Aspects of warped extra-dimensions models

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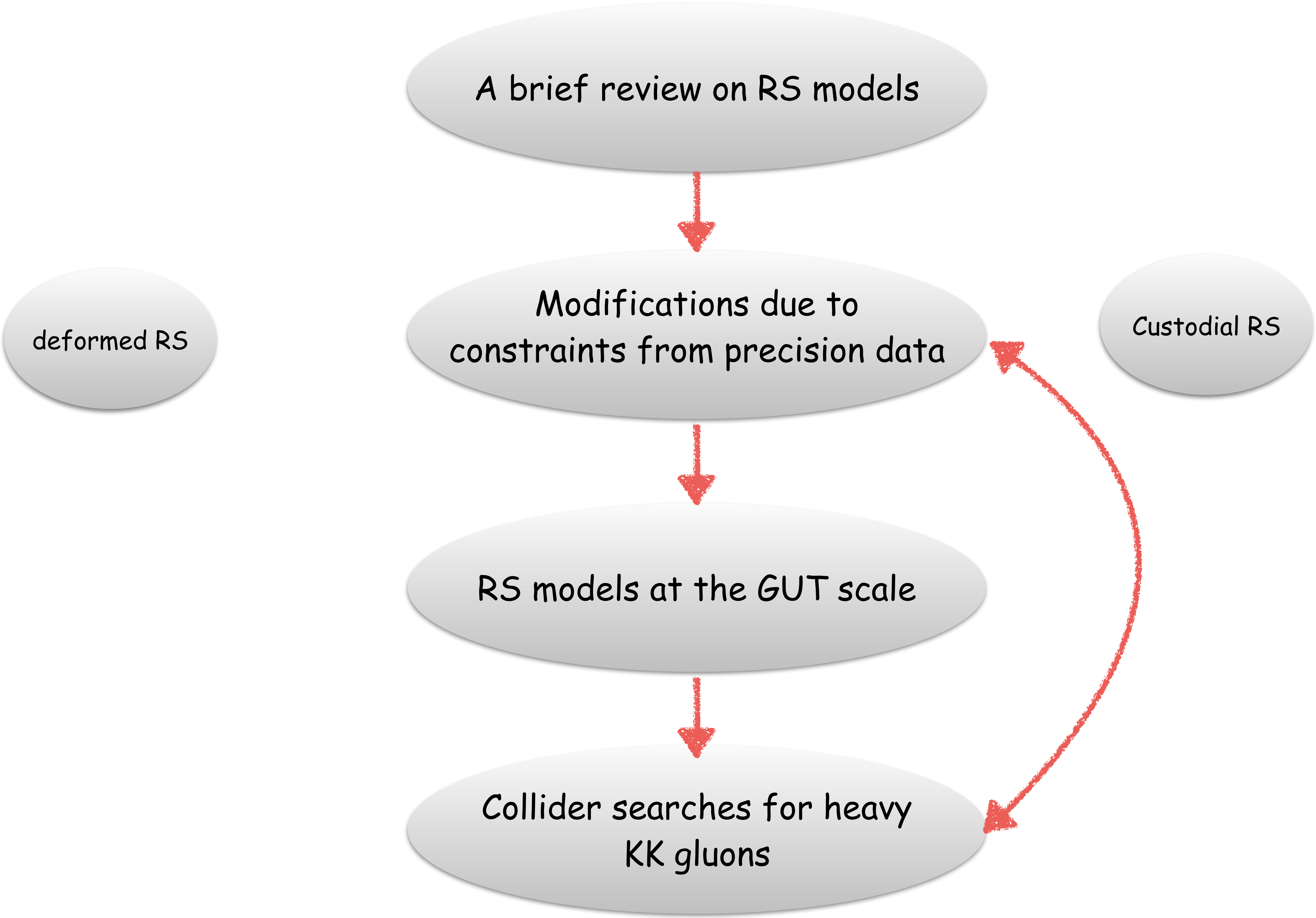


A brief review on RS models

deformed RS

Modifications due to constraints from precision data

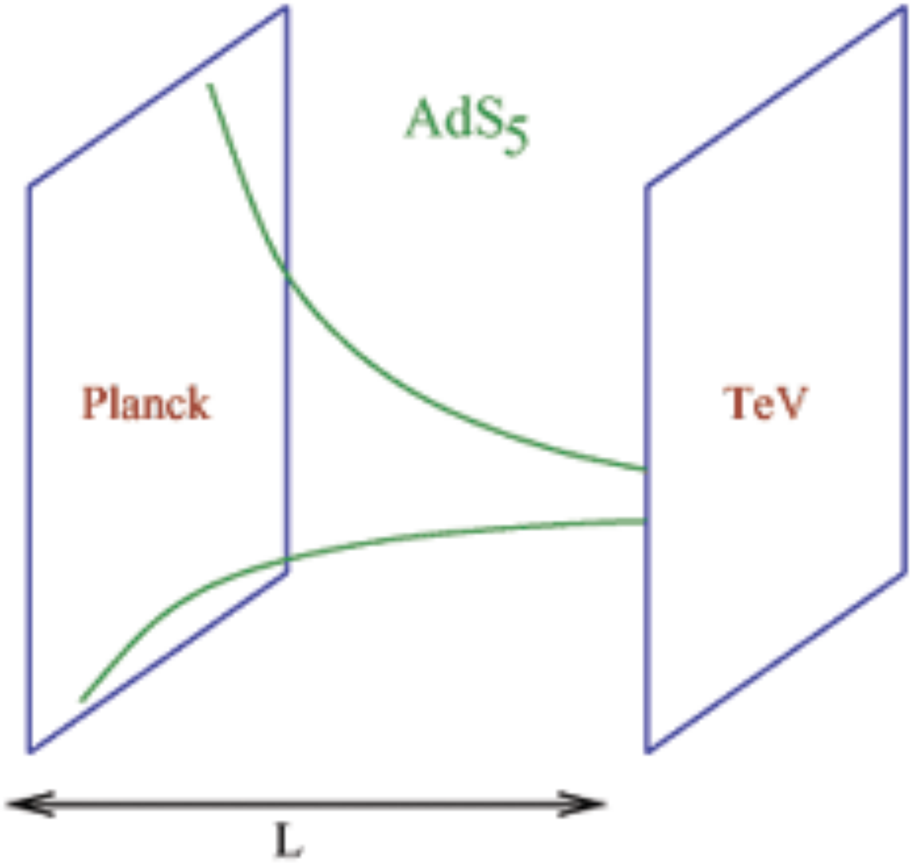
Custodial RS

RS models at the GUT scale

Collider searches for heavy KK gluons

Randall Sundrum Model

Randall, Sundrum '99



S_1/Z_2 compactified

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

effective 4D scale depends on the position in the bulk

One Fundamental gravity scale!!

Hierarchy problem Solved!!

$$M_{ew} = e^{-kL}M_{Pl}$$

- Solution to the Yukawa hierarchy problem #win

Gherghetta, Pomarol

Fermions in RS

Bulk femionic lagrangian in a warped background is written as

$$\mathcal{L}_{\text{fermion}} \quad = \quad e^{-3\sigma}\overline{\Psi}\left[i\gamma^{\mu}\partial_{\mu} - \gamma_5\, e^{-\sigma}\left(\partial_5 - 2\sigma'\right)\right]\Psi$$

where $\sigma = k|y|$. Expanding the bulk field as

$$\Psi(x,y) = \frac{1}{\sqrt{\pi R}}\sum_n \left[\psi_L^{(n)}(x) f_L^{(n)}(y) + \psi_R^{(n)}(x) f_R^{(n)}(y)\right]$$

But

5D theory is non-chiral

How do we reproduce chiral SM ?

$$\Big\downarrow Z_2$$

$$\Psi = \begin{bmatrix} \psi_L(+) \\ \psi_R(-) \end{bmatrix}$$

even -massless zero mode

odd -no zero mode

Zero mode for the Z_2 even field say $f_L^{(0)}$ satisfies

Localized profiles!!

$$e^{-\sigma}\left(\partial_y - 2\sigma'\right) f_L^{(0)} = 0 \quad \xrightarrow[\text{field re-definitions}]{\text{Using orthonormality}} \quad f_L^{(0)} = Ne^{k0.5(y-\pi R)}$$

Introducing a bulk mass term $m_{1/2} = c\sigma' = ck$ modifies the solution to

$$f_L^{(0)} = Ne^{(0.5-c)\sigma(y)}$$

Bulk Fields in RS

Gherghetta, Pomarol ‘00

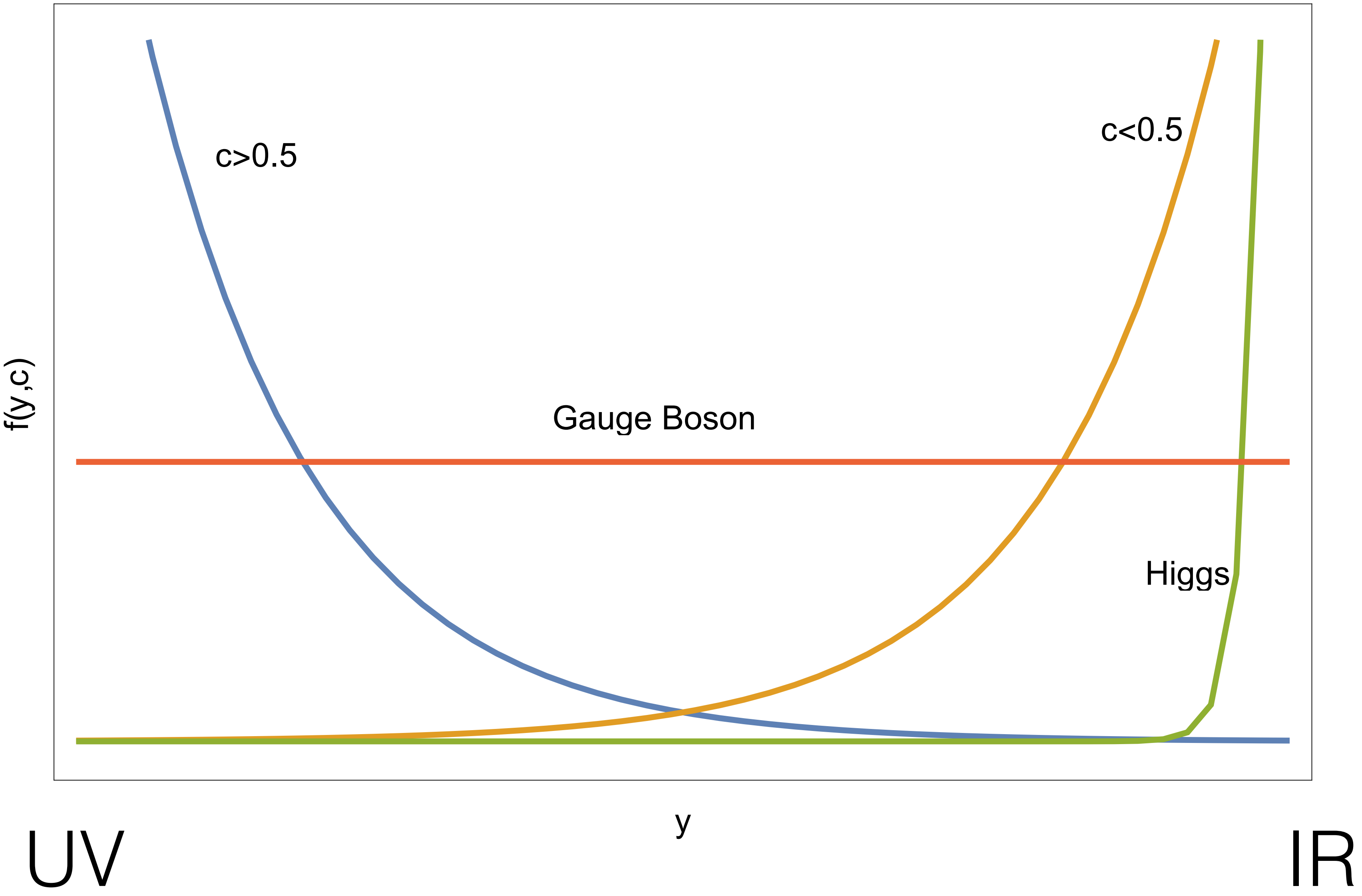
The normalised zero mode profiles are given as

SM fields!!

$$
\begin{aligned}
f_0^{(0)}(b,y) &= \sqrt{\frac{2(b-1)kR\pi}{e^{2(b-1)kR\pi}-1}}e^{(b-1)ky} \\
f_{1/2}^{(0)}(c,y) &= \sqrt{\frac{(1-2c)kR\pi}{e^{(1-2c)kR\pi}-1}}e^{(0.5-c)ky} \\
f_1^{(0)}(y) &= 1
\end{aligned}
$$

Like the `c' parameter, the `b' parameter for the scalar field controls localisation of its zero mode in the bulk

limit b $\rightarrow \infty$ is the TeV brane localised limit



SM Couplings are given by the \`overlap' of these profiles:

Yukawa hierarchy solved!!

