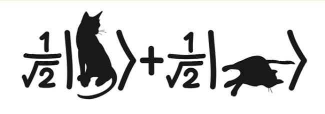
When the Schrödinger's Cat comes out of the box · · ·



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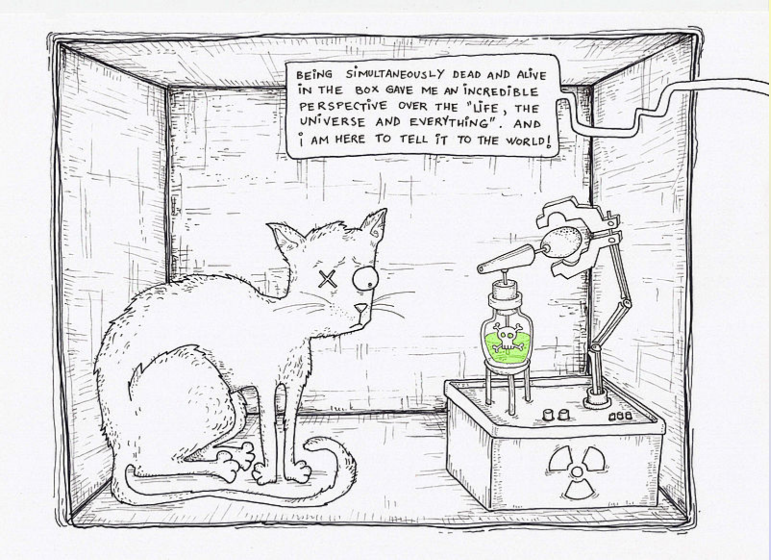
Listening to the Schrödinger's Cat

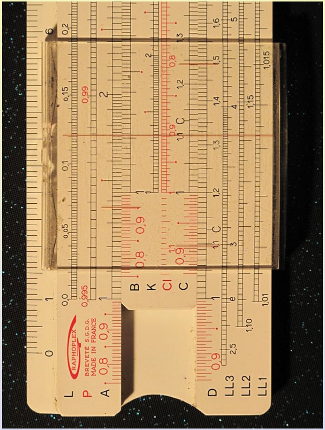


What is this perspective?

Outline

Classical computation: analog machines

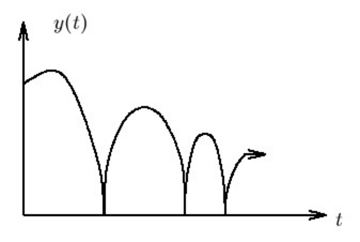
The Antikythera machine

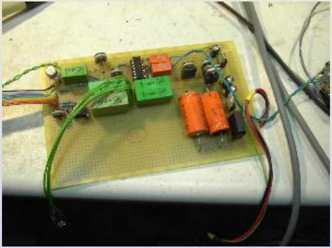


The Slide Rule

An analog computer: bouncing ball

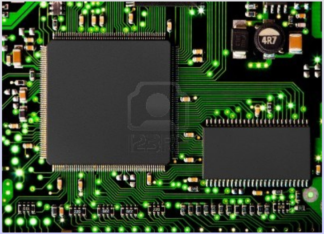
The motion of a bouncing ball can be simulated by an electrical circuit that models the kinematics. Variables are gravity, damping due to air friction and elasticity of the ball.

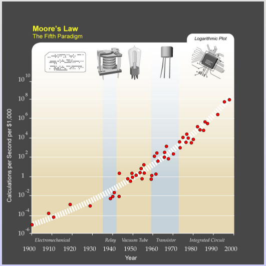




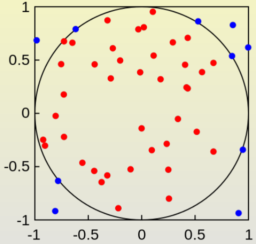
Digital computers: old and new

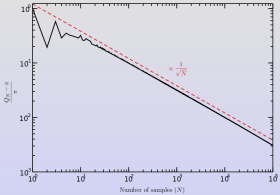
Driven by a revolution in transistors and integrated circuits development





Monte-Carlo methods: when random numbers have a meaning



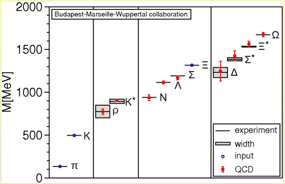


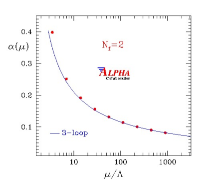
- $w(x, y) = 1$ if $x^2 + y^2 \leq 1$
  $= 0$ otherwise
- Area $Q = \int w(x, y)\, dx\, dy = \pi$
- Pick N random numbers in the set $x \in [-1, 1]$ and $y \in [-1, 1]$
- Calculate the sum $Q_N = 4 \frac{1}{N} \sum_{i=1}^{N} w(x_i, y_i)$
- The larger the N, the more accurate the result is!
- Estimate of error

