Determining the relative phases of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ and $\bar{D}^0 \to \pi^+\pi^-\pi^+\pi^-$ amplitudes from charm threshold data for a model-independent determination of γ

CKM 2016

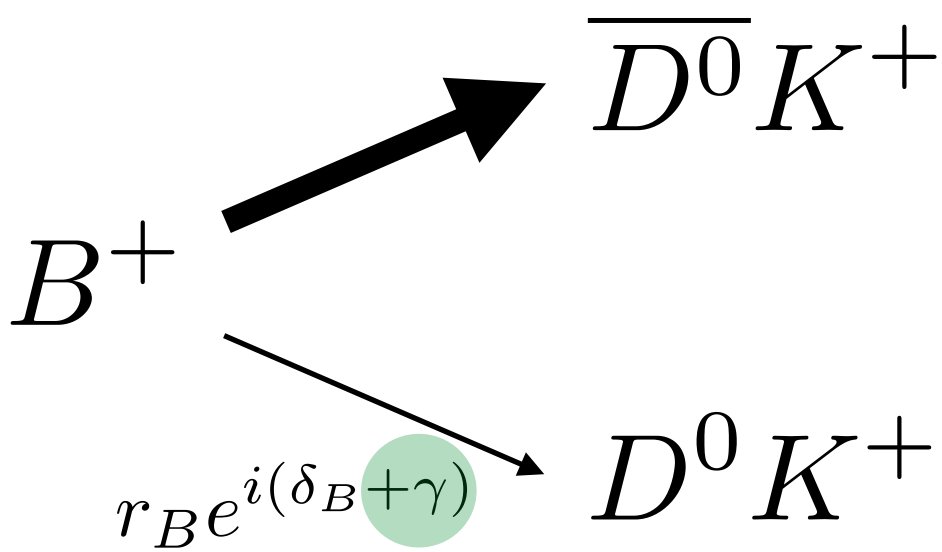
Sam Harnew, Claire Prouve, Jonas Rademacker

University of BRISTOL

Outline

- Measuring γ using $B^{\pm} \to DK^{\pm}$ decays
- GGSZ method
- Why $D \to \pi^+\pi^-\pi^+\pi^-$?
- Adaptive binning technique for $D \to \pi^+\pi^-\pi^+\pi^-$
- Measuring the relative phase of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ and $\bar{D}^0 \to \pi^+\pi^-\pi^+\pi^-$ using CLEO-c data
- The $D \to K^+\pi^-\pi^+\pi^-$ final state
- Conclusions

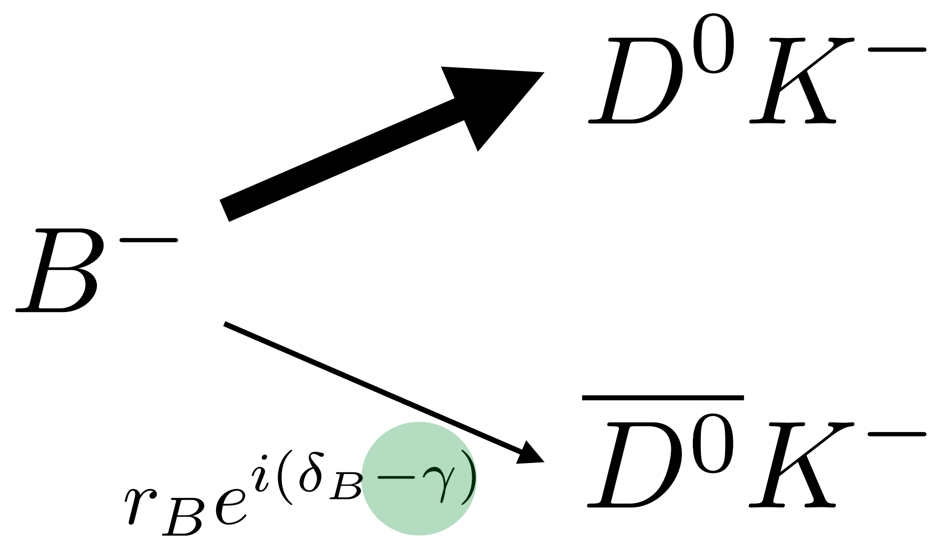
CKM phase γ from $B^{\pm} \rightarrow DK^{\pm}$



- Two amplitudes have relative:
  - Strong phase δ_B
  - Weak phase $+\gamma$
  - Magnitude r_B

$$|D\rangle = |\overline{D^0}\rangle + r_B e^{i(\delta+\gamma)}|D^0\rangle$$

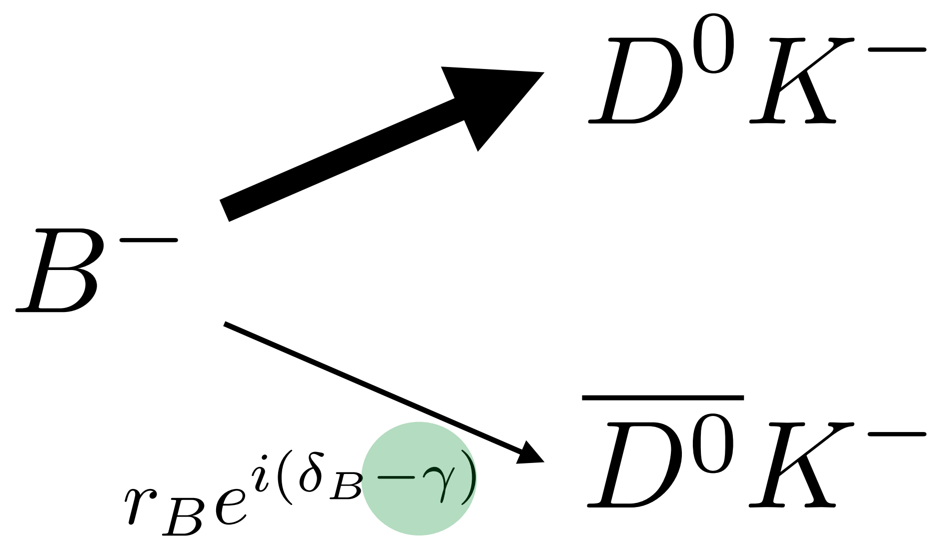
CKM phase γ from $B^{\pm} \rightarrow DK^{\pm}$



- Two amplitudes have relative:
  - Strong phase δ_B
  - Weak phase $-\gamma$
  - Magnitude r_B

$$|D\rangle = |D^0\rangle + r_B e^{i(\delta - \gamma)} |\overline{D^0}\rangle$$

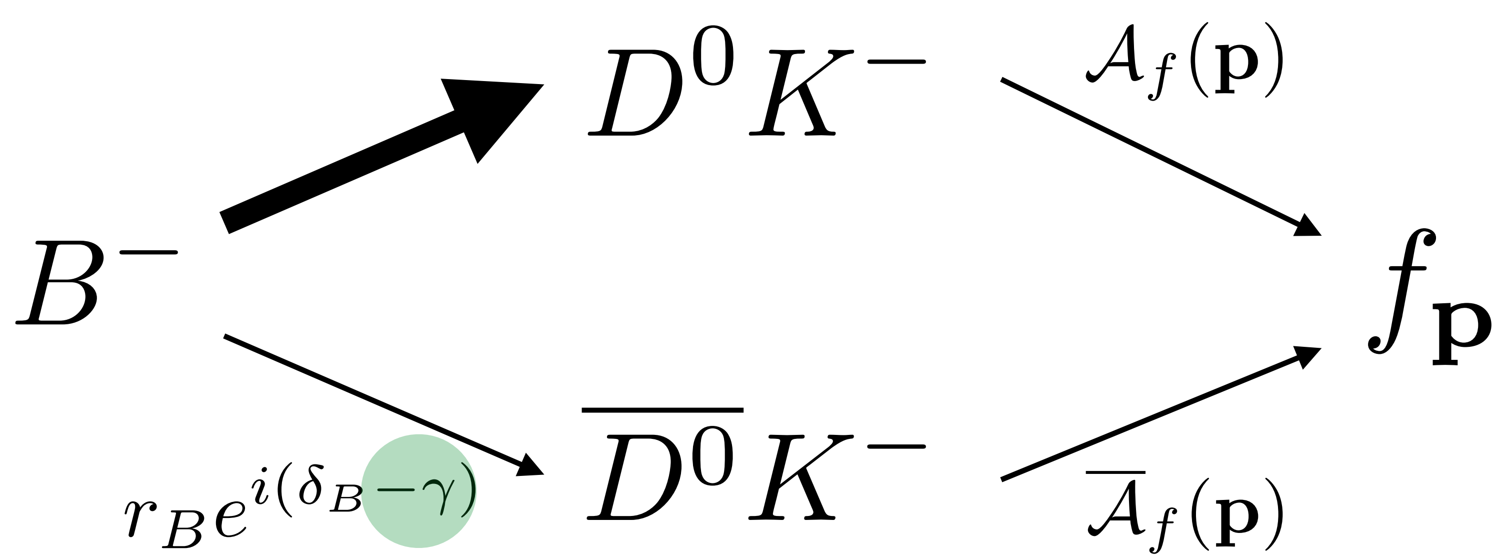
CKM phase γ from $B^{\pm} \rightarrow DK^{\pm}$



- γ is a phase, therefore can only measure through interference!
- Need a final state f that is accessible from D^0 and $\overline{D}^0$

$$|D\rangle = |D^0\rangle + r_B e^{i(\delta-\gamma)}|\overline{D^0}\rangle$$

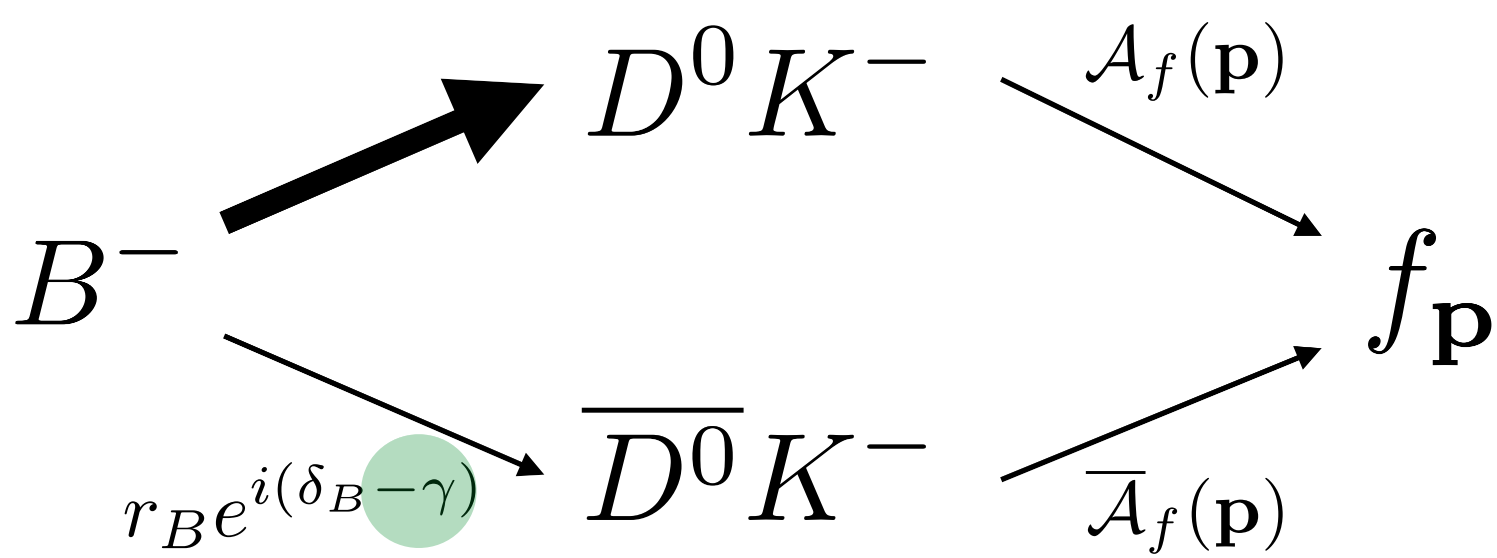
CKM phase γ from $B^{\pm} \to DK^{\pm}$



- If the final state is multi-body **p** describes a point in the phase space of the decay

$$\langle f_{\mathbf{p}} | \hat{\mathcal{H}} | D^0 \rangle = \mathcal{A}_f(\mathbf{p}) \qquad \langle f_{\mathbf{p}} | \hat{\mathcal{H}} | \overline{D^0} \rangle = \overline{\mathcal{A}}_f(\mathbf{p})$$

CKM phase γ from $B^{\pm} \to DK^{\pm}$



- If the final state is multi-body **p** describes a point in the phase space of the decay

$$\delta_D^f(\mathbf{p}) \equiv \arg(\overline{\mathcal{A}}_f(\mathbf{p})\mathcal{A}_f(\mathbf{p})^*)$$

$$\Gamma(B^- \to DK^-, D \to f_{\mathbf{p}}) \propto r_B^2|\overline{\mathcal{A}}_f(\mathbf{p})|^2 + |\mathcal{A}_f(\mathbf{p})|^2 + r_B|\mathcal{A}_f(\mathbf{p})\overline{\mathcal{A}}_f(\mathbf{p})|\cos(\delta_B + \delta_D^f(\mathbf{p}) - \gamma)$$

CKM phase γ from $B^\pm \to DK^\pm$

In order to measure γ one must have external input for the magnitude and the relative phase of the $D^0 \to f$ and $\bar{D}^0 \to f$ amplitudes!

lti-

B^-

$r_B e^{i(\delta_B - \gamma)}$ $D^0 K^-$ $\mathcal{A}_f(\mathbf{p})$

$$\delta_D^f(\mathbf{p}) \equiv \arg(\overline{\mathcal{A}}_f(\mathbf{p})\mathcal{A}_f(\mathbf{p})^*)$$

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CKM phase γ from $B^{\pm} \to DK^{\pm}$

In order to measure γ one must have external input for the magnitude and the relative phase of the $D^0 \to f$ and $\bar{D}^0 \to f$ amplitudes!

B^- ... lti-

$r_B e^{i(\delta_B - \gamma)}$ $D^0 K^-$ $\mathcal{A}_f(\mathbf{p})$

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Easy

Difficult!

GGSZ Method

GGSZ method concerns final states that are self conjugate e.g.

- $K_{S/L}\pi^+\pi^-$
- $K_{S/L}K^+K^-$

A. Giri, Yu. Grossman, A. Soffer and J. Zupan, Phys. Rev. D 68, 054018 (2003).

self-conjugate

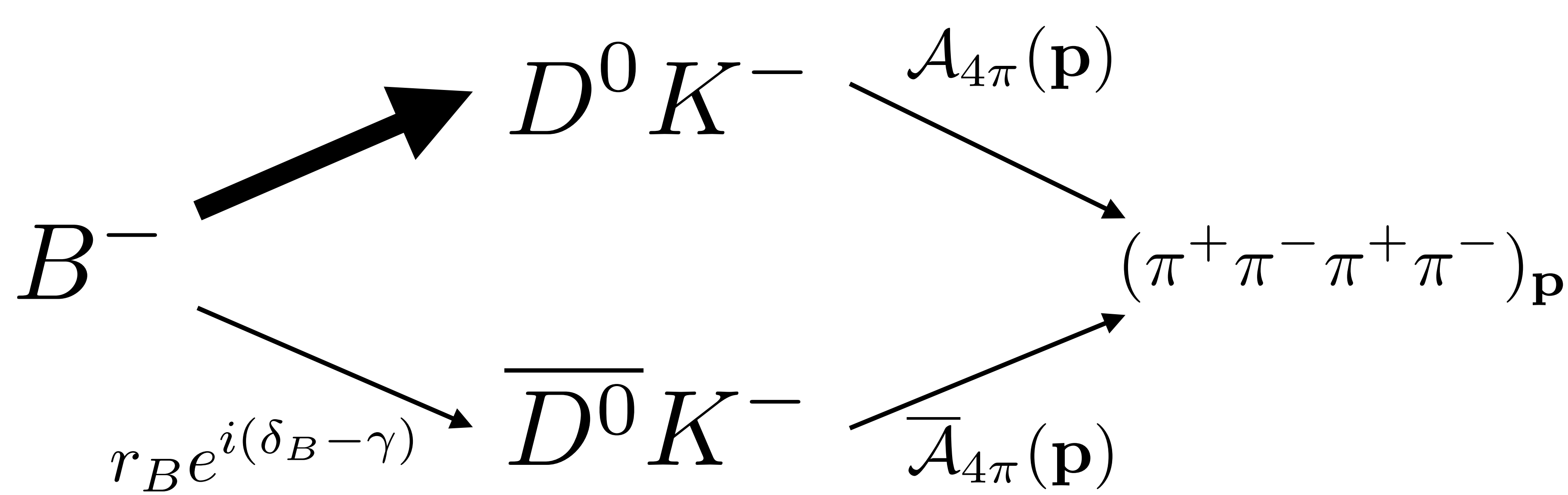
$$\mathcal{A}_{4\pi}(\mathbf{p}) \equiv \overline{\mathcal{A}}_{4\pi}(\overline{\mathbf{p}})$$

All **charges** and **momenta** of final state reversed

To date only 3 body modes have been exploited! Exciting 4-body opportunities:

- $\pi^+\pi^-\pi^+\pi^-$
- $K_S\pi^+\pi^-\pi^0$

See next talk by Resmi!



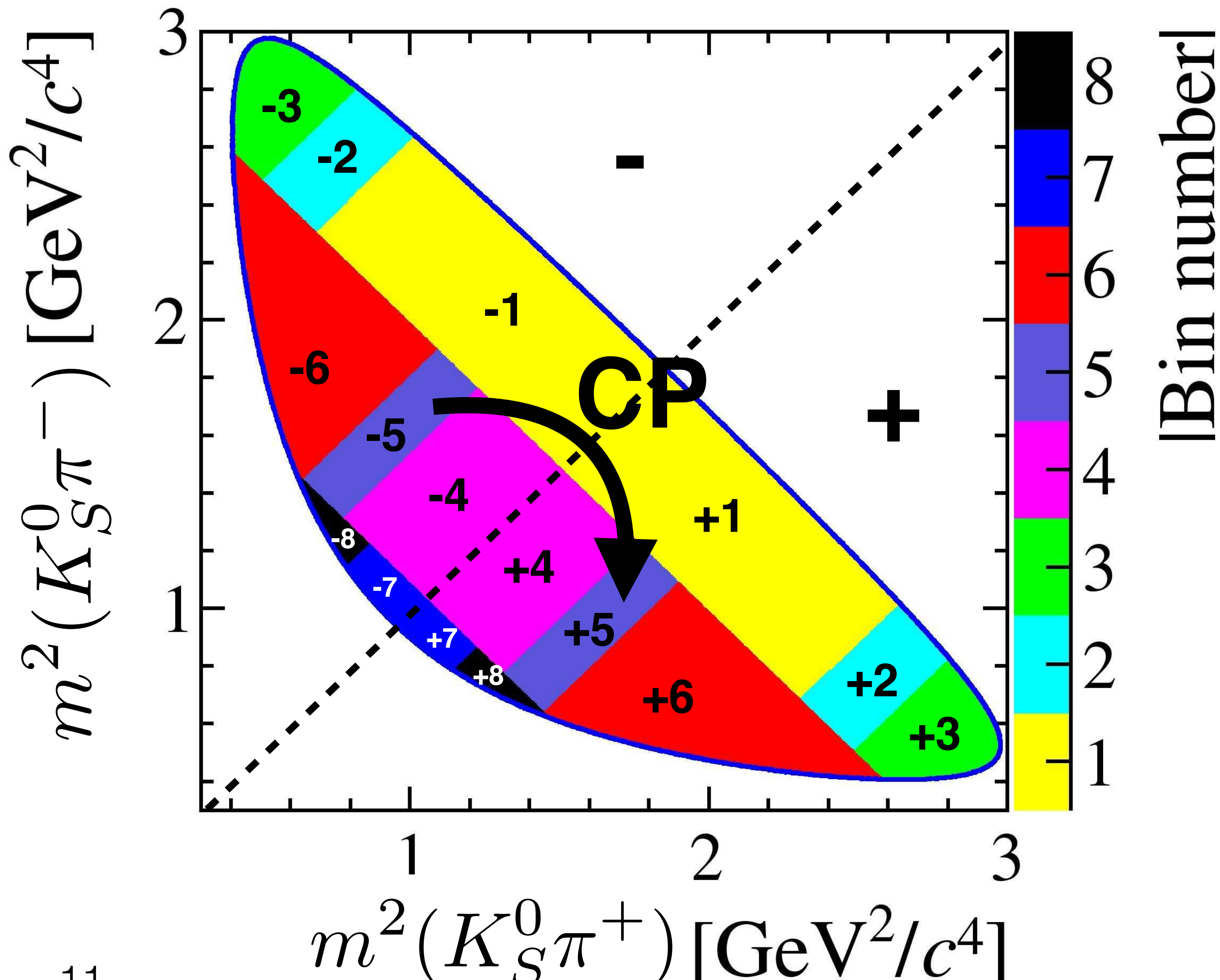
GGSZ Method

- GGSZ method involves integrating over bins of phase space

- Exploit the fact that the decay is self-conjugate by choosing bins that map to one another via CP

$$D \to K_S^0 \pi^+ \pi^-$$

JHEP 10 (2014) 097
(https://arxiv.org/abs/1408.2748)



GGSZ Method

- In each bin the D decay amplitudes are described by 4 parameters:

$$K_i = \int_i |\mathcal{A}_f(\mathbf{p})|^2 \, \mathrm{d}\mathbf{p}$$

$$\overline{K_i} = \int_i |\overline{\mathcal{A}}_f(\mathbf{p})|^2 \, \mathrm{d}\mathbf{p}$$

$$c_i + is_i = \frac{\int_i \mathcal{A}_f(\mathbf{p})\overline{\mathcal{A}}_f(\mathbf{p})^* \mathrm{d}\mathbf{p}}{\sqrt{K_i \overline{K_i}}}$$

$D \to K_S^0 \pi^+ \pi^-$

JHEP 10 (2014) 097
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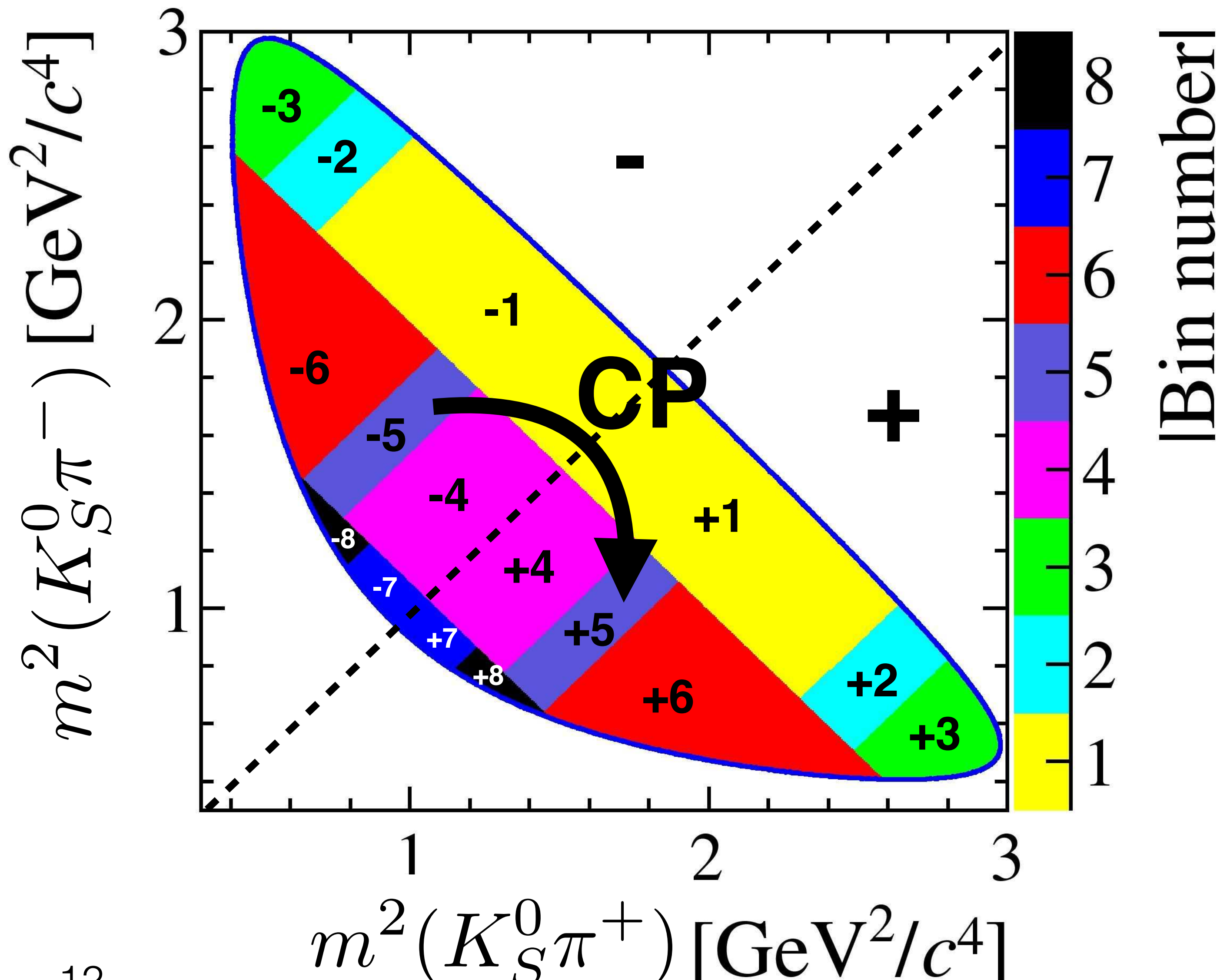
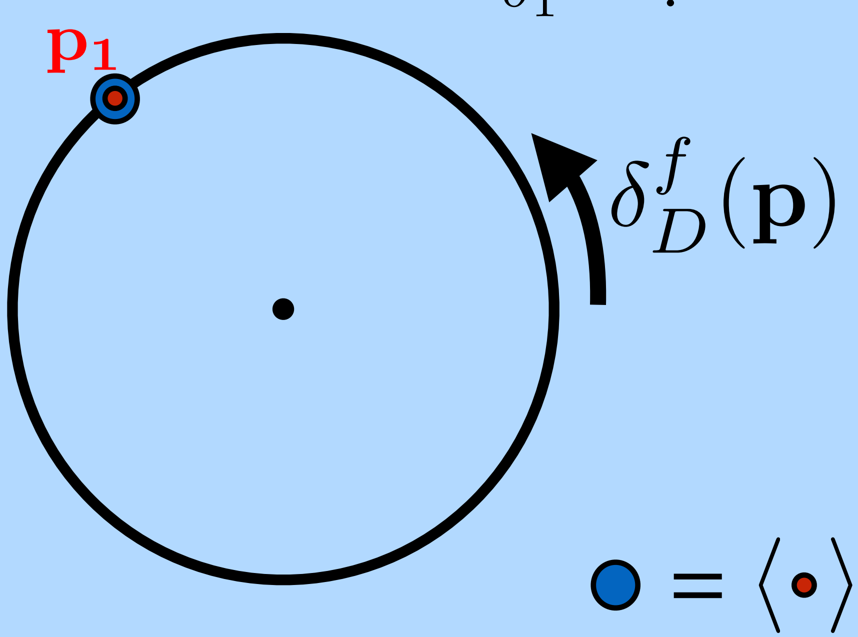


