The relic density

of heavy neutralinos

Aoife Bharucha

to appear soon on the arXiv

Free Meson Seminar, TIFR
14^{th} January 2016

In collaboration with Martin Beneke, Charlotte Hellmann, Francesco Dighera, Andrzej Hryczuk, Stefan Recksiegel and Pedro Ruiz-Femenia

An introduction to neutralinos

The mass matrix

$$\begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

where $s_\beta/c_\beta = \sin\beta/\cos\beta$, $\tan\beta$ being the ratio of the vevs of the two MSSM Higgs doublets, $M_{W/Z}$ is the mass of the W/Z boson, and $s_W \equiv \sin\theta_W$ for θ_W

Diagonalising this matrix results in four neutralinos, the lightest of which is a candidate for dark matter, the nature of which depends on the ordering of the bino, wino and higgsino parameters M_1, M_2 and μ

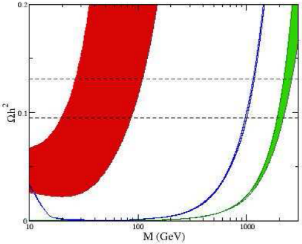
Which one could be a thermal relic?

Impressive experimental precision on relic abundance:

$$\Omega_{\rm CDM}^{\rm Planck} h^2 = 0.1198 \pm 0.0026$$

How does $\tilde{\chi}^0$ relic density compare?[1]

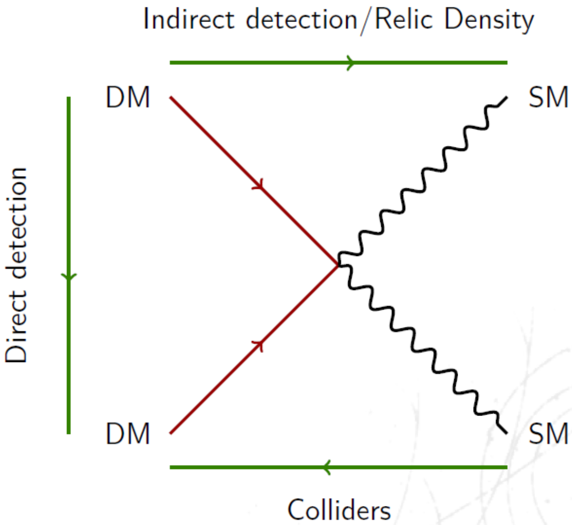
- $\Omega_{\tilde{B}} h^2 = 0.01 \left(\frac{m_{\tilde{e}_R}}{100\,\mathrm{GeV}} \right)^2 \frac{(1+r)^4}{r(1+r^2)}$
- $\Omega_{\tilde{H}} h^2 = 0.1 \left(\frac{\mu}{1\,\mathrm{TeV}} \right)$
- $\Omega_{\tilde{W}} h^2 = 0.1 \left(\frac{M_2}{2.2\,\mathrm{TeV}} \right)^2$



Only way to get correct relic density at EW scale: well-tempered, i.e. mixture of bino+wino or higgsino; coannihilation or Higgs funnels. Precise degeneracies except maybe bino-Higgsino.

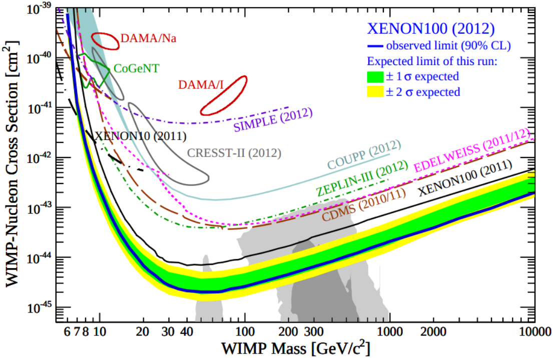
[1] The Well-tempered neutralino, N. Arkani-Hamed, A. Delgado, G.F. Giudice, hep-ph/0601041, Nucl.Phys. B741 (2006) 108-130

Detecting dark matter



The Relic density of heavy neutralinos

Why?