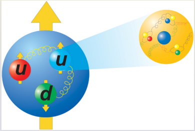
$$\frac{\Delta Q}{Q} = \frac{\sqrt{\langle w^2 \rangle - \langle w \rangle^2}}{\langle w \rangle \sqrt{N}}$$

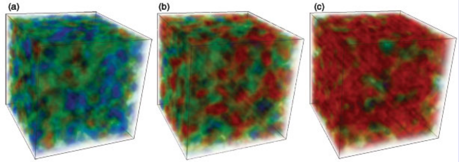
- **P** (polynomial time) problem!

What can random numbers do for you?





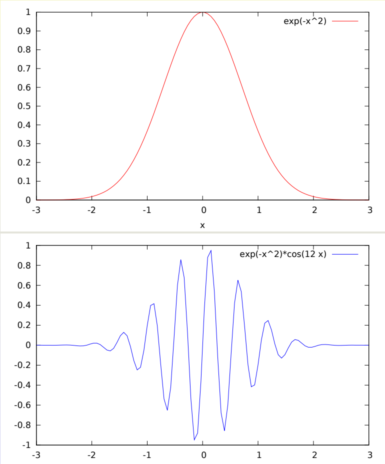




Properly harnessed (= good algorithms + digital computers), they can:

- Reproduce the meson and baryon spectrum (pions, kaons, protons, $\cdots$)
- A truly remarkable feat, given that the building blocks of matter, quarks and gluons are confined within the hadrons by the strong force. This theory, unlike the theory describing photons and electrons, (Quantum Electrodynamics) is strongly interacting.
- Because quarks and gluons carry color charges, this theory is often called Quantum Chromodynamics or QCD in short.
- MC predict that if quarks and gluons are heated to 2 trillion = 2×10^{12} degrees, the hadrons would melt setting them free!
- Currently being verified at experiments at CERN and BNL.

A (classical) nightmare



- Another way to calculate π
  $\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}$
- Weighting factor $f(x)$ essential to make calculation efficient

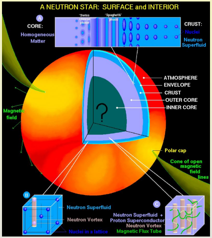
$$Q = \int w(x)dx = \frac{1}{N}\sum_{i=1}^{N} w_i$$

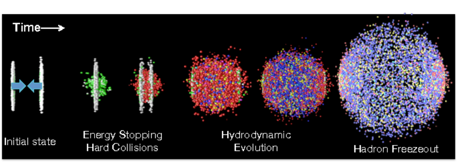
w_i drawn from a Gaussian distribution.

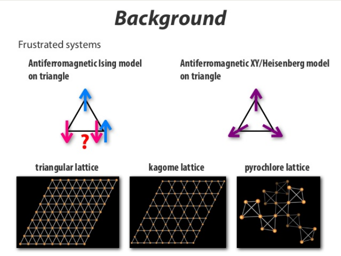
- For an oscillatory integral, MC is difficult:
  $\int_{-\infty}^{\infty} \exp(-x^2)\cos(kx)dx = \sqrt{\pi}\exp(-\frac{k^2}{4})$
- This means that $\frac{\Delta Q}{Q} \sim \exp(c\, k^2)$
- One needs $N \sim \exp(d\, k^2)$ to have the same signal-to-noise ratio!
- Suddenly, the problem becomes exponentially, i.e., **NP** (non-polynomial) hard! Technically called sign problem.

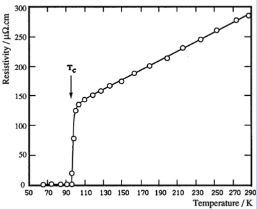
How does it hurt physics?

A zoo of interesting problems in strongly correlated systems, all reliant on non-perturbative methods (Monte-Carlo simulations).



