Using HQET to study form-factors in semi-leptonic decays

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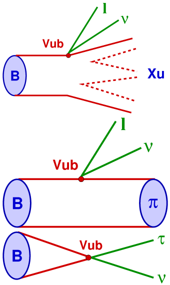
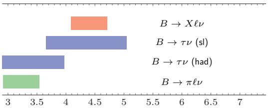
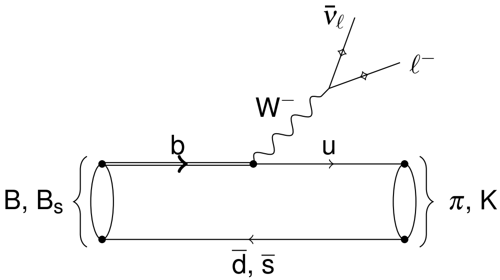
Motivation

- Understanding of *CP*-violation within the S(tandard) M(odel) + new physics needs a good understanding of flavor physics, CKM matrix elements.
- Precise (non-perturbative, first principles) determination of $|V_{ub}|$, currently the least well determined.
- $\sim 3\sigma$ discrepancy [PDG] :
  - Inclusive $B \to X_u \ell \nu$: $V_{ub} = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$
  - Exclusive $B \to \pi \ell \nu$: $V_{ub} = (3.28 \pm 0.29) \times 10^{-3}$
  - leptonic $B \to \tau \nu$ via f_B : $V_{ub} = (4.22 \pm 0.42) \times 10^{-3}$
- theoretical and experimental input needed
- This talk: Non-perturbative determination of form factors for $B_s \to K \ell \nu$ decay





Semi-leptonic decays $\mathrm{B} \to \pi \ell \nu$, $\mathrm{B_s} \to \mathrm{K} \ell \nu$



$\mathrm{B_s} \to \mathrm{K}$:

- no experimental data *yet* – predictions
- easier on the lattice (valence $m_\mathrm{K} = m_\mathrm{K}^\mathrm{phys}$ computationally less expensive than for the π)
- not far from $\mathrm{B} \to \pi$

$$\langle \mathrm{K}(p_\mathrm{K}^\mu) | V^\mu | \mathrm{B_s}(p_{\mathrm{B_s}}^\mu) \rangle = f_+(q^2) \left[p_{\mathrm{B_s}}^\mu + p_\mathrm{K}^\mu - \frac{m_{\mathrm{B_s}}^2 - m_\mathrm{K}^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_{\mathrm{B_s}}^2 - m_\mathrm{K}^2}{q^2} q^\mu$$

Form factor

B rest frame: $p_{B_s} = m_{B_s} v_\mu = m_{B_s}(1,0,0,0)$.

$$\langle \mathrm{K}| V^0 |\mathrm{B_s}\rangle = \sqrt{2m_{\mathrm{B_s}}} f_\parallel$$

$$\langle \mathrm{K}| V^j |\mathrm{B_s}\rangle = \sqrt{2m_{\mathrm{B_s}}} p^j_{\mathrm{K}} f_\perp$$

The vector current is $V_\mu = \bar{\psi}_l(x)\gamma_\mu \psi_h(x)$.

$$f_+ = \frac{1}{\sqrt{2m_{\mathrm{B_s}}}} f_\parallel + \frac{1}{\sqrt{2m_{\mathrm{B_s}}}} (m_{\mathrm{B_s}} - E_{\mathrm{K}}) f_\perp$$

$$f_0 = \frac{\sqrt{2m_{\mathrm{B_s}}}}{m^2_{\mathrm{B_s}} - m^2_{\mathrm{K}}} [(m_{\mathrm{B_s}} - E_{\mathrm{K}}) f_\parallel + (E^2_{\mathrm{K}} - m^2_{\mathrm{K}}) f_\perp]$$