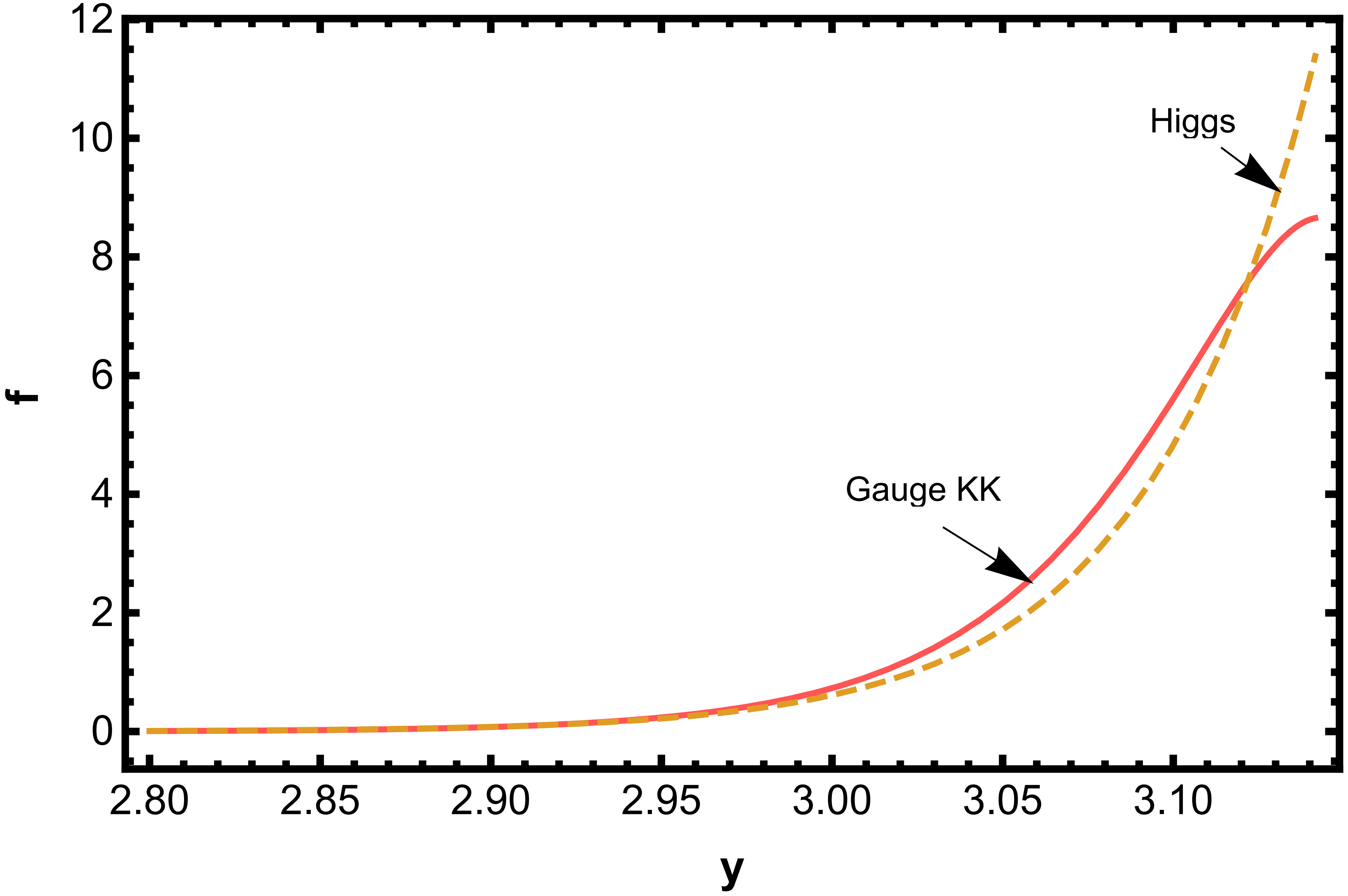
$$Y^{(4)} = Y^{(5)} \int_0^{\pi R} dy \; f_0^{(0)}(b,y) f_{1/2}^{(0)}(c_L,y) f_{1/2}^{(0)}(c_R,y)$$

Massive KK modes spoil the party!!

KK modes of all fields are localised near the IR brane

Mixing of SM states (zero modes) with massive KK states can give rise to potentially large contributions to various observables

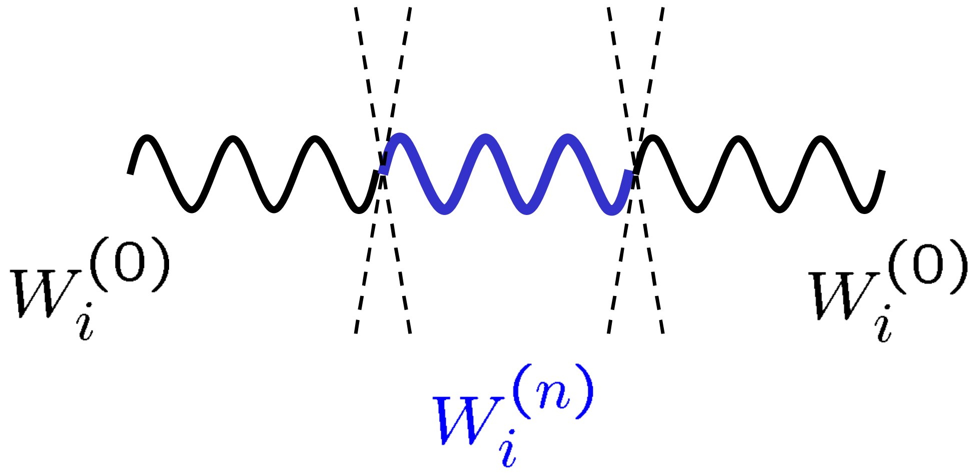
Higgs (vev) and first gauge KK mode



Large
overlap!!
Not good.

SM gauge states mix with their KK counterparts.

Mixing through Higgs vev.

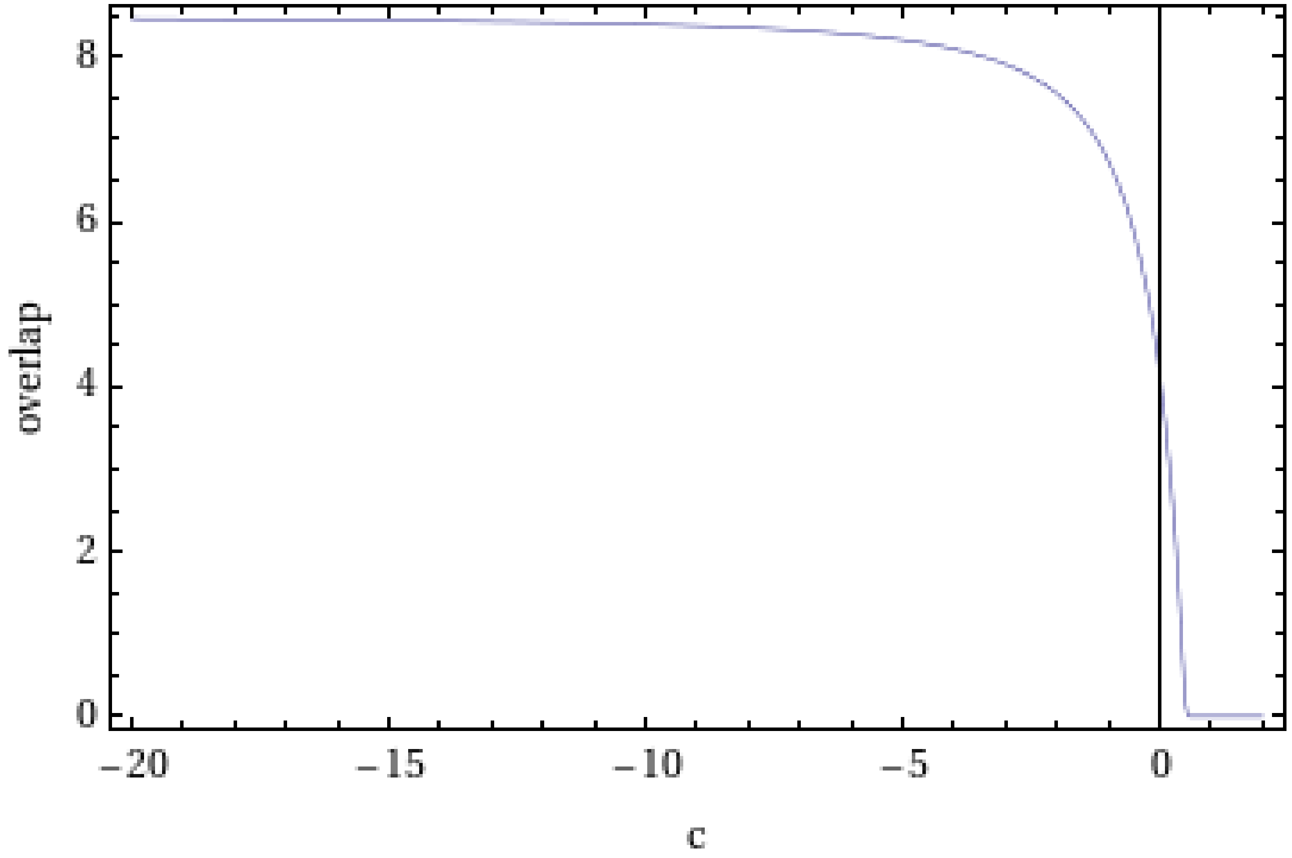


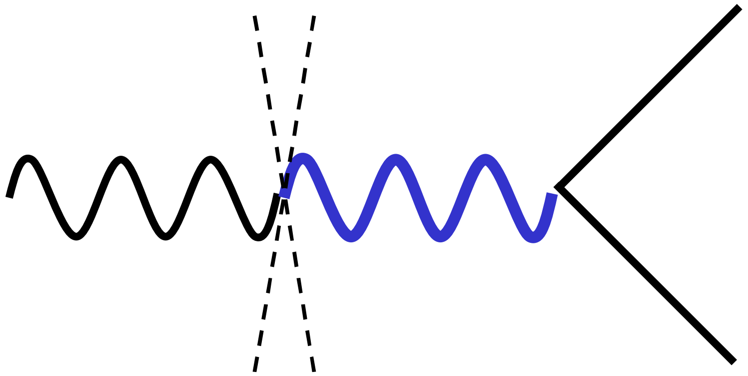
Higgs and the KK modes localised near IR brane!!

Large T parameter!!

The fermion coupling to the gauge bosons also get modified

Also induced due to mixing of gauge boson with corresponding KK states



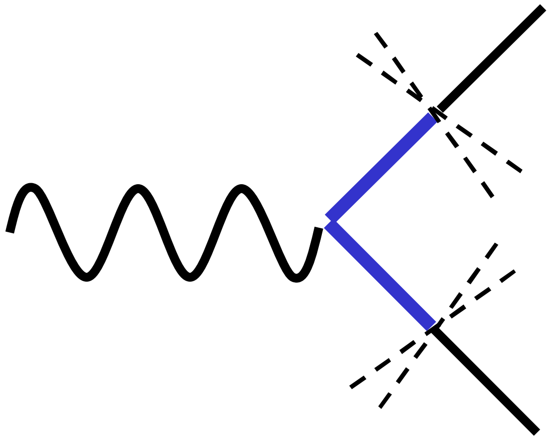


If all fermions near UV brane-universal corrections to the gauge couplings-S parameter

But Wait!! What about top mass?

The top doublet and the singlet must be localised close to Higgs

Large overlap of the doublet with the KK states



Contributions to Zbb!!

