From Cold Atoms to Complex Materials: Opportunities and Challenges

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Acknowledgements

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Outline

- ✓ Cold Atoms : A Brief Primer
- ✓ Optical Lattices and Quantum Simulators
- ✓ New Probes : Optical Lattice modulation
- ✓ Eqbm vs. Non Eqbm : challenges and opportunities

Outline

✓ Cold Atoms : A Brief Primer

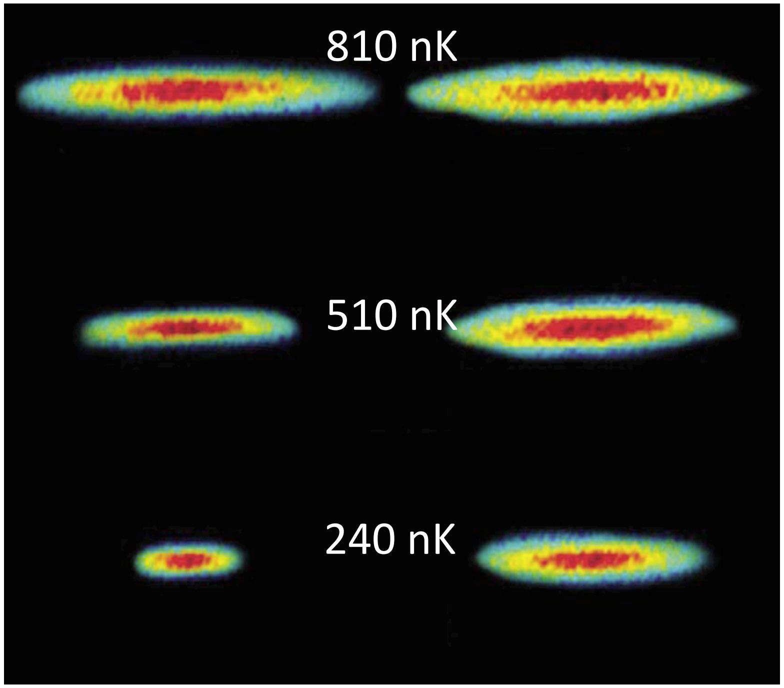
✓ Optical Lattices and Quantum Simulators

✓ New Probes : Optical Lattice modulation

✓ Eqbm. vs. Non-Eqbm : Challenges and Opportunities

Cold Atoms

- Alkali Atoms Cooled to ultralow temperatures
- Temperature ~ 10 nK (lowest 500 pK)
- Atom No. ~ $10^5 - 10^7$
- Atoms Trapped by Magnetic/Optical Potential
- Low Energy short range s-wave interaction



Boson Fermion

Bosons : Rb^{87}, Li^7, Na^{23}

Fermions : Li^6, K^{40}

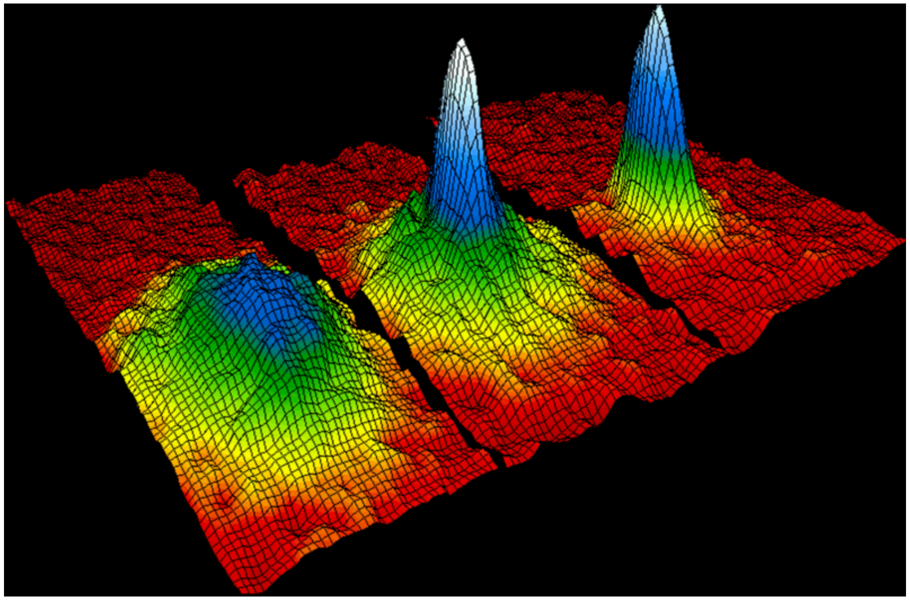
More Interesting Systems :

Cr^{52} Bosons with long range dipolar interactions

K^{40} Rb^{87} Fermionic molecules with dipolar interactions

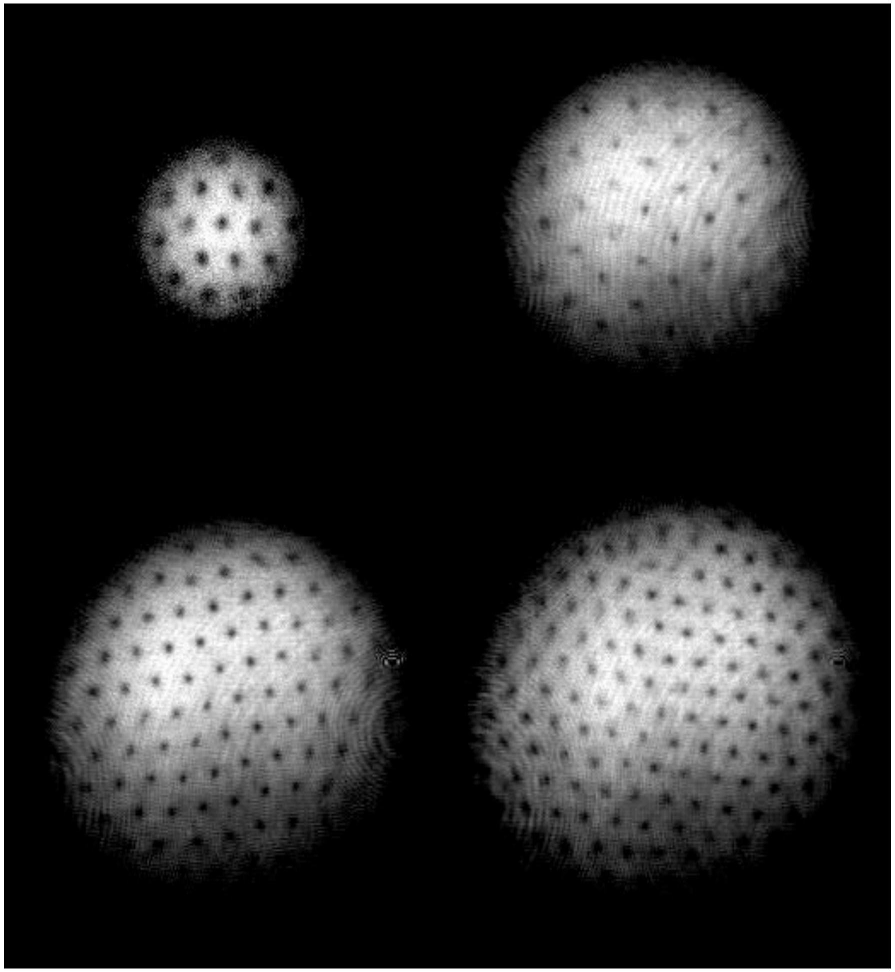
Yb^{176} Bosons with large spins

Superfluidity in Ultracold Bosons



M. H. Anderson et al.
Science, ***269***, *198 (1995)*

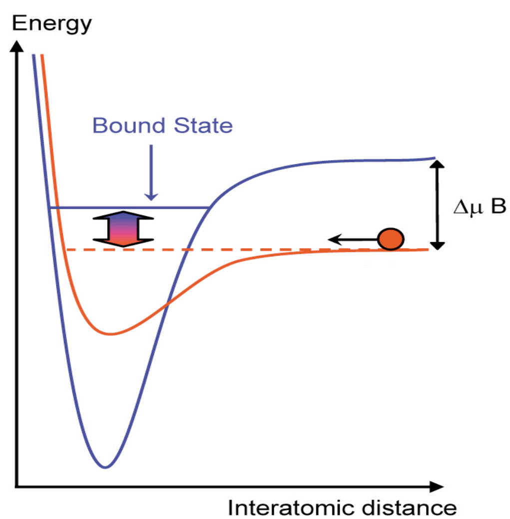
Cornell Group JILA

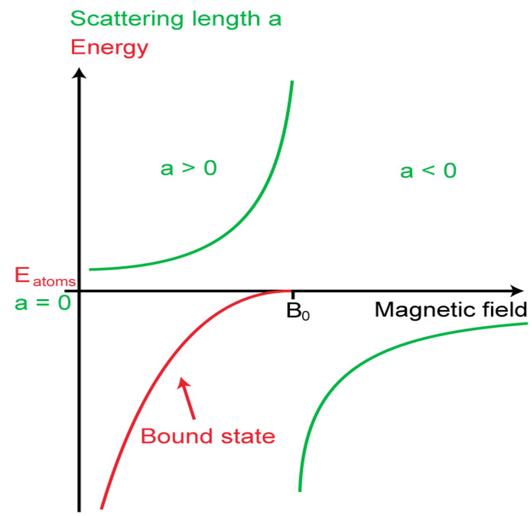


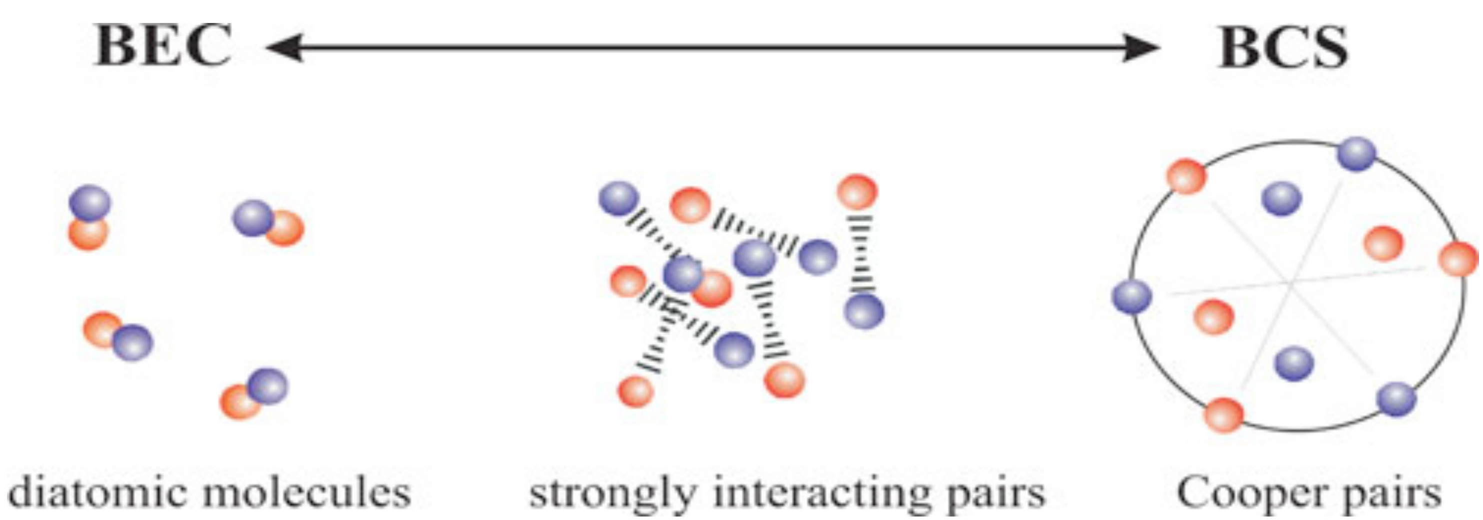
J. R. Abo-Shaeer et al.
Science, ***292***, *476 (2001)*

Ketterle Group, MIT

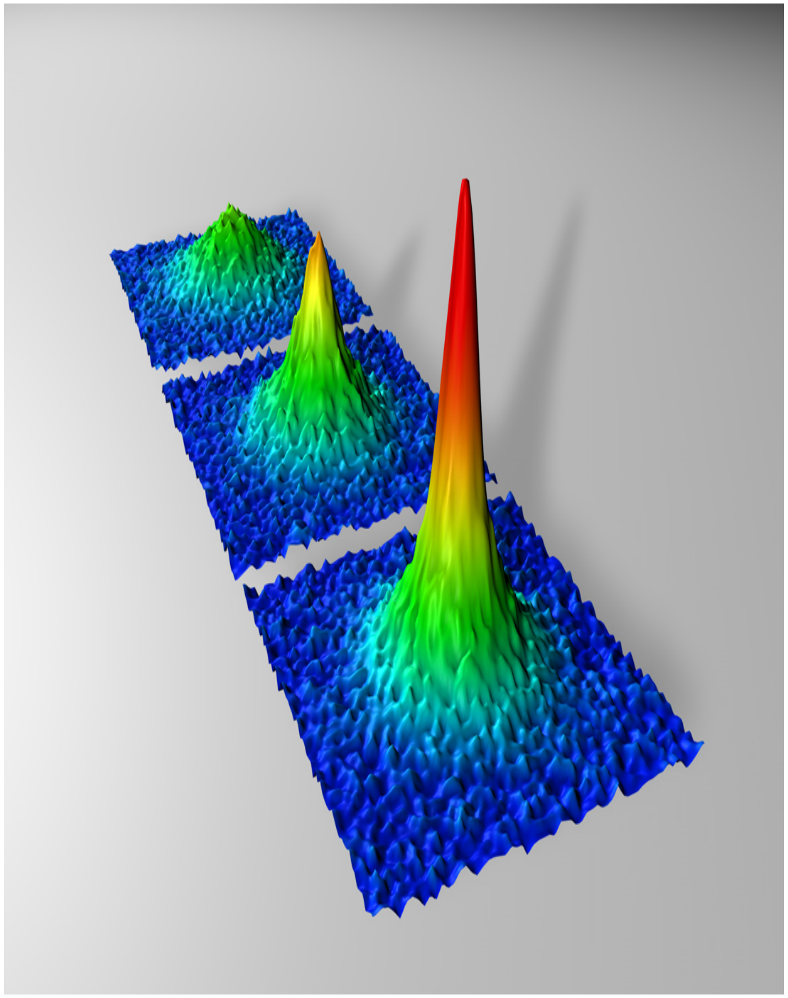
Tuning Interactions : BCS-BEC Crossover (Fermions)



