Lattice Study of Symmetry Breaking (?) in Planar QED

Nikhil Karthik* and Rajamani Narayanan

Department of Physics
Florida International University, Miami

Free Meson Seminar, TIFR

Nov 23, 2015

Table of Contents

QED in 2+1 dimensions

$$L = \overline{\psi}\sigma_\mu\left(\partial_\mu + iA_\mu\right)\psi + m\overline{\psi}\psi + \frac{1}{2g^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2$$

- $\psi \rightarrow$ 2-component fermion field
- $g^2 \rightarrow$ coupling constant of dimension $[\text{mass}]^1 \Rightarrow$ Super-renormalizable
- Define the UV regulated theory on Euclidean l^3 torus
- Scale setting $\Rightarrow g^2 = 1$

- Notation: $C = \sigma_\mu\left(\partial_\mu + iA_\mu\right)$
- A special property in 3d: $C^\dagger = -C$

- Aside from field theoretic interest, QED_3 relevant to high-T_c cuprates.

Parity, a defining feature

- Under parity

$$x_\mu \to -x_\mu$$

  the fields transform as

$$\begin{aligned} A_\mu &\to - & A_\mu \\ \psi &\to & \psi \\ \overline{\psi} &\to - & \overline{\psi}. \end{aligned}$$

- $m\overline{\psi}\psi \to -m\overline{\psi}\psi \Rightarrow$ Mass term breaks parity.

Phase of Fermion Determinant and Parity Anomaly

- Effective gauge action induced by the fermion:

$$\int \mathcal{D}\overline{\psi}\mathcal{D}\psi e^{-\overline{\psi}C(m,A)\psi} = \left|\det C(m,A)\right| e^{i\Gamma(A)},$$

  on fixed gauge field background A_μ
- $\Gamma_{\rm odd} \to -\Gamma_{\rm odd}$ under parity.
- $\Gamma_{\rm odd} \neq 0$ when $m \to \infty$ and $m \to 0$ (Niemi & Semenoff '83, Redlich '84).
- Reason in perturbation theory: Induced local Chern-Simons action

$$\Gamma_{\rm CS} = \frac{\kappa}{4\pi} \int F^*_\mu A_\mu d^3x.$$

- What are the non-perturbative aspects of Γ? Is there a parity-even phase? Is the phase still a local $\Gamma_{\rm CS}$ at finite m with $\kappa = \kappa(m)$?
- Importance: The only way to introduce Chern-Simons term on the lattice is through fermions.

Even number of flavors

- Two flavors of two component fermions: ψ_1 and ψ_2.
- Define parity transformation:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \sigma_1 \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$
$$\begin{pmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \end{pmatrix} \rightarrow -\sigma_1 \begin{pmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \end{pmatrix}$$

- Parity-even mass terms:

$$\begin{pmatrix} \overline{\psi}_1 & \overline{\psi}_2 \end{pmatrix} [m\sigma_3 + M\sigma_2] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Connection to 4-component fermions

- Construct Dirac fermions using the ansatz: $\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ and $\overline{\Psi} = \begin{pmatrix} \overline{\psi}_2 & \overline{\psi}_1 \end{pmatrix}$
- "Chiral" U(2) symmetry in the massless limit.

Dirac operator

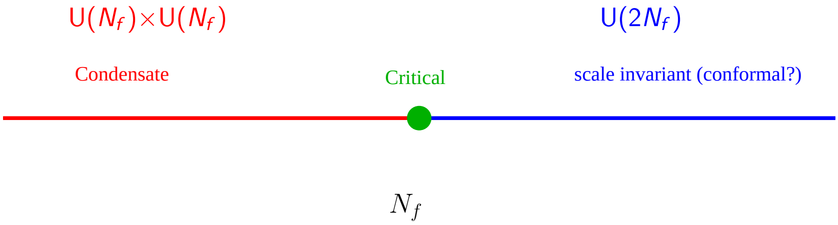
$$D = \begin{bmatrix} M & C + m \\ -(C+m)^\dagger & M \end{bmatrix}$$

- As long as $\sqrt{m^2 + M^2}$ is the same, all combinations are equivalent. We will use this in our lattice formulation.
- (Aside: by appropriately redefining parity, in general, the mass term has terms proportional to 1, $i\gamma_4$ and $i\gamma_5$.)

