CPV results from time-dependent analysis of $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$

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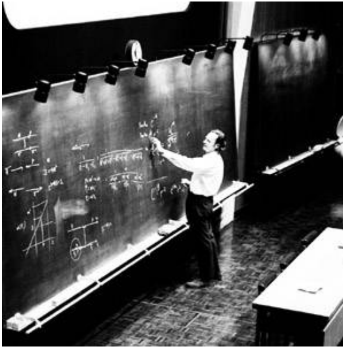
CKM 2016, 9th International Workshop on the CKM Unitarity Triangle
TIFR, Mumbai (India), 1st December 2016

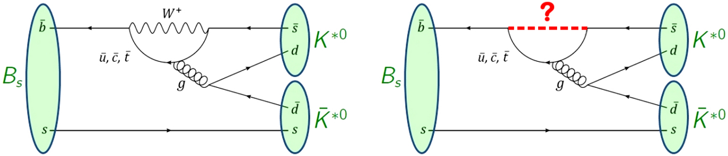
Outline

1. Theoretical overview
2. Time integrated studies
3. Time-dependent study
4. Conclusions



Theoretical overview

- The $\mathbf{B_s^0 \to K^{*0}\overline{K}^{*0}}$ decay is sensitive to the ***CP*-violating phase $\phi_s^{d\bar{d}}$**, arising from the interference between the decay of oscillated and not-oscillated $\mathrm{B_s^0}$ mesons.



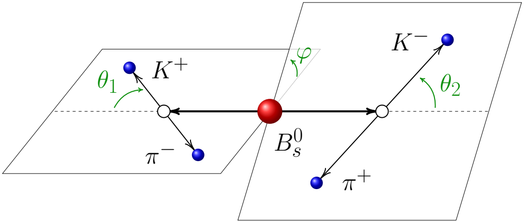
- The decay proceeds at leading order in the Standard Model (SM) through a **gluonic loop (penguin) diagram**.
  - * Within the SM, $\phi_s^{d\bar{d}} \approx 0$.
  - * New particles entering the loop may change the value of $\phi_s^{d\bar{d}}$, hinting the presence of **New Physics**,

- The **theoretical uncertainty** on $\phi_s^{d\bar{d}}$ due to next to leading order contributions can be estimated using the **U-spin related channel $\mathbf{B^0 \to K^{*0}\overline{K}^{*0}}$** (Phys. Rev. D 60 (1999) 073008, Phys. Rev. D 76 (2007) 074005, Phys. Rev. Lett. 100 (2008) 031802).

Kinematics of the decay

P → VV decay

- Three different polarisation amplitudes: A_0^{VV}, $A_\parallel^{VV}$, $A_\perp^{VV}$.
- Angular analysis needed to disentangle between CP-even, $\eta_h^{VV} = 1$, and CP-odd, $\eta_h^{VV} = -1$, polarisation states, with $h \in \{0, \parallel, \perp\}$.



$$A = \sum_h A_h^{VV} \cdot \Theta_h^{VV}(\cos\theta_1, \cos\theta_2, \varphi)$$

$$\overline{A} = \sum_h \eta_h^{VV} \overline{A}_h^{VV} \cdot \Theta_h^{VV}(\cos\theta_1, \cos\theta_2, \varphi)$$

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   - First observation
   - Polarisation fractions and *CP* asymmetries
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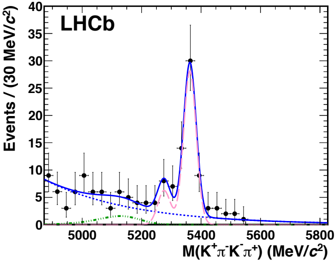
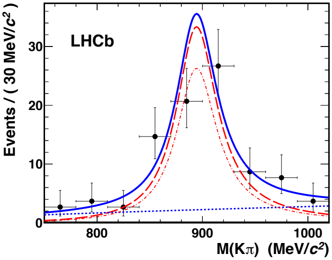
First observation

Physics Letters B 709 (2012) 50–58

- **First observation** of the decay $B_s^0 \to K^{*0}\bar{K}^{*0}$ with $K^{*0} \to K^+\pi^-$ and $\bar{K}^{*0} \to K^-\pi^+$ reported by LHCb, using **35 pb^{-1}** of data from p-p collisions at $\sqrt{s} = 7$ TeV.