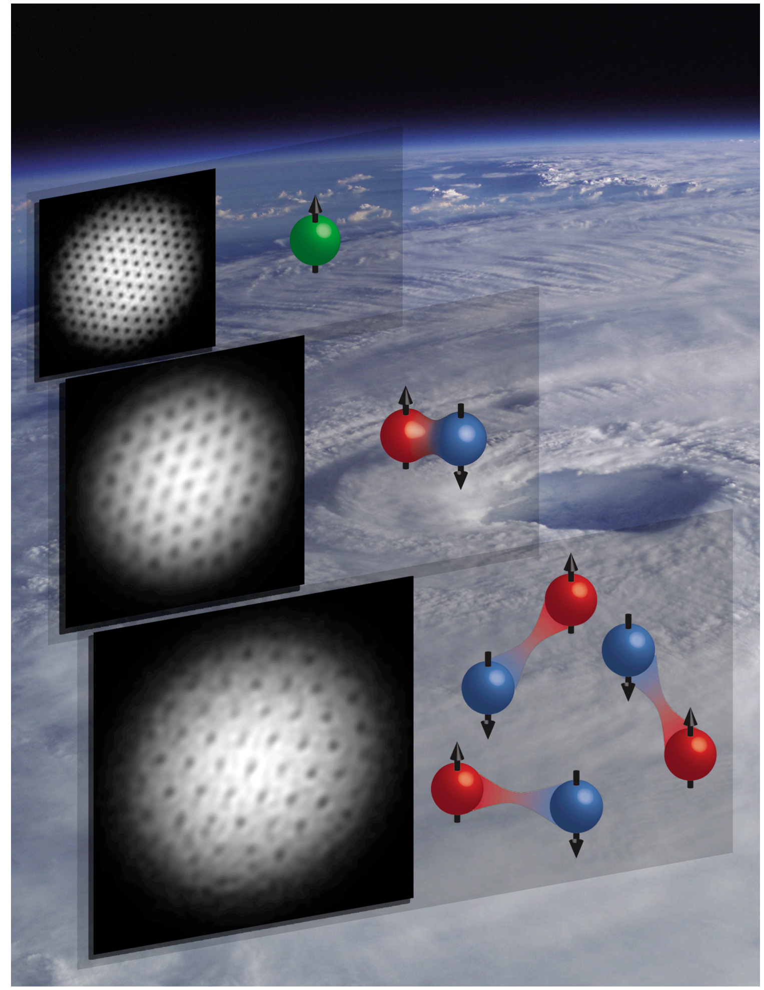




Observation of Fermion Superfluidity



C. A. Regal, M. Greiner and D. S. Jin
*PRL, **92**, 040403 (2004)*
Jin Group, JILA



M. W. Zwierlein et al.
*Nature, **435**,1047 (2005)*
Ketterle Group, MIT

New Directions

- ✓ Polarized Fermi Gases and BCS-BEC Crossover (MIT, Rice)

- ✓ Spinor Bose Condensates and Spin Textures (S=1 Bosons)
  (Berkeley)

- ✓ Dipolar Bose and Fermi gases (Long Range Interaction)
  (JILA, Boulder)

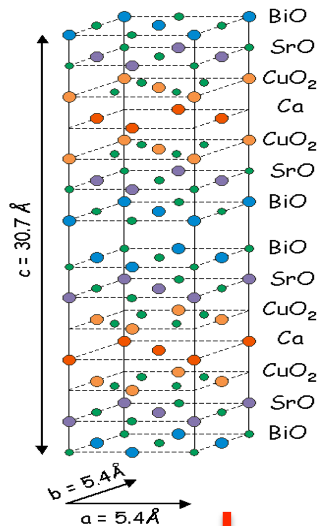
- ✓ Creating artificial disorder to study disorder effects
  (Paris)

Outline

- ✓ Cold Atoms : A Brief Primer
- ✓ Optical Lattices and Quantum Simulators
- ✓ New Probes : Optical Lattice modulation
- ✓ Mott Insulators, Antiferromagnets and Superfluids

Quantum Simulators

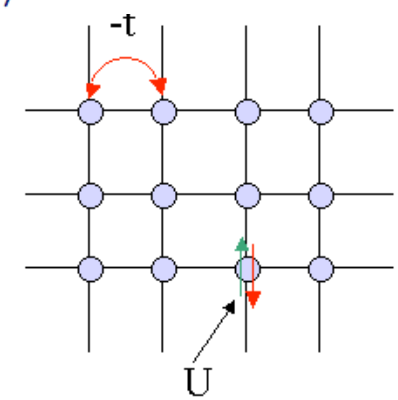
Cuprate Superconductors



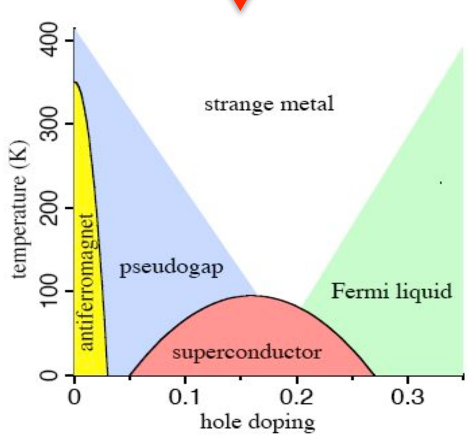
Minimal Model

One Band Hubbard Model
on Square Lattice

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



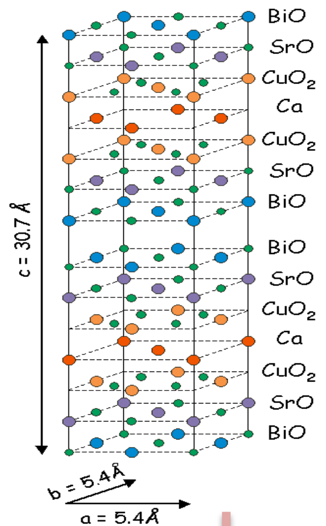
Experiments



Various
Approximation
Schemes

Quantum Simulators

Cuprate Superconductors



Minimal Model

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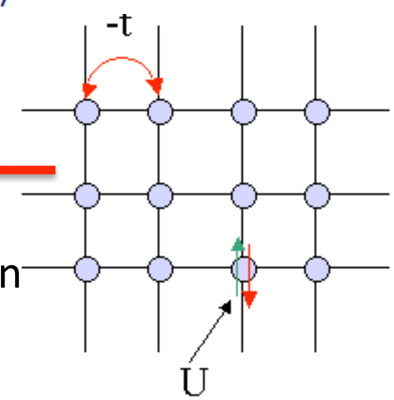
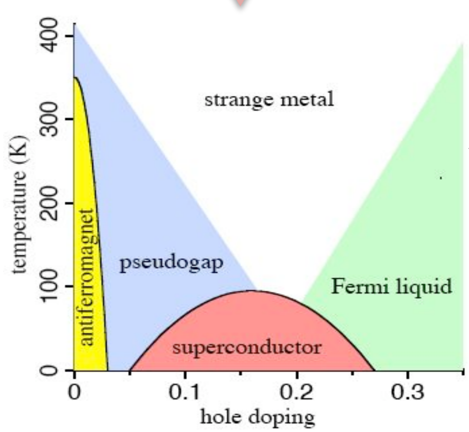


QUANTUM SIMULATOR

Experimental Implementation Of Model

Experiments



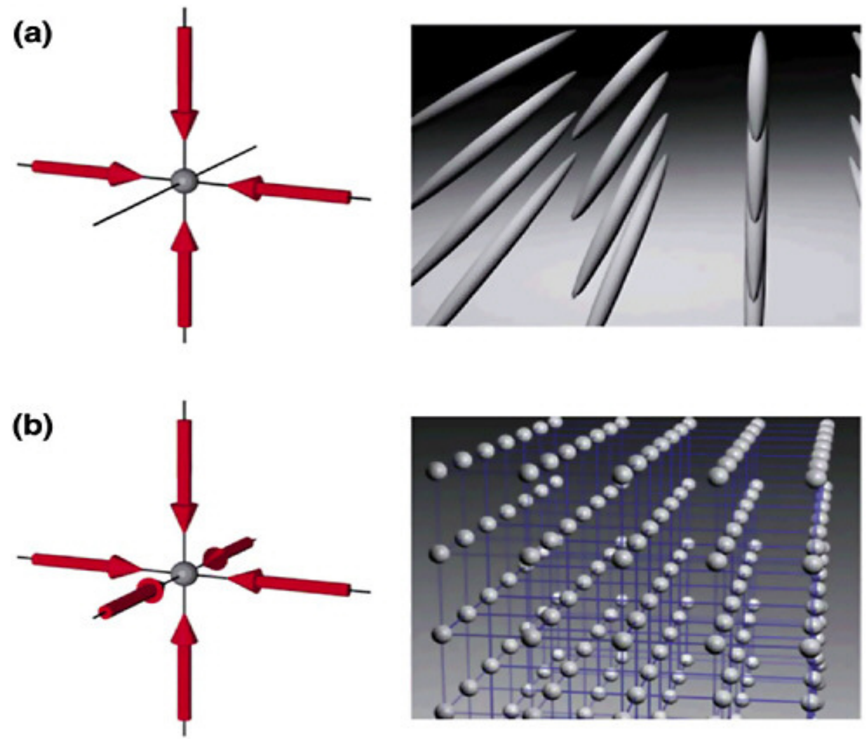
Validity of Model

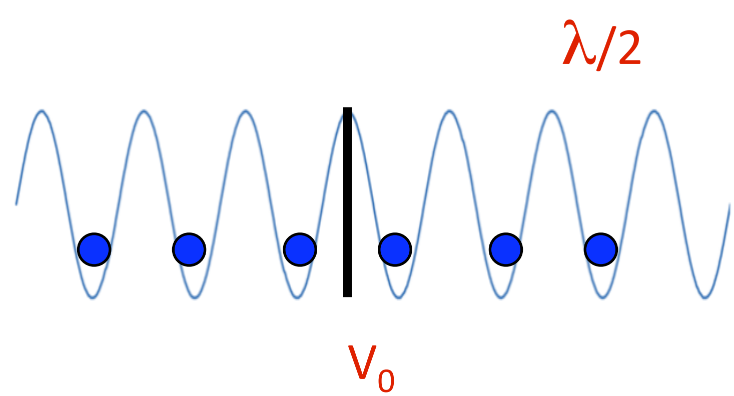
Validity of Approx.

Various Approximation Schemes

Cold Atoms on Optical Lattice

- Counter-propagating laser beams create periodic potential
- Can create 3D, 2D or 1D lattices
- Laser beams at suitable angles can create other lattice geometries e.g. Triangular lattice
- Lattice depth can be tuned by changing beam intensity
- Lattice spacing can be tuned by changing wavelength of the light used
- Superlattices can be created by using more than one beam with different wavelengths
- Spin dependent lattices





Cold Atoms on Optical Lattice

Bands around single well states with width z t

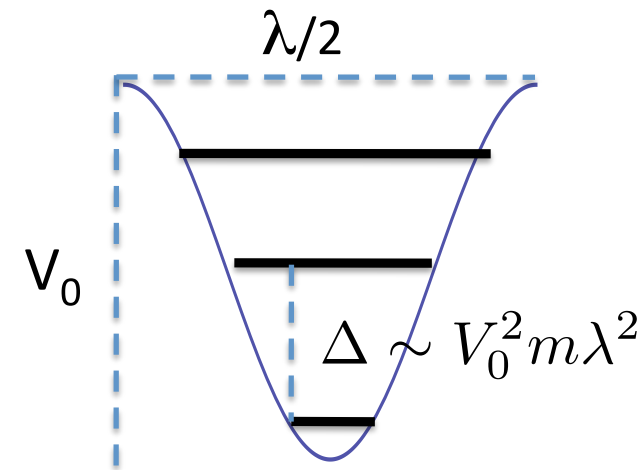
Tunneling : $t = \frac{2}{\sqrt{\pi}} E_R \left(\frac{V_0}{E_R}\right)^{\frac{3}{2}} e^{-2\sqrt{\frac{V_0}{E_R}}}$

