Lattice developments for $\Delta M_{d,s}$

Elvira Gámiz (on behalf of Fermilab Lattice-MILC)

(with C. Bouchard and E. Freeland)

Universidad de Granada / CAFPE

· **9th International Workshop on the CKM Unitarity Triangle**,
TIFR, Mumbai, Nov 28-Dec 2 2016 ·

Neutral B mixing

* Particularly interesting process for indirect NP searches and constraining BSM theories.
  * ** Tension between $\Delta M_{s,d}$ and ε_K. See **M. Blanke** talk
  * ** Main physical observables $\Delta M_{s,d}$ measured at the subpercent level.

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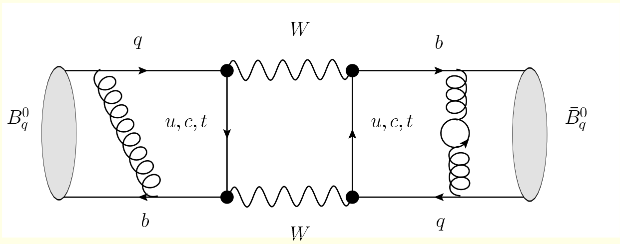
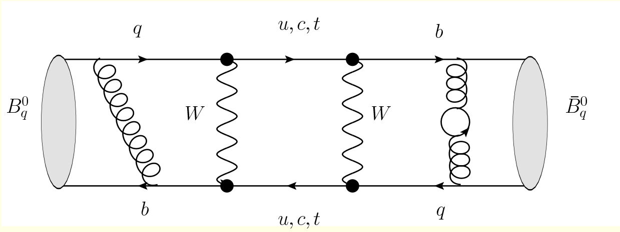
* Using experimental measurements of $\Delta M_{s,d}$ and theoretical determinations of the relevant hadronic matrix elements
  $\rightarrow$ extract the CKM matrix elements $|V_{ts}|$, $|V_{td}|$.

Neutral B mixing

In the Standard Model, the neutral $B - \bar{B}$ mixing occurs at leading order in the EW interactions via the box diagrams





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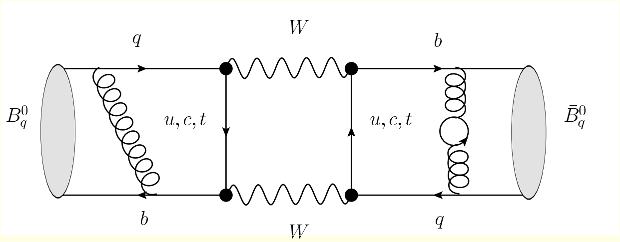

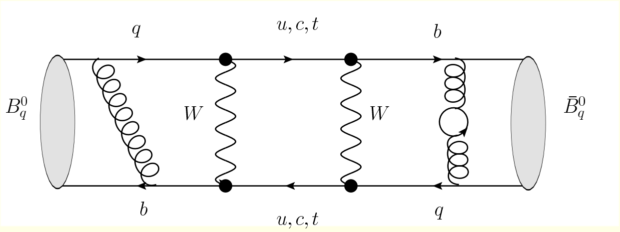

In extensions of the SM, other particles can appear

* In the boxes

* At tree level (flavour changing neutral currents)

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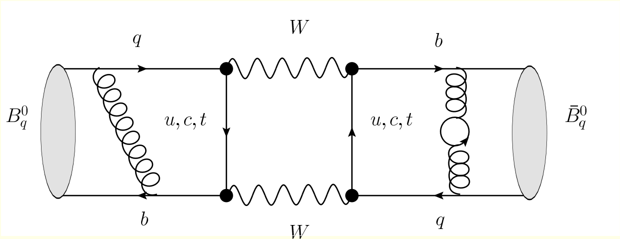
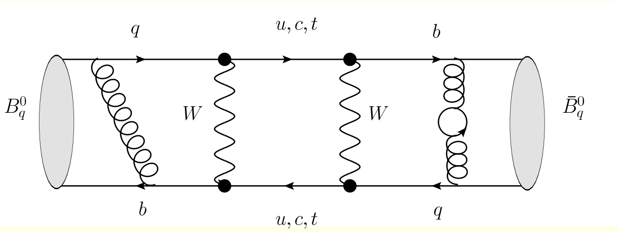
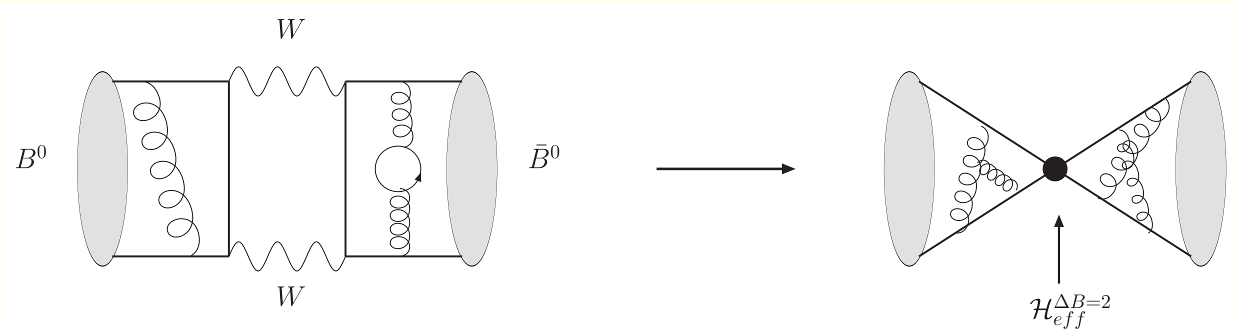




In extensions of the SM, other particles can appear

* In the boxes
* At tree level (flavour changing neutral currents)

Through a combination of GIM mechanism and Cabibbo suppression, the top dominates quark loop contributions