Wells, Zhang ‘14

Global fit

Input

Output

Input observables	m_Z	$91.1876(21)$	G_F	$1.1663787(6) \times 10^{-5}$
	$\alpha(m_Z)$	$7.81592(86) \times 10^{-3}$	$m_t(m_t)$	$173.20(87)$
	$\alpha_s(m_Z)$	$0.1185(6)$	m_H	$125.9(4)$
Output observables	m_W	$80.385(15)$	Γ_Z	$2.4952(23)$
	σ_{had}	$41.541(37)$	R_e	$20.804(50)$
	R_μ	$20.785(33)$	R_{tau}	$20.764(45)$
	R_b	$0.21629(66)$	R_c	$0.1721(30)$
	$sin^2\theta_e$	$0.23153(16)$	$sin^2\theta_b$	$0.281(16)$
	$sin^2\theta_c$	$0.2355(59)$	A^e_{FB}	$0.0145(25)$
	A^b_{FB}	$0.0992(16)$	A^c_{FB}	$0.0707(35)$
	A_b	$0.923(20)$	A_c	$0.670(27)$

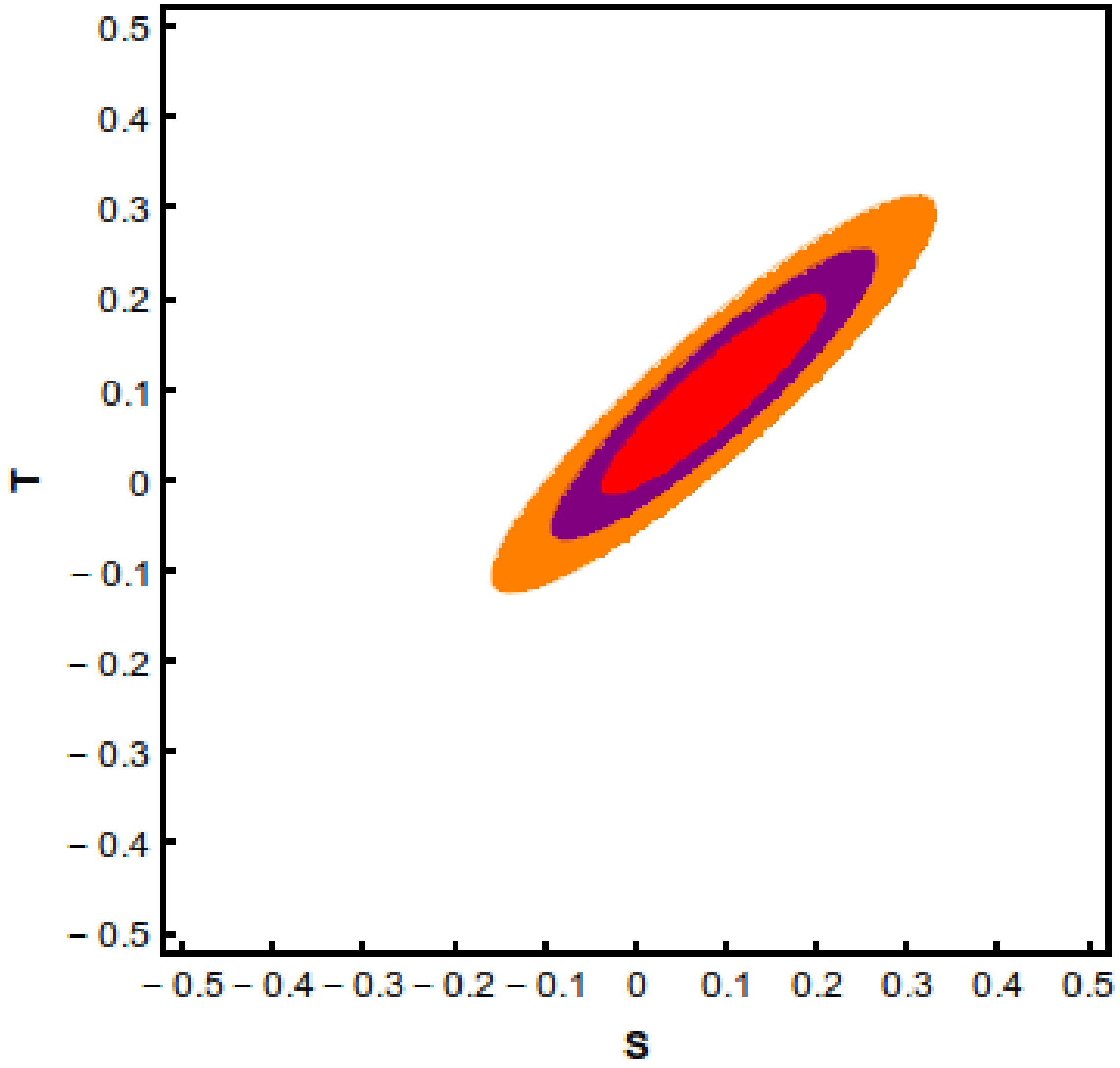
$$\hat{O}_i^{SM}(\{\hat{O}_{k'}\}) = \hat{O}_i^{ref} + \sum_{k'} \frac{\partial \hat{O}_i^{SM}}{\partial \hat{O}_{k'}} (\hat{O}_{k'} - \hat{O}_{k'}^{ref}) + \dots$$

New Physics

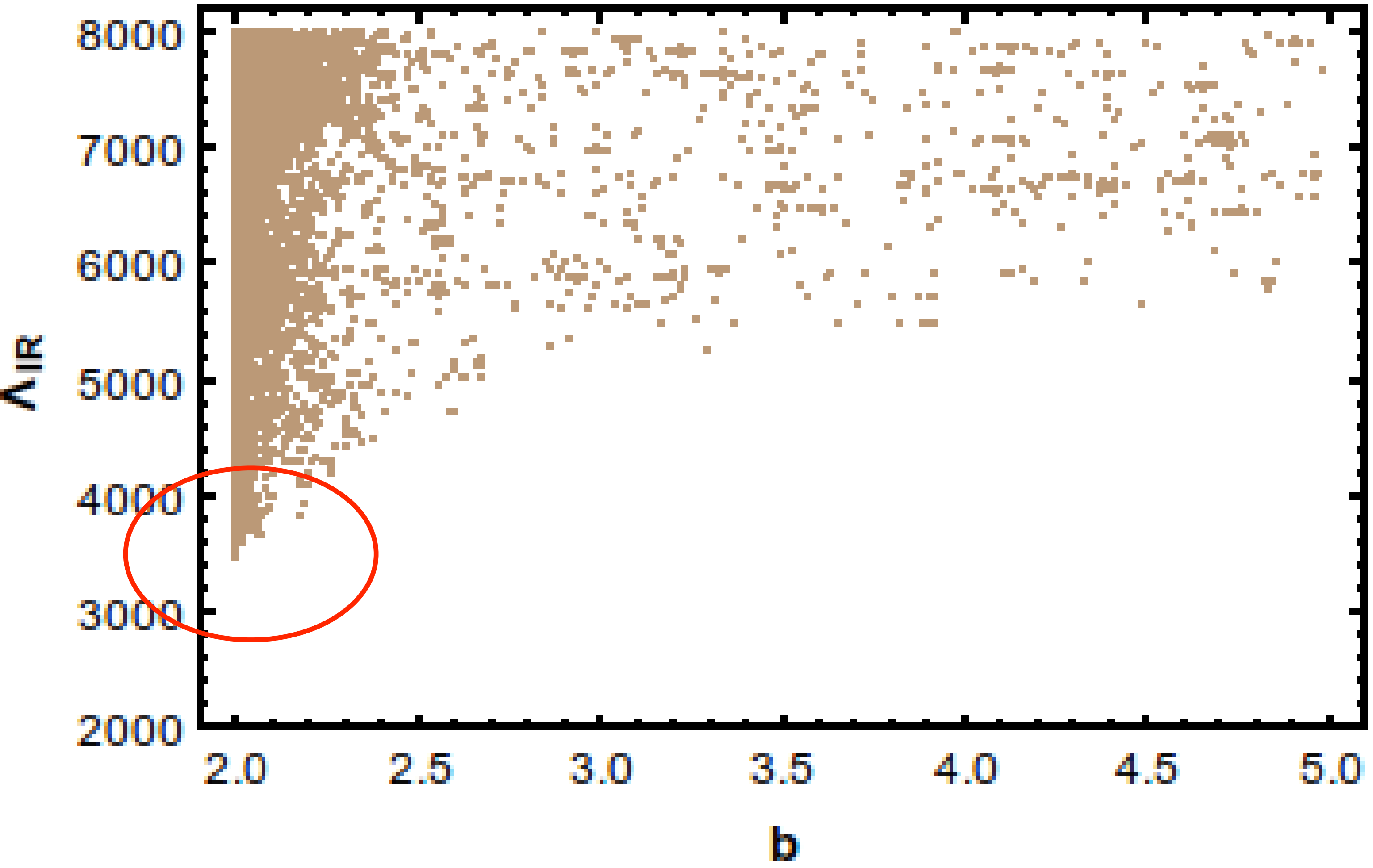
‘Universal’ effects can be captured by S and T parameters

Construct a chi-sq for all the SM observables including NP

$$\chi^2 = 25.0898 + 1102.39\ S^2 + 28.746\ S - 72.0085\ T - 2256.69\ ST + 1377.07\ T^2$$



Iyer, Sridhar, Vempati ‘15



$$\Lambda_{IR} \sim e^{-kR\pi} M_{Pl}$$

$$m_{KK}^{(1)} \sim 2.44\Lambda_{IR}.$$

$$\sim 9\ \mathrm{TeV}$$

Lowest KK modes are decoupled!!

The culprit: T parameter due to large coupling of Higgs to gauge KK modes

Is there a way to minimize/neutralize this effect?

Solution # 1

Is it possible to reduce the coupling of the KK modes to the Higgs?

The profiles are determined by the background geometry.
Change the metric?

Cabrer, Gersdorff, Quiros ‘ 10

$$A(y) = ky - \frac{1}{\nu^2}\log(1 - \frac{y}{y_s})$$

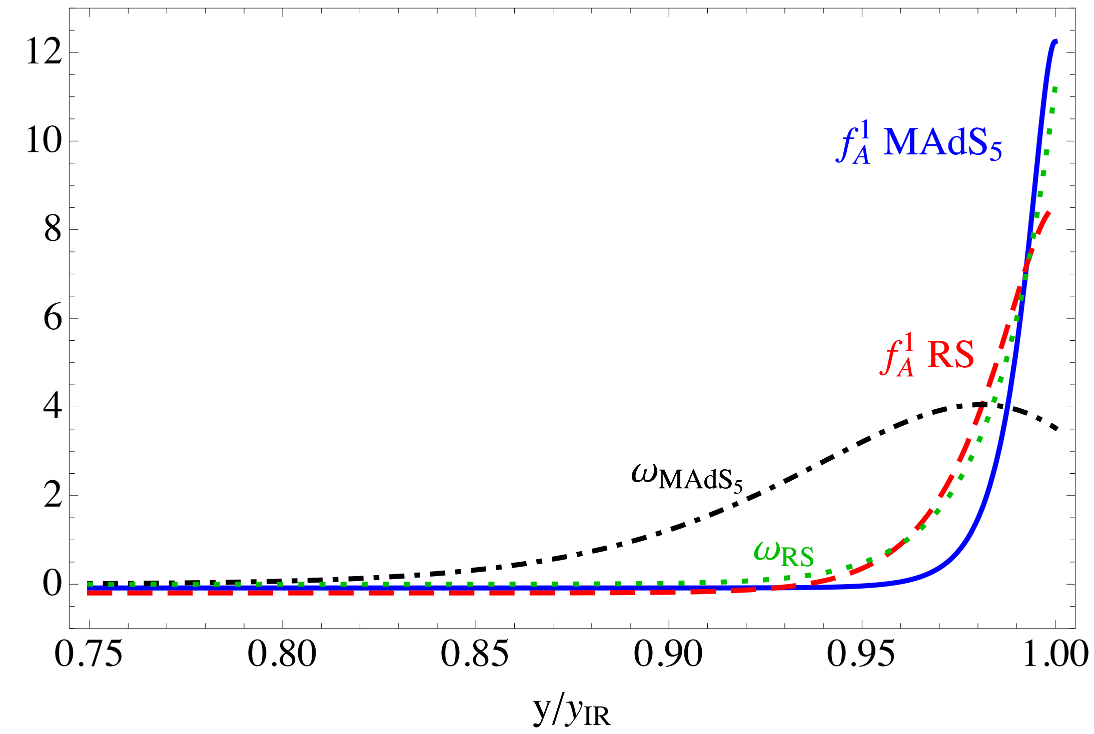
The position of the singularity is

$$y_s = y_1 + \Delta$$

outside the domain of integration!!

