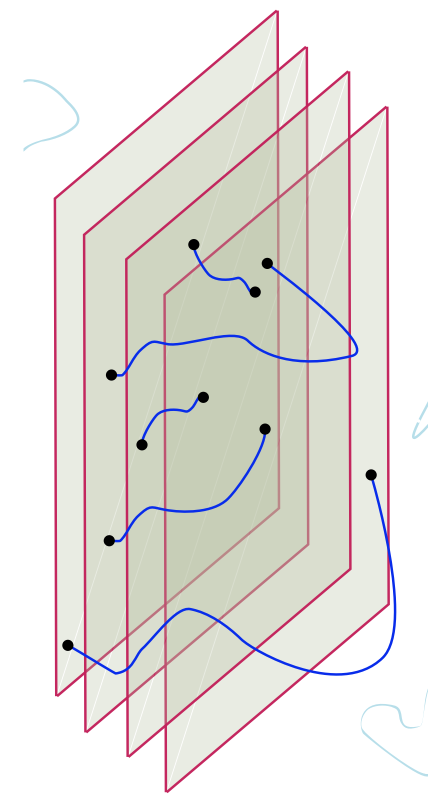
Bredberg, Keeler, Lysov, Strominger (2011)

Charged branes: Decoupling

- One can study more interesting systems, by considering black branes carrying charges. e.g., D-branes in string theory.
- Charges $\Longrightarrow$ one has a new scale in the problem, which one can use to tune the temperature of the black hole (independently of its size).
- In the near-extremal (vanishing temperature) limit one encounters an infinite throat: the proper distance to the horizon diverges.
- The dynamics in the throat in the low energy limit decouples from the asymptotic dof: this is the basis of the famous AdS/CFT correspondence.
- Of course, in string theory one can identify the decoupled theory which describes the physics of the throat region.

From D-branes to geometry

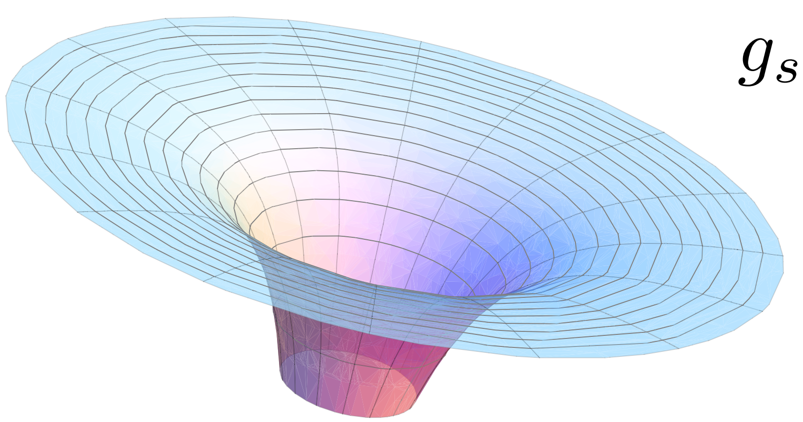
At weak coupling D-brane dynamics is well described in field theory language. A stack of *N* D-branes has a world-volume non-Abelian *SU(N)* gauge dynamics.



$g_s \ll 1$

Smoothly interpolate by dialing string interaction strength

However, at strong coupling, it is more useful to pass to a geometric picture, where the branes are replaced by an effective geometry supported by fluxes.