GGSZ Method

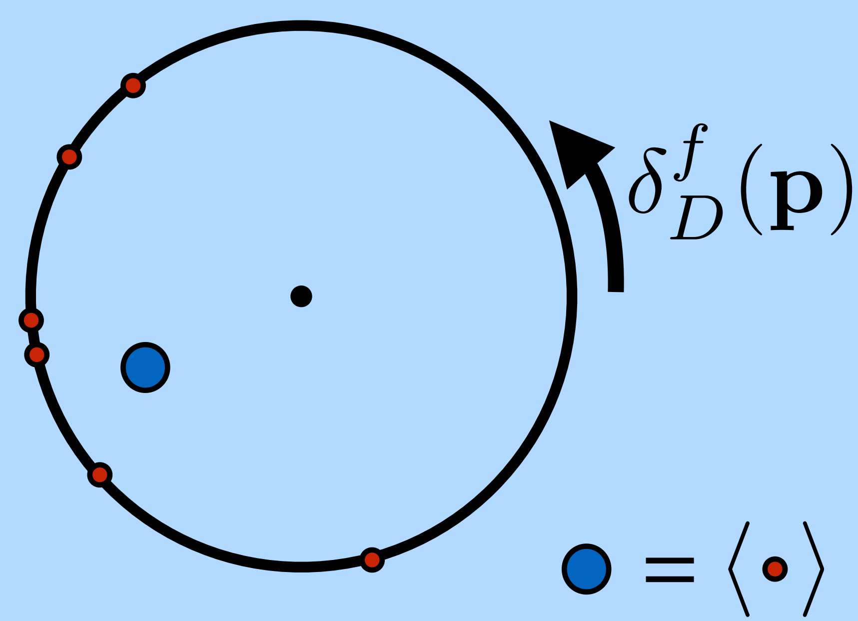
describe a single phase space point $\mathbf{p}_1$ with a phase δ_1

$$c_1 = \cos\delta_1$$
$$s_1 = \sin\delta_1$$
$$\delta_1 = ?$$



when summing over several phase space points, need two independent parameters!

$$c = ?$$
$$s = ?$$

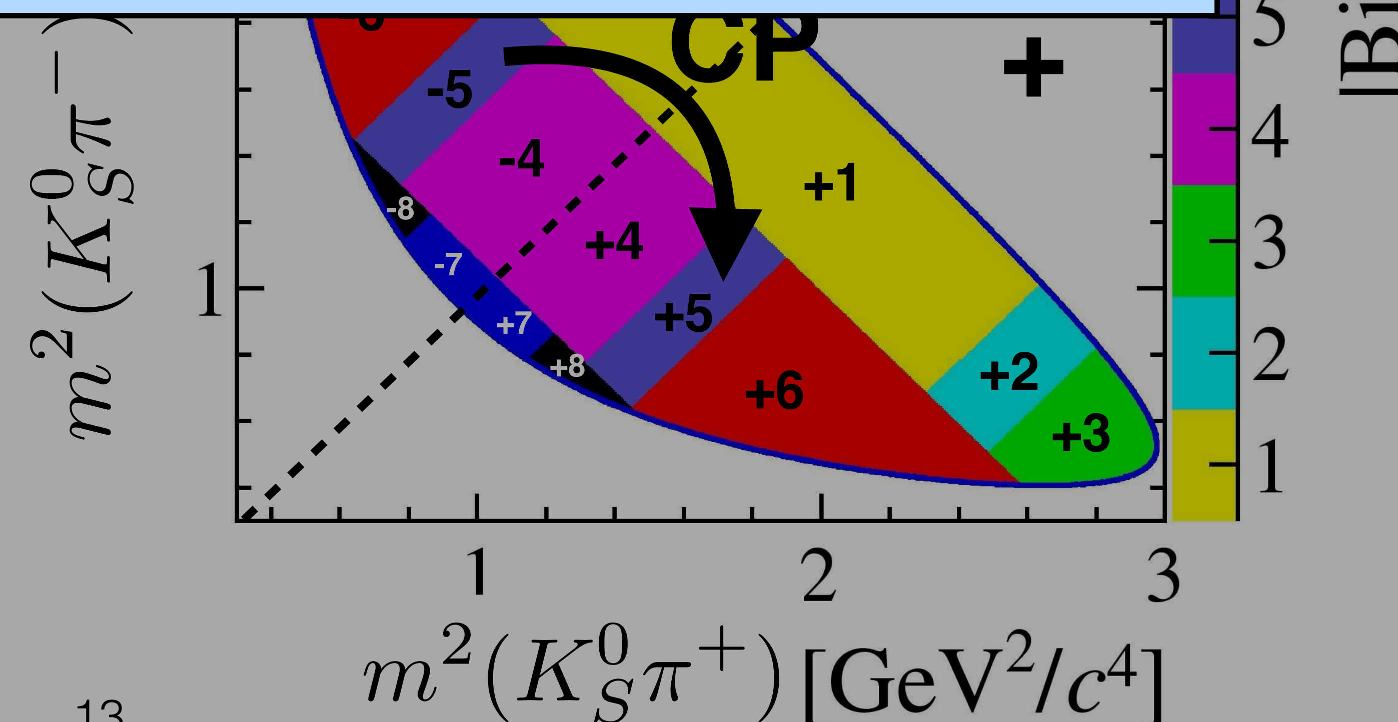


- In each b
amplitud
by 4 para

$$K_i =$$

$$\overline{K_i} = \int_i \left|\overline{\mathcal{A}}_f(\mathbf{p})\right|^2 \mathrm{d}\mathbf{p}$$

$$c_i + i s_i = \frac{\int_i \mathcal{A}_f(\mathbf{p})\overline{\mathcal{A}}_f(\mathbf{p})^* \mathrm{d}\mathbf{p}}{\sqrt{K_i \overline{K_i}}}$$



GGSZ Method

- Symmetric binning choice leads to relations between CP mapped bins

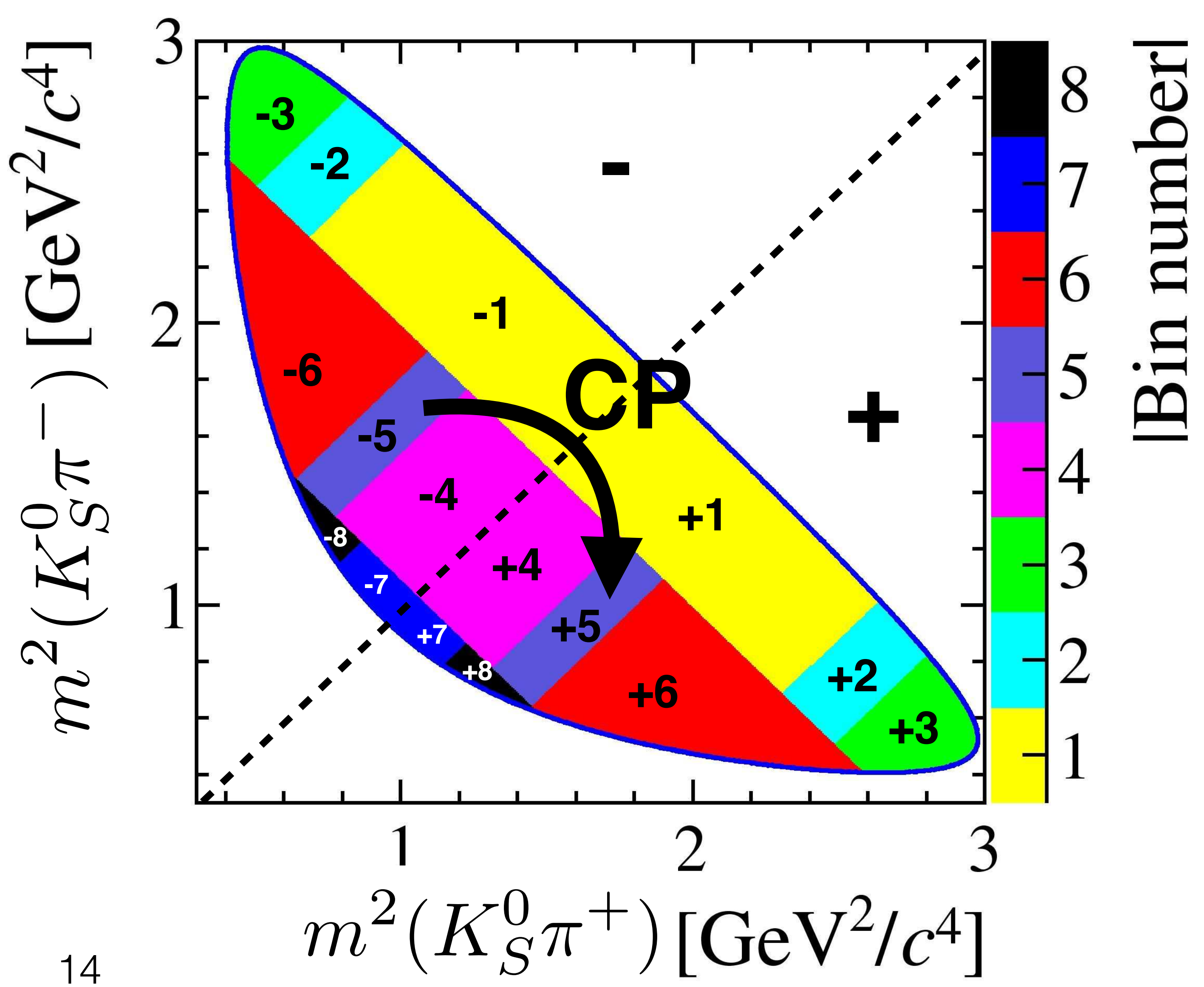
$$K_{+i} \equiv \overline{K}_{-i}$$

$$c_{+i} \equiv c_{-i}$$

$$s_{+i} \equiv -s_{-i}$$

$D \to K_S^0 \pi^+ \pi^-$

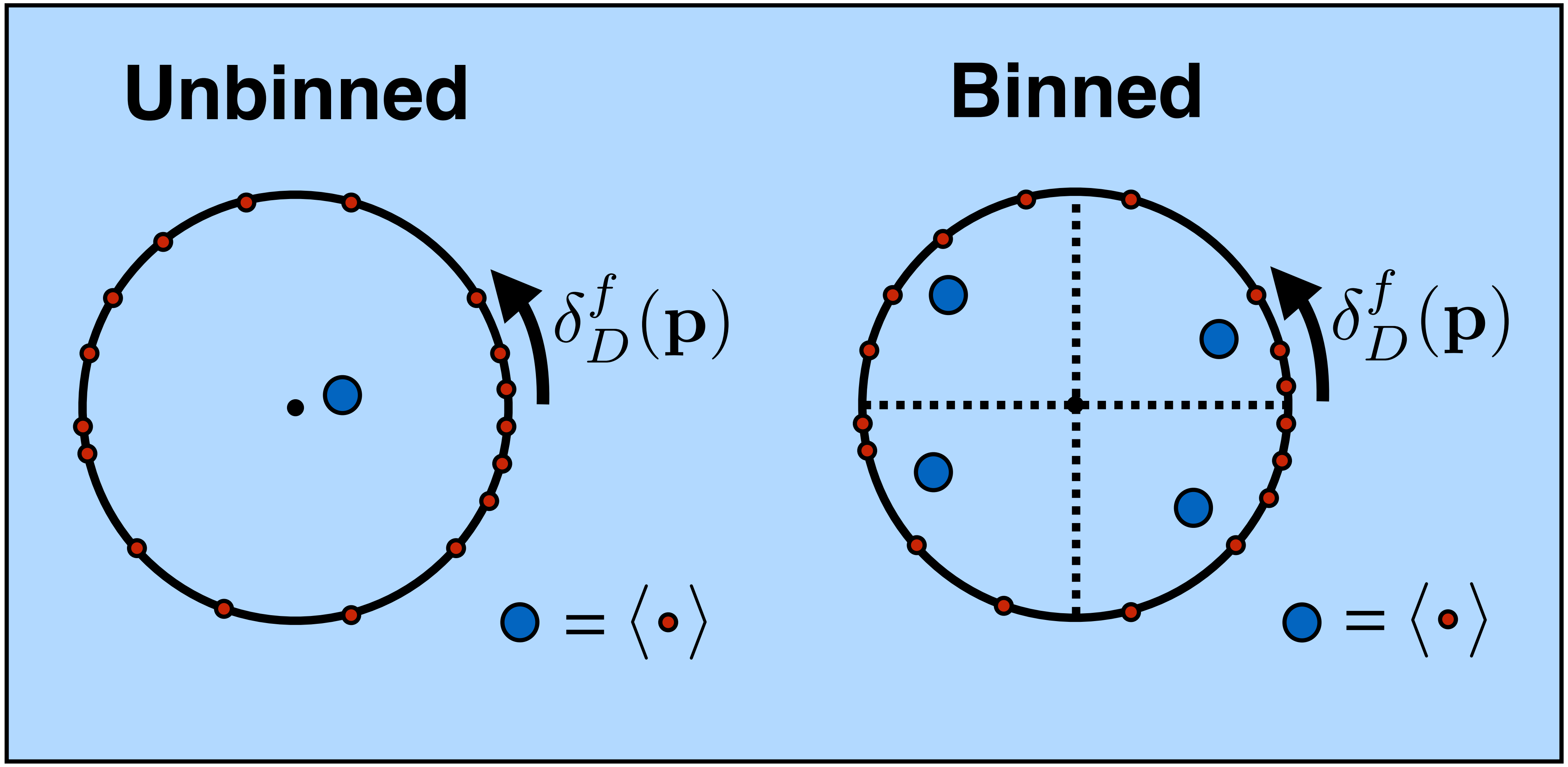
JHEP 10 (2014) 097
(https://arxiv.org/abs/1408.2748)



Model Inspired GGSZ Binning

- Sensitivity to γ is ~ proportional to $\sqrt{c_i^2 + s_i^2}$
- Want to choose a binning scheme such that this is as large as possible in each bin!

SOLUTION: Use an amplitude model to assign each event a δ_D

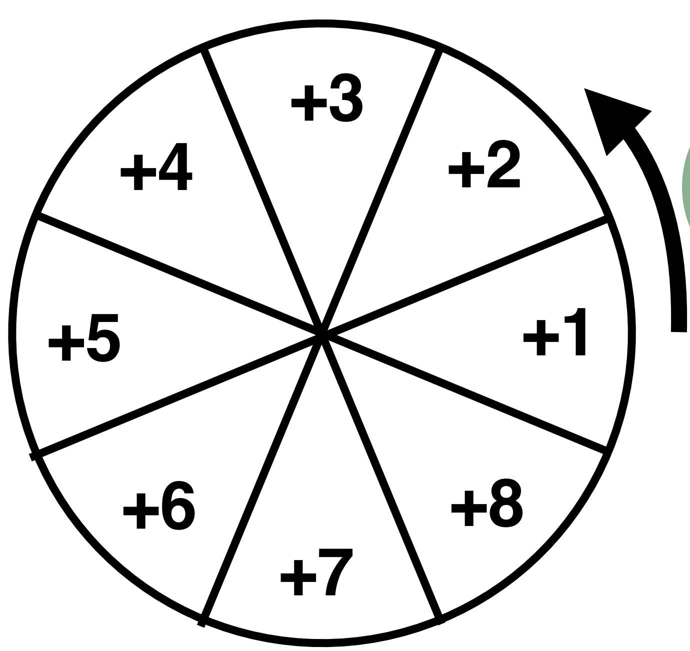


Model Inspired GGSZ Binning

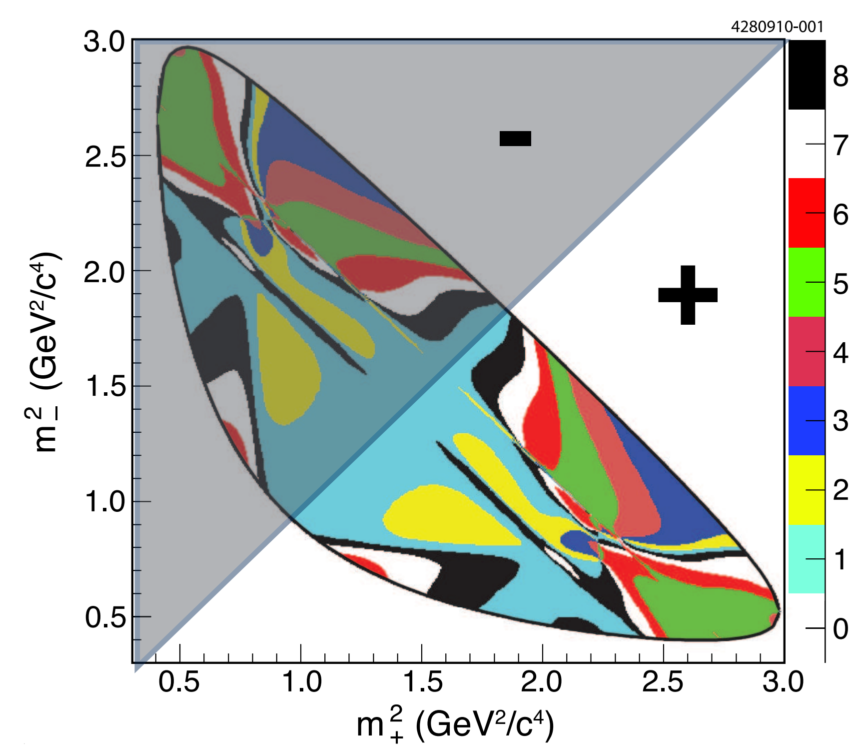
$\sqrt{c_i^2 + s_i^2}$ is maximised when the phase difference between amplitudes is constant

$$c_i + i s_i = \frac{\int_i \mathcal{A}_f(\mathbf{p}) \overline{\mathcal{A}}_f(\mathbf{p})^* \mathrm{d}\mathbf{p}}{\sqrt{K_i \overline{K_i}}} = \frac{\int_i |\mathcal{A}_f(\mathbf{p})| |\overline{\mathcal{A}}_f(\mathbf{p})| e^{i\delta_D^f(\mathbf{p})} \mathrm{d}\mathbf{p}}{\sqrt{K_i \overline{K_i}}}$$

Take from amplitude model



$\delta_D^f(\mathbf{p})$

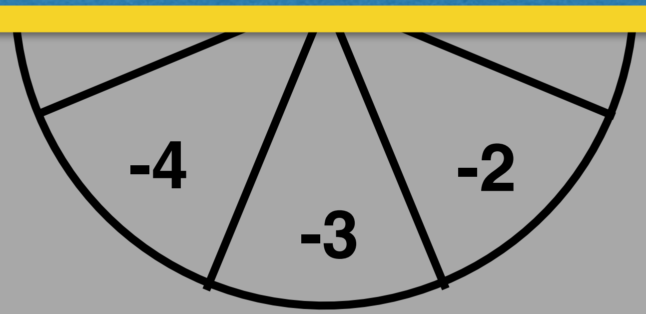


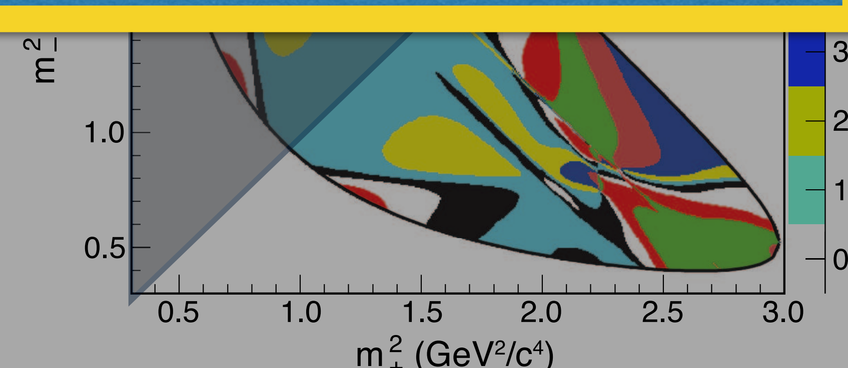
Model Inspired GGSZ Binning

$\sqrt{c_i^2 + s_i^2}$ is maximised when the phase difference between

an

Model is used to define the binning, but the measurement of c_i and s_i in each bin is still model-independent. An incorrect model just leads to a reduced **statistical** uncertainty.