First calculate $f_\perp$ and $f_\parallel$ and then relate to f_0 and f_+.
In the static limit:

$$f_+(q^2) = \sqrt{\frac{m_{\mathrm{B_s}}}{2}} f_\perp(q^2)$$

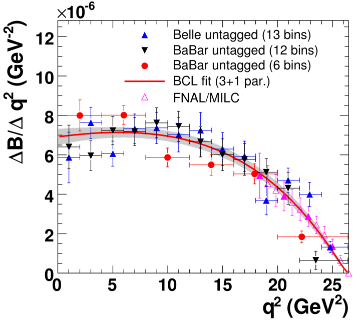
corrections $O(10\%)$

Experimental decay rates

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B_s}^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

$$\lambda(q^2) = (m_{B_s}^2 + m_K^2 - q^2)^2 - 4 m_{B_s}^2 m_K^2$$

- experimentally measured decay rate
- form factor $f_+(q^2)$ computed in LQCD
- $\Rightarrow$ determine V_{ub}
- The so-called BCL (Bourelly, Caprini, Lellouch) parametrization can be used to obtain results for a whole range of q^2.



Challenges in form factor computations

ALPHA Collaboration

- $m_b\, a > 1 \;\rightarrow$ effective field theories — HQET

- eft's $\rightarrow$ non-trivial renormalization
  higher dimensional operators, e.g.
  $\delta V_k = c_1 \overline{\psi}_{\rm h} \gamma_k \gamma_l D_l \psi$
  present for HQET, relativistic heavy quark action, NRQCD — NPR by matching

- $a \rightarrow 0, \;\; m_\pi \rightarrow m_\pi^{\rm phys}$ — extrapolations

- $|\mathbf{p}| \neq 0 \;\rightarrow$ signal/noise degradation — discuss here

Heavy Quark Effective Theory I

- Problem: $L^{-1} \ll m_\pi \approx 140\,\mathrm{MeV}, \ldots, m_\mathrm{B} \approx 5\,\mathrm{GeV} \ll a^{-1}$
- Eg. A (charm) quark of mass $\approx$ 1 GeV with lattice spacings a $\approx 0.1 \cdots 0.05$ fm would need lattices $L/a \approx 60 \cdots 120$.

Solution: Heavy Quark Effective Theory (HQET) [ALPHA collab. '01-'13]

- *Non-perturbative* effective theory treating the heavy quarks in the background of a sea of strongly interacting quarks and gluons.
- $\langle O \rangle_\mathrm{LO} = Z^{-1} \int_\mathrm{fields} \mathrm{e}^{-S_\mathrm{LO}} O$
- For lattice formulation $\longrightarrow$ continuum limit exists, and is unique (for a finite number of renormalized parameters).
- Expansion parameter: $1/m_\mathrm{h}$.
- Higher order terms carry higher mass dimensions $L^\mathrm{NLO} = \sum_i \omega_i \mathcal{O}_i, \quad \omega_i = \frac{1}{m_\mathrm{h}} \tilde{\omega}_i$
- *Non-perturbatively* renormalisable (order by order in $1/m_\mathrm{h}$).
- well-defined continuum limit with $1/m_\mathrm{h}$ insertions in correlation functions.
- valid for kaon momenta $p_\mathrm{K} \ll m_\mathrm{b}$.
- in practice $p_\mathrm{K} \lesssim 1\,\mathrm{GeV} \Rightarrow q^2$ close to q^2_max.

HQET II

For *smooth* fields, $\mathcal{L}_{\text{Dirac}} = \bar{\psi}(m_{\text{h}} + D_\mu \gamma_\mu)\psi$ can be split order by order in $1/m_{\text{h}}$:

$$
\begin{aligned}
\mathcal{L} &= L_{\text{h}}^{\text{stat}} + L_{\text{h}}^{(1)} + L_{\bar{\text{h}}}^{\text{stat}} + L_{\bar{\text{h}}}^{(1)} + O(\frac{1}{m_{\text{h}}^2}) \\
L_{\text{h}}^{\text{stat}} &= \bar{\psi}_h(m_h + D_0)\psi_h; \;\; L_{\bar{\text{h}}}^{\text{stat}} = \bar{\psi}_{\bar{h}}(m_h - D_0)\psi_{\bar{h}} \\
L_{\text{h}}^{(1)} &= -\frac{1}{2m_{\text{h}}}(O_{\text{kin}} + O_{\text{spin}}) \\
O_{\text{kin}}(x) &= \bar{\psi}_h(x) D^2 \psi_h(x); \;\; O_{\text{spin}} = \bar{\psi}_h(x)\sigma.B(x)\psi_h(x) \\
\sigma_k &= \frac{1}{2}\varepsilon_{ijk}\sigma_{ij}; \;\; B_k = i\frac{1}{2}\varepsilon_{ijk}[D_i, D_j]
\end{aligned}
$$

- smooth $\rightarrow D_K\psi = O(1) = G_\mu; \;\; D_0\psi = O(m_{\text{h}})$
- The heavy quark is thus, treated non-relativistically.
- In contrast to QCD, the renormalizability to all orders in the expansion has not been “proved”.
- Matching to observables in QCD can be performed fully non-perturbatively.

$$S_h(x,y) = \Theta(x_0 - y_0)\delta(\vec{x} - \vec{y})\,\mathrm{e}^{-m(x_0 - y_0)}\,\mathcal{P}\exp\left\{-\int dt\, A_0(t,\vec{x})\right\} P_+; \;\; E_{\text{h}}^{\text{QCD}} = E_h^{\text{stat}}|_{m=0} + m$$

HQET III: Additional symmetries

- **Flavor** : For F heavy quarks, there is an additional symmetry

$$\psi_h(x) \to V\psi_h(x); \quad \bar{\psi}_h(x) \to \bar{\psi}_h(x)V^\dagger; \quad V \in SU(F)$$

  Emerges in the large mass limit, but not so much useful phenomenologically

- **Spin** : $SU(2)$ spin rotations on the two (non-relativistic) Dirac components

$$\psi_h(x) \to \mathrm{e}^{i\sigma_k\alpha_k}\,\psi_h(x); \quad \bar{\psi}_h(x) \to \bar{\psi}_h(x)\,\mathrm{e}^{-i\sigma_k\alpha_k}$$

  Relates the vector and the axial-vector components, and important for renormalization properties. Can be used to classify the spectrum, and/or predict relations between different masses e.g.,

$$m^2_{\mathrm{B}^\star} - m^2_{\mathrm{B}} = m^2_{\mathrm{D}^\star} - m^2_{\mathrm{D}}; \quad m^2_{\mathrm{B}'} - m^2_{\mathrm{B}} = m^2_{\mathrm{D}'} - m^2_{\mathrm{D}}$$

- **Local flavor-number**: No space derivatives in the lagrangian, implying

$$\psi_h(x) \to \mathrm{e}^{i\eta(x)}\,\psi_h(x); \quad \bar{\psi}_h(x) \to \bar{\psi}_h(x)\,\mathrm{e}^{-i\eta(x)}$$

  Local quark number $Q_h(x) = \bar{\psi}_h\psi_h(x)$ is conserved.

HQET IV: Predictions, an example

- Consider the leptonic decay of B-meson: $B^- \to \tau^- \bar{\nu}_\tau$
- To a good approximation, the transition amplitude $\mathcal{A}$ is given in terms of the effective weak Hamiltonian, which factorizes into a hadronic and a leptonic part:

$$\mathcal{A} \propto \langle \tau\bar{\nu}|\tau(x)\gamma_\mu(1-\gamma_5)\bar{\nu}(x)|0\rangle\langle 0|\bar{u}(x)\gamma_\mu(1-\gamma_5)b(x)|B^-\rangle$$

Using parity and Lorenz invariance for the hadronic part,

$$\langle 0|\bar{u}(x)\gamma_\mu(1-\gamma_5)b(x)|B^-(\vec{p})\rangle = \langle 0|A_\mu(x)|B(\vec{p})\rangle = p_\mu f_{\rm B}\, e^{ipx}$$

where $A_\mu(x) = \bar{u}(x)\gamma_\mu\gamma_5 b(x)$ is the flavored axial current.