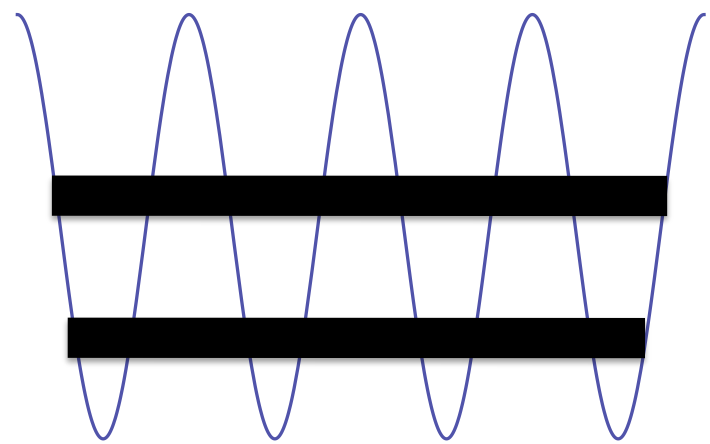
On Site Interaction : $U = \frac{2\pi^2 a_s}{3\lambda} E_R \left(\frac{V_0}{E_R}\right)^{\frac{3}{4}}$

$$t, U \ll \Delta$$

Simulating the one band Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$





- Hamiltonian parameters known to good accuracy
- Off site Interactions, next neighbour hopping etc
  Small but known quantities
- Clean Implementation : No disorder, No long range forces

Cold Atoms on Optical Lattice

Simulating the one band Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Tunneling :

$$t = \frac{2}{\sqrt{\pi}} E_R \left(\frac{V_0}{E_R} \right)^{\frac{3}{2}} e^{-2\sqrt{\frac{V_0}{E_R}}}$$

On Site Interaction :

$$U = \frac{2\pi^2 a_s}{3\lambda} E_R \left(\frac{V_0}{E_R} \right)^{\frac{3}{4}}$$

Tuning t/U by : (i) Changing V_0 (Beam Intensity)

(ii) Changing a_s (Magnetic Field)

Precise tunability :

Controlled access to both weakly interacting and strongly interacting regimes

Optical Lattice Systems as Quantum Simulators

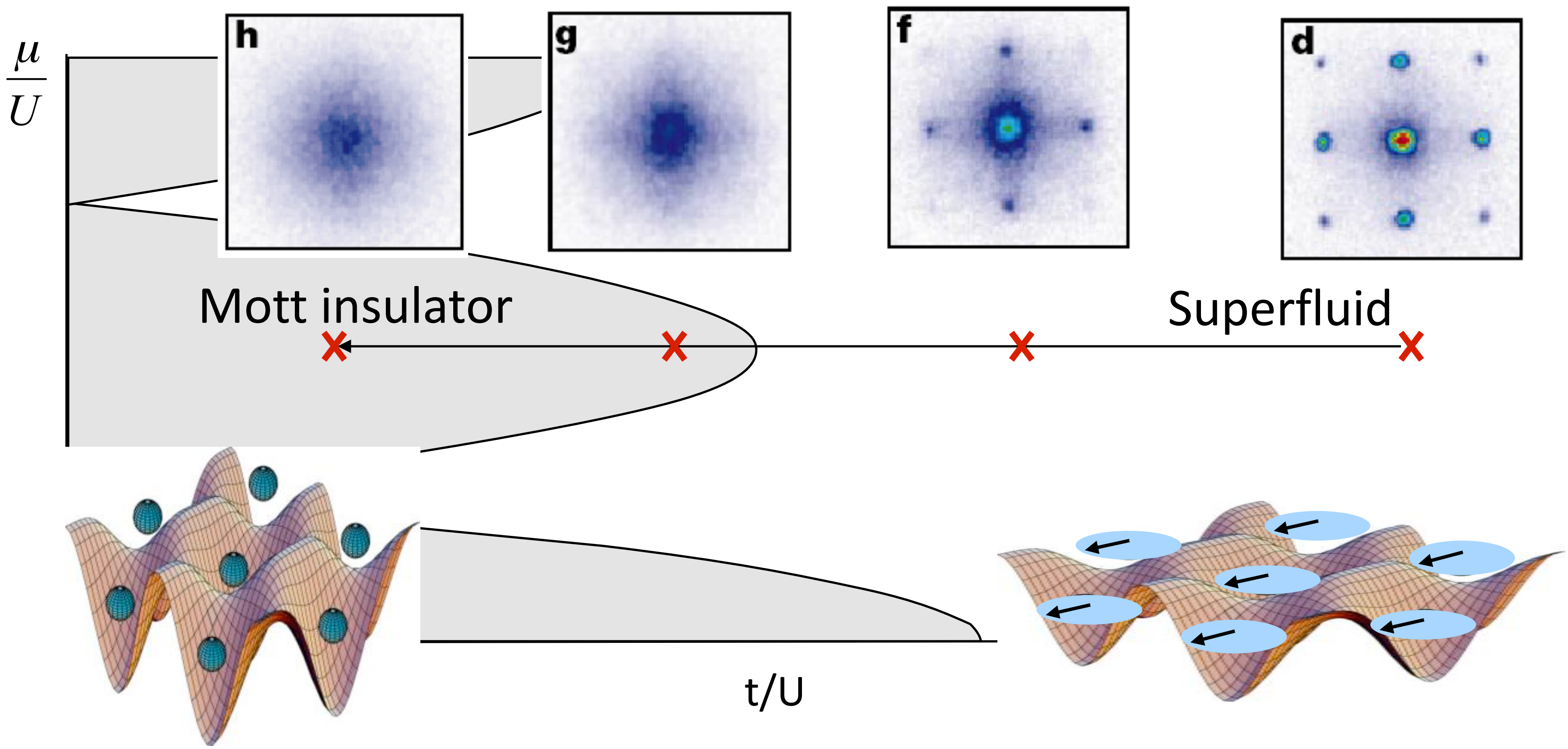
- ✧ Clean and precise implementation of model Hamiltonians

- ✧ Do not need to think about disorder or long range Coulomb interactions.

- ✧ Hamiltonian parameters can be tuned in a controlled manner. One can access everything between the weakly interacting limit to a strongly interacting limit.

Superfluid-insulator transition in Bose Hubbard Model

M. Greiner et al., Nature 415 (2002)

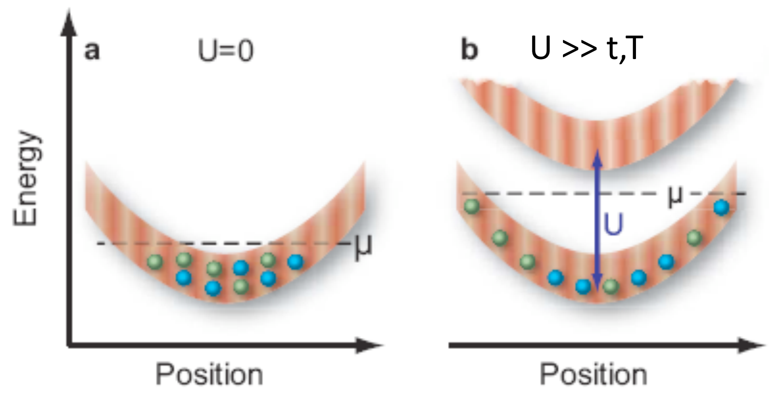


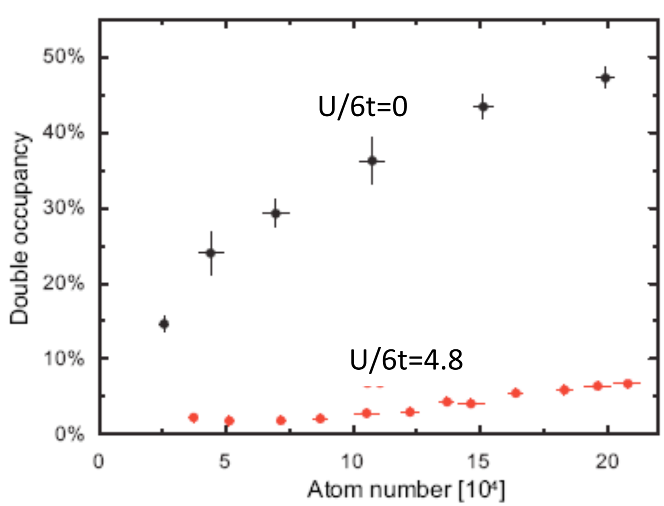
Signatures of incompressible Mott state of fermions in optical lattice

Suppression of double occupancies

*R. Jordens et al. Nature, **455**, 204(2008)*

Esslinger Group, ETH Zurich

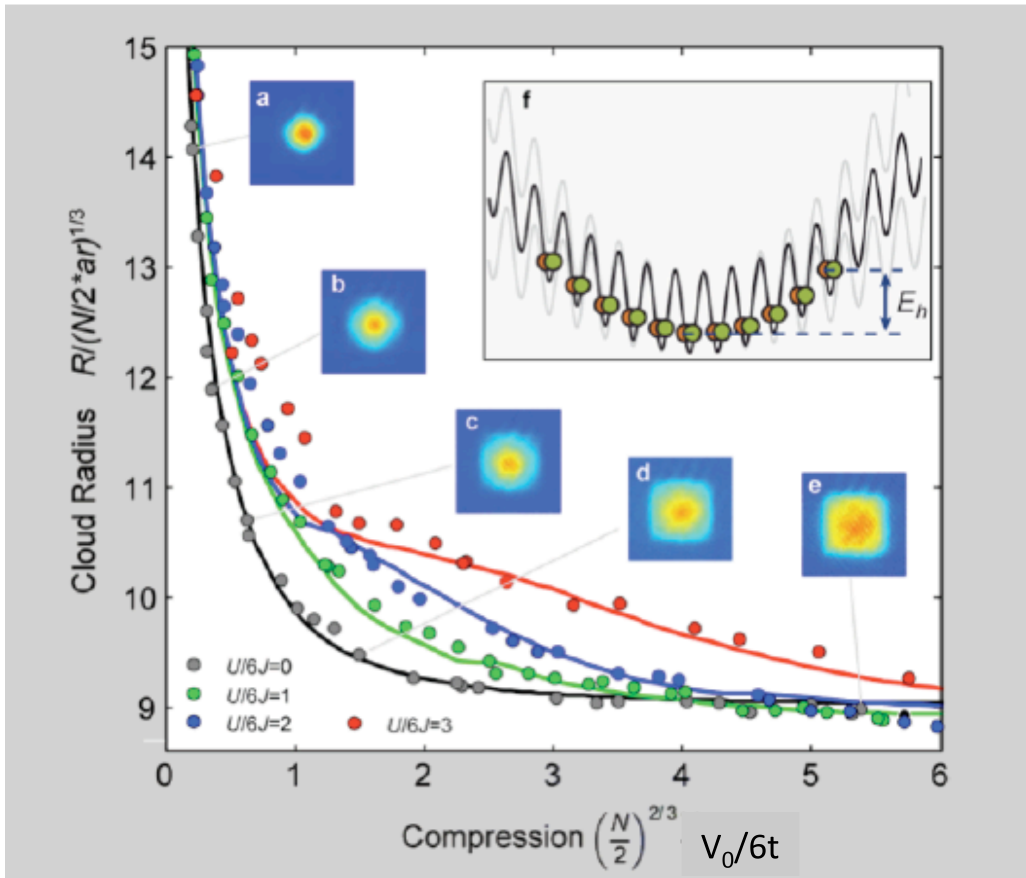




Compressibility measurements

*U. Schneider et al. Science, **322**, 1520 (2008)*

Bloch Group, Mainz/Munich



Challenges with Optical Lattice Systems

✧ Measurement of temperature in the lattice

Current Experiments :

Measure temperature without the lattice and assume Adiabatic turning on of lattice

Turn on the lattice and turn it off and look at temperature changes. Mean is taken as temperature in lattice.

✧ Interesting low temperature regimes yet to be achieved.

Hubbard Model energy scales : U, t, $J=4t^2/U$

Current temperatures $\sim t$

Need temperature $\sim J$ to see ordering phenomena like Antiferromagnetism etc.

Challenges with Optical Lattice Systems

✧ New experimental probes to measure many-body correlations

Need to develop cold-atom analogue of

- Thermodynamic probes : Magnetization, susceptibility, specific heat etc

- Spectroscopic probes : ARPES, STM, neutron scattering, optics etc

- Non-equilibrium probes : Transport, NMR etc.

Challenges with Optical Lattice Systems

✧ Attaining Equilibrium after turning on the optical lattice

- Quantum Simulators need to attain eqbm to compare with cond-mat systems.

- Ramping up the lattice is a dynamic process which drives the system out of eqbm.

- One needs to understand the relaxation of the system and corr. timescales.

- Sample lifetime is finite due to 3-body losses (typically ~ 1-10 sec)

- This places constraints on the regime of interaction parameters where quantum simulation is possible.

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✓ Optical Lattices and Quantum Simulators

✓ New Probes : Optical Lattice modulation

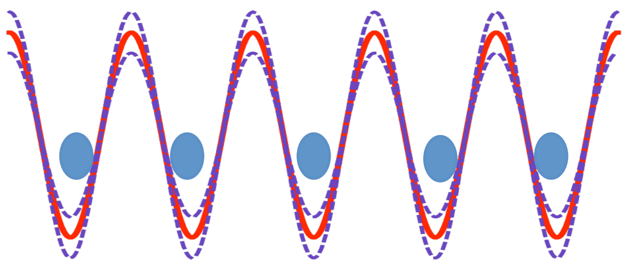
Mott Insulators, Antiferromagnets and d-wave Superfluids

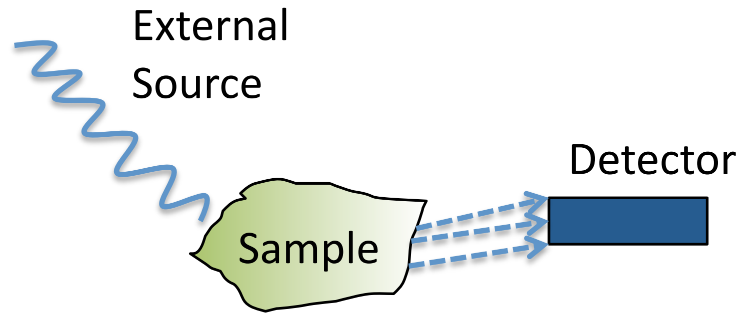
✓ Eqbm vs. Non-Eqbm. : Challenges and Opportunities

Ref : *R. Sensarma, D. Pekker, M. Lukin and E. Demler, PRL **103** 035303 (2009)*

D. Pekker, R. Sensarma and E. Demler, cond-mat/ 0906.0931

Optical Lattice Modulation





Typical Cond-Mat Expt.