Table of Contents

Conjectured phase diagram

$N_f \rightarrow$ No. of 4-component fermions



Parity-even condensates: $\overline{\psi}_1\psi_1 - \overline{\psi}_2\psi_2, \quad \overline{\psi}_2\psi_1 - \overline{\psi}_1\psi_2, \quad \overline{\psi}_2\psi_1 + \overline{\psi}_1\psi_2$

Plausibility arguments

Large-N_f gap equation: $N_{fc} \approx 4$ (Appelquist *et al.* '88)

Assumptions: no wavefunction renormalization, and $\Sigma(p) \ll p$

Free energy argument: $N_{fc} = 1.5$ (Appelquist *et al.* '99)

- Contribution to free energy: bosons$\rightarrow$ 1 and fermions$\rightarrow$ 3/4
- IR $\Rightarrow 2N_f^2$ Goldstone bosons + 1 photon
- UV $\Rightarrow$ 1 photon + N_f fermions
- Equate UV and IR free energies

Previous Lattice Results

(Hands *et al.* '04) using square-rooted staggered fermions provides an upper-limit on condensate.

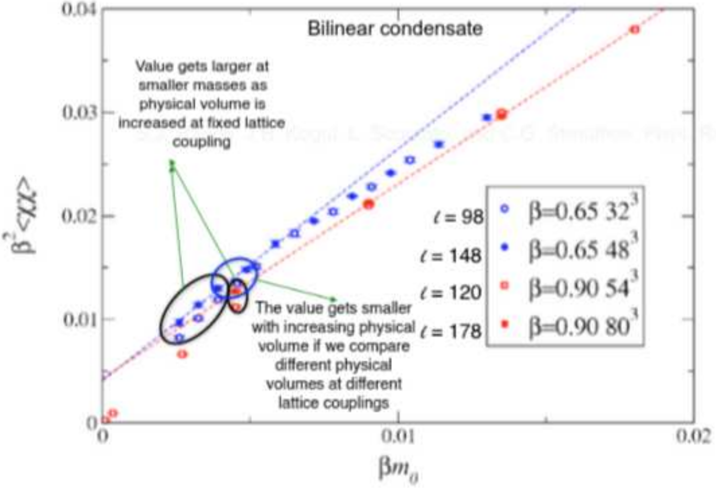


Table of Contents

Wigner's RMT

- Let a system with Hamiltonian H be chaotic at classical level.
- Let random matrix T, and H have same symmetries: UHU^{-1}
- H and T share universal features in the unfolded spectrum :

$$\lambda_i^{(u)} = \int_0^{\lambda_i} \langle \rho(\lambda) \rangle d\lambda$$

  Unfolding : Scale λ by the local eigenvalue density
- Universal features:
  - splitting s= $\lambda_{i+1}^{(u)} - \lambda_i^{(u)}$
  - eigenvalue correlations: number variance Σ_2.
- Unfolding loses information on the scales present in H.
- Eigenvalue repulsion is a typical feature.

RMT and Broken phase

- System with broken symmetry $\Rightarrow$ condensate Σ
- Banks-Casher $\Rightarrow$ Low-lying eigenvalues

$$\lambda \sim \frac{1}{\rho(0)} = \frac{\pi}{\Sigma V} \qquad \text{(level repulsion)}$$

- Natural to scale λ by ΣV rather than with $\rho \Rightarrow$ Microscopic quantities

$$z = \lambda V \Sigma \qquad \text{and} \qquad \rho_S(z) = \frac{1}{\Sigma V}\rho\left(\frac{z}{\Sigma V}\right).$$

- $\rho_S(z)$ is universal and reproduced by random T with the same symmetries as that of Dirac operator D. (Shuryak and Verbaarschot '93)
- Rationale: Reproduces the Leutwyler-Smilga sum rules from the zero modes of Chiral Lagrangian.
- Eigenvalues for which agreement with RMT is expected / momentum scale upto which only the fluctuations of zero-mode of Chiral Lagrangian matters:

$$\lambda < \frac{F_\pi}{\Sigma V^{2/3}} \qquad \text{(Thouless energy)}$$

RMT and Broken phase: Salient points

- Scaling of eigenvalues:

$$\lambda l \sim l^{-2}$$

- Look at ratios $\lambda_i/\lambda_j = z_i/z_j$. Agreement with RMT has to be seen without any scaling.