- **Untagged and time-integrated** angular analysis yielded:

$$|A_0^{VV}|^2 = 0.30 \pm 0.12(\text{stat.}) \pm 0.04(\text{syst.})$$
$$|A_\perp^{VV}|^2 = 0.38 \pm 0.11(\text{stat.}) \pm 0.04(\text{syst.})$$

- **Visible S-wave** component in the $M(K\pi)$ distribution, yet difficult to estimate.

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Angular analysis

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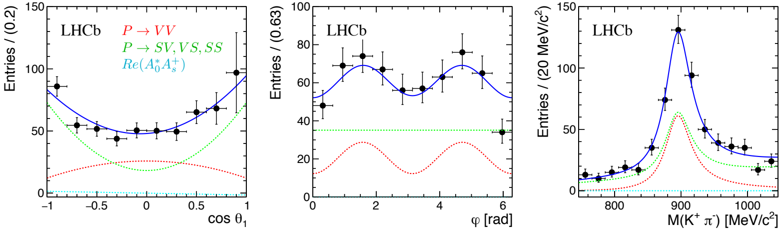
- Untagged and time-integrated angular analysis based on 697 ± 33 events observed in LHCb in **1 fb^{-1}** of data from p-p collisions at $\sqrt{s} = 7$ TeV.
- Three S-wave states included in the analysis (**6 amplitudes in total**): A^{SS}, $A^{S+} = 1/\sqrt{2}(A^{VS} + A^{SV})$ and $A^{S-} = 1/\sqrt{2}(A^{VS} - A^{SV})$.



Parameter	Value ± stat. ± syst.	Parameter	Value ± stat. ± syst.
f_L^{VV}	$0.201 \pm 0.057 \pm 0.040$	$\delta_{\parallel}^{VV}$	$5.31 \pm 0.24 \pm 0.14$
$f_{\parallel}^{VV}$	$0.215 \pm 0.046 \pm 0.015$	$\delta_{\perp}^{VV} - \delta^{S+}$	$1.95 \pm 0.21 \pm 0.04$
$\lvert A^{S+}\rvert^2$	$0.114 \pm 0.037 \pm 0.023$	δ^{S-}	$1.79 \pm 0.19 \pm 0.19$
$\lvert A^{S-}\rvert^2$	$0.485 \pm 0.051 \pm 0.019$	δ^{SS}	$1.06 \pm 0.27 \pm 0.23$
$\lvert A^{SS}\rvert^2$	$0.066 \pm 0.022 \pm 0.007$		

CP asymmetries

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Accessible time-integrated CP asymmetries:

- $\mathcal{T}$-odd Triple Product correlations, $q \cdot (\epsilon_1 \times \epsilon_2)$:

$$\mathcal{A}_T^i \propto Im(A_f^* A_\perp^{VV} - \bar{A}_f^* \bar{A}_\perp^{VV})$$

- S-wave induced direct CP asymmetries:

$$\mathcal{A}_D^i \propto Re(A_f^* A^{S+} - \bar{A}_f^* \bar{A}^{S+})$$

Computed as:

$$\mathcal{A}_{T,D}^i = \frac{N(g_{T,D}^i(\cos\theta_1, \cos\theta_2, \varphi) < 0) - N(g_{T,D}^i(\cos\theta_1, \cos\theta_2, \varphi) > 0)}{N(g_{T,D}^i(\cos\theta_1, \cos\theta_2, \varphi) < 0) + N(g_{T,D}^i(\cos\theta_1, \cos\theta_2, \varphi) > 0)}$$

Parameter	Value $\pm$ stat. $\pm$ syst.	Parameter	Value $\pm$ stat. $\pm$ syst.
$\mathcal{A}_T^1$	$0.003 \pm 0.041 \pm 0.009$	$\mathcal{A}_D^1$	$-0.061 \pm 0.041 \pm 0.012$
$\mathcal{A}_T^2$	$0.009 \pm 0.041 \pm 0.009$	$\mathcal{A}_D^2$	$0.081 \pm 0.041 \pm 0.008$
$\mathcal{A}_T^3$	$0.019 \pm 0.041 \pm 0.008$	$\mathcal{A}_D^3$	$-0.079 \pm 0.041 \pm 0.023$
$\mathcal{A}_T^4$	$-0.040 \pm 0.041 \pm 0.008$	$\mathcal{A}_D^4$	$-0.081 \pm 0.041 \pm 0.010$

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2. Time integrated studies
3. Time-dependent study **(New, LHCb preliminary)**
   - Extended range approach
   - Event selection
   - Flavour tagging
   - Time-dependent fit
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Extended range approach

- The **3 fb^{-1}** corresponding to the **full LHC Run 1** allow for the measurement of the *CP*-violating phase $\phi_s^{d\bar{d}}$.