Perturbation :

Tunneling : $t = \frac{2}{\sqrt{\pi}} E_R \left(\frac{V_0}{E_R}\right)^{\frac{3}{2}} \boxed{e^{-2\sqrt{\frac{V_0}{E_R}}}}$

$$V(\tau) = V_0 + \Delta V \sin \omega\tau \rightarrow t(\tau) = t + \lambda t \sin \omega\tau$$

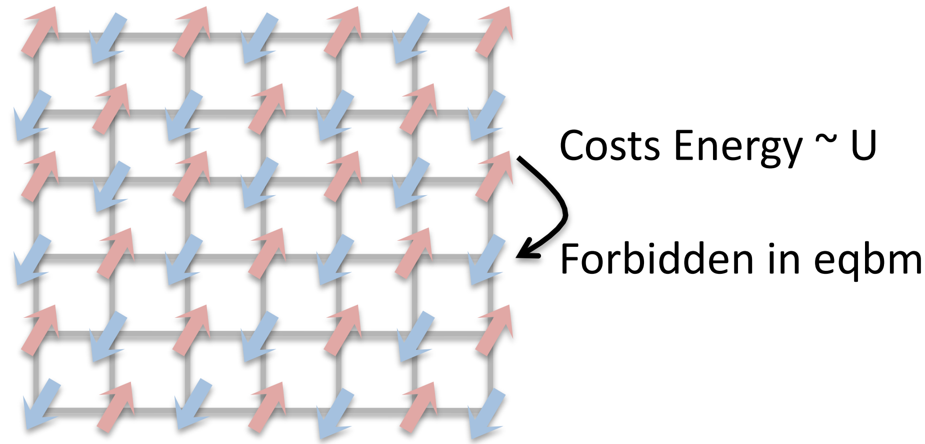
Modulation of effective mass couples energy to the system

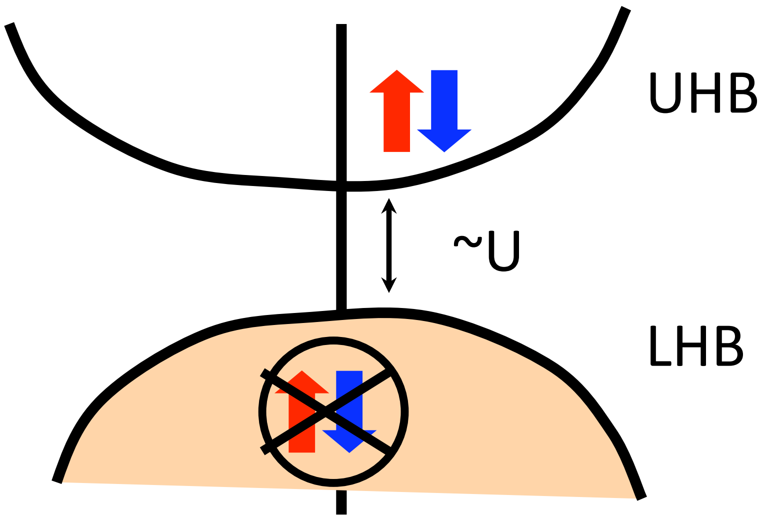
Creates excitations for any interacting system

Optical Lattice modulation can be used as a global external source

What happens in a Mott Insulator ? (Large U/t)

Fermions are localized due to strong interaction



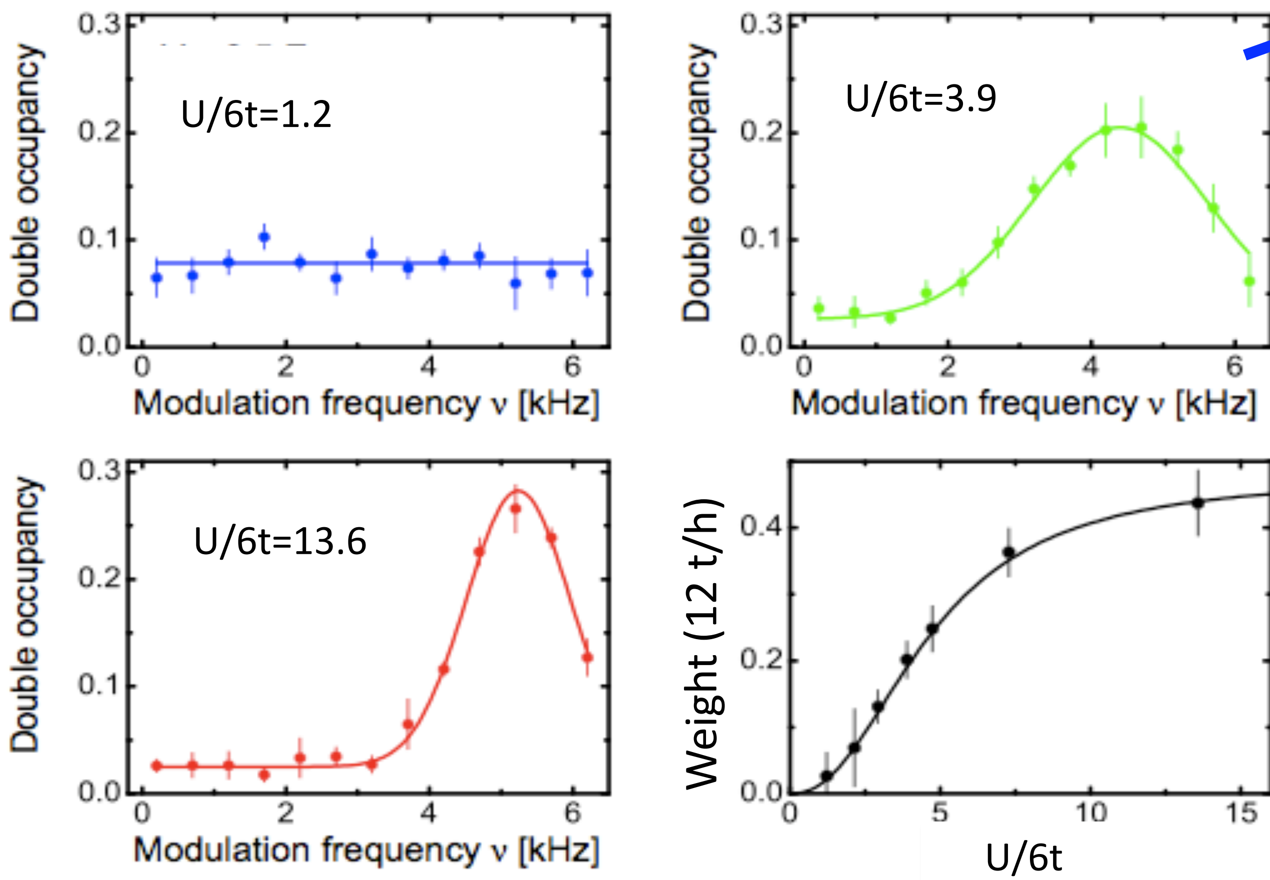


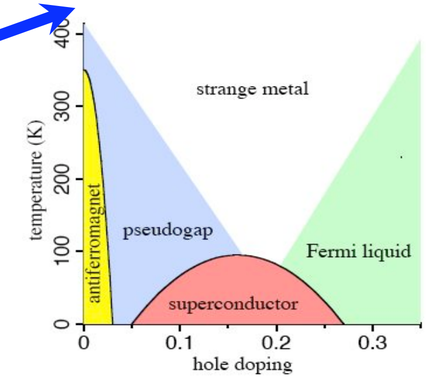
- ❑ Upper Hubbard Band has double occupancy
- ❑ Lower Hubbard Band has ~no D.O.

Lattice Modulation can create double occupancies if frequency exceeds Mott gap

Signal : Density of Double Occupancies

What really happens in a Mott Insulator ?





R. Jordens et al (Esslinger Group) Nature 455 , 204 (2008)

Parameters :

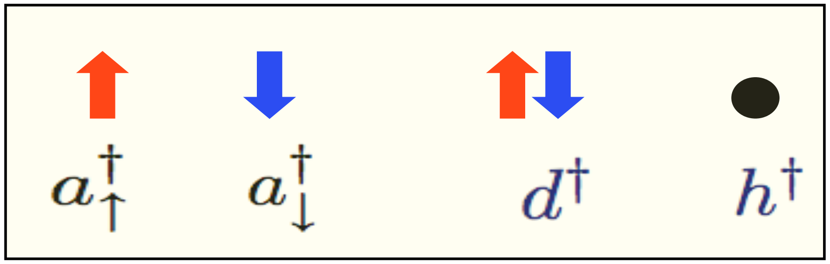
Mod Ampl. : 10 %

Temp : 0.15 T_F

Duration : 50 cycles

- What Correlation function is being measured ?
- What information about the system can be obtained from this?

Modelling the System



Bosons Fermions

Schwinger Bosons
and
Slave Fermions

$$c_{i\sigma}^{\dagger} = a_{i\sigma}^{\dagger} h_i + \sigma a_{i-\sigma} d_i^{\dagger}$$

Constraint :

$$a_{i\sigma}^{\dagger} a_{i\sigma} + d_i^{\dagger} d_i + h_i^{\dagger} h_i = 1$$

Singlet Creation $A_{ij}^{\dagger} = a_{i\uparrow}^{\dagger} a_{j\downarrow}^{\dagger} - a_{i\downarrow}^{\dagger} a_{j\uparrow}^{\dagger}$

Boson Hopping $F_{ij}^{\dagger} = a_{i\uparrow}^{\dagger} a_{j\uparrow} + a_{i\downarrow}^{\dagger} a_{j\downarrow}$

$$H_0 = t \sum_{\langle ij \rangle} \underbrace{[h_i^{\dagger} h_j + d_i^{\dagger} d_j] F_{ij}}_{\text{Hopping of doublons and holes}} + \underbrace{[d_i^{\dagger} h_j^{\dagger} A_{ij} + h.c.]}_{\text{Creation of doublon-hole pair}} + U \sum_i d_i^{\dagger} d_i$$

Response of the System

Lattice Modulation : $H_1(\tau) = t\lambda \sin[\omega\tau] \sum_{\langle ij \rangle} d_i^\dagger h_j^\dagger A_{ij} + h.c.$

Assumptions :
- T << U
- Initial system at half-filling

Other Approaches :
Huber & Ruegg PRL 09
Kollath et al PRA 06

2nd Order Perturbation Theory : Rate of Doublon production :

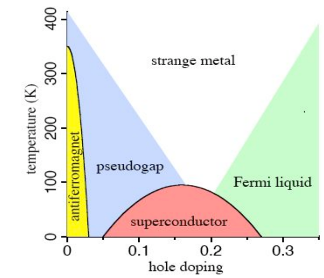
$$P_d(\omega) = \frac{\pi}{2} t^2 \lambda^2 \int d\omega_1 \int d\omega_2 \sum_{\langle ij \rangle \langle lm \rangle} \mathcal{A}^d_{il}(\omega_1) \ \mathcal{A}^h_{jm}(\omega_2) \ \mathcal{A}^s_{ijlm}(\omega - \omega_1 - \omega_2)$$

Spectral fn. of Doublon

Spectral fn. of Hole

Singlet Spectral fn.

High Temperature Limit (U >> T ~t >> J)



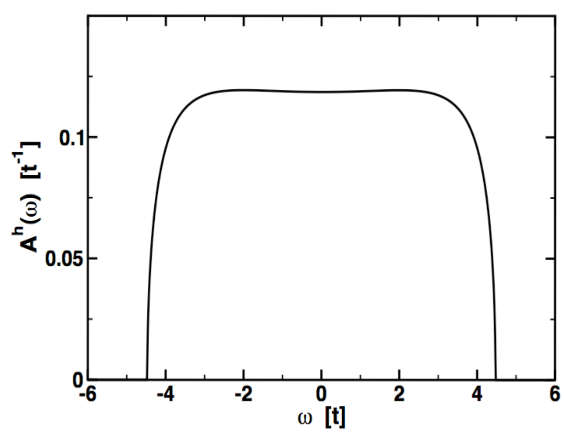
Disordered spins ---> All spin config. equally probable

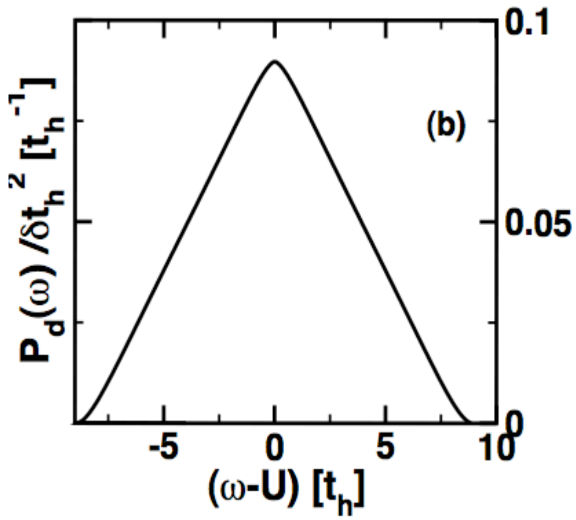
$\mathcal{A}^{s}_{ijlm}$ replaced by probability of singlet P_s