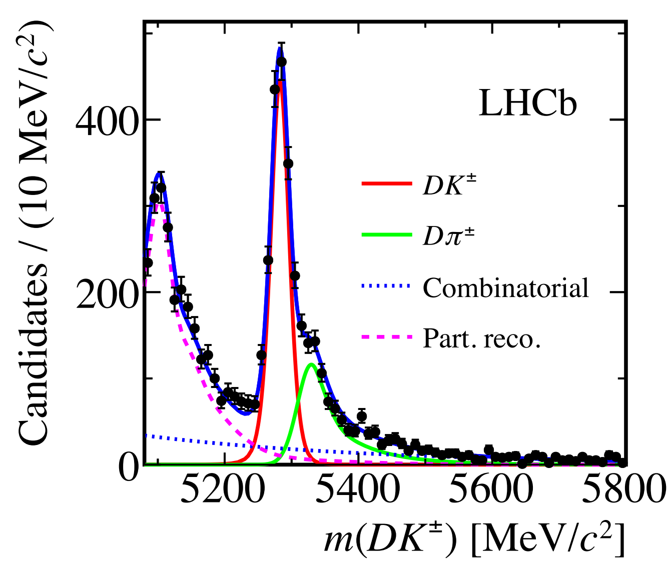
Why $\pi^+\pi^-\pi^+\pi^-$

- The single most precise γ measurement comes from the $K_S\pi^+\pi^-$ final state ($\sigma(\gamma)$ ~ 15°). **JHEP 10 (2014) 097 (https://arxiv.org/abs/1408.2748)**

- Similar numbers of $K_S\pi^+\pi^-$ and 4π reconstructed at LHCb with 3.0 fb^{-1}

$$K_S^0\pi^+\pi^-$$

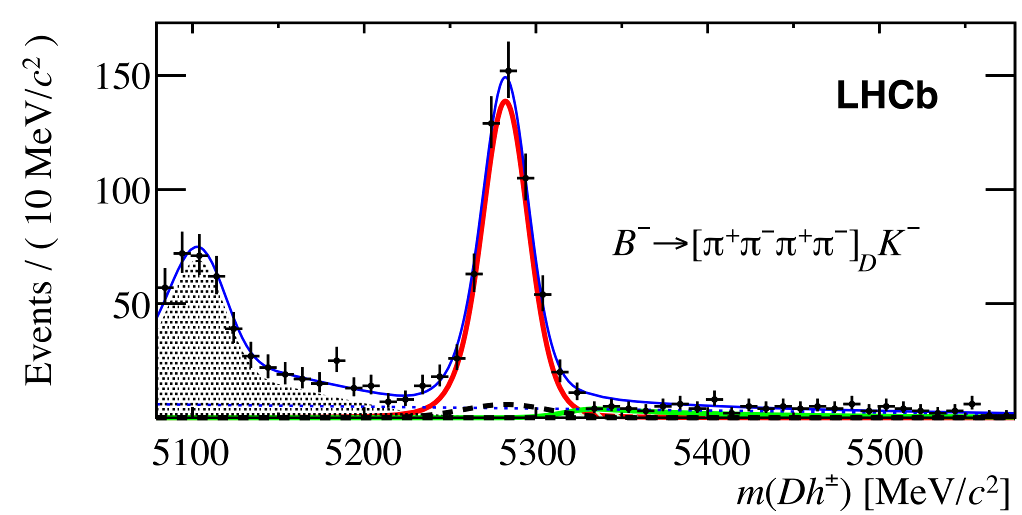
JHEP 10 (2014) 097
(https://arxiv.org/abs/1408.2748)



2257 ± 43

$$\pi^+\pi^-\pi^+\pi^-$$

Phys. Let. B 760 (2016)117-131
(https://arxiv.org/abs/1603.08993)



1497 ± 60

- Therefore, one would expect to obtain a similar sensitivity to γ

Current $\pi^+\pi^-\pi^+\pi^-$ status

- The $\pi^+\pi^-\pi^+\pi^-$ mode is already used to help constrain γ at LHCb - but only a phase space integrated measurement i.e. GLW(ish) rather than GGSZ **JHEP 10 (2014) 097 arxiv:1408.2748, arXiv:1611.03076**
  - Requires the $\pi^+\pi^-\pi^+\pi^-$ CP even fraction F+ which has already been measured at CLEO-c (directly related to c_i) **Phys. Let. B 05 (2015) 043**
  - Need input from other B→DK decays to constrain γ

$$F_+^{4\pi} = \frac{\mathcal{B}(D_{\mathrm{CP}+} \to 4\pi)}{\mathcal{B}(D_{\mathrm{CP}+} \to 4\pi) + \mathcal{B}(D_{\mathrm{CP}-} \to 4\pi)}$$

$$c_{\mathrm{ALL}}^{4\pi} \equiv 2F_+^{4\pi} - 1$$

$$c_{\mathrm{ALL}}^{4\pi} = 0.474 \pm 0.056$$

$$s_{\mathrm{ALL}}^{4\pi} \equiv 0.0$$

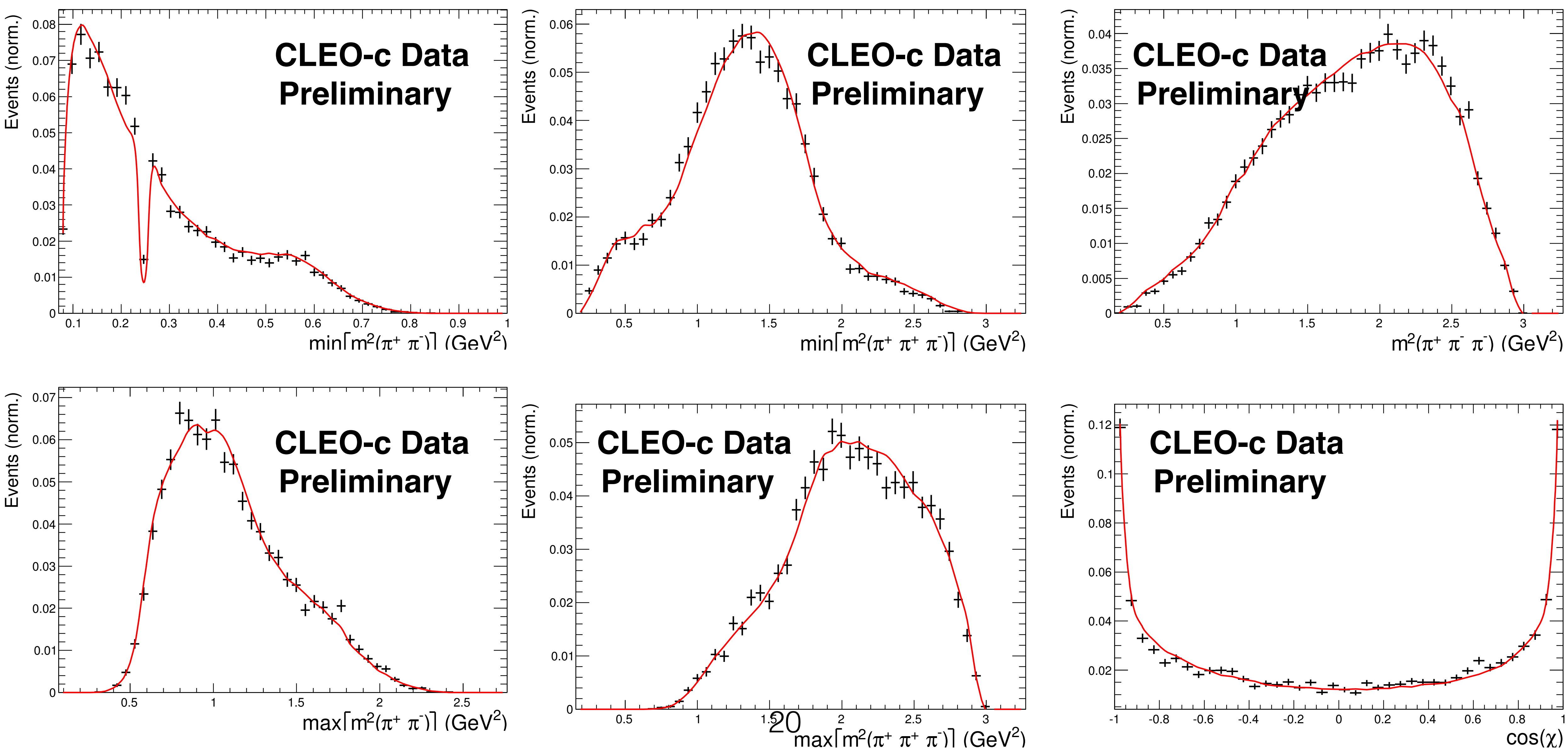
← **Potential to increase sensitivity by ~2x**

Phys. Let. B 05 (2015) 043 (https://arxiv.org/abs/1504.05878)

$\pi^+\pi^-\pi^+\pi^-$ Model

- A $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ model, based on CLEO-c data, is nearing completion…

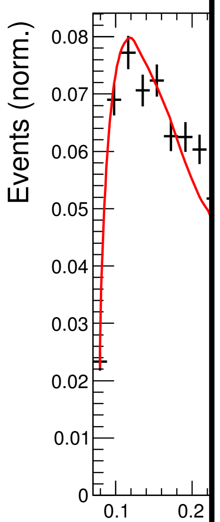
See Philippe d'Argent's proceedings from CHARM 2016 (arXiv:1611.09253)

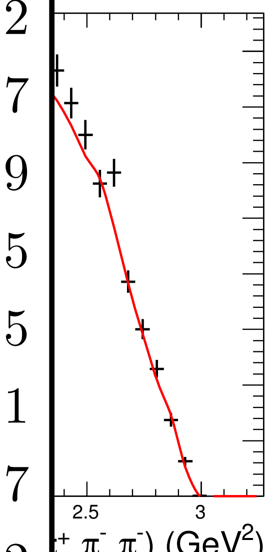


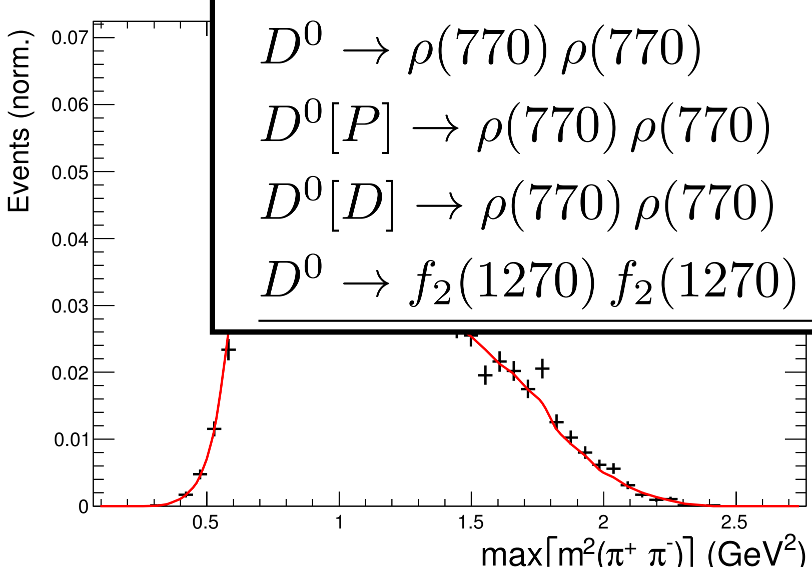
Decay mode	Re a_i	Im a_i	$F_i(\%)$
$D^0 \to \pi^- \; [a_1(1260)^+ \to \pi^+ \, \rho(770)]$	100.0 (fixed)	0.0 (fixed)	$36.7 \pm 2.4 \pm 2.3$
$D^0 \to \pi^- \; [a_1(1260)^+ \to \pi^+ \, \sigma]$	43.8 ± 4.5	35.5 ± 4.2	$10.9 \pm 1.5 \pm 2.9$
$D^0 \to \pi^+ \; [a_1(1260)^- \to \pi^- \, \rho(770)]$	31.9 ± 3.7	10.7 ± 2.8	$4.1 \pm 0.5 \pm 1.9$
$D^0 \to \pi^+ \; [a_1(1260)^- \to \pi^- \, \sigma]$			$1.2 \pm 0.2 \pm 0.5$
$D^0 \to \pi^- \; [\pi(1300)^+ \to \pi^+ \, (\pi^+ \pi^-)_P]$	-17.2 ± 2.7	-37.3 ± 5.0	$6.1 \pm 0.7 \pm 2.2$
$D^0 \to \pi^- \; [\pi(1300)^+ \to \pi^+ \, \sigma]$	-33.4 ± 4.4	5.6 ± 3.5	$4.2 \pm 1.0 \pm 2.0$
$D^0 \to \pi^+ \; [\pi(1300)^- \to \pi^- \, (\pi^+ \pi^-)_P]$	19.6 ± 11.5	-59.0 ± 7.4	$2.3 \pm 0.5 \pm 1.2$
$D^0 \to \pi^+ \; [\pi(1300)^- \to \pi^- \, \sigma]$			$1.6 \pm 0.4 \pm 0.7$
$D^0 \to \pi^- \; [a_1(1640)^+[D] \to \pi^+ \, \rho(770)]$	-16.2 ± 4.5	28.1 ± 8.9	$3.6 \pm 0.6 \pm 0.9$
$D^0 \to \pi^- \; [a_1(1640)^+ \to \pi^+ \, \sigma]$	0.1 ± 0.4	-18.3 ± 5.1	$1.2 \pm 0.5 \pm 0.5$
$D^0 \to \pi^- \; [\pi_2(1670)^+ \to \pi^+ \, f_2(1270)]$	0.2 ± 2.6	21.0 ± 2.7	$1.5 \pm 0.3 \pm 0.5$
$D^0 \to \pi^- \; [\pi_2(1670)^+ \to \pi^+ \, \sigma]$	-15.0 ± 2.7	-27.1 ± 3.5	$3.3 \pm 0.6 \pm 1.1$
$D^0 \to \sigma \, f_0(1370)$	28.3 ± 3.4	69.8 ± 5.9	$18.4 \pm 1.4 \pm 3.7$
$D^0 \to \sigma \, \rho(770)$	34.8 ± 4.4	-9.5 ± 4.0	$4.4 \pm 1.0 \pm 2.2$
$D^0 \to \rho(770) \, \rho(770)$	1.0 ± 3.0	15.1 ± 3.7	$0.9 \pm 0.3 \pm 0.6$
$D^0[P] \to \rho(770) \, \rho(770)$	-4.1 ± 2.7	-41.6 ± 2.6	$7.1 \pm 0.5 \pm 1.8$
$D^0[D] \to \rho(770) \, \rho(770)$	-66.4 ± 5.1	0.1 ± 3.1	$15.5 \pm 1.2 \pm 3.1$
$D^0 \to f_2(1270) \, f_2(1270)$	-7.9 ± 2.5	-15.4 ± 2.3	$1.1 \pm 0.3 \pm 0.5$

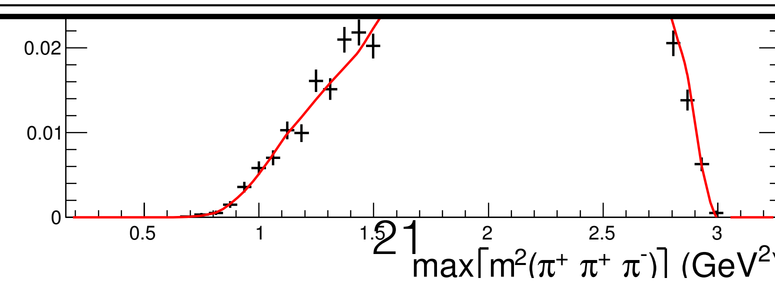
- A ...
  n...

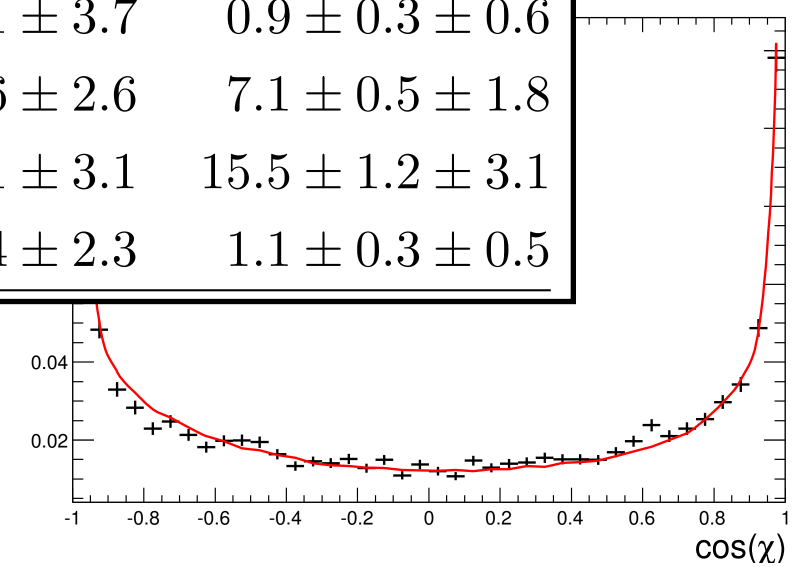
See ... 09253)





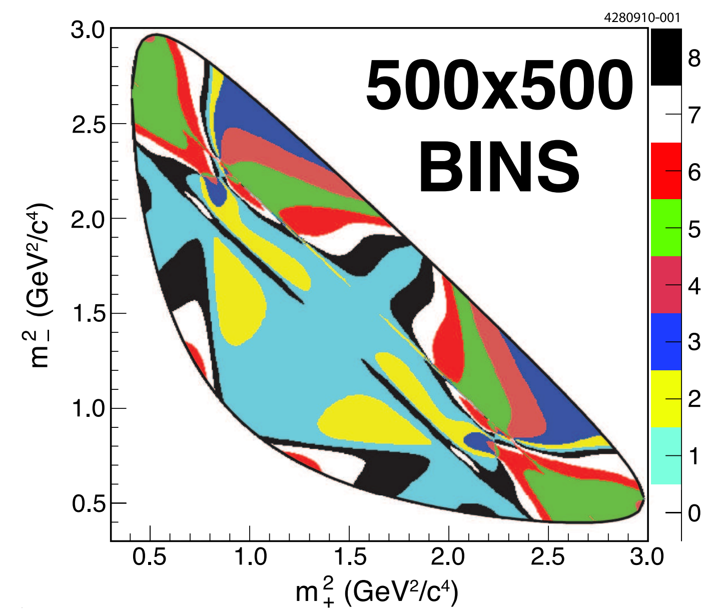






$\pi^+\pi^-\pi^+\pi^-$ Binning

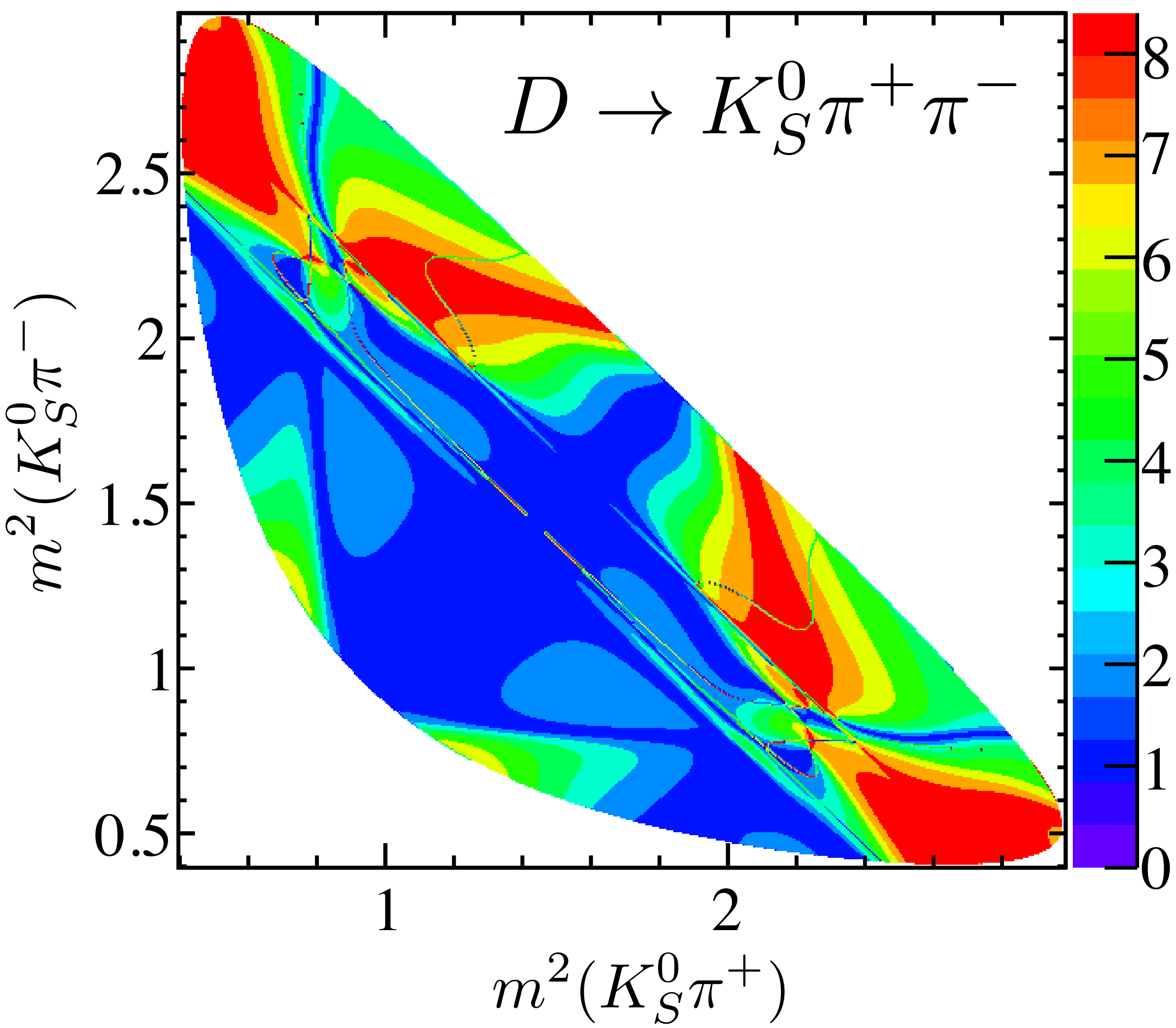
- One way to perform the binning is to use the model directly to assign each event a δ_D
    - In reality, this is not good for reusability - amplitude models can be tricky to reproduce.
- Solution for $K_S\pi^+\pi^-$ is to split 2D phase space into a 500x500 grid → 250,000 bins



- The phase space of the $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decay is 5D - what do we do? $500^5 \sim 3 \times 10^{13}$
    - 500 per dimension is probably overkill, but even 100 would give 10^{10} bins!
- Solution - adaptive binning…

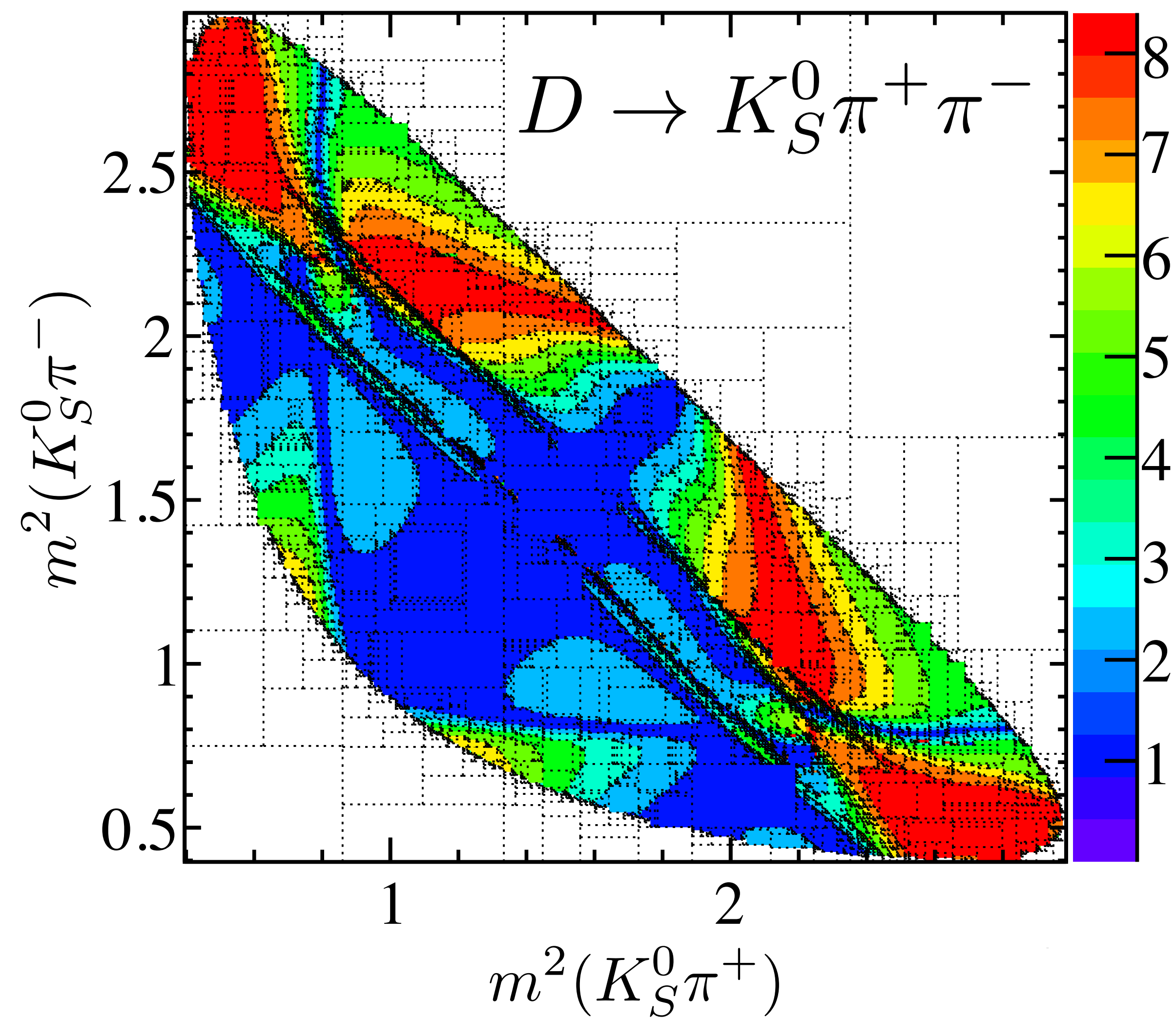
$\pi^+\pi^-\pi^+\pi^-$ Binning

Phys. Rev. D 82 (2010) 112006
(https://arxiv.org/abs/1010.2817)



500x500 = 250,000 Bins

https://github.com/samharnew/HyperPlot.git



7945 Bins

~30x less bins and negligible loss of resolution!

