# On the theory prediction of $R_K$ and $R_{K^*}$

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# Lepton Flavour Universality

**Lepton Flavour Universality**: weak interactions do not distinguish between lepton families.

How do we test LFU?

We use **ratios** of semileptonic decays

- in the ratios hadronic uncertainties are lower
- we can easily compare different leptonic families

# LFU ratios

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\overline{\nu})}{\mathcal{B}(B \to D^{(*)}\mu\overline{\nu})}$$

- Charged current
- $\tau$ vs $\mu$
- in the SM tree level process
- $4\sigma$ discrepancy

$$R_K = \frac{\mathcal{B}(B \to K\mu\mu)}{\mathcal{B}(B \to Kee)}$$

- FCNC
- $\mu$ vs $e$
- in the SM arises at loop level
- $2.6\sigma$ discrepancy

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- New particles, e.g. leptoquark, vector boson, heavy fermion, scalars
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