The background is AdS near the Planck brane.

moving away from the Planck brane results in departure from AdS



k ν chosen such that $A(y_1) \simeq 36$

Smaller bulk volume

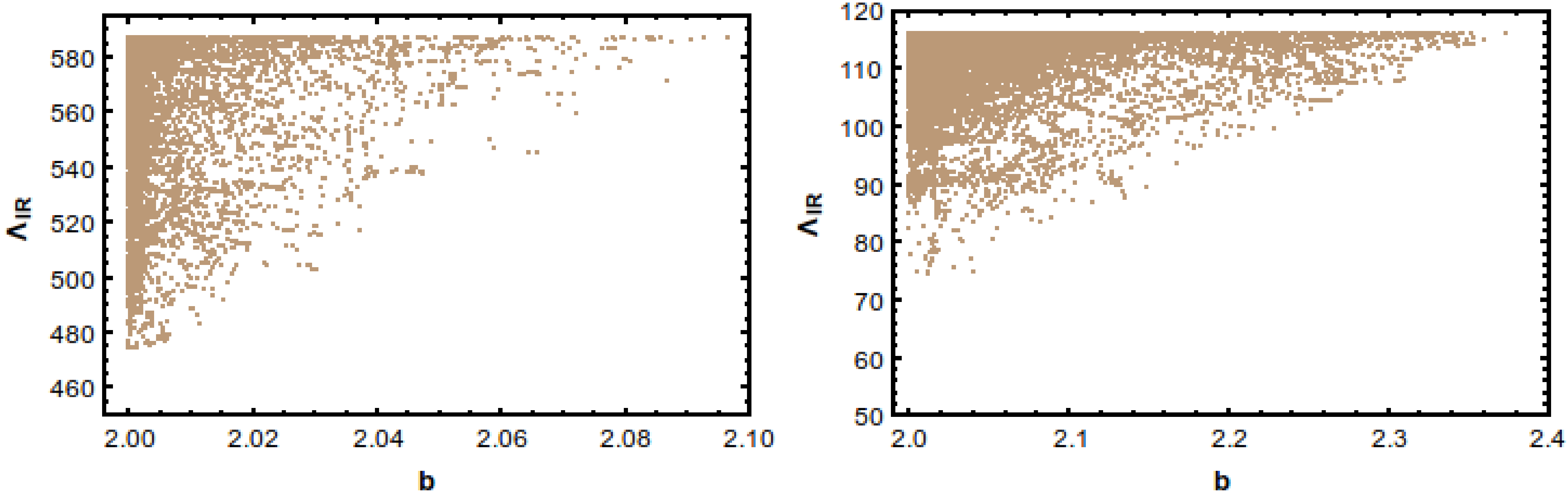


FIG. 4: Allowed parameter space in the $b - \Lambda_{IR}$ plane for deformed metric. Λ_{IR} is in GeV. The left panel corresponds to $\nu = 0.8$ and $\Delta = 1$ while the right panel corresponds to $\nu = 1$ and $\Delta = 0.1$

$$m^1_{kk} \sim j_{0,1} \frac{A'(y_1)}{k} \Lambda_{IR}$$

$$b \sim 2 \text{ and } m_{KK} \sim 2.3 \text{ TeV}$$

Agashe, Delgado, May, Sundrum '00

Introduce a bulk gauge symmetry

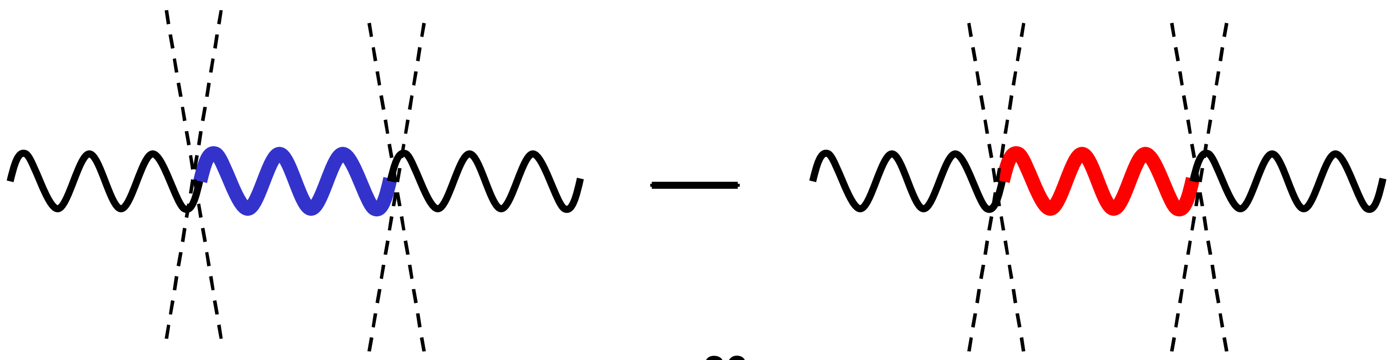
$$SU(2)_L \times SU(2)_R \times U(1)_X$$

$$\Big\downarrow \text{ broken to SM}$$

$$W^{1,2,3}_{L\mu}(++) \quad B_\mu(++) \quad W^{1,2}_{R\mu}(-+) \quad Z'_\mu(-+)$$

S parameter unchanged but T parameter receives new contributions

$T \propto$



$- \quad \sim 0$

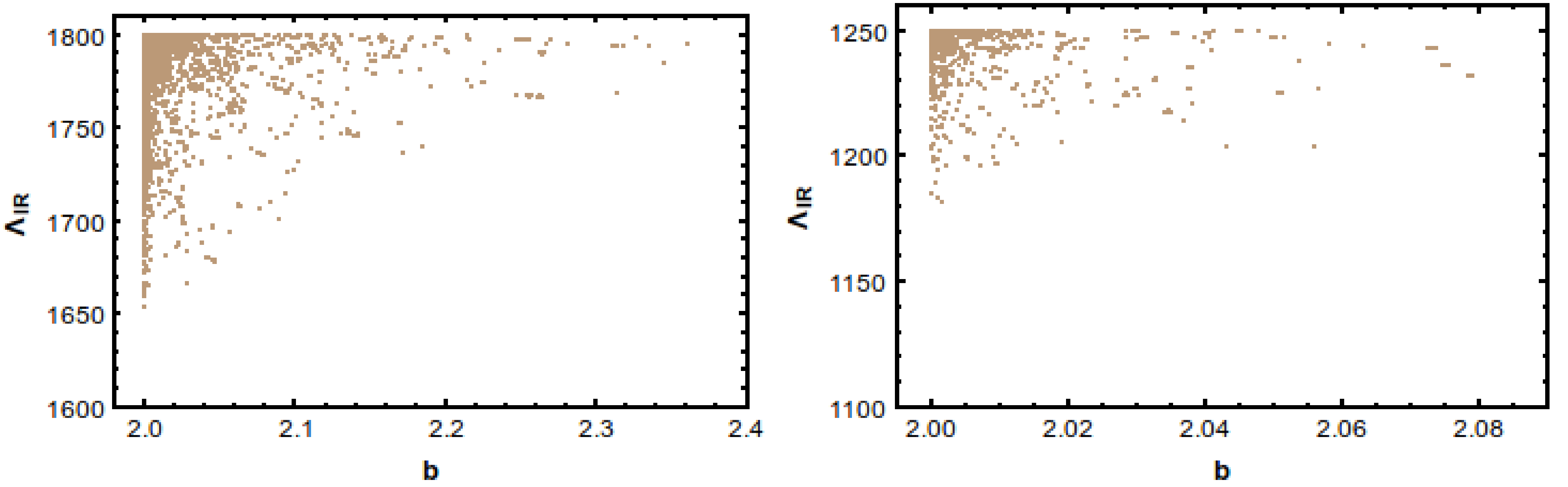


FIG. 5: Left panel shows the $b - \Lambda_{IR}$ parameter space when just the treel level computations of $S - T$ are taken into account. In the right panel, the loop contributions to the T parameter are also included. Λ_{IR} is in GeV.

Iyer, Sridhar, Vempati ‘15

Thus far- we looked at looked at simple modifications of the the RS setup to resolve the tension with EWPD

These modifications however do not address large contribution to FCNC especially in the lepton sector

Solutions have been proposed by the addition of flavour symmetries-MFV

Over Tea!!

A brief pause:

Question: Is it possible to have a scenario where the KK scales are naturally large?

What if..

Choi et al., Dudas Gersdorff, Iyer Vempati..

R is reduced to R/6 → warp factor $\epsilon = 0.01$ → Scale of physics on IR brane is GUT scale

Features of GUT RS model

Lowest KK scale is GUT scale

RS is no longer solution to hierarchy problem

Supersymmetrize!!

Next question: How does one break SUSY?

Contact interactions
on the IR (GUT) brane

Brane localized
Interaction

$$\mathcal{L}^{(4)}_{breaking} = \delta(y - \pi R)\left[d^4\theta e^{-2k\pi R} k^{-2} X^\dagger X \left(\beta_{\Phi,ij}\Phi_i^\dagger \Phi_j\right) + d^2\theta k^{-1} X \mathcal{W}_{A\alpha}\mathcal{W}_A^\alpha + d^2\theta e^{-3ky} k^{-1} X \left(\tilde{A}^u_{ij} H_u Q_i u_j + \ldots\right)\right]$$

SUSY Breaking spurion $X = \theta^2 \; F$

F term of X develops a vev giving a gravitino mass

$$m_{3/2} = \frac{\langle F \rangle}{k} \sim TeV$$

In the canonical basis

$$
\begin{aligned}
m_{1/2} &= f m_{3/2} \\
(m^2_{\tilde{f}})_{ij} &= m^2_{3/2}\ \hat{\beta}_{ij}\ e^{(1-c_i-c_j)kR\pi}\xi(c_i)\xi(c_j) \\
A^{u,d}_{ij} &= m_{3/2}A'_{ij}e^{(1-c_i-c'_j)kR\pi}\xi(c_i)\xi(c'_j)
\end{aligned}
$$

where $\hat{\beta}_{ij}, A'$ are dimensionless $\mathcal{O}(1)$ parameters.

Some features:

Structure of the soft masses is predicted by the fits to the fermion masses.

Soft masses are flavourful but FCNC under control!!

The trilinear coupling for the third generation is naturally large.

Structure of soft mass matrix

Typical soft mass matrix for the up type squarks looks like

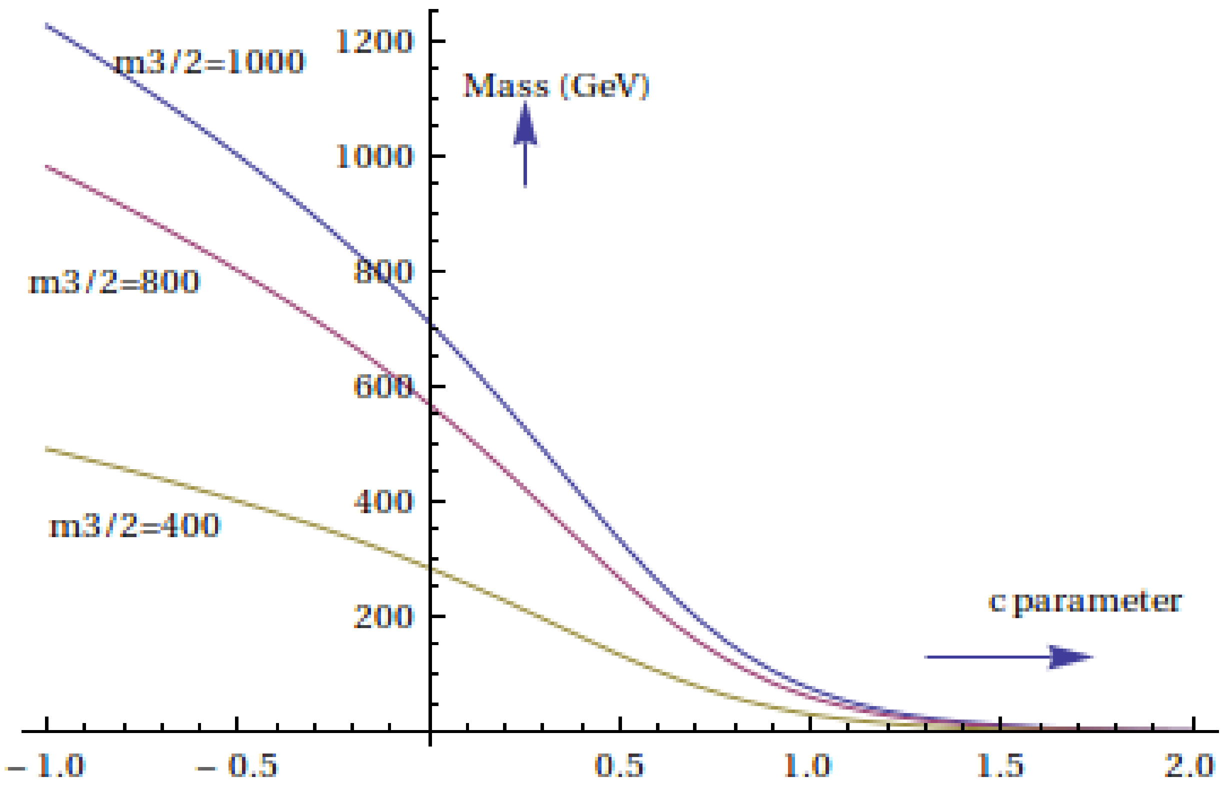
$$\tilde{M}^2_{Q,U} = m^2_{3/2}(0.5 - c_{Q_3,U_3}) \begin{pmatrix} \epsilon^{\alpha} & \epsilon^{\gamma} & \epsilon^{\frac{\alpha}{2}} \\ \epsilon^{\gamma} & \epsilon^{\beta} & \epsilon^{\frac{\beta}{2}} \\ \epsilon^{\frac{\alpha}{2}} & \epsilon^{\frac{\beta}{2}} & 1 \end{pmatrix}$$

$\alpha = 2c_1 - 1,\ \beta = 2c_2 - 1,\ \gamma = c_2 + c_1 - 1.$ c_1 and c_2 are bulk mass parameters for first two generation squarks.