$\pi^+\pi^-\pi^+\pi^-$ Binning

- To describe a point in the $\pi^+\pi^-\pi^+\pi^-$ phase space we use:

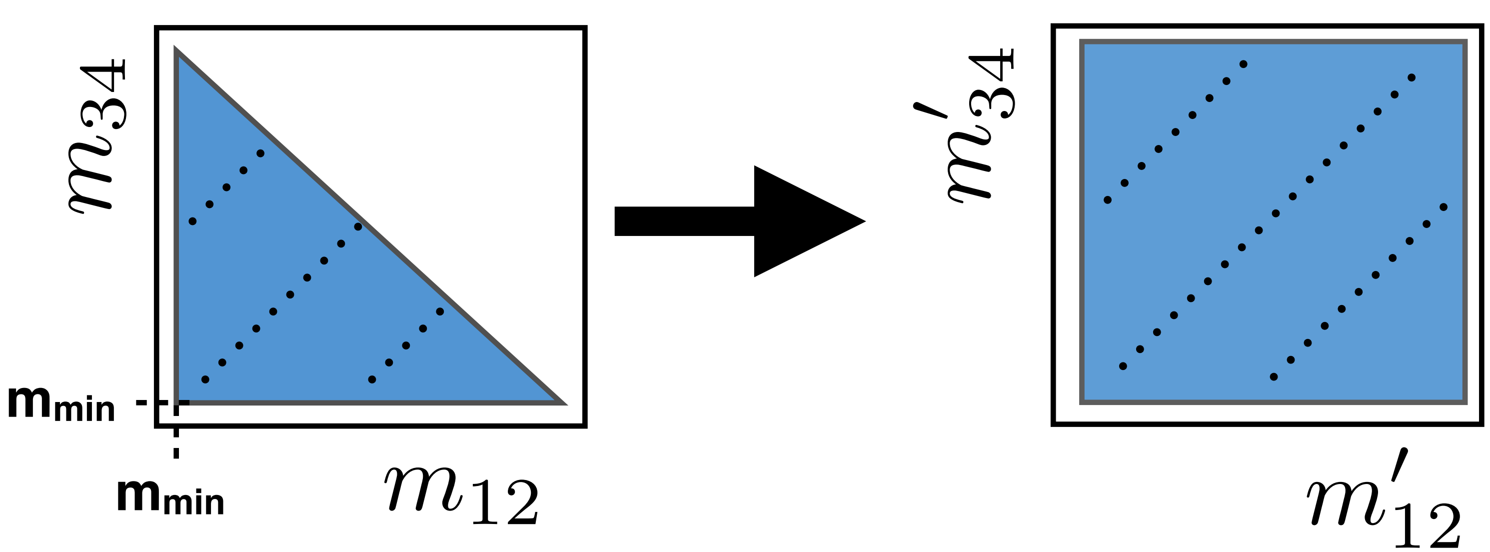
1 2 3 4

$$\mathbf{p} = (m'_{12}, m'_{34}, \cos\theta_{12}, \cos\theta_{34}, \phi)$$

Transformation ensures the phase space has rectangular boundaries

Helicity angle of the ij pair

Angle between the decay planes of 12 and 34



$$m'_{12} = m_{12} + \delta$$
$$m'_{34} = m_{34} + \delta$$
$$\delta = \min\{m_{12}, m_{34}\} - m_{\min}$$

$\pi^+\pi^-\pi^+\pi^-$ Binning

- This set of variables also has nice transformation properties under C and P

$$\mathbf{p} = (m'_{12}, m'_{34}, \cos\theta_{12}, \cos\theta_{34}, \phi)$$

$$C : \mathbf{p} = (m'_{12}, m'_{34}, -\cos\theta_{12}, -\cos\theta_{34}, +\phi)$$

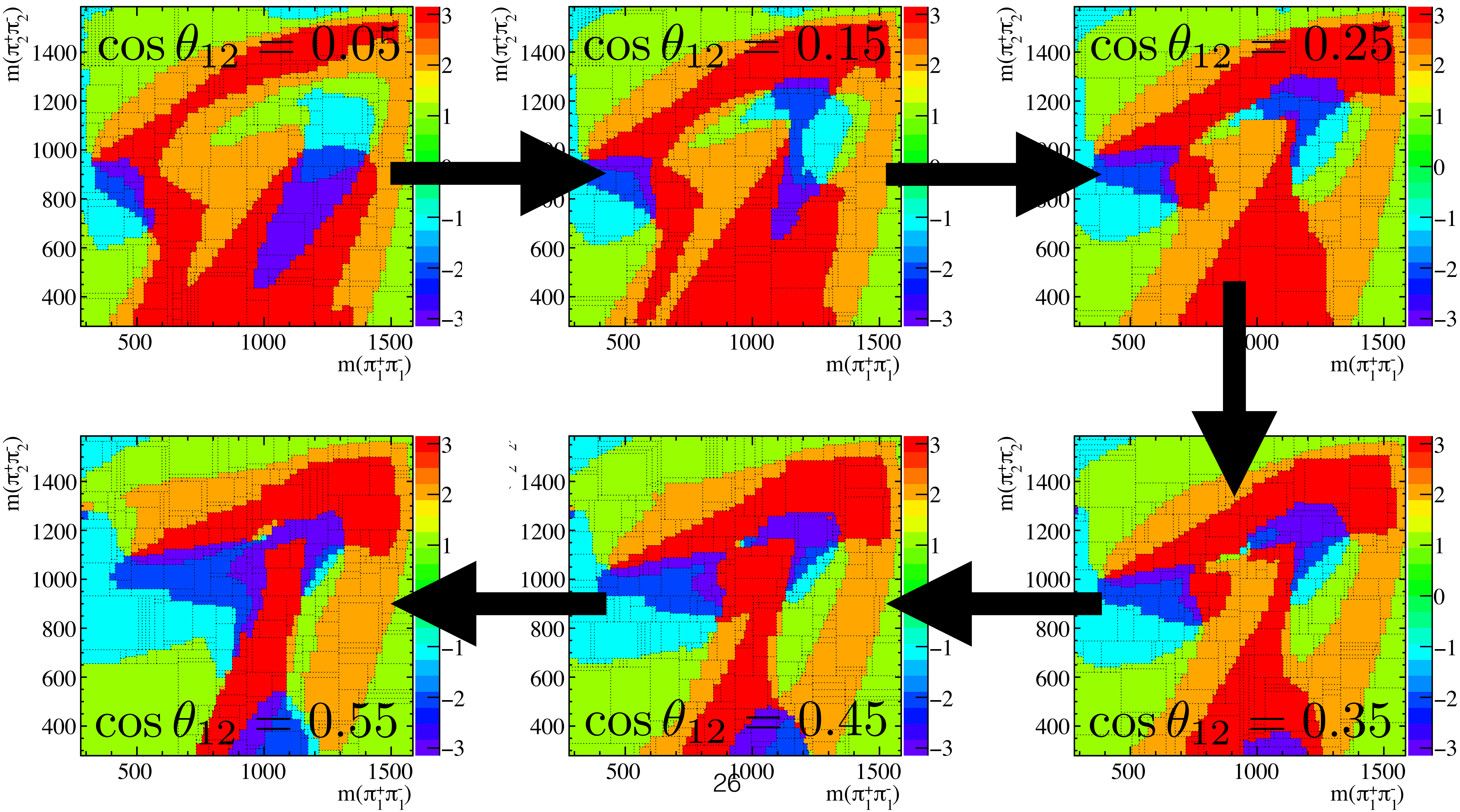
$$P : \mathbf{p} = (m'_{12}, m'_{34}, +\cos\theta_{12}, +\cos\theta_{34}, -\phi)$$

$$CP : \mathbf{p} = (m'_{12}, m'_{34}, -\cos\theta_{12}, -\cos\theta_{34}, -\phi)$$

- This means the binning only has to be defined in $\phi > 0$ then can be reflected to get the remaining bins

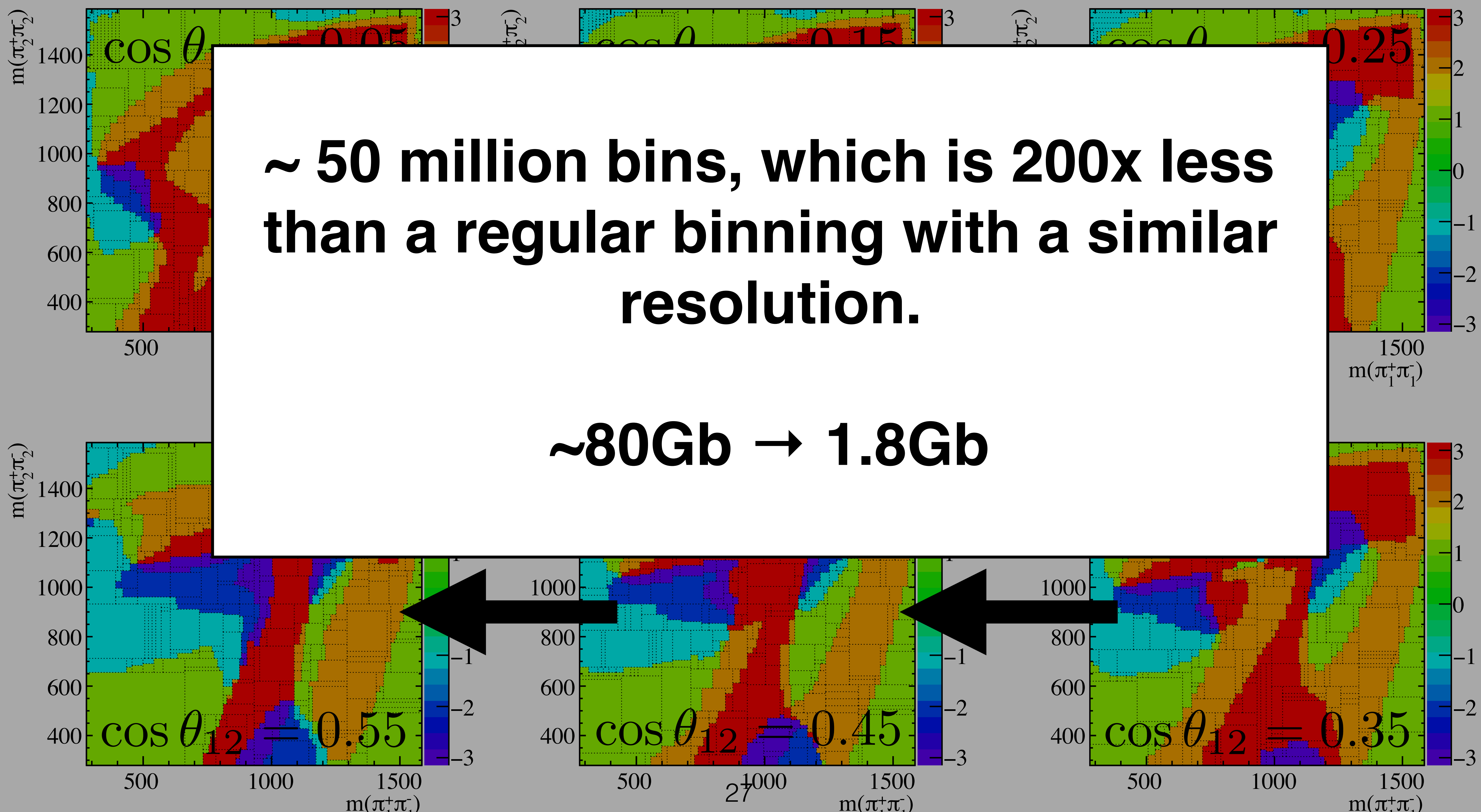
$\pi^+\pi^-\pi^+\pi^-$ Binning

$$\cos\theta_{34} = 0 \qquad \phi = \pi/2$$



$\pi^+\pi^-\pi^+\pi^-$ Binning

$$\cos\theta_{34} = 0 \qquad \phi = \pi/2$$



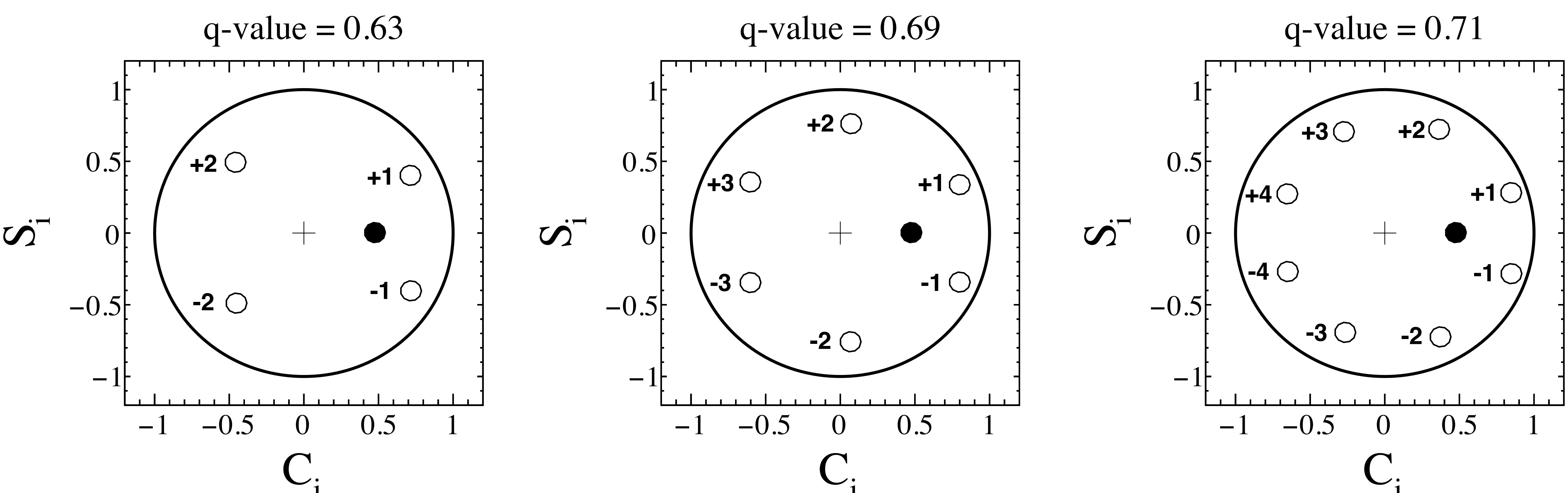
~ 50 million bins, which is 200x less than a regular binning with a similar resolution.

~80Gb → 1.8Gb

$\pi^+\pi^-\pi^+\pi^-$ Binning

- From the preliminary $D \rightarrow 4\pi$ model it is possible to calculate the expected values of the c_i and s_i parameters in each bin.
  - Clearly something to be gained though a binned analysis!
  - Remember, sensitivity is ~ proportional to $\sqrt{c_i^2 + s_i^2}$

 # bin pairs = 2 **# bin pairs = 3** **# bin pairs = 4**



Model-Independent c_i and s_i

- Quantum correlated $\psi(3770) \rightarrow D_1 D_2$ decays from CLEO-c can be used to determine c_i and s_i model-independently

We thank the former CLEO collaboration for the privilege of being able to use their data!

$$J^{PC} = 1^{--}$$

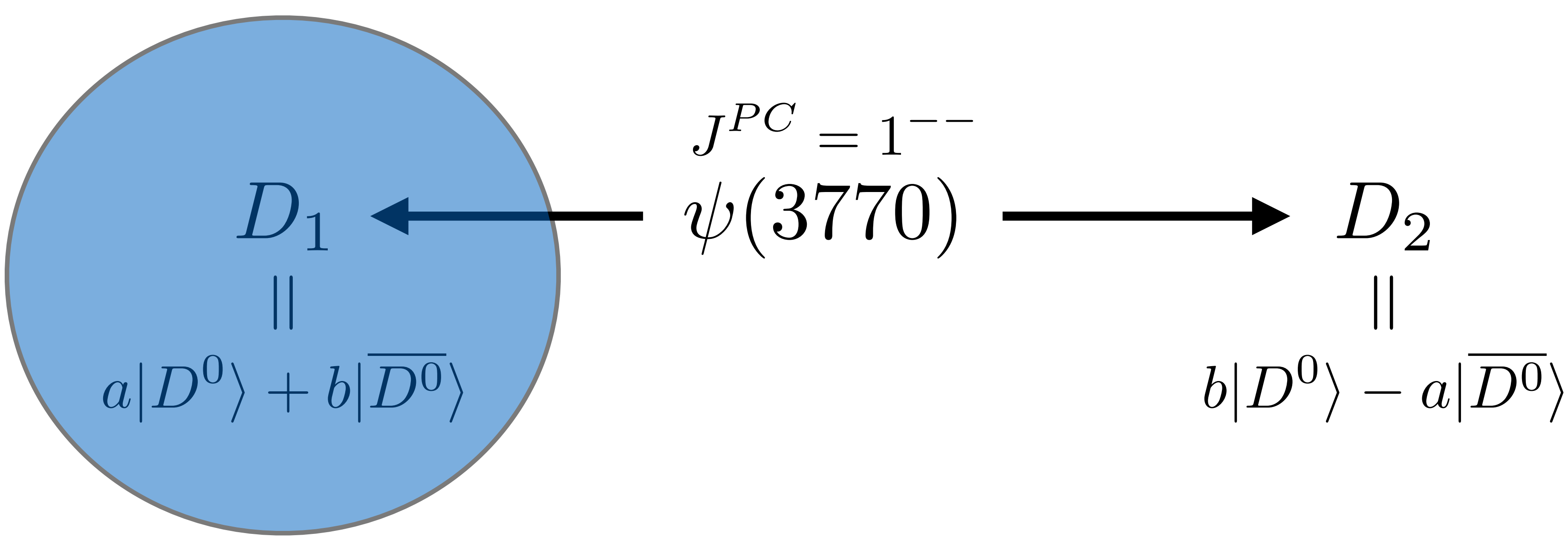
$$D_1 \longleftarrow \psi(3770) \longrightarrow D_2$$

$$D_1 = a|D^0\rangle + b|\overline{D^0}\rangle \qquad D_2 = b|D^0\rangle - a|\overline{D^0}\rangle$$

Model-Independent c_i and s_i

- Quantum correlated $\psi(3770) \rightarrow D_1 D_2$ decays from CLEO-c can be used to determine c_i and s_i model-independently

We thank the former CLEO collaboration for the privilege of being able to use their data!



'Tag' D_1 with a final state of known D^0 $\bar{D}^0$ content

CP+ tags

$$\pi^+\pi^- \longleftarrow D_1 \longleftarrow \underset{J^{PC}=1^{--}}{\psi(3770)} \longrightarrow D_2$$

$$D_1 = |D^0\rangle + |\overline{D^0}\rangle \qquad D_2 = |D^0\rangle - |\overline{D^0}\rangle$$

$$\left|\left\langle \pi^+\pi^-\pi^+\pi^- \middle| \mathcal{H} \middle| D_2 \right\rangle\right|^2 \propto K_i + \overline{K}_i - 2c_i\sqrt{K_i\overline{K}_i}$$

CP+ tags used: K^+K^- $\pi^+\pi^-$ $K_L\pi^0$ $K_L\omega$ $K_L\pi^0\pi^0$

CP- tags

$$K_S^0\pi^0 \longleftarrow D_1 \longleftarrow \overset{J^{PC}=1^{--}}{\psi(3770)} \longrightarrow D_2$$

$$D_1 = |D^0\rangle - |\overline{D^0}\rangle \qquad D_2 = |D^0\rangle + |\overline{D^0}\rangle$$

$$\left|\langle\pi^+\pi^-\pi^+\pi^-|\mathcal{H}|D_2\rangle\right|^2 \propto K_i + \overline{K}_i + 2c_i\sqrt{K_i\overline{K}_i}$$

CP- tags used: $K_S^0\omega$ $K_S^0\eta$ $K_S^0\eta'$ $K_S^0\pi^0$

D^0 tags

$$J^{PC} = 1^{--}$$

$$K^- e^+ \nu \longleftarrow D_1 \longleftarrow \psi(3770) \longrightarrow D_2$$

$$D_1 = |D^0\rangle \qquad D_2 = |\overline{D^0}\rangle$$

$$\left|\left\langle \pi^+\pi^-\pi^+\pi^- |\mathcal{H}| D_2 \right\rangle\right|^2 \propto \overline{K}_i$$

D^0 tags used: $K^-\pi^+$ $\quad K^-\pi^+\pi^0$ $\quad K^-\pi^+\pi^-\pi^+$ $\quad K^-e^+\nu$

$\bar{D}^0$ tags

$$J^{PC} = 1^{--}$$

$$K^+ e^- \nu \longleftarrow D_1 \longleftarrow \psi(3770) \longrightarrow D_2$$

$$D_1 = |\overline{D^0}\rangle \qquad D_2 = |D^0\rangle$$

$$\left|\left\langle \pi^+\pi^-\pi^+\pi^- | \mathcal{H} | D_2 \right\rangle\right|^2 \propto K_i$$

$\bar{D}^0$ tags used: $K^+\pi^-$ $\quad K^+\pi^-\pi^0$ $\quad K^+\pi^-\pi^+\pi^-$ $\quad K^+e^-\nu$

Mixed tags

$$J^{PC} = 1^{--}$$

$$(K_S^0\pi^+\pi^-)_i \longleftarrow D_1 \longleftarrow \psi(3770) \longrightarrow D_2$$

$$\left|\langle \pi^+\pi^-\pi^+\pi^- |\mathcal{H}|D_2\rangle\right|^2 \propto K_i\overline{K'_i} + \overline{K}_i K'_i - 2\sqrt{K_i\overline{K}_i K'_i\overline{K}'_i}\,(c_i c'_i + s_i s'_i)$$

Phys. Rev. D 82 (2010) 112006
(https://arxiv.org/abs/1010.2817)

Mixed tags used: $K_S^0\pi^+\pi^-$ $K_L^0\pi^+\pi^-$ $\pi^+\pi^-\pi^0$ $\pi^+\pi^-\pi^+\pi^-$

Mixed tags

$(K_S^0\pi$

Mixed tags not yet included in this analysis, so preliminary result presented is only sensitive to c_i

$$\left|\langle\pi^+\pi^-\pi^+\pi^-|\mathcal{H}|D_2\rangle\right|^2 \propto K_i\overline{K'_i} + \overline{K}_i K'_i - 2\sqrt{K_i\overline{K}_i K'_i\overline{K'}_i}(c_i c'_i + s_i s'_i)$$

Phys. Rev. D 82 (2010) 112006
(https://arxiv.org/abs/1010.2817)

Mixed tags used: $K_S^0\pi^+\pi^-$ $K_L^0\pi^+\pi^-$ $\pi^+\pi^-\pi^0$ $\pi^+\pi^-\pi^+\pi^-$

Event Yields

Where possible, single tags used for normalisation

- Background subtracted yields for each reconstructed decay

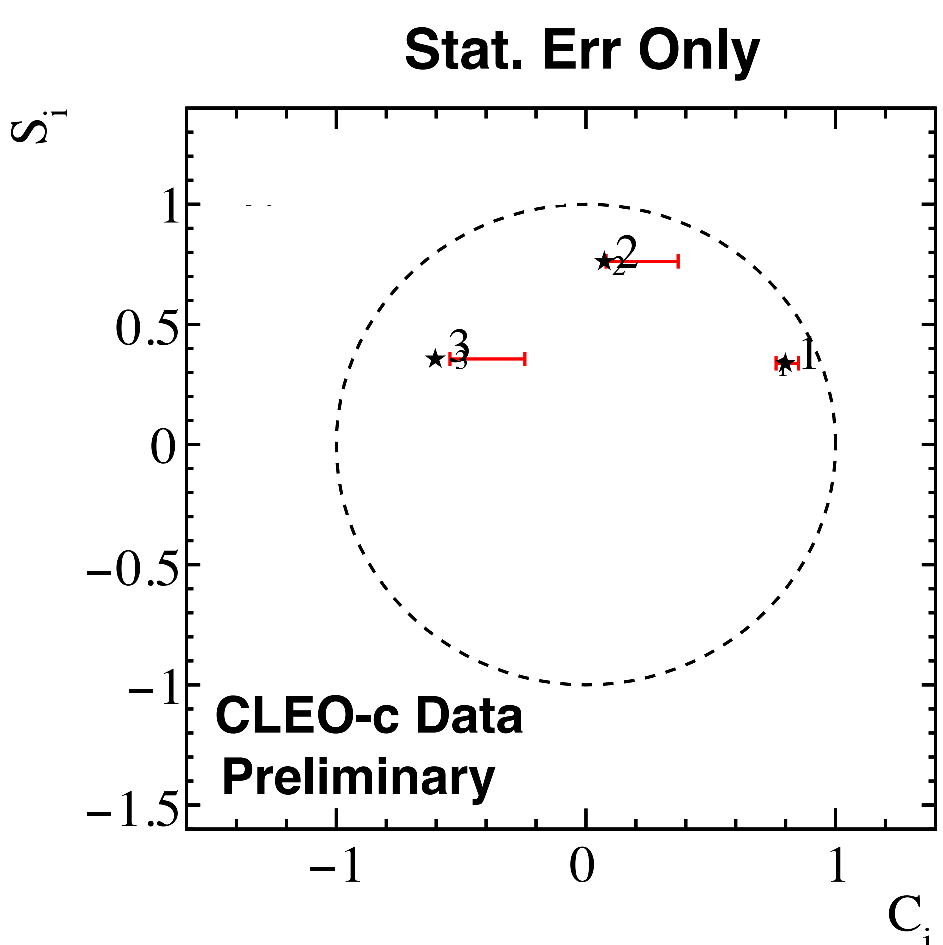
	Decay Mode	$\pi^+\pi^-\pi^+\pi^-$	All
CP-	$K^0_S\eta'$	5.7 ± 2.9	1310 ± 44
	$K^0_S\eta(\pi^+\pi^-\pi^0)$	5.7 ± 2.7	1269 ± 45
	$K^0_S\eta(\gamma\gamma)$	18.0 ± 5.0	2859 ± 80
	$K^0_S\omega$	49.8 ± 8.0	8064 ± 101
	$K^0_S\pi^0$	108 ± 12	19946 ± 156
CP+	$K^0_S\pi^0\pi^0$	14.3 ± 6.0	6465 ± 110
	$\pi^+\pi^-$	1.7 ± 8.7	5620 ± 97
	K^+K^-	12.7 ± 7.3	11899 ± 115
	$K^0_L\pi^0$	47.5 ± 12	–
	$K^0_L\omega$	17.7 ± 6.7	–
~ CP+ (F+ = 0.973 ± 0.017)	$\pi^+\pi^-\pi^0$	73.6 ± 15.4	30107 ± 286
Mixed	$K^0_L\pi^+\pi^-$	486 ± 28	–
	$K^0_S\pi^+\pi^-$	193 ± 18	–
	$\pi^+\pi^-\pi^+\pi^-$	47 ± 17	–
Pseudo Flavour	$K^\pm\pi^\mp$	545 ± 28	–
	$K^\pm\pi^\mp\pi^0$	1120 ± 41	–
	$K^\pm\pi^\mp\pi^\pm\pi^\mp$	802 ± 41	–
Flavour	$K^\pm e^\mp\nu$	444 ± 26	–

