

Neutral charm mixing results from the UTfit collaboration



Marcella Bona

मार्चेल्ला बोना

QMUL 

*heavily inspired by talks by
Marco Ciuchini and Luca Silvestrini*

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Neutral meson mixing

Time evolution of an unstable two-states system of CP-conjugated mesons:

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} M \\ \bar{M} \end{pmatrix} = \left[\begin{pmatrix} m & m_{12} \\ m_{12}^* & m \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix} \right] \begin{pmatrix} M \\ \bar{M} \end{pmatrix}$$

with eigenvectors:

$$|M_{1,2}\rangle = p|M\rangle \pm q|\bar{M}\rangle, \quad \frac{q}{p} = -\sqrt{\frac{m_{12}^* - \frac{i}{2}\Gamma_{12}^*}{m_{12} - \frac{i}{2}\Gamma_{12}}}, \quad |p|^2 + |q|^2 = 1$$

and eigenvalues:

$$m_{1,2} - \frac{i}{2}\Gamma_{1,2} = m - \frac{i}{2}\Gamma \mp \sqrt{\left(m_{12} - \frac{i}{2}\Gamma_{12}\right) \left(m_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

Thus connecting with the now usual formalism with x and y parameters

$$\Gamma(x - iy) = 2\sqrt{\left(m_{12} - \frac{i}{2}\Gamma_{12}\right) \left(m_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

$$\begin{aligned} \Gamma x &= \Delta m = m_2 - m_1 \\ 2\Gamma y &= \Delta\Gamma = \Gamma_2 - \Gamma_1 \end{aligned}$$

Neutral meson mixing: formalism for D mesons

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$$\begin{aligned}\Gamma x &= \Delta m = m_2 - m_1 \\ 2\Gamma y &= \Delta\Gamma = \Gamma_2 - \Gamma_1\end{aligned}$$

Meson mixing is characterized by three parameters

$$|m_{12}|, |\Gamma_{12}|, \Phi_{12} = \arg\left(\frac{\Gamma_{12}}{m_{12}}\right)$$

or

$$x, y, \left|\frac{q}{p}\right| \simeq 1 + \frac{1}{2}a_{\text{SL}}$$

Neutral meson mixing: formalism for D mesons

The two parametrizations are equivalent: one can pass from one to the other (with some effort)

$$x = \frac{\text{sgn}(\cos \Phi_{12})}{\sqrt{2}\Gamma} \sqrt{4|m_{12}|^2 - |\Gamma_{12}|^2 + \sqrt{(4|m_{12}|^2 + |\Gamma_{12}|^2)^2 - 16|m_{12}|^2|\Gamma_{12}|^2 \sin^2 \Phi_{12}}}$$

$$y = \frac{1}{2\sqrt{2}\Gamma} \sqrt{|\Gamma_{12}|^2 - 4|m_{12}|^2 + \sqrt{(4|m_{12}|^2 + |\Gamma_{12}|^2)^2 - 16|m_{12}|^2|\Gamma_{12}|^2 \sin^2 \Phi_{12}}}$$

$$\left| \frac{q}{p} \right| = \sqrt{\frac{1 - \delta}{1 + \delta}}, \quad \delta = \frac{2|m_{12}||\Gamma_{12}| \sin \Phi_{12}}{(\Gamma x)^2 + |\Gamma_{12}|^2}$$

CP conservation in mixing corresponds to

$$|q/p| = 1 \leftrightarrow \Phi_{12} = 0, \pi$$

Notice that the phases of m_{12} (or Γ_{12}) and q/p are phase-convention dependent, thus not observable

Neutral meson mixing: time-dependent rates

Meson mixing produces time-dependent decay rates

$$\Gamma(M(t) \rightarrow f) = \frac{1}{2} e^{-\tau} |A_f|^2 \left[(1 + |\lambda_f|^2) \cosh(y\tau) + (1 - |\lambda_f|^2) \cos(x\tau) + 2 \operatorname{Re}(\lambda_f) \sinh(y\tau) - 2 \operatorname{Im}(\lambda_f) \sin(x\tau) \right]$$

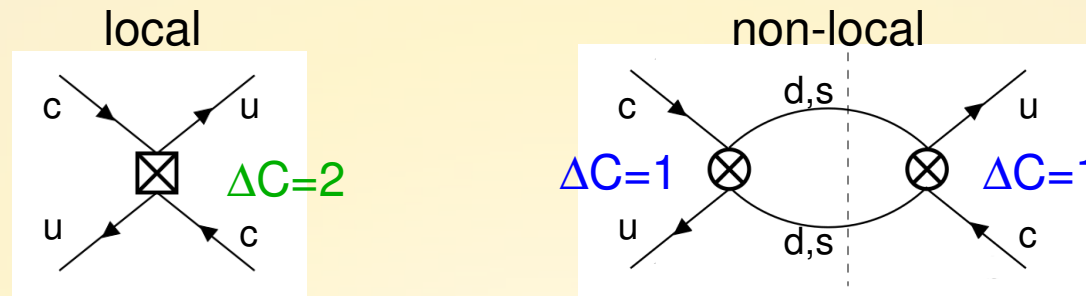
In general, there are 4 rates $M \rightarrow f, \bar{M} \rightarrow f, M \rightarrow \bar{f}, \bar{M} \rightarrow \bar{f}$