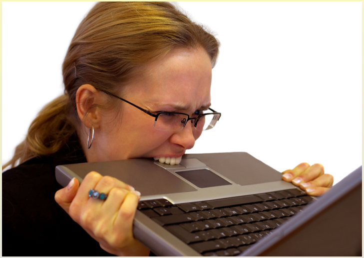


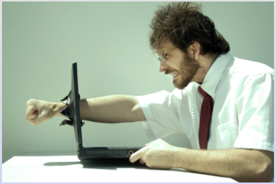


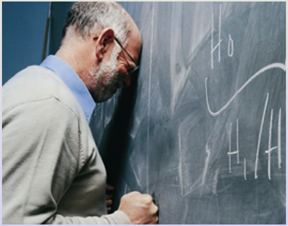


ALL suffer from severe sign problems!

... and make us frustrated ...







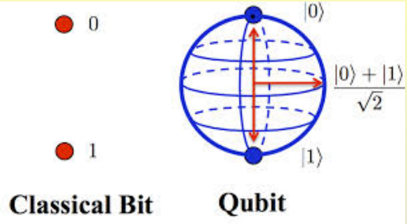
New interdisciplinary tools needed for a breakthrough.

What’s the way out?

Richard Feynman’s vision from 1982

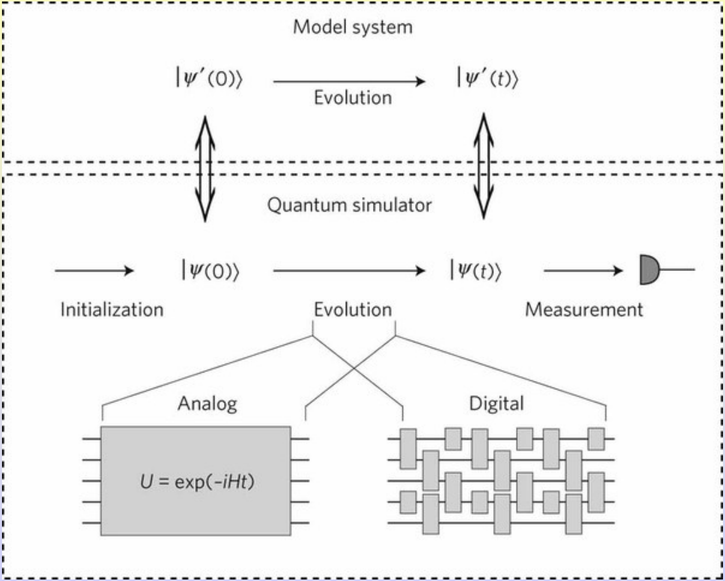
I’m not happy with all the analyses that go with just the classical theory, because nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy.

The Qubit



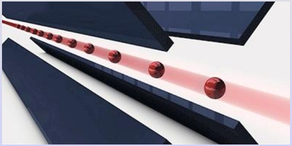
- While a classical bit can only take two values, the quantum bit can be in a superposition state: $|\psi_1\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$ where $\alpha_1, \alpha_2 \in \mathbb{C}, \quad |\alpha_1|^2 + |\alpha_2|^2 = 1$
- A two-qubit system can be represented as: $|\psi_2\rangle = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle$, with $|\alpha_1|^2 + \cdots + |\alpha_4|^2 = 1$
- Storing a n qubit already requires 2^n complex coefficients on a classical computer, but only n quantum mechanical degrees of freedom (spins, atoms, ions, molecules, $\cdots$.)
- For $n = 500$, this number is more than the number of atoms in the Universe!

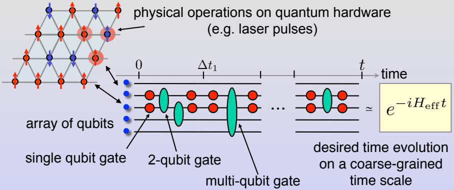
The basic principle of Quantum Computers



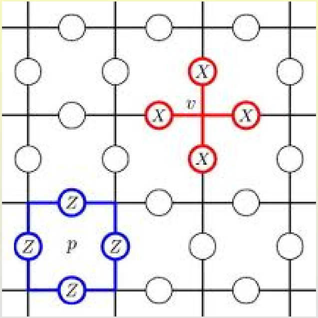
Ion-traps as digital quantum computers

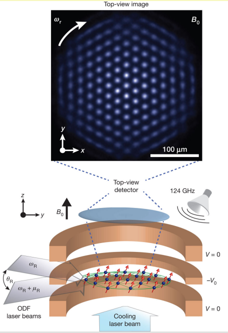
- Well controlled strings of trapped atomic ions held in linear radiofrequency traps.
- Qubits are encoded in one ion each.
- Manipulated and made to interact with other qubit by laser pulses or microwave radiation.
- Coherence can be preserved for a few milliseconds upto seconds.
- Quantum error correction techniques required.