- As DD limits improve, masses below and above $\mathcal{O}(100 \text{ GeV})$ more likely[2], bino-Higgsino ruled out
- Heavier neutralinos are difficult to detect both at the LHC and at indirect and direct detection (ID/DD) experiments
- However situation can change if the annihilation is enhanced by the Sommerfeld effect (Hisano 2004,6)

[2] We will not discuss sfermion coannihilation in this talk

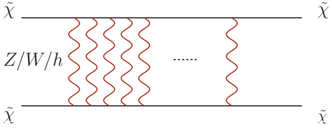
Relic density of heavy neutralinos

The Sommerfeld enhancement (Hisano et al. 2004,6, Arkani-Hamed et al. 2008)

- The Sommerfeld effect may have a large effect on the annihilation rate of SU(2) charged neutralinos, already studied to a great extent[3]
- Enhancement factor $S(v)$ for charged particle annihilation due to Coulomb potential if $v \lesssim \pi\alpha$ well known (Sommerfeld '31),

$$S(v) = \frac{\pi\alpha}{v} \frac{1}{1 - e^{-\pi\alpha/v}}$$

- Large corrections also occur in general if also mass of mediator (M_W) such that $M_W < \alpha M_{\tilde{\chi}} \Rightarrow$ Yukawa potential $V(r) = -\frac{\alpha}{r} e^{-M_W r}$

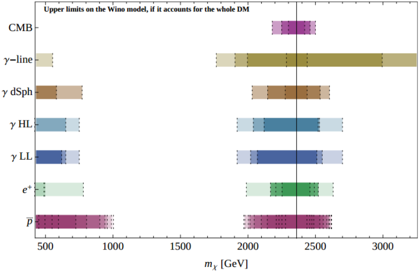


Aim: to calculate the relic density in general MSSM in wino-like region including the full effect of the Sommerfeld enhancement

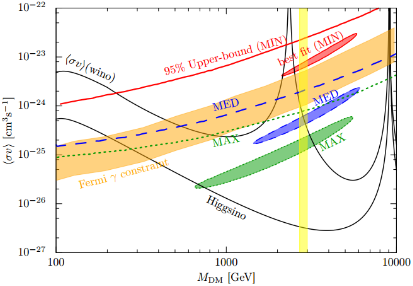
[3] Previous work in MSSM for pure winos/Higgsino includes Cirelli et al. 2007,8,9, Hryczuk et al. 2010,14, Slatyer (et al) 2008,13,14, Fan et al 2013, Cabrera et al 2015

Indirect detection

Limits on the wino and wino-like DM



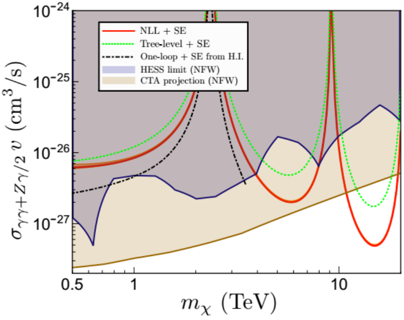
Hryczuk, Cholis, Iengo, Tavakolie, Ullio '14