Incoherent holes : Retraceable Path Approx

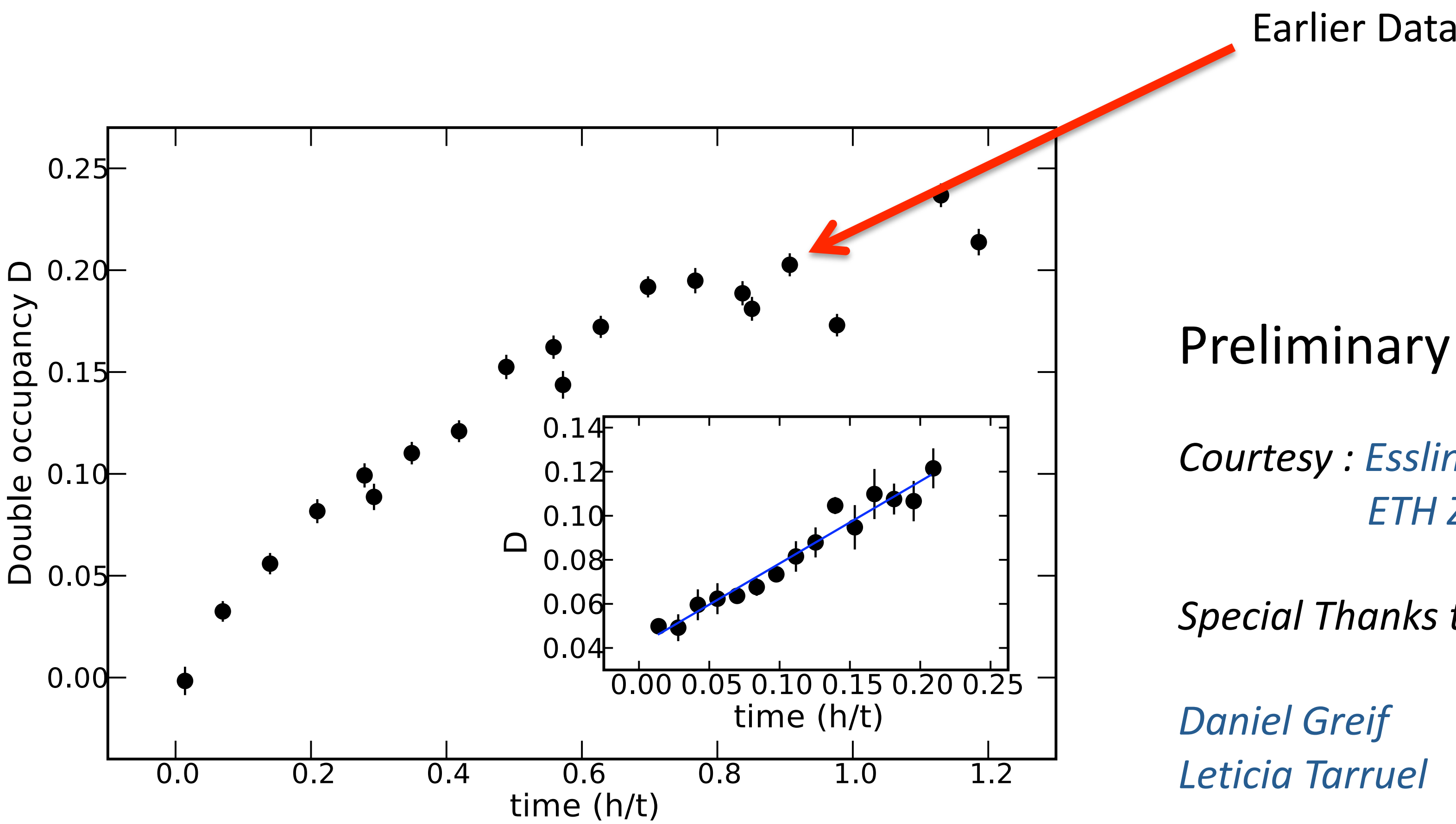
Brinkman & Rice, 1970

$$\mathcal{A}^{h}_{ii}(\omega) = \frac{1}{\pi z t} \frac{(5 - 9\omega^2/z^2t^2)^{\frac{1}{2}}}{1 - \omega^2/z^2t^2} \qquad \mathcal{A}^{d}_{ii}(\omega + U) = \mathcal{A}^{h}_{ii}(\omega)$$





Linear Regime



Preliminary Data

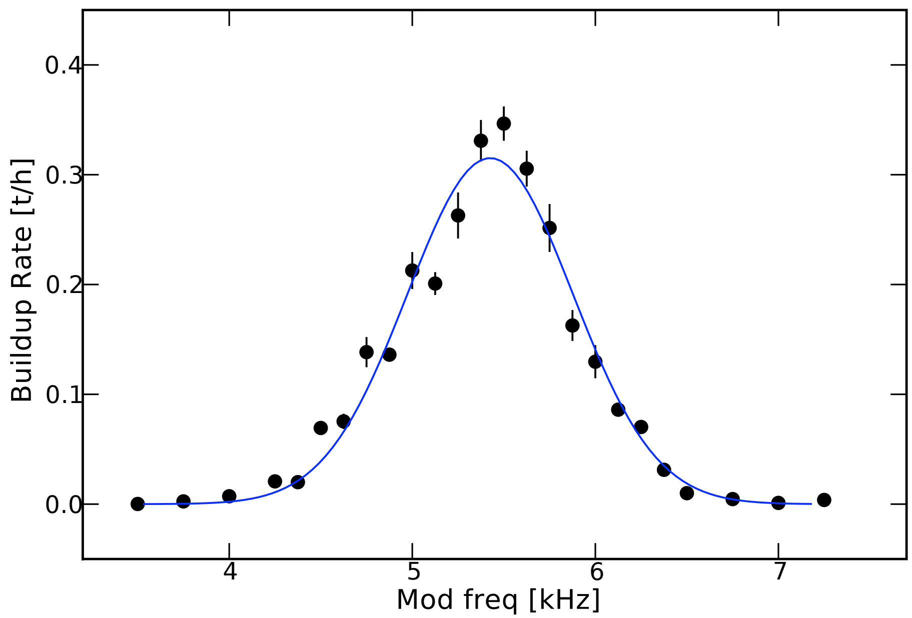
Courtesy : Esslinger Group
ETH Zurich

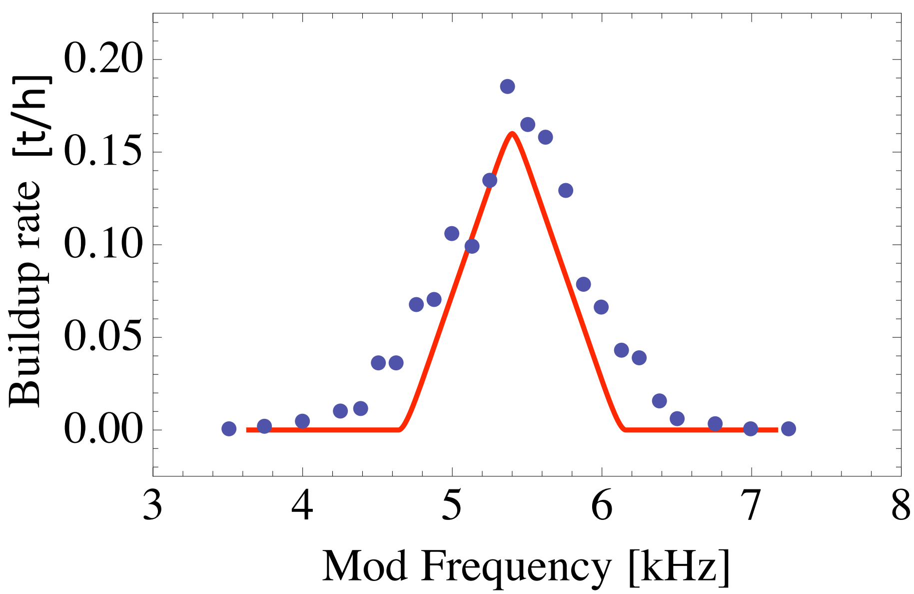
Special Thanks to :

Daniel Greif
Leticia Tarruel

Measurement in the linear regime : raw data

Doublon Production Rate





Preliminary Data

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Special Thanks to :

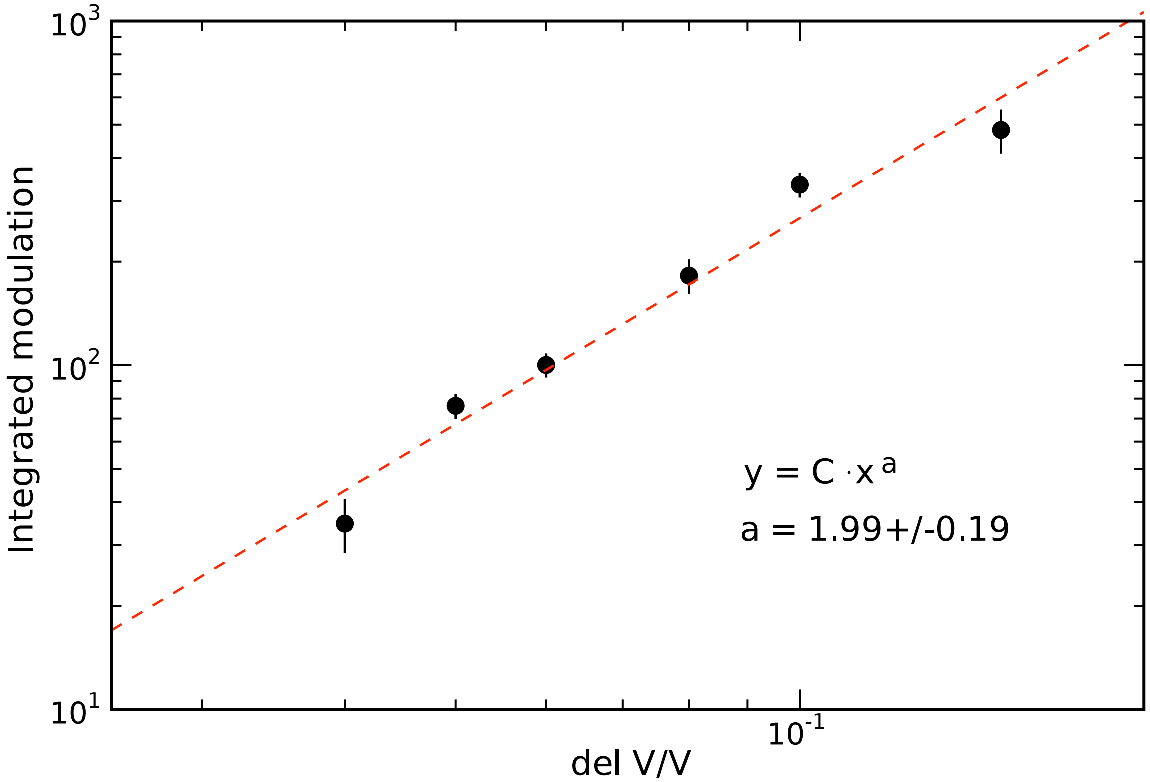
Daniel Greif
Leticia Tarruel

U= 5400 Hz

t = 85 Hz

Atom No = 80000

2^{nd} Order Response



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Special Thanks to :

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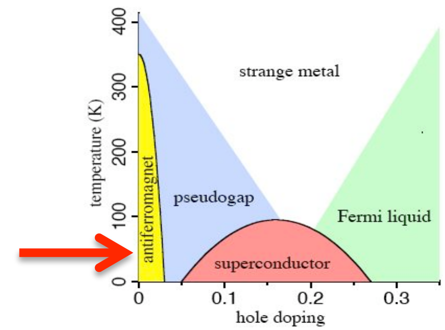
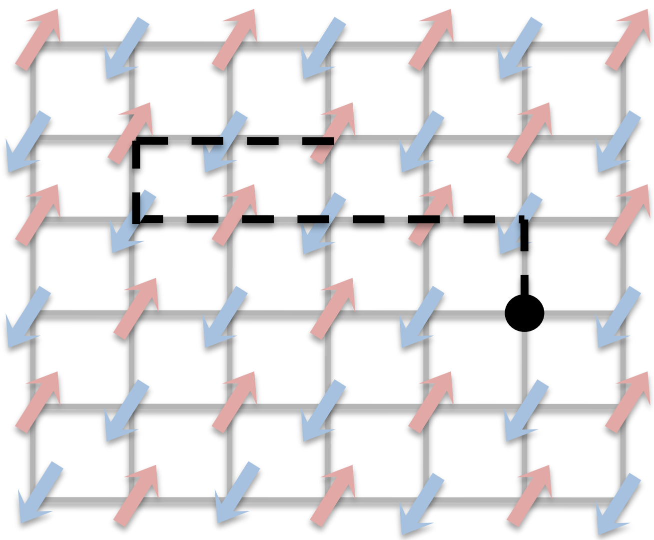
Leticia Tarruel

Low Temperature Limit (T << J << t << U)



AF Ordered state



Excitations : Spin wave Spectrum

$$\omega_k = Jz(1-\gamma_k^2)^{\frac{1}{2}}$$

$$\gamma_k = (2/z)\sum_i \cos k_i$$

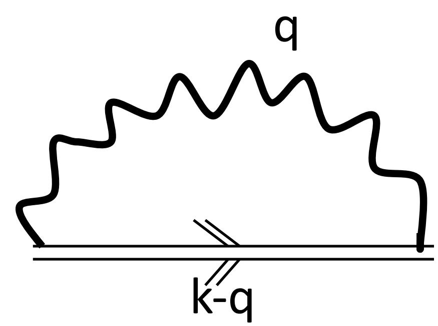
Propagation of hole is accompanied by creation of spin waves