- A single hadronic parameter f_B parameterizing the decay.
- HQET can determine the asymptotic mass dependence of $f_{\rm B}$.
- In the leading order, $A_0^{\rm HQET}(x) = A_0^{\rm stat} + O(1/m)$, $\quad A_0^{\rm stat} = \bar{u}(x)\gamma_0\gamma_5\psi_{\rm h}(x)$
- $\langle 0|A_0^{\rm stat}(0)|B^-(\vec{p}=0)\rangle = \Phi^{\rm stat}$ with a mass-independent $\Phi^{\rm stat} = m_B^{-1/2} p_0 f_B = m_B^{1/2} f_B$, non-relativistic normalization of states use $|\mathbf{p}\rangle_{\rm rel} = \sqrt{E(\mathbf{p})}|\mathbf{p}\rangle$
- $$f_{\rm B} = \frac{\Phi^{\rm stat}}{\sqrt{m_{\rm B}}} + O(1/m_b), \quad \frac{f_{\rm B}}{f_{\rm D}} = \frac{\sqrt{m_{\rm D}}}{\sqrt{m_{\rm B}}} + O(1/m_c)$$

HQET: Form Factors

$$V_0^{\rm stat} = \bar\psi_u\gamma_0\psi_h + a c_{\rm V_0}(g_0)\bar\psi_l \sum_l \overleftarrow{\nabla}_l^S \gamma_l\psi_h$$

$$V_k^{\rm stat} = \bar\psi_u\gamma_k\psi_h - a c_{\rm V_k}(g_0)\bar\psi_l \sum_l \overleftarrow{\nabla}_l^S \gamma_l\gamma_k\psi_h$$

- Improvement coefficients $c_{\rm V_0}, c_{\rm V_k}$ known to 1-loop order.
- The (multiplicative) renormalization of the currents are expressed as:

$$V_{0,k}^{\rm stat,RGI} = Z^{\rm stat,RGI} V_{0,k}^{stat}$$

- HQET parameters $(Z_i, c_{\rm V_i}, \omega_i)$ determined non-perturbatively:

$$\Phi_i^{\rm QCD}(L, m_{\rm h}, 0) = \Phi_i^{\rm HQET}(L, m_{\rm h}, a)$$

- Matching HQET and QCD for certain (finite L) “observables” Φ_i [Della Morte et al. ’13]
- The matrix elements obtained in HQET can be related to those in QCD via the so-called matching coefficients:

$$f_{\perp,\parallel} = C_{\rm V_0,V_k}(M_b/\Lambda_{\overline{\rm MS}}) f_{\perp,\parallel}^{\rm stat,RGI}$$

- At the moment $C_{\rm V_0,V_k}$ are known upto 2-loop order, but will be obtained non-perturbatively fully in the future.

Extrapolations

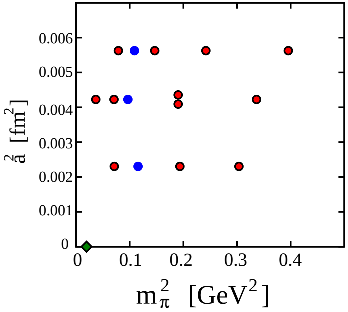
At fixed q^2, achieved by "twisting" [Bedaque '04] the s quark:
$\psi(x+L\hat{k}) = \mathrm{e}^{\mathrm{i}\,\theta_k}\,\psi(x)$ $\vec{p}^{\theta} = (2\pi\vec{n}+\vec{\theta})/L$ freely tuneable $\rightarrow$ heavy quark twisting

(keep B_s in rest frame)

- continuum, $a \rightarrow 0$
- chiral, $m_\pi \rightarrow m_\pi^{\text{phys}}$

Ensembles and simulation

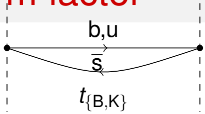
- non-perturbatively $O(a)$ improved Wilson fermions
- $N_\mathrm{f} = 2$ CLS ensembles
- scale setting via f_K [Fritzsch et al. '12]
- $m_\pi L \gtrsim 4$
- Error estimates taking into account autocorrelations [Schaefer et al. '12]