Big quantum computers

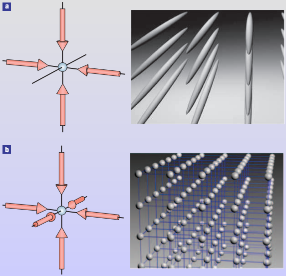


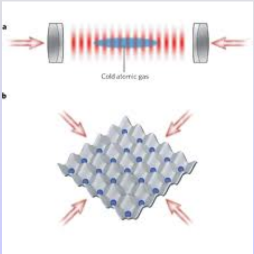


- The toric code (lattice gauge theory with a $\mathbb{Z}(2)$ gauge group) digitally simulated in an ion-trap. A prescribed sequence of quantum gate operations implements the interaction.
  Kitaev (2003); Lanyon et. al. (2011)
- Triangular lattice with spin-$\frac{1}{2}$ in a Penning trap: upto 300 spins.
  Britton et. al. (2012)

Analog Quantum Simulators

- A well-controlled quantum system: $H_{\text{sim}} \longleftrightarrow H_{\text{sys}}$.
- Ultra-cold atoms in optical lattices turn out to be ideal candidates.
- Make an optical (super)-lattice by shining lasers on each other.
- Different lattices, different geometries, different dimensions.
- Tunable parameters include electric fields, magnetic fields, microwaves, lasers, atomic species.
- ultracold temperatures in nanokelvin range maintain long coherence.
- Simpler interactions but more scalability.

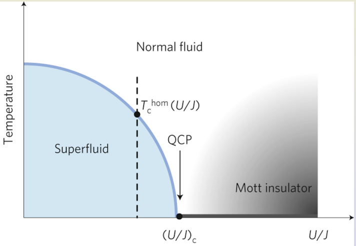


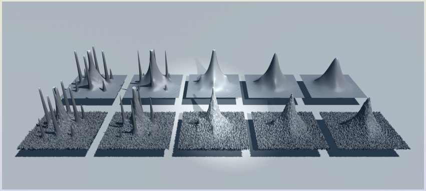


Validating a Quantum Computer

The Bose-Hubbard model $\longrightarrow$ analog quantum simulator validated.

$$H = -J \sum_{<i,j>} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i(n_i - 1)$$





comparison of Monte-Carlo and experiment with $\sim 3 \times 10^5$ particles

Prokofiev, Svistunov, Troyer et. al.(MC); Bloch et. al. (Expt); Nature 2010

Superiority of quantum computers

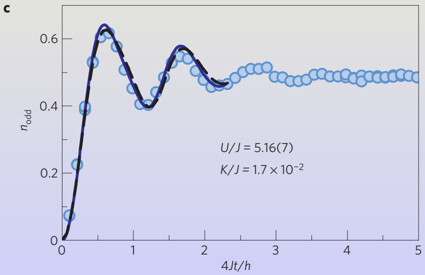
Quantum computers would be absolutely essential in studying real-time evolution in quantum systems and non-equilibrium physics.

Example of a quantum quench in a strongly correlated Bose gas.

S. Trotzky et. al., Nature Physics (2012).

$$H = \sum_j \left[-J(a_j^\dagger a_{j+1} + \text{h.c.}) + \frac{U}{2} n_j(n_j - 1) + \frac{K}{2} n_j j^2 \right]$$

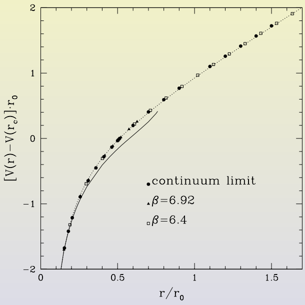
Start the system in the state $|\psi(t=0)\rangle = |\cdots, 1, 0, 1, 0, 1, \cdots\rangle$ and then study the evolution by the Hamiltonian



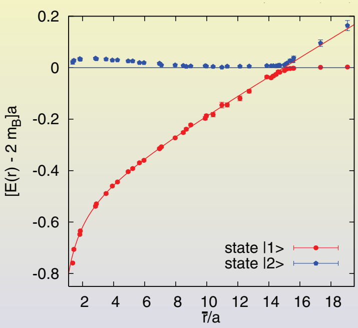
Measured: no of bosons on odd lattices. Solid curves are from DMRG (Density Matrix Renormalization Group, a numerical variational procedure)

What to *see* in *real* time?