Not yet included → Mixed

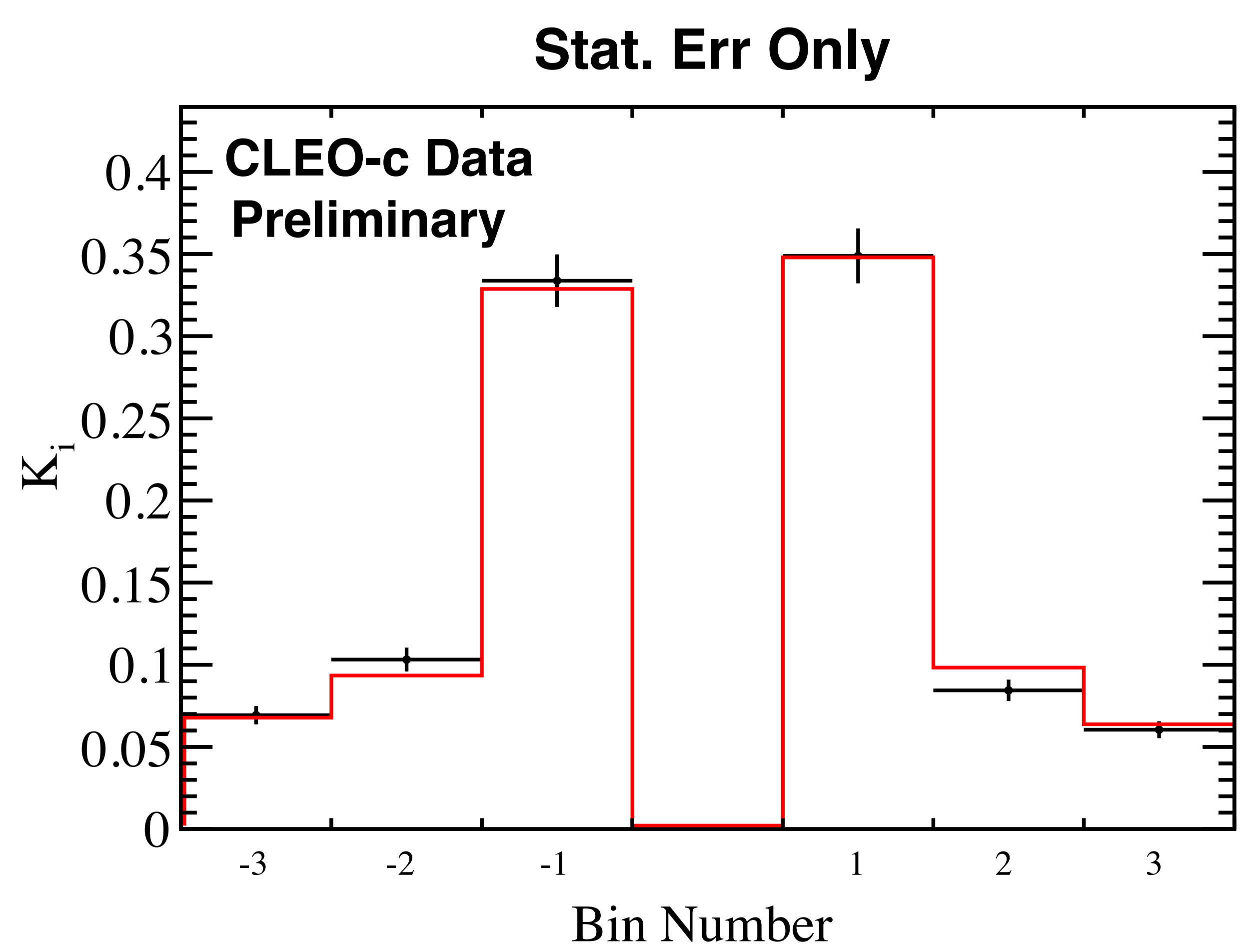
CLEO-c Data Preliminary

Preliminary Results

- From a fit to CP and flavour tags we get the following results.
  - First model-independent test of a D→4h amplitude model
  - From these preliminary results it looks promising!

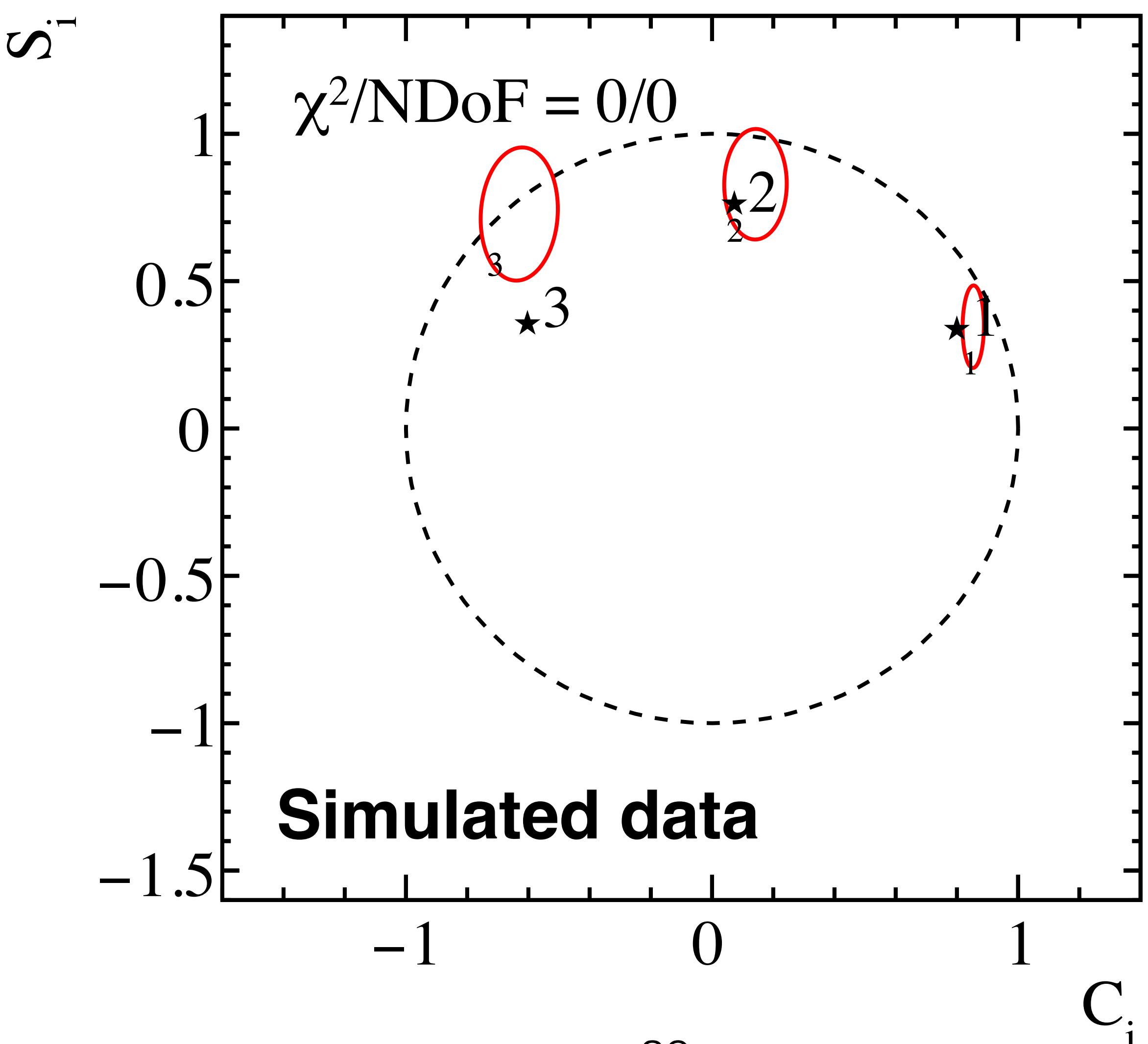


Currently no sensitivity to si



Simulated Tests

- Simulation study used to estimate the sensitivity once $K_S\pi^+\pi^-$, $K_L\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ tags are added.



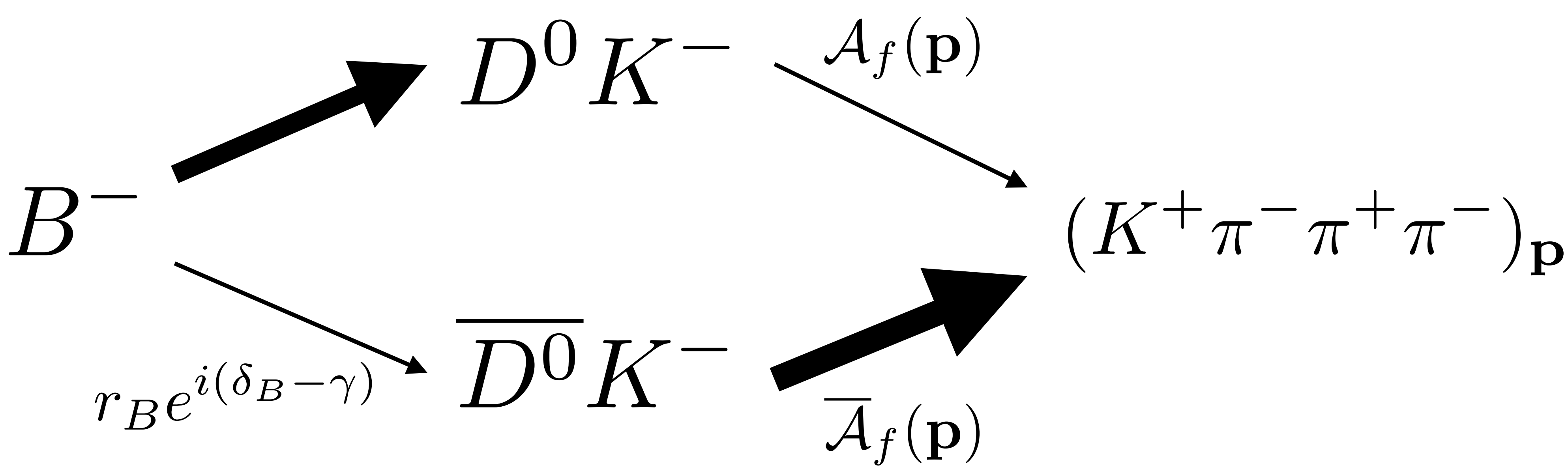
$B^- \to DK^-$, $D \to K^-\pi^+\pi^-\pi^+$

$B^- \to DK^-$, $D \to K^+\pi^-\pi^+\pi^-$

Doubly Cabibbo Suppressed

- $K^-\pi^+\pi^-\pi^+$ is an ADS mode: $$\frac{\langle |\mathcal{A}_{K^+3\pi}(\mathbf{p})|^2 \rangle}{\langle |\overline{\mathcal{A}}_{K^+3\pi}(\mathbf{p})|^2 \rangle} = (r_D^{K3\pi})^2 \sim \frac{1}{300}$$

Cabibbo Favoured



- Larger interference, at the expense of less statistics!

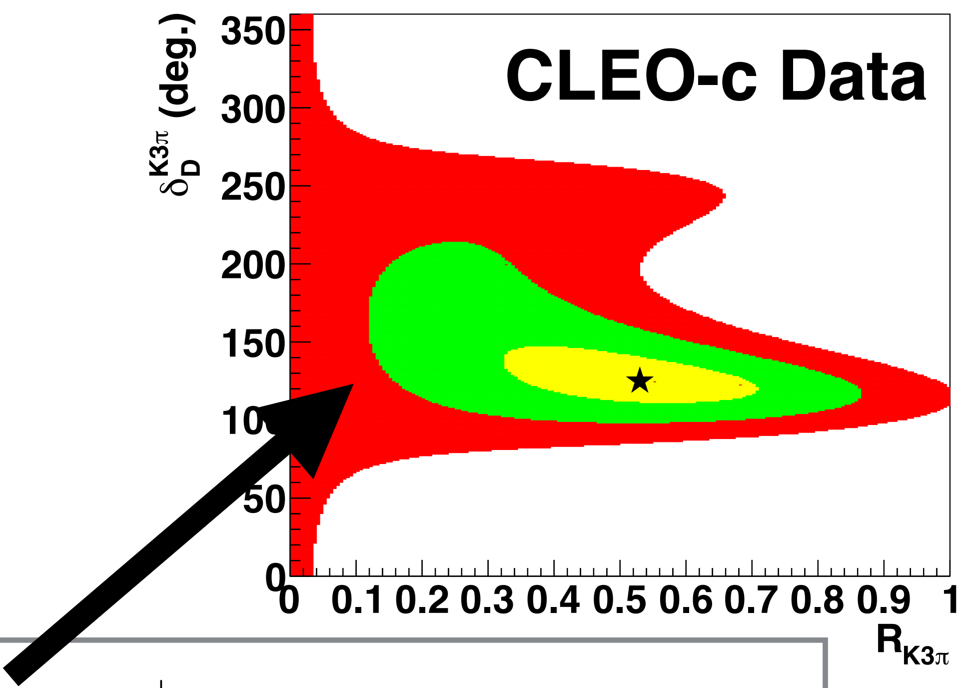
Current status $D \to K^- \pi^+ \pi^- \pi^+$

- As for $D \to \pi^+ \pi^- \pi^+ \pi^-$, only a phase space integrated measurement has been performed, which contributes to the LHCb γ combination.

arXiv:1611.03076

- The D decay parameters have also been determined at CLEO-c

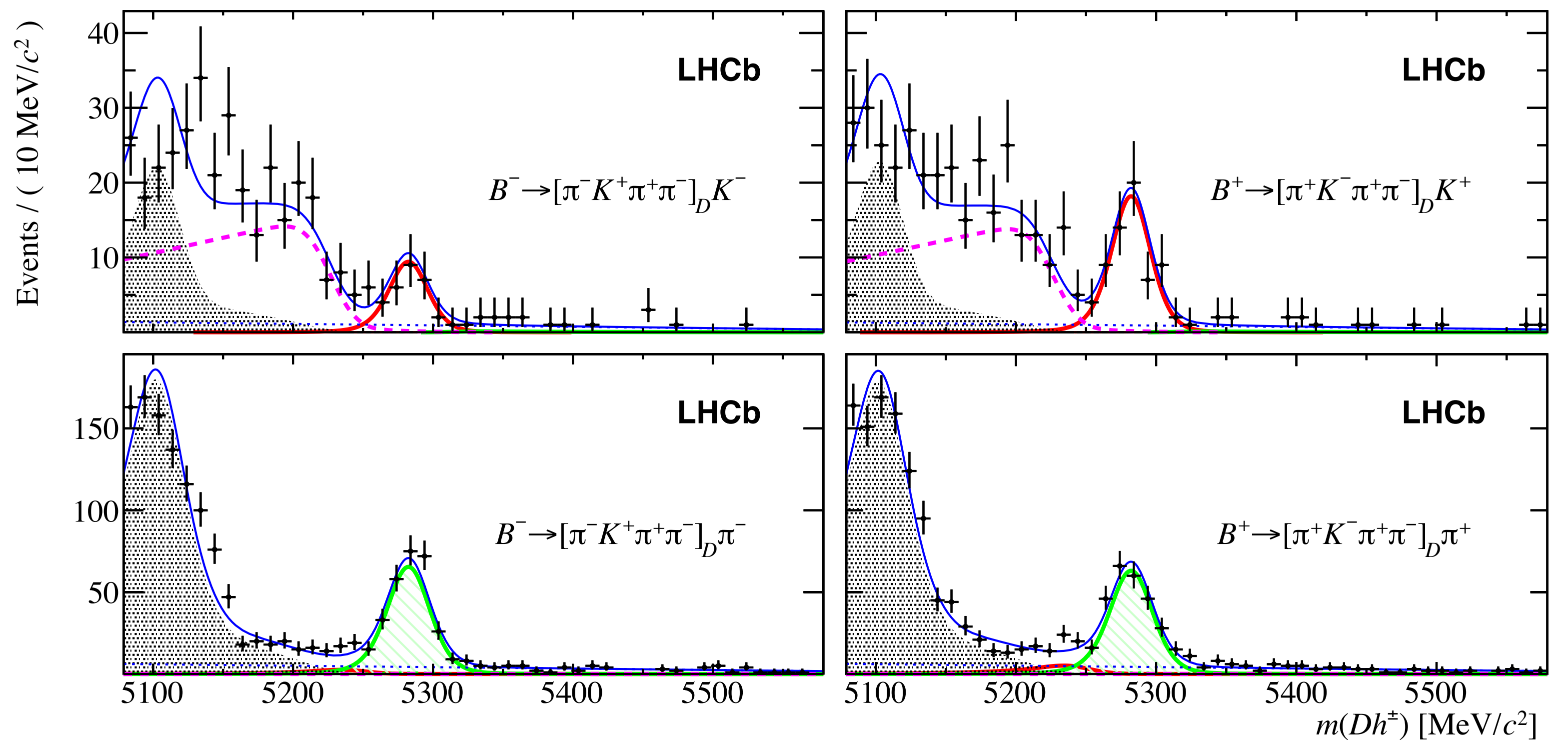
Phys. Let. B 757 (2016) 520-527
(https://arxiv.org/abs/1602.07430)



$$c_{\text{ALL}}^{K^+3\pi} = R_{K3\pi} \cos \delta_{K3\pi}$$
$$s_{\text{ALL}}^{K^+3\pi} = R_{K3\pi} \sin \delta_{K3\pi}$$

Just ci and si in a different parameterisation

Phys. Lett. B 76 (2016) 117-131
(https://arxiv.org/abs/1603.08993)

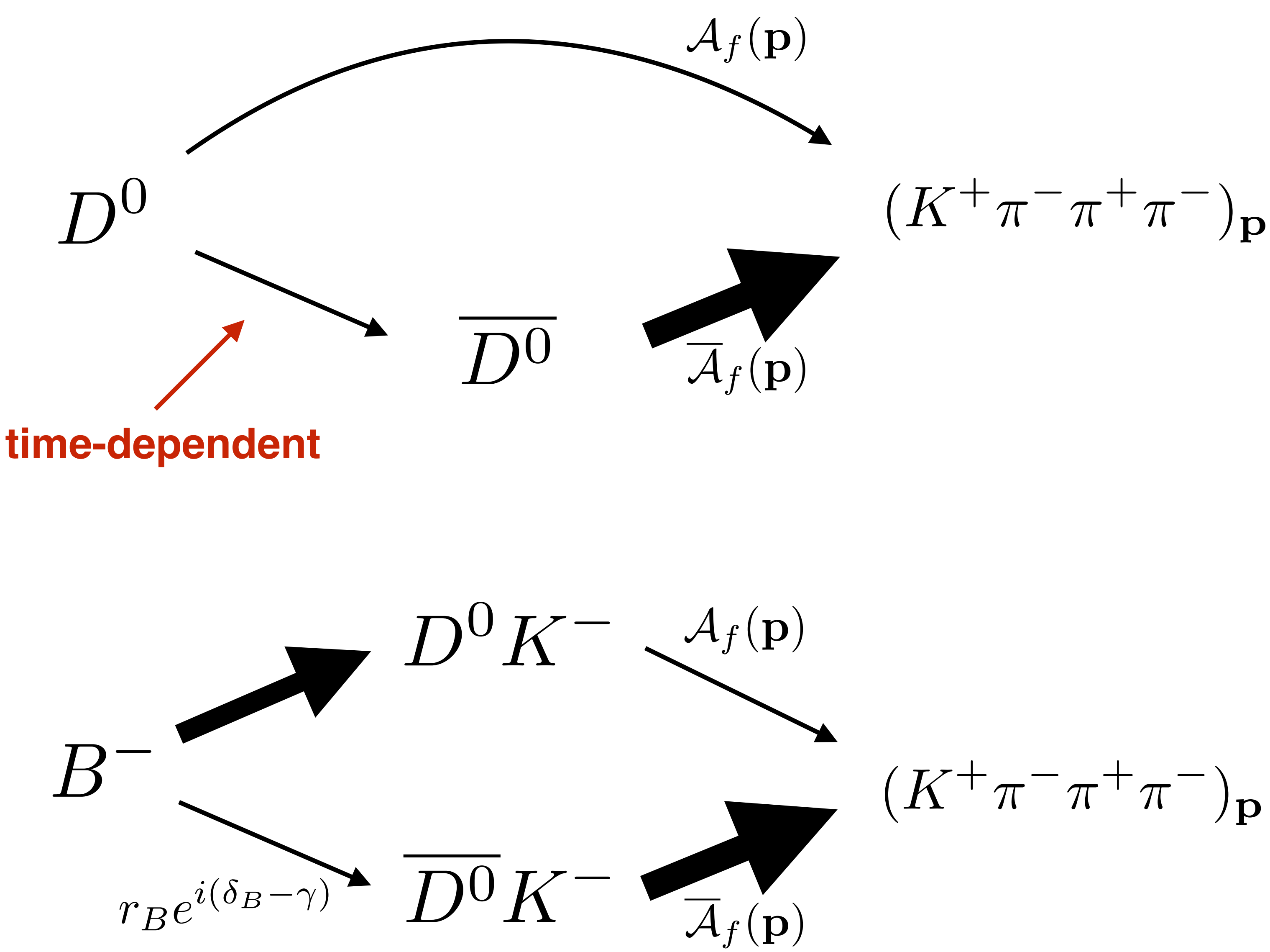


Current status $D \to K^-\pi^+\pi^-\pi^+$

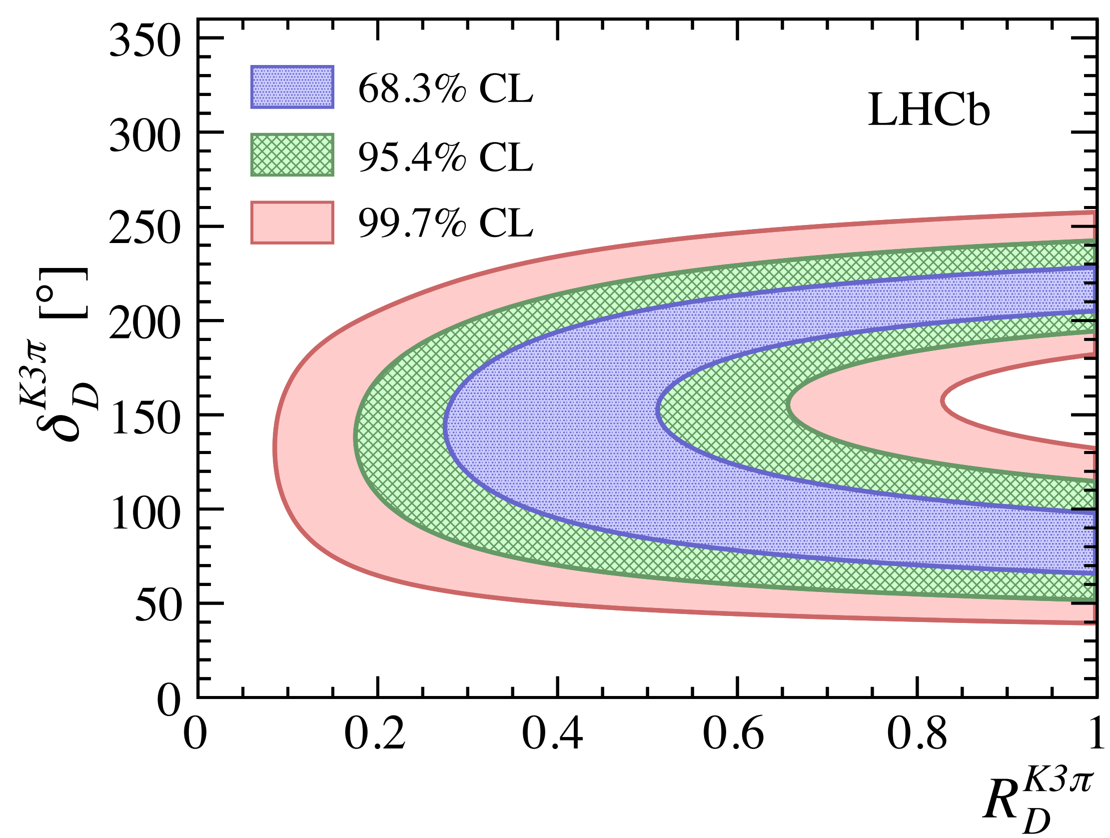
- For $D \to K^-\pi^+\pi^-\pi^+$ the D decay parameters also come from D-mixing!