Neutral B mixing

And the mixing is described to a good approximation by the effective hamiltonian

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i \mathcal{O}_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{\mathcal{O}}_i \qquad \text{with}$$

$$\mathcal{O}_1^q = \left(\bar{b}^i \gamma^\nu (1-\gamma_5) q^i\right)\left(\bar{b}^j \gamma^\nu (1-\gamma_5) q^j\right) \qquad \textbf{SM}$$

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$$\tilde{\mathcal{O}}_{1,2,3}^q = \mathcal{O}_{1,2,3}^q \text{ with the replacement } (1 \pm \gamma_5) \rightarrow (1 \mp \gamma_5)$$

(for BSM theories with new heavy particles scale $\geq$ TeV, the local effective four-quark operator remains a convenient description)

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In **this talk**:

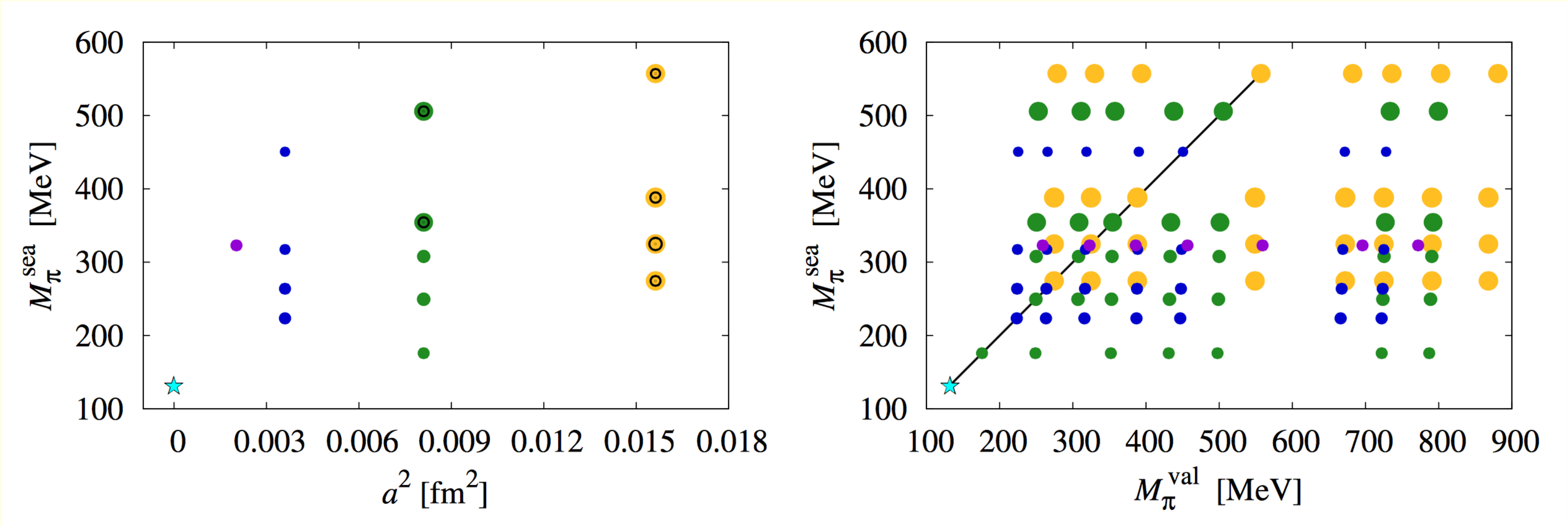
Calculation of the five hadronic matrix elements (and combinations of them) using three-flavour lattice QCD **FNAL-MILC** 1602.03560 (SM prediction of $\Delta M_{d,s}$ and ξ)

1.1 Simulation details

MILC $N_f = 2+1$ asqtad ensembles

* 600-2000 gauge fields per ensemble

* pions as light as 177 MeV



1.2 Matching and renormalization

* Mostly non-perturbative renormalization (mNPR).

$$\mathcal{O}_i = Z_{V^4_{bb}} Z_{V^4_{dd}} \rho_{ij} O_j + \mathrm{O}(\alpha_s a, a^2)$$

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* Two-loop corrections are incorporated in the chiral+continuum fit.

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($O_{1,2,3}$ mix under renormalization, as well as $O_{4,5}$)

1.3 Chiral-Continuum extrapolation

Extrapolate the lattice data to the continuum and infinite volume limits, and physical light quark masses in the Heavy Meson (HM)ChPT framework:

* Including dominant light quark discretization effects (NLO Staggered HMChPT) and NNLO ChPT analytic terms

* Gluon and light-quark discretization effects a la Symanzik

* Heavy-quark discretization effects (derived in HQET)

* Fine tuning m_b.

* Include higher order renormalization effects, $\mathcal{O}(\alpha_s^2)$ in the fit.

$$F_i = F_i^{\rm logs} + F_i^{\rm analytic} + F_i^{\alpha_s a^2} + F_i^{\rm HQ\, disc.} + F_i^{m_b\, \rm tune} + F_i^{\rm renor.}$$