Hence we can define two more observable parameters λ characterising the CP violation in interference between mixing and decay

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}, \quad \lambda_{\bar{f}} = \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}$$

$\arg(\lambda_f, \lambda_{\bar{f}}) \neq 0, \pi \leftrightarrow$ CP violation

Computing charm mixing



$$2M_D \left(m_{12} - \frac{i}{2} \Gamma_{12} \right) = \langle D | H_{\text{eff}}^{\Delta C=2} | \bar{D} \rangle + \int d^4x \langle D | T (H_{\text{eff}}^{\Delta C=1}(x) H_{\text{eff}}^{\Delta C=1}(0)) | \bar{D} \rangle$$

Γ_{12} comes from the non-local term, while both contribute to m_{12}

$H_{\text{eff}}^{\Delta C=2}$ contains the contribution of heavy particles (b, NP)

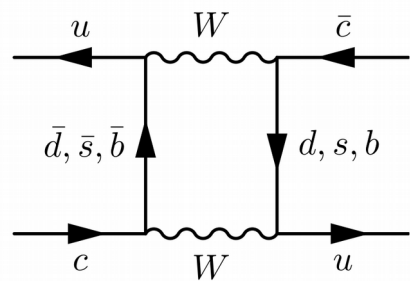
Attempts at computing x and y:

- OPE in Λ_{QCD}/m_c , but $m_c \sim \Lambda_{\text{QCD}}$: relies on local quark-hadron duality and Λ_{QCD}/m_c being small
- sum over hadronic states: too many states, needs assumptions

$$x, y \leq 10^{-2} \div 10^{-3} \ll 1$$

A. Petrov, hep-ph/0611361

Flavour structure of the amplitude



$$= \lambda_q \lambda_{q'} D_{qq'}$$

$$\lambda_q = V_{cq}^* V_{uq}$$

$$\lambda_s \sim \sin \theta_c$$

$$\lambda_b \sim (\sin \theta_c)^5$$

$$\lambda_d = -\lambda_s - \lambda_b$$

$$A^{\Delta C=2} = \sum_{q, q'=d, s, b} \lambda_q \lambda_{q'} D_{qq'} = \lambda_s^2 (D_{ss} + D_{dd} - 2D_{sd}) + 2\lambda_s \lambda_b (D_{bs} + D_{dd} - D_{bd} - D_{sd}) + \mathcal{O}(\lambda_b^2)$$

GIM and SU(3) suppression coincide: the leading term is quadratic in the SU(3) breaking ε , while the subleading one is linear

$$A^{\Delta C=2} \sim \lambda_s^2 \varepsilon^2 + 2\lambda_s \lambda_b \varepsilon + \mathcal{O}(\lambda_b^2)$$

A. Falk et al.,
hep-ph/0110317

The leading term has no weak phase: m_{12} and Γ_{12} are real up to corrections suppressed by $\lambda_b/\lambda_s \times 1/\varepsilon \sim 10^{-3}$.

SM CP violation in mixing is negligible
(yet enhanced by SU(3) breaking w.r.t. the leading term)

Measuring the mixing parameters

The smallness of x , y , $\delta=(1-|q/p|^2)/(1+|q/p|^2)$, λ_b , $\arg(m_{12})$ and $\arg(\Gamma_{12})$ makes charm mixing and CP violation rather peculiar:

- the relations between two sets of parameters become:

$$|m_{12}| = 2\Gamma x + \mathcal{O}(\delta^2), \quad |\Gamma_{12}| = \Gamma y + \mathcal{O}(\delta^2), \quad \sin \Phi_{12} = \frac{x^2 + y^2}{xy} \delta + \mathcal{O}(\delta^2)$$

- the decay rate time dependence can be expanded to the first few orders: for ex., time-dependent decays to CP eigenstates become:

$$\Gamma(D(t)/\bar{D}(t) \rightarrow f) = e^{-\tau} |A_f|^2 \left[1 + \eta_{CP} \left| \frac{q}{p} \right|^{\pm 1} (y \cos \phi \mp x \sin \phi) \tau \right] = |A_f|^2 e^{-\Gamma_{D/\bar{D} \rightarrow f} t} + \mathcal{O}(x^2, y^2)$$

i.e. an exponential decay in terms of the effective rates

$$\Gamma_{D/\bar{D} \rightarrow f} = \Gamma \left[1 + \eta_{CP} \left| \frac{q}{p} \right|^{\pm 1} (y \cos \phi \mp x \sin \phi) \right]$$

which in turn are used to define the observables (e.g. $f=K^+K^-$, $\pi^+\pi^-$)

$$y_{CP} = \eta_{CP} \frac{\Gamma_{\bar{D} \rightarrow f} + \Gamma_{D \rightarrow f}}{2\Gamma} - 1, \quad A_\Gamma = \frac{\Gamma_{D \rightarrow f} - \Gamma_{\bar{D} \rightarrow f}}{2\Gamma}$$

