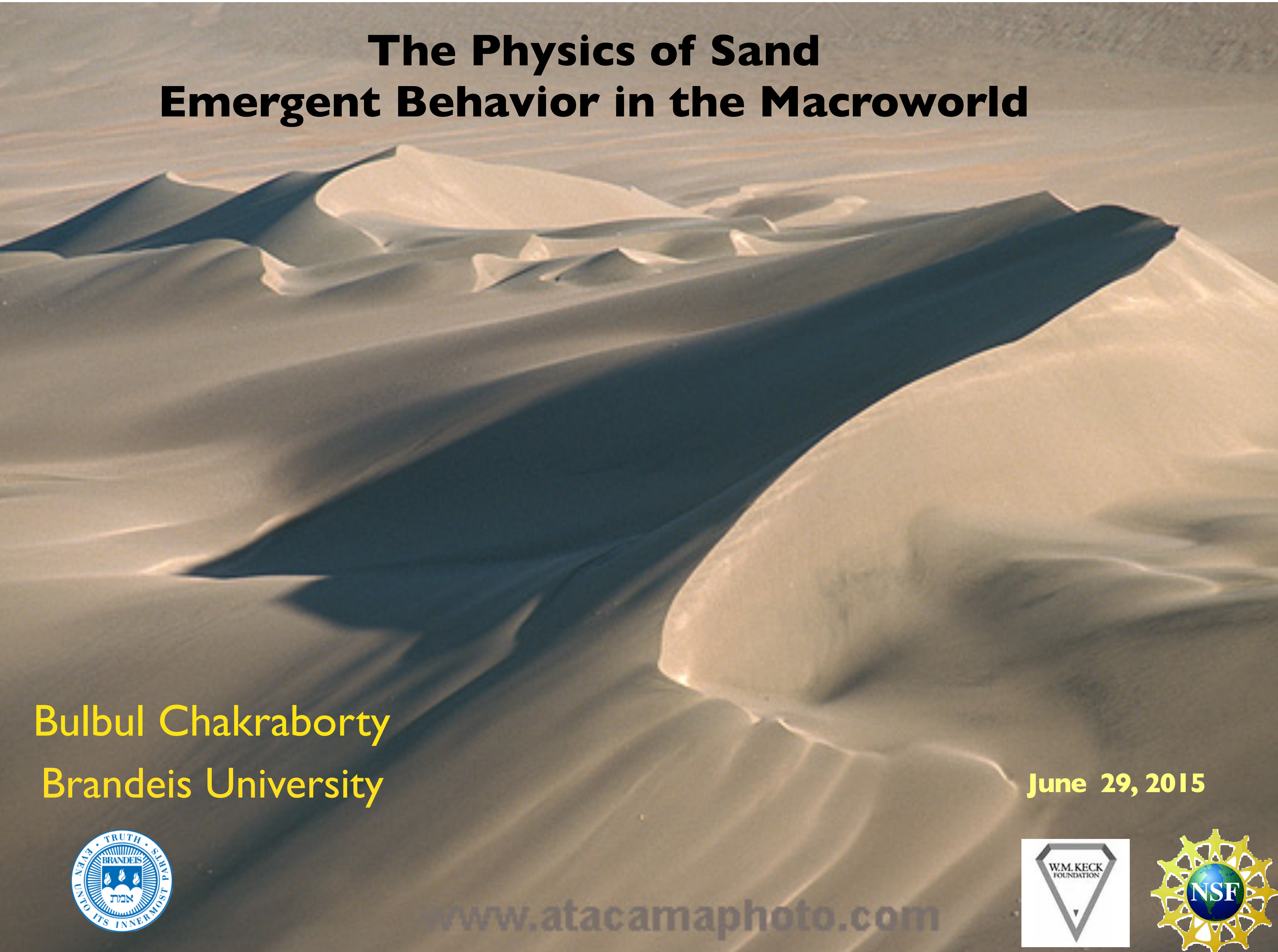
The Physics of Sand
Emergent Behavior in the Macroworld



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June 29, 2015

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Bob Behringer, Jie Zhang,
Jie Ren,
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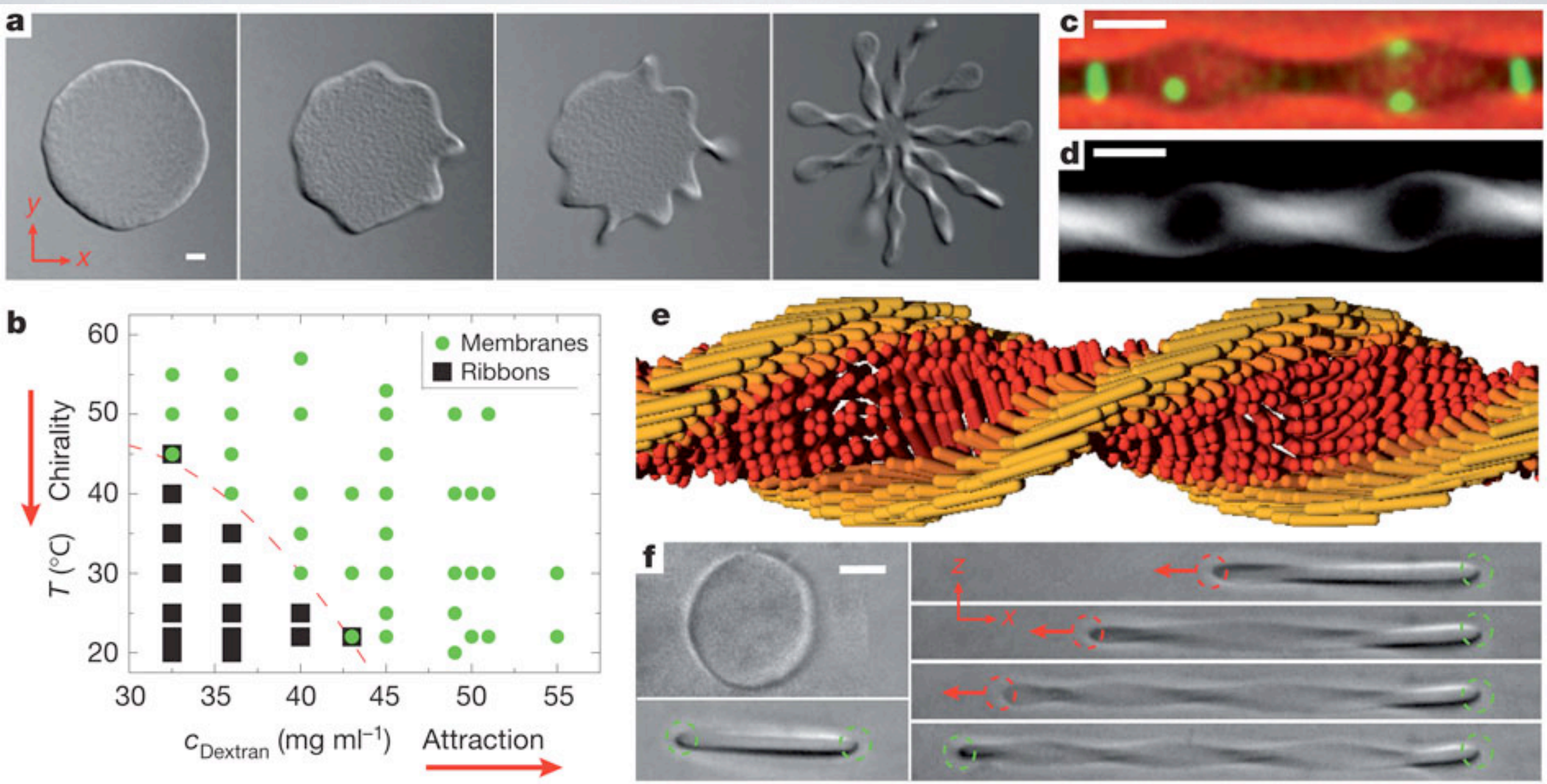
Kabir Ramola

COLLOIDAL WORLD

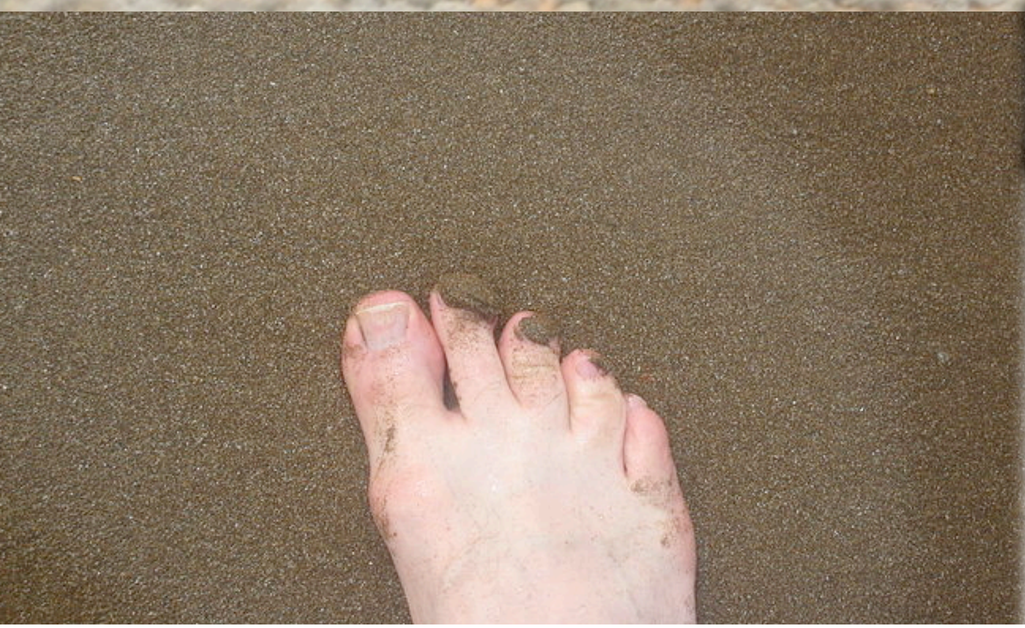


Z. Dogic, Brandeis U.

GRANULAR WORLD

- No thermal fluctuations
- Purely repulsive, contact interactions, friction
- States controlled by driving at the boundaries
- Non-ergodic in the extreme sense: stays in one configuration unless driven