- **Extra considerations:**

  ↓ Still small statistics compared to similar channels like $\mathrm{B}_s^0 \to \phi\phi$.

  ↓ Large S-wave component, difficult to estimate in the $\bar{\mathrm{K}}^{*0}$ window.

  ↑ Decays to similar $(\bar{s}d)(\bar{d}s)$ quark states going to $(K^+\pi^-)(K^-\pi^+)$ can be assumed to share the same $\phi_s^{d\bar{d}}$ phase.

- **Followed approach:**

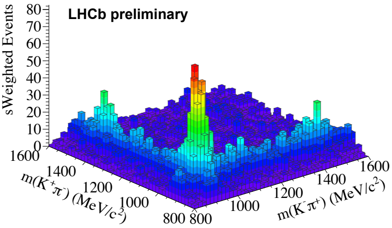
  Perform a **tagged and time-dependent study of $\mathrm{B}_s^0 \to (K^+\pi^-)(K^-\pi^+)$** decays, with $\boldsymbol{M(K^\pm\pi^\mp) \in [750, 1600]\,\mathrm{MeV}/c^2}$, so as to measure the $\boldsymbol{\phi_s^{d\bar{d}}}$ phase common to those channels.

Extended range approach

Dominant $K\pi$ components:

- **Scalar comp.**: $K_0^*(1430)^0$ + Non Res.
- **Vector comp.**: $K^*(892)^0$
- **Tensor comp.**: $K_2^*(1430)^0$

This leads to $3\times3 = $ **9 decay channels** with **19 polarisation amplitudes**.



Channel	Decay	Polarisation states
Channel #1	$B_s^0 \to (K^+\pi^-)_0^*(K^-\pi^+)_0^*$	SS
Channel #2	$B_s^0 \to (K^+\pi^-)_0^*\bar{K}^*(892)^0$	SV
Channel #3	$B_s^0 \to K^*(892)^0(K^-\pi^+)_0^*$	VS
Channel #4	$B_s^0 \to (K^+\pi^-)_0^*\bar{K}_2^*(1430)^0$	ST
Channel #5	$B_s^0 \to K_2^*(1430)^0(K^-\pi^+)_0^*$	TS
Channel #6	$B_s^0 \to K^*(892)^0\bar{K}^*(892)^0$	VV0, VV$\parallel$, VV$\perp$
Channel #7	$B_s^0 \to K^*(892)^0\bar{K}_2^*(1430)^0$	VT0, VT$\parallel$, VT$\perp$
Channel #8	$B_s^0 \to K_2^*(1430)^0\bar{K}^*(892)^0$	TV0, TV$\parallel$, TV$\perp$
Channel #9	$B_s^0 \to K_2^*(1430)^0\bar{K}_2^*(1430)^0$	TT0, TT$\parallel$1, TT$\perp$1, TT$\parallel$2, TT$\perp$2

Outline

Event selection

Selection strategy:

1. **Loose cut-based selection**, using topological and kinematic variables, to remove the bulk of the background.
2. **Boosted-Decision-Tree**, using topological and kinematic variables, against combinatorial background.
3. **Particle identification cuts on kaons and pions**, against background from particle misidentification (reflections).
4. **Mass vetoes on $M(K^+K^-\pi^\pm)$, $M(K^+K^-)$ and $M(\pi^+\pi^-)$**, against alternative $K^+\pi^-K^-\pi^+$ combinations.
5. **sWeighting procedure**, using $M(K^+\pi^-K^-\pi^+)$ as the discriminant variable, to remove remaining background contributions. $\rightarrow$ **See next slide**