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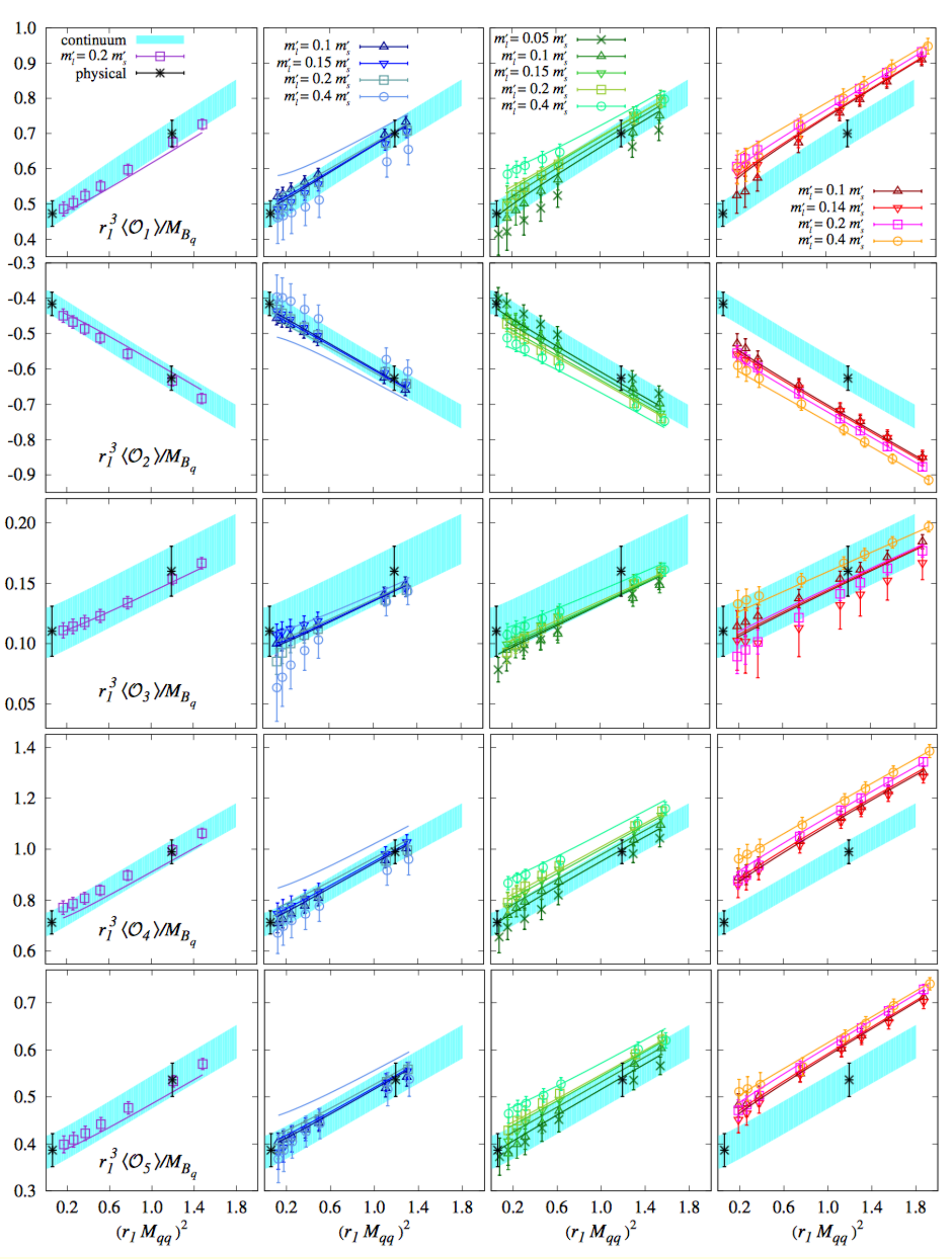
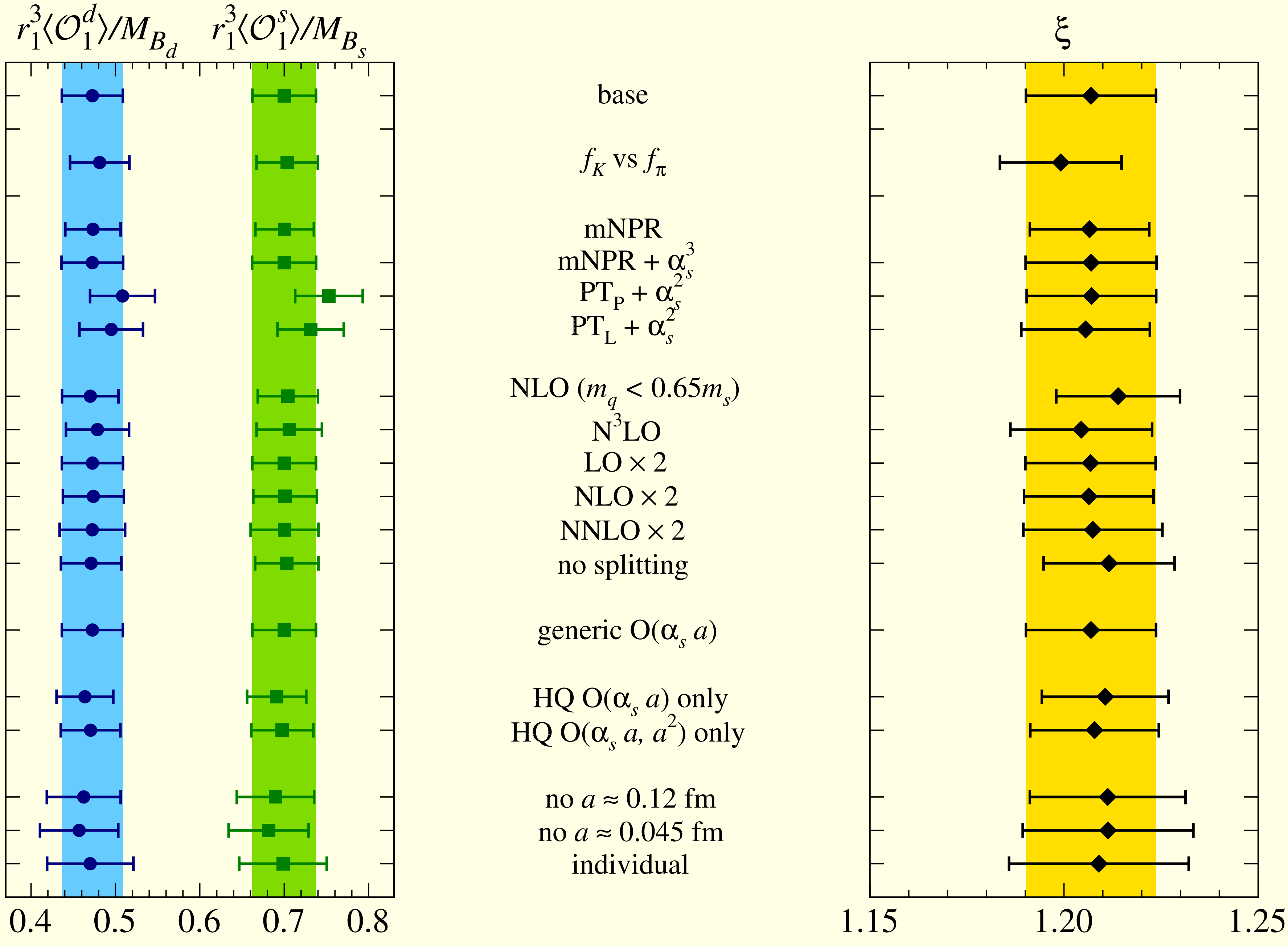