Measuring the mixing parameters

We are in the approximation and phase convention for which

$$\frac{\bar{A}_f}{A_f} = 1, \quad \lambda_f \rightarrow \frac{q}{p}$$

so that CP violation in the interference of mixing and decay is controlled by

$$\phi = \arg\left(\frac{q}{p}\right)$$

In the same approximation, assuming that Γ_{12} is dominated by Cabibbo-allowed decays, $\arg(\Gamma_{12}) = 0$, hence the relation

$$\arg\left(\Gamma_{12} \frac{q}{p}\right) = \arg(y + i\delta x) \rightarrow \phi = \arg(y + i\delta x)$$

Ciuchini et al, hep-ph/0703204
Kagan&Sokoloff, 0907.3917

CP violation in the interference between mixing and decay is directly related to CP violation in mixing. In particular, one finds that

$$\arg(\phi) = 0 \quad \Leftrightarrow \quad \arg(\Phi_{12}) = 0 \quad \Leftrightarrow \quad \left|\frac{q}{p}\right| = 1$$

Fit to D mixing data

- Given present experimental errors, it is perfectly adequate to assume that SM contributions to both M_{12} and Γ_{12} are real
- all decay amplitudes relevant for the mixing analysis can also be taken real
- NP could generate a non-vanishing phase for M_{12}
- Fit all data with universal parameters:
 - x, y, and $|q/p|$
 - fit assuming **no direct CP violation**
 - inputs mostly from HFAG 2015
 - symmetrised errors
 - Bayesian fit
 - floated parameters:
 - x, y, $|q/p|$, $R_{\kappa\pi}$, $\delta_{\kappa\pi}$, $\delta_{\kappa\pi\pi}$

Fit to D mixing data

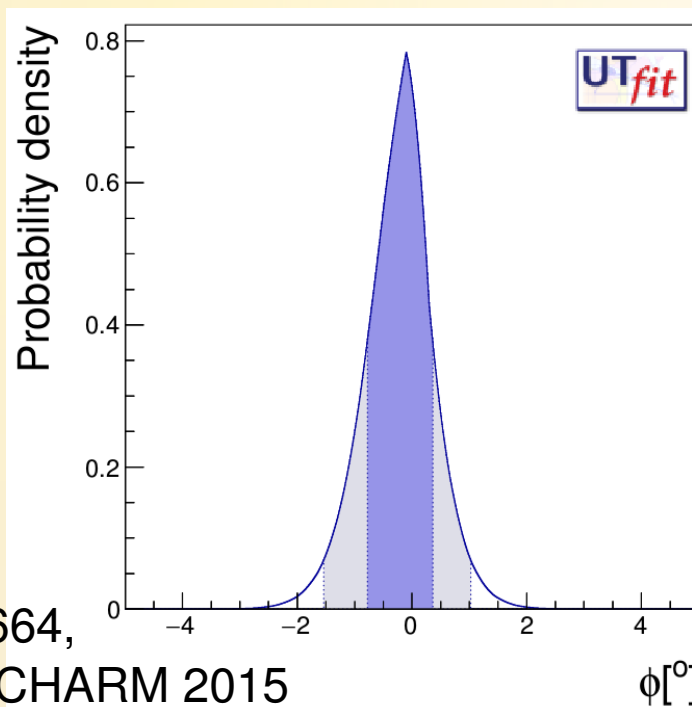
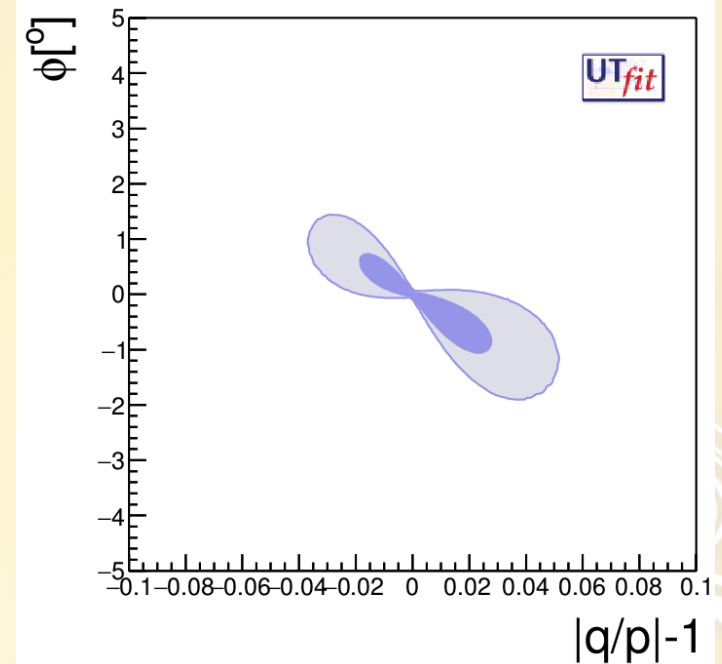
Summer15 UFit average:

$$x = (3.5 \pm 1.5) 10^{-3}$$

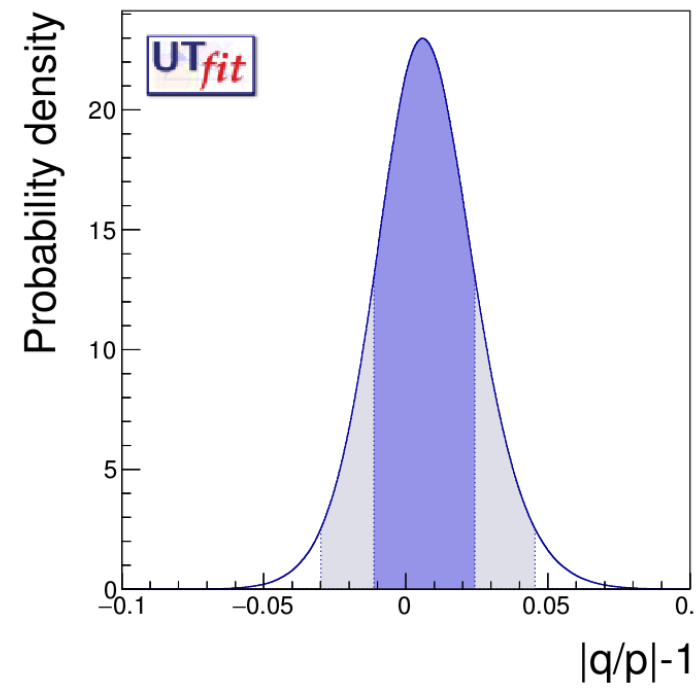
$$y = (5.8 \pm 0.6) 10^{-3}$$

$$|q/p|-1 = (0.7 \pm 1.8) 10^{-2}$$

$$\phi = \arg(q/p) = (-0.21 \pm 0.57)^\circ$$



UTfit, 1402.1664,
L. Silvestrini, CHARM 2015



Fit to D mixing data

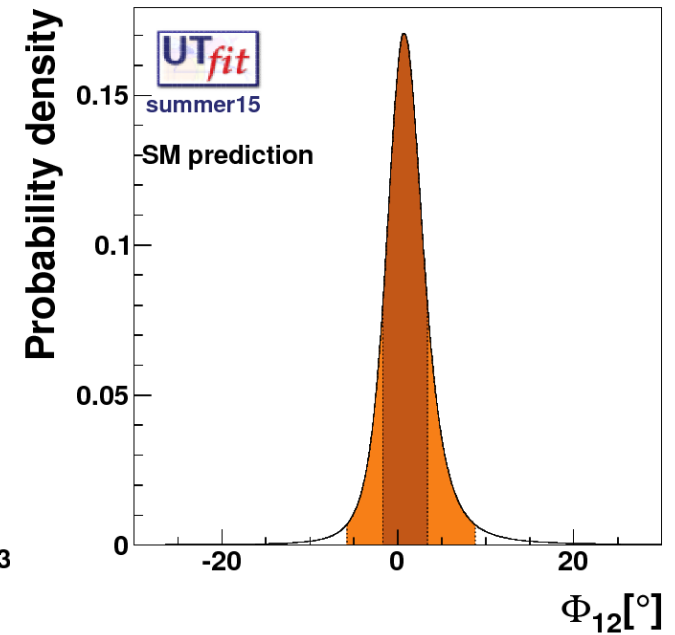
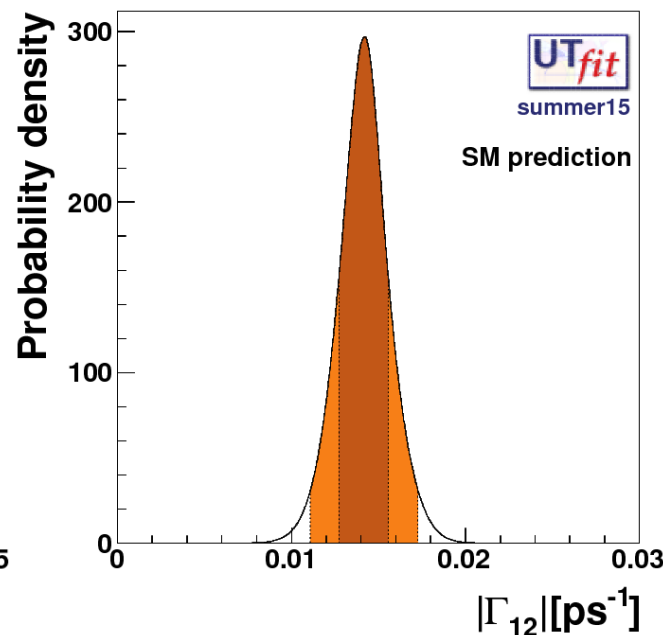
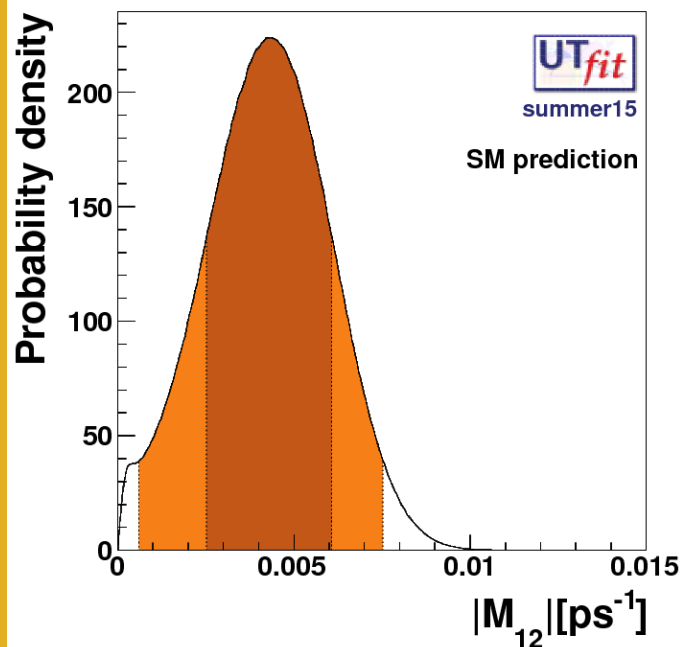
The corresponding results on fundamental parameters are