Confinement in QCD is phenomenologically described by a “string”
String breaking from a study of the spectrum:



S Necco, R Sommer (2001);



Bali et. al. (2005).

- string breaking in real time, as a quench, as the analog for the above example in condensed matter physics.
- Simpler models with similar physics needed.

Schwinger Model: QED in $(1+1)-$d

- The lattice Hamiltonian:

$$H = \frac{g^2}{2} \sum_x e^2_{x,x+1} - t_F \sum_x \left[\psi^\dagger_x u_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi^\dagger_x \psi_x$$

- Fermions anti-commute. Link fields unitary : $u^\dagger u = 1$; satisfy:

$$[e_{x,x+1}, u_{x,x+1}] = u_{x,x+1}; \quad [e_{x,x+1}, u^\dagger_{x,x+1}] = -u^\dagger_{x,x+1}; \quad [u_{x,x+1}, u^\dagger_{x,x+1}] = 0$$

- Gauss' law generates gauge transformations, $\Longrightarrow [G_x, H] = 0$, and selects the physical states: $G_x|\Psi\rangle = (e_{x,x+1} - e_{x-1,x})|\Psi\rangle = \rho_x|\Psi\rangle$

- Compact QED: $u_{x,x+1} = \exp(iA_{x,x+1}), \quad A_{x,x+1} \in [0, 2\pi)$.
  $e_{x,x+1}$ has only integer quantum numbers:$0, \pm 1, \pm 2, \cdots, \pm\infty$.

- Infinite dimensional Hilbert space for the gauge field non-trivial to implement in quantum simulators with finite degrees of freedom.

- Alternate formulations of gauge theories with finite dimensional Hilbert space needed! Use Quantum Link Models (QLMs).

Abelian Quantum Link model

- QLMs have discrete Hilbert spaces at each link, but generate continuous gauge transformations Horn (1981); Orland(1990); Chandrasekharan, Wiese (1996)
- At each link, use a quantum spin $\vec{S} = (S^1, S^2, S^3)$ with spin-S. The Hilbert space is then automatically $(2S+1)$-dimensional.

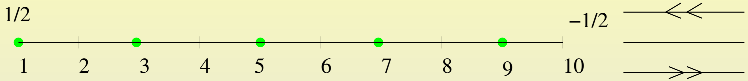
$$H = \frac{g^2}{2} \sum_x E^2_{x,x+1} - t_F \sum_x \left(\psi^\dagger_x U_{x,x+1} \psi_{x+1} + \mathrm{h.c.} \right) + \sum_x (-1)^x \psi^\dagger_x \psi_x$$

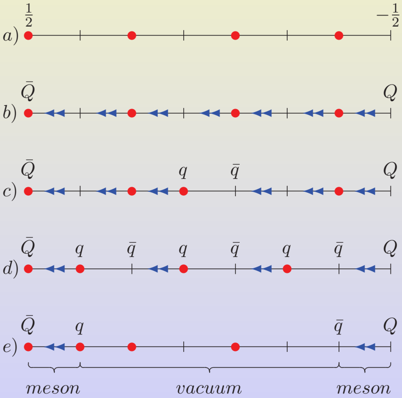
- Gauge fields:
  $U_{x,x+1} = S^+_{x,x+1} = S^1_{x,x+1} + iS^2_{x,x+1}, \quad U^\dagger_{x,x+1} = S^-_{x,x+1} = S^1_{x,x+1} - iS^2_{x,x+1}$
- Electric field: $E_{x,x+1} = S^3_{x,x+1}$ with eigenvalues $-S, \ldots, S$.
- Constructed this way, gauge fields satisfy:
  $[E_{x,x+1}, U_{x,x+1}] = U_{x,x+1}; \quad [E_{x,x+1}, U^\dagger_{x,x+1}] = -U^\dagger_{x,x+1}$
- $G_x|\Psi\rangle = (E_{x,x+1} - E_{x-1,x})\,|\Psi\rangle = \psi^\dagger_x \psi_x |\Psi\rangle$;
  As usual G_x generates Gauge transformations and $[H, G_x] = 0$.
- However, $[U, U^\dagger] = 2E$. Possibility of new physics as well.

DB, Bögli, Dalmonte, Rico Ortega, Stebler, Wiese, Zoller (2012)

The String and its breaking

Consider the model with spin S=1, in the electric flux basis.