- The number of eigenvalues with agreement with RMT has to increase as

$$\frac{\text{(Thouless Energy)}}{\text{(Splitting)}} \sim l$$

Non-chiral RMT

- RMT partition function that has the same symmetries as that of QED_3 (Verbaarschot and Zahed '94)

$$Z_{\mathrm{RMT}} = \int \mathcal{D}T \begin{vmatrix} 0 & iT+m \\ -(iT+m)^\dagger & 0 \end{vmatrix}^{N_f} e^{-\mathrm{Tr}\,V(T^2)}$$

with Hermitean random matrix $T = T^\dagger$.

- Simple way to study this: Simulate Z_{RMT} with Hybrid Monte Carlo (HMC).

Table of Contents

Gauge-invariant lattice regularization of two-component fermions

- Choice: preserve either parity (Narayanan & Nishimura '97) or gauge symmetry (Kikukawa & Neuberger '98).
- This work: Wilson-Dirac fermions which are gauge-invariant (Coste & Lüscher '89).
- Wilson-Dirac operator chosen such that

$$D = \begin{cases} C_n - B + m & \text{if} \quad m > 0 \\ C_n + B - m & \text{if} \quad m < 0. \end{cases}$$

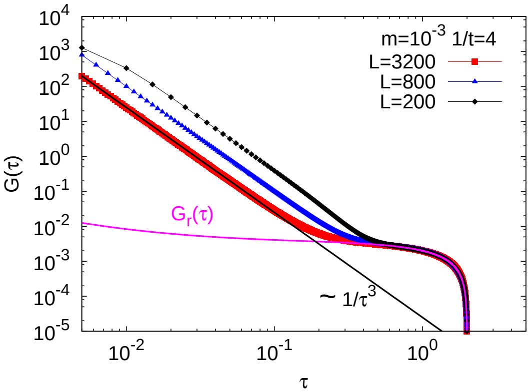
$C_n \rightarrow$ Naive Dirac operator
$B \rightarrow$ Wilson term which removes fermion doublers

- Parity covariant regularization: $\Gamma(-m) = -\Gamma(m)$.

Parity anomaly from the Wilson Term

Non-perturbative
Background fields

$$A_1 = A_1^p(\tau) - \frac{2\pi q_2}{l^2}\tau$$
$$A_2 = \frac{2\pi q_3}{l^2}x + A_2^p(\tau)$$
$$A_3 = \frac{2\pi h_3}{l}$$



$$\Gamma = \pi\,(q_2 + q_3 + q_2 q_3) - 2\pi h_3 q_3 - \int d\tau d\tau' G(\tau - \tau') A_1^p(\tau) A_2^p(\tau').$$

$$G(\tau) = G_r(\tau) + \frac{1}{L}\left(\frac{k}{\tau^3}\right)$$ (N.K. and Narayanan '15)

Wilson-Dirac Formulation

- Continuum:

$$S_{\mathrm{F}} = \overline{\psi}_1 \, (C + m) \, \psi_1 + \overline{\psi}_2 \, (C - m) \, \psi_2$$

- Parity-covariant Wilson-Dirac fermions$\Rightarrow$ Add Wilson term $\pm B$ to $\mp m$.

$$S_{\mathrm{F}} = \overline{\psi}_1 \, (C_n - B + m) \, \psi_1 + \overline{\psi}_2 \, (C_n + B - m) \, \psi_2$$

Four-component Wilson-Dirac operator along with diagonal mass term:

$$D = \begin{bmatrix} M & C_n - B + m \\ -(C_n - B + m)^{\dagger} & M \end{bmatrix}$$