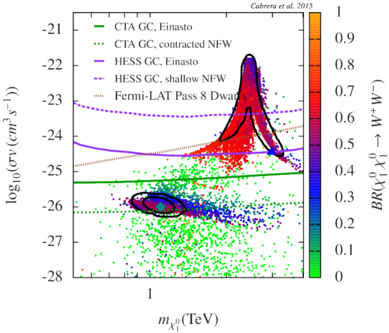
Ibe, Matsumoto, Shirai, Yanagida '15

Indirect detection

Limits on the wino and wino-like DM



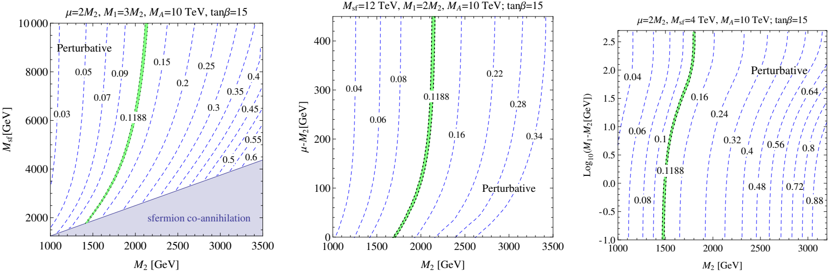
Ovanesyan, Slatyer, Stewart '14



Cabrera, Ando, Weniger, Zandanel '15

Relic density of heavy neutralinos

The perturbative wino-like case



- As the sfermion mass decreases the annihilation rate is suppressed due to t-channel interference and therefore the correct relic abundance is obtained for lighter neutralino masses of $\sim$ 1400 GeV
- As the Higgsino and bino annihilate less strongly, they dilute the wino annihilation and reduces the mass providing the correct relic density to 1700 and 1500 GeV respectively for the chosen set of parameters

Outline

- The Sommerfeld enhancement
- Mixing in the neutralino sector
- Details of the calculation
- The Sommerfeld enhanced relic density for:
  - the pure wino LSP with non-decoupled sfermions masses
  - the mixed wino-Higgsino LSP
  - the mixed wino-bino LSP
  - the mixed wino-bino LSP/effect of additional parameters
- Summary

Introducing the non-relativistic Lagrangian

Beneke, Hellmann, Ruiz-Femenia 2014

$$\mathcal{L}^{\text{NRMSSM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \delta\mathcal{L}_{\text{ann}} + \ldots\ ,$$

$$\mathcal{L}_{\text{kin}} = \sum_{i=1}^{n_0} \xi_i^\dagger \left(i\partial_t - \delta m_i + \frac{\vec{\partial}^{\,2}}{2m_{\text{LSP}}} \right) \xi_i + \sum_{\psi=\eta,\zeta} \sum_{j=1}^{n_+} \psi_j^\dagger \left(i\partial_t - \delta m_j + \frac{\vec{\partial}^{\,2}}{2m_{\text{LSP}}} \right) \psi_j\,.$$

where $\delta m_i = m_i - m_{\text{LSP}}, \delta m_j = m_j - m_{\text{LSP}}$

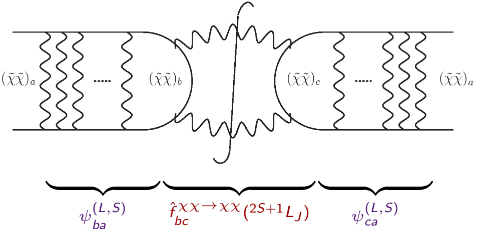
$$\mathcal{L}_{\text{pot}} = - \sum_{\chi\chi\to\chi\chi} \int d^3\vec{r}\ V^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(r)\, \chi^\dagger_{e_4}(t,\vec{x})\chi^\dagger_{e_3}(t,\vec{x}+\vec{r}\,)\chi_{e_1}(t,\vec{x})\chi_{e_2}(t,\vec{x}+\vec{r}\,)\,.$$

$$V^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(r) = \left(A_{e_1e_2e_4e_3}\delta_{\alpha_4\alpha_1}\delta_{\alpha_3\alpha_2} + B_{e_1e_2e_4e_3}\left(\vec{S}^2\right)_{\alpha_4\alpha_1,\alpha_3\alpha_2} \right) \frac{e^{-m_\phi r}}{r}\,,$$

$$\delta\mathcal{L}^{d=6}_{\text{ann}} = \sum_{\chi\chi\to\chi\chi} \sum_{S=0,1} \frac{1}{4}\, f^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}\left({}^{2S+1}S_S\right)\ \mathcal{O}^{\chi\chi\to\chi\chi}_{\{e_4e_3\}\{e_2e_1\}}\left({}^{2S+1}S_S\right)\,,$$