Energetics in $t_F \to 0$ limit easy to analyze:

$E_0 = -m\frac{L}{2}$

$E_{\text{string}} - E_0 = \frac{g^2}{2}(L-1)$

$E_{\text{mesons}} - E_0 = 2(\frac{g^2}{2} + m)$

$E_{\text{string}} - E_{\text{mesons}} = \frac{g^2}{2}(L-3) - 2m$

$E_{\text{string}} - E_{\text{mesons}} = 0$

$\Longrightarrow L = \frac{4m}{g^2} + 3$

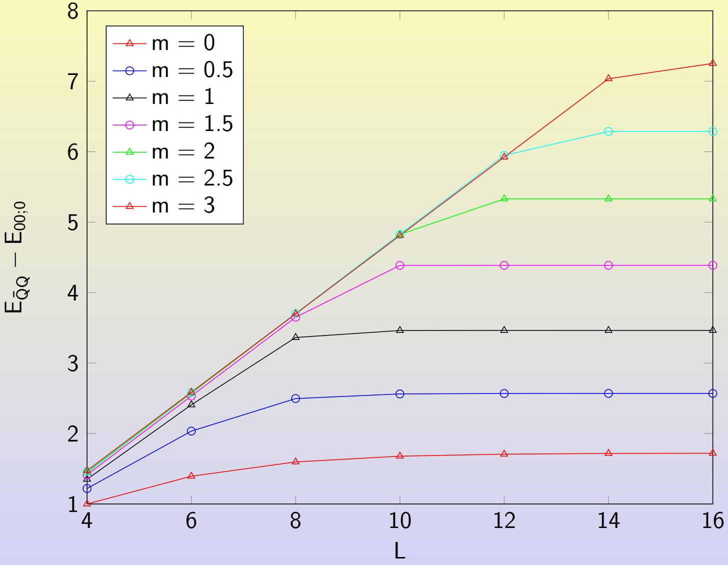
Note that the Gauss' Law here is actually:

$G_x + \frac{1}{2}[(-1)^x - 1]$ because of the

staggered occupation of the vacuum.

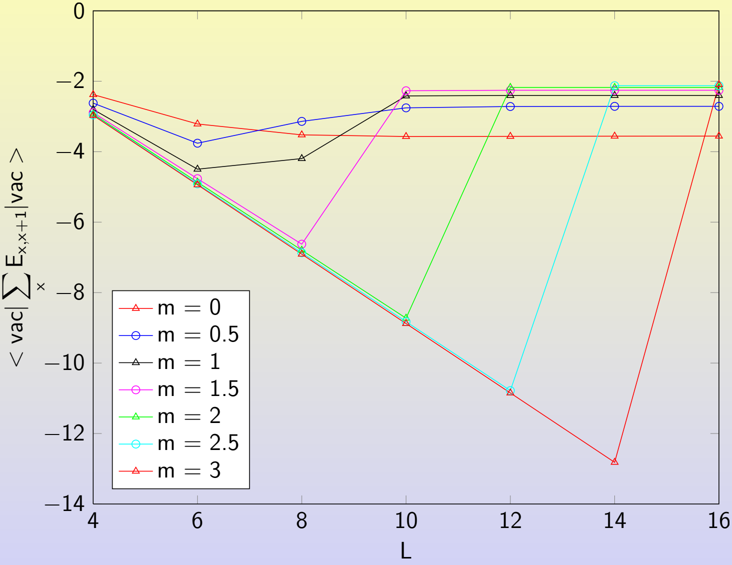
DB, Bögli, Dalmonte, Rico Ortega, Stebler, Wiese, Zoller (2012)

Static Properties



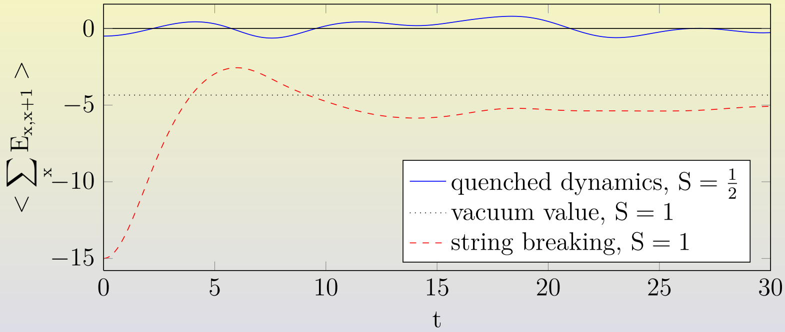
Results obtained by exact diagonalization.

Static Properties



Results obtained by exact diagonalization.

Dynamic properties



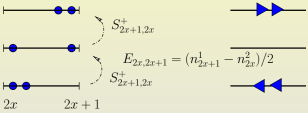
Results obtained by exact diagonalization.