* $O_{1,2,3}$ and $O_{4,5}$ also mix within ChPT.

* All operators are correlated via common gauge fields and valence quarks.

* Perform a simultaneous (Bayesian) fit to all five operators.

1.4 Stability under fit variations



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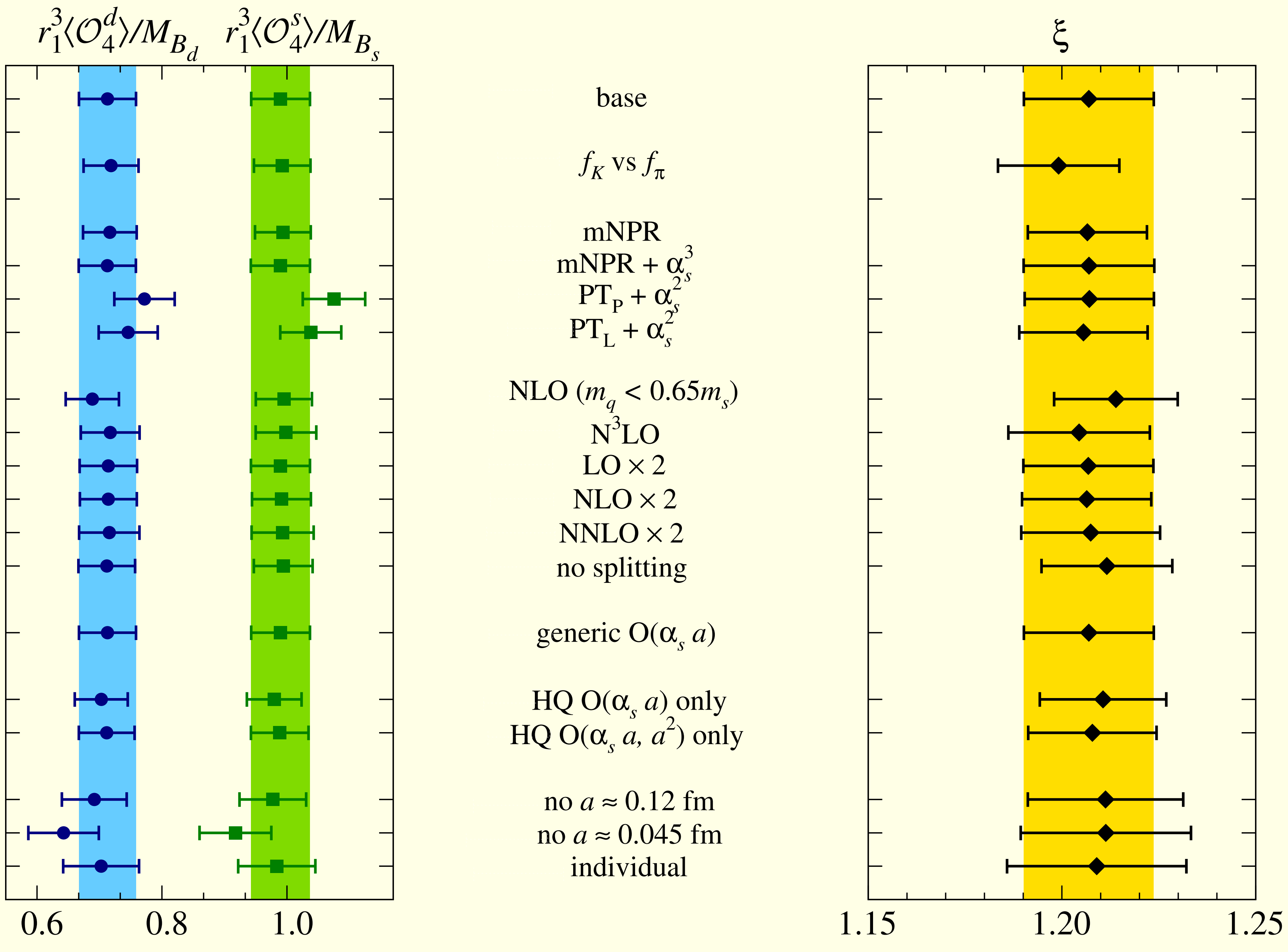
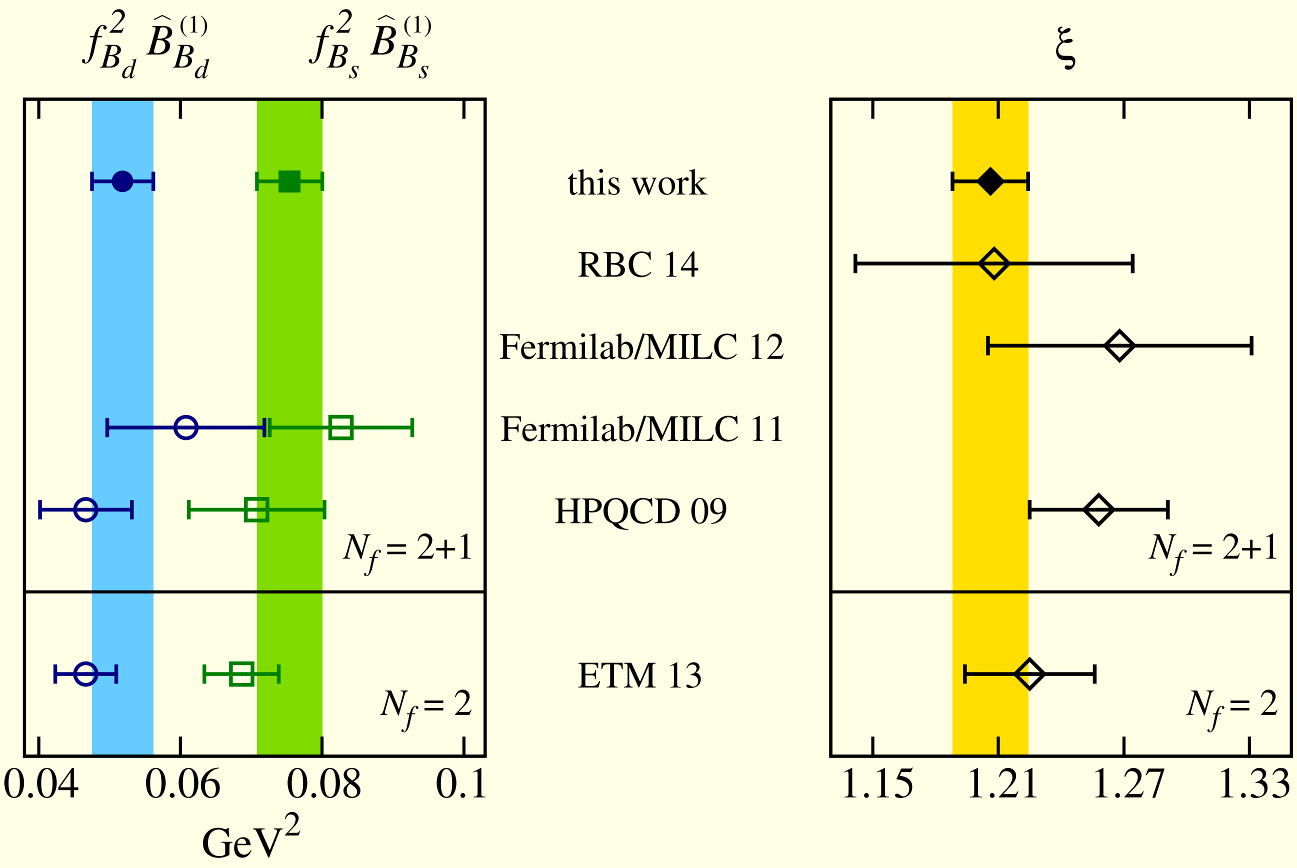