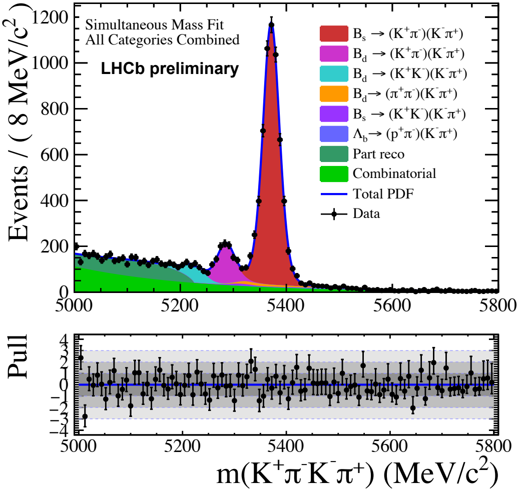
Four-body invariant mass spectrum

Model components:

- $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$ (signal, all channels)
- $B^0 \to (K^+\pi^-)(K^-\pi^+)$
- Reflection bkgs.
- Partially reconstructed bkg.
- Combinatorial bkg.

Simultaneous fit in year (2011/2012) and hardware trigger (independent on signal or not) categories.

More than 6000 signal events are found (LHCb preliminary).



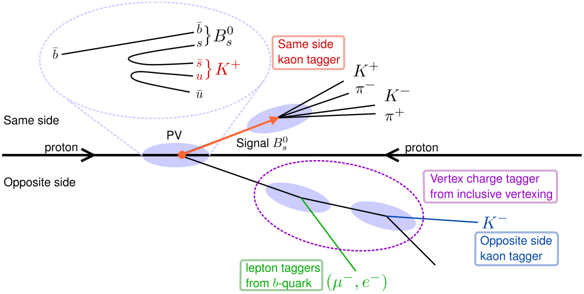
Outline

Flavour tagging

The b quarks are produced in $b\bar{b}$ pairs at the LHC.

$\Longrightarrow$ Two algorithms:

- **Opposite Side (OS) tagger combination**
- **Same Side (SS) kaon**



Tagging information, per algorithm:

- **Tagging decision, q** ($q = 1$ for B_s^0, $q = -1$ for $\bar{B}_s^0$ and $q = 0$ for untagged). The fraction of events with $q \neq 0$ gives the tagging efficiency, ϵ_{tag}.
- An **estimated mistag probability, η**, which is calibrated on data to obtain the true mistag probability, $\omega(\eta)$.

The OS and the SS tagger decisions are combined on-the-fly during fitting.
The effective tagging power is $\epsilon_{eff} = \epsilon_{tag}(1 - 2\omega)^2 \sim 5\%$ (LHCb preliminary).

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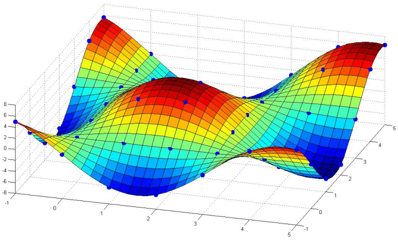

Phenomenological model

Analysis variables: **time**, **angles**, **two-body invariant masses**.

Time evolution of B_s^0 and $\overline{\mathrm{B}}_s^0$ mesons decaying to the final state $f \equiv (K^+\pi^-)(K^-\pi^+)$:

$$\begin{aligned}\langle f|\mathrm{B}_s^0(t)\rangle &= g_+(t)\langle f|\mathrm{B}_s^0(0)\rangle + \frac{q}{p}g_-(t)\langle f|\overline{\mathrm{B}}_s^0(0)\rangle, \\ \langle f|\overline{\mathrm{B}}_s^0(t)\rangle &= \frac{p}{q}g_-(t)\langle f|\mathrm{B}_s^0(0)\rangle + g_+(t)\langle f|\overline{\mathrm{B}}_s^0(0)\rangle\end{aligned}$$

Amplitudes at $t = 0$:
($j_1 \equiv J(K^+\pi^-)$, $j_2 \equiv J(K^-\pi^+)$, $h \equiv$ helicity)

$$\begin{aligned}\langle f|\mathrm{B}_s^0(0)\rangle &= \sum_{j_1,j_2,h} A_h^{j_1j_2} \cdot \Theta_h^{j_1j_2}(\cos\theta_1, \cos\theta_2, \varphi) \cdot \mathcal{H}_h^{j_1j_2}(m_1, m_2), \\ \langle f|\overline{\mathrm{B}}_s^0(0)\rangle &= \sum_{j_1,j_2,h} \eta_h^{j_1j_2}\overline{A}_h^{j_1j_2} \cdot \Theta_h^{j_1j_2}(\cos\theta_1, \cos\theta_2, \varphi) \cdot \mathcal{H}_h^{j_1j_2}(m_1, m_2)\end{aligned}$$