Wilson-Dirac Formulation

- Continuum:

$$S_{\mathrm{F}} = \overline{\psi}_1 \, (C + m) \, \psi_1 + \overline{\psi}_2 \, (C - m) \, \psi_2$$

- Parity-covariant Wilson-Dirac fermions⇒ Add Wilson term $\pm B$ to $\mp m$.

$$S_{\mathrm{F}} = \overline{\psi}_1 \, (C_n - B + m) \, \psi_1 + \overline{\psi}_2 \, (C_n + B - m) \, \psi_2$$

Four-component Wilson-Dirac operator along with diagonal mass term:

$$D = \begin{bmatrix} M & C_n - B + m \\ -(C_n - B + m)^{\dagger} & M \end{bmatrix}$$

Wilson-Dirac Formulation

- Continuum:

$$S_{\mathrm{F}} = \overline{\psi}_1 \left(C + m\right) \psi_1 + \overline{\psi}_2 \left(C - m\right) \psi_2$$

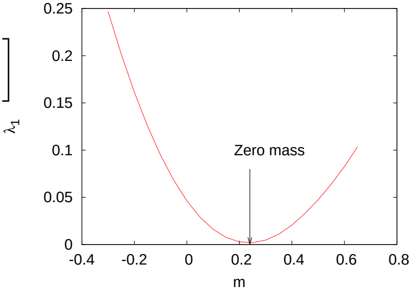
- Parity-covariant Wilson-Dirac fermions⇒ Add Wilson term $\pm B$ to $\mp m$.

$$S_{\mathrm{F}} = \overline{\psi}_1 \left(C_n - B + m\right) \psi_1 + \overline{\psi}_2 \left(C_n + B - m\right) \psi_2$$

Four-component Wilson-Dirac operator along with diagonal mass term:

$$D = \begin{bmatrix} M & C_n - B + m \\ -\left(C_n - B + m\right)^{\dagger} & M \end{bmatrix}$$

m $\rightarrow$ tune mass to zero as Wilson fermion has additive renormalization



Wilson-Dirac Formulation

- Continuum:

$$S_{\mathrm{F}} = \overline{\psi}_1 \left(C + m\right) \psi_1 + \overline{\psi}_2 \left(C - m\right) \psi_2$$

- Parity-covariant Wilson-Dirac fermions$\Rightarrow$ Add Wilson term $\pm B$ to $\mp m$.

$$S_{\mathrm{F}} = \overline{\psi}_1 \left(C_n - B + m\right) \psi_1 + \overline{\psi}_2 \left(C_n + B - m\right) \psi_2$$

Four-component Wilson-Dirac operator along with diagonal mass term:

$$D = \begin{bmatrix} M & C_n - B + m \\ -\left(C_n - B + m\right)^\dagger & M \end{bmatrix}$$

M $\rightarrow$ fermion mass

Simulation details

Parameters

- L^3 lattice of physical volume l^3
- Non-compact gauge-action: $\beta = \frac{2L}{l}$

Improvements

- 1 level of HYP smeared links (on gauge fields instead of gauge-links) used in Dirac operator
- Clover term with κ_{SW} to bring the tuned mass m closer to zero