Shear: a dominant driving force in the athermal world



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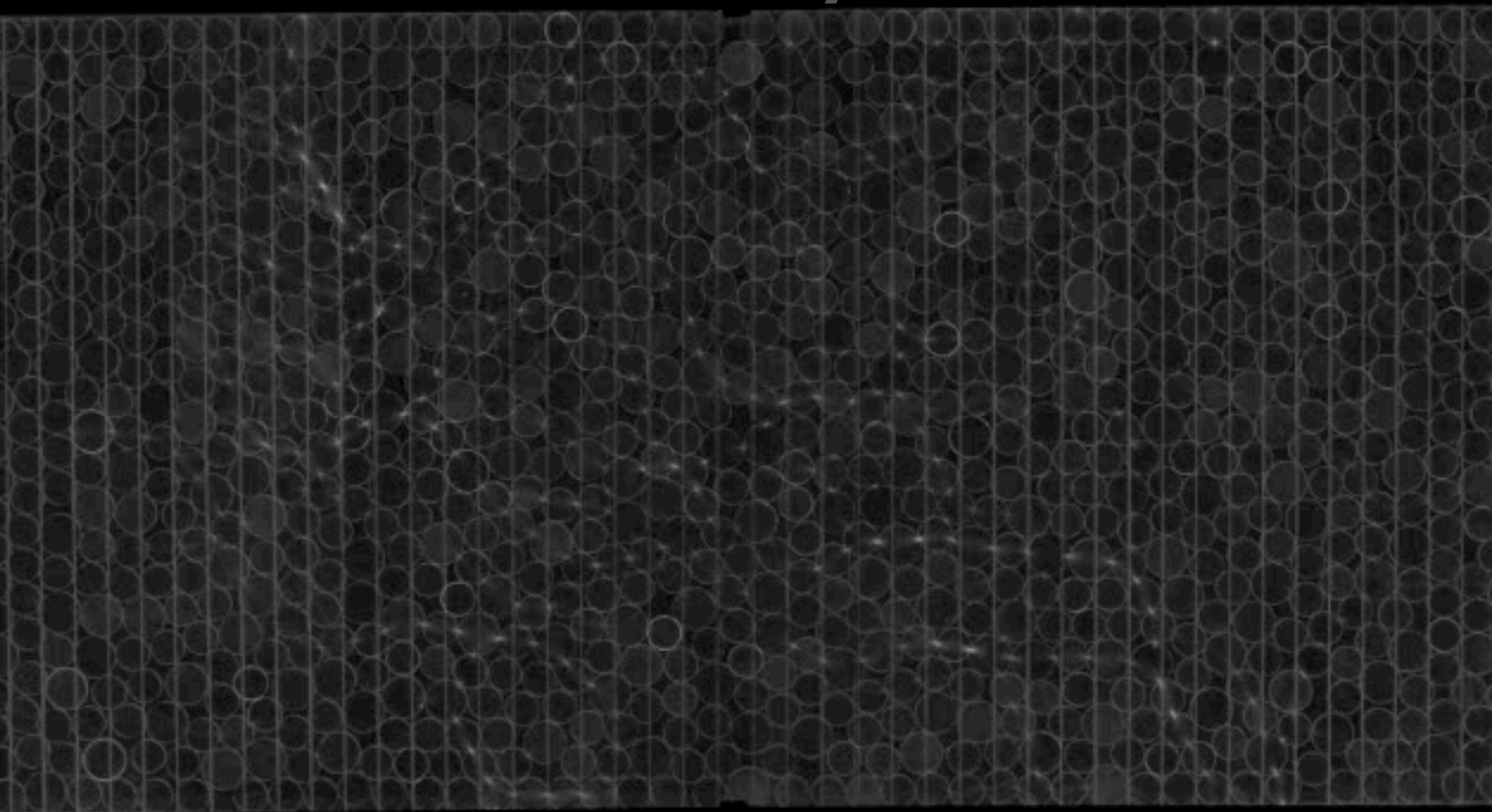
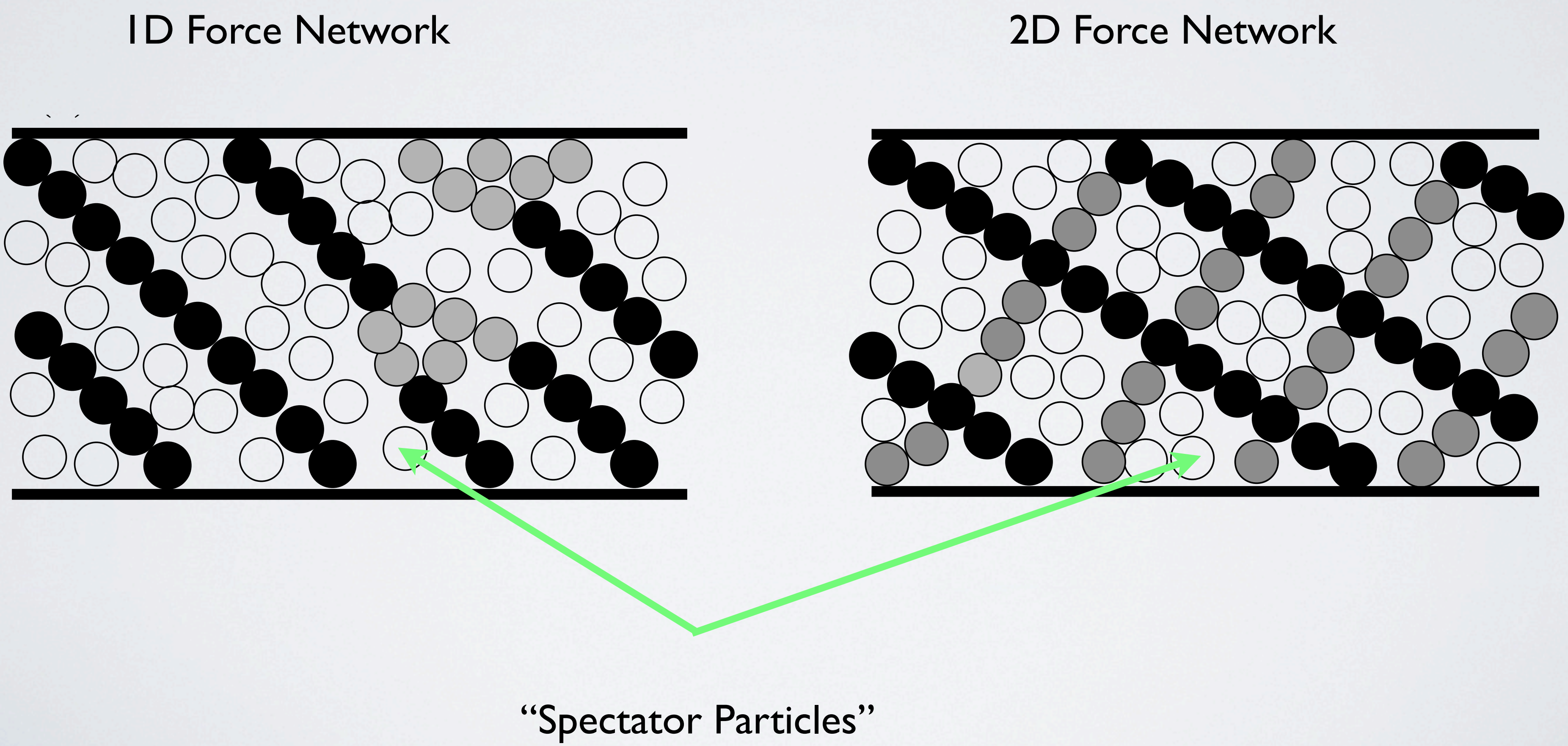
Shear-induced Solidification in a Model Granular System

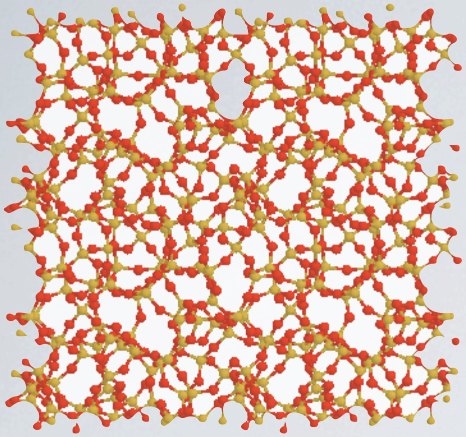


Shear-Jamming

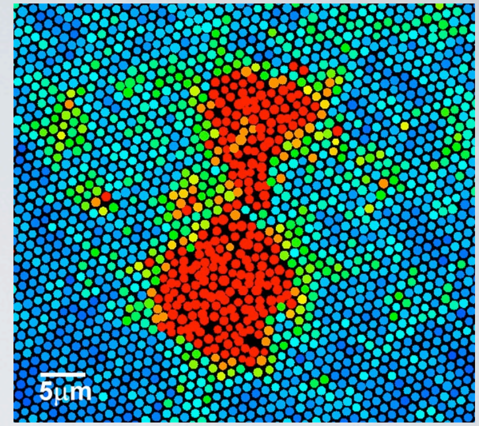
The governing principle is that force chains emerge to support the imposed external stress. Question is whether they are rigid in the sense that they can support additional shear stress, even small.

Cates et al: Phys Rev. Lett 81, 1841 (1998)





Rigidity of amorphous solids



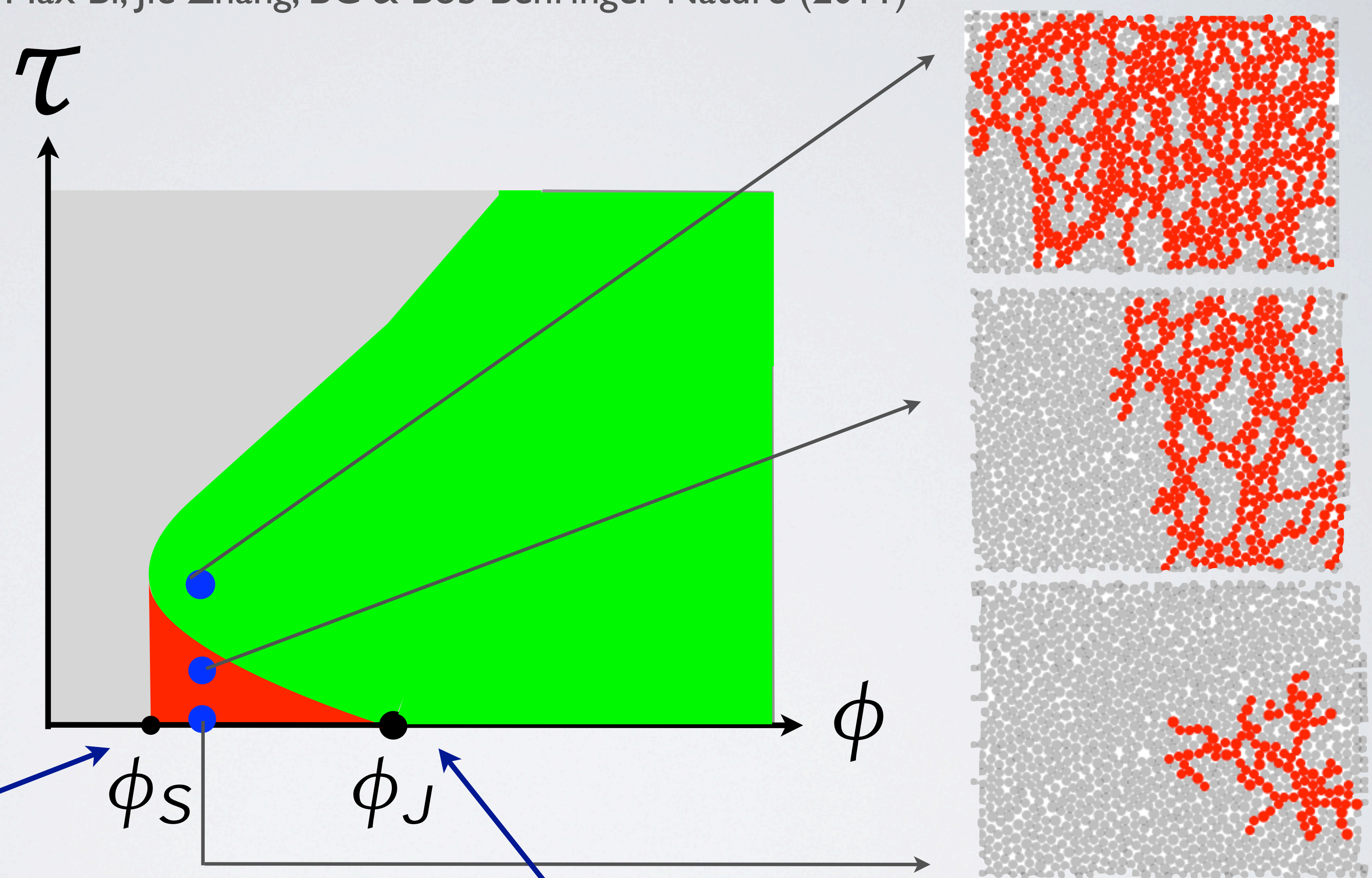
- Fundamental Principle underlying Condensed Matter Physics: rigidity is associated with broken symmetry
- Amorphous solids: Broken translational symmetry. No obvious order parameter but patterns of particles not destroyed by small thermal fluctuations **Overlap of configurations is a commonly used measure**.
- Traditionally: energy or entropy gain leads to solidification
- Dry grains: no cohesive interactions and no thermal fluctuations.