Phenomenological parameterisation for the mass-dependent terms:

$$\mathcal{H}_h^{j_1j_2}(m_1, m_2) = \mathcal{F}_h^{j_1j_2}(m_1, m_2) \cdot \mathcal{M}_{j_1}(m_1) \cdot \mathcal{M}_{j_2}(m_2)$$

$\mathcal{F}_h^{j_1j_2}(m_1, m_2) \rightarrow$ Angular momentum barrier factors. $\mathcal{M}_j(m) \rightarrow$ Two-body decay mass amplitudes, including the $\mathrm{K}_0^*(1430)^0$, $\mathrm{K}^*(892)^0$ and $\mathrm{K}_2^*(1430)^0$ resonances.

Acceptance and resolution effects

Acceptance

The geometry of the LHCb detector and the event selection process induce variable-dependent efficiencies, that are studied in simulated data.

- Time and kinematic (angles and masses) acceptances assumed to factorise.
- **Time acceptance** introduced analytically, parameterised with cubic splines.
- **Kinematic acceptance** introduced via weights in the normalisation of the fitting model, accounting for correlations among the angles and masses.

Resolution

The **time resolution** associated to the event reconstruction process impacts the sensitivity on $\phi_s^{d\bar{d}}$, so it has to be accounted for.

- Estimated per-event decay time error, δ_t, obtained from the reconstruction fits.
- Calibration of δ_t using simulated data, to obtain the true decay time resolution per event, $\sigma_t(\delta_t)$.

Current status

- The B_s^0 and $\overline{\mathrm{B}}_s^0$ time, angular and mass-dependent decay amplitudes, including the 19 polarisation states, are squared and combined according to tagging, accounting for acceptance and resolution effects.
- The PDF is fitted to the background subtracted data sample, so as to measure:
  - * **Polarisation fractions** and **strong phases** for all the states.
  - * A $\phi_s^{d\bar{d}}$ **phase** and a **direct *CP* asymmetry** common to the channels.
- The analysis is currently in internal review and the measured value for $\phi_s^{d\bar{d}}$ is kept blind.
  - * A preliminary estimate of the **statistical uncertainty** on $\phi_s^{d\bar{d}}$ is found to be **less than 0.2 rad** (LHCb preliminary).
    For comparison with similar measurements:
    - The precision on the world average measurement of $\phi_s^{c\bar{c}}$ ($\mathrm{B}_s^0 \to J/\psi K^+K^-$ and $\mathrm{B}_s^0 \to J/\psi\pi^+\pi^-$) is 0.033 rad (arXiv:1412.7515).
    - The precision on the LHCb measurement of $\phi_s^{s\bar{s}}$ ($\mathrm{B}_s^0 \to \phi\phi$) is 0.15 rad (Phys. Rev. D 90, 052011).

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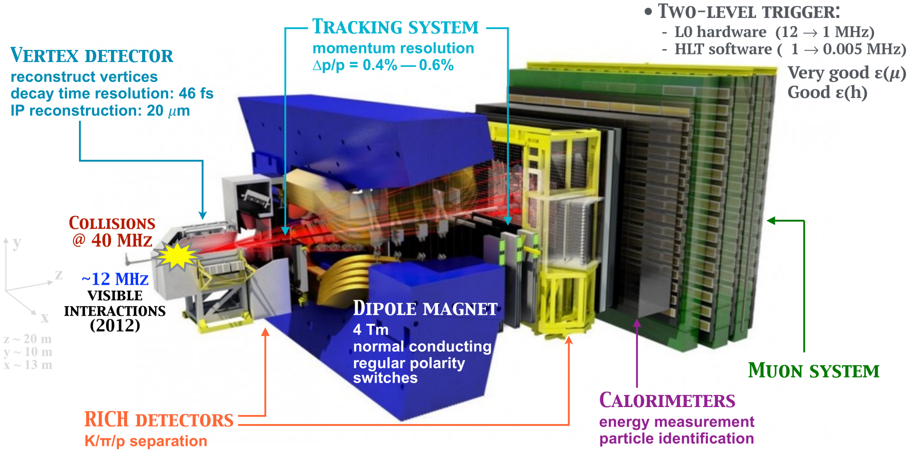
Conclusions