$$|M_{12}| = (4.3 \pm 1.8)/\text{fs},$$

$$|\Gamma_{12}| = (14.1 \pm 1.4)/\text{fs}$$

$$\Phi_{12} = (0.8 \pm 2.6)^\circ$$

- y well determined, x still uncertain
 - $|q/p|$ is well compatible with 1
 - phases are all compatible with 0
- no evidence for CPV, yet..



New Physics in the $\Delta C=2$ amplitude

NP enters through the local ME $\langle D | H_{eff}^{\Delta C=2} | \bar{D} \rangle$

$$H_{eff}^{\Delta C=2} = \sum_{i=1}^5 C_i(\mu) Q_i(\mu) + \sum_{i=1}^3 \tilde{C}_i(\mu) \tilde{Q}_i(\mu) + H.c.$$

$$Q_1 = \bar{u}_L^\alpha \gamma_\mu c_L^\alpha \bar{u}_L^\beta \gamma^\mu c_L^\beta \quad (\text{SM})$$

$$Q_2 = \bar{c}_R^\alpha u_L^\alpha \bar{c}_R^\beta u_L^\beta \quad Q_3 = \bar{c}_R^\alpha u_L^\beta \bar{c}_R^\beta u_L^\alpha$$

$$Q_4 = \bar{c}_R^\alpha u_L^\alpha \bar{c}_L^\beta u_R^\beta \quad Q_5 = \bar{c}_R^\alpha u_L^\beta \bar{c}_L^\beta u_R^\alpha$$

$$\tilde{Q}_1 = \bar{c}_R^\alpha \gamma_\mu u_R^\alpha \bar{c}_R^\beta \gamma^\mu u_R^\beta$$

$$\tilde{Q}_2 = \bar{c}_L^\alpha u_R^\alpha \bar{c}_L^\beta u_R^\beta \quad \tilde{Q}_3 = \bar{c}_L^\alpha u_R^\beta \bar{c}_L^\beta u_R^\alpha$$

7 new operators beyond SM involving
quarks with different chiralities

New Physics in the $\Delta C=2$ amplitude and scale analysis

H_{eff} can be recast in terms of the high-scale $C_i(\Lambda)$
 $C_i(\Lambda)$ can be extracted from the data (one by one)

the NP scale Λ is defined as:

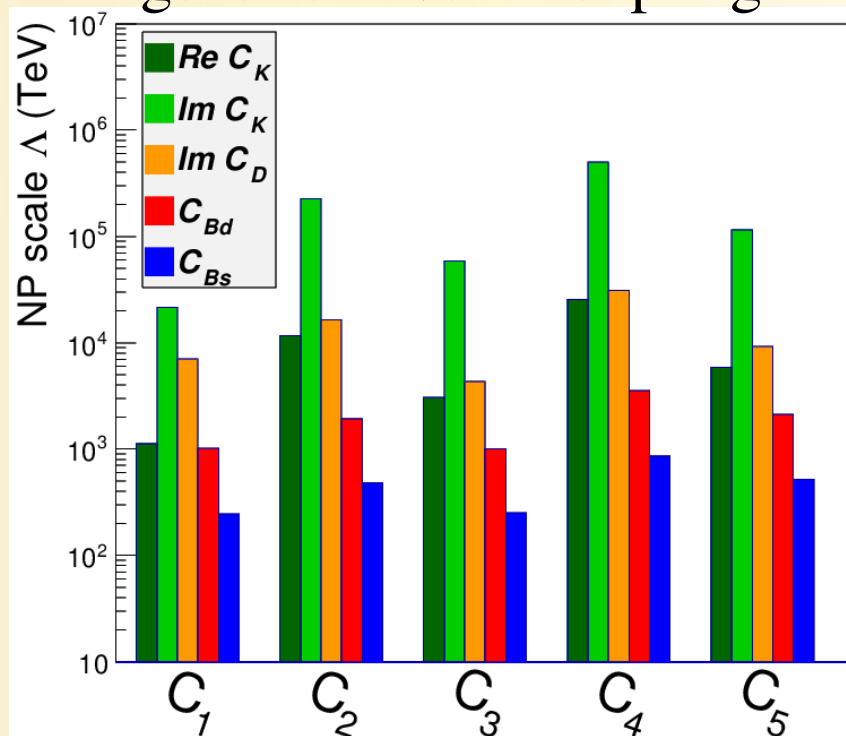
$$\Lambda = \sqrt{\frac{L \times F_i}{C_i(\Lambda)}}$$