$g_s \gg 1$

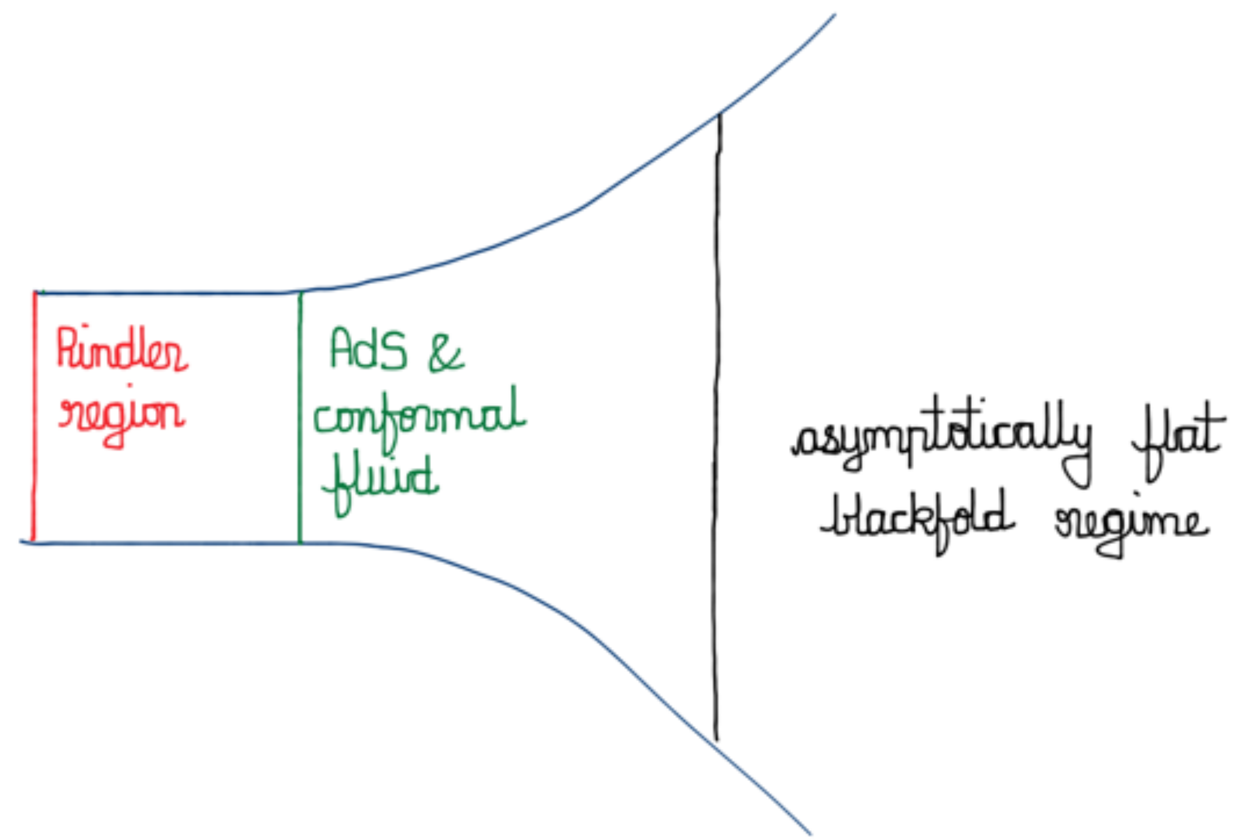
D-branes and membranes

- In classical gravity one can explore the hypersurface dynamics in various regimes of charged black branes.

- Asymptotic region: Blackfold fluid with features described earlier.

- Throat region: Conformal fluid dual to AdS geometries. Low energy limit of a QFT.

- Rindler region: incompressible fluid

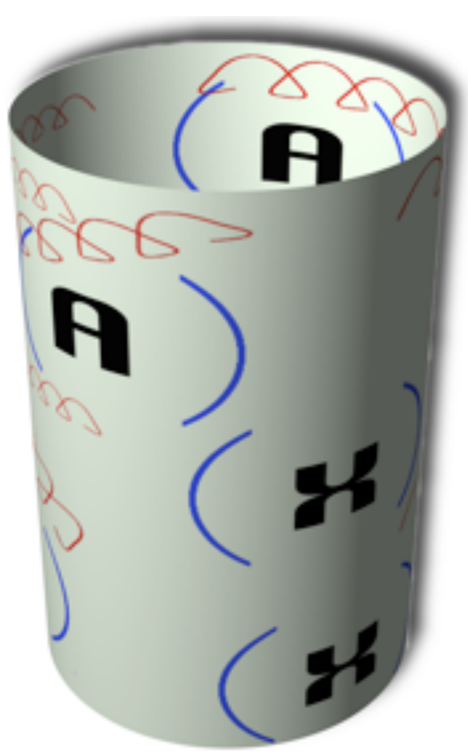


- Monitor the variation of transport properties of the fluid across the regimes: transport coefficients are pretty much determined by the throat dynamics.

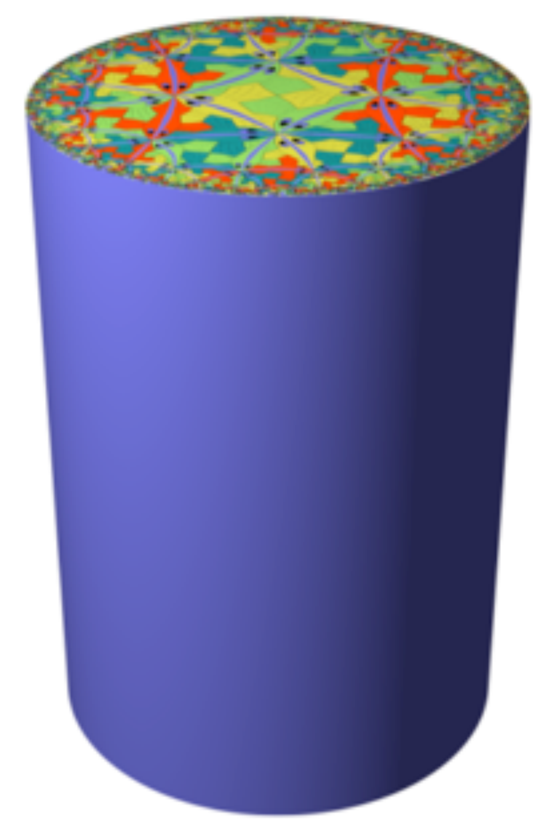
Emparan, Hubeny, MR (wip)   Brattan, Camps, Loganayagam, MR (2011)

The gauge/gravity correspondence

String theory which includes quantum gravity is exactly equivalent (or dual) to a non-gravitational quantum theory (gauge theory).