Idea proposed in: **Phys. Let. B 728 (2014) 296-302 (https://arxiv.org/abs/1602.07430)**

In principle could also do the same for $D \to \pi^-\pi^+\pi^-\pi^+$



Phys. Rev. Let. 116 (2016) 241801 (https://arxiv.org/abs/1602.07430)

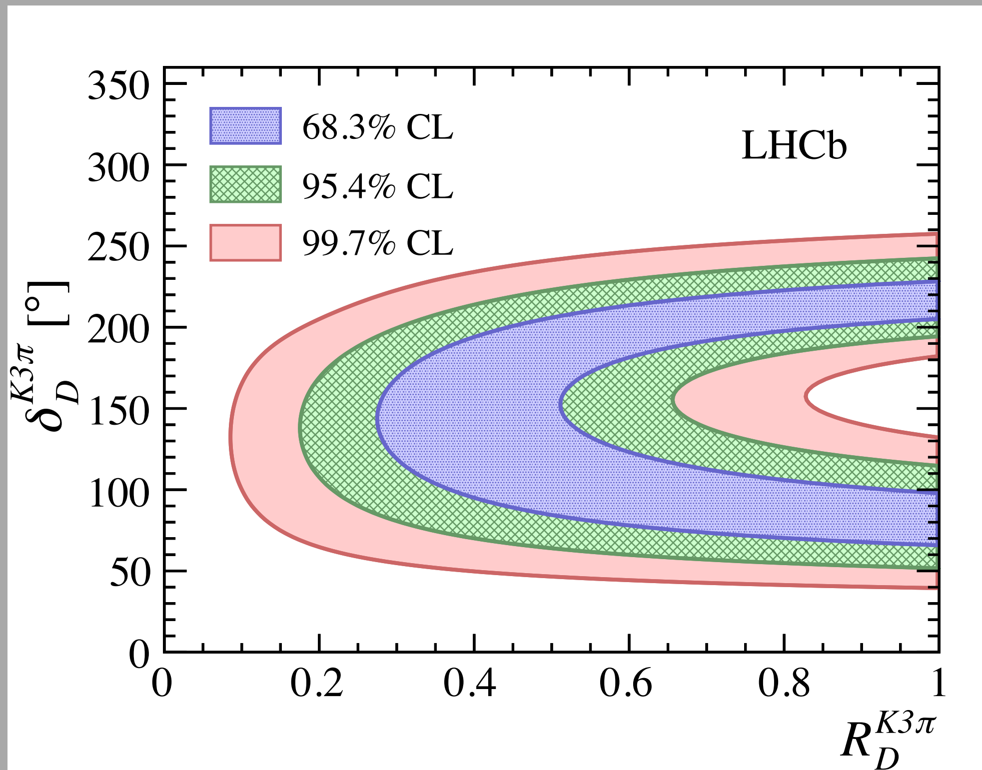


tatus D

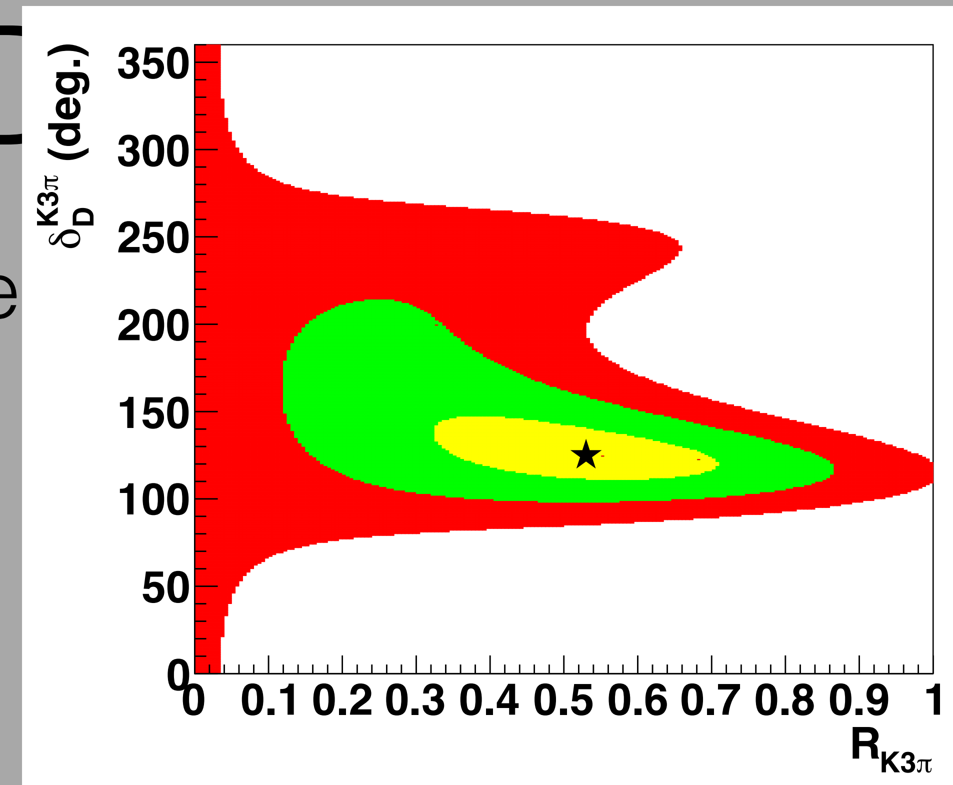
- ay ... ete ... g!

(20 ... /abs/...) ... 01

(https://arxiv.org/abs/1602.07430)



+



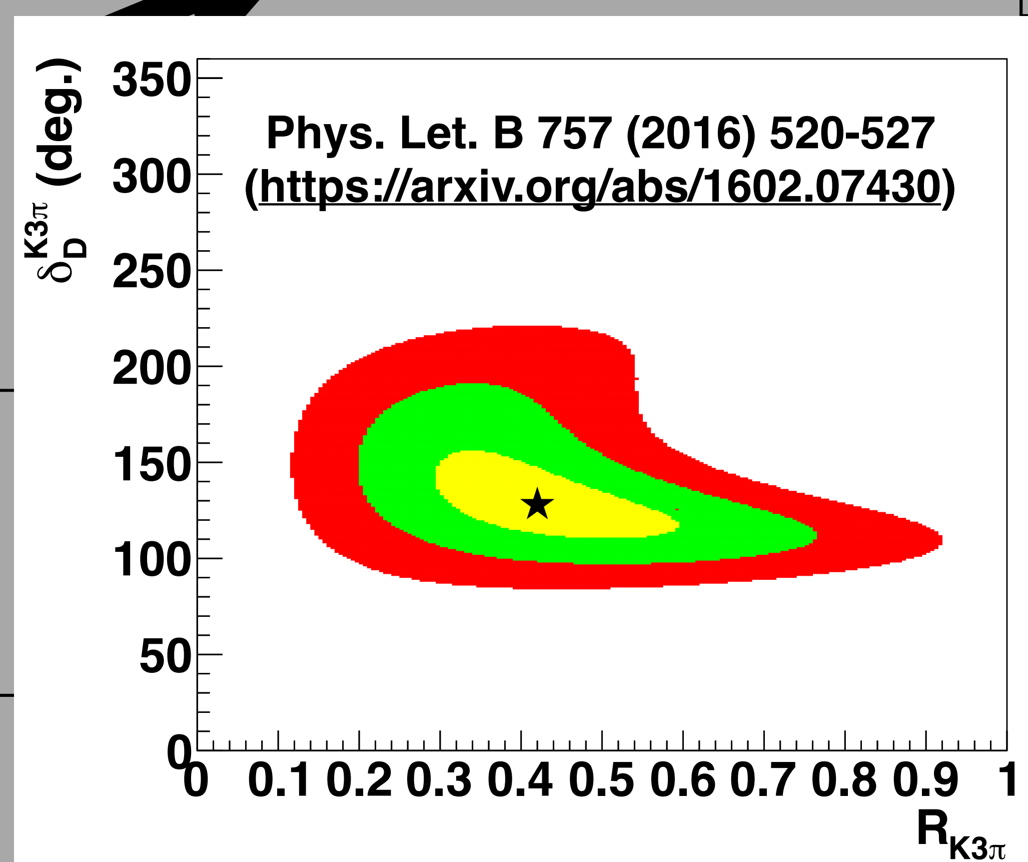
D^0 → $\overline{D^0}$ → $(K^+\pi^-\pi^+\pi^-)_\mathbf{p}$

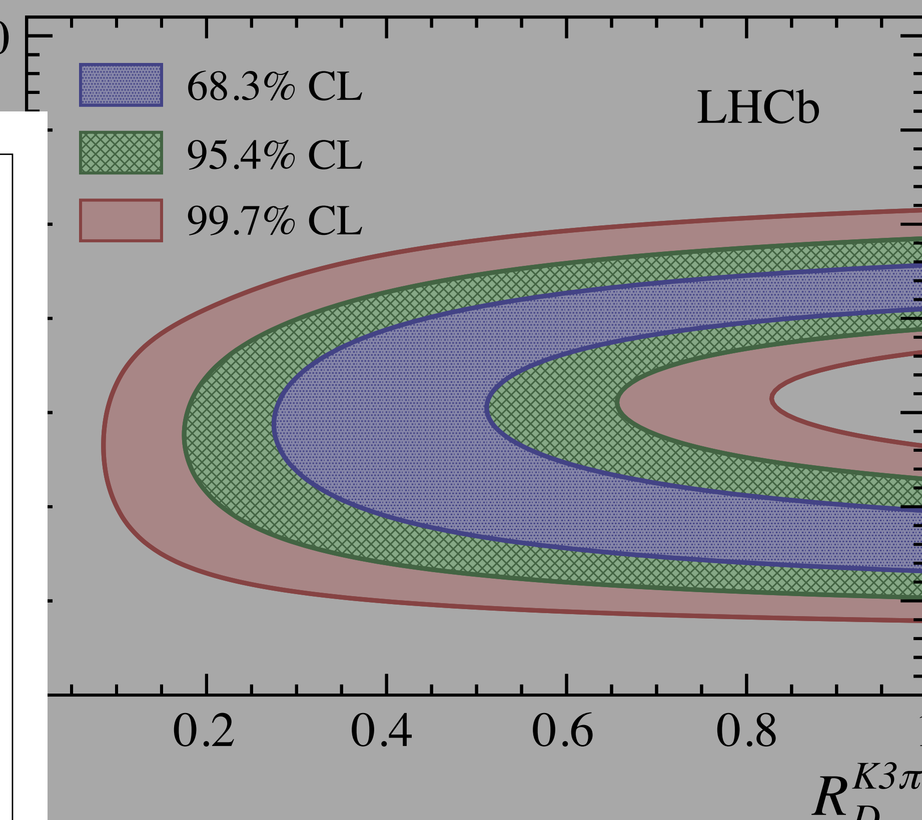
time-dependent

B^- → K^-

$r_B e^{i(\delta_B-\gamma)}$ → $\overline{D^0}K^-$

=





Future $D\to K^-\pi^+\pi^-\pi^+$

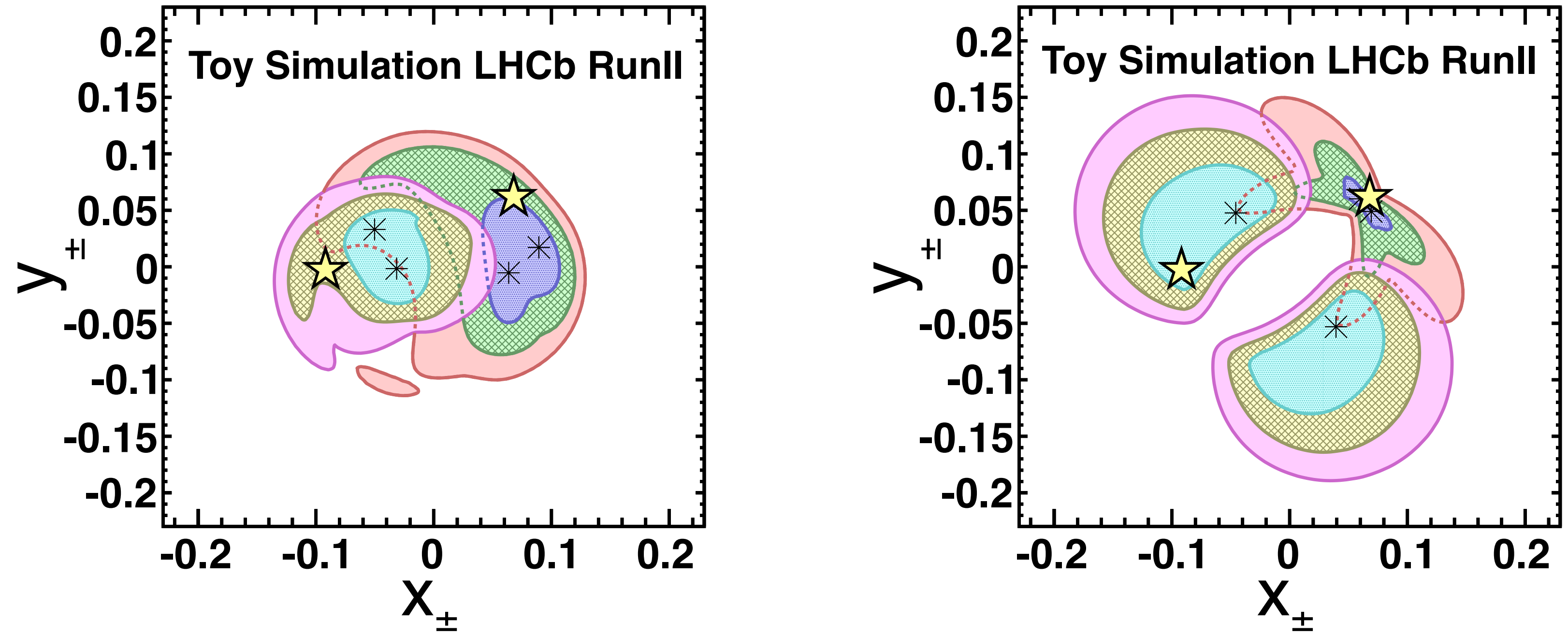
- Ideally the next step is a binned $D\to K^-\pi^+\pi^-\pi^+$ measurement
  - Amplitude model needed to inspire the binning (in progress at LHCb)

Toy simulation with expected LHCB statistics at the end of RunII

JHEP 03 (2015) 169 (https://arxiv.org/abs/1412.7254)

Simulated Expected BESIII Input

Simulated Expected LHCb D-Mixing Input

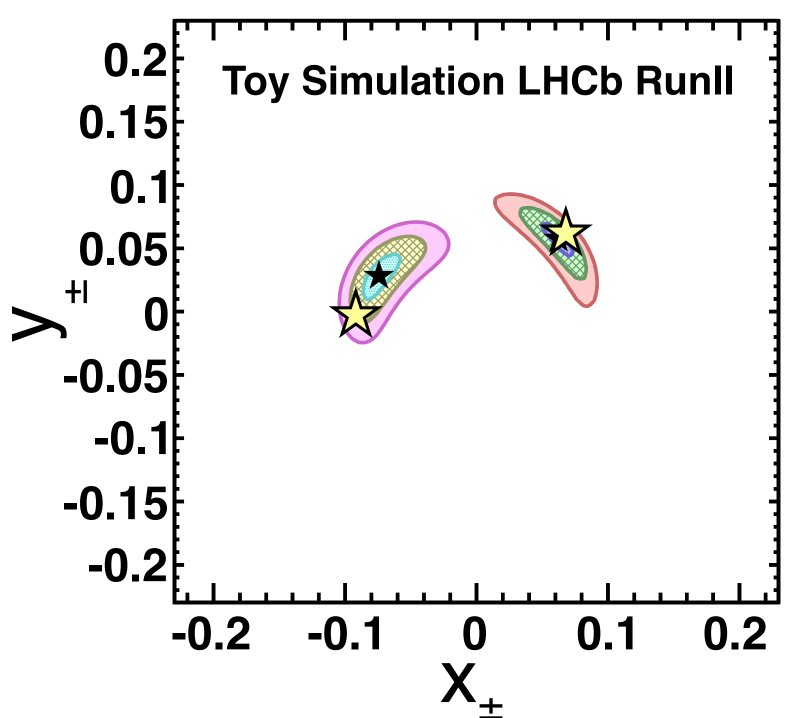


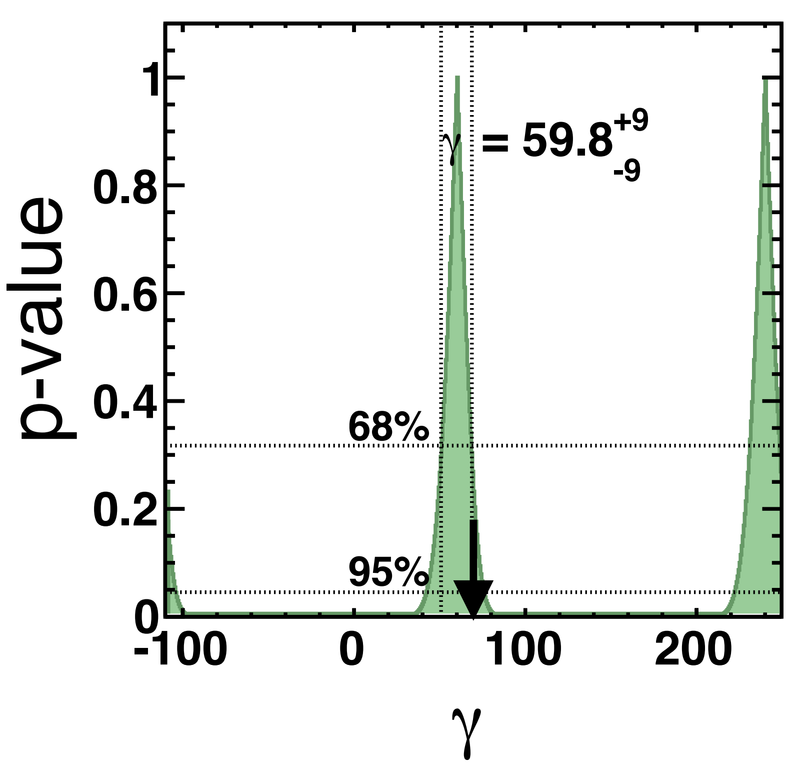
$\sigma_\gamma \sim 30$ $\sigma_\gamma \sim 24$

Future $D \to K^- \pi^+ \pi^- \pi^+$

- Ideally the next step is a binned $D \to K^- \pi^+ \pi^- \pi^+$ measurement
  - Best sensitivity to γ when using input from both BESIII and D-Mixing

Simulated Expected BESIII and LHCb (RunII) D-Mixing Input





RunII LHCb simulation studies indicate γ could be measured to ~9° with combined input! (or 12° with BESIII→CLEO)

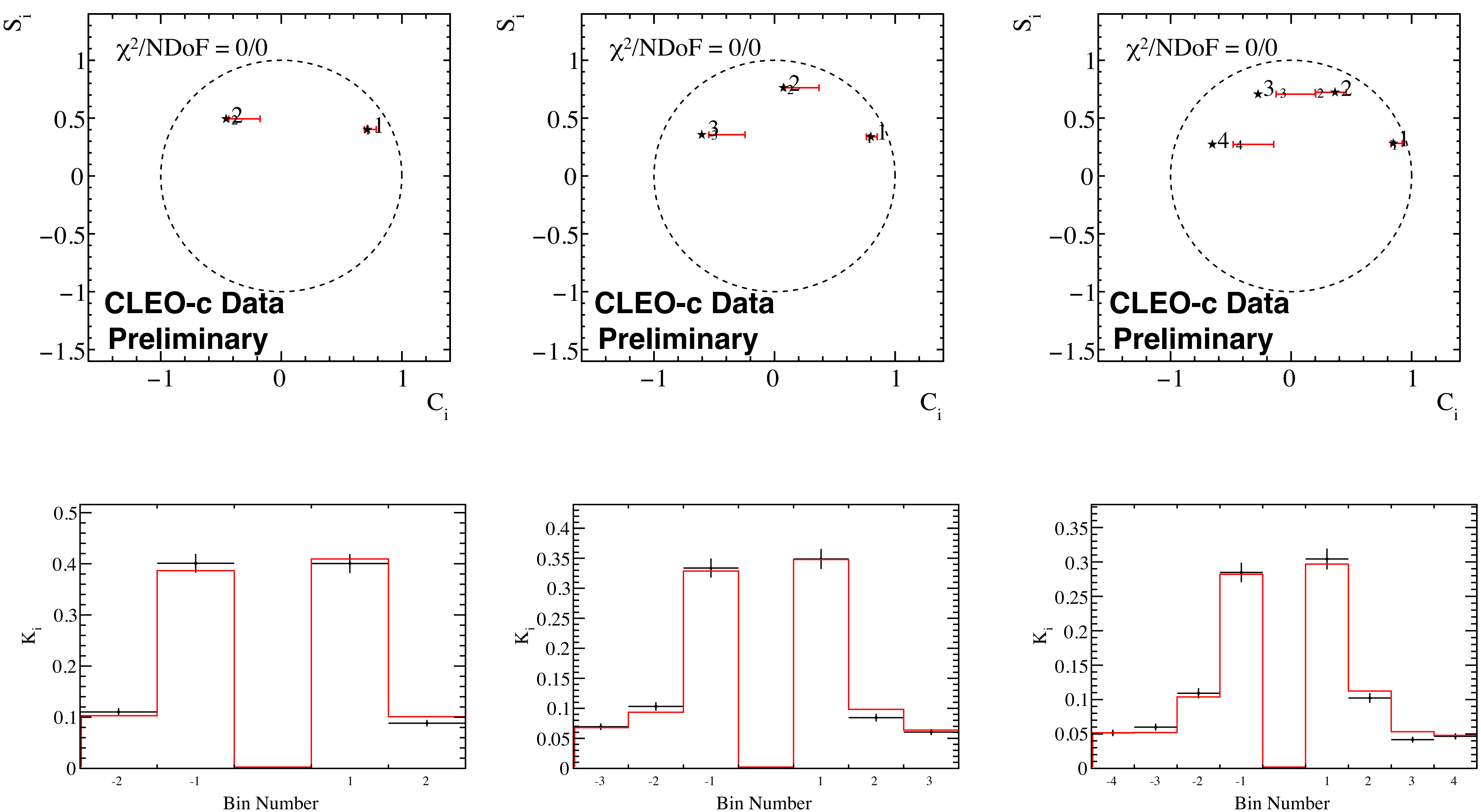
JHEP 03 (2015) 169
(https://arxiv.org/abs/1412.7254)

Conclusions

- GGSZ measurement of $D \to \pi^+\pi^-\pi^+\pi^-$ final state could give one of the most precise single measurements of γ.
    - Measurement needs external input to describe the D decay amplitudes
- First measurement of binned c_i (and soon s_i) in the $D \to \pi^+\pi^-\pi^+\pi^-$ decay.
    - Adaptive binning scheme developed to describe 5D phase space bins
    - Preliminary results show good agreement with the model predictions
    - Good news for four-body amplitude analyses!
- With combined input from BESIII / CLEO-c + D-Mixing, the four-body $D^0 \to K^+\pi^-\pi^+\pi^-$ final state offers another γ measurement with similar/better sensitivity to $D \to \pi^+\pi^-\pi^+\pi^-$.