NP loop (L) and flavour (F_i) couplings
are model dependent: shown NP
scales assume $L=1$ and $|F_i|=1$ with
arbitrary phases

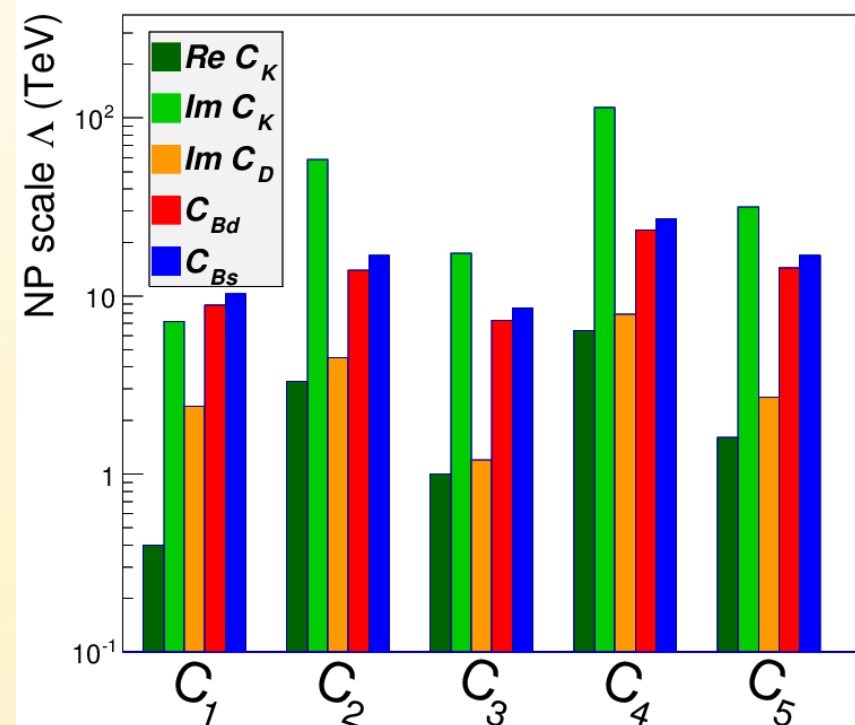
New Physics in the $\Delta C=2$ amplitude and scale analysis

- CP violation in $\Delta F=2$ processes is the most sensitive probe of new physics, reaching scales of $O(10^5)$ TeV
- CPV in D-D mixing shows its NP sensitivity by putting a strong lower bound (best after ε_K) on the scale of NP with generic flavour structure and $O(1)$ couplings

generic flavour coupling



NMFV



Conclusions

- Need to update to the latest results: plan to do it asap
- A technique for computing long-distance contributions is eagerly awaited.
- Yet the flavour structure of mixing and decay amplitudes is such that (i) SM CPV in mixing is very small, (ii) CPC measurements, CPV in mixing, and CPV in the interference between mixing and decay can all be combined to constrain the mixing parameters
- No sign of deviation from the SM so far (i.e. no CPV - in mixing found).
- Yet CPV in D-D mixing shows its NP sensitivity by putting a strong lower bound on the scale of NP with generic flavour structure and $O(1)$ couplings