These results serve as benchmark for the quantum simulators.

Implementation in optical lattices

How to implement all these constraints in optical lattices?

⋆ Different states of atoms in optical lattices represent different states:



$n_{2x} + n_{2x+1} = 2S = 2$

No. of particles fixed within a link for a given spin.

$U_{2x,2x+1} = S^{+} = b_{2x} b^{\dagger}_{2x+1}$;

$E_{2x,2x+1} = S^{z} = \frac{1}{2}(b^{\dagger}_{2x+1} b_{2x+1} - b^{\dagger}_{2x} b_{2x})$

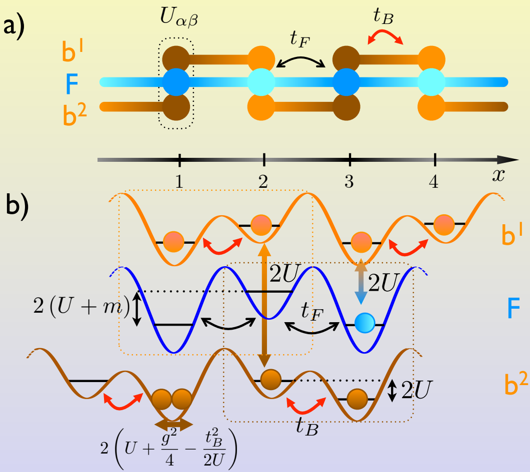
⋆ Gauss Law: $\tilde{G}_x = (\nabla \cdot E)_x - \rho_x = n^F_x + n^1_x + n^2_x - 2S + \frac{1}{2}[(-1)^x - 1]$

⋆ In optical lattices, realized using a microscopic Hubbard-type Hamiltonian

$$\begin{aligned}\tilde{H} &= \sum_x h_B^{x,x+1} + \sum_x h_F^{x,x+1} + m \sum_x (-1)^x n^F_x + V \sum_x \tilde{G}^2_x \\ &= -t_B \sum_{x \in \mathrm{odd}} b^{1\dagger}_x b^1_{x+1} - t_B \sum_{x \in \mathrm{even}} b^{2\dagger}_x b^2_{x+1} - t_F \sum_x \psi^{\dagger}_x \psi_{x+1} + \mathrm{h.c.} \\ &+ \sum_{x,\alpha,\beta} n^{\alpha}_x V_{\alpha\beta} n^{\beta}_x + \sum_{x,\alpha} (-1)^x V_{\alpha} n^{\alpha}_x\end{aligned}$$

DB, Dalmonte, Müller,Rico Ortega, Stebler, Wiese, Zoller (2012)

Optical lattice setup



Because of energy constraint, only correlated hops are allowed.

This gives rise to terms like:

$\psi_2^\dagger b_2^{(2)} b_3^{(2)\dagger} \psi_3 \quad \sim \quad \psi_2^\dagger U_{2,3} \psi_3$

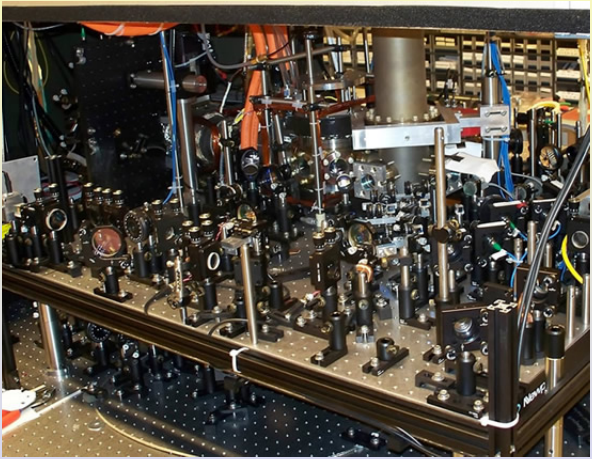
and

$\psi_3 b_3^{(1)\dagger} b_4^{(1)} \psi_4^\dagger \quad \sim \quad \psi_3 U_{3,4}^\dagger \psi_4$,

the fermion-gauge field coupling.