2.1. Matrix elements relevant for SM $\Delta M_{s,d}$

In the SM, $\Delta M_q \propto \left|V_{tq}^* V_{tb}\right|^2 f_{B_q}^2 \hat{B}_{B_q}^{(1)}$, where $\frac{8}{3} f_{B_q}^2 B_{B_q}^{(1)}(\mu) M_{B_q}^2 = \langle \mathcal{O}_1^q \rangle(\mu)$



This work: 1602.03560, **RBC 14:** 1406.6192, **Fermilab/MILC 12:** 1205.7013, **Fermilab/MILC 11:** 1112.5642 (proceedings), **HPQCD 09:** 0902.1815, **ETM 13:** 1308.1851

In the $SU(3)$-breaking ratio $\xi = \sqrt{\frac{f_{B_s}^2 \hat{B}_{B_s}^{(1)}}{f_{B_d}^2 \hat{B}_{B_d}^{(1)}}}$, statistical and systematic uncertainties largely cancel (1.5% error dominated by statistics and HQ disc.)

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In the SM, using tree-level inputs for the CKM matrix elements CKMfitter and Fermilab-MILC 1602.03560 results

$$f_{B_d}\sqrt{\hat{B}^{(1)}_{B_d}} = 227.7(9.5)(2.3)\ \mathrm{MeV}\,,\ f_{B_s}\sqrt{\hat{B}^{(1)}_{B_s}} = 274.6(8.4)(2.7)\ \mathrm{MeV} \quad ,$$

$$\xi = 1.206(18)(6)$$

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$$\xi = 1.206(18)(6)$$

we get

$$\Delta M_d^{SM} = 0.630(53)(42)(5)(13)\, ps^{-1} \qquad \Delta M_d^{expt,HFAG} = 0.5064(19)\, ps^{-1}$$

$$\Delta M_s^{SM} = 19.6(1.2)(1.0)(0.2)(0.4)\, ps^{-1} \qquad \Delta M_s^{expt,HFAG} = 17.757(21)\, ps^{-1}$$