back-up



Observable	Value	Correlation Coeff.				
y_{CP}	$(0.835 \pm 0.155)\%$					
A_{Γ}	$(-0.059 \pm 0.040)\%$					
x	$(0.53 \pm 0.19 \pm 0.06 \pm 0.07)\%$	1	0.054	-0.074	-0.031	
y	$(0.28 \pm 0.15 \pm 0.05 \pm 0.05)\%$	0.054	1	0.034	-0.019	
$ q/p $	$(0.91 \pm 0.16 \pm 0.5 \pm 0.6)$	-0.074	0.034	1	0.044	
ϕ	$(-6 \pm 11 \pm 3 \pm 4)^{\circ}$	-0.031	-0.019	0.044	1	
x	$(0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$	1	0.0615			
y	$(0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$	0.0615	1			
R_M	$(0.0130 \pm 0.0269)\%$					
$(x'_+)_{K\pi\pi}$	$(2.48 \pm 0.59 \pm 0.39)\%$	1	-0.69			
$(y'_+)_{K\pi\pi}$	$(-0.07 \pm 0.65 \pm 0.50)\%$	-0.69	1			
$(x'_-)_{K\pi\pi}$	$(3.50 \pm 0.78 \pm 0.65)\%$	1	-0.66			
$(y'_-)_{K\pi\pi}$	$(-0.82 \pm 0.68 \pm 0.41)\%$	-0.66	1			
R_D	$(0.533 \pm 0.107 \pm 0.045)\%$	1	0	0	-0.42	0.01
x^2	$(0.06 \pm 0.23 \pm 0.11)\%$	0	1	-0.73	0.39	0.02
y	$(4.2 \pm 2 \pm 1)\%$	0	-0.73	1	-0.53	-0.03
$\cos \delta_{K\pi}$	$(0.84 \pm 0.2 \pm 0.06)$	-0.42	0.39	-0.53	1	0.04
$\sin \delta_{K\pi}$	$(-0.01 \pm 0.41 \pm 0.04)$	0.01	0.02	-0.03	0.04	1
R_D	$(0.3030 \pm 0.0189)\%$	1	0.77	-0.87		
$(x'_+)_{K\pi}^2$	$(-0.024 \pm 0.052)\%$	0.77	1	-0.94		
$(y'_+)_{K\pi}$	$(0.98 \pm 0.78)\%$	-0.87	-0.94	1		
A_D	$(-2.1 \pm 5.4)\%$	1	0.77	-0.87		
$(x'_-)_{K\pi}^2$	$(-0.020 \pm 0.050)\%$	0.77	1	-0.94		
$(y'_-)_{K\pi}$	$(0.96 \pm 0.75)\%$	-0.87	-0.94	1		
R_D	$(0.364 \pm 0.018)\%$	1	0.655	-0.834		
$(x'_+)_{K\pi}^2$	$(0.032 \pm 0.037)\%$	0.655	1	-0.909		
$(y'_+)_{K\pi}$	$(-0.12 \pm 0.58)\%$	-0.834	-0.909	1		
A_D	$(2.3 \pm 4.7)\%$	1	0.655	-0.834		
$(x'_-)_{K\pi}^2$	$(0.006 \pm 0.034)\%$	0.655	1	-0.909		
$(y'_-)_{K\pi}$	$(0.20 \pm 0.54)\%$	-0.834	-0.909	1		
R_D	$(0.351 \pm 0.035)\%$	1	-0.967	0.900		
$(y'_{CPA})_{K\pi}$	$(0.43 \pm 0.43)\%$	-0.967	1	-0.975		
$(x'_{CPA})_{K\pi}^2$	$(0.008 \pm 0.018)\%$	0.900	-0.975	1		
R_D	$(0.3568 \pm 0.0058 \pm 0.0033)\%$	1	-0.894	0.77	-0.895	0.772
$(y'_+)_{K\pi}$	$(0.48 \pm 0.09 \pm 0.06)\%$	-0.894	1	-0.949	0.765	-0.662
$(x'_+)_{K\pi}^2$	$(6.4 \pm 4.7 \pm 3)10^{-5}$	0.77	-0.949	1	-0.662	0.574
$(y'_-)_{K\pi}$	$(0.48 \pm 0.09 \pm 0.06)\%$	-0.895	0.765	-0.662	1	-0.95
$(x'_-)_{K\pi}^2$	$(4.6 \pm 4.6 \pm 3)10^{-5}$	0.772	-0.662	0.574	-0.95	1

Another example: time-dependent decays to “wrong-sign” non-CP eigenstate final states f . In general, 4 amplitudes:

$$A_f = T_f e^{i\phi_f} [1 + r_f e^{i(\Delta_f + \Phi_f)}], \quad \bar{A}_{\bar{f}} = T_{\bar{f}} e^{-i\phi_{\bar{f}}} [1 + r_{\bar{f}} e^{i(\Delta_{\bar{f}} - \Phi_{\bar{f}})}],$$

$$A_{\bar{f}} = T_{\bar{f}} e^{i(\delta_{\bar{f}} + \phi_{\bar{f}})} [1 + r_{\bar{f}} e^{i(\Delta_{\bar{f}} + \Phi_{\bar{f}})}], \quad \bar{A}_f = T_f e^{i(\delta_f - \phi_f)} [1 + r_f e^{i(\Delta_f - \Phi_f)}]$$

Consider CF A_f and DCS $A_{\bar{f}}$ (e.g. $K\bar{p}^+$): neglecting subleading terms ($r_f, r_{\bar{f}} = 0$), the weak phase difference of leading amplitudes ($\phi_{\bar{f}} = \phi_f$) and expanding for small $x, y, R_f = |T_{\bar{f}}/T_f|, \sin q_c$, the time-dependent decay rates become:

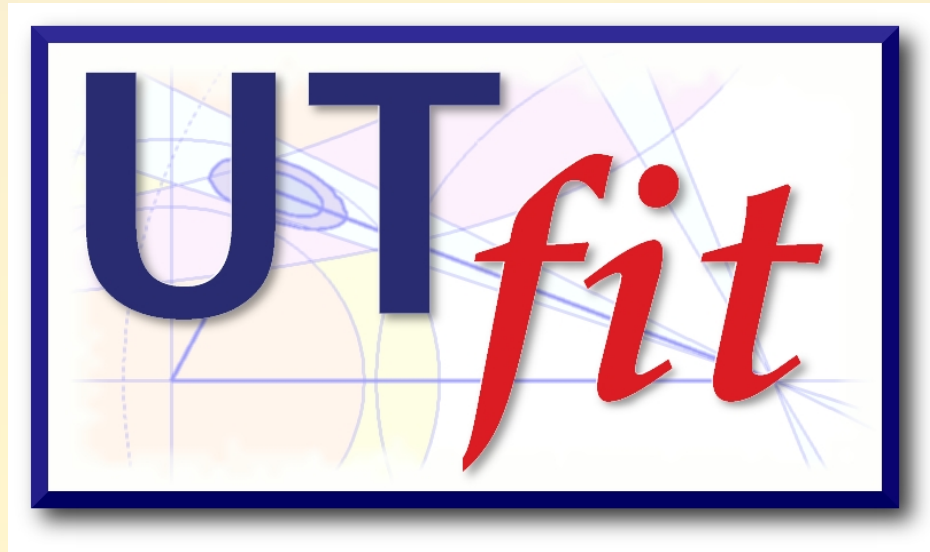
$$\Gamma(D(t) \rightarrow f) = e^{-\tau} |A_f|^2, \quad \Gamma(D(t) \rightarrow \bar{f}) = e^{-\tau} |A_{\bar{f}}|^2 \left(R_f^2 + R_f (y'_+) \tau + \frac{(x'_+)^2 + (y'_+)^2}{4} \tau^2 \right),$$

$$\Gamma(\bar{D}(t) \rightarrow \bar{f}) = e^{-\tau} |\bar{A}_{\bar{f}}|^2, \quad \Gamma(\bar{D}(t) \rightarrow f) = e^{-\tau} |\bar{A}_f|^2 \left(R_f^2 + R_f (y'_-) \tau + \frac{(x'_-)^2 + (y'_-)^2}{4} \tau^2 \right)$$

where $(x'_{\pm})_f = \left| \frac{q}{p} \right|^{\pm 1} (x'_f \cos \phi \pm y'_f \sin \phi), \quad (y'_{\pm})_f = \left| \frac{q}{p} \right|^{\pm 1} (y'_f \cos \phi \mp x'_f \sin \phi)$ and

$$\begin{pmatrix} x'_f \\ y'_f \end{pmatrix} = \begin{pmatrix} \cos \delta_f & \sin \delta_f \\ -\sin \delta_f & \cos \delta_f \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Constrain the mixing parameters, but need two additional inputs: R_f, δ_f



www.utfit.org

C. Alpigiani, A. Bevan, M.B., M. Ciuchini,
D. Derkach, E. Franco, V. Lubicz, G. Martinelli,
F. Parodi, M. Pierini, C. Schiavi, L. Silvestrini,
A. Stocchi, V. Sordini, C. Tarantino and V. Vagnoni

Other UT analyses exist, by:

CKMfitter (<http://ckmfitter.in2p3.fr/>),

Laiho&Lunghi&Van de Water (<http://latticeaverages.org/>)

Lunghi&Soni (1010.6069)

D mixing

- D mixing is described by:
 - Dispersive $D \rightarrow \bar{D}$ amplitude M_{12}
 - SM: long-distance dominated, not calculable
 - NP: short distance, calculable with lattice
 - Absorptive $D \rightarrow \bar{D}$ amplitude Γ_{12}
 - SM: long-distance, not calculable
 - NP: negligible
 - Observables: $|M_{12}|$, $|\Gamma_{12}|$, $\Phi_{12} = \arg(\Gamma_{12}/M_{12})$



GIM \leftrightarrow SU(3) (U-spin)

- Use CKM unitarity
 - $V_{cd} V_{ud}^* + V_{cs} V_{us}^* + V_{cb} V_{ub}^* = \lambda_d + \lambda_s + \lambda_b = 0$
 - eliminate λ_d and take λ_s real (all physical results are convention independent)
 - imaginary parts suppressed by $r = \text{Im } \lambda_b / \lambda_s = 6.5 \cdot 10^{-4}$
 - M_{12} and Γ_{12} have the following structure:
 - $\lambda_s^2 (f_{dd} + f_{ss} - 2f_{ds}) + 2\lambda_s \lambda_b (f_{dd} - f_{ds} - f_{db} + f_{sb}) + O(\lambda_b^2)$

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- Use CKM unitarity
 - $V_{cd} V_{ud}^* + V_{cs} V_{us}^* + V_{cb} V_{ub}^* = \lambda_d + \lambda_s + \lambda_b = 0$
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 - M_{12} and Γ_{12} have the following structure:
 - $\lambda_s^2 (f_{dd} + f_{ss} - 2f_{ds}) + 2\lambda_s \lambda_b (f_{dd} - f_{ds} - f_{db} + f_{sb}) + O(\lambda_b^2)$
- Write long-distance contributions to M_{12} and Γ_{12} in terms of U-spin quantum numbers:
 - $\lambda_s^2 (\Delta U=2) + \lambda_s \lambda_b (\Delta U=2 + \Delta U=1) + O(\lambda_b^2) \sim \lambda_s^2 \varepsilon^2 + \lambda_s \lambda_b \varepsilon$
- CPV effects at the level of $r/\varepsilon \sim 2 \cdot 10^{-3} \sim 1/8^\circ$
for “nominal” SU(3) breaking $\varepsilon \sim 30\%$