Broken translational symmetry in position space is a necessary but not a sufficient condition for rigidity

Rigidity also requires broken translational symmetry in a space of forces

Shear-Jamming Experiments (Quasistatic Forward + Cyclic Shear)

Max Bi, Jie Zhang, BC & Bob Behringer Nature (2011)

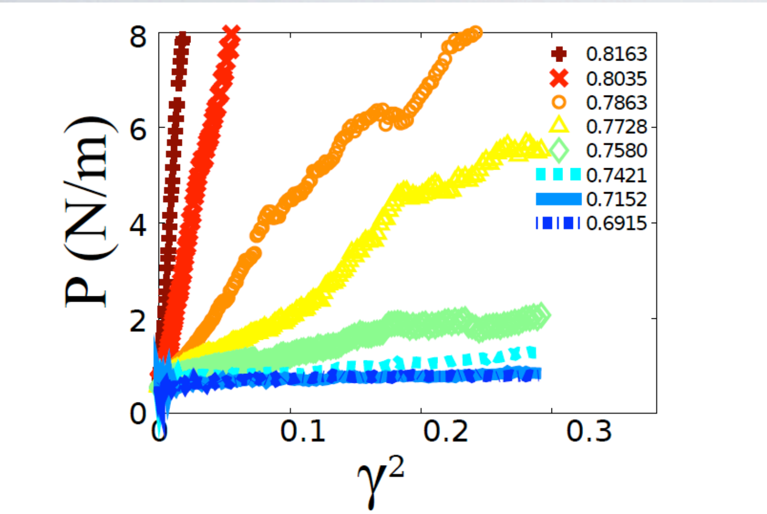


Onset density for broken translational symmetry in position space

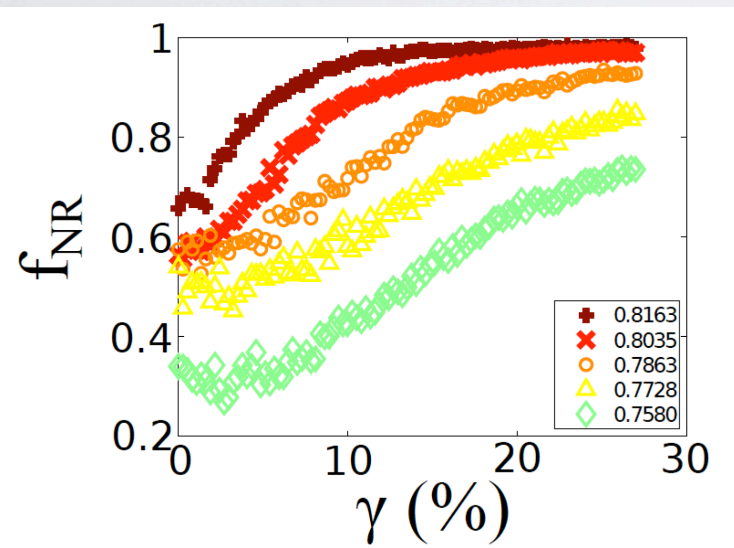
Lowest density for jamming without shear

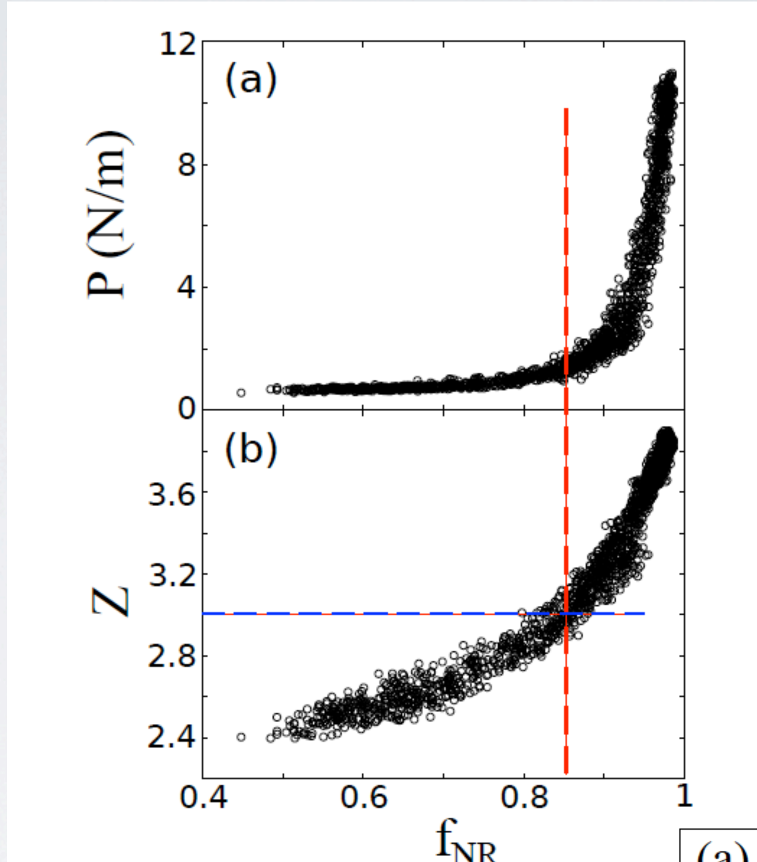
Tour of Shear Jamming Experiments

Reynolds Pressure

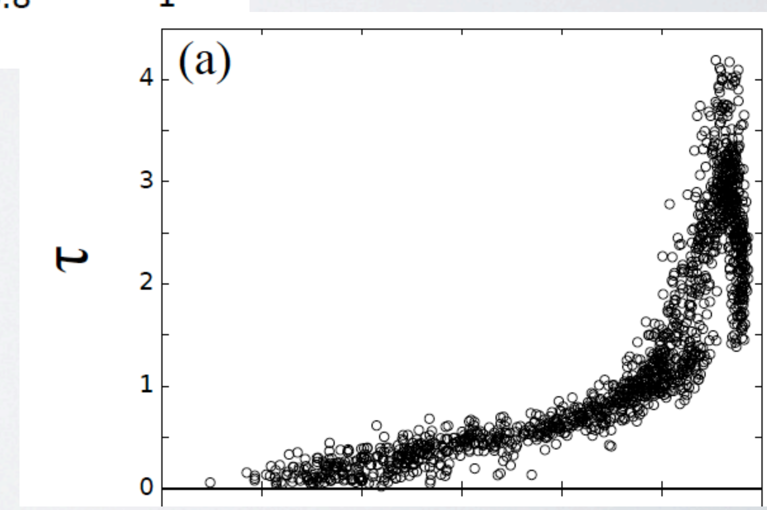


Non-rattler Fraction (Non-Spectators)

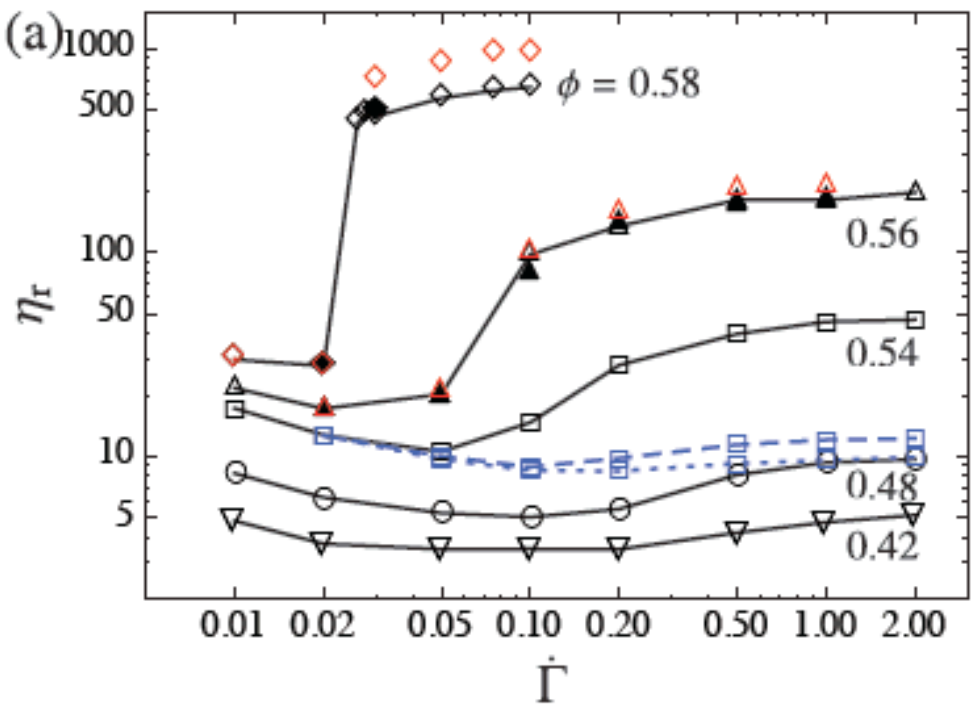


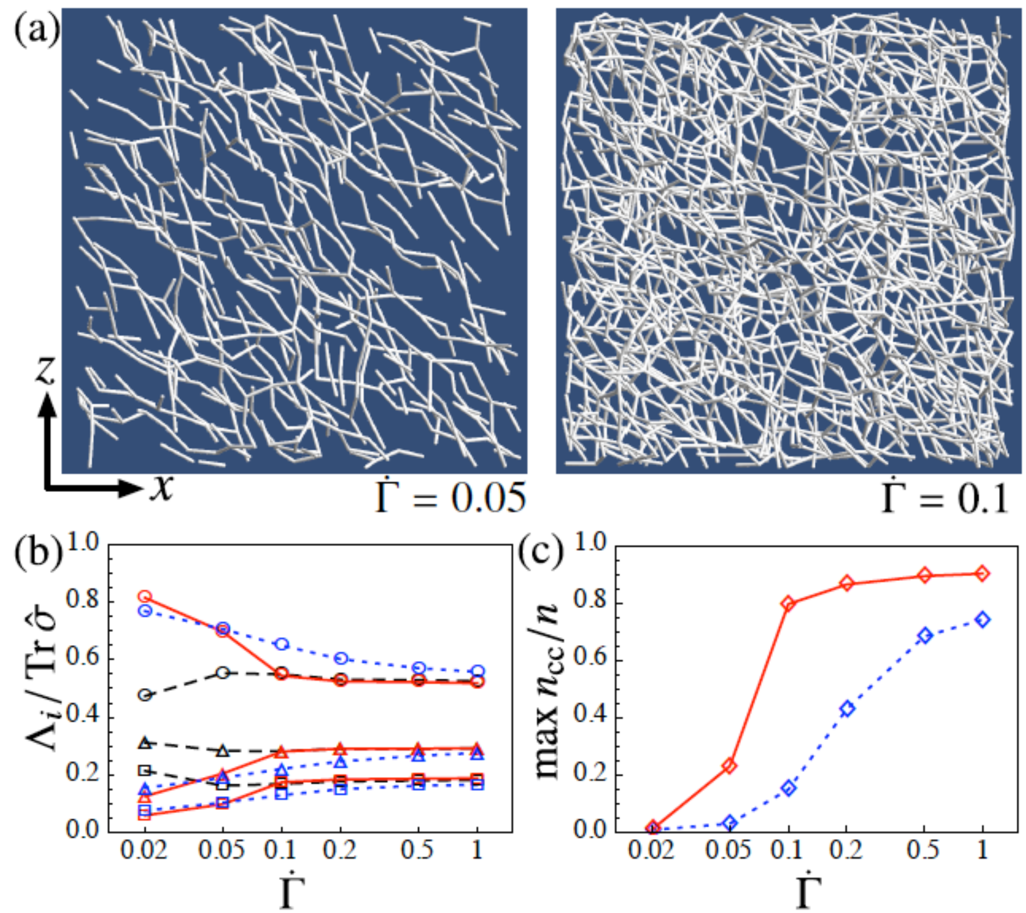


Can think of this as increasing an effective friction coefficient



Tour of Discontinuous Shear Thickening





Seto, Mari, Morris,Denn (simulations)

Network of frictional contacts evolves with shear rate: similar to force chain evolution in grains