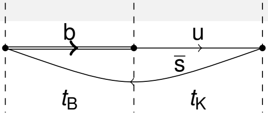


id	$T \times L^3$	a [fm]	m_π [MeV]	$m_\pi L$	# meas.	# target
A5	64×32^3	0.0749(8)	330	4.0	500	500
F6	96×48^3	0.0652(6)	310	5.0	254	500
N6	96×48^3	0.0483(4)	340	4.0	220	500

- keep $m_\mathrm{K}/f_\mathrm{K} = \text{phys.}$
- for now: one value of q^2 only, $q^2 = 21.23\,\mathrm{GeV}^2$

Obtaining the form factor

$\mathcal{C}^{\{B,K\}} \sim$ 

$\mathcal{C}^{3} \sim$ 

Ratio – plateaux

$$\langle K(p_K^\theta)|V^\mu|B_s(0)\rangle = \lim_{T,t_B,t_K\to\infty} \frac{\mathcal{C}^3_\mu(t_K,t_B)}{\sqrt{\mathcal{C}^K(t_K)\mathcal{C}^B(t_B)}} e^{E_K t_K/2} e^{E_B t_B/2} \equiv \lim_{T,t_B,t_K\to\infty} f_\mu^{\rm ratio}(q^2)$$

Factorising Fit

Combined fit to ground and first two excited states of $\mathcal{C}^3, \mathcal{C}^B$

$$\begin{cases} \mathcal{C}^3_{\mu i}(t_B,t_K) & = \sum_{n,m} \beta_i^{(n)} \varphi_\mu^{(n,m)} \kappa^{(m)} e^{-E_B^{(n)} t_B} e^{-E_K^{(m)} t_K}, \qquad \varphi_\mu^{(1,1)} \sim f_+(q^2) \\ \mathcal{C}^B_{ij}(t_B) & = \sum_n \beta_i^{(n)} \beta_j^{(n)} e^{-E_B^{(n)} t_B} \\ \mathcal{C}^K(t_K) & = \sum_m (\kappa^{(m)})^2 e^{-E_K^{(m)} t_K} \end{cases}$$

- Gaussian smearing, $\psi_l^{\rm sm}(x) = (1+\kappa\Delta)^{N_{\rm it}} \psi_l(x)$, $N_{\rm it} \leftrightarrow$ wavefunctions
- random noise sources, full time dilution

Results: Effective Masses

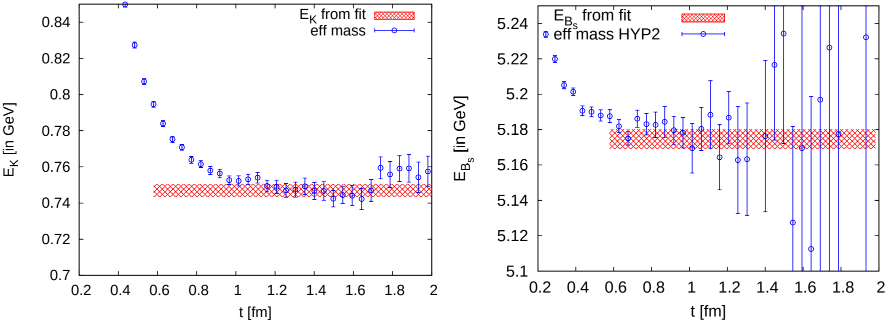


Figure : Effective energy of C_K (left) and C_{Bs} (right) on ensemble N6. Note that both panels have equal ranges on the corresponding axes. One can identify reasonable plateaus with small errors even though the Kaon carries a non-vanishing momentum and we have a static B_s-meson. The value for the ground state energy as obtained from a two-exponential fit is shown as a red band. Uncertainties shown here are only those of E^{stat} and E_K in lattice units, not the ones of m_{bare} and the lattice spacing. The data points are shown for the case of maximum smearing, while the fit involves all the smearing levels.