- The $\mathbf{B_s^0 \to K^{*0}\overline{K}^{*0}}$ decay is a golden channel in the search for New Physics through the measurement of the mixing-induced *CP*-violating phase $\phi_s^{d\bar{d}}$.
  - The U-spin related mode $\mathbf{B^0 \to K^{*0}\overline{K}^{*0}}$ can be used to estimate the theoretical uncertainty on $\phi_s^{d\bar{d}}$ due to higher order SM contributions (Phys. Rev. D 60 (1999) 073008, Phys. Rev. D 76 (2007) 074005, Phys. Rev. Lett. 100 (2008) 031802).
- LHCb has already presented the measurements of the *CP* asymmetries accessible with an **untagged and time-integrated** analysis, obtaining results compatible with the SM (JHEP 07 (2015) 166).
- The LHCb **tagged and time-dependent** analysis of $\mathbf{B_s^0} \to (K^+\pi^-)(K^-\pi^+)$ decays, with $M(K^\pm\pi^\mp) \in [750, 1600]\,\mathrm{MeV}/c^2$, is currently under internal review and waiting for the unblinding of $\phi_s^{d\bar{d}}$.

  Stay tuned!

Backup slides

The LHCb detector



Time-dependent model

Conditional PDF used for fitting:

$$PDF_{fit}(t,\Omega) = \frac{\sum_{i=1}^{19}\sum_{j\leq i}\Re e[K_{ij}(t)\cdot F_{ij}(\Omega)]}{\sum_{i'=1}^{19}\sum_{j'\leq i'}\Re e[(\int dt'\, K_{i'j'}^{untag}(t')\cdot\epsilon_t(t'))\cdot\xi_{i'j'}]}$$

Time-dependent terms and physical parameters:

$$\begin{aligned} K_{ij}(t) &= R(t,\delta_t)\otimes\{e^{-\Gamma_s t}\cdot[\zeta_+\cdot(a_{ij}\cosh(1/2\Delta\Gamma_s t)+b_{ij}\sinh(1/2\Delta\Gamma_s t)) \\ &+ \zeta_-\cdot(c_{ij}\cos(\Delta M_s t)+d_{ij}\sin(\Delta M_s t))]\} \end{aligned}$$

K_{ij}^{untag} are obtaining by summing K_{ij} over the tagging decisions.
Kinematic terms (making the identifications $i\equiv\{j_1,j_2,h\}$ and $j\equiv\{j_1',j_2',h'\}$):

$$\begin{aligned} F_{ij}(\Omega) &= (2-\delta_{ij})\cdot\Theta_h^{j_1j_2}(\cos\theta_1,\cos\theta_2,\varphi)\cdot\Theta_{h'}^{j_1'j_2'*}(\cos\theta_1,\cos\theta_2,\varphi) \\ &\cdot \mathcal{M}_{j_1}(m_1)\cdot\mathcal{M}_{j_2}(m_2)\cdot\mathcal{M}_{j_1'}^*(m_1)\cdot\mathcal{M}_{j_2'}^*(m_2) \\ &\cdot \mathcal{F}_h^{j_1j_2}(m_1,m_2)\cdot\mathcal{F}_{h'}^{j_1'j_2'}(m_1,m_2)\cdot\Phi_4(m_1,m_2) \end{aligned}$$

Angular terms (I)