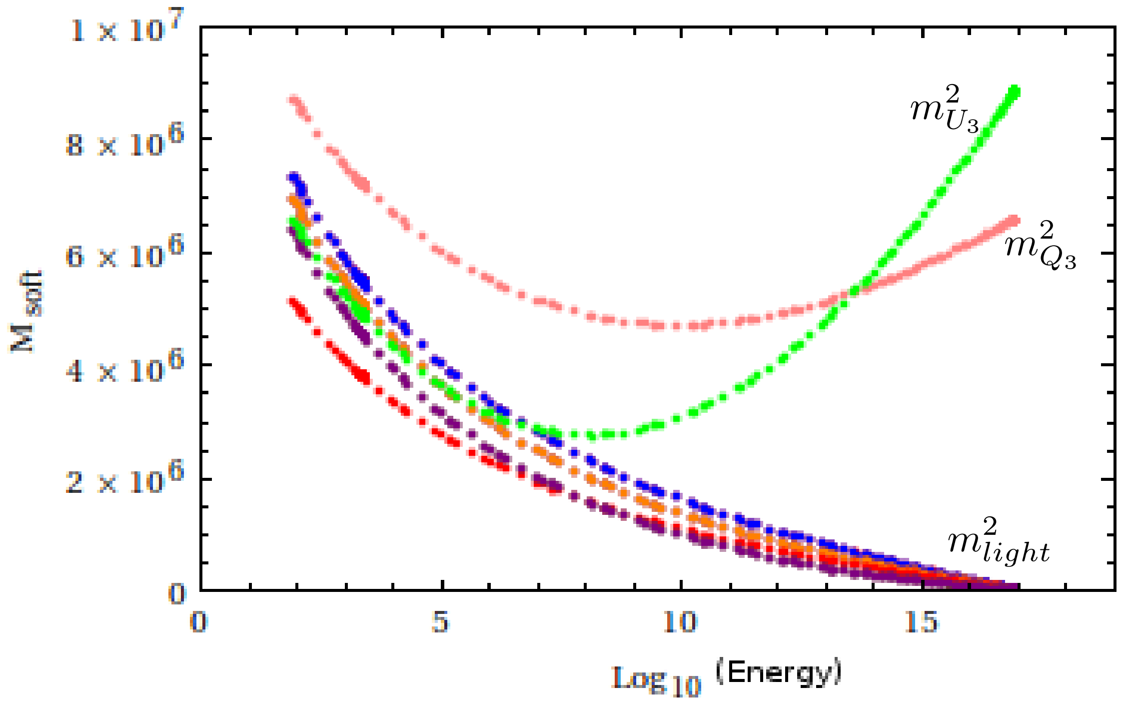
Significant amount of flavour violation present at the high scale!!