F. Marsiglio et. al, PRB ***43****, 10882 (1991)*

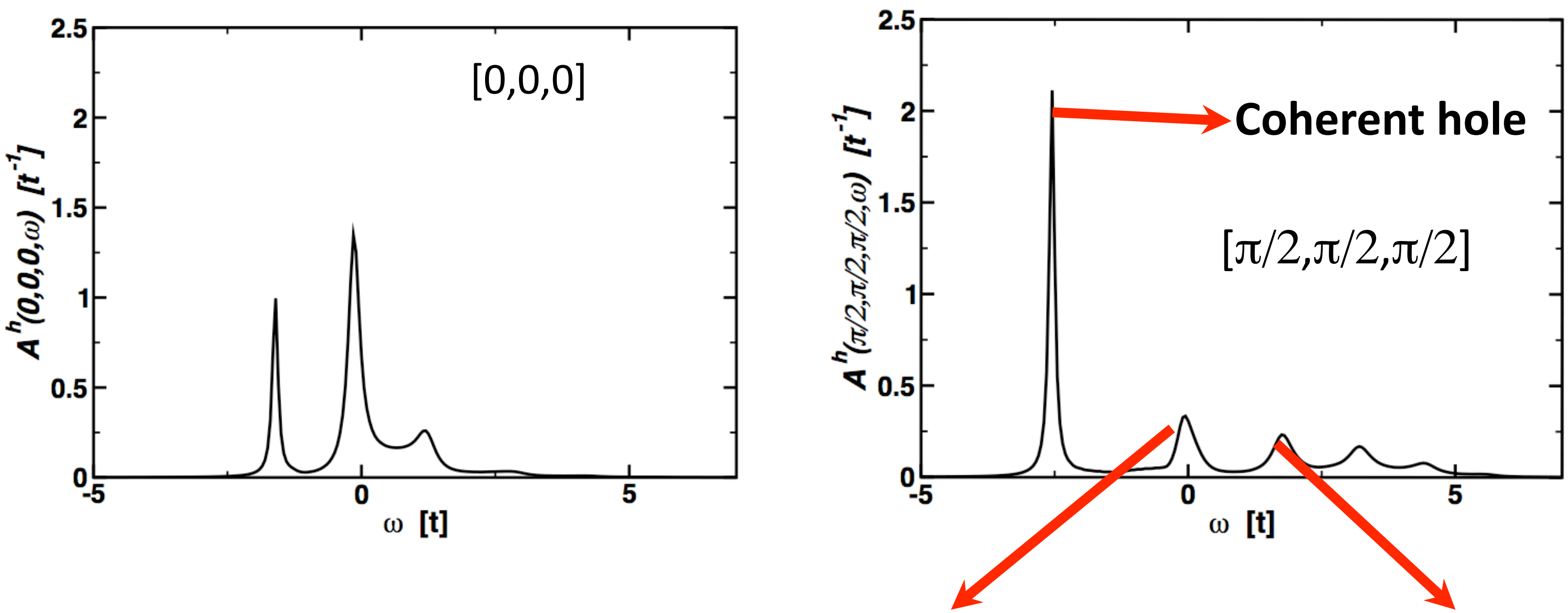
Spectral Fn. of holes calculated within Self Consistent Born Approx. at T=0

Singlet Spectral Fn. leads to vertex renormalization

$$\Sigma(k) =$$



Hole Spectral Function

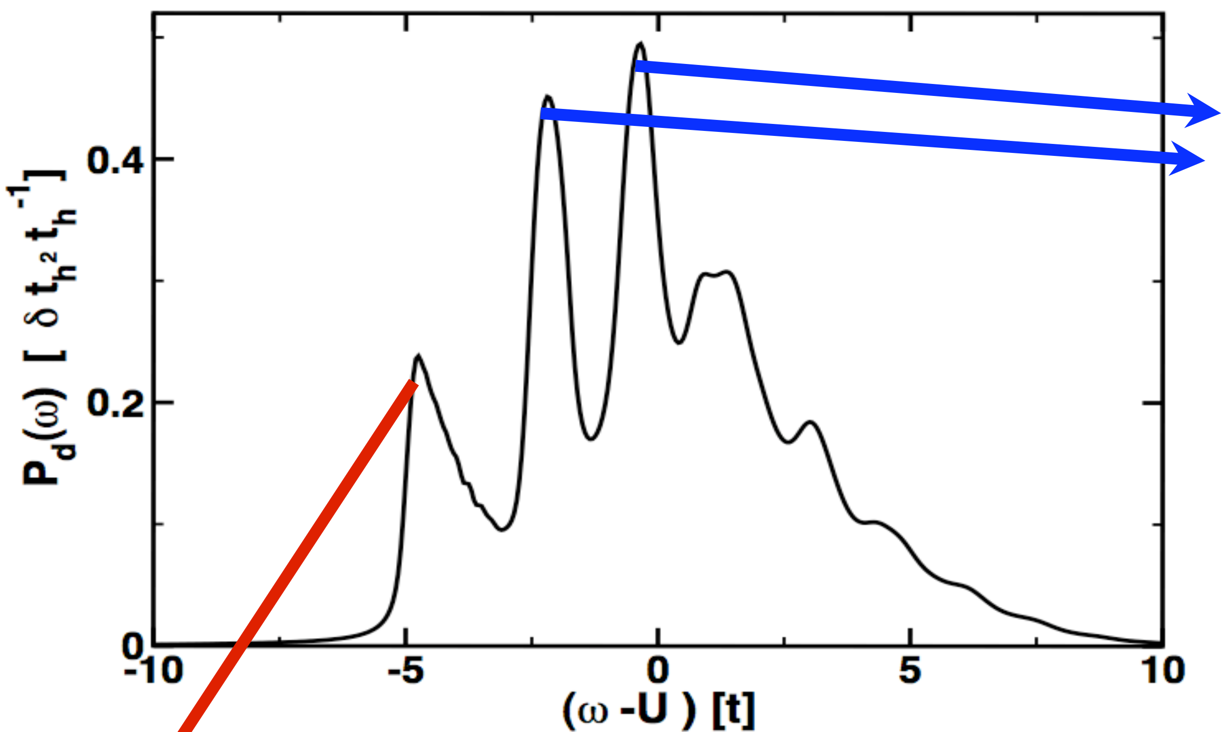


Broad Satellite peaks due to spin-wave shake off

- Coherent Dispersion : $-\epsilon_0 + J\gamma_k^2$
- Band bottom at [π/2,π/2,π/2] : Large Coherent peak
- Near [0,0,0] Sp. Fn. is incoherent

Response in AF phase

J/t=0.2



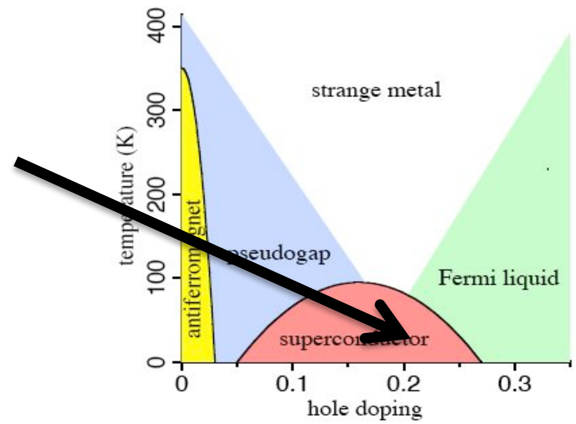
Convolution of coherent peaks and shake off peaks

Sharp edge due to coherent peaks

Signature of AF order in the sharp edge and oscillations

d-wave SF

BCS State $|\Psi\rangle = \prod_k [u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}]|0\rangle$



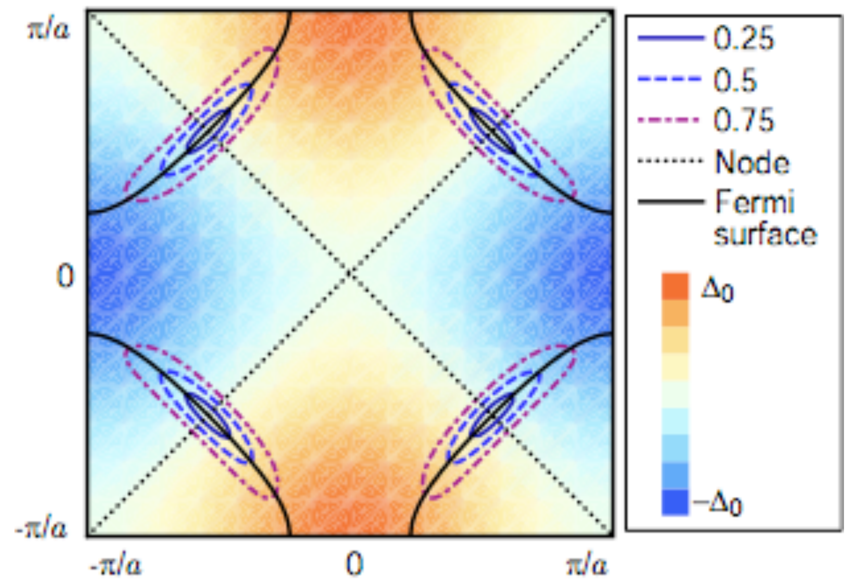
Bogoliubov QP $\Gamma^\dagger_{k\sigma} = u_k c^\dagger_{k\sigma} + \sigma v_k c_{-k-\sigma}$

BCS Excitation Spectrum

$$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$$

$$\xi_k = -2t(\cos k_x + \cos k_y) - \mu$$

$$\Delta_k = \Delta(\cos k_x - \cos k_y)$$



Perturbation :

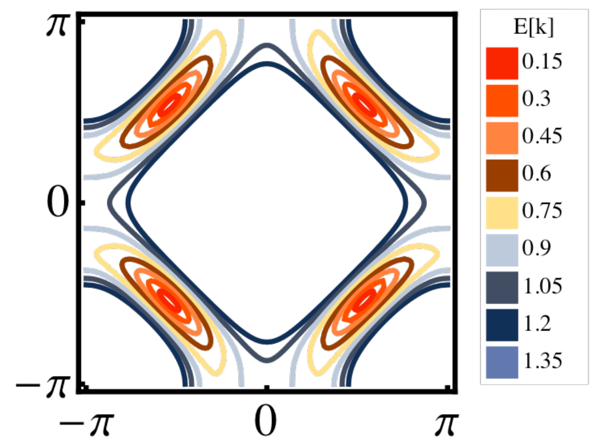
$$\begin{aligned} H_1(\tau) &= -2t\lambda \sin(\omega\tau) \sum_k \gamma_k c^\dagger_{k\sigma} c_{k\sigma} \\ &= -2t\lambda \sin(\omega\tau) \sum_k \gamma_k [(u_k^2 - v_k^2)\Gamma^\dagger_{k\sigma}\Gamma_{k\sigma} + 2u_k v_k (\Gamma^\dagger_{k\uparrow}\Gamma^\dagger_{-k\downarrow} + h.c.)] \end{aligned}$$