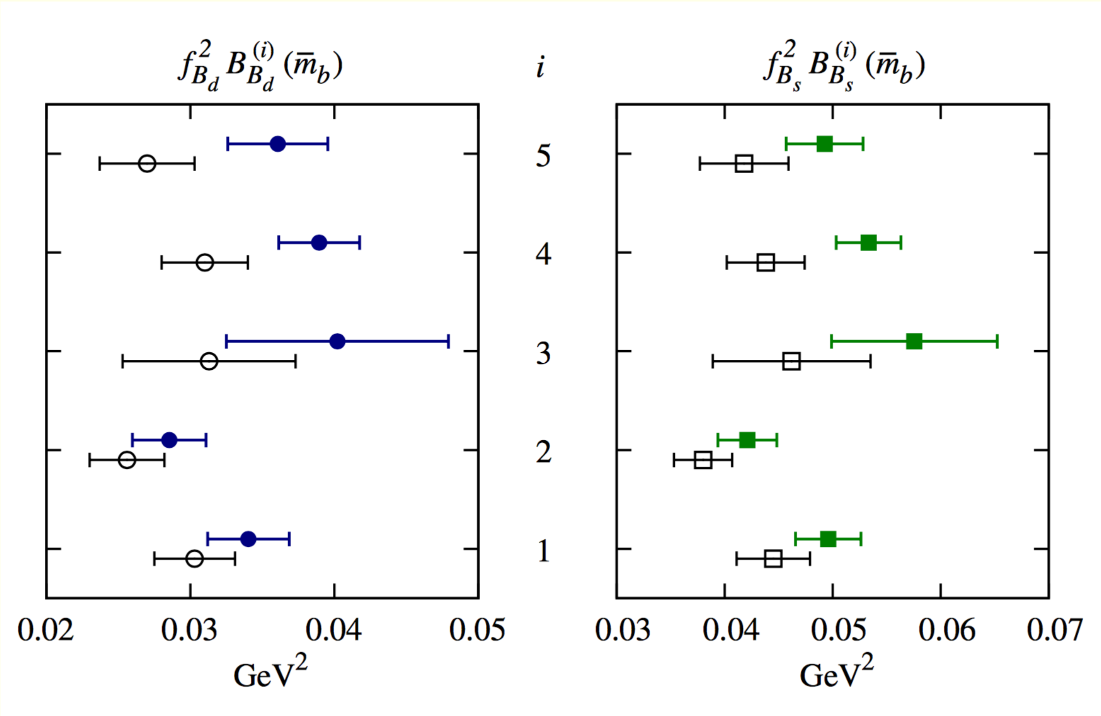
$$(\Delta M_d/\Delta M_s)^{SM} = 0.0321(10)(15)(0)(3)\, ps^{-1}$$

(where the errors are from lattice, CKM matrix elements, other inputs in SM expression, omission of charm quark on the sea, respectively)

* These amount to tensions of 2.1σ, 1.3σ and 2.9σ, respectively.

2.2 Matrix elements relevant for BSM physics

Comparison with $N_f = 2$ ETM collaboration 1308.1851 results



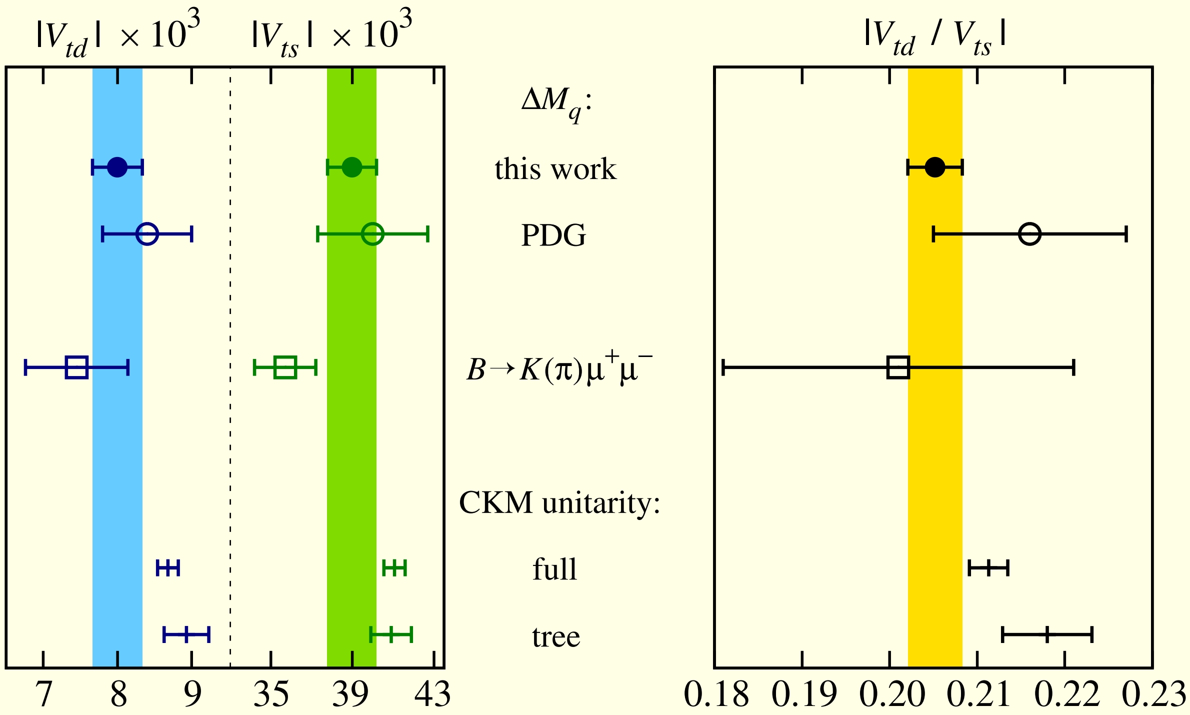
Open symbols: ETM

Full symbols: our results

* Errors range from $\sim 5 - 15\%$, larger for B_d matrix elements.