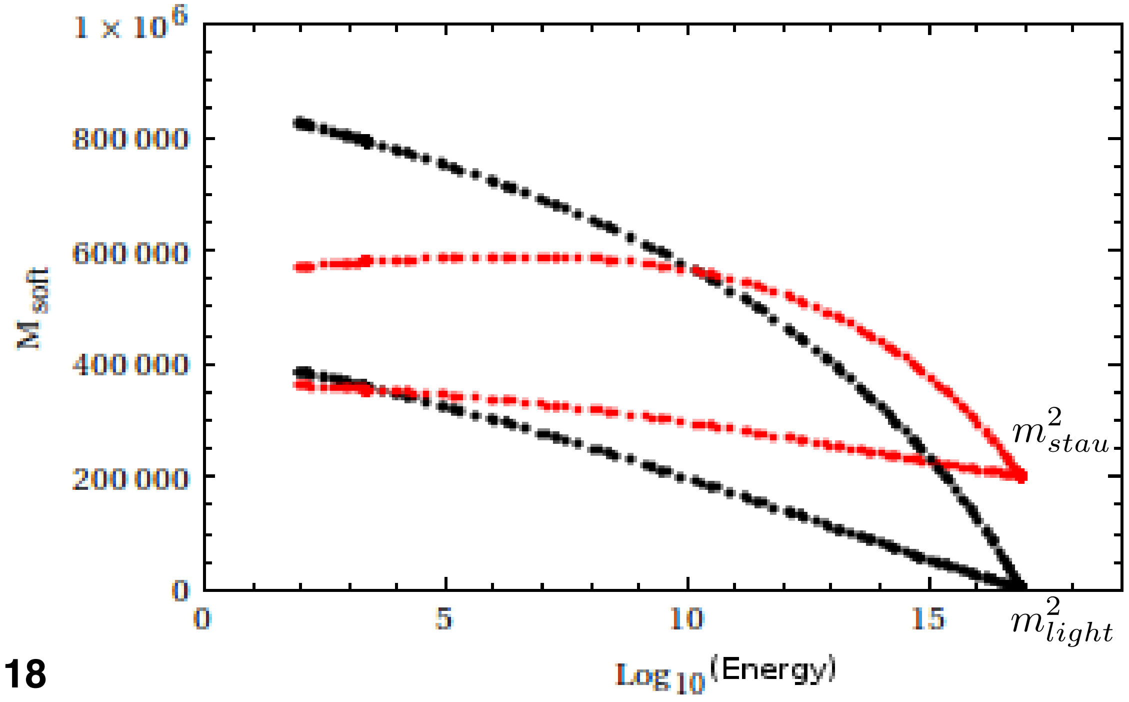
Soft masses at High scale

Iyer Vempati

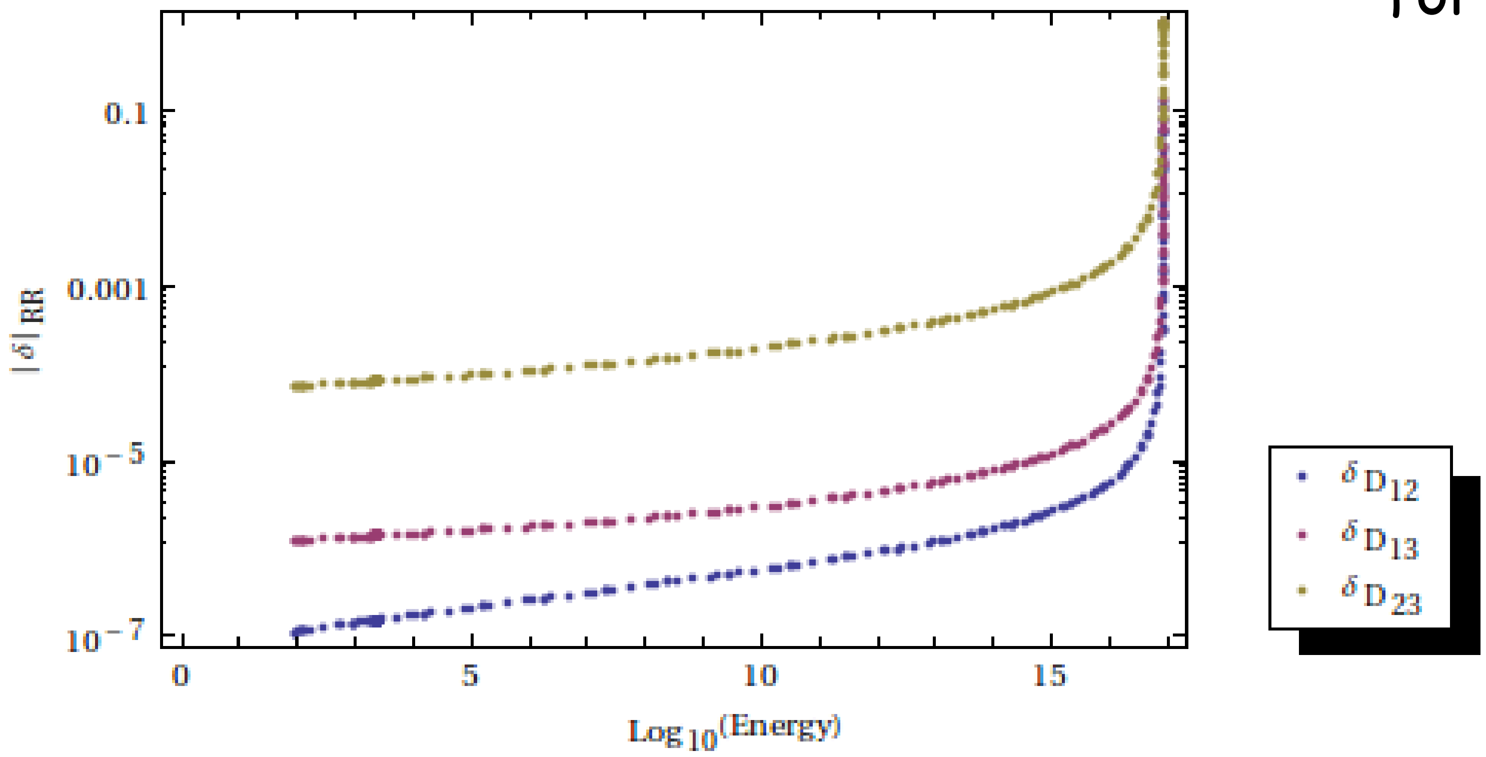


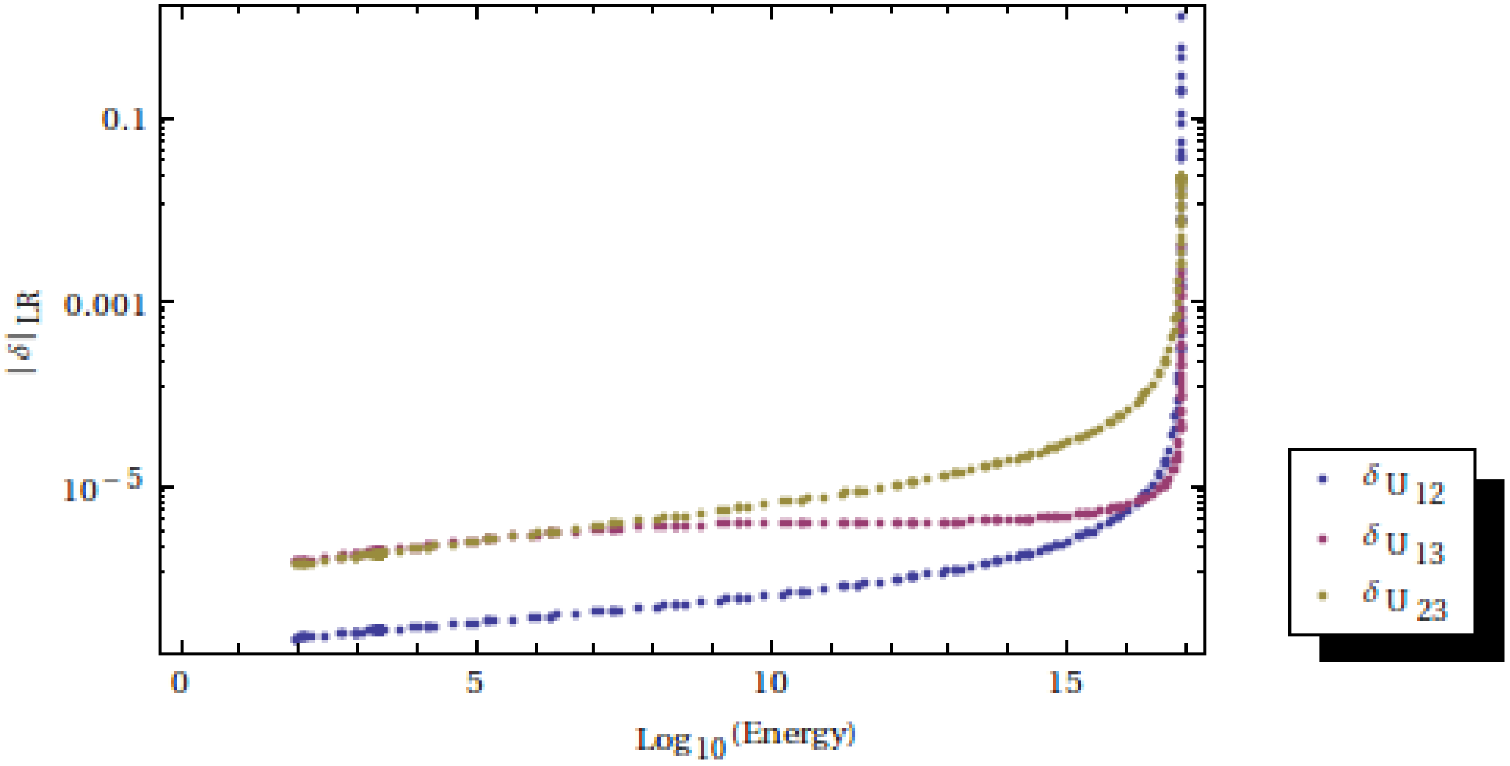
Running of masses





The value of δ is only for illustration





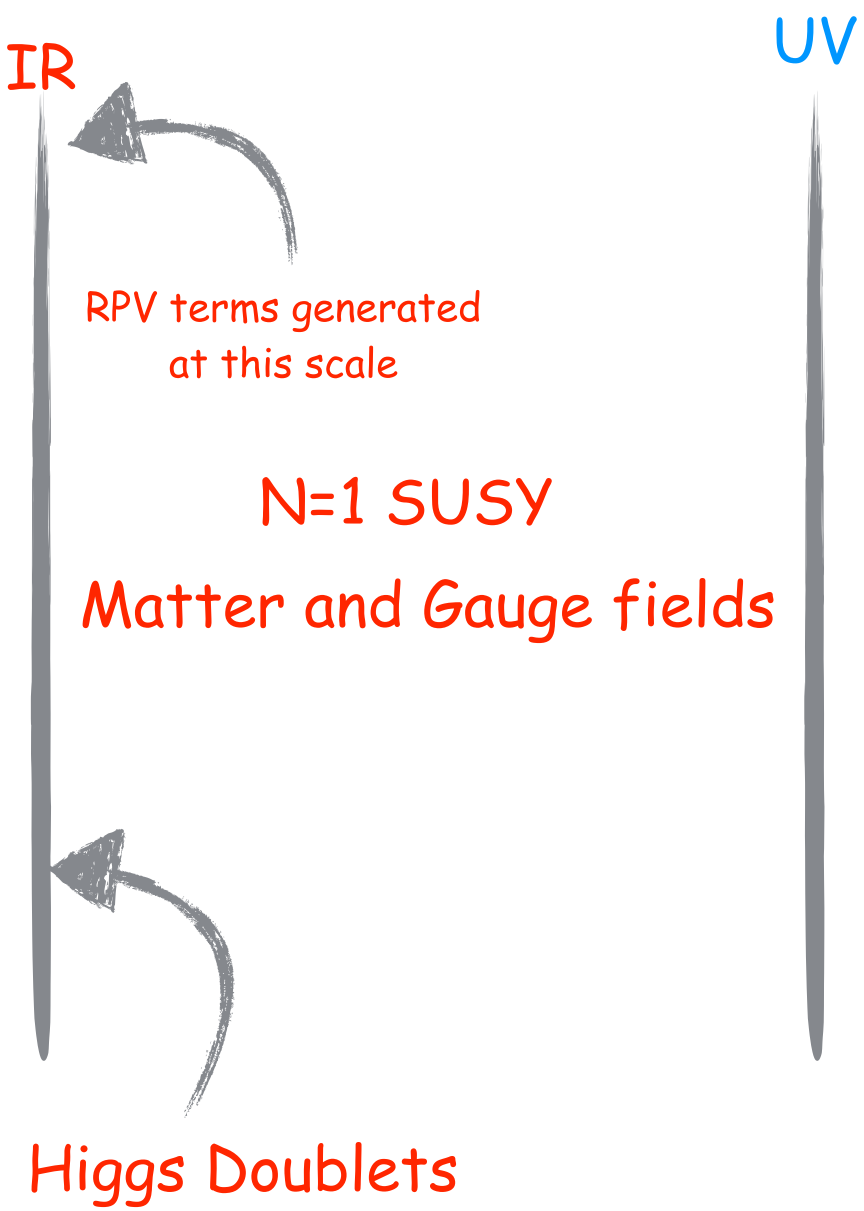
So far:

- We have a model of flavourful supersymmetry, where soft terms are predicted by the same mechanism which explains the hierarchy of Yukawa couplings.

- The structure of the soft masses were such that the contributions to the flavour processes were within control

- The supersymmetric lagrangian, however was not the most general one could have started with.

- Just like the soft masses, one can also have a prediction for the sizes of the L violating and B violating terms


IR

UV

RPV terms generated
at this scale

N=1 SUSY

Matter and Gauge fields

Higgs Doublets

Global symmetries are not the
holiest of symmetries!!

Write down the
most general RPV lagrangian-with B and L violating
terms.

But proton decay constraints are too strong-
may play spoilsport!

Let's see how well we do
without imposing any
symmetries

What is R parity- Z_2 subgroup of continuous $U(1)_R$ transformations

- R symmetry or its subgroup R parity serve the purpose of preventing unwanted scalar exchange diagrams
  - R symmetry however forbids mass terms for the gaugino even in the presence of broken supersymmetry

Thus the discrete subgroup was chosen and thus:

$$\boxed{R\text{-parity} \quad R_p = (-1)^R = \begin{cases} +1 & \text{for ordinary particles,} \\ -1 & \text{for their superpartners.} \end{cases}}$$

$$\big\downarrow$$

$$\boxed{R\text{-parity} = (-1)^{2S}\,(-1)^{3B+L}\ .}$$

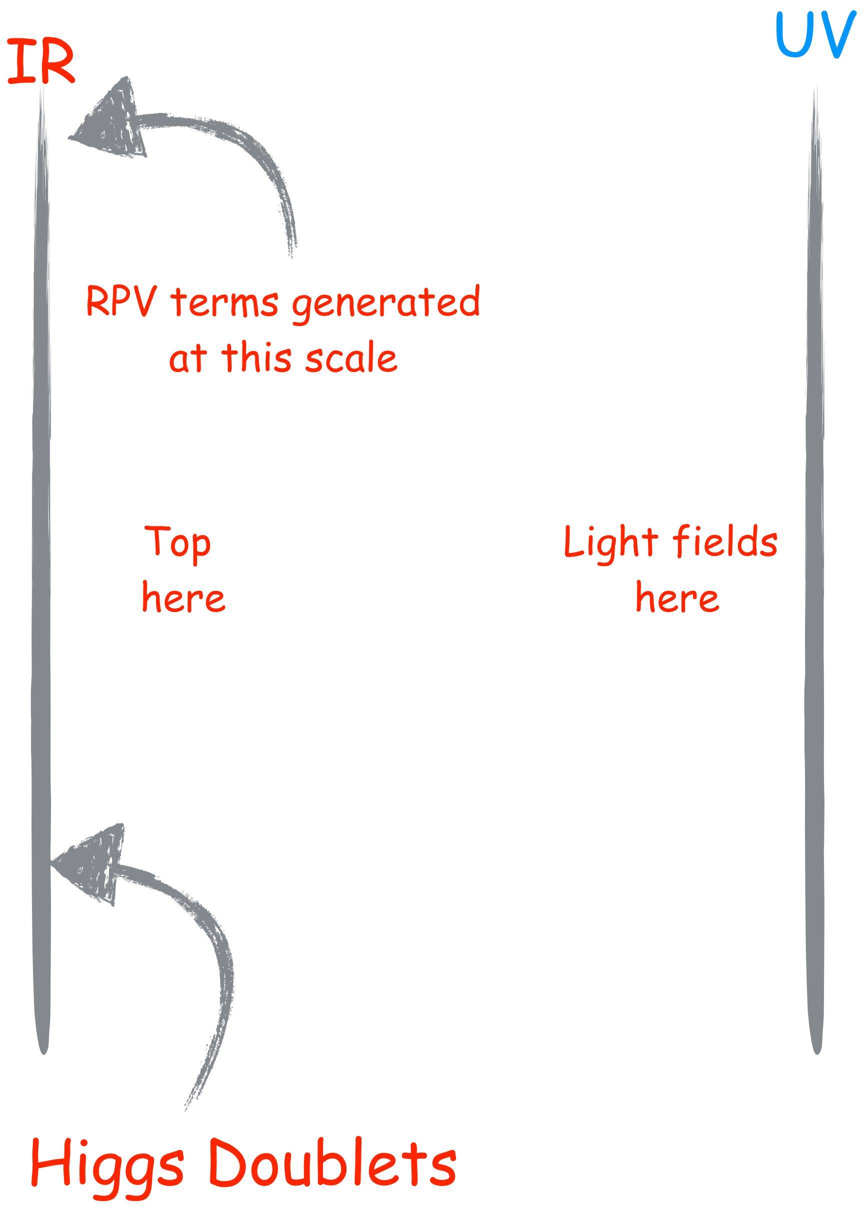
Allanach,Iyer, Sridhar, '16

The RPV terms on the IR brane correspond to

$$
\begin{aligned}
W^{(5)}_{\Delta \mathrm{L}=1} &= \frac{1}{2}\lambda^{ijk} L_i L_j \overline{e}_k + \lambda'^{ijk} L_i Q_j \overline{d}_k + \mu'^{i} L_i H_u \\
W^{(5)}_{\Delta \mathrm{B}=1} &= \frac{1}{2}\lambda''^{ijk} \overline{u}_i \overline{d}_j \overline{d}_k
\end{aligned}
$$

The terms are in general higher dimensional operators as the chiral super-fields are bulk fields.

Allanach,Iyer, Sridhar, ‘16



IR

UV

RPV terms generated at this scale

Top here

Light fields here

Higgs Doublets

It is important the RPV terms and the Higgs doublets are on the same boundary.

Like the soft mass terms, the magnitude of effective 4D magnitude also depends on the magnitude of the wavefunction at the boundary.

If the RPV terms and Higgs doublets are at different boundaries, the light fields are naturally close to source of RPV lagrangian.

Catastrophic due to strong flavour bounds!

Allanach,Iyer, Sridhar, ‘16

The trilinear couplings

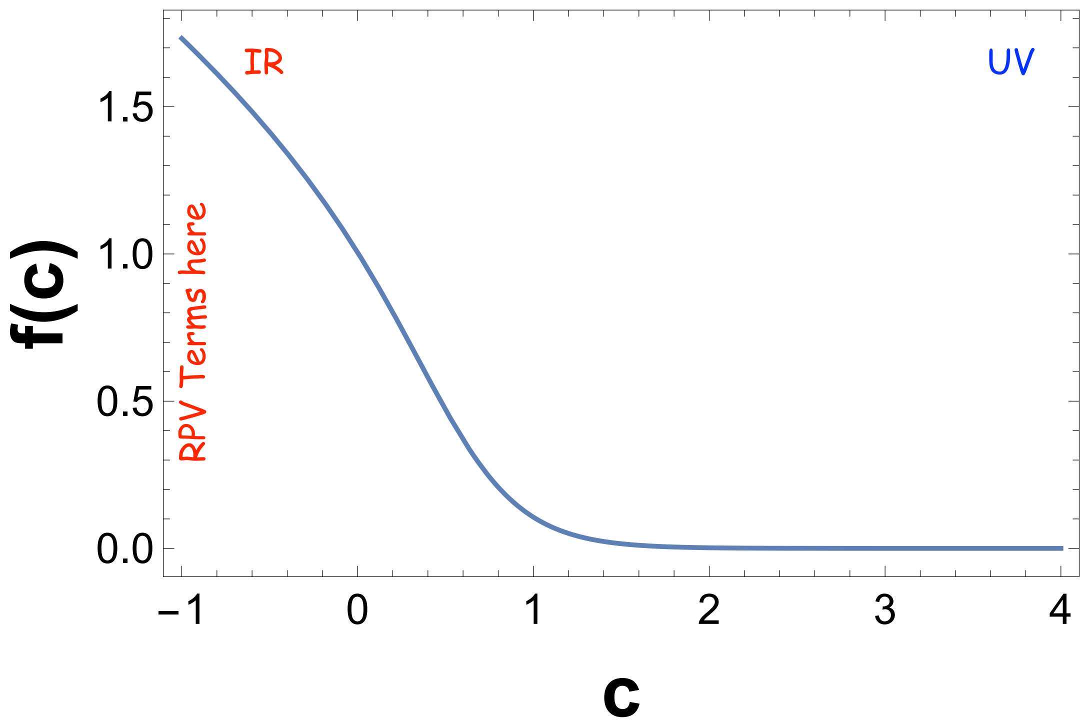
The bilinears

$$\lambda_{ijk} = \hat{\lambda}_{ijk} f(c_i) f(c_j) f(c_k)$$

$$\mu_i = \hat{\mu}_i \; \mu f(c_{L_i}) e^{-kR\pi}$$

Dimensionless
O(1) parameters

μ is typically the EW scale.