Preliminary results

- Final quoted values from the fits.
- GEVP is used to guess the B_s energies.
- GEVP guesses used in the combined fit to the C^K, C^{B_s} and the C^3-point functions.
- Large t_{min}^K is used to get rid of the excited state contribution but the wrappers also need to be accounted for.
- Two excited state parameters for the B_s state considered.
- $f_{\parallel}^{\text{bare}} = \varphi_0^{(0,0)} \sqrt{2E_{\text{K}}}; \quad f_{\perp}^{\text{bare}} = \frac{\varphi_0^{(0,0)}}{p_{\text{K}}^k} \sqrt{2E_{\text{K}}}.$

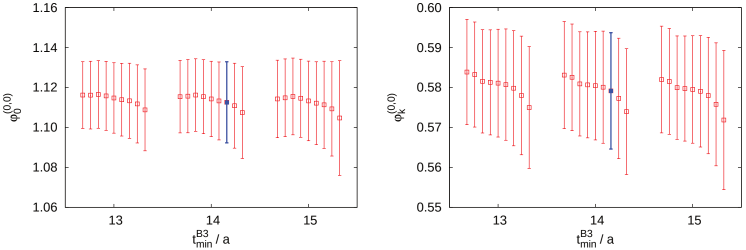


Figure : Stability of fit parameters with the choice of fit range

Form factors from ratios

Different ratios can be defined to extract the form factors:

$$
\begin{aligned}
\mathcal{R}_I(t_K,t_B) &= \frac{C^3_{ij}(t_K,t_B)}{\left[C^K_{ii}(t)C^B_{jj}(t)\right]^{1/2}} \exp\left((E^{\rm eff}_B(t)-E^{\rm eff}_K(t))(t_B-t_K)/2\right) \\
\mathcal{R}_f(t_K,t_B) &= \frac{C^3_{ij}(t_K,t_B)}{\left[C^K_{ii}(t_K)C^B_{jj}(t_B)\right]^{1/2}} \left[\frac{C^K_{ii}(t_B)C^B_{jj}(t_K)}{C^K_{ii}(t_B)C^B_{jj}(t_B)}\right] \\
\mathcal{R}_{II}(t_K,t_B) &= \frac{C^3_{ij}(t_K,t_B)}{\left[C^K_{ii}(t_K)C^B_{jj}(t_B)\right]^{1/2}} \exp\left(E^{\rm eff}_B(t)t_B/2+E^{\rm eff}_K(t)t_K/2\right) \\
\mathcal{R}_{III}(t_K,t_B) &= \frac{C^3_{ij}(t_K,t_B)}{C^K_{ii}(t_K)C^B_{jj}(t_B)}
\end{aligned}
$$

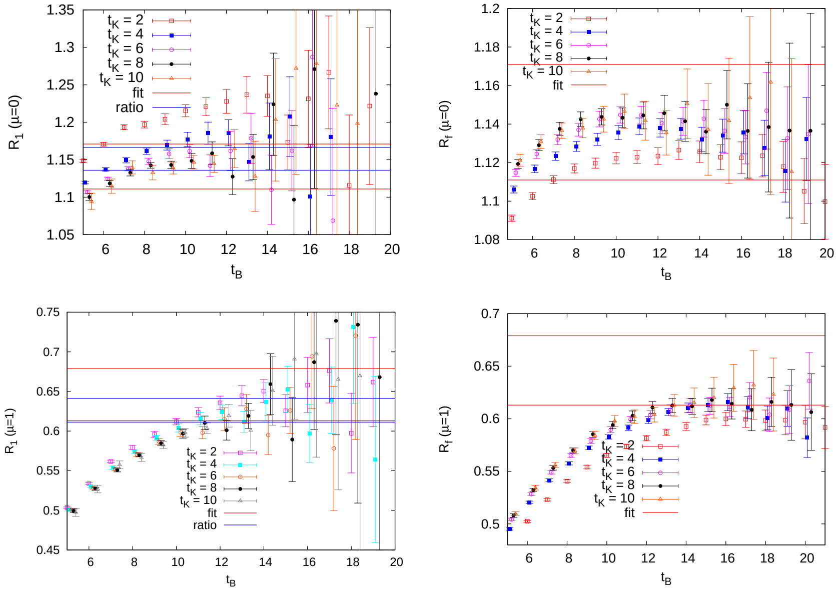
The ratios asymptote to the respective form factors, the last one however, has a non-trivial normalization.