2.3 Extraction of CKM matrix elements

Alternatively, use ΔM_q^{expt} **HFAG 2014** and determine CKM factors



* $B \to K(\pi)\mu^+\mu^-$ results from **D. Du et al**, 1510.02349

* Full/tree CKM unitarity results come from CKMfitter's fit using all inputs/only observable mediated at tree level of weak interactions.

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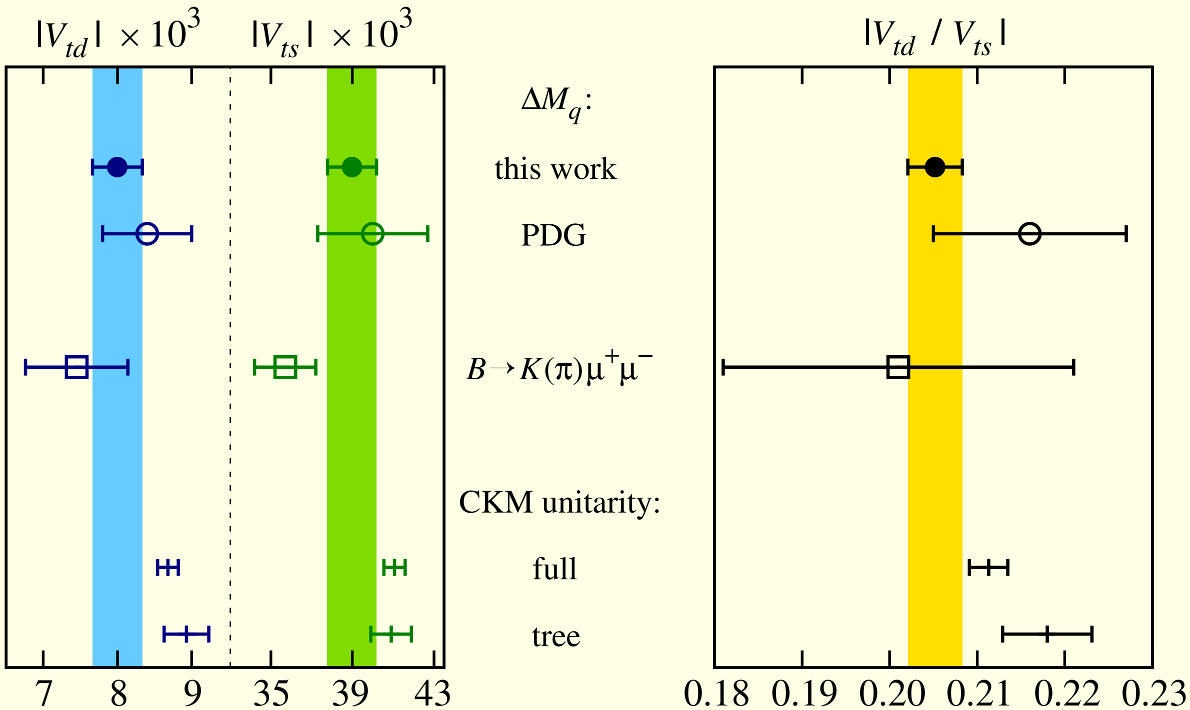

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Our results for $|V_{td}|$, $|V_{ts}|$ are 2σ, 2.9σ below the CKM tree-fit results

* Errors dominated by lattice mixing matrix elements

2.4 Bag parameters

Matrix elements of four fermion operators are often recast in terms of bag parameters: $\langle \bar{B}_q | \mathcal{O}_i | B_q \rangle \propto f_{B_q}^2 B_{B_q}^{(i)}$

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** For the SM RGI bag parameters we get

$$\hat{B}_{B_d}^{(1)} = 1.38(12)(6), \quad \hat{B}_{B_s}^{(1)} = 1.443(88)(48), \quad \frac{\hat{B}_{B_d}^{(1)}}{\hat{B}_{B_d}^{(1)}} = 1.033(31)(26)$$

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In progress: Correlated calculation of decay constants → decrease bag parameters errors.

2.5 Rare decays $B \to \mu^+\mu^-$

Bag parameters $\hat{B}_{B_{d,s}}$ describing B-meson mixing in the SM can be used for (indirect) theoretical predictions of $\mathcal{B}(B \to \mu^+\mu^-)$ **Buras** hep-ph/0303060,**Bobeth et al** 1311.0903

$$\left(\frac{\Gamma(B_q \to \mu^+\mu^-)}{\Delta M_q}\right)^{\rm SM} = \frac{3}{\pi^3} \frac{(G_F M_W m_\mu)^2}{\eta_{2B} S_0(x_t)} \frac{C_A^2(\mu_b)}{\hat{B}^{(1)}_{B_q}} \sqrt{1 - \frac{4m_\mu^2}{M_{B_q}^2}}$$

(with $C_A(\mu_b)$ including NLO EW and NNLO QCD corrections)

Herman,Misiak,Steinhauser 1311.1347,**Bobeth,Gorbahn,Stamou**, 1311.1348

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(with $C_A(\mu_b)$ including NLO EW and NNLO QCD corrections)

Herman,Misiak,Steinhauser 1311.1347,**Bobeth,Gorbahn,Stamou**, 1311.1348

* Using our $N_f = 2+1$ $\hat{B}_{B_{d,s}}$, including the effects of a non-vanishing $\Delta\Gamma_s$ to compute the time-averaged branching fractions $\overline{\mathcal{B}}$ measured in experiment ($\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)^{\rm SM} = \tau_{H_s}\Gamma(B_s \to \mu^+\mu^-)^{\rm SM}$, $\overline{\mathcal{B}}(B_d \to \mu^+\mu^-) = \mathcal{B}(B_d \to \mu^+\mu^-)$) and the experimental ΔM_q **HFAG 2014**

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)^{\rm SM} = 9.06(85)(4)(16)\cdot 10^{-11}$$