Creates Pair of Bogoliubov QP preferentially at $\omega = 2E_k$

Momentum Distribution

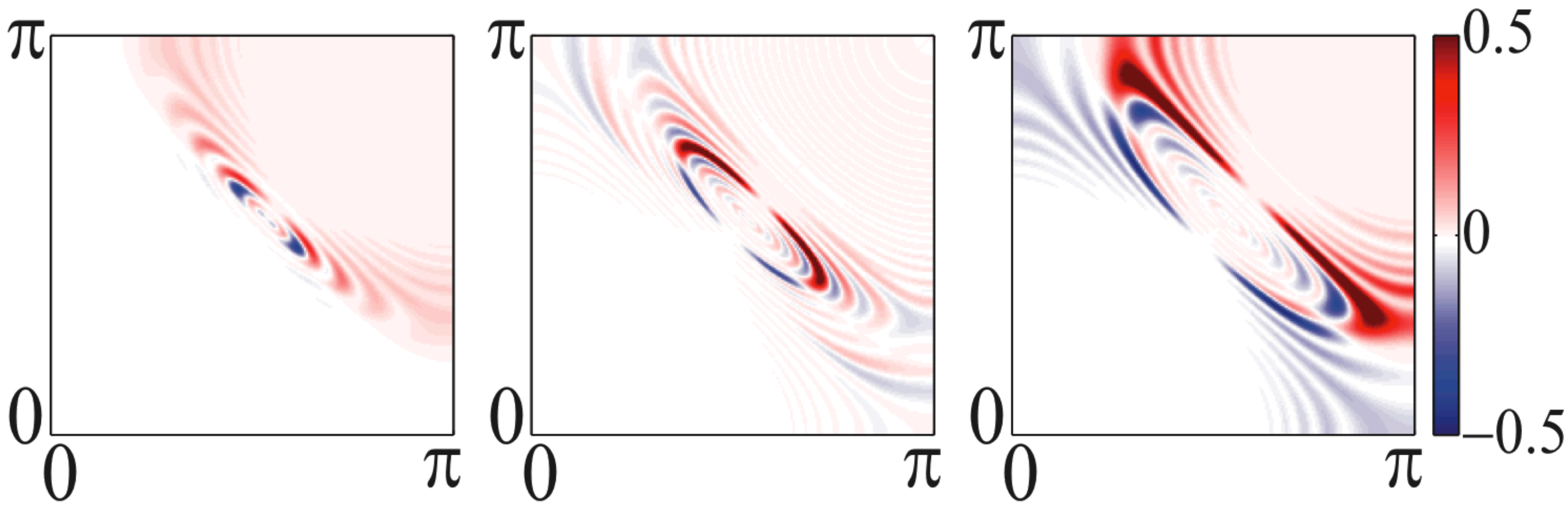
$$n_k(\tau) = 2|v_k(\tau)|^2$$

Ballistic Expansion



$\delta n(k)$ measures the density of QPs

Modulation creates pair of QP at (k,-k) with $\omega = 2\ E_k$



ω=0.5 Δ ω=1.0 Δ ω=1.5 Δ

Maps out equal energy contours

Dispersion with point nodes show k dependence of gap

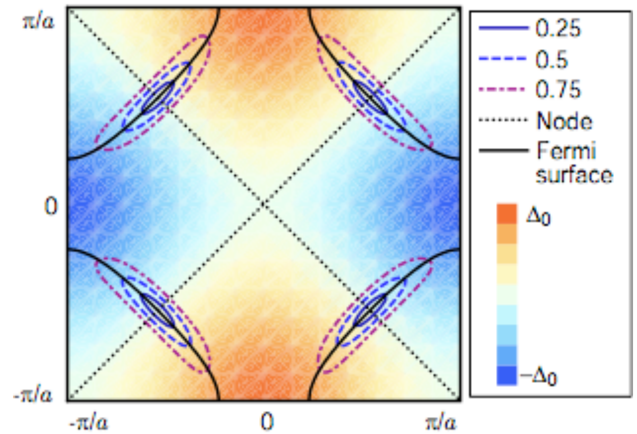
Indirect signature of d-wave

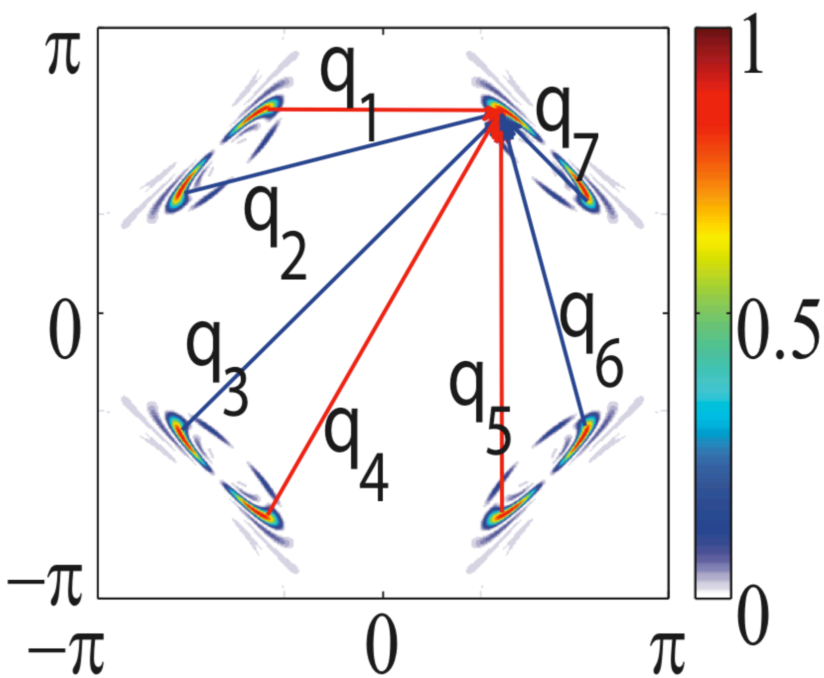
Noise Correlations

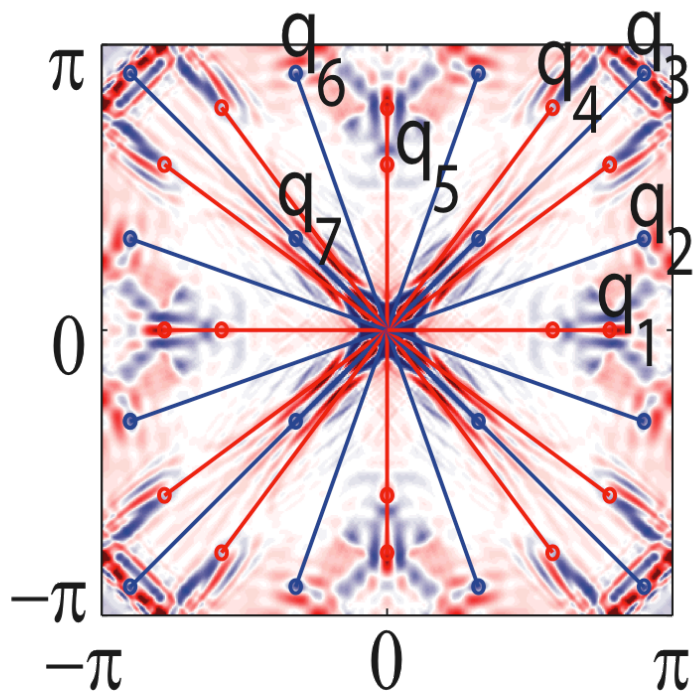
$$\rho_{q\uparrow}\rho_{q\downarrow}(\tau) = \sum_k v_k^*(\tau)u_k(\tau)u_{k+q}^*(\tau)v_{k+q}(\tau)$$

FT of real space noise correlations

measures the interference of QPs







Phase sensitive measurement shows d-wave symmetry

Outline

- ✓ Cold Atoms : A Brief Primer

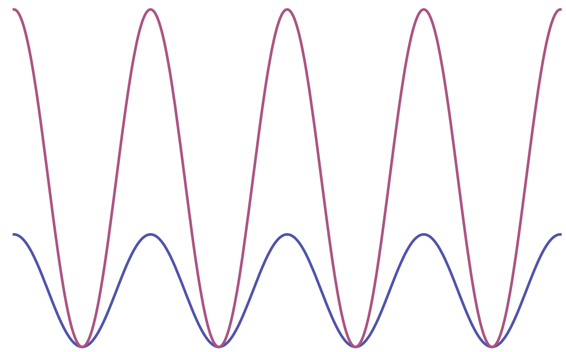
- ✓ Optical Lattices and Quantum Simulators

- ✓ New Probes : Optical Lattice modulation

- ✓ Eqbm vs. Non-Eqbm. : Challenges and Opportunities

Ref : *N. Strohmaier et. al cond-mat/ 0905.2963*

Non-Equilibrium dynamics with Cold Atoms



- Lattice Depth ramped up with finite rate
- Optical Modulation pumps energy into system

  Drives system out of equilibrium

  How does the system relax ?

Advantages of Cold Atom systems

System almost completely decoupled from environment (Intrinsic dynamics)

Easy to drive system out of Eqbm. Characterization of Initial States

Low energy scales imply large time scales

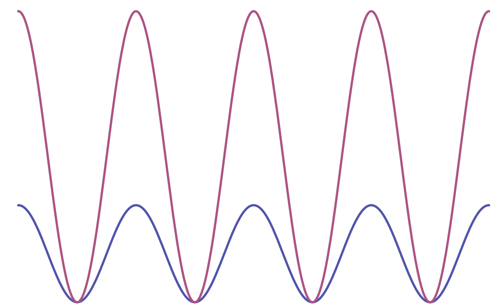
Can follow the system in real time without ultrafast techniques

Decay of Doublons

Hubbard Model $$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Low Lattice Depth → Low U/t → Large Doublon density

High Lattice Depth → High U/t → Low Doublon density

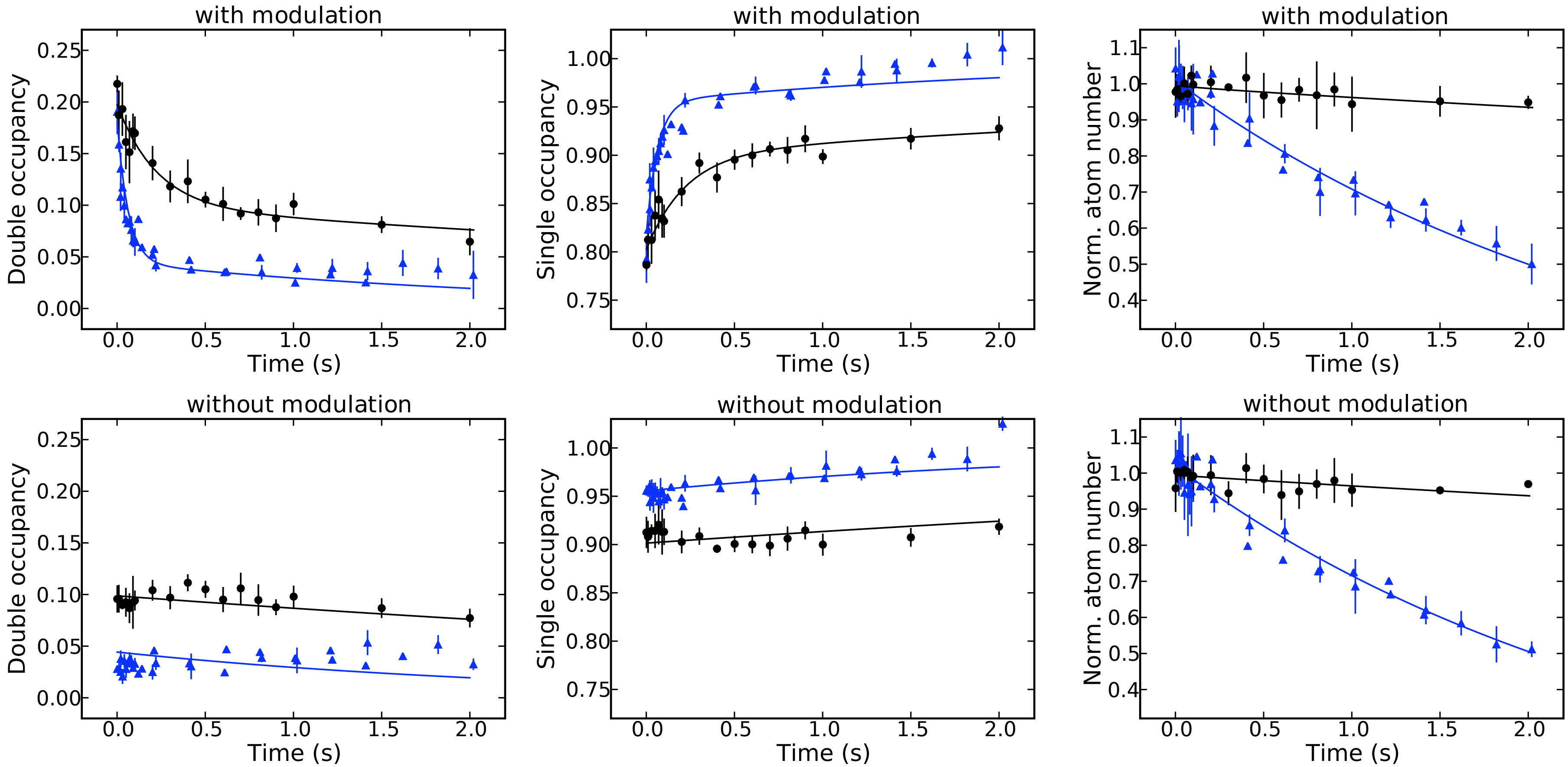


Doublons need to decay to attain equilibrium at large U/t

Doublons are high energy excitations ~ U → Slow Decay

Limiting timescale for equilibriation

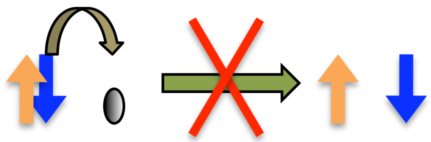
Experimental Observation of Doublon Decay



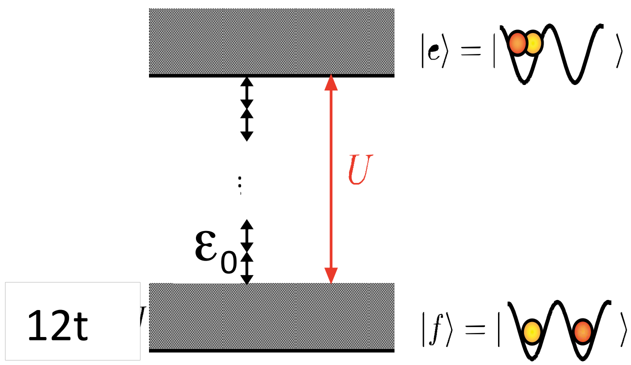
$$\Delta\dot{N}_d = -[\Gamma_D + \Gamma_{in} + \Gamma_{loss}]\Delta N_d \qquad \dot{N}_d = -[\Gamma_{in} + \Gamma_{loss}]N_d$$

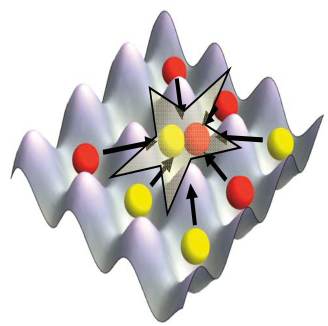
$$\dot{N}_s = \Gamma_D \Delta N_d - \Gamma_{loss} N_s$$

Scaling Argument for Doublon Lifetime



Forbidden by energy Conservation





Typical Energy of excitations ε_0

Doublon needs to create

n = U/ε_0 excitations to decay

Matrix Element for decay process in n^{th} order Perturbation Theory

$$M = t\frac{t}{\epsilon_0}\frac{t}{2\epsilon_0}\frac{t}{3\epsilon_0}\cdots\frac{t}{(n-1)\epsilon_0}\frac{t}{n\epsilon_0} \sim t\left(\frac{t}{U}\right)^{\frac{U}{\epsilon_0}}$$