To get improved convergence, used summed ratios:

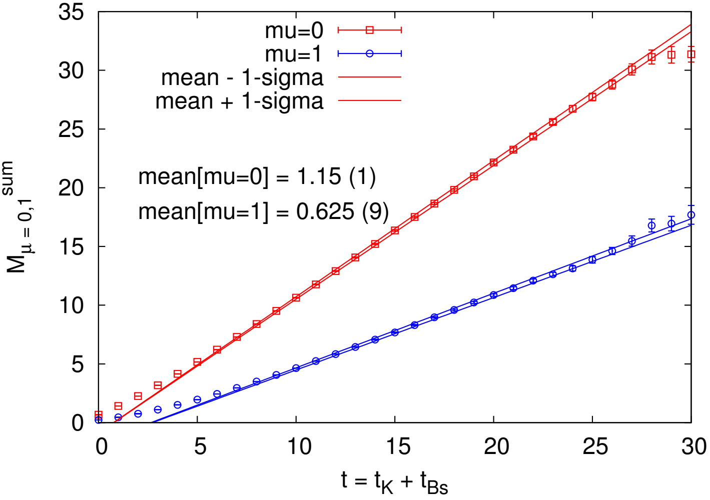
$$\mathcal{S}_i = \sum_{t_B} \mathcal{R}_i(t-t_B,t_B)$$

Obtain form factors using a fit or a numerical derivative.

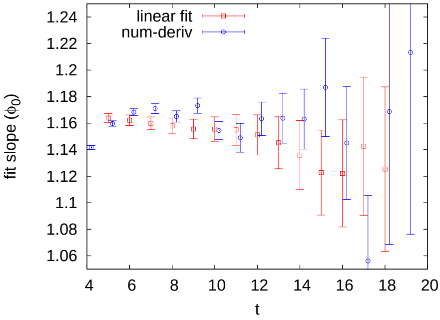
Ratios: R_l and R_f (A5 ensembles)

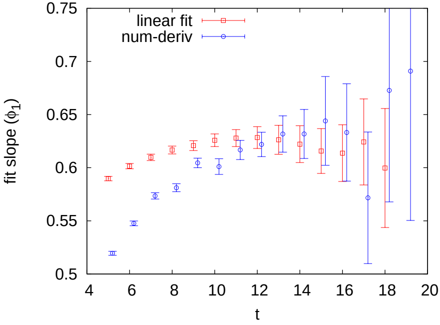


Summed ratios (A5 ensembles)

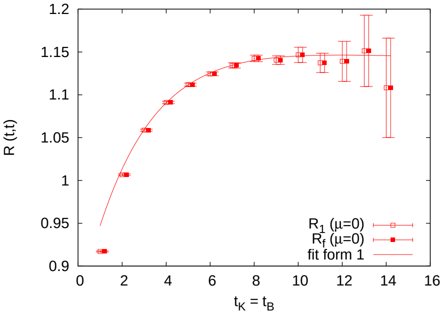


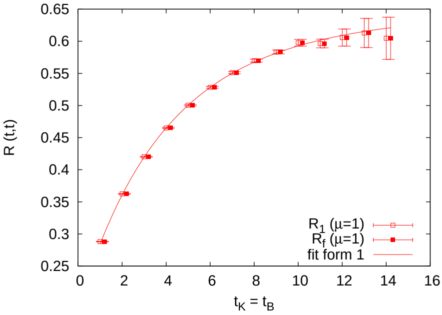
Form factors from summed ratios (prelim.)