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)^{\rm SM} = 3.22(22)(0)(6)\cdot 10^{-9}$$

$$\left(\frac{\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)}{\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)}\right)^{\rm SM} = 0.02786(109)(12)(19)$$

(with errors coming from bag parameters, experimental ΔM_q and others, respectively)

2.5 Rare decays $B \to \mu^+\mu^-$

* SM predictions using

$$\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)^{\rm SM} = 9.06(85)(4)(16) \cdot 10^{-11}$$

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)^{\rm SM} = 3.22(22)(0)(6) \cdot 10^{-9}$$

$$\left(\frac{\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)}{\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)}\right)^{\rm SM} = 0.02786(109)(12)(19)$$

To be compared with the experimental averages from **LHCb** and **CMS** 1411.4413

$$\begin{aligned} \overline{\mathcal{B}}(B_d \to \mu^+\mu^-)^{\rm exp} &= 3.9(^{+1.6}_{-1.4}) \times 10^{-10} \\ \overline{\mathcal{B}}(B_s \to \mu^+\mu^-)^{\rm exp} &= 2.8(^{+0.7}_{-0.6}) \times 10^{-9} \\ \left(\frac{\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)}{\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)}\right)^{\rm exp} &= 0.14(^{+0.08}_{-0.06}) \end{aligned}$$

$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ agrees with experiment, $\overline{\mathcal{B}}(B_d \to \mu^+\mu^-)$ is 2σ above (symmetrizing exp. errors), and the ratio 1.6σ below.

3. Matrix elements contributing to $\Delta\Gamma_{d,s}$

At NLO in the heavy quark expansion $\Delta\Gamma_q^{\rm SM}$ depends on

$$\langle\mathcal{O}_1\rangle,\ \langle\mathcal{O}_3\rangle,\ \langle R_0\rangle,\ \langle R_{1,2,3}\rangle$$

* With **FNAL/MILC** 1602.03560: $\langle\mathcal{O}_1\rangle$ and $\langle\mathcal{O}_3\rangle$ known with 6% and 13% error.
* $R_0 = \mathcal{O}_1 + \alpha_1\mathcal{O}_2 + \frac{\alpha_2}{2}\mathcal{O}_3$ and $R_1 = \frac{m_q}{m_b}\mathcal{O}_4$ calculated in **FNAL/MILC** 1602.03560
* VSA estimates for dimension-7 operators R_2 and R_3 (50% error)

$$\begin{aligned}\langle R_2\rangle &= \frac{1}{m_b^2}\left(\bar{b}^i\overleftarrow{D}_\alpha\gamma^\nu(1-\gamma_5)D^\alpha q^i\right)\left(\bar{b}^j\gamma^\nu(1-\gamma_5)q^j\right)\\ \langle R_3\rangle &= \frac{1}{m_b^2}\left(\bar{b}^i\overleftarrow{D}_\alpha(1-\gamma_5)D^\alpha q^i\right)\left(\bar{b}^j(1-\gamma_5)q^j\right)\end{aligned}$$

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On-going: $N_f = 2+1+1$ **HPQCD** calculation of dimension-7 operators R_2 (and R_3)

(see **M. Wingate** talk at Lattice 2016)

* **Goal**: Reduce error in $\langle R_2 \rangle$ to $25\% \Longrightarrow \Delta\Gamma_s$ error $19\% \to 14\%$

4. Conclusions and outlook

* First three-flavor results for full set of $B_{s,d}$ mixing matrix elements

  ** All source of systematic uncertainty controlled.

  ** Most precise determination (1.6% error) of ξ and $\langle \mathcal{O}_1^{d,s} \rangle$.

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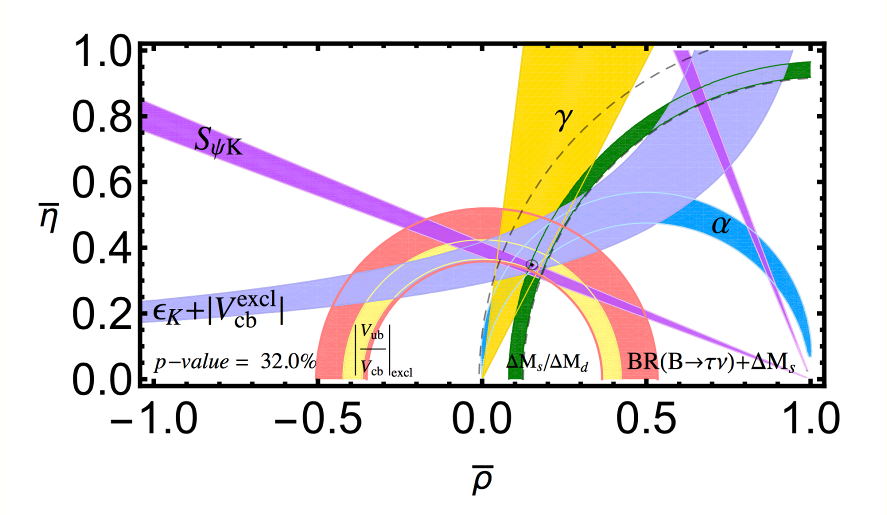
* Using Fermilab-MILC results for $B_{s,d}$-meson mixing parameters 1602.03560, V_{cb} 1403.0635, 1503.07237 and V_{ub} 1503.07839



Compatible with SM at $p = 0.32$, but still ample room for BSM flavor-changing neutral currents

Plot by E. Lunghi

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On-going Fermlab-MILC

Combined analysis of matrix elements and decay constants $\rightarrow$ correlations $\rightarrow$ reduction of errors for bag parameters

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On-going: another Lattice collaborations

* $N_f = 2+1+1$ HPQCD with HISQ light quarks and non-relativistic b. Preliminary results 1411.6989