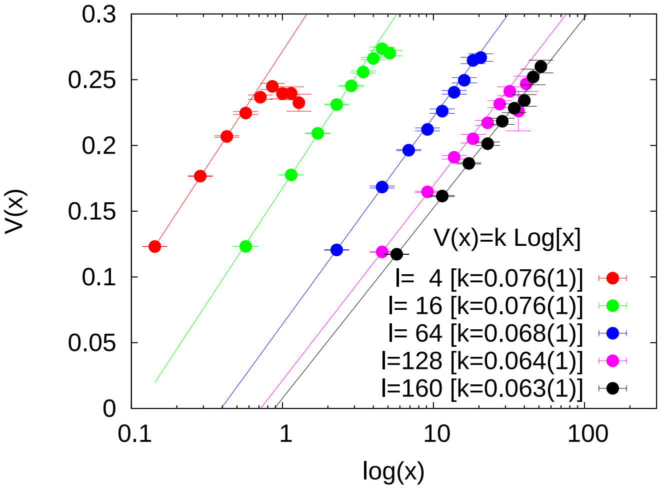
Statistics

- Standard Hybrid Monte-Carlo with N_f flavors of pseudo-fermions
- 14 different l from $l = 4$ to $l = 250$
- 4 different lattice spacings: $L = 16, 20, 24$ and 28
- $\sim$15k trajectories at 14 different $l \Rightarrow 500 - 1000$ independent cnfgs

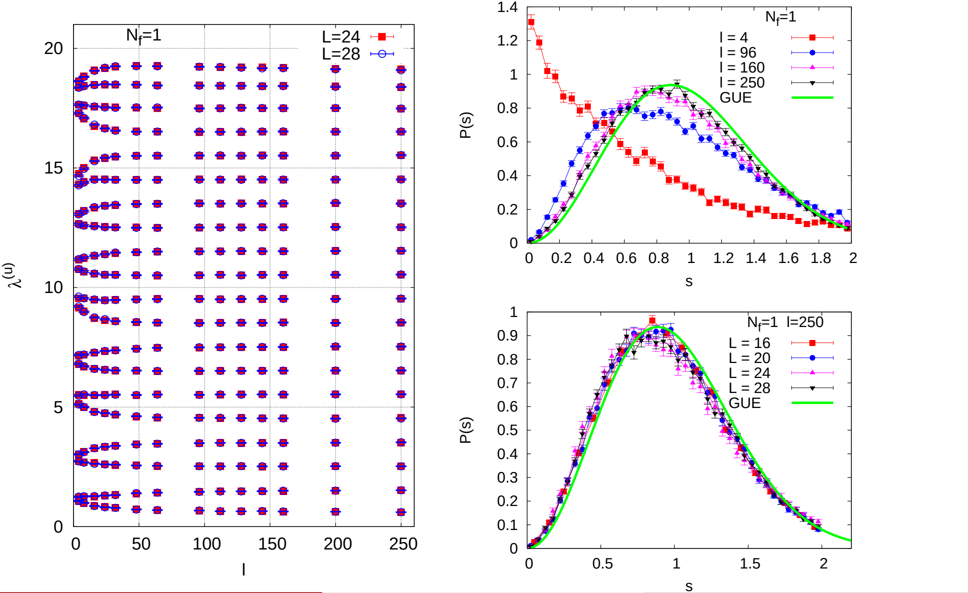
Table of Contents

Sanity check: Logarithmic confinement

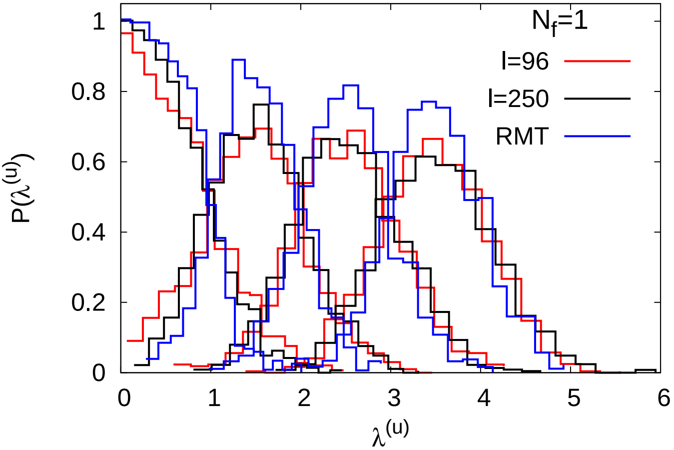
$t \times x$ Wilson loop $\rightarrow \log(\mathcal{W}) = A + V(x)t$



Unfolded eigenvalue spacing (Volumes are large enough)