j_1	j_2	h	$Y_{\ell_1}^{m_1}(\theta_1, -\varphi) Y_{\ell_2}^{m_2}(\pi - \theta_2, 0)$	$\Theta_{j_1 j_2 h}(\cos\theta_1, \cos\theta_2, \varphi)$
0	0	0	$\sqrt{\pi} Y_0^0 Y_0^0$	$\frac{1}{2\sqrt{2\pi}}$
0	1	0	$\sqrt{\pi} Y_0^0 Y_1^0$	$-\frac{\sqrt{3}}{2\sqrt{2\pi}} \cos\theta_2$
1	0	0	$\sqrt{\pi} Y_1^0 Y_0^0$	$\frac{\sqrt{3}}{2\sqrt{2\pi}} \cos\theta_1$
0	2	0	$\sqrt{\pi} Y_0^0 Y_2^0$	$\frac{\sqrt{5}}{4\sqrt{2\pi}}(3\cos^2\theta_2 - 1)$
2	0	0	$\sqrt{\pi} Y_2^0 Y_0^0$	$\frac{\sqrt{5}}{4\sqrt{2\pi}}(3\cos^2\theta_1 - 1)$
1	1	0	$\sqrt{\pi} Y_1^0 Y_1^0$	$-\frac{3}{2\sqrt{2\pi}} \cos\theta_1 \cos\theta_2$
1	1	$\parallel$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_1^{-1} Y_1^{+1} + Y_1^{+1} Y_1^{-1})$	$-\frac{3}{4\sqrt{\pi}} \sin\theta_1 \sin\theta_2 \cos\varphi$
1	1	$\perp$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_1^{-1} Y_1^{+1} - Y_1^{+1} Y_1^{-1})$	$-i\frac{3}{4\sqrt{\pi}} \sin\theta_1 \sin\theta_2 \sin\varphi$

Angular terms (II)

j_1	j_2	h	$Y_{\ell_1}^{m_1}(\theta_1, -\varphi) Y_{\ell_2}^{m_2}(\pi - \theta_2, 0)$	$\Theta_{j_1 j_2 h}(\cos\theta_1, \cos\theta_2, \varphi)$
1	2	0	$\sqrt{\pi} Y_1^0 Y_2^0$	$\frac{\sqrt{15}}{4\sqrt{2\pi}} \cos\theta_1 (3\cos^2\theta_2 - 1)$
1	2	$\parallel$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_1^{-1} Y_2^{+1} + Y_1^{+1} Y_2^{-1})$	$\frac{3\sqrt{5}}{4\sqrt{\pi}} \sin\theta_1 \sin\theta_2 \cos\theta_2 \cos\varphi$
1	2	$\perp$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_1^{-1} Y_2^{+1} - Y_1^{+1} Y_2^{-1})$	$i \frac{3\sqrt{5}}{4\sqrt{\pi}} \sin\theta_1 \sin\theta_2 \cos\theta_2 \sin\varphi$
2	1	0	$\sqrt{\pi} Y_2^0 Y_1^0$	$-\frac{\sqrt{15}}{4\sqrt{2\pi}} (3\cos^2\theta_1 - 1) \cos\theta_2$
2	1	$\parallel$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_2^{-1} Y_1^{+1} + Y_2^{+1} Y_1^{-1})$	$-\frac{3\sqrt{5}}{4\sqrt{\pi}} \sin\theta_1 \cos\theta_1 \sin\theta_2 \cos\varphi$
2	1	$\perp$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_2^{-1} Y_1^{+1} - Y_2^{+1} Y_1^{-1})$	$-i \frac{3\sqrt{5}}{4\sqrt{\pi}} \sin\theta_1 \cos\theta_1 \sin\theta_2 \sin\varphi$
2	2	0	$\sqrt{\pi} Y_2^0 Y_2^0$	$\frac{5}{8\sqrt{2\pi}} (3\cos^2\theta_1 - 1)(3\cos^2\theta_2 - 1)$
2	2	$\parallel 1$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_2^{-1} Y_2^{+1} + Y_2^{+1} Y_2^{-1})$	$\frac{15}{4\sqrt{\pi}} \sin\theta_1 \cos\theta_1 \sin\theta_2 \cos\theta_2 \cos\varphi$
2	2	$\perp 1$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_2^{-1} Y_2^{+1} - Y_2^{+1} Y_2^{-1})$	$i \frac{15}{4\sqrt{\pi}} \sin\theta_1 \cos\theta_1 \sin\theta_2 \cos\theta_2 \sin\varphi$
2	2	$\parallel 2$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_2^{-2} Y_2^{+2} + Y_2^{+2} Y_2^{-2})$	$\frac{15}{16\sqrt{\pi}} \sin^2\theta_1 \sin^2\theta_2 \cos(2\varphi)$
2	2	$\perp 2$	$\frac{\sqrt{\pi}}{\sqrt{2}}(Y_2^{-2} Y_2^{+2} - Y_2^{+2} Y_2^{-2})$	$i \frac{15}{16\sqrt{\pi}} \sin^2\theta_1 \sin^2\theta_2 \sin(2\varphi)$