DB, Dalmonte, Müller,Rico Ortega, Stebler, Wiese, Zoller (2012)

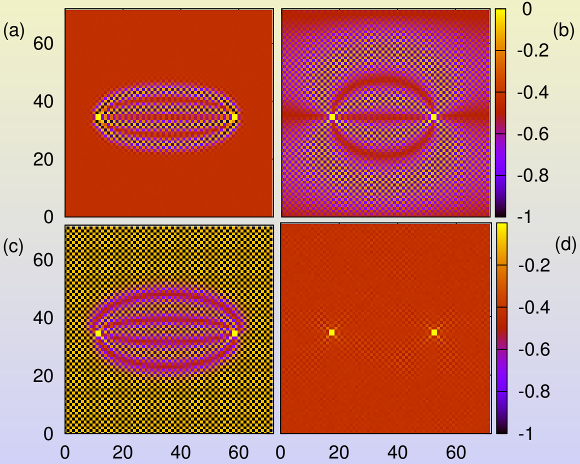
What does the simulator *really* look like?



Basic elements for the implementation proposal already exist in some labs. They need to combine the different experimental techniques.

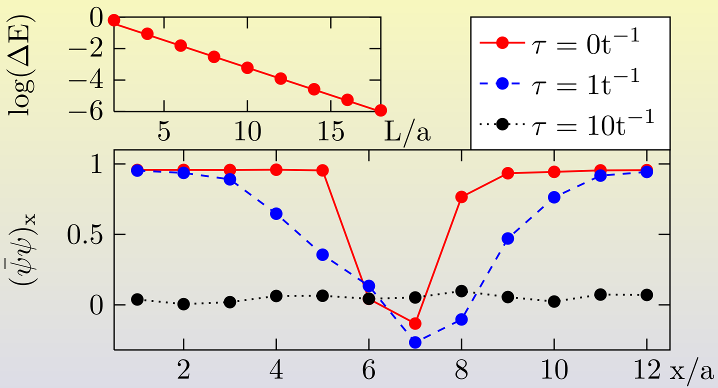
Other novel physics: Crystalline Confinement

A pure-gauge U(1) quantum link model in $(2+1)-$ dimensions with quantum spin $\frac{1}{2}$ shows novel confined phases. The flux is quantized in half-integer units! DB, Jiang, Widmer, Wiese (2013)



Energy density $\langle H_J \rangle$ of two charges $Q = \pm 2$ placed in along the axis in $L = 72$ lattice, results from a Monte Carlo simulation with an efficient cluster algorithm.

Chiral Dynamics: Expansion of a ”fireball”



Top: χ-SB in a $U(2)$ QLM with $m = 0$ and $V = -6t$.

Bottom: Real-time evolution of the order parameter profile $(\overline{\psi}\psi)_x(\tau) = s_x \langle \psi_x^{i\dagger} \psi_x^i - \frac{N}{2} \rangle$ for $L = 12$, mimicking the expansion of a hot quark-gluon plasma.
All results from exact diagonalization.

DB, Bögli, Dalmonte, Rico, Stebler, Wiese, Zoller (2014)

Where do we stand?

Basic ingredients presented in our proposals exist in several quantum optics labs, methods need to be combined for the full implmentation.

Note that other groups from Barcelona, Tel-Aviv and Munich have also proposed meaningful quantum simulator construction in particle physics contexts. A widely developing field!

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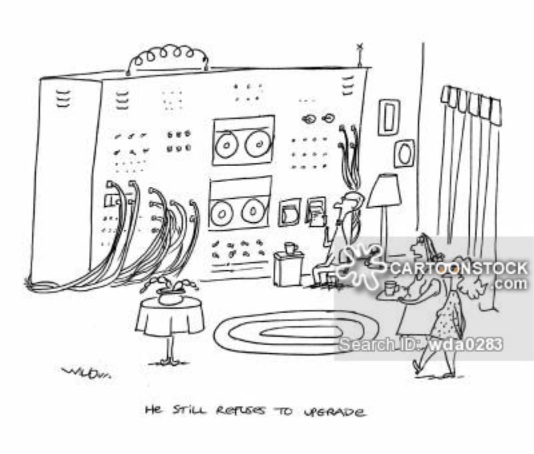
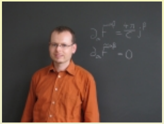


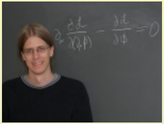
Quantum simulators arise as a supplement to high-precision classical simulators. When they start becoming available we should not be afraid to upgrade!

Collaborators from Uni Bern (AEC):

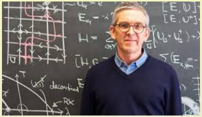
Michael Bögli



Pascal Stebler



Philippe Widmer



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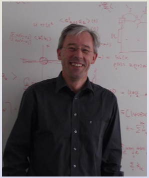

Peter Zoller

Thank you for your attention!