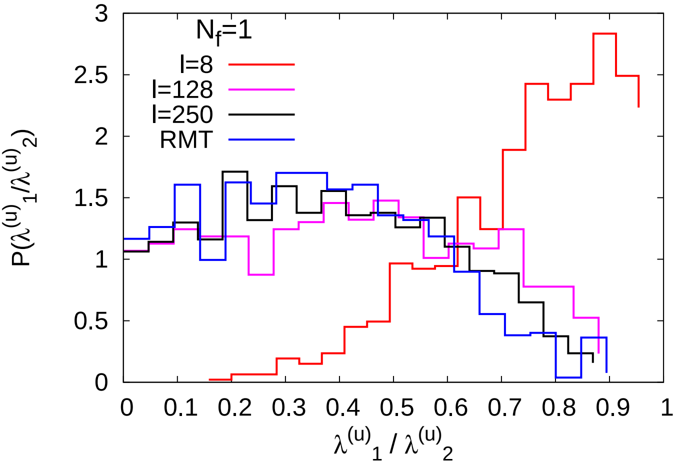


Unfolded Eigenvalue Distributions



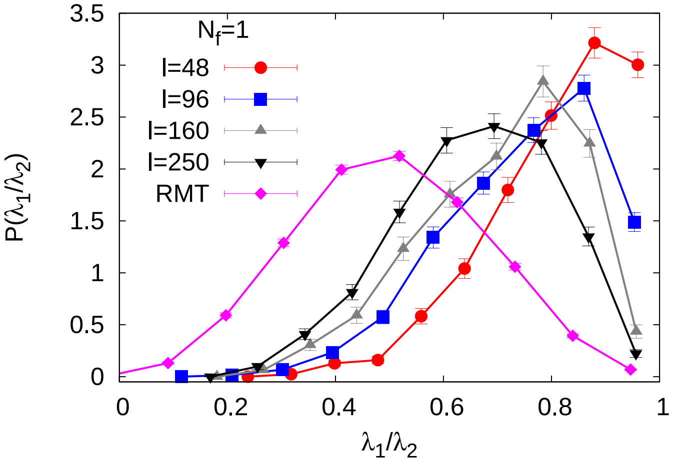
Correlations in unfolded spectrum

Agreement with non-chiral RMT at unfolded level.



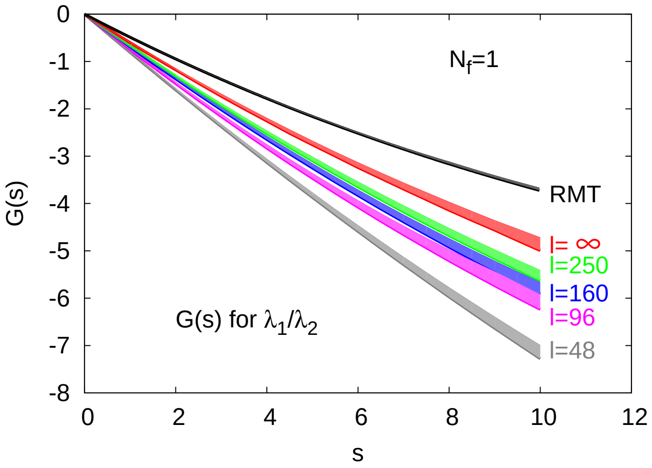
Distribution of ratio λ_1/λ_2

Large-volume dependence is seen.



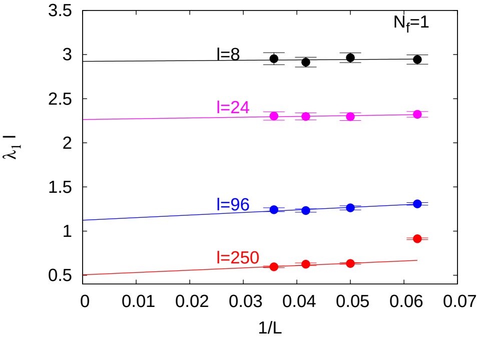
Distribution of ratio λ_1/λ_2: $l \to \infty$

Cummulant generating function $G(s) = \int P(x) e^{-sx} dx$

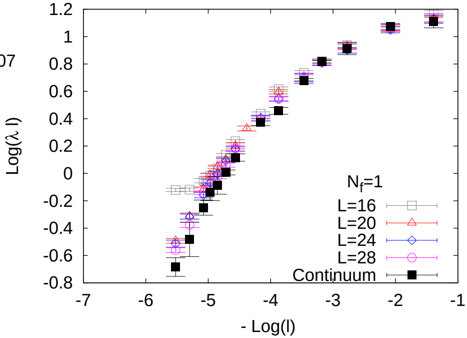


Use [1/1] Padé for extrapolation. No agreement with non-chiral RMT is seen.