The Sommerfeld Enhancement

(Beneke, Hellmann, Ruiz-Femenia 2012,14, Hellmann, Ruiz-Femenia 2013)



$\hat{f}_{ab}(^{2S+1}L_J)$, $\hat{g}_{ab}(^{2S+1}L_J)$: absorptive part of Wilson coefficients of local four-fermion operators.
Sommerfeld factors computed by solving Coupled S.E. for $\psi_{ba}^{(L,S)}$[4]:

$$\left(\left[-\frac{\vec{\nabla}^2}{2\mu_a} - E\right]\delta^{ab} + V^{ab}(r)\right)[\psi_E(\vec{r})]_{b,ij} = 0$$

$$\sigma^{(\chi\chi)_a\to \text{light}}\, v_{\text{rel}} = S_a[\hat{f}_h(^1S_0)]\,\hat{f}_{aa}(^1S_0) + S_a[\hat{f}_h(^3S_1)]\,3\,\hat{f}_{aa}(^3S_1) + \frac{\vec{p}_a^{\,2}}{M_a^2}\Big(S_a[\hat{g}_\kappa(^1S_0)]\,\hat{g}_{aa}(^1S_0)$$
$$+ S_a[\hat{g}_\kappa(^3S_1)]\,3\,\hat{g}_{aa}(^3S_1) + S_a\left[\frac{\hat{f}(^1P_1)}{M^2}\right]\hat{f}_{aa}(^1P_1) + S_a\left[\frac{\hat{f}(^3P_{\mathcal{J}})}{M^2}\right]\hat{f}_{aa}(^3P_{\mathcal{J}})\Big)\,,$$

Sommerfeld factors ($S_a[\hat{f}(^{2S+1}L_J)]$) given by:

$$S_a[\hat{f}(^{2S+1}L_J)] = \frac{\left[\psi_{ca}^{(L,S)}\right]^*\hat{f}_{bc}^{\chi\chi\to\chi\chi}(^{2S+1}L_J)\,\psi_{ba}^{(L,S)}}{\hat{f}_{aa}^{\chi\chi\to\chi\chi}(^{2S+1}L_J)}\,.$$

4 $\psi_{ba}^{(L,S)}$ is the $(\chi\chi)_b$ component of the scattering wavefunction for incoming state $(\chi\chi)_a$ with quantum nos. L, S evaluated for zero relative distance and normalized to the free scattering solution.

How do we obtain our results?

Based on Beneke, Hellmann, Ruiz-Femenia (2012,13,14):

- Mixed neutralinos possible (beyond pure wino- or higgsino scenarios), including off-diagonals in potentials and annihilation matrices
- Partial wave separation for P- and $\mathcal{O}(v^2)$ S-wave (beyond leading order S-wave included)

In addition, the (to become public) code includes:

- Include exp.constraints on MSSM parameter space: $b \to s\gamma$, m_h, $(g-2)_\mu$, ρ, DD and theoretical constraints on Higgs potential
- Include exact 1-loop on-shell mass splittings and running couplings
- Allow extraction of separate exclusive final states to obtain indirect detection results
- Accuracy at $\mathcal{O}(\%)$, dominated by theoretical uncertainties due to EFT

A brief detour

Mixing in the neutralino sector

The mass matrix for the charginos is given by

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix},$$

The mass matrix for the neutralinos is given by

$$\begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}.$$

where $s_\beta/c_\beta = \sin\beta/\cos\beta$, $\tan\beta$ being the ratio of the vevs of the two MSSM Higgs doublets, $M_{W/Z}$ is the mass of the W/Z boson, and $s_W \equiv \sin\theta_W$ for θ_W.