Emergence of Rigidity: Story of Constraints

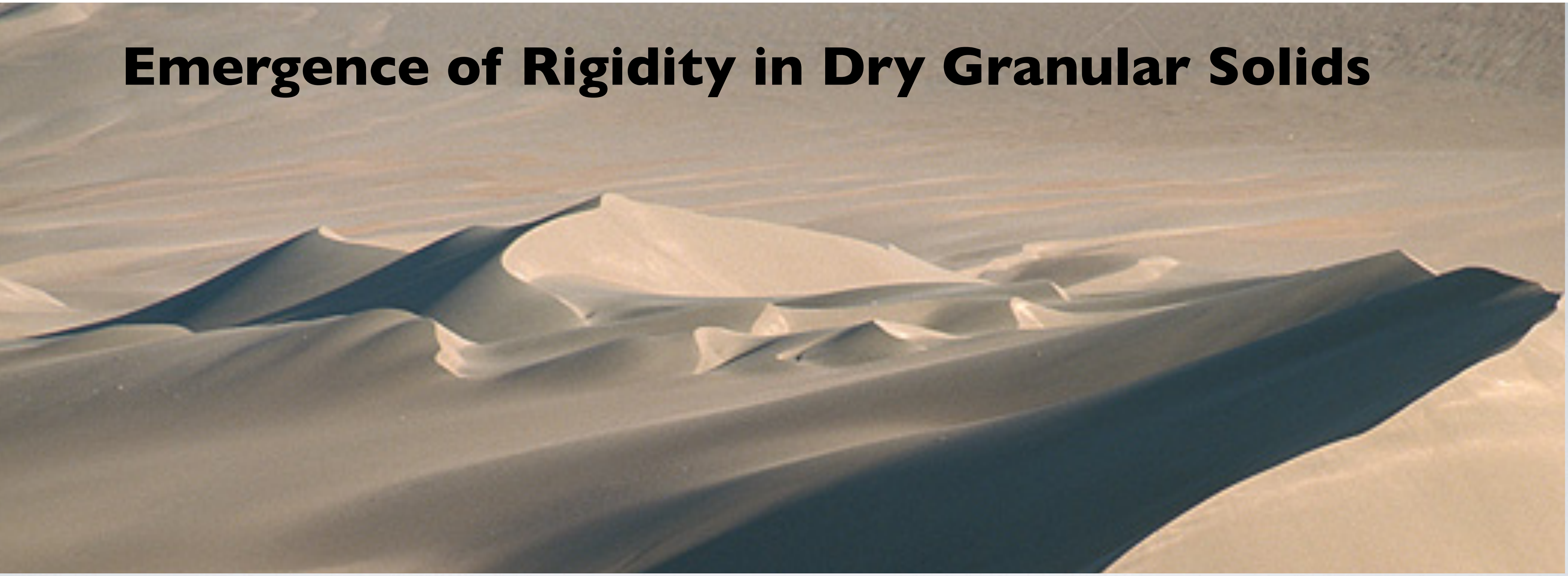
Local force & torque balance satisfied for every grain

Friction law on each contact $f_t \leq \mu f_N$

Positivity of all forces $f_N \geq 0$

Imposed stresses determine sum of stresses over all grains

Emergence of Rigidity in Dry Granular Solids



- Present a representation that captures the essential physics: “force” space
- In this representation there is a qualitative difference between frictionless and frictional grains
- Objective: Construct a rigorous theory of rigidity in athermal systems
- Results for shear-jammed experimental states: shear-induced broken symmetry in “force” space

Imposing the conditions through gauge potentials (2D)

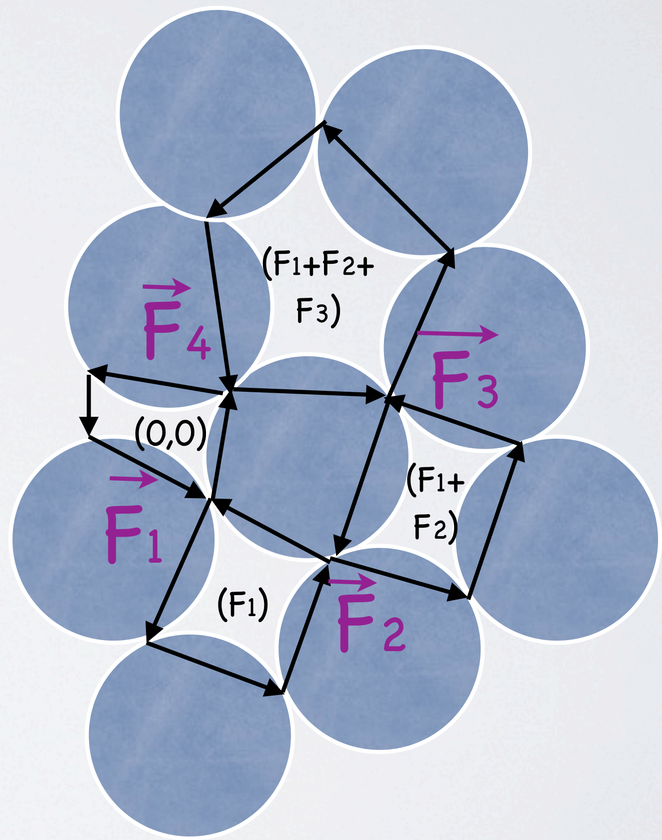
Ball & Blumenfeld (2002), Henkes, Bi, & BC (2007---), DeGuili (2011--)

- Vector fields enforce force balance constraint
- Additional scalar field enforces torque balance
- There is a relation between the two

The vector fields:

We refer to them as heights: like a vector height field familiar in the context of groundstates of some frustrated magnets

Here the fields are continuous



Imposing the conditions through gauge potentials (2D)

Ball & Blumenfeld (2002), Henkes, Bi, & BC (2007---), DeGuili (2011--)

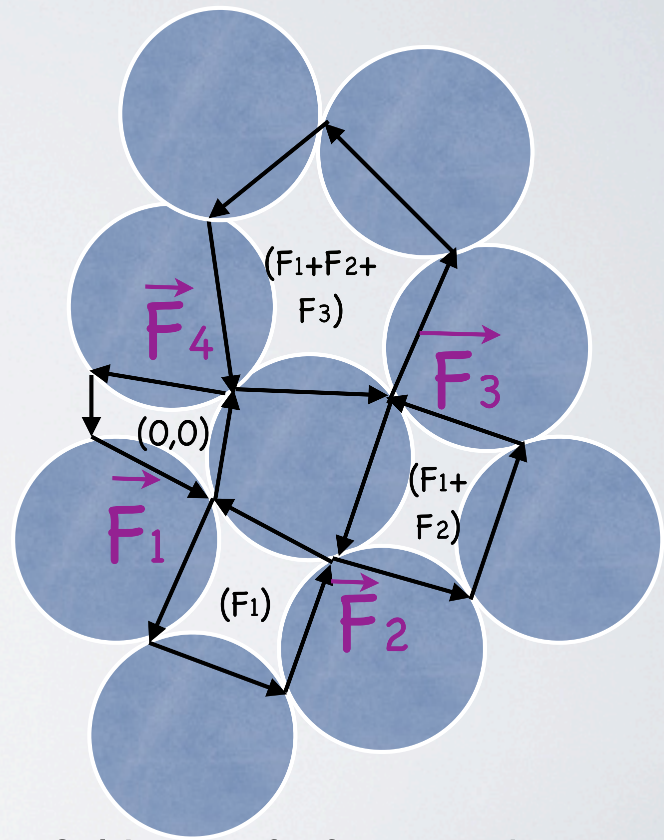
loops enclosing voids

$$\boldsymbol{f}_g^c = \mathrm{h}^{\ell'} - \mathrm{h}^{\ell},$$

$$\boldsymbol{r}^c \times \boldsymbol{f}_g^c = \varphi^{\ell'} - \varphi^{\ell} + \boldsymbol{r}^{\ell'} \times \mathrm{h}^{\ell'} - \boldsymbol{r}^{\ell} \times \mathrm{h}^{\ell}$$

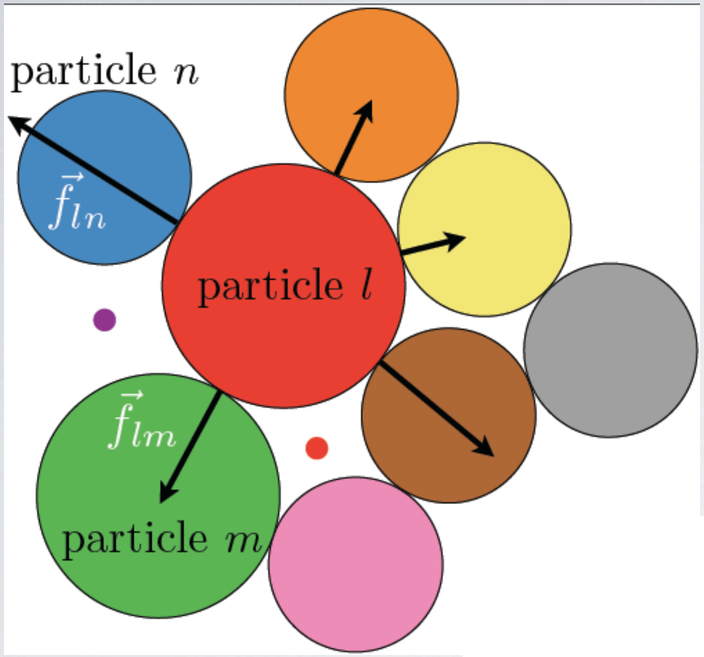
scalar potential

height vector



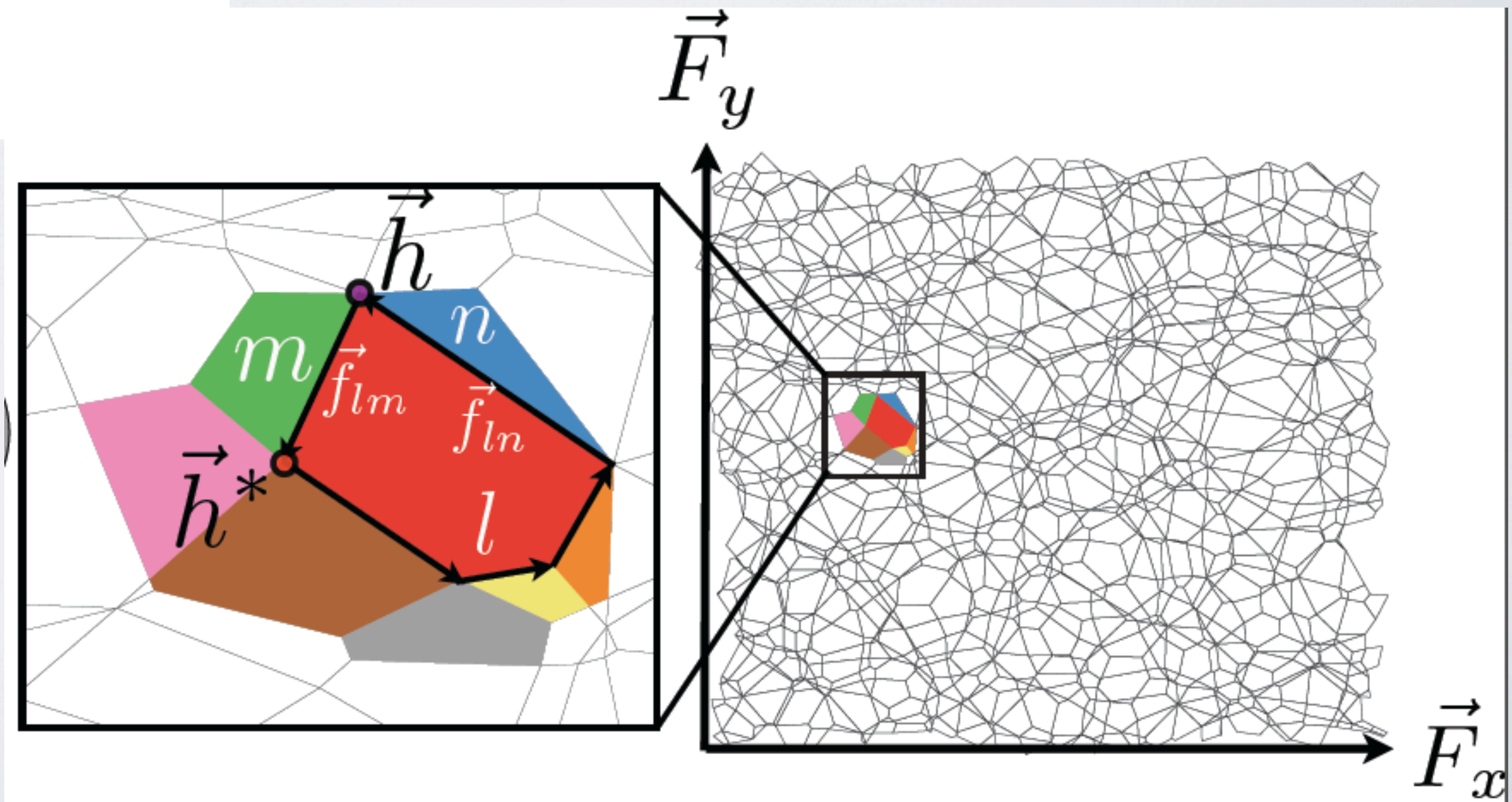
Gauge potentials: irrelevant additive constant. Any set of these fields satisfy force and torque balance. There are constraints relating the two potentials, which depend on real-space geometry.