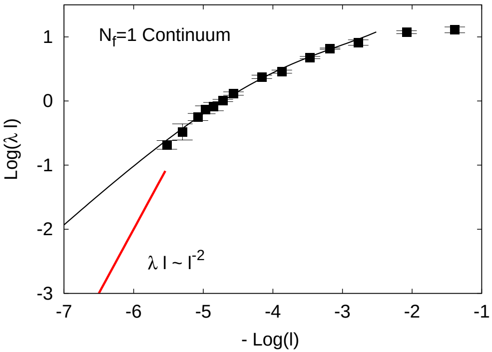
Lattice spacing effect in low eigenvalues



Small $1/L$ lattice corrections seen.



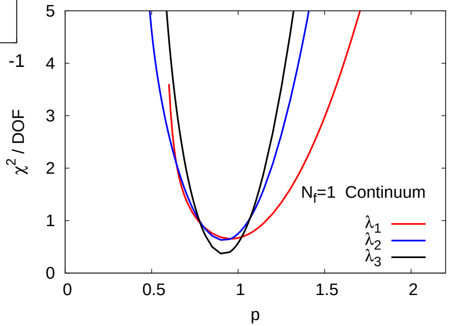
Volume dependence of low eigenvalues



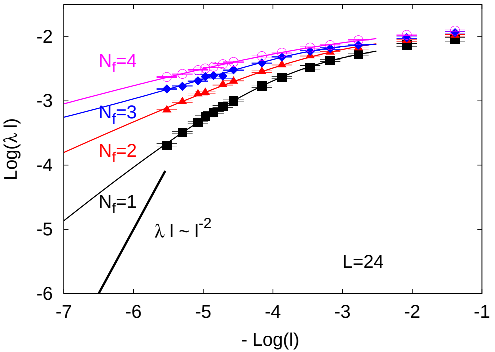
Ansatz:

$$\log(\lambda l) = \frac{A + (p + \frac{B}{l})\log(l)}{1 + \frac{C}{l}}$$

- $\lambda l \propto l^{-1}$ seems to be prefered.
- The condensate scenario, $\lambda l \propto l^{-2}$ seems to be ruled out.

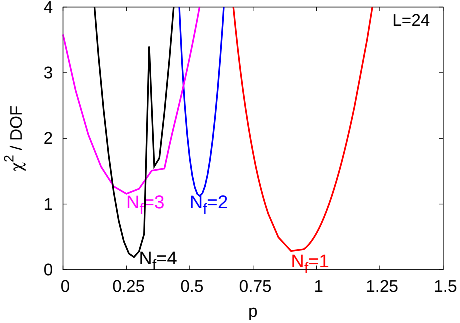


l-dependence for $N_f = 1 \dots 4$



Ansatz:

$$\log(\lambda l) = \frac{A + (p + \frac{B}{l}) \log(l)}{1 + \frac{C}{l}}$$



- Large-N_f limit: Comparable physics at same $N_f l$
- p decreases with N_f ($p \sim \frac{1}{N_f}$?)