Mixing

Wino-Higgsino mixing depends on if $\theta_h < 1$ where $\theta_h = \frac{(c_\beta + s_\beta) c_W M_Z}{\sqrt{2}(\mu - M_2)}$,

$\delta m_{\tilde{\chi}_1^+} \equiv m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}$ is given by

$$\delta m_{\tilde{\chi}_1^+} \simeq \frac{1}{2} \frac{m_W^4 M_2 \, (c_\beta^2 - s_\beta^2)^2}{(\mu^2 - {}^2)^2} \, ,$$

$$\delta m_{\tilde{\chi}_1^+} \simeq \frac{m_Z^2}{8 M_2} \left(c_W^2 \, (1 \mp s_{2\beta}) \left(1 - \frac{\delta\mu}{\sqrt{2} \, (s_\beta \pm c_\beta) \, m_W} \right) + 2 \, s_W^2 \, (1 \pm s_{2\beta}) \frac{M_2}{M_1} \right) \, ,$$

Wino-bino mixing depends on if $\theta_b < 1$ where $\theta_b = \frac{s_{2\beta} s_{2W} M_Z^2}{2\mu(M_1 - M_2)}$

$$\delta m_{\tilde{\chi}_1^+} \simeq \theta_b^2 \, \delta M_1 \left(1 + \frac{2 M_2}{s_{2\beta} \, \mu} \right) \quad \text{or}$$

$$\delta m_{\tilde{\chi}_1^+} \simeq \begin{cases} s_W^2 \, \frac{m_Z^2}{\mu} \left(s_{2\beta} + \frac{M_2}{\mu} \right) - s_W^2 \delta M_1 , & \text{if } \mu > 0 \text{ or } \frac{s_{2\beta} |\mu|}{M_2} < 1 \\ c_W^2 \, \frac{m_Z^2}{|\mu|} \left(s_{2\beta} + \frac{M_2}{\mu} \right) - c_W^2 \delta M_1 , & \text{otherwise} \end{cases}$$

The position of the resonance

Mixed neutralinos can be modeled by two state $\tilde{\chi}_1^0\ \tilde{\chi}_1^+$ system, with splitting δ and the $\tilde{\chi}_1^0\ \tilde{\chi}_1^+ W^-$ coupling α.

Defining $\epsilon_v = (v/c)/\alpha$, $\epsilon_\delta = \sqrt{2\delta/m_{\tilde{\chi}^0}}/\alpha$ and $\epsilon_{\tilde{\chi}^+} = (m_{\tilde{\chi}^+}/m_{\tilde{\chi}^0})/\alpha$

Resonances appear at[5]

$$\epsilon_{\tilde{\chi}^+} \sim \frac{2}{\pi n^2} - \frac{\epsilon_\delta}{cn}$$

Since we are interested in the first resonance that means

$$\sqrt{\frac{2\delta}{m_{\tilde{\chi}^0}} \frac{3}{\pi^2} \frac{m_{\tilde{\chi}^0}}{m_{\tilde{\chi}^+}}} = \sqrt{\frac{2\alpha m_{\tilde{\chi}^0}}{\pi m_{\tilde{\chi}^+}}} - 1$$

Therefore as the splitting increases the mass of the LSP increases.

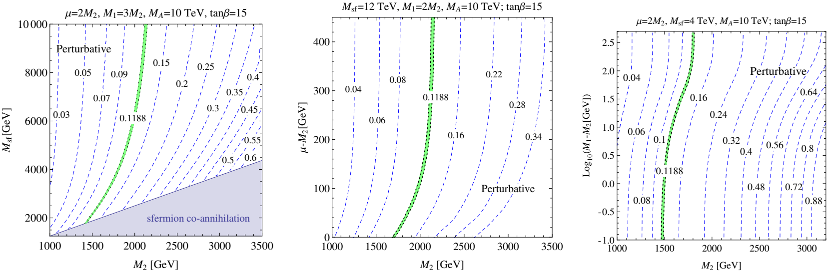
[5] The Sommerfeld enhancement for dark matter with an excited state, Tracy R. Slatyer, arXiv:0910.5713 [hep-ph] JCAP 1002 (2010) 028.