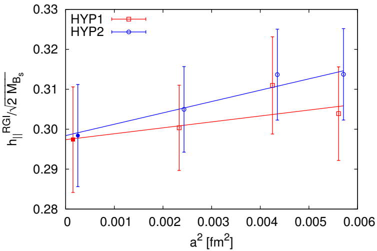


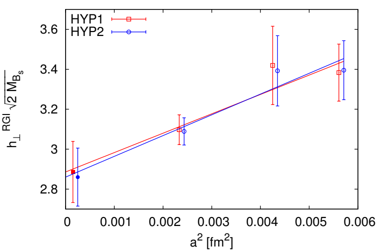
Ratio fits (prelim.)



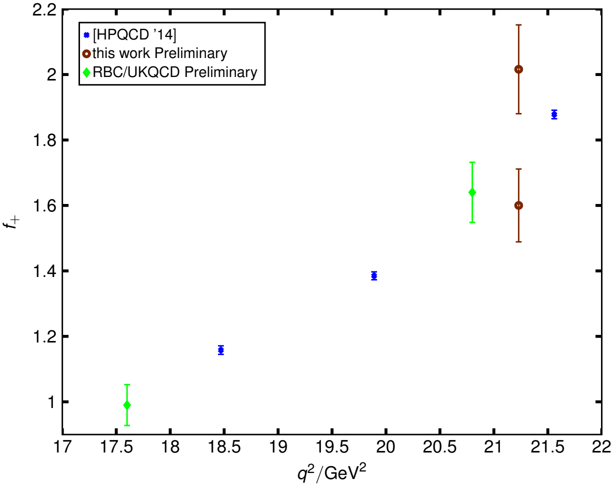


Towards the continuum limit





A little comparison



- blue: [HPQCD '14] , $a = 0.09$ fm, $m_\pi = 320\,\mathrm{MeV}$ Pert. renormalisation
- brown: this work, continuum, static, $m_\pi = 340\,\mathrm{MeV}$, NP renormalisation [Della Morte et al. '07] . Preliminary.
- green: RBC/UKQCD Preliminary, chiral, continuum. Pert. Renormalisation

Error budget – rough estimates

- extraction of FF through fits / ratios ($\approx 2\%$)
- lattice spacing (scale setting): determination of q^2 ($\approx 1\%$)
- continuum extrapolations ($2 \ldots 5\%$)
- chiral extrapolations (seems flat: small)
- BCL parameterisation, experimental data (none yet, for $B \to \pi \approx 10\%$)
- $N_f = 2$ (*"To date, no significant differences between results with different values of* N_f *have been observed."* [FLAG '13])
- HQET truncation (static: $\sim 10\%$, at $O(1/m_h)$: $\sim 1\%$; [$< 1\%$ for f_{B_s} [Bernardoni et al. '14]])

Conclusions and Outlook

Conclusions

- $f_+(q^2)$ for $B_s \to K$ in HQET
- *fully non-pertubative* renormalisation setup (at LO, soon at NLO in $1/m_h$)
- *small discretisation errors*
- rough agreement with recent HPQCD results $\longrightarrow V_{ub}$ puzzle remains

Outlook

- Chiral extrapolation: $m_\pi \to m_\pi^{\text{phys}}$.
- Inclusion of $O(1/m_b)$ effects in analysis (matching to be done, large volume measurements available).
- Measure at one or two more q^2.
- $N_f = 2 + 1$, open BC, wrappers gone.
- $B \to \pi$.

Parameterisation of $f(q^2) \times V_{\mathrm{ub}}$

Our ultimate plan:
BCL-Parameterisation [Bourrely, Caprini, Lellouch '09] :

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{B_s^*}} \sum_{k=0}^{K-1} b_k \left[z^k(q^2) - (-1)^{k-K} \frac{k}{K} z^K(q^2) \right]$$

- Correlated, combined fit of our data and experimental data
- Minimise $\chi^2 = \chi^2_{\mathrm{th}} + \chi^2_{\mathrm{exp}}$
- fit parameters b_k, V_{ub}