- The quantum theory lives on the boundary of the spacetime where gravity reigns.
- All the gravitational action is captured completely on the boundary.
- Boundary dynamics holographically captures gravitational physics.



The fluid/gravity correspondence

- The fluid/gravity correspondence establishes a correspondence between Einstein's equations with a negative cc and those of relativistic conformal fluids.

Einstein's eqn with negative cc

$$E_{MN} = R_{MN} - \frac{1}{2} G_{MN} R - \frac{d(d-1)}{2} G_{MN} = 0$$

$$\updownarrow$$

Relativistic ideal fluid equations

$$(\rho + P)\, \nabla_\mu u^\mu + u^\mu \nabla_\mu \rho = 0$$

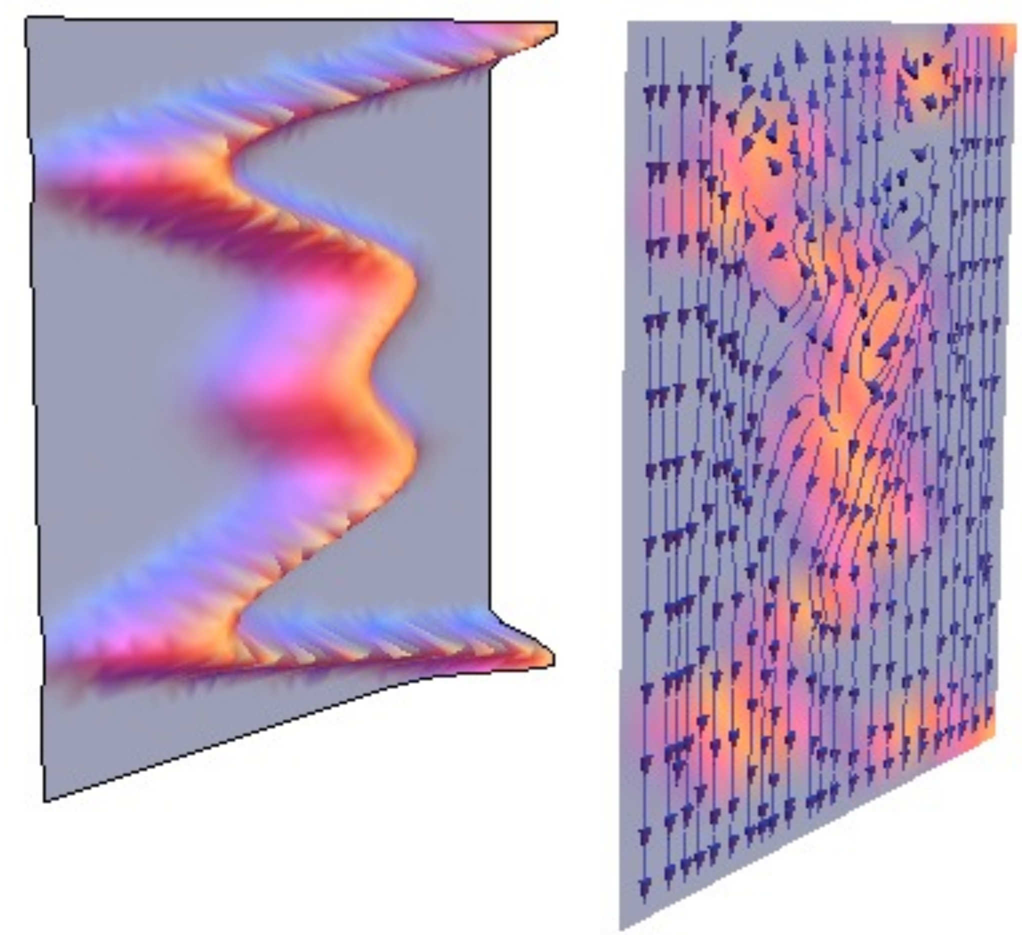
$$P_\alpha{}^\mu \nabla_\mu P + (\rho + P)\, P_{\nu\alpha}\, u^\mu\, \nabla_\mu u^\nu = 0$$

- Given any solution to the hydrodynamic equations, one can construct, in a gradient expansion, an approximate *inhomogeneous, dynamical black hole* solution in an asymptotically AdS spacetime.

Bhattacharyya, Hubeny, Minwalla, MR (2007)

Black branes as lumps of fluid

- Black branes really behave as lumps of fluid in the low energy limit.
- In the fluid/gravity correspondence, the fluid lives at the end of the universe, on the asymptotic boundary of the spacetime where the black hole resides.
- Here the fluid is a hologram, honestly capturing all the low energy physics of the entire geometry.



Black branes as lumps of fluid

- More generally, the blackfold approach allows us to isolate the fluid regime associated with a black brane.
- Based on where one chooses to put the hypersurface demarcating the near and far zones, one obtains different constitutive relations.
- However, there is an universality of the very near horizon region: this Rindler region is described by a nearly ideal, non-relativistic, incompressible viscous fluid.