Ranges of parameters

Parameter	Range
M_2	$1 - 5$ TeV
$\|M_1\| - M_2$	$0 - 500$ GeV
$\|\mu\| - M_2$	$0 - 500$ GeV
$M_{\rm sf}$	$1.25\ M_2 - 12$ TeV
M_{A_0}	$0.5 - 10$ TeV
$\tan\beta$	$5 - 30$
$\|A_f\|$	$0 - 8$ TeV
M_3	$1.25\ M_2 - 8$ TeV

Relic density of heavy neutralinos

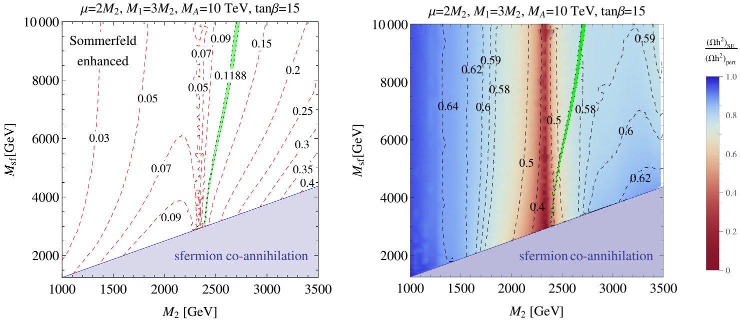
The perturbative wino-like case



- As the sfermion mass decreases the annihilation rate is suppressed due to t-channel interference and therefore the correct relic abundance is obtained for lighter neutralino masses of $\sim$ 1400 GeV
- As the Higgsino and bino annihilate less strongly, they dilute the wino annihilation and reduces the mass providing the correct relic density to 1700 and 1500 GeV respectively for the chosen set of parameters

Results

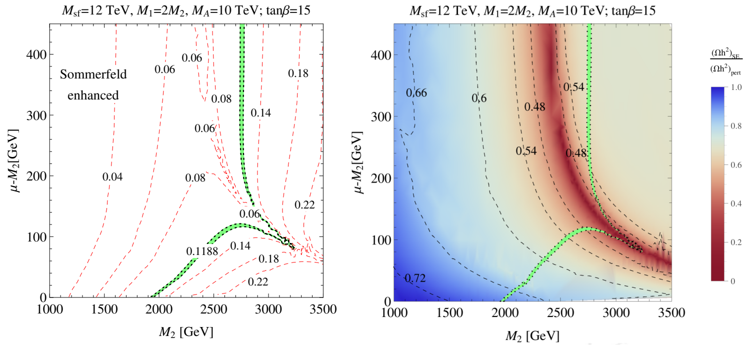
the pure wino with non-decoupled sfermions



- The correct relic density is moved from 1.5-2.1 TeV up to 2.4-2.7 TeV
- The resonance of the Sommerfeld effect is seen at around 2.4 TeV leading to the largest effects for the lightest sfermion masses

Results

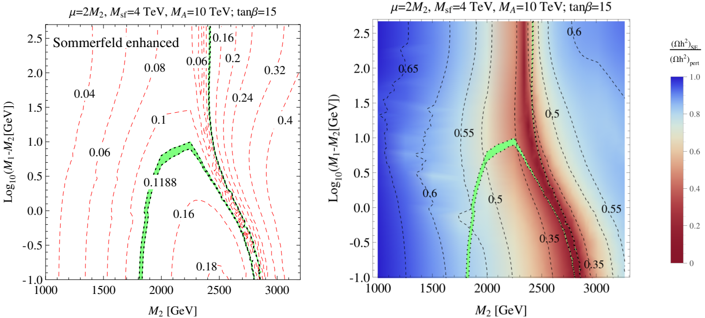
the wino-higgsino admixture



- The correct relic density is moved from 1.7-2.2 TeV up to 2-3.3 TeV
- The position of the resonance in the Sommerfeld effect is seen to be a function of μ, and largest effects are seen for $\mu - M_2 = 100$ GeV for the chosen parameters