BACKUP

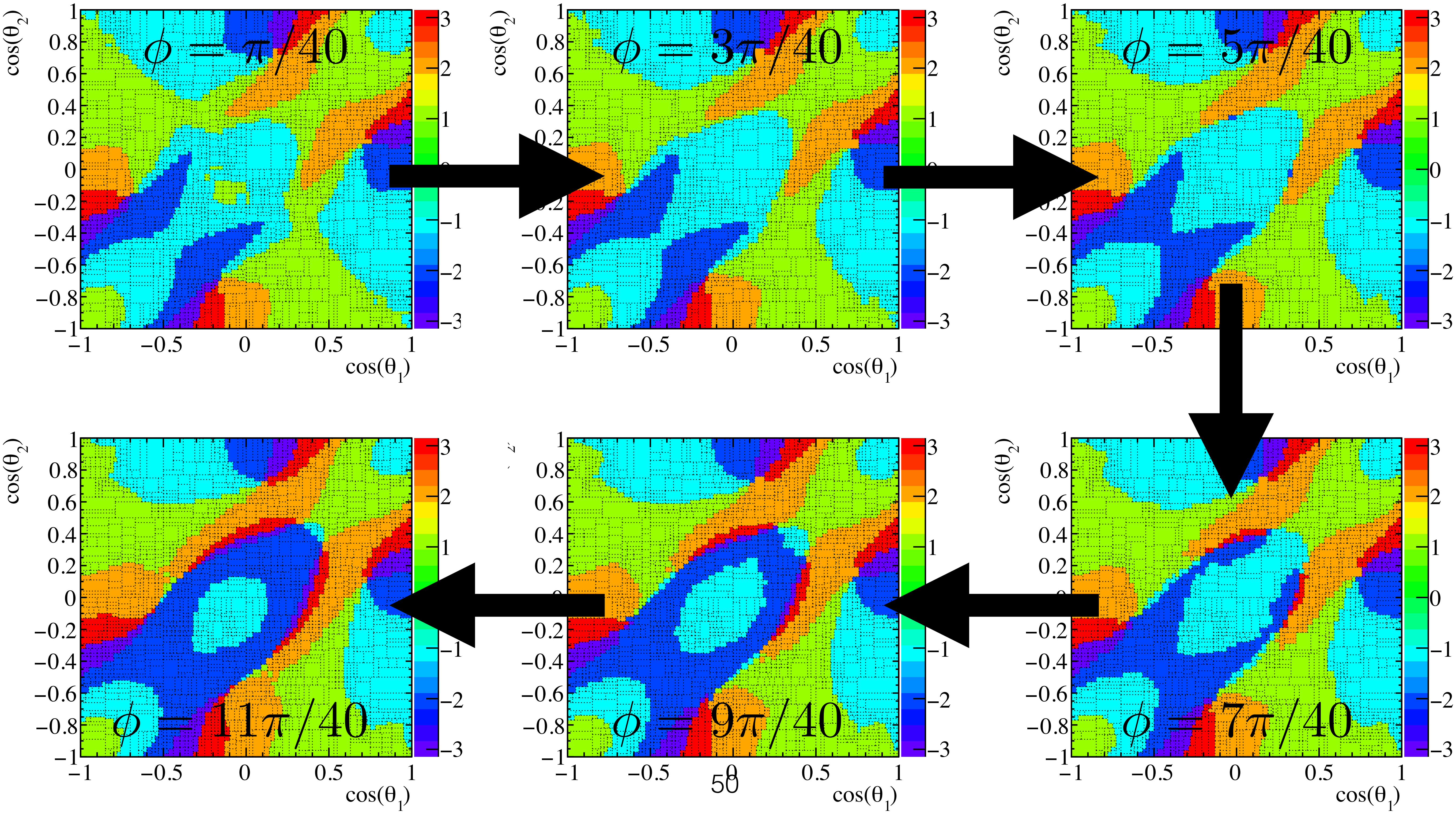
#bin pairs = 2 | **#bin pairs = 3** | **#bin pairs =4**



$\pi^+\pi^-\pi^+\pi^-$ Binning

 # bin pairs = 3

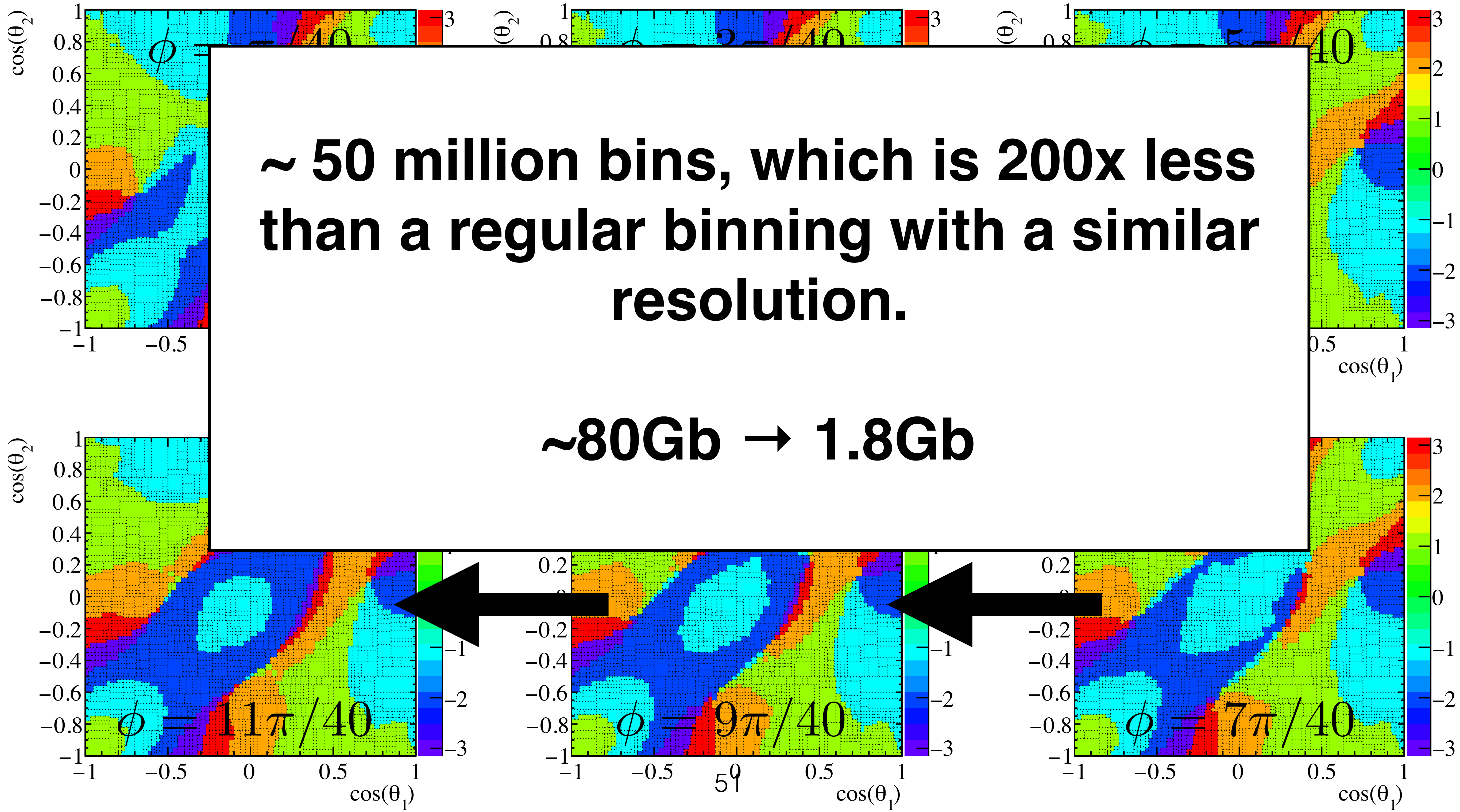
$$m_{12} = m_{34} = 1\text{GeV}/c^2$$



$\pi^+\pi^-\pi^+\pi^-$ Binning

 # bin pairs = 3

$$m_{12} = m_{34} = 1\text{GeV}/c^2$$



~ 50 million bins, which is 200x less than a regular binning with a similar resolution.

~80Gb → 1.8Gb

Model Inspired GGSZ Binning

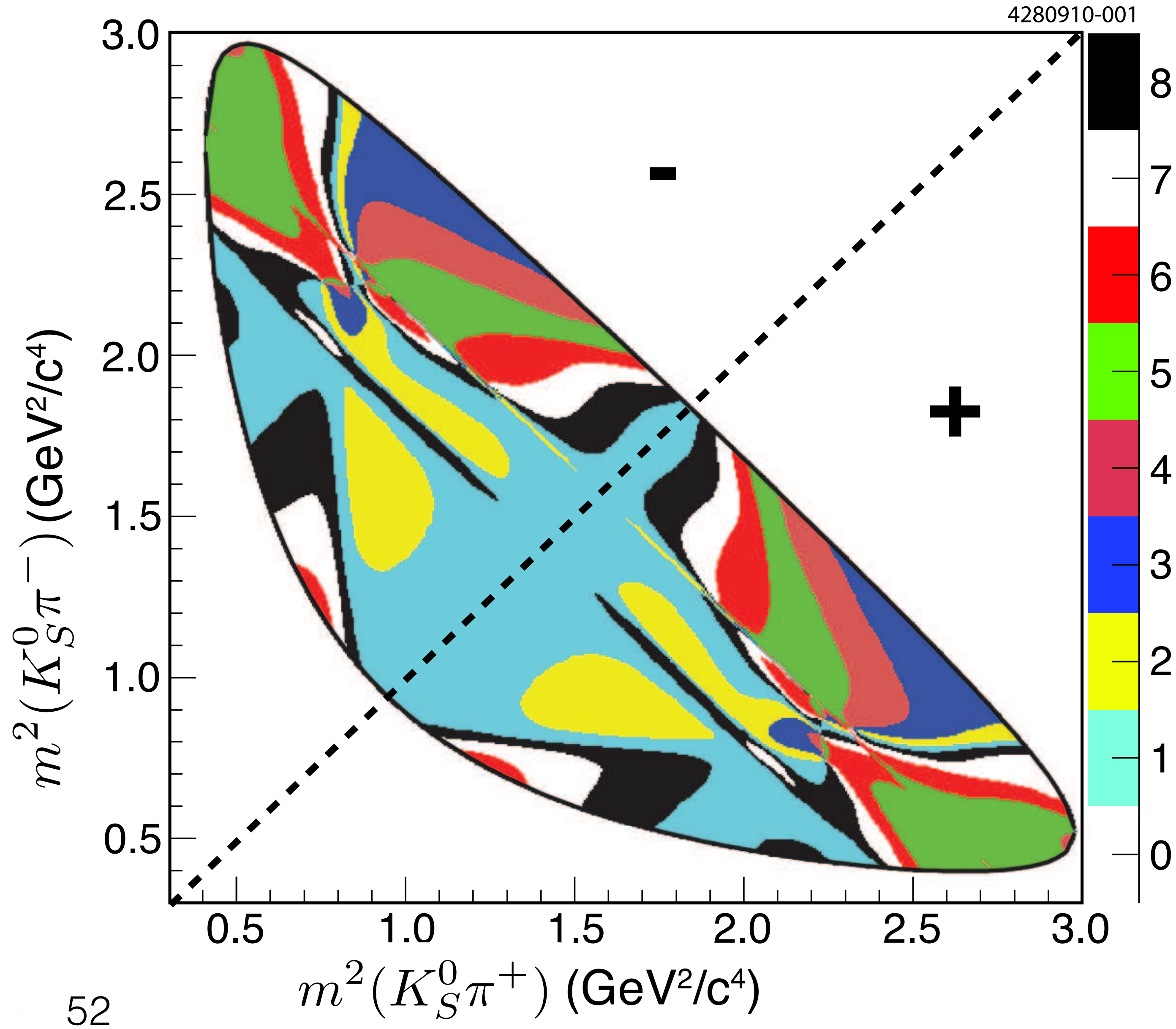
Sensitivity to γ is ~ proportional to

$$\sqrt{c_i^2 + s_i^2}$$

Want to choose a binning scheme such that this is as large as possible in each bin!

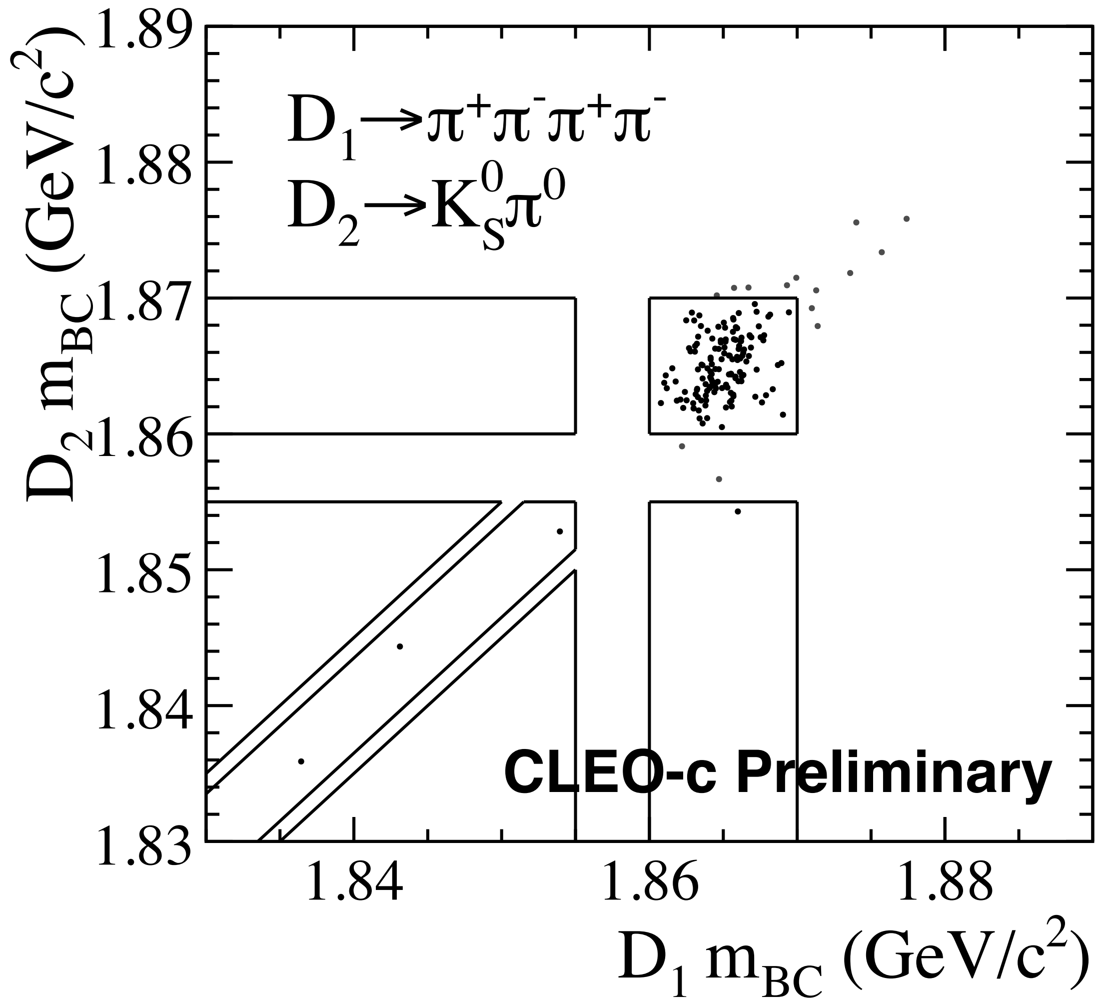
$$D \to K_S^0 \pi^+ \pi^-$$

Phys. Rev. D 82 (2010) 112006 (https://arxiv.org/abs/1010.2817)



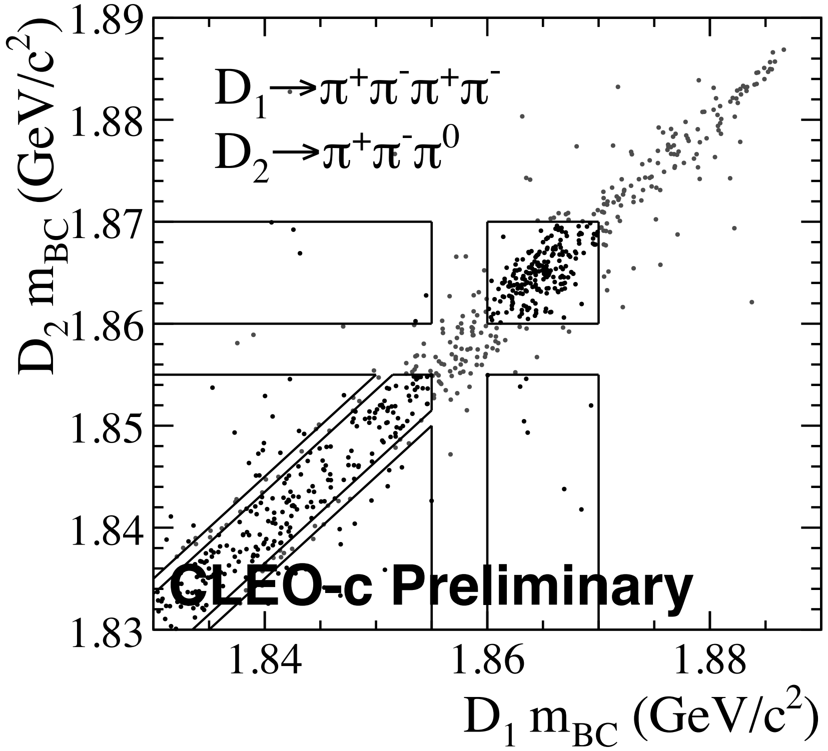
Fully Reconstructed Tags

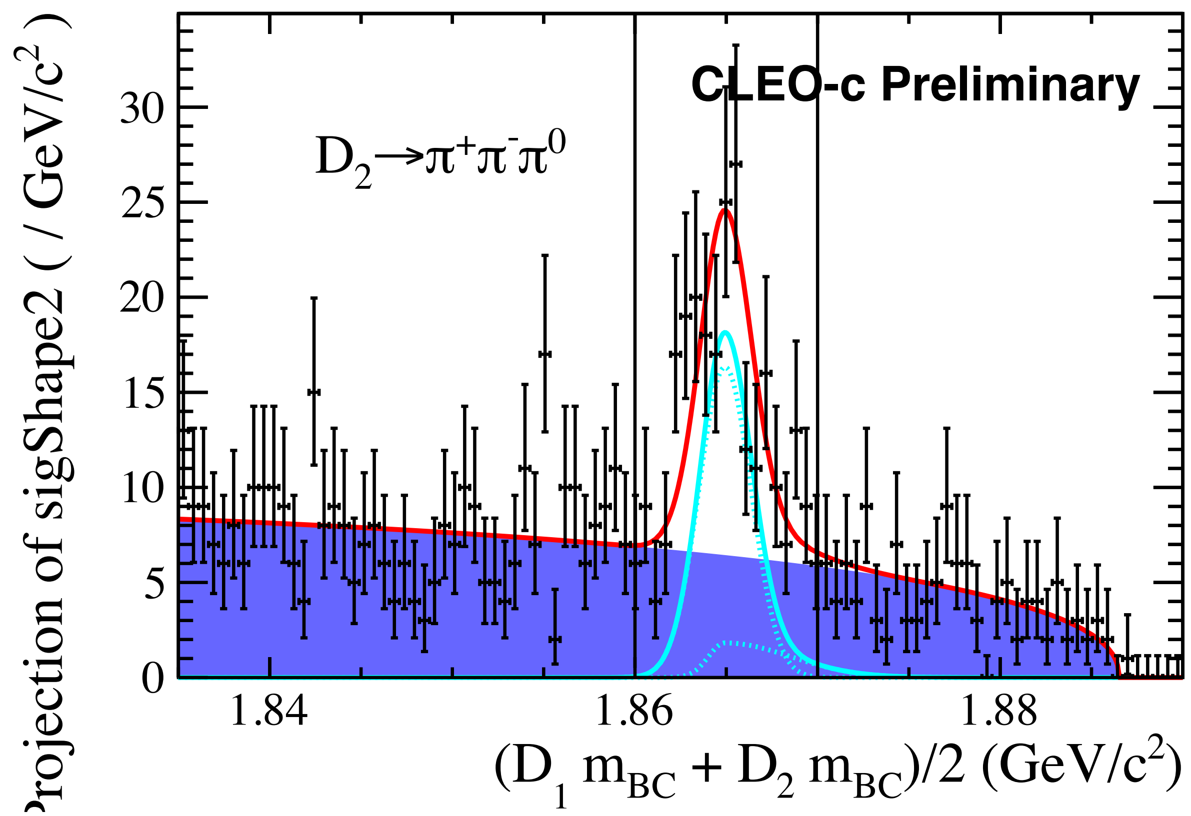
- Plot the beam constrained mass of each reconstructed D meson
- Define different sideband regions to determine flat background contributions



Continuum Dom. Tags

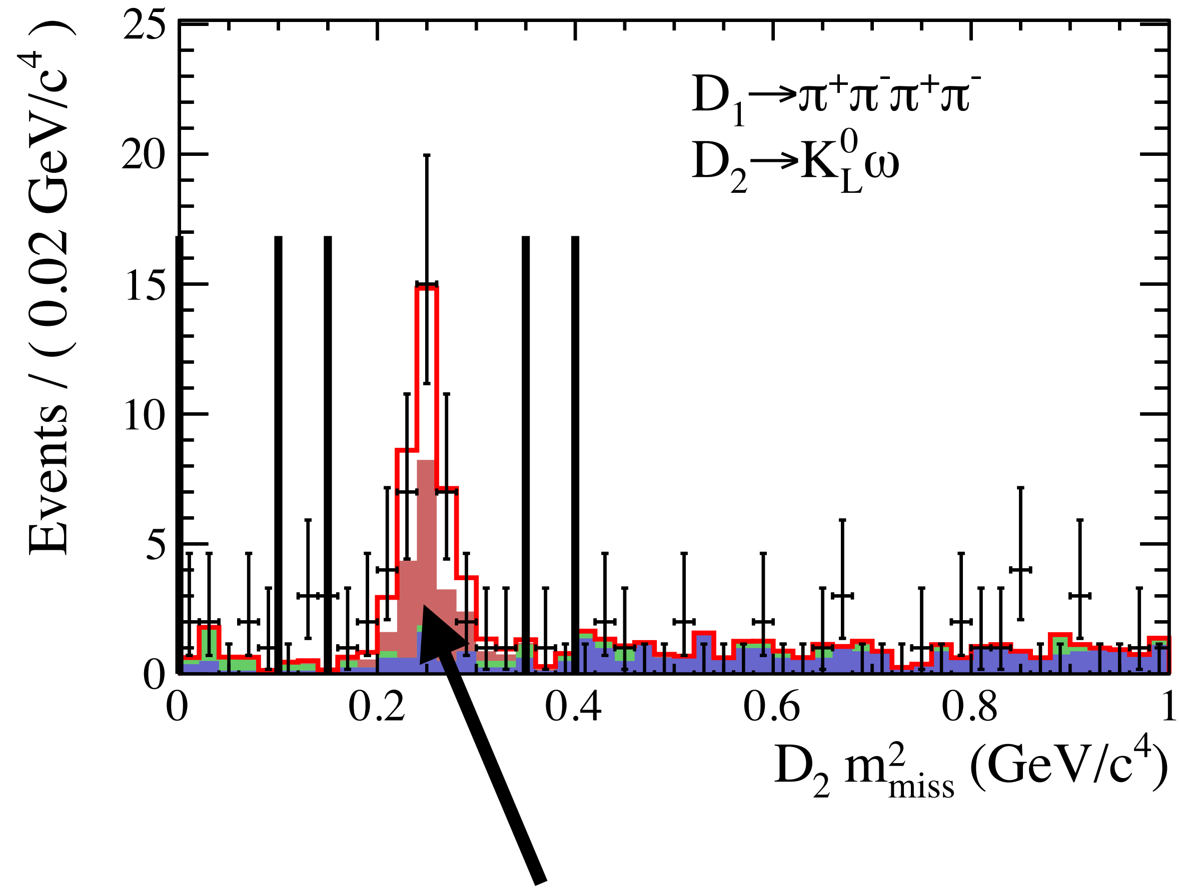
- For continuum dominated tags we fit the average of the two beam constrained masses