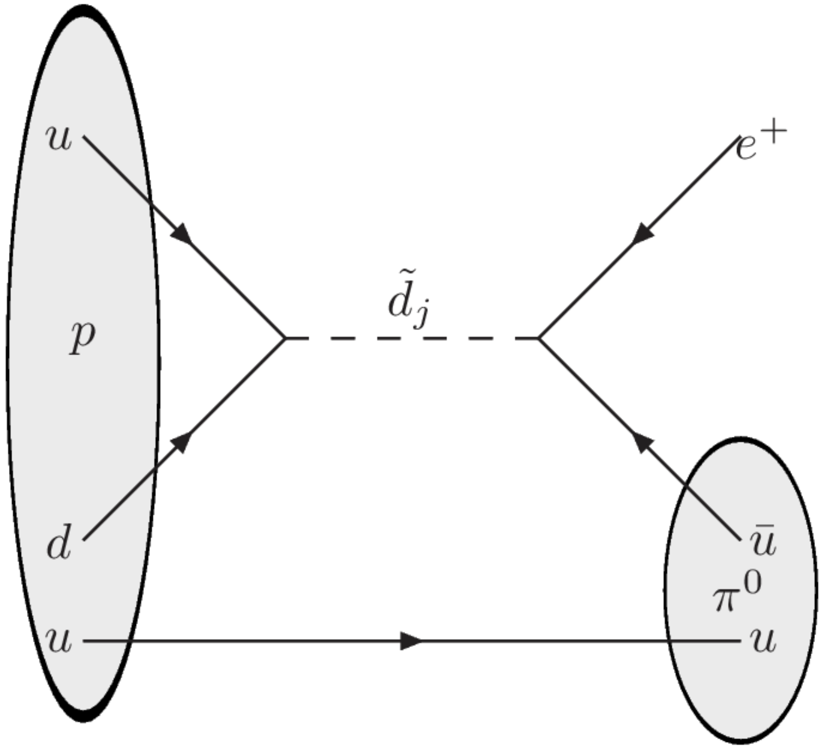
Thus one can see the most of the RPV terms are suppressed naturally by wave function overlap

But again is it enough?

Allanach,Iyer, Sridhar, '16

Dominant constraints come from proton decay

Both $\not{B}$ and $\not{L}$ $\Rightarrow$ **Proton Decay**



Allanach,Iyer, Sridhar, '16

Possibilities

The most dominant constraints to proton decay come from couplings of the form

$$\lambda'_{i31}\lambda''_{123} < 3 \times 10^{-25}$$

In our analysis the RPV O(1) couplings were chosen to be 1

With this choice the best one can do is to have $\lambda'_{i31}\lambda''_{123} \sim 10^{-18}$

In principle one can utilise some freedom in the choice of the O(1) couplings ONLY making then as small as $\sim 10^{-4}$

Smaller than fine tuning required in models to suppress flavour!!

Allanach,Iyer, Sridhar, ‘16

Only lepton number violation

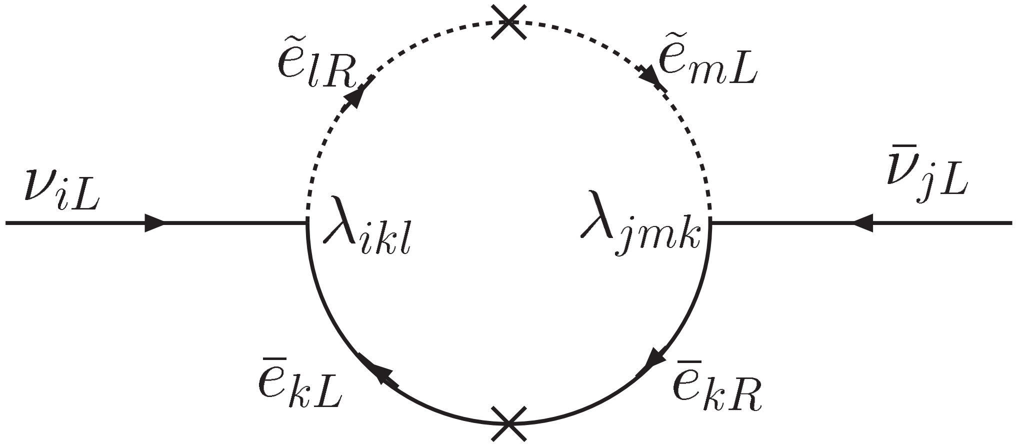
As a toy model we study a scenario where Baryon number is conserved-No proton decay

Contraints on this scenario include-FCNC and possibly large neutrino masses

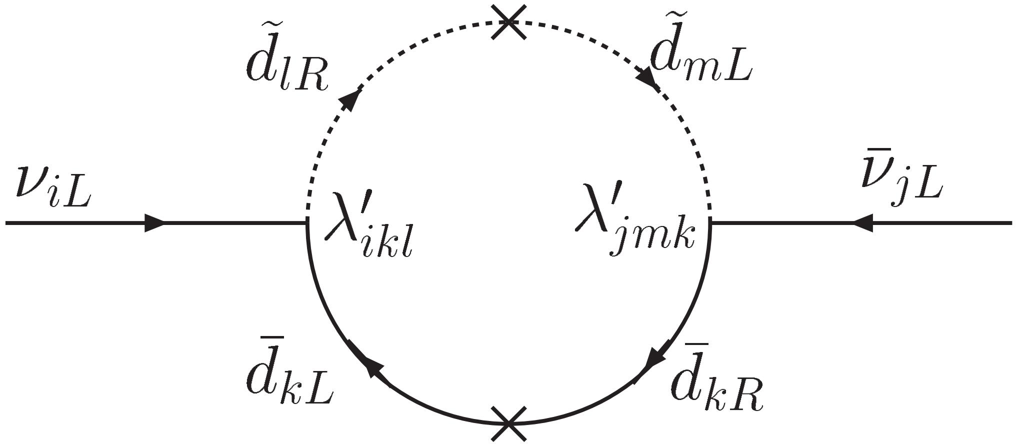
In an RPV scenario it is more challenging to get neutrino oscillation data just right.

Luckily-we can generate it from wave function overlap from extra-dimension

Generated at loop level due to trilinear



(a)



(b)

$$M^{\nu}_{ij}|_{\lambda} \simeq \frac{1}{8\pi^2}\left\{\lambda_{i33}\lambda_{j33}\,\frac{m_\tau^2}{\tilde{m}} + (\lambda_{i23}\lambda_{j32}+\lambda_{i32}\lambda_{j23})\,\frac{m_\mu m_\tau}{\tilde{m}} + \lambda_{i22}\lambda_{j22}\,\frac{m_\mu^2}{\tilde{m}}\right\},$$

$$M^{\nu}_{ij}|_{\lambda'} \simeq \frac{3}{8\pi^2}\left\{\lambda'_{i33}\lambda'_{j33}\,\frac{m_b^2}{\tilde{m}} + (\lambda'_{i23}\lambda'_{j32}+\lambda'_{i32}\lambda'_{j23})\,\frac{m_s m_b}{\tilde{m}} + \lambda'_{i22}\lambda'_{j22}\,\frac{m_s^2}{\tilde{m}}\right\},$$

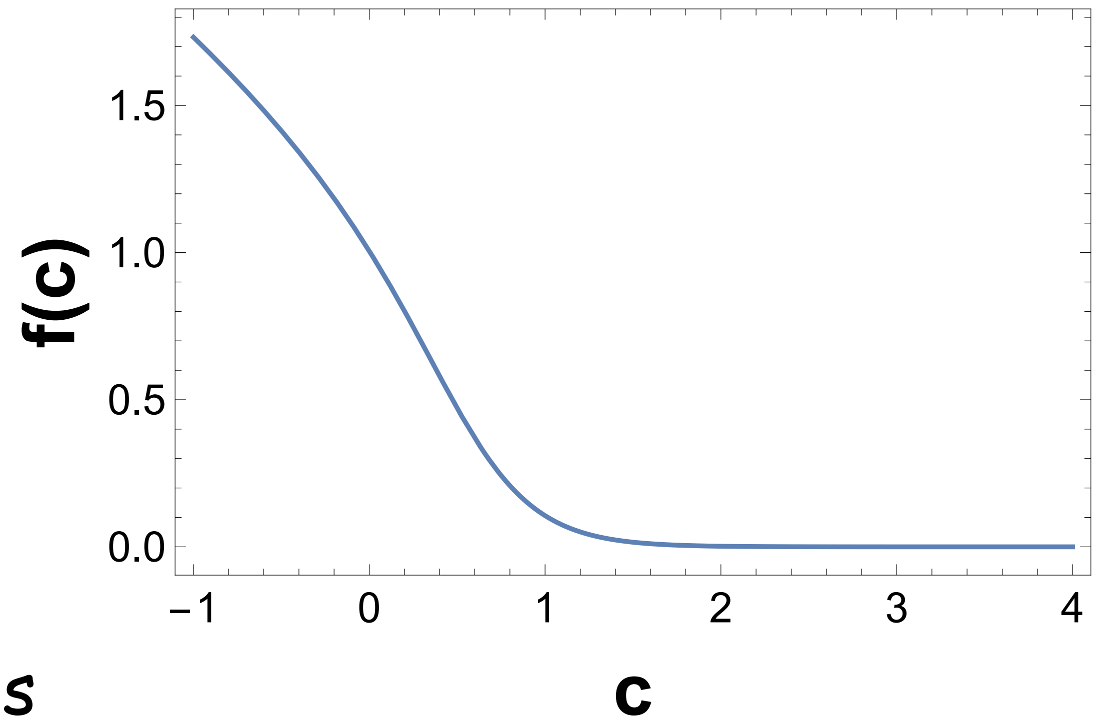
Allanach,Iyer, Sridhar, ‘16

We can have a scenario where the lepton number contributions to the neutrino masses are suppressed

Neutrino Masses = RS (Dirac like) + RPV

The couplings involving third generation fermions are large

ν_3 is likely to have a fairly heavier mass



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Some Numbers

Neutrino mass eigenstates due to RS (in eV)

$$m_{\nu_3} \sim 0.05 \quad m_{\nu_2} \sim 0.008 \quad m_{\nu_1} \sim 0$$

Neutrino mass eigenstates due RPV (in eV) with all order one parameters set to one except

$$\hat{\lambda}_{i,3,3} = 0.1 \quad \hat{\mu}_3 = 0.1$$

$$m_{\nu_3} \sim 0.005 \quad m_{\nu_2} \sim 10^{-6} \quad m_{\nu_1} \sim 0$$

Thus with a slight modification of the order one parameters, we can adjust the masses due to wave function overlap to be dominant

Allanach,Iyer, Sridhar, ‘16

What about other lepton number violating operators

$$\mathcal{W}_{\Delta L=2} = \frac{\kappa_{ij}}{M_{Pl}}(L_i H_u)\cdot(L_j H_u),$$

The contribution to neutrino masses then become

$$(m_\nu)_{ij} = \hat{\kappa}_{ij}\frac{v_u^2}{2M_{Pl}}e^{kR\pi}f(c_{L_i})f(c_{L_j})$$

For $c_{L_i} > 0.5$

$$(m_\nu)_{ij} \sim \frac{v_u^2}{2M_{Pl}}e^{(2-2c_L)kR\pi},$$

$c_{L_i} > 1.5$

from the fits

Parameter	Mass/TeV	Parameter	Mass/TeV	Parameter	Mass/TeV	Parameter	Mass/TeV
$\tilde{t}_1$	1.8	$\tilde{b}_1$	2.2	$\tilde{\tau}_1$	1.1	$\tilde{\nu}_\tau$	1.6
$\tilde{t}_2$	2.3	$\tilde{b}_2$	2.3	$\tilde{\tau}_2$	1.6	$\tilde{\nu}_\mu$	1.6
$\tilde{c}_1$	2.2	$\tilde{s}_1$	2.2	$\tilde{\mu}_R$	1.2	$\tilde{\nu}_e$	1.6
$\tilde{c}_2$	2.7	$\tilde{s}_2$	2.7	$\tilde{\mu}_L$	1.6	$\tilde{g}$	2.6
$\tilde{u}_1$	2.2	$\tilde{d}_1$	2.2	$\tilde{e}_R$	1.1	$\chi_1^\pm$	2.0
$\tilde{u}_2$	2.7	$\tilde{d}_2$	2.7	$\tilde{e}_L$	1.6	$\chi_2^\pm$	2.3
m_{A^0}	3.1	$m_H^\pm$	3.1	m_h	0.121	m_H	3.1
χ_1^0	1.1	χ_2^0	2.0	χ_3^0	2.3	χ_4^0	2.4

A final pause:

After discussing this radical RS solution, we go back to scenario where the KK modes are light.