Results

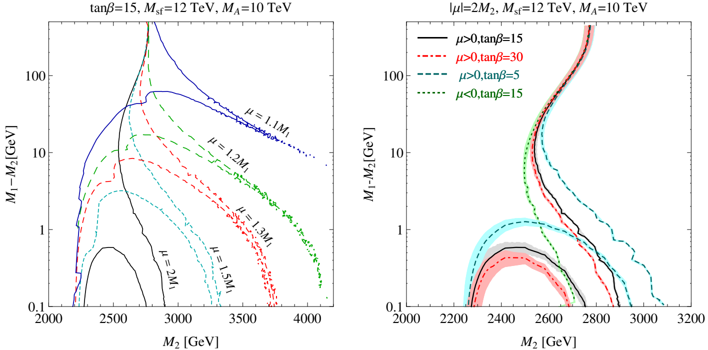
the wino-bino admixture



- The correct relic density is moved from 1.5-1.8 TeV up to 1.8-2.9 TeV
- The position of the resonance in the Sommerfeld effect is seen to be a function of M_1, and largest effects are seen for smallest splittings between M_2 and M_1

Results

the wino-bino admixture-the affect of additional parameters

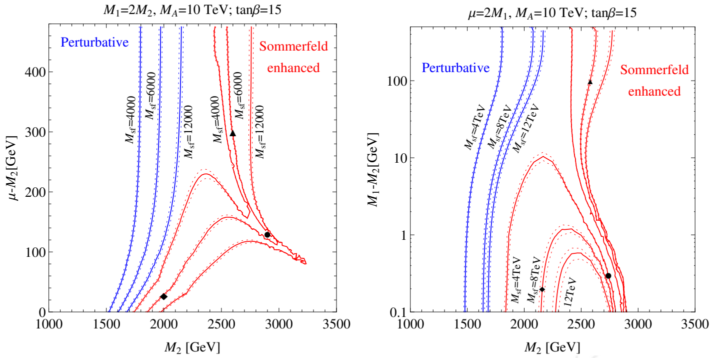


- The position of the resonance for bino-wino case is in fact strongly dependent on choice of parameters controlling mixing, i.e. μ, $\tan\beta$
- As the mixing is increased the effect is enhanced, i.e. when $|\mu|$ decreases, $\tan\beta$ decreases or when $\mu < 0$