Inverse Participation Ratio (IPR)

- For normalize eigenvectors of D

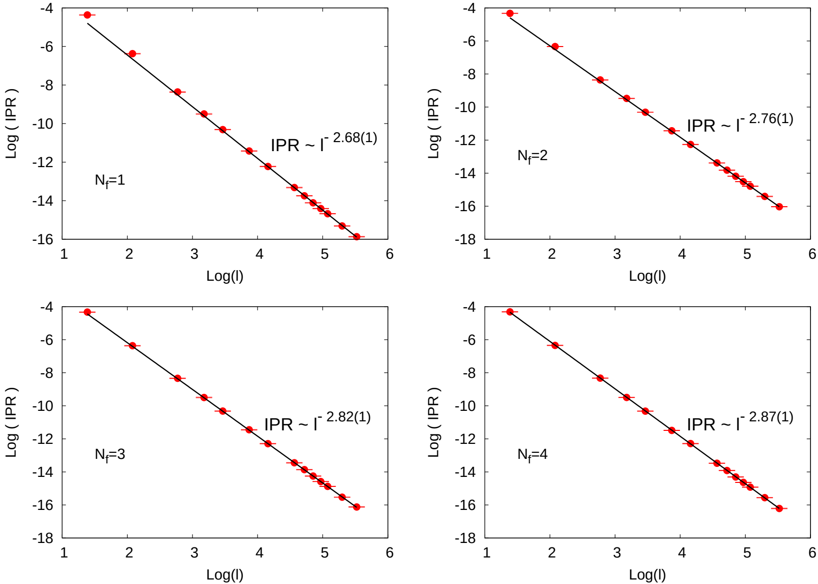
$$\mathrm{IPR} = \int |\psi(x)|^4 d^3x$$

- Volume scaling

$$\mathrm{IPR} \propto l^{-(3-\eta)}$$

- RMT $\rightarrow \eta = 0$.
- Localized eigenvectors $\rightarrow \eta = 3$.
- Eigenvector is multi-fractal for other values.

Inverse Participation Ratio (IPR)



Number Variance

Let $n(\lambda)$ be the number of eigenvalues below λ, then

$$\Sigma_2(n) = \mathrm{Var}(n)$$

- Metal: $\Sigma_2(n) \sim \log(n)$.
- Thouless energy $\Rightarrow$ Number of eigenvalues showing this behavior will increase linearly with l.

- Insulator: Poisson distribution has the property $\Sigma_2(n) = n$.

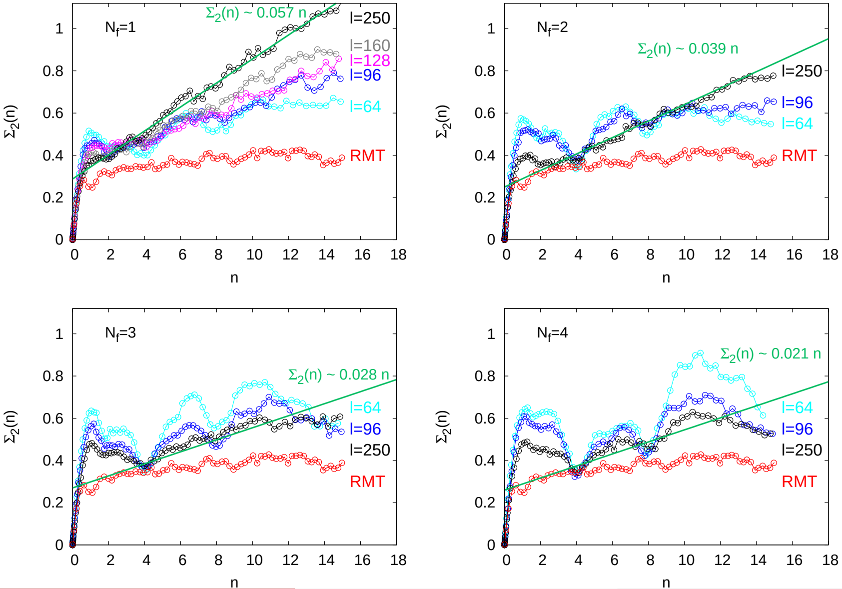
- Metal-insulator critical point (Altshuler *et al.* '88) :

$$\Sigma_2(n) = \chi n \quad \text{with} \quad \chi \ll 1$$

- Relationship to multi-fractal index of the eigenvector (Chalker *et al.* '96):

$$\eta = 6\chi$$

Number variance: Critical?



Conclusions