Decay Rate

$$\Gamma \sim M^2 \sim C\exp[-\alpha(U/\epsilon_0)\ln(U/t)]$$

Scaling Argument for Doublon Lifetime

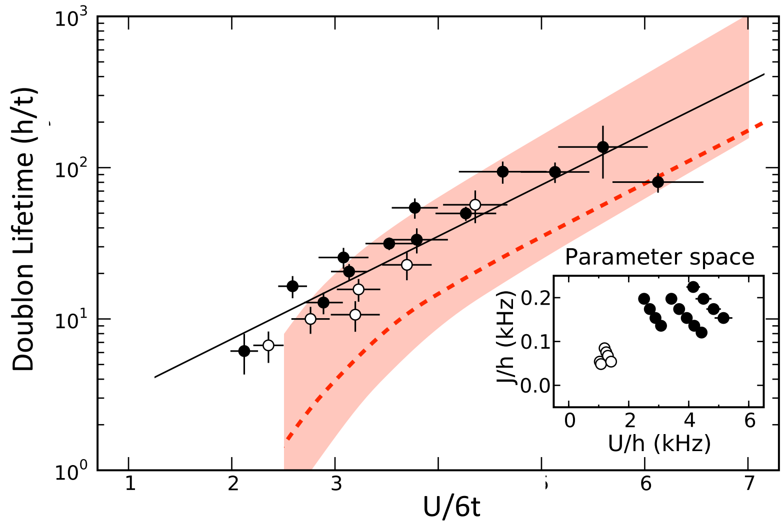
Background State : Mott Insulator Spin Excitations with $\epsilon_0 \sim J$

$$\Gamma_d \sim Ct \exp[-\alpha U^2/t^2 \ln(U/t)]$$

Background State : Compressible State with Holes (Relevant for Experiments)

$\epsilon_0 \sim t$

$$\Gamma_d \sim Ct \exp[-\alpha U/t \ln(U/t)]$$



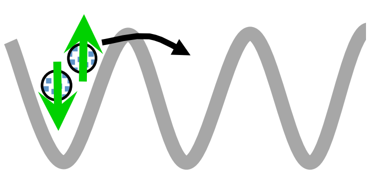
Doublon decay in a compressible state

Background of projected Fermions $\mathcal{H}_t = -t \sum_{\langle ij \rangle \sigma} P\, c_{i\sigma}^\dagger\, c_{j\sigma}\, P$

Projection induces interaction between Fermions $= -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} - c_{i\sigma}^\dagger c_{j\sigma} n_{j\overline{\sigma}} - n_{i\overline{\sigma}} c_{i\sigma}^\dagger c_{j\sigma}$

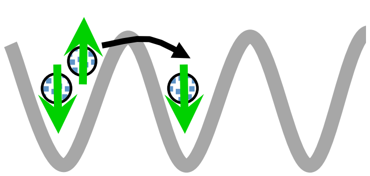
Doublons : $\mathcal{H}_d = U \sum_i d_i^\dagger\, d_i$

Doublon-Fermion Interaction



$$\mathcal{H}_{\mathrm{int1}} = -t \sum_{\langle ij \rangle \sigma} d_i\, c_{i\sigma}^\dagger\, c_{j-\sigma}^\dagger$$

Decay



$$\mathcal{H}_{\mathrm{int2}} = -t \sum_{\langle ij \rangle \sigma} d_i^\dagger\, c_{j\sigma}^\dagger\, d_j\, c_{i\sigma}$$

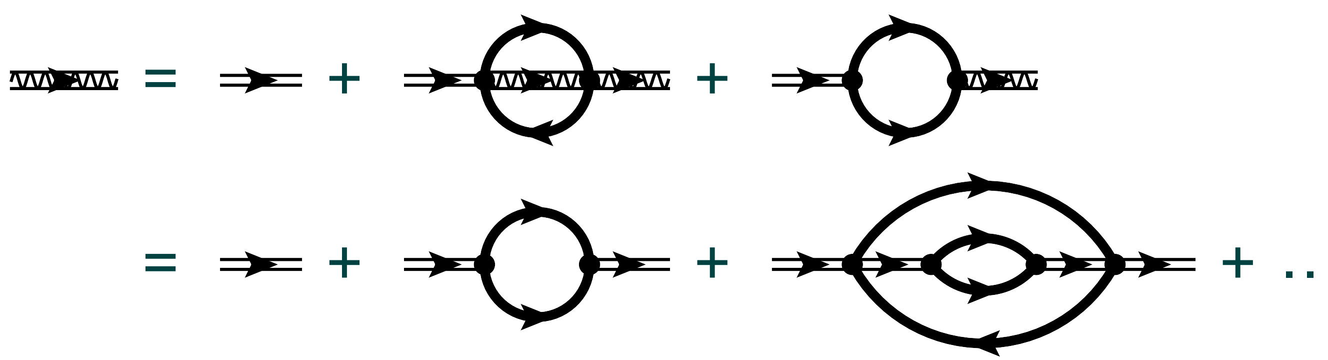
Scattering

Calculation of Doublon Lifetime

Doublon Green Function
$$G_d(\omega) = \frac{1}{\omega - U - \Sigma_d(\omega)}$$

Doublon Decay Rate
$$\Gamma_d = Im\ \Sigma_d(U)$$

Doublon Self Energy in Non-Crossing Approximation



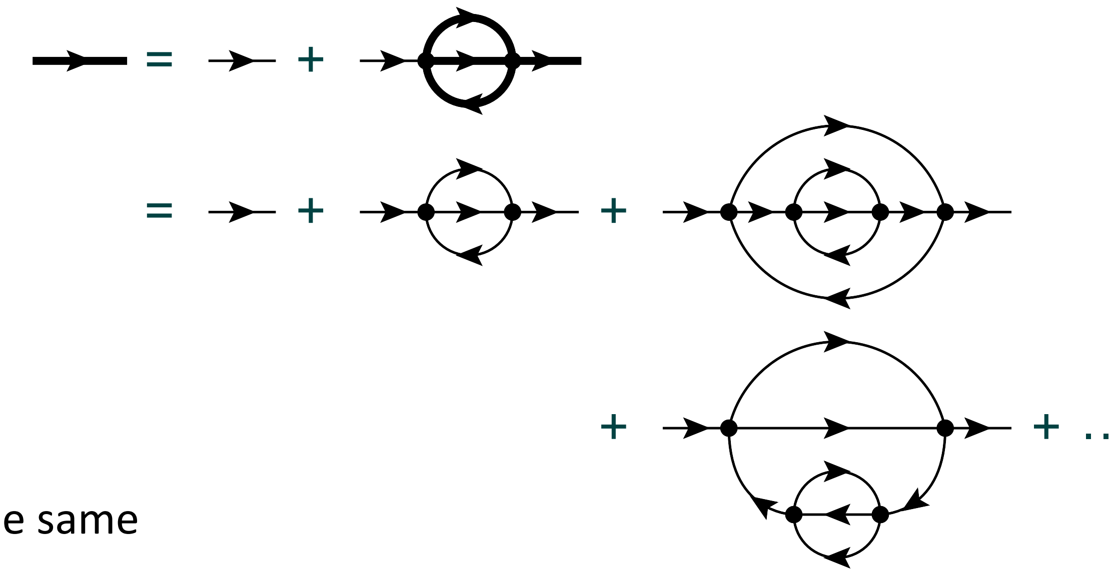
Calculation of Doublon Lifetime

Fermion Green Function $$G_f(k,\omega) = \frac{1}{\omega - \epsilon_k + \mu - \Sigma_f(k,\omega)}$$

1) Non-interacting Fermions

2) Non-Crossing Approx

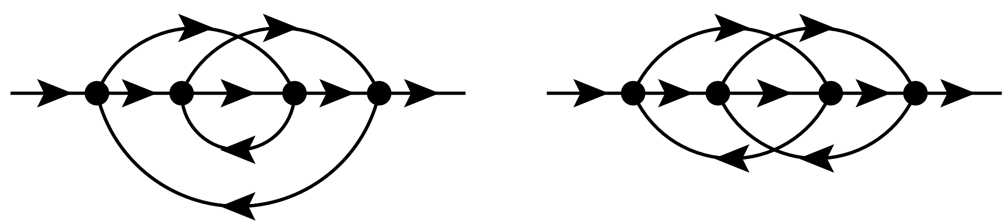
3) Modified Crossing Approx



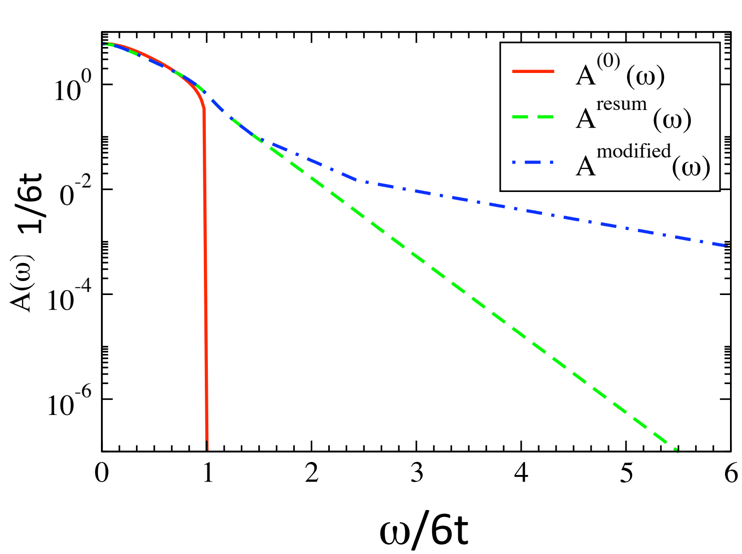
Crossing Diagrams contribute the same as non crossing ones

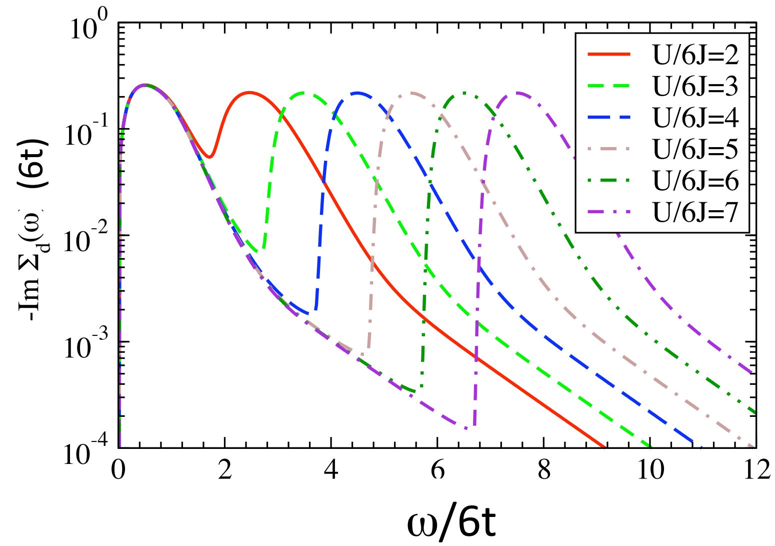
Disregard sign of diagrams

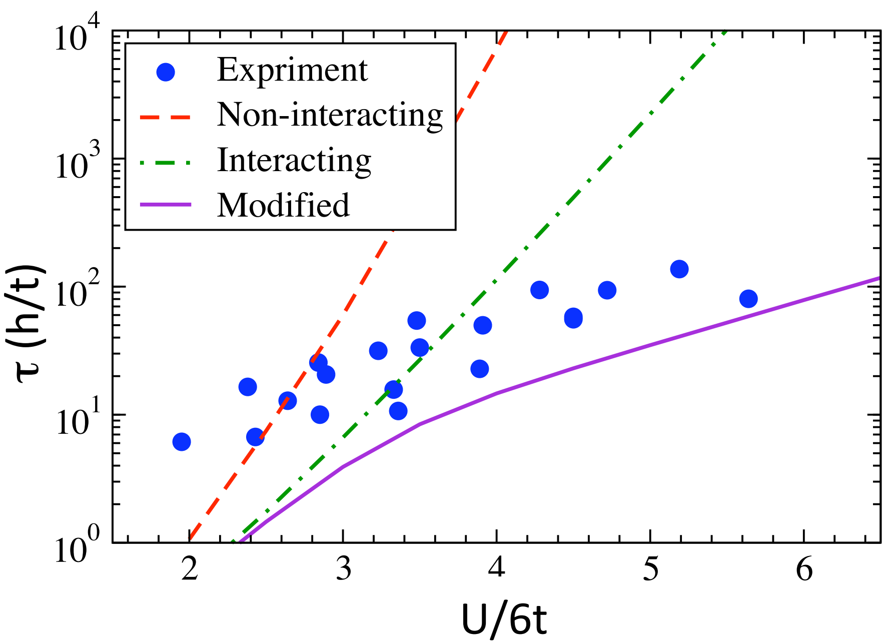
Over-estimate of decay rates



Doublon decay in a compressible state







Two Channels for Decay :

1) Create a large number of low energy p-h pairs

2) Create a high energy p-h pair which decays into a shower of low energy excitations

Conclusions

- Ultracold atoms on optical lattices present a new system to study lattice models relevant to cond-mat. Systems.

- Optical lattice modulation can be used as a spectroscopic tool to probe these systems.

- One can use optical lattice modulation to probe interesting phases like Antiferromagnets and d-wave superfluids

- Non-equilibrium dynamics of these systems are now experimentally accessible. This opens up new opportunities.

- Slow decay of high energy excitations (doublons) sets equilibriation timescales for these systems

Ref : *R. Sensarma, D. Pekker, M. Lukin and E. Demler, PRL **103** 035303 (2009)*
D. Pekker, R. Sensarma and E. Demler, cond-mat/ 0906.0931
N. Strohmaier et al. cond-mat/0905.2963