Wino-like LSPs can give correct RD up to and beyond 4 TeV

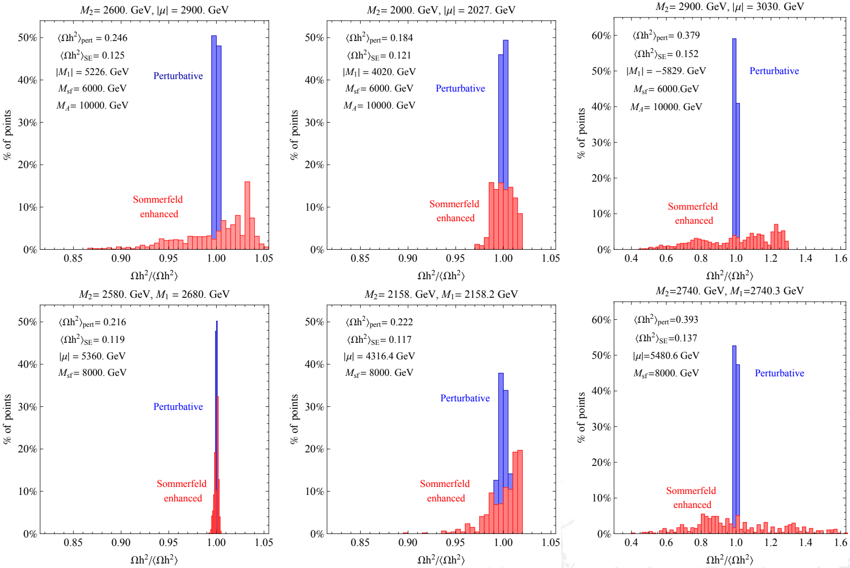
Results

the wino-bino admixture-the effect of additional parameters

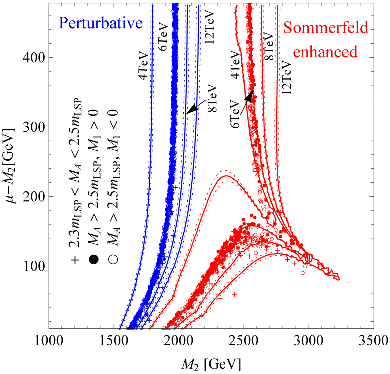


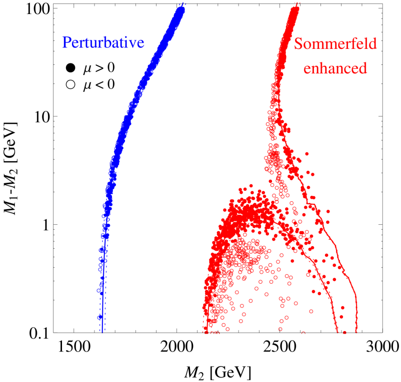
- Here we look at the effect of the sfermion mass on the M_1-M_2 vs M_2 and μ-M_2 vs M_2 planes
- For the three marked points we will study the effect of the remaining parameters

Effect of remaining parameters



Effect of remaining parameters





Plots showing the impact of the remaining parameters on the relic density for wino-like LSPs with varying Higgsino (left) and bino (right) admixtures

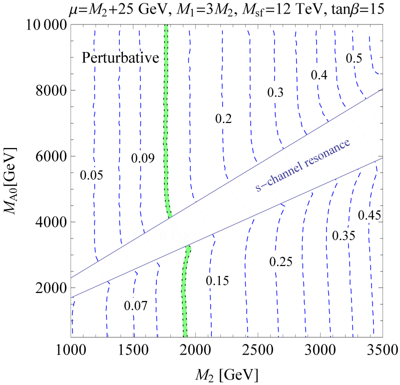
Summary

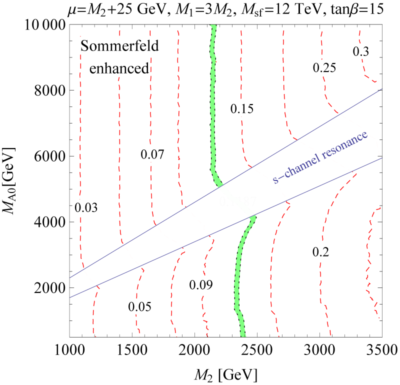
- Developed code and performed scan for relic density (RD) including the full Sommerfeld effect (SE) for wino-like region in general MSSM with accuracy $\mathcal{O}(\%)$ and running time $\mathcal{O}(\mathrm{mins})$
- For the pure wino the effect is $\sim$ 600 GeV but the sfermions alone can change value of M_2 giving the correct wino mass by several hundred GeV
- *For mixed wino-Higgsino scenarios:*
  - For $\mu - M_1 \sim 0.1$ GeV, M_2 down to 1.75 TeV can give correct RD and SE of 30%
  - Maximum effect seen for $\mu - M_1 \sim 100$ GeV and $M_2 \sim 3.3$ TeV where resonance produces effect on the RD of$\sim$ 90%
- *For mixed wino-bino scenarios:*
  - For low $M_1 - M_2$, M_2 down to 1.8 TeV can give correct RD and SE of 45% but also resonance also allows M_2 up to 2.9 TeV to satisfy RD constraint, with SE$\sim$ 80%.
  - Maximum possible LSP mass > 4 TeV, dependent on μ and $\tan\beta$, with maximum values arising when μ is small and positive and $\tan\beta$ is small.

Outlook: Public code to become available with and indirect detection rates including full SE

Results

the effect of the heavy Higgs bosons





Results

the effect of the heavy Higgs bosons