The hierarchy problem and Physics Beyond the Standard Model

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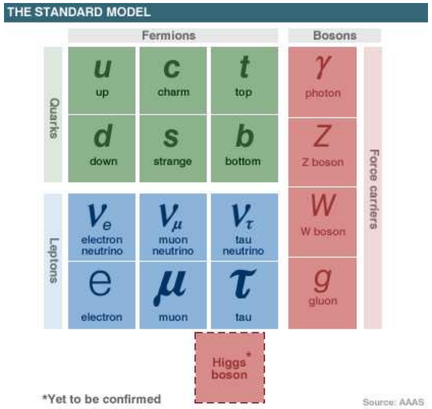
The Standard Model

Gauge group: $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y \to U(1)_{EM}}$

$$\mathcal{L}_{\rm SM} = \overline{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + y \overline{\Psi} \Psi \Phi + |D_\mu \Phi|^2 - V(\Phi)$$

where $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$



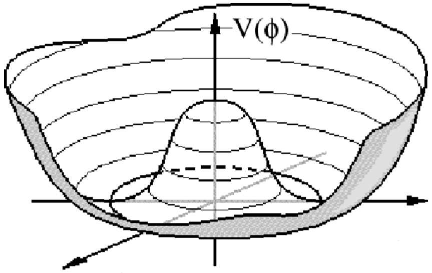
THE STANDARD MODEL

	Fermions			Bosons	
Quarks	u up	c charm	t top	γ photon	Force carriers
	d down	s strange	b bottom	Z Z boson	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
	e electron	μ muon	τ tau	g gluon	
		Higgs* boson			

*Yet to be confirmed

Source: AAAS

Precision: gauge: (0.1-0.5)%, flavor: %, scalar: %



SSB and stability: $\mu^2 < 0, \lambda > 0 \quad \rightsquigarrow \quad v^2 = \frac{|\mu^2|}{\lambda}$

$M_W = \frac{1}{2} g v, \quad m_f = \frac{1}{\sqrt{2}} y_f v, \quad m_h = \sqrt{4\lambda}\, v$

Spontaneous Symmetry Breaking $\quad E = p$

Examples: Superconductivity, Ferromagnet, Pions

SSB of Global symmetry of potential $\rightsquigarrow$ (i) Goldstone bosons, (ii) order parameter v, (iii) massive scalar (Higgs), W/Z become massive by ‘eating up’ the GBs.

Do we need to go beyond the SM?

SM rules supreme at electroweak scale. A few hiccups, but no real threat! Why go beyond?

Experimental requirements: Dark matter and neutrino mass require BSM physics.

Theoretical inadequacy: Hierarchy problem which originates in the electroweak symmetry breaking sector.

Leading BSM candidates: Supersymmetry and Extra Dimension. These are two classes of scenarios which include specific models.

Measurements at LEP

LEP: Large Electron Positron Collider at CERN, which operated during 1989-95 at c.m. energy of $\sim$ 90 GeV (Z pole) in the first phase. In the second phase, the c.m. energy was increased in steps to about 205 GeV.

Z-pole Observables: Cross section (σ_f), Fwd-Bwd asymmetry ($a^f_{\rm FB}$).

$$\sigma_f = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}, \quad \text{where} \;\; \Gamma_f \propto (v_f^2 + a_f^2), \quad a^f_{\rm FB} = f\left(\frac{v_f}{a_f}\right)$$

Here

$$v_f = \left[t_3^f - 2Q_f \sin^2\theta_W\right], \qquad a_f = t_3^f$$

- $\Gamma_b^{\rm SM} = 376$ MeV, $\Gamma_b^{\rm top-less} = 23.5$ MeV. $\Gamma_b^{\rm exp}$ closer to SM value.
- $a^b_{\rm FB}$ non-vanishing $\Rightarrow$ $\boxed{\text{Top has to be there!}}$

Measurements at LEP (contd..)