We discuss them both in the context of custodial RS model and deformed RS model.

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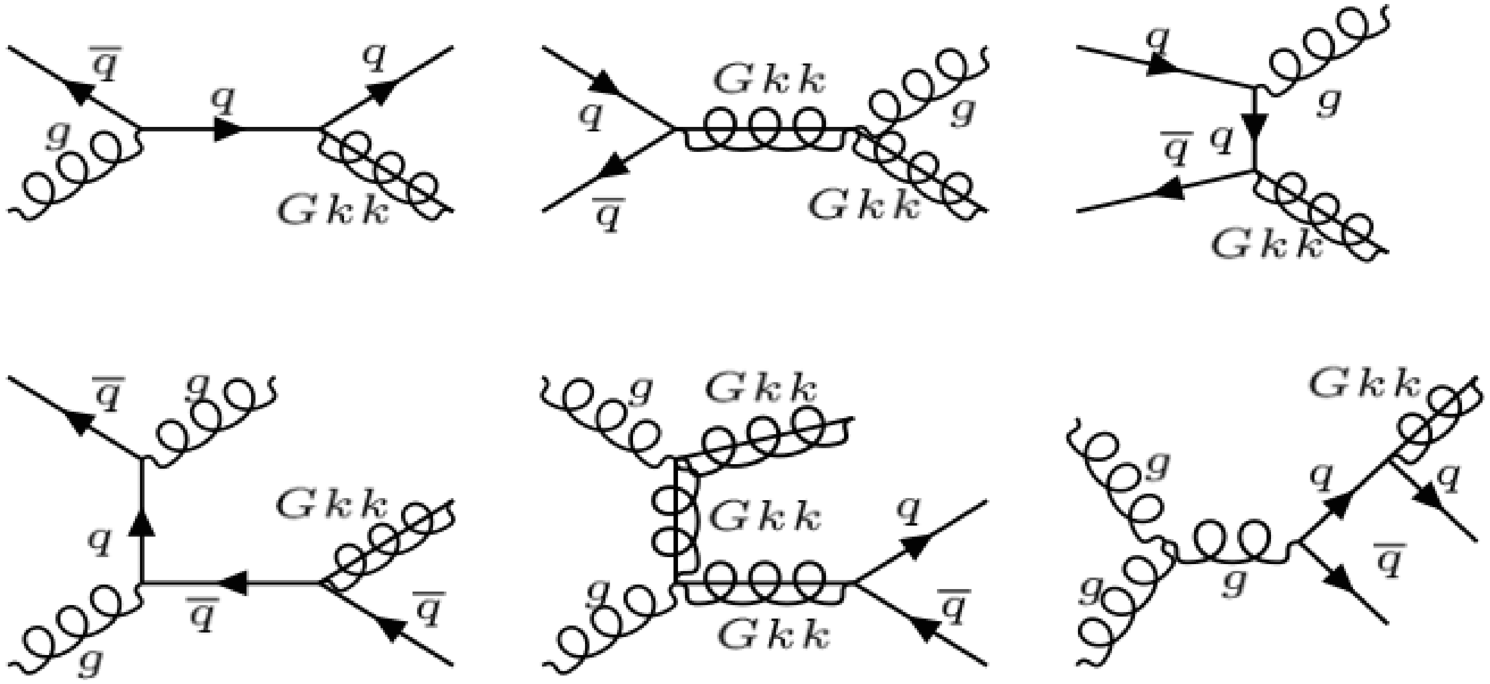
Search for KK-gluons

Ordinarily a KK-gluon can not couple to a pair of gluons-orthonormality

A simple s-channel production will have diagrams only due to quark annihilation

We consider a case where the KK-gluons can be produced in association with a hard patron-number of contributing diagrams increases from 12 to 36!!



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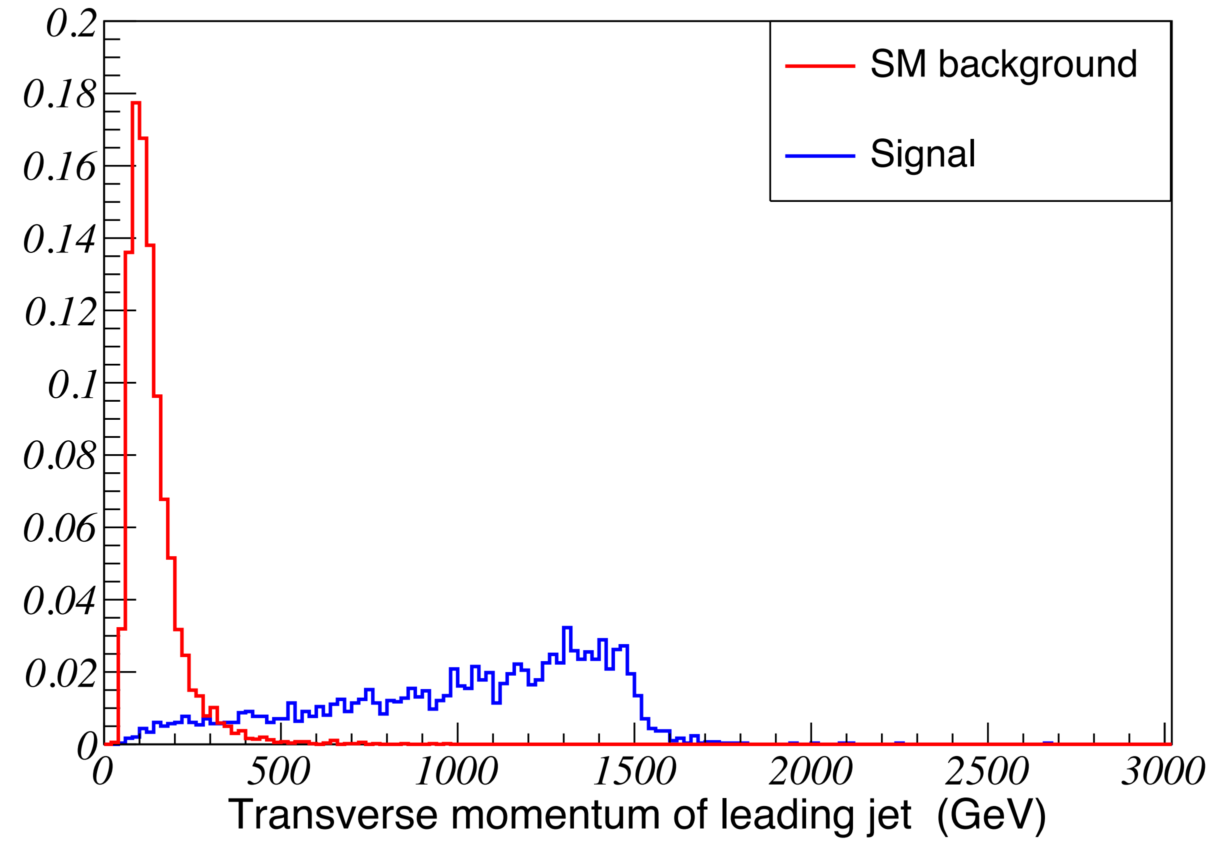

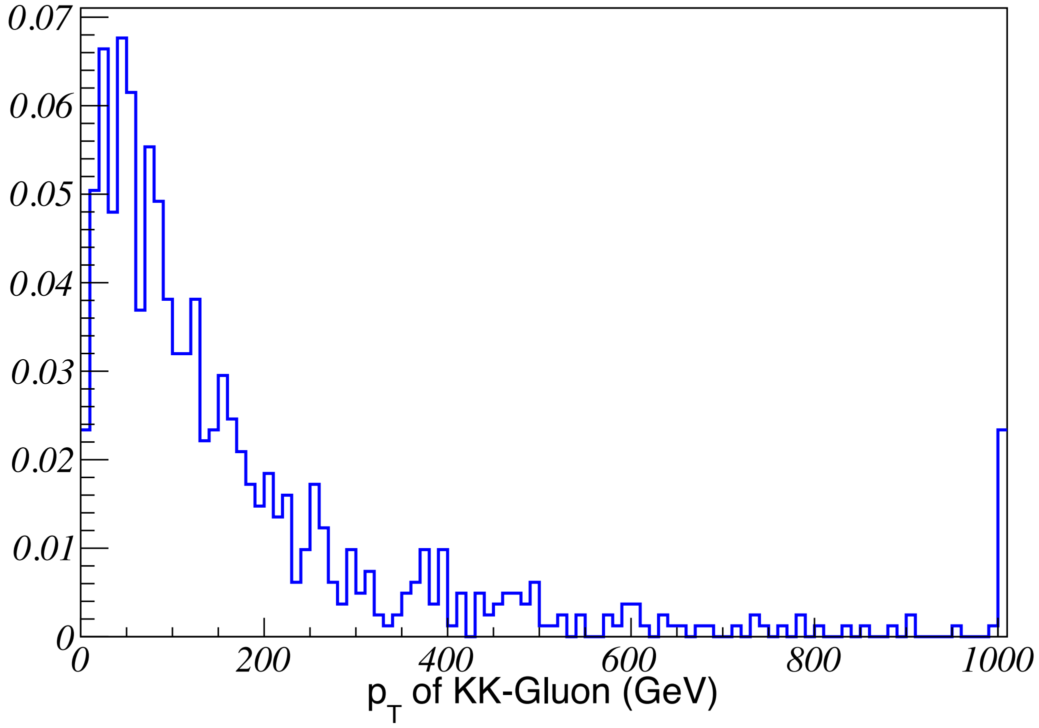

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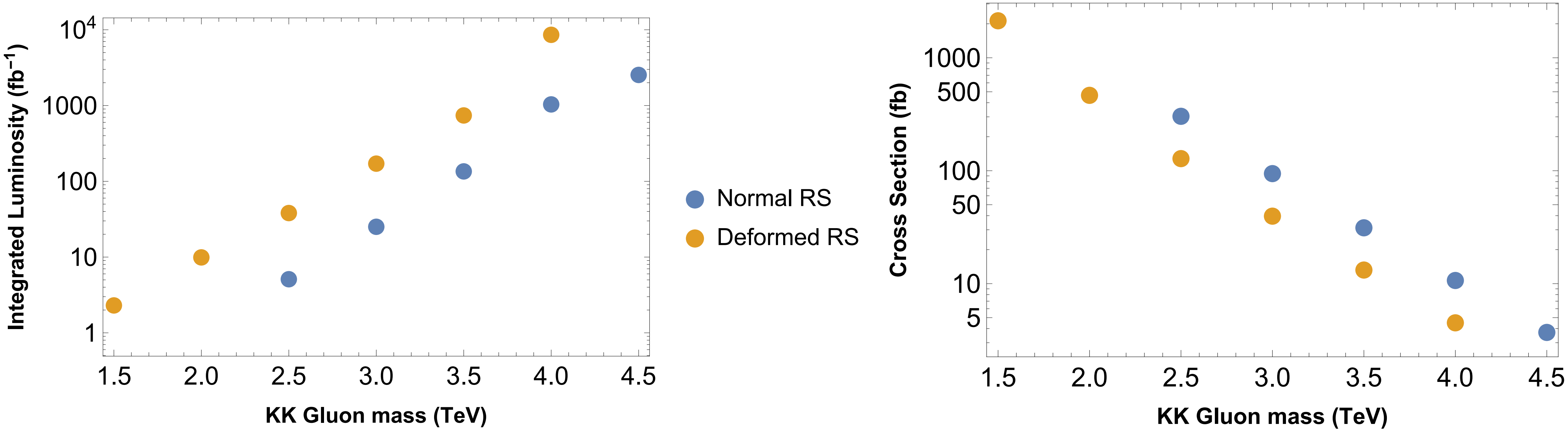

FIG. 5: Minimum luminosity required for a 5σ sensitivity for normal RS (blue) and deformed RS (orange). The right plot shows the production cross-section for the different masses.

To Summarise..

RS is an interesting model, but needs to be supplemented by additional features

The geometry of RS can be put to good effects-predicting several unknown parameters in SUSY extensions

Finis