ONLY FORCE BALANCE

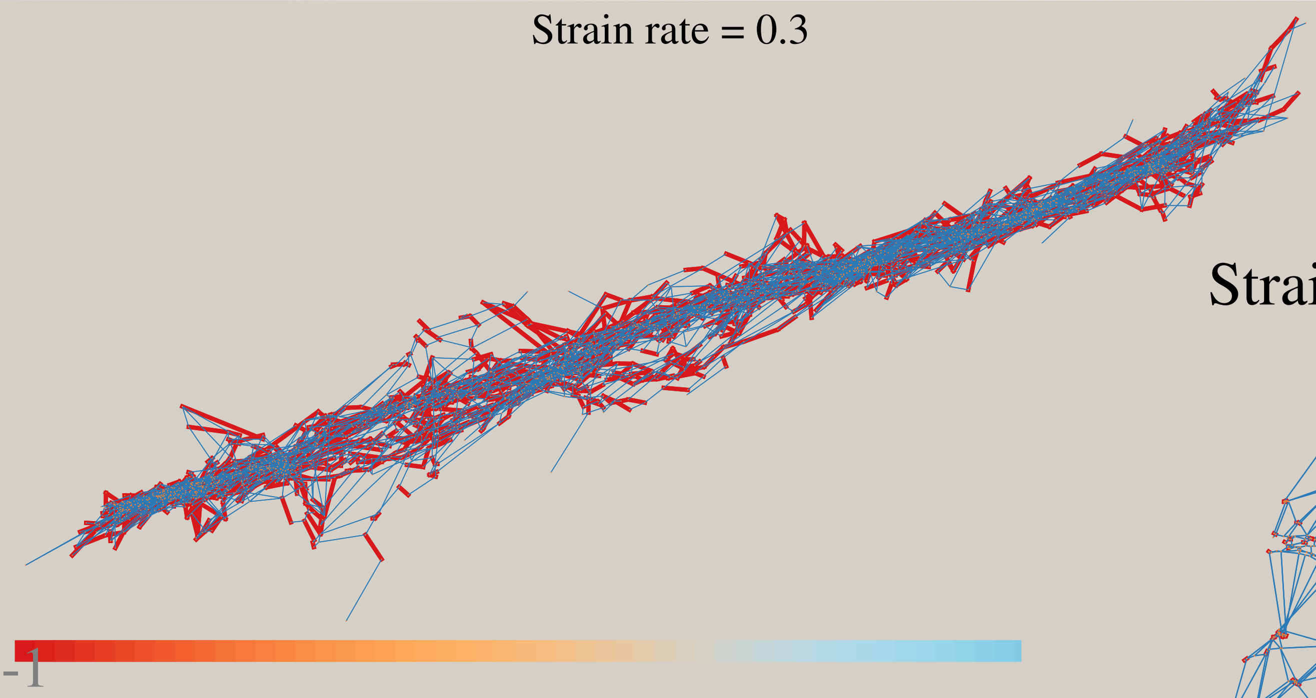


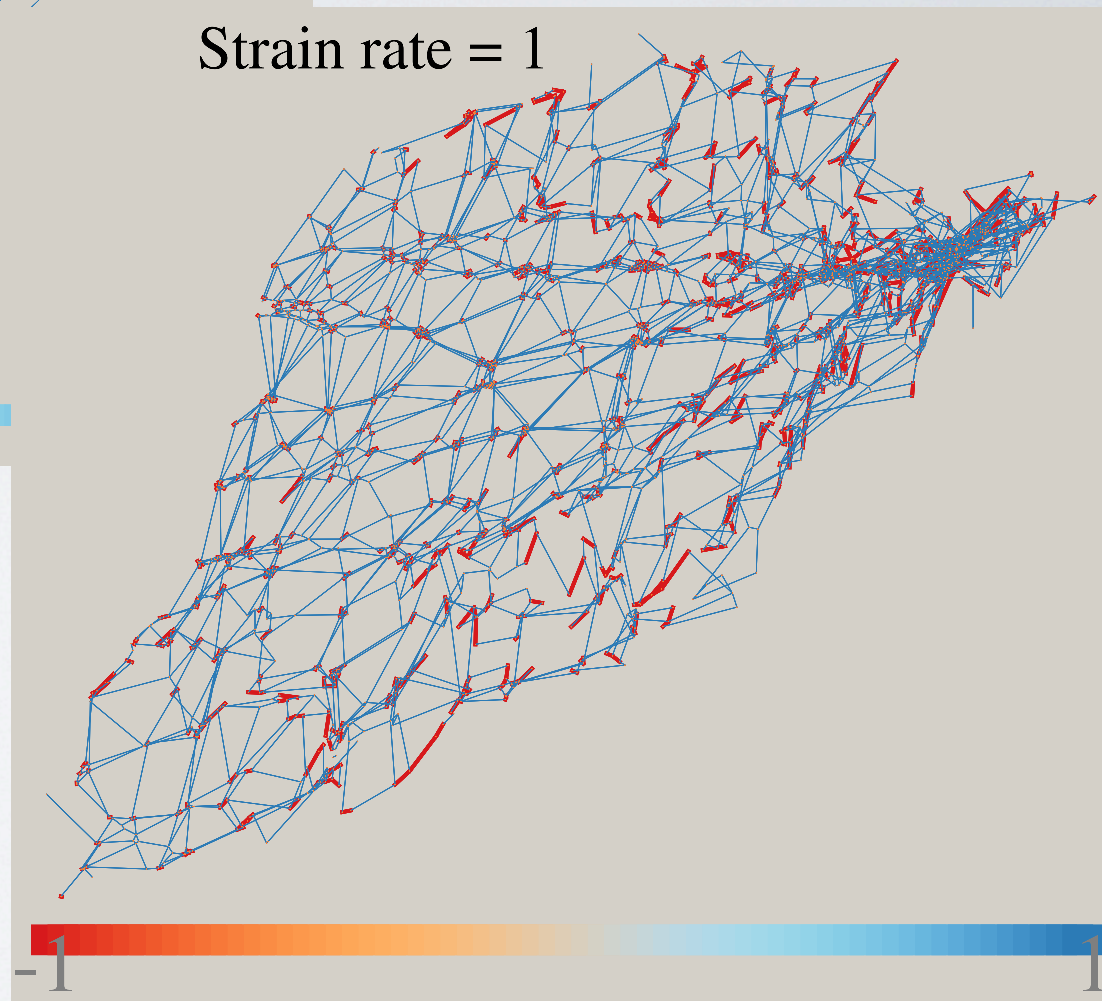
Force Tilings (Maxwell-Cremona Tiles)

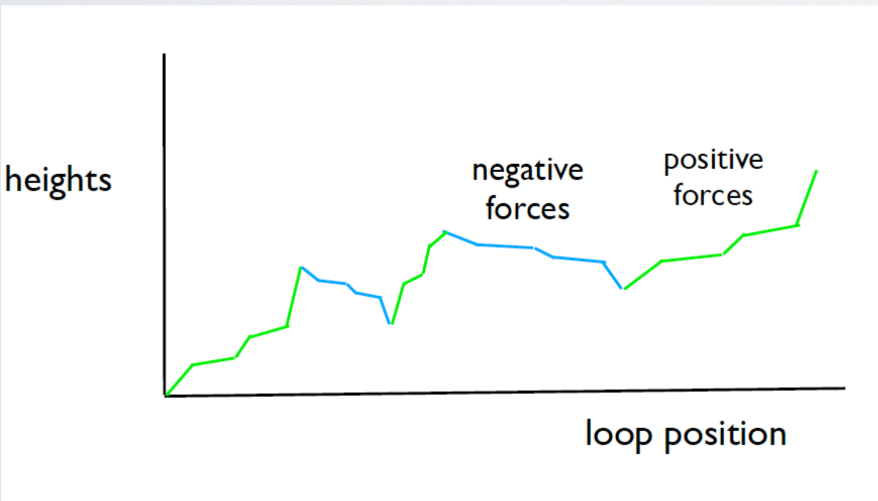
for systems where all normal forces are repulsive, we have a single sheet



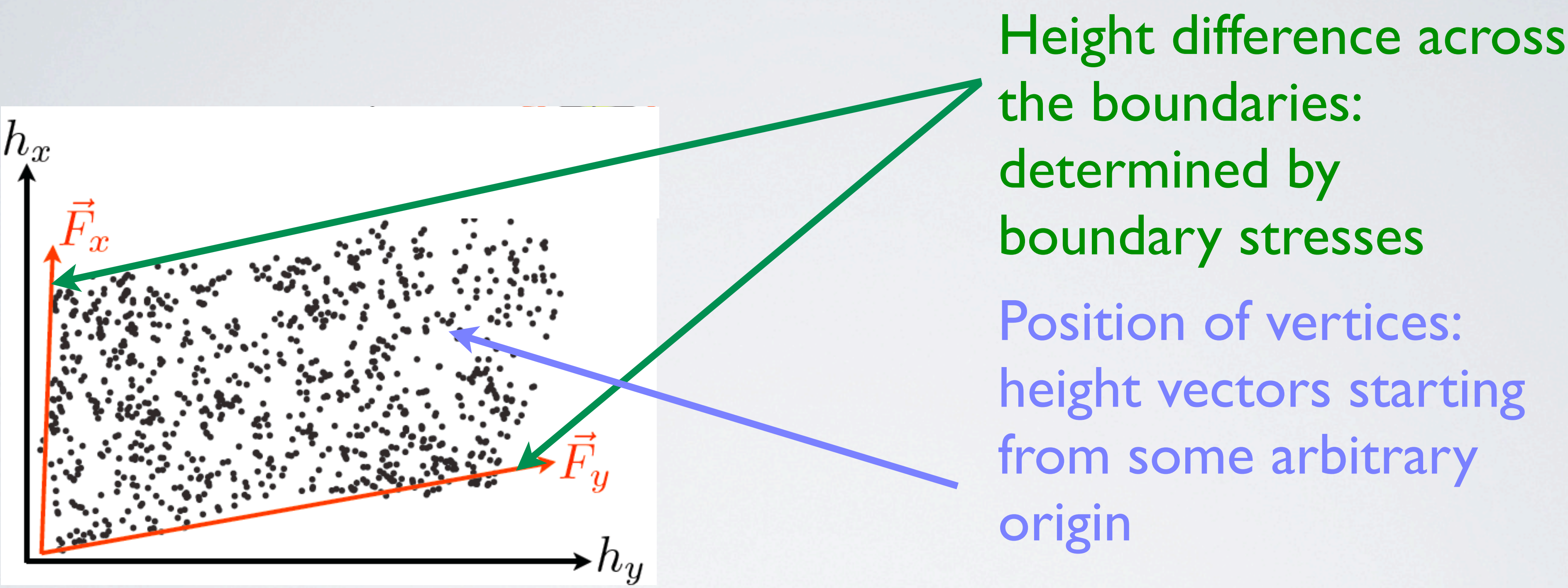
SHEAR-THICKENING





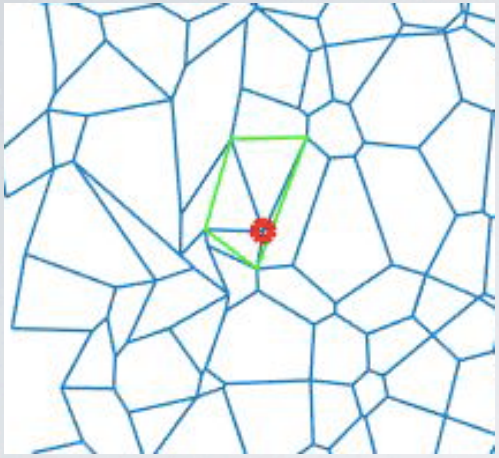


Ensemble of tilings at a given external stress: Statistical properties, correlations, order parameters. Devise a Monte Carlo Metropolis scheme, for example.



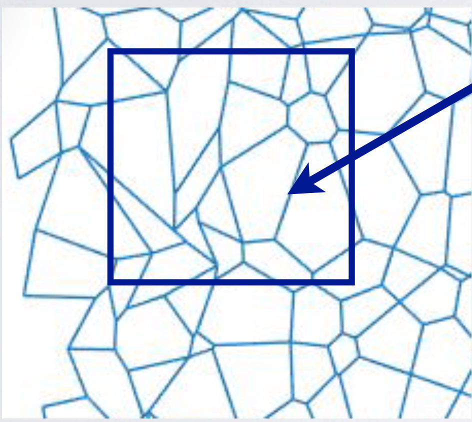
Force space: height vectors play the role of position vectors (of grains, atoms...)

Torque Balance | Friction law on each contact



★Do these introduce correlations ?
★Example: Polygons have to be convex for frictionless, convex-shaped grains

$$\langle \rho(\vec{h}) \rangle \neq const$$

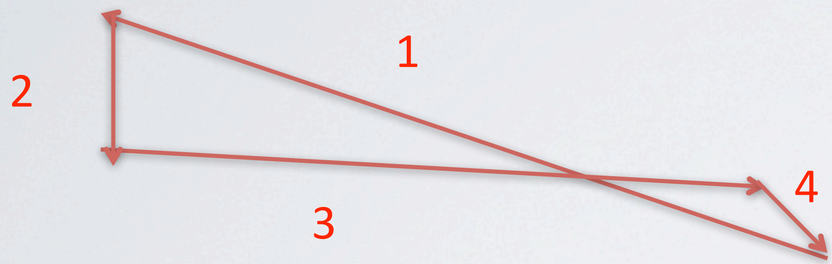


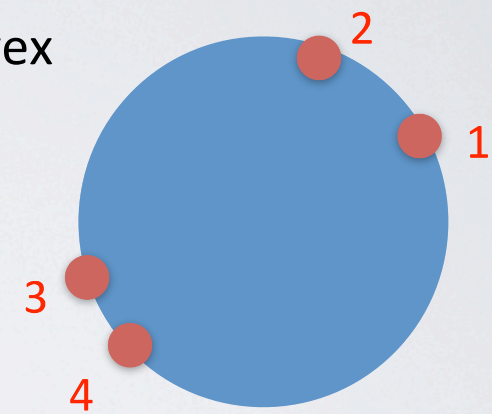
RIGIDITY

★Changing shape of bounding box is the analog of straining
★Does the pattern persist ?
★Alternatively, create an ensemble with a given stress tensor (flat measure) and measure density pattern/overlap

Torque Balance | **Friction law on each contact**

With friction: force tiles can be convex and non-convex



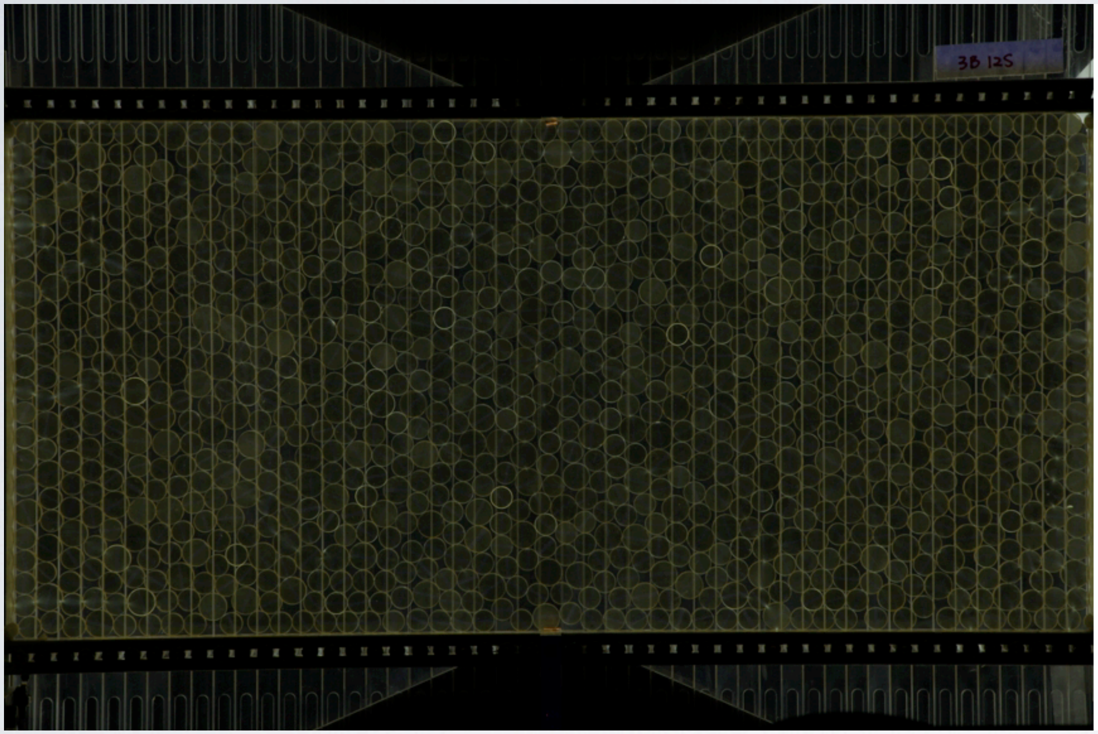


Torque balance from 2 large and two small forces

- ★**Do not have a way of implementing these constraints, rigorously yet since they involve coupling of real and force/gauge space.**

- ★**We have analyzed experiments to determine the statistics of tilings**

EVOLUTION IN RECIPROCAL SPACE



http://www.aps.org/meetings/march/vpr/2015/videogallery/index.cfm

TEST OF PERSISTENT PATTERN IN HEIGHT SPACE

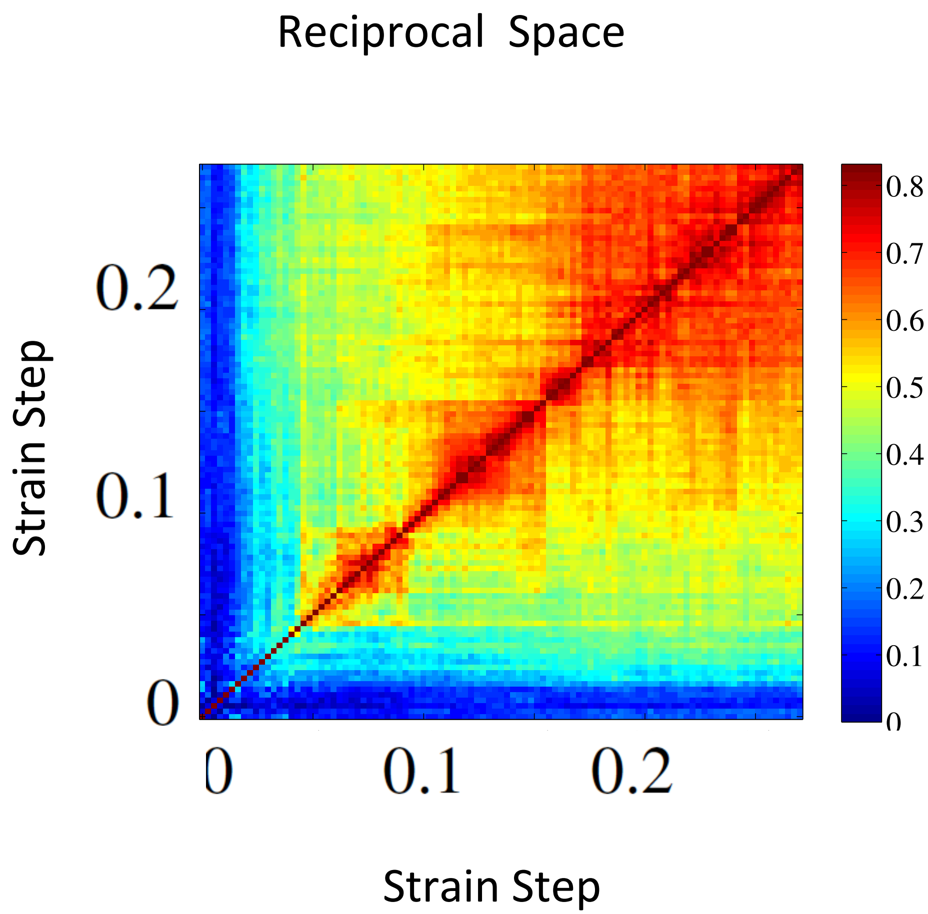
Overlap between two configurations

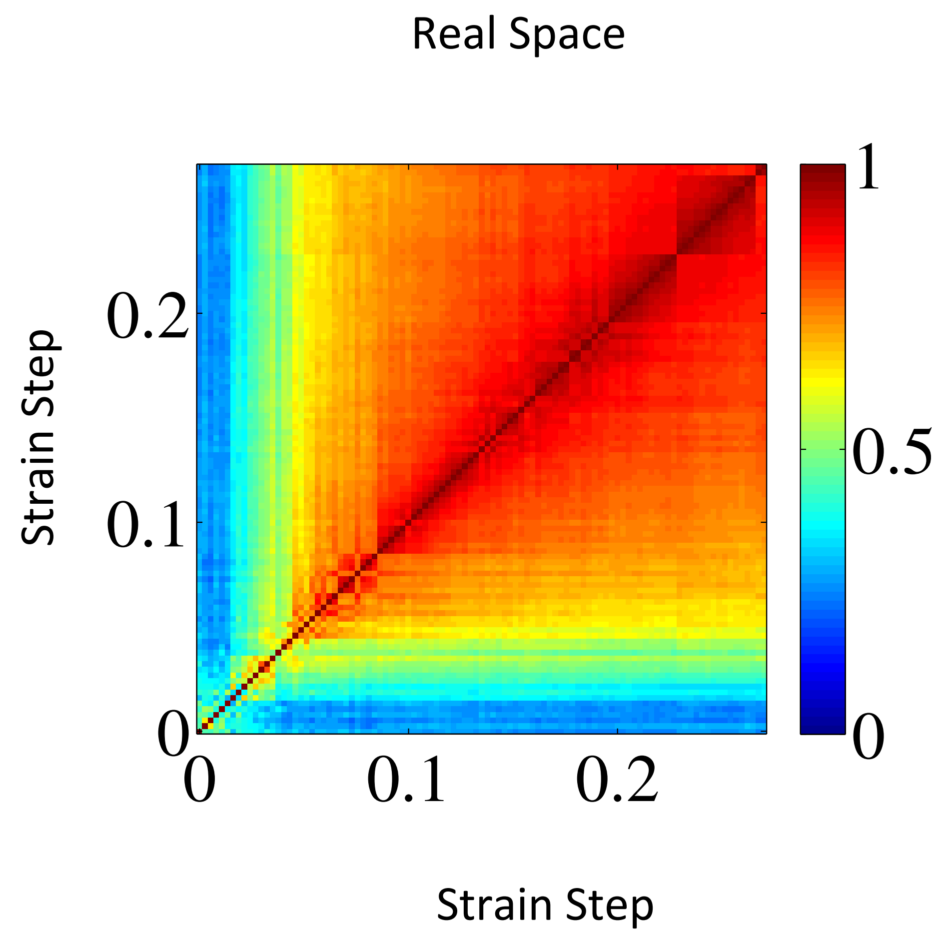
Grid stretched affinely with bounding box

$$d^{\alpha,\beta} = \sum_{m,n} \rho^{\alpha}_{m,n} \rho^{\beta}_{m,n}$$

If height pattern evolves affinely, large overlap
The stress-generated pattern can sustain further loading