- Summer 1992: v_l^{LEP} and v_l^{SM} showed 13σ discrepancy. When $\sin^2\theta_W$ extracted using $\alpha(M_Z) \approx 1/128$, discrepancy reduced to 1σ. Much later, genuinely weak radiative effects of order $G_F m_t^2$ was felt.
- weak radiative effects captured in $\Delta\rho(=\alpha T)$, S, $Zb\bar{b}$ vertex.

$$v_f = \sqrt{\rho}\left[t_3^f - 2Q_f \sin^2\bar{\theta}_W\right] , \qquad a_f = \sqrt{\rho}\, t_3^f$$

- $\rho^{\text{SM}} = 1$, $\rho^{\text{LEP}} \approx 1$
  $\Rightarrow$ This attests the doublet structure of Higgs.

$\Gamma(Z \to \text{ inv}) \Rightarrow N_\nu = 2.984 \pm 0.008 \Rightarrow$ Three light families

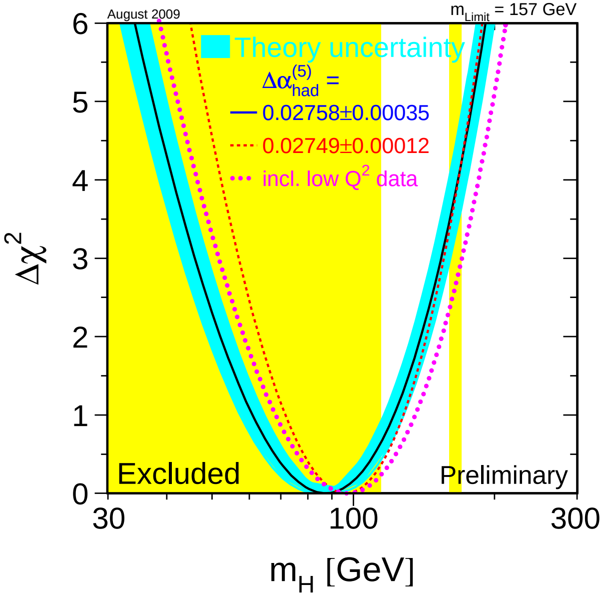
S (together with T) $\Rightarrow$ At most one more *heavy* chiral family.

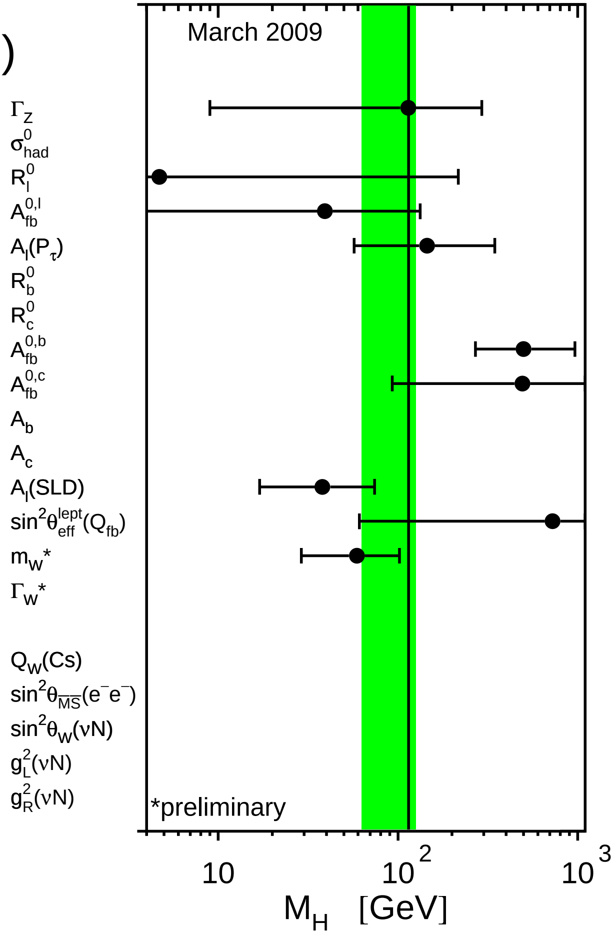
EWPT Constraints on m_h in SM

- Electroweak precision tests

$$\Delta\rho \simeq \frac{3G_F}{8\pi^2\sqrt{2}}\left[m_t^2 - (M_Z^2 - M_W^2)\ln\left(\frac{m_h^2}{M_Z^2}\right)\right] \quad , \quad S = \frac{1}{6\pi}\ln\left(\frac{m_h}{M_Z}\right)$$