Partially Reco Tags

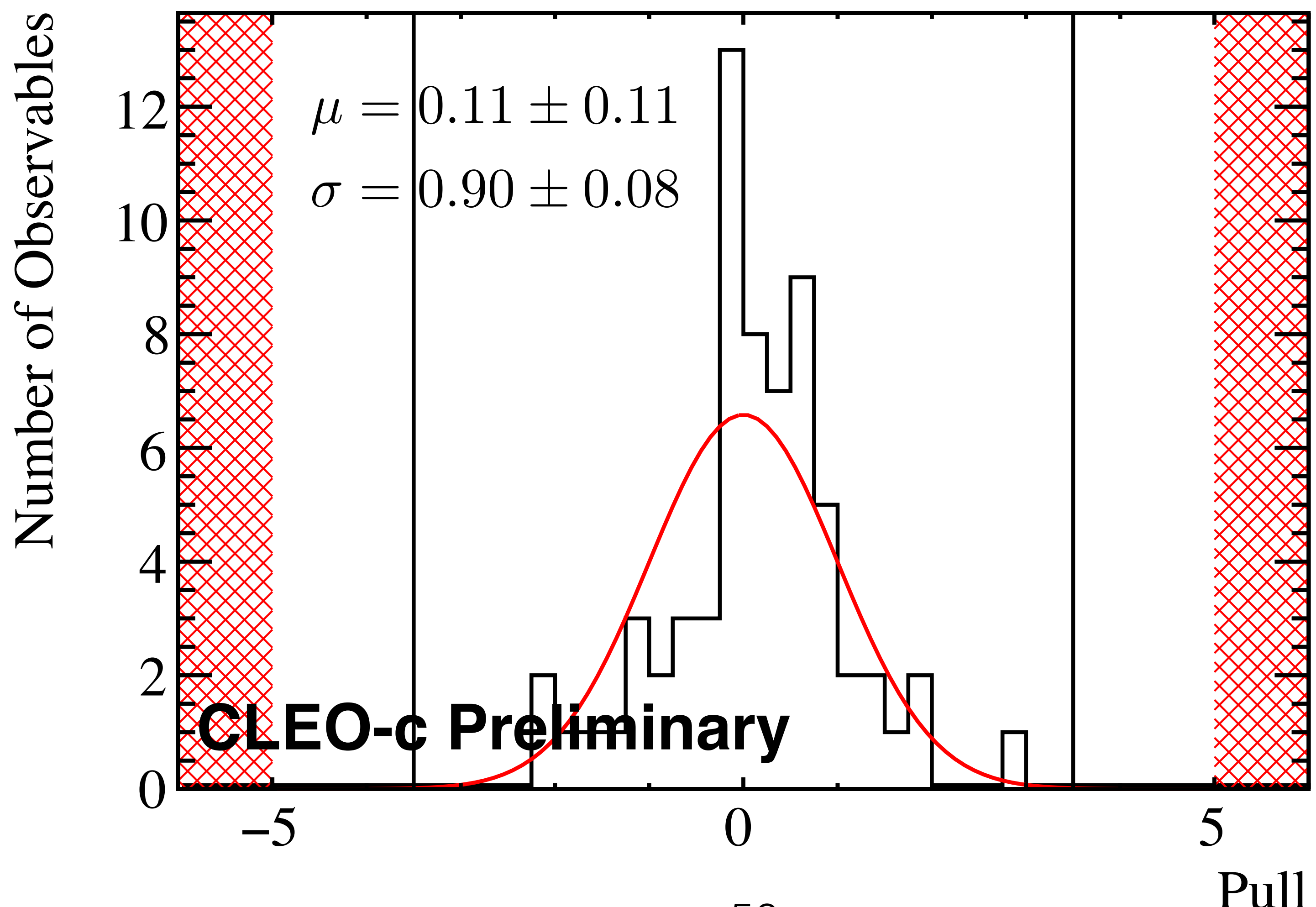
- For partially reconstructed tags (missing K_L or ν) we fit the missing mass squared.
    - Shapes taken from MC samples



Red component is peaking background ($K_S\pi\pi$). Majority of this is later removed by a K_S veto

Preliminary Results

- The agreement between the expected and measured observables is good



LHCb scenario	D^0 mix?	charm threshold?	$\sigma(\gamma)$ [°]	$\sigma(\delta_B)$ [°]	$\sigma(r_B)$ $\times 10^2$	$\sigma(x_+)$ $\times 10^2$	$\sigma(y_+)$ $\times 10^2$	$\sigma(x_-)$ $\times 10^2$	$\sigma(y_-)$ $\times 10^2$
run I			26	47	1.6	8.7	9.1	8.8	8.2
run II	Y	none	22	29	1.4	7.6	6.9	4.5	4.0
upgr			15	14	0.17	4.7	5.2	0.56	0.98
run I			20	29	0.82	6.4	5.7	6.6	5.9
run II	Y	CLEO global	15	19	0.62	5.4	3.9	2.5	2.7
upgr			11	10	0.16	3.8	2.8	0.44	0.50
run I			19	25	0.78	6.4	5.5	6.5	5.8
run II	Y	BESIII global	14	18	0.57	5.4	3.9	2.4	2.7
upgr			9.0	8.2	0.15	3.7	2.7	0.43	0.48
run I			46	35	3.2	6.9	6.5	8.6	10
run II	N	CLEO binned	50	34	3.3	6.9	6.7	8.9	11
upgr			52	35	3.3	7.6	6.7	8.9	11
run I			40	24	2.6	4.1	5.0	5.7	6.2
run II	N	BESIII binned	34	17	2.5	3.6	4.1	5.0	5.1
upgr			39	14	2.9	3.9	4.1	4.3	5.6
run I			16	18	0.78	2.1	3.5	2.6	3.1
run II	Y	CLEO binned	12	13	0.53	1.7	3.1	1.7	2.0
upgr			7.8	7.2	0.15	1.1	2.6	0.40	0.46
run I			12	14	0.68	1.6	2.6	2.0	2.5
run II	Y	BESIII binned	8.6	9.6	0.47	0.90	2.1	1.5	1.5
upgr			4.1	3.9	0.14	0.53	1.3	0.35	0.38

Table 2. Uncertainties on key parameters, obtained based on the default amplitude model in different configurations, averaged over 50 simulated experiments. All results are for the binned approach applied to $B^\mp \to DK^\mp$ and, where used, charm mixing data. The first column refers to the scenarios defined in Tab. 1. The second column defines whether charm mixing input was used (Y), or not (N). The third column describes additional input from the charm threshold. "CLEO global" refers to the phase-space integrated input from [14]. "BES III global" is the same, but uses the uncertainties predicted in [14] for a data sample 3.5 times as large as that collected by CLEO-c. "CLEO binned" and "BES III binned" extrapolate to a potential binned analysis of the charm threshold data described in Sec. 4.6.3.

	$B^\pm \to D(K3\pi)K^\pm$ suppressed	$B^\pm \to D(K3\pi)K^\pm$ favoured	$D^{*\pm} \to D(K3\pi)\pi^\pm$
LHCb run I ($3\,\text{fb}^{-1}$ @ $7-8\,\text{TeV}$)	120	10k	8M
LHCb run II ($8\,\text{fb}^{-1}$ @ $13\,\text{TeV}$)	800	60k	50M
LHCb upgrade ($50\,\text{fb}^{-1}$ @ $13\,\text{TeV}$)	9000	700k	600M

Table 1. Event yields assumed in the simulation studies, based on reported event yields for $1\,\text{fb}^{-1}$ at LHCb [31, 33]. The event yields are inclusive, for example, LHCb run II yields includes those from LHCb run I. The fraction of WS events in $D^{*\pm} \to D(K3\pi)\pi^\pm$ depends on the input variables; typically it is 0.38%.

	$B^\pm \to D(K3\pi)K^\pm$ suppressed	favoured	$D^{*\pm} \to D(K3\pi)\pi^\pm$
LHCb run I ($3\,\text{fb}^{-1}$ @ $7-8\,\text{TeV}$)	120	10k	8M
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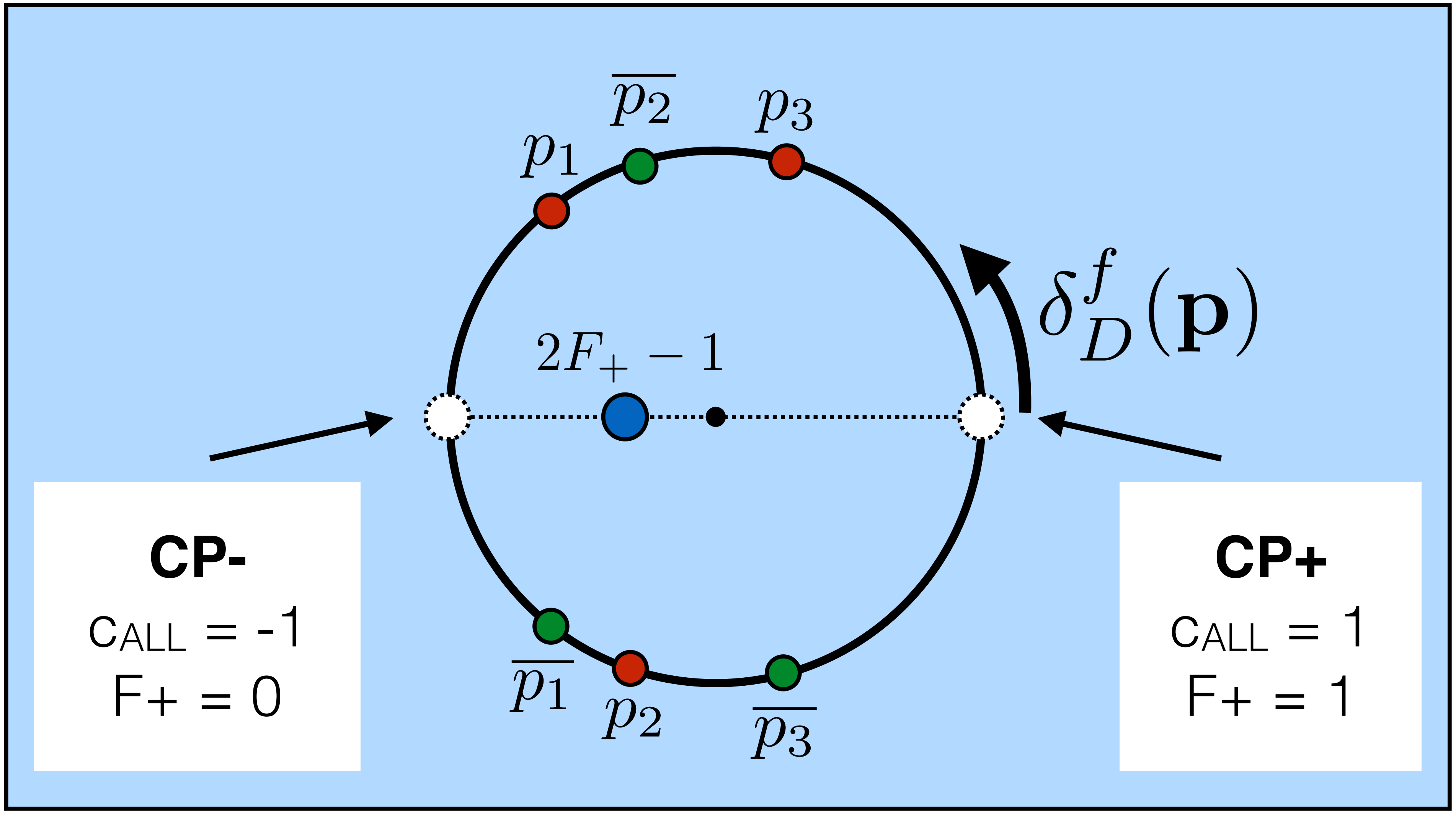


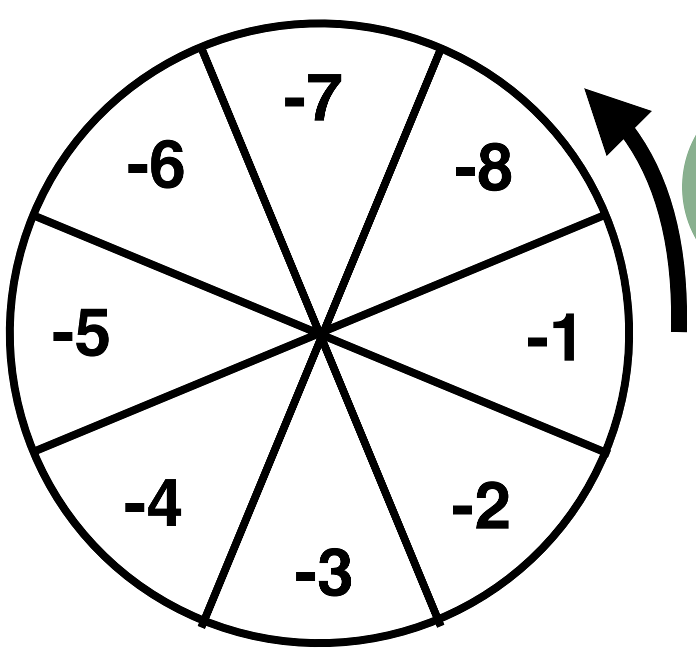
$\overline{p_2}$
p_3
p_1
$\delta_D^f(\mathbf{p})$
$2F_+ - 1$
CP-
cALL = -1
F+ = 0
CP+
cALL = 1
F+ = 1
$\overline{p_1}$
p_2
$\overline{p_3}$

Model Inspired GGSZ Binning

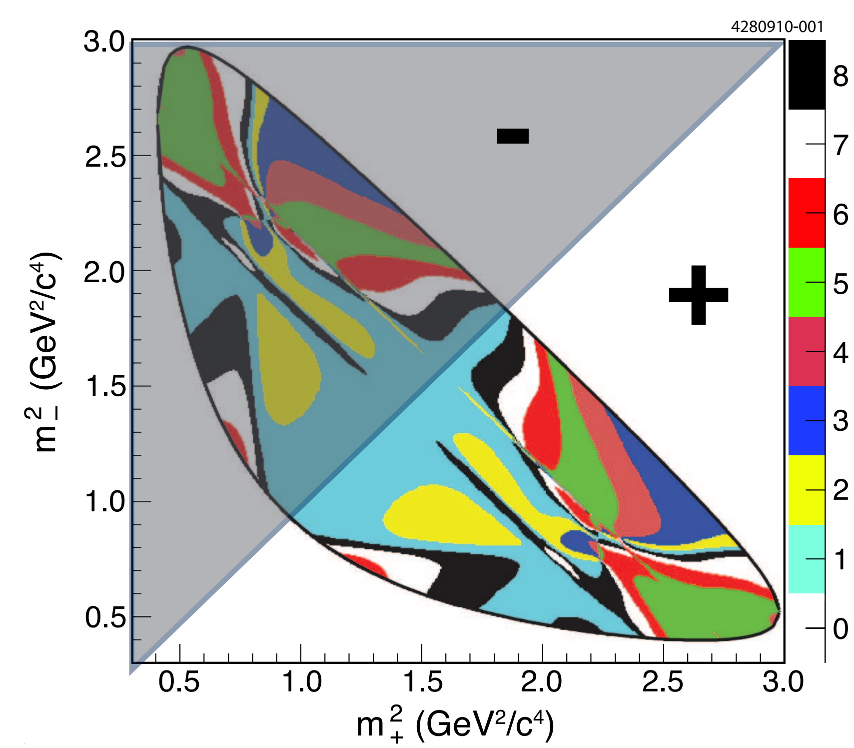
$\sqrt{c_i^2 + s_i^2}$ is maximised when the phase difference between amplitudes is constant

$$c_i + i s_i = \frac{\int_i \mathcal{A}_f(\mathbf{p}) \overline{\mathcal{A}}_f(\mathbf{p})^* \mathrm{d}\mathbf{p}}{\sqrt{K_i \overline{K_i}}} = \frac{\int_i |\mathcal{A}_f(\mathbf{p})| |\overline{\mathcal{A}}_f(\mathbf{p})| e^{i\delta_D^f(\mathbf{p})} \mathrm{d}\mathbf{p}}{\sqrt{K_i \overline{K_i}}}$$

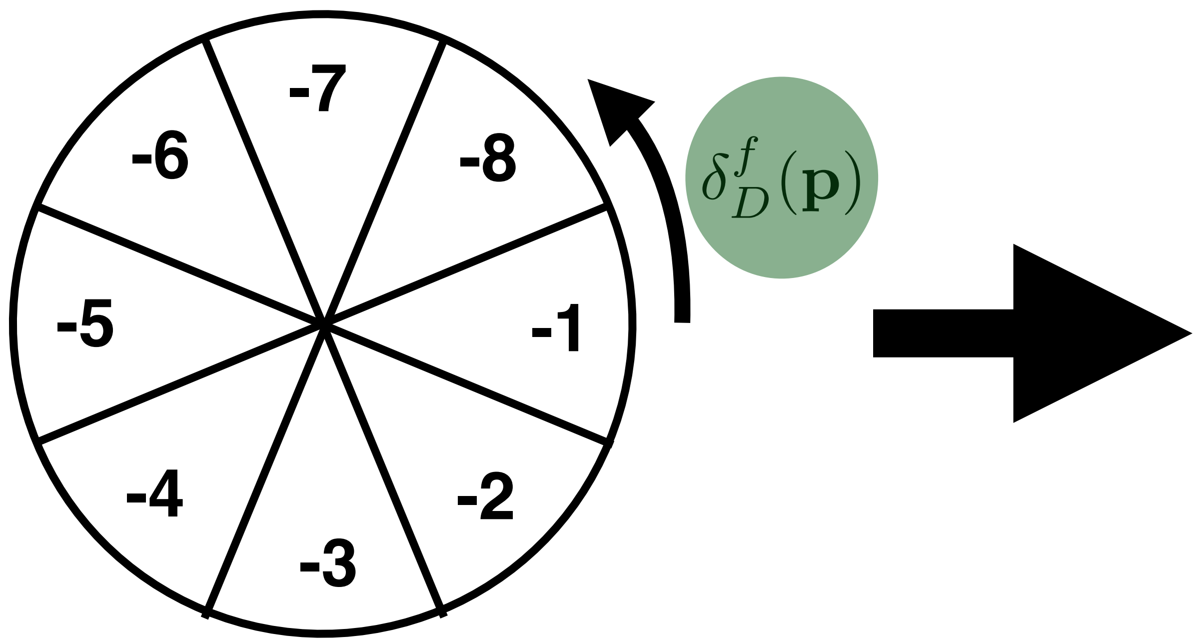
Take from amplitude model

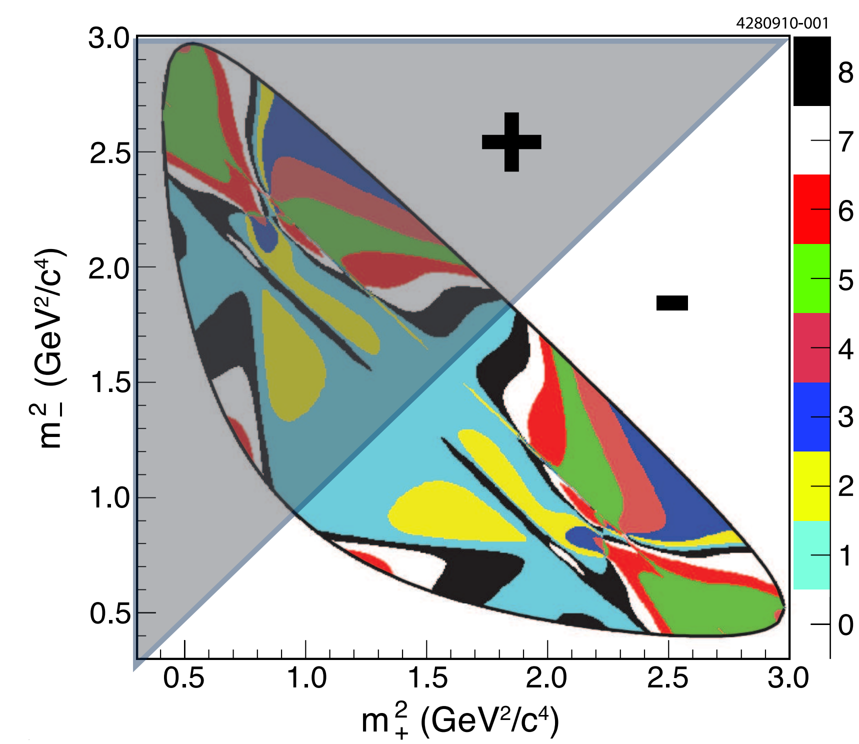


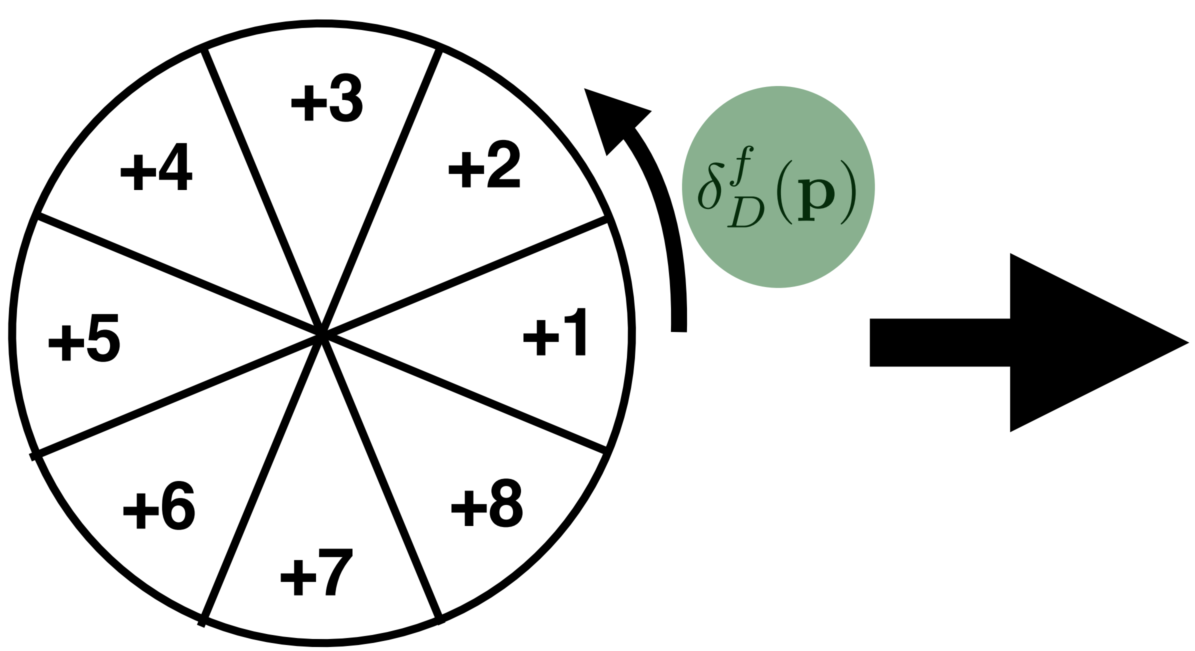
$\delta_D^f(\mathbf{p})$

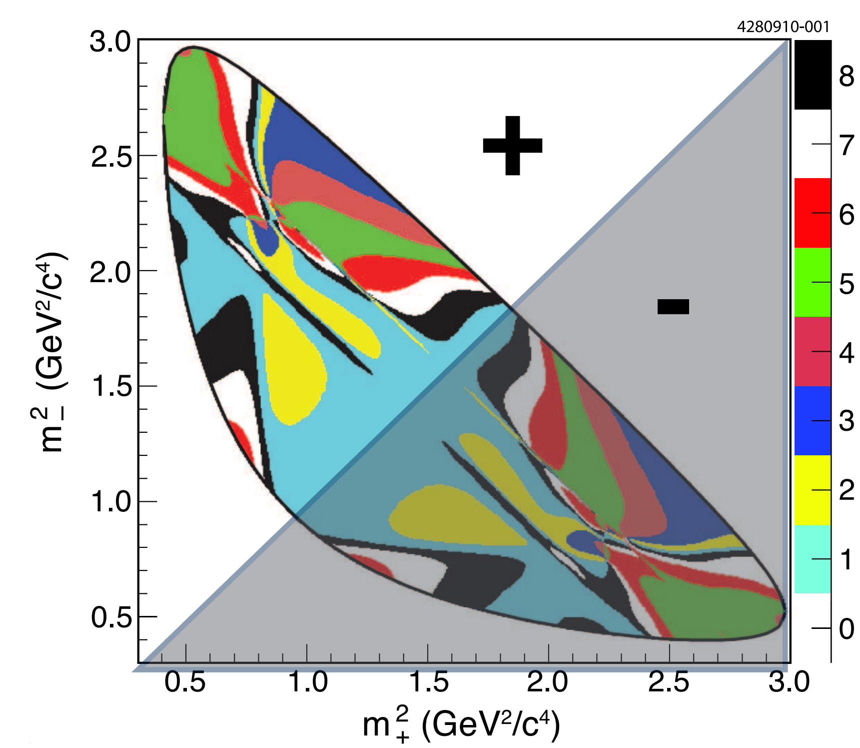


$K_S^0\pi^+\pi^-$


-7
-6
-8
-5
-1
-4
-2
-3
$\delta_D^f(\mathbf{p})$


4280910-001
+
-
m$^2_-$ (GeV^2/c^4)
m$^2_+$ (GeV^2/c^4)
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5
6
7
8


+3
+4
+2
+5
+1
+6
+8
+7
$\delta_D^f(\mathbf{p})$


4280910-001
+
-
m$^2_-$ (GeV^2/c^4)
m$^2_+$ (GeV^2/c^4)
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5
6
7
8

$\pi^+\pi^-\pi^+\pi^-$