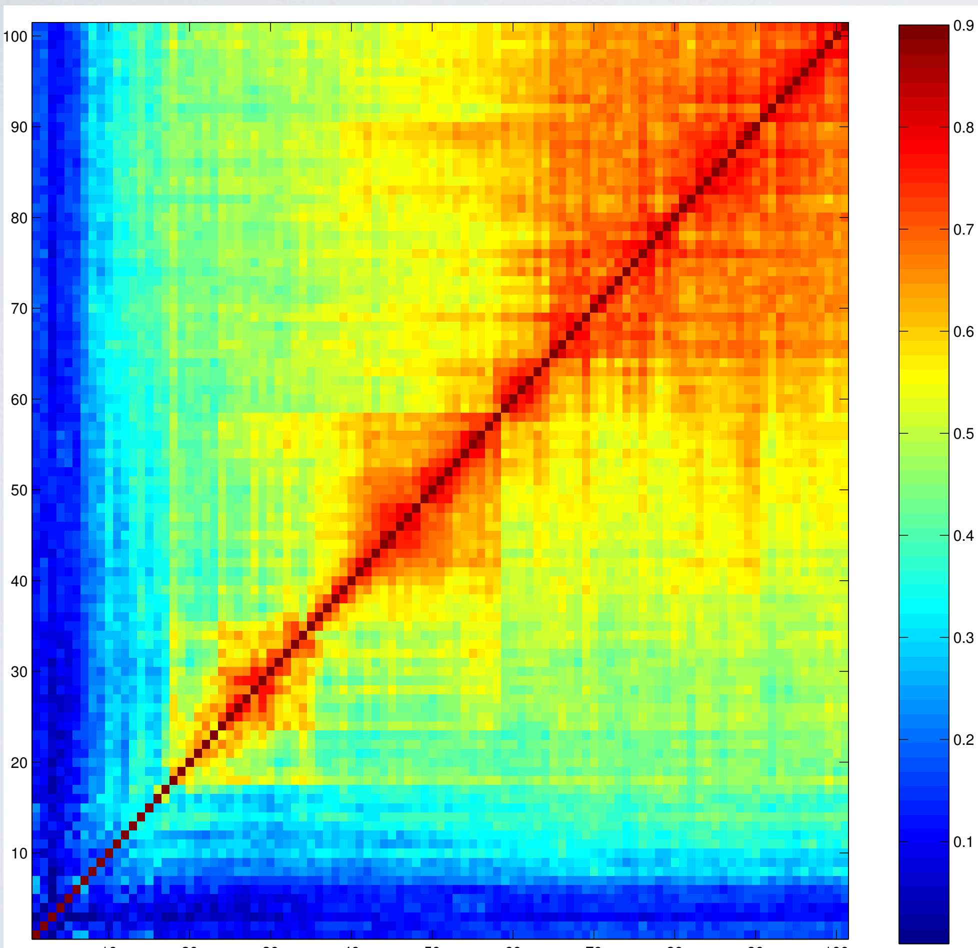
Persistence of Structure: Overlap



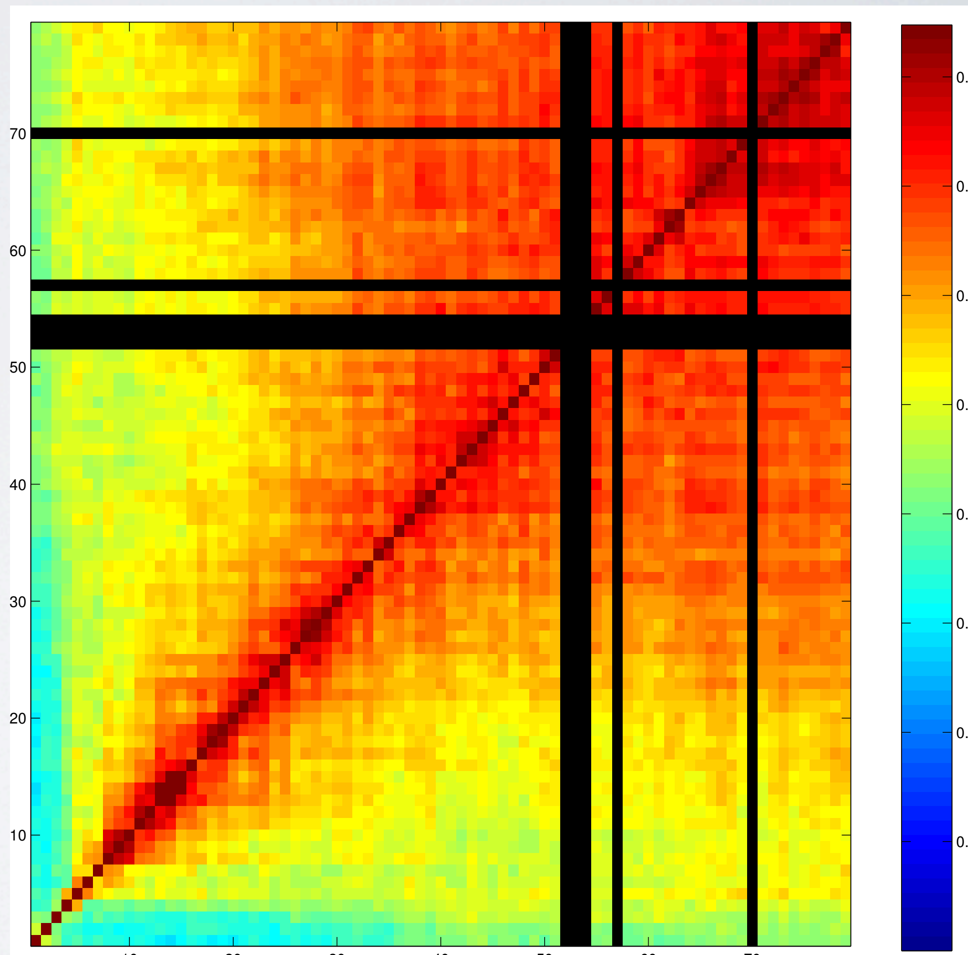


PERSISTENCE OF PATTERNS

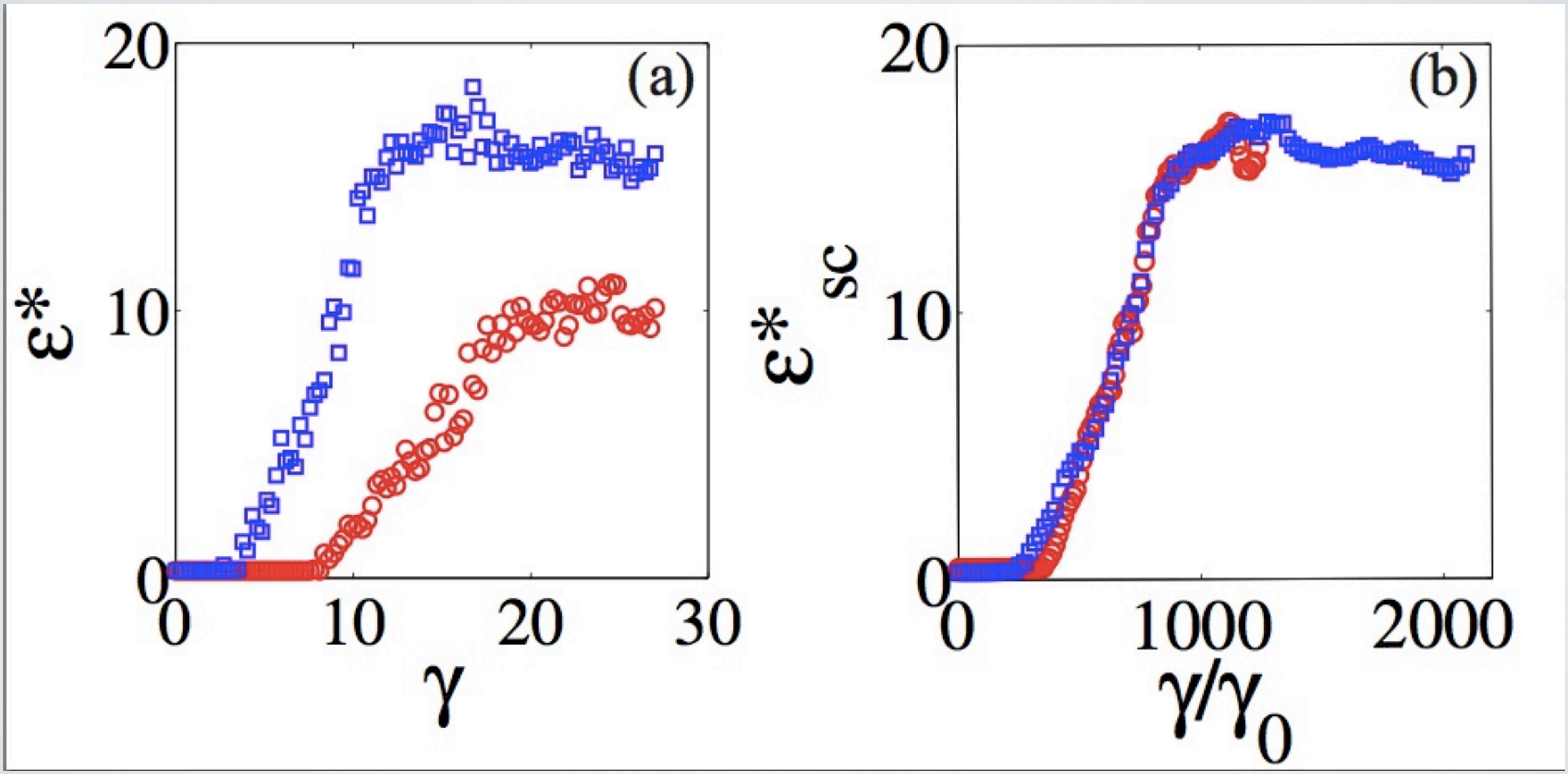
Low density



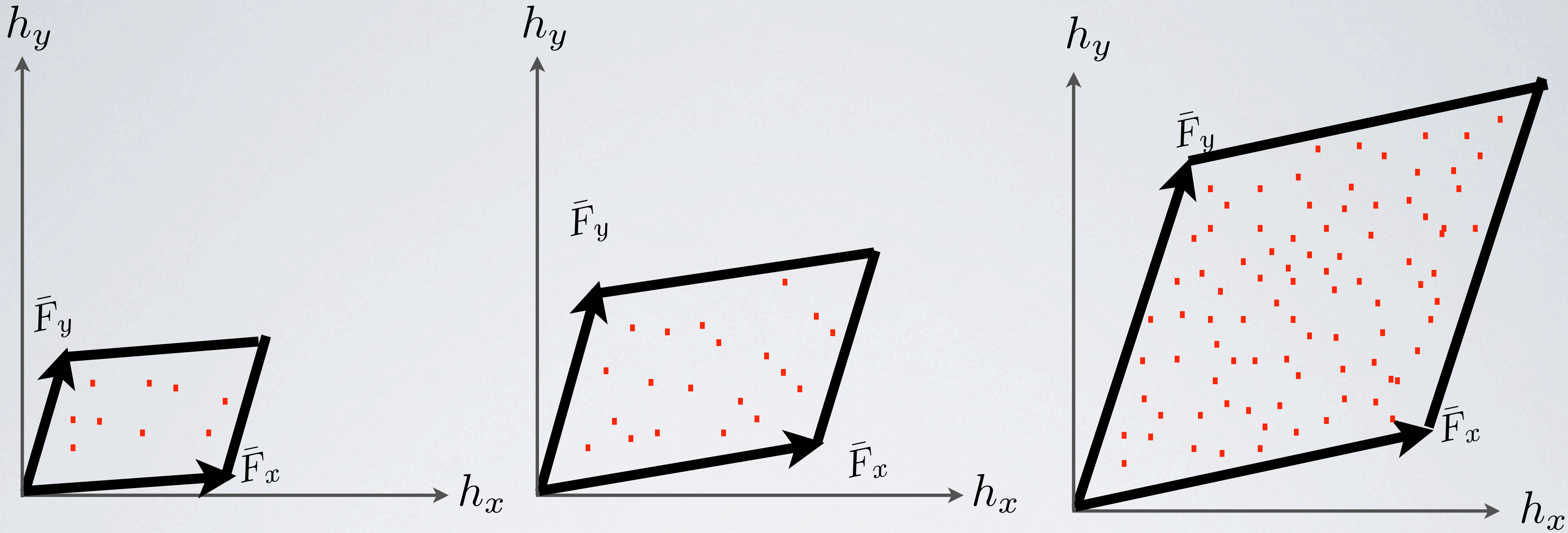
High density



ORDER PARAMETER



Competition



Under shear: Box shape changes and point density increases (depends on protocol)

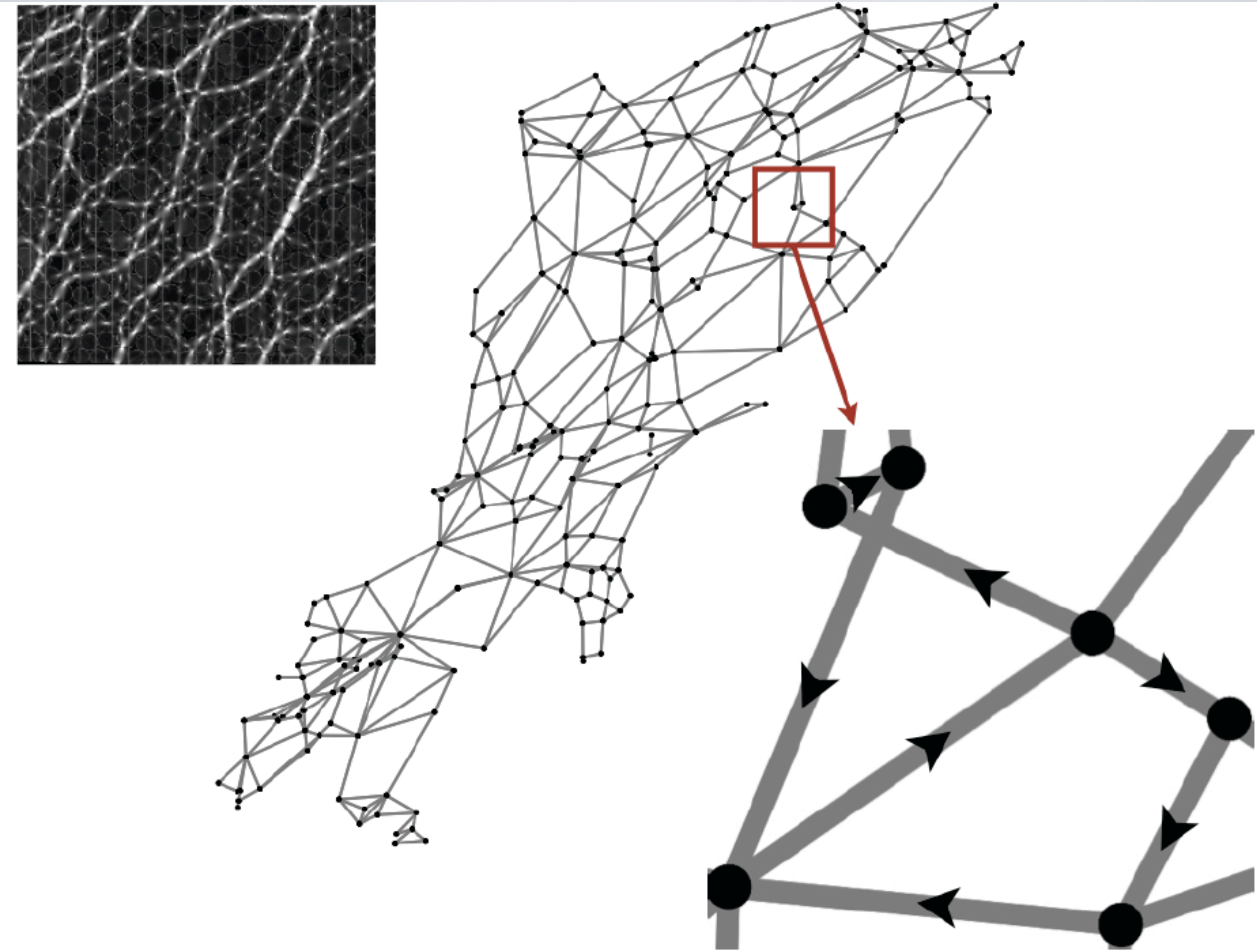
Shear favors: non-convex polygons

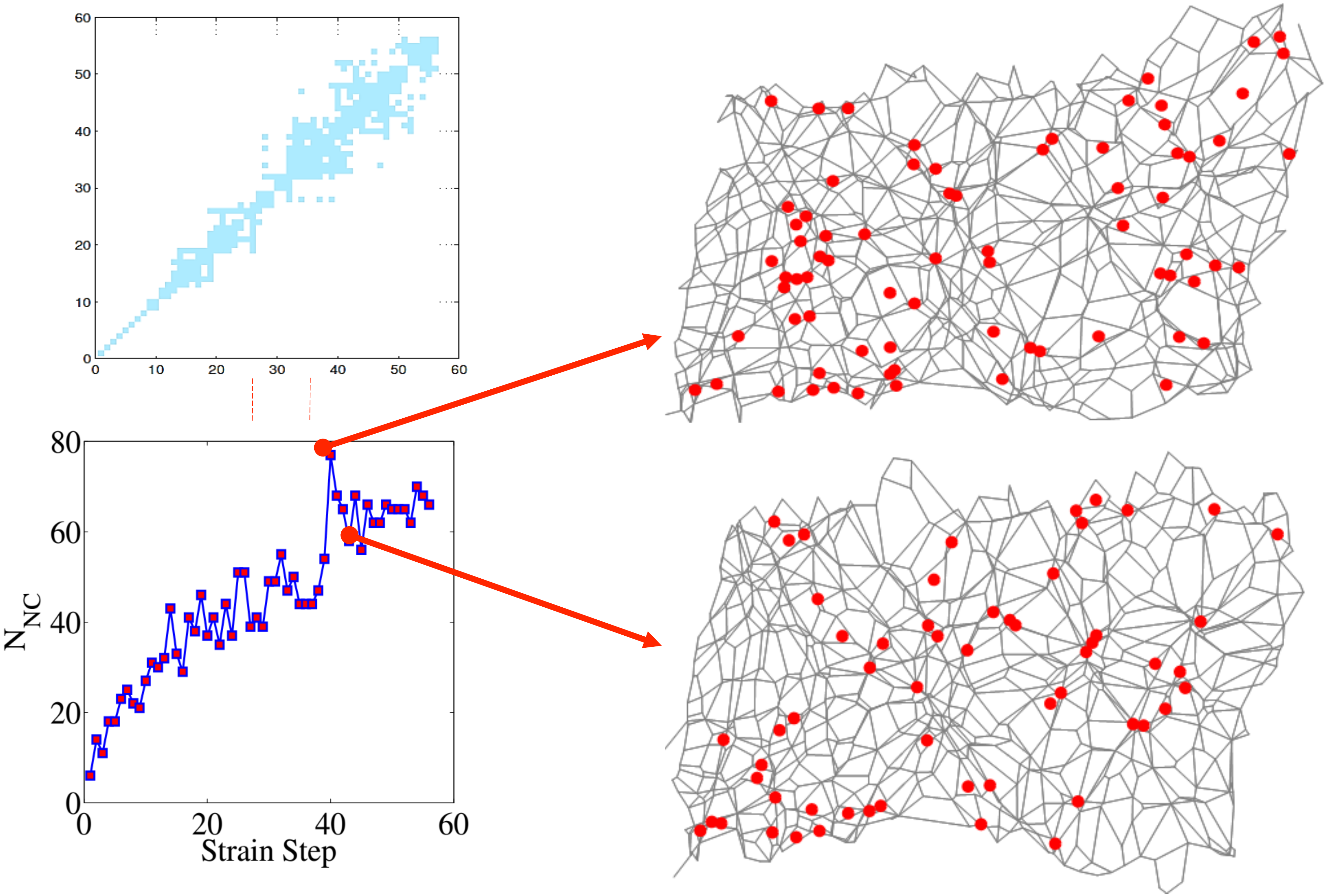
Increasing number of contacts favors convex polygons

Competition drives transition (much like entropy vs energy ?)

Transition driven by density of points in this reciprocal space

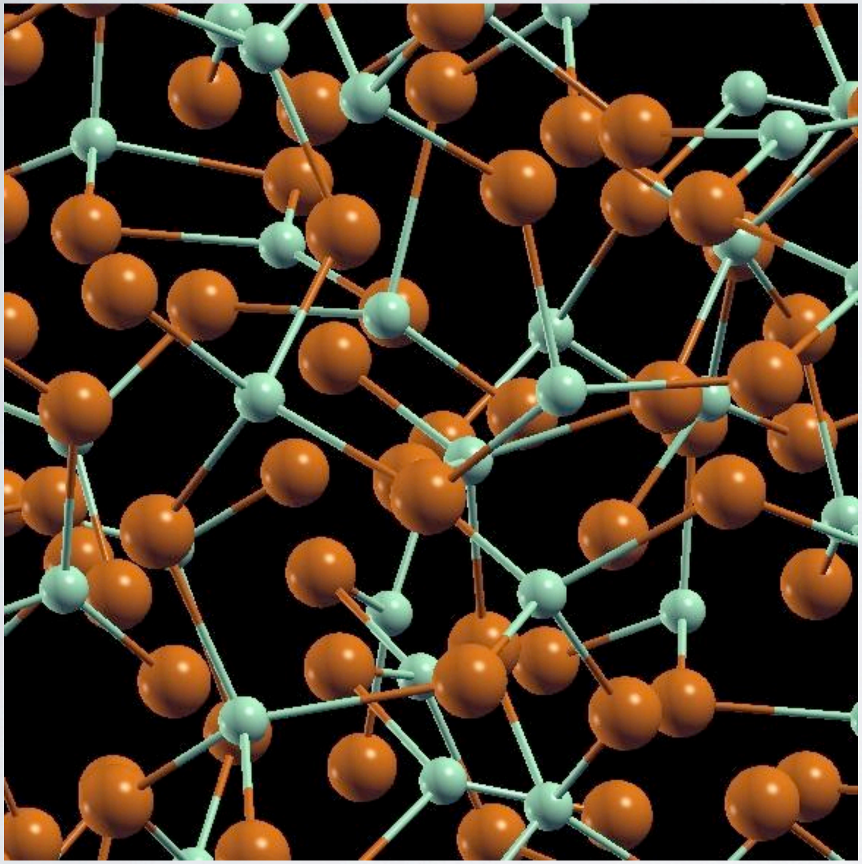
Broken Translational Symmetry in Height Space



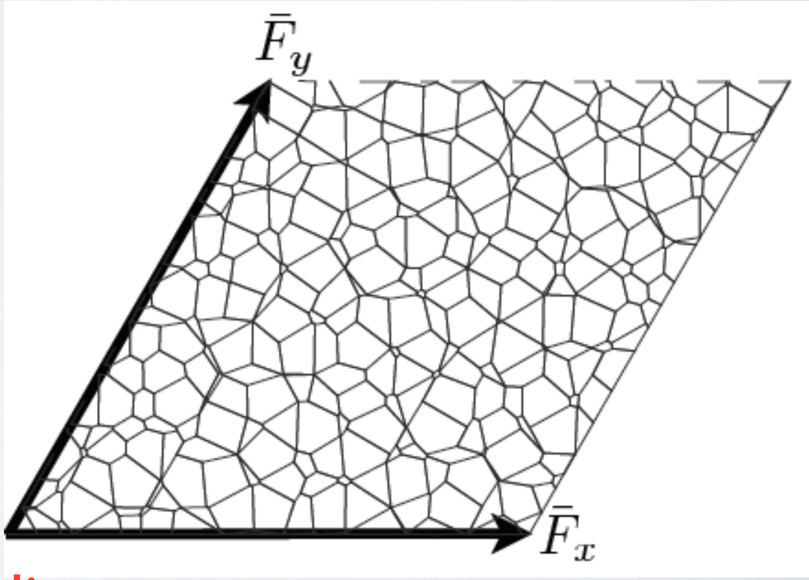


0
10
20
30
40
50
60
N_{NC}
80
Strain Step

RIGIDITY



Amorphous Solids:
Broken Translational Symmetry
Straining costs energy/entropy



T=0 Granular Solids:
Broken Translational Symmetry(height space)
No concept of strain
States created by load can sustain further loading