- Combined EW fit $m_h < 186$ GeV @ 95% C.L. LEPEWWG
- Direct search: $m_h > 114.4$ GeV (full strength ZZh)





The custodial symmetry

- The SM Higgs is a complex scalar doublet ⇒ 4 real fields. Both kinetic and potential terms have $\mathrm{O}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry.
- One of them is $\mathrm{SU}(2)_{\mathrm{L}}$. Usually, the other is called $\mathrm{SU}(2)_{\mathrm{R}}$. Higgs carries a (2,2) rep.
- When Higgs gets a vev, $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \to \mathrm{SU}(2)_{\mathrm{V}}$. This is called 'custodial SU(2)'. So what?
- Consider the Lagrangian for gauge bosons after EWSB.
  $L = \Pi_{\pm} W^+ W^- + \Pi_{33} W^3 W^3 + \Pi_{3B} W^3 B + \Pi_{BB} BB$ where $\Pi_{ab} \sim < J_a J_b >$.
  Also $\Pi(p^2) = \Pi(0) + p^2 \Pi'(0)$.
- Each J transforms as (3,1) under $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ or as 3 under $\mathrm{SU}(2)_{\mathrm{V}}$. Recall $3 \times 3 = 1 + 3 + 5$.
  $A_i B_j : A_i B_i + (A_i B_j - A_j B_i) + \frac{1}{2}(A_i B_j + A_j B_i) - \frac{1}{3}(A.B)\delta_{ij}$
- $\Delta\rho = \alpha T \propto (\Pi_{\pm}(0) - \Pi_{33}(0))$, symmetric under indices and traceless, so transforms as 5 of $\mathrm{SU}(2)_{\mathrm{V}}$.
- Since $\mathrm{SU}(2)_{\mathrm{V}}$ is a symmetry of the vacuum, only singlets of $\mathrm{SU}(2)_{\mathrm{V}}$ can have non-vanishing expectation value. Hence $T = 0$ at leading order.

Another reason for the Higgs!

Dipankar Das PhD thesis: arXiv: 1511.02195 [hep-ph]

Unitarity: $W_L^+W_L^- \to W_L^+W_L^-$ scattering: The amplitude grows as $a(E/M_W)^4 + b(E/M_W)^2 + c$. The E^4 divergence cancels between the contact term and other gauge boson mediated graphs. Cancellation of E^2 divergence requires the presence of a scalar. The scattering amplitude is

$$A = 16\pi \sum_{J=0}^{\infty} (2J+1) P_J(\cos\theta) a_J$$

Using the orthogonality of the Legendre polynomials and optical theorem,

$$\sigma = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1)|a_J|^2 = \frac{1}{s}|A(\theta=0)| = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1)\ \mathrm{Im}\ a_J$$

$$\therefore |a_J|^2 = \mathrm{Re}\ (a_J)^2 + \mathrm{Im}\ (a_J)^2 = \mathrm{Im}\ a_J \rightsquigarrow |\mathrm{Re}(a_J)| \leq \frac{1}{2}$$

For $s \gg m_h^2$, the $J = 0$ mode is given by (at tree level), $a_0 = -\frac{m_h^2}{8\pi v^2}$. Therefore,

$$\boxed{m_h < 2\sqrt{\pi}v = 870 \text{ GeV}}$$

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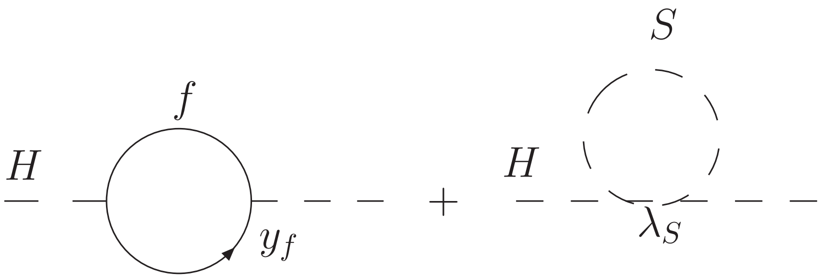
Quantum fluctuations



- No symmetry protects the Higgs mass, unlike in QED:
  $\Delta m_e = m_e \frac{\alpha}{4\pi} \ln(\Lambda)$, as $m_e \to 0$ gives enhanced symmetry (chiral).
  Photon mass is zero due to EM gauge symmetry.
- A quantity is naturally small is setting it to zero increases the symmetry of the theory.
- ΔM_h^2 is quadratically divergent $\left(\int d^4k/k^2\right)$
  $\Delta M_h^2(f) = -\frac{y_f^2}{16\pi^2} 2\Lambda^2 \quad ; \quad \Delta M_h^2(S) = \frac{\lambda_S}{16\pi^2}\Lambda^2$. Here Λ is the highest scale of the theory.
- Thus physics at several orders of magnitude shorter distances is influencing weak scale dynamics.

Unnatural cancellation

- Since Higgs mass is not protected, the order parameter v is also not protected, which destabilizes the entire theory.
- Quad. div. cancels if $\lambda_S = 2y_f^2$. Fine-tuning has to be done order by order in perturbation theory.

Hierarchy problem

What guarantees the stability of v against quantum fluctuations?
$\Rightarrow$ Physics Beyond the Standard Model

Supersymmetry

- Fermions $\leftrightarrow$ bosons; relates matter and force particles.
- Symmetry protects the scalar mass. Quad. div. cancels between diagrams with different spin particles even when SUSY is broken.
- The three gauge couplings unify at $M_G \sim 10^{16}$ GeV.
- Two Higgs doublets. 5 physical scalars: $h, H, A, H^{\pm}$

Quartic coupl related to gauge coupls $\Rightarrow M_h$ predictive

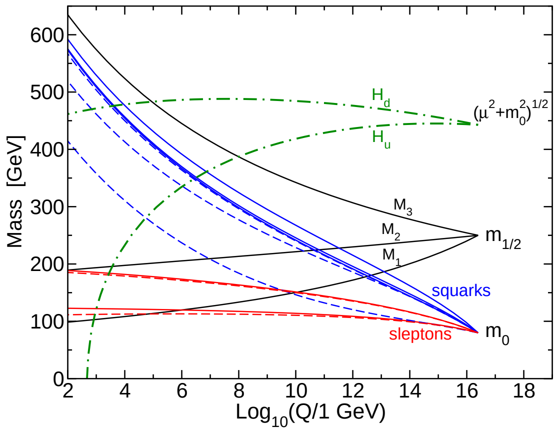
$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3m_t^4}{\sqrt{2}\pi^2 v^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Relation valid irrespective of SUSY breaking mechanism.

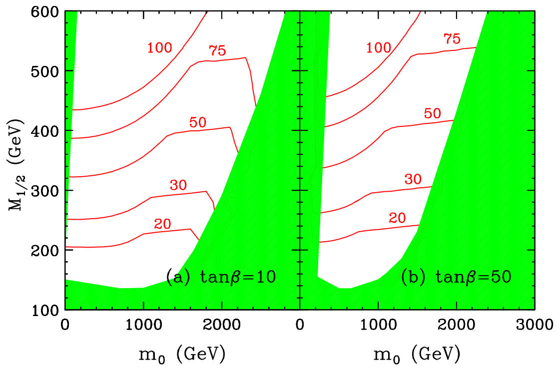
$m_h \simeq 125$ GeV requires the stop mass in TeV range.

Radiative EWSB & Fine-tuning

Large m_t drives $M_{H_u}^2$ negative. EWSB dynamically triggered by RG.



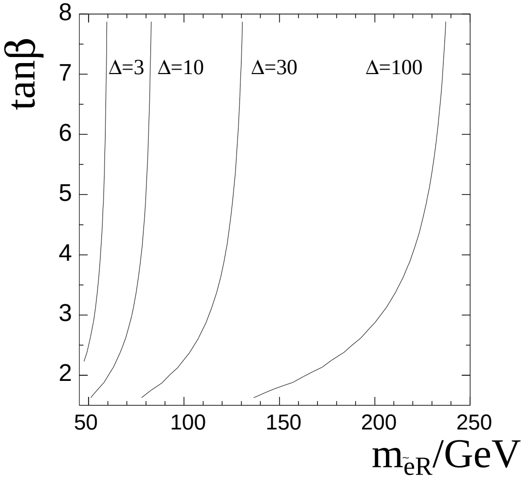
Fine-tuning : $\Delta = \left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2}{\partial a_i} \right|$

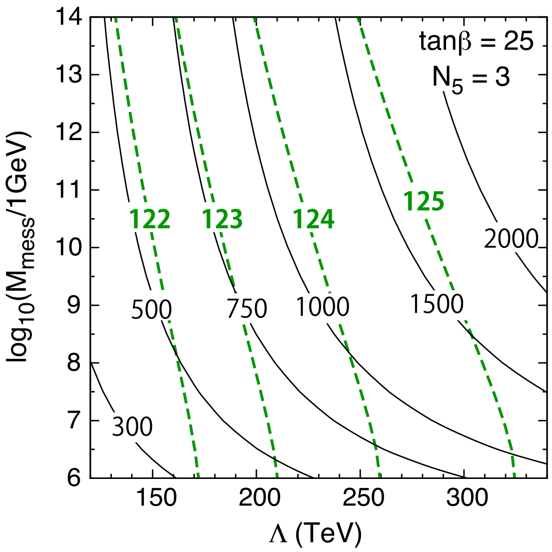


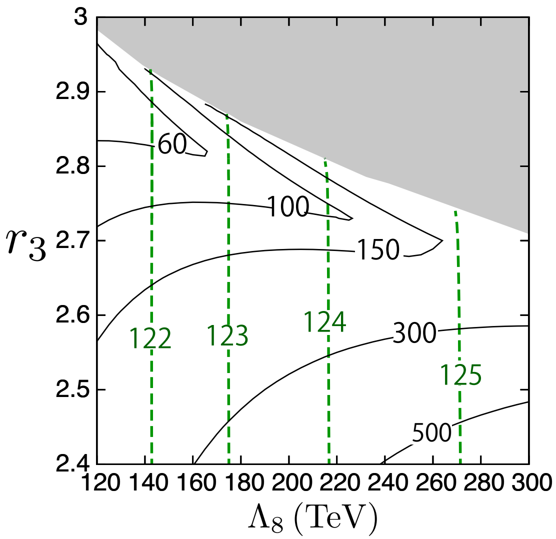
$$0.5\, M_Z^2 \simeq -|\mu|^2 - M_{H_u}^2 \simeq -|\mu^2| + \mathcal{O}(1)\, m_{\tilde{t}}^2$$

$m_h \simeq 125$ GeV $\Rightarrow m_{\tilde{t}} \sim$ few TeV $\Rightarrow$ large cancellation $\Rightarrow$ little hierarchy problem.

F.T. ‘then’ and ‘now’







- Years ago, $\Delta \sim 50$ for $M \sim 10^5$ TeV in minimal GMSB. It was worse than mSUGRA then (GB, Romanino 1997) than now. It can be substantially improved by choosing an unconventional set of messengers (GB, Yanagida, Yokozaki 2015).
- In 20 years it has gone up by a factor of ~ 20.

In general, in SUSY models, little hierarchy is getting increasingly worse!

Little Higgs

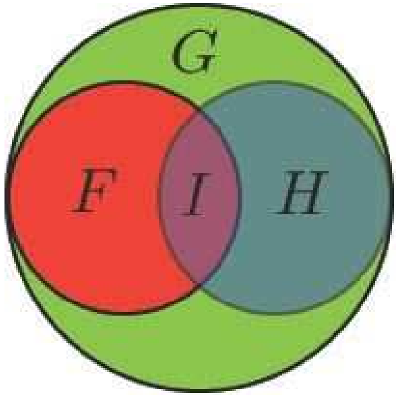
(Cohen, Arkani-Hamed, Georgi, Schmaltz, $\cdots$)

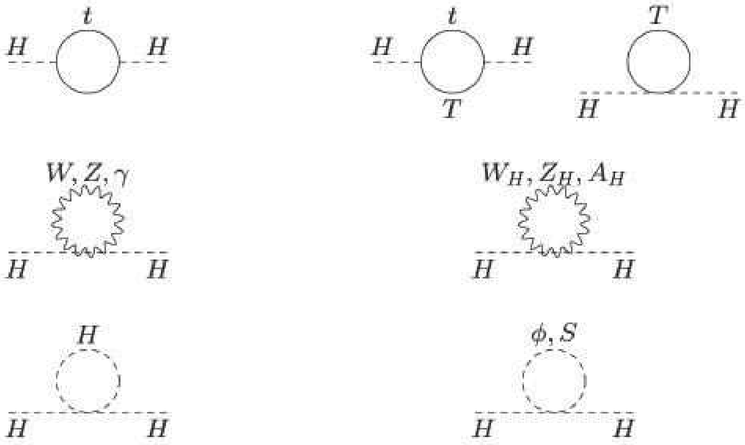
- Little Higgs is a pseudo-NGB of a spontaneously broken global symmetry ($\mathrm{G} \to \mathrm{H}$). Pions are pseudo-NGB of $\mathrm{SU(2)_L \times SU(2)_R / SU(2)_{Isospin}}$.
- Quark masses & electromagnetic interactions explicitly break chiral symmetry: $m^2_{\pi^+} - m^2_{\pi^0} \sim \frac{e^2}{16\pi^2}\Lambda^2_{\mathrm{QCD}}$.

  Gauge/Yukawa interactions *explicitly* break G. $\boxed{m_h^2 \sim \frac{g^2}{16\pi^2}\Lambda^2 \rightsquigarrow \Lambda \sim 1 \text{ TeV}}$.

  Too low Λ, disfavored!!
- If we can arrange, $m_h^2 \sim \frac{g_1^2 g_2^2}{(16\pi^2)^2}\Lambda^2$, then $\Lambda \sim 10$ TeV. Little hierarchy problem is solved without paying the price of fine-tuning. The idea of little Higgs is all about achieving this extra suppression factor.
- *Collective* symmetry breaking ($g_1 \neq 0, g_2 \neq 0$) $\Rightarrow$ Cutoff *postponed* to 10 TeV. UV completions can be weakly OR strongly coupled (composite, like pions).

Little Higgs potential



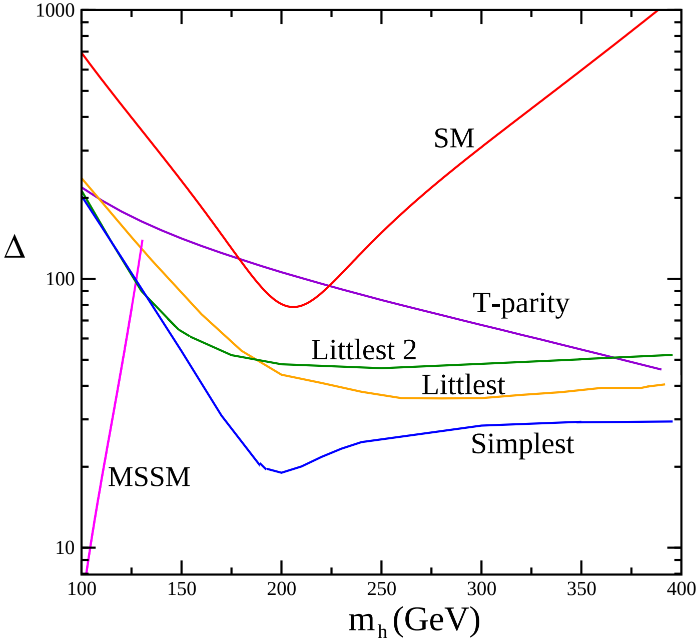


- $\mathrm{G} = \mathrm{SU}(5), \mathrm{H} = \mathrm{SO}(5), \mathrm{F} = [\mathrm{SU}(2) \times \mathrm{U}(1)]^2$: Littlest
  $G = [\mathrm{SU}(3) \times \mathrm{U}(1)]^2, \mathrm{H} = [\mathrm{SU}(2) \times \mathrm{U}(1)]^2, \mathrm{F} = \mathrm{SU}(3) \times \mathrm{U}(1)$: Simplest
- $V(h) = -\frac{g_{\mathrm{SM}}^4 f^2}{16\pi^2}(h^\dagger h) + g_{\mathrm{SM}}^2 (h^\dagger h)^2$.
  Large top quark Yukawa coupling responsible for generating the ‘minus’ sign.
- $m_h^2 \sim \frac{g_{\mathrm{SM}}^4}{16\pi^2} f^2 \ln\left(\frac{\Lambda^2}{f^2}\right), \quad f^2 \to F^2 = f^2 + \frac{\Lambda^2}{16\pi^2}$
- Same statistics cancellation of Λ^2 divergence in m_h^2 at one loop.

Little Higgs - EWPT & Fine-tuning

(Csaki,Hubisz,Kribs,Meade,Terning,Hewett,Petriello,Rizzo,Chen,Dawson, Noble,Perelstein)

- EWPT: $\mathcal{O}_T = \left|H^\dagger D_\mu H\right|^2$ and $\mathcal{O}_S = \left(H^\dagger \sigma^a H\right) W^a_{\mu\nu} B_{\mu\nu}$. $\boxed{S \sim v^2/f^2}$.
- $f > (2-5)$ TeV in a general class of little Higgs models due to tree level mixing of SM particles with the new particles. In littlest Higgs model large contribution to $\mathcal{O}_T$ from $H^T \Phi H$, where Φ is a triplet.
- With T-parity ($H \to H$ but $\Phi \to -\Phi$), it is possible to allow $f \sim 500$ GeV. (Cheng,Low)



Casas, Espinosa, Hidalgo (2005)

EWPT vs Naturalness

- To keep the Higgs quartic coupling to be $\mathcal{O}(1)$ requires tuning.
- Fine-tuning in little Higgs larger than in MSSM.

Little Higgs - Collider signatures

- New scalars: Han et al (2003), Hektor et al (2007)
  Doubly charged scalar as a component of a complex triplet scalar, decaying into like-sign dileptons ($\Phi^{++} \to \ell^+\ell^+$).
  Resonant enhancement of $W_L W_L \to W_L W_L$ by Φ^{++} mediation. Search up to $m_{\Phi^{++}} \sim 1.5$ TeV with 300 fb^{-1}.

- New fermions: Hubisz et al (2006)
  Colored vector-like T quarks: $\Gamma(T \to th) \approx \Gamma(T \to tZ) \approx \frac{1}{2}\Gamma(T \to bW)$.
  When T-parity is conserved, both $t_+ \equiv T$ and t_- exist.
  $\sigma(gg \to t_- t_-) \approx 0.3$ pb for $m_{t_-} = 800$ GeV. Decay $t_- \to A_H t$, where A_H is stable and a DM candidate.

- New gauge bosons: Han et al (2003), Burdman et al (2003)
  Heavy gauge bosons would decay as $Z_H \to W_L^+ W_L^-, W_H \to W_L Z_L, Z_H \to Z_L h$.
  Brs will follow definite pattern. About 30000 Z_H can produced with 100 fb^{-1} data.

Composite Higgs

(Agashe,Contino,Pomarol,Nomura,Barbieri,Rattazzi,Grojean,Espinosa, Muehlleitner,$\cdots$)
Better realization of little Higgs: Composite bound state from a strongly interacting sector.

- Strong sector: $\mathrm{G} \to \mathrm{H}$ at a scale $f(> v)$. G/H contains Higgs. Ex: $\mathrm{SO}(5)/\mathrm{SO}(4)$.
- Holographic description: $A_5^{(0)}$ of a 5d warped model can be the Higgs, which is massless at tree level and acquires finite mass at one-loop (Serone 2009).

Collider test of compositeness

- $g_{hff} = g_{hff}^{\mathrm{SM}}\left(1 - C_f \xi\right)\,,\quad g_{hVV} = g_{hVV}^{\mathrm{SM}}\left(1 - C_V \xi\right)$,

  where $\boxed{\xi \equiv \frac{v^2}{f^2}}$. $\xi \sim (20-30)\%$ (from EWPT).

  $\sigma_h \times (\mathrm{Br})_h$ can be measured with 20% precision at LHC (Duhrssen et al).
- Scattering amplitude $A(VV) \sim \frac{s}{f^2}$ $\Rightarrow$ Excess events in $V_L V_L \to V_L V_L$ scattering.
- $q\bar{q}, gg \to q^*_{5/3}\bar{q}^*_{5/3} \to W^+ t W^+ t \to W^+ W^+ b W^- W^- \bar{b}$. Highly energetic same sign leptons, plus 6 jets two of which two are tagged b jets.

$$\boxed{\textbf{SM}} \xleftarrow{0 \leftarrow \xi} \boxed{\textbf{Composite}} \xrightarrow{\xi \to 1} \boxed{\textbf{Higgsless/TC RIP!}}$$

Few things to ponder

- Is the End of the World Near?? Vacuum metastable! $\Lambda \sim 10^{10-12}$ GeV. But $\tau_{vac} > \tau_{\rm univ}$. Why didn't we slip into true vacuum in early universe?
- Hierarchy problem can be addressed by 'relaxion' mechanism. A somewhat unusual axion and inflation together solve this problem *without* any weak scale dynamics.

- $(g-2)_\mu$ is still an enigma!
- Branching ratio of $h \to \mu\tau$ is $(0.84 \pm 0.38)\%$ (CMS) and $(0.77 \pm 0.63)\%$ (ATLAS).
- W' explanation for excess events in ATLAS and CMS ??

Does Nature respect Naturalness?

Appearance of new states restoring naturalness!

$m_{\pi^+}^2 - m_{\pi^0}^2 \sim \frac{\alpha}{\pi}\Lambda^2 < (4\ \mathrm{MeV})^2$, where Λ is the maximum energy of the effective theory of pions. This demands $\Lambda < 850$ GeV. The ρ meson appears at 750 GeV, and the composite structure of pion softens the EM contribution. $m_{\pi^+}^2 - m_{\pi^0}^2 \sim \frac{\alpha}{\pi}\frac{m_\rho^2 m_{a_1}^2}{m_{a_1}^2 - m_\rho^2}\ln\left(\frac{m_{a_1}^2}{m_\rho^2}\right)$

Cancellation does not necessarily mean fine-tuning!

$\frac{\Delta m_K}{m_K} = \frac{G_F^2 f_K^2}{6\pi^2}\sin^2\theta_c\Lambda^2$. LHS $= 7 \times 10^{-15}$ tells us $\Lambda < 2$ GeV. Mass of charm quark was deduced through the implementation of GIM mechanism.

Outlook

GB, Rep. Prog. Phys. 74 (2011) 026201.

- Which symmetry protects m_h? SUSY? NGB? gauge symmetry?
- All models based on calculability $\Rightarrow$ $\boxed{M_Z = \Lambda_{\rm NP} f(a_i)}$
- Is Higgs strictly 'elementary' or 'composite'? SUSY vs Extra-dim?
- Revival of interest in strongly interacting light Higgs models.
- Goal: 3-fold. (i) Unitarize, (ii) check EWPT, (iii) Naturalness?
- Naturalness or observability vs EWPT? Tension!
- Tension between hierarchy and flavor theories!

Future agenda will be set by the next few years of LHC run. Look for new resonances, rare decays of Higgs and top, more precise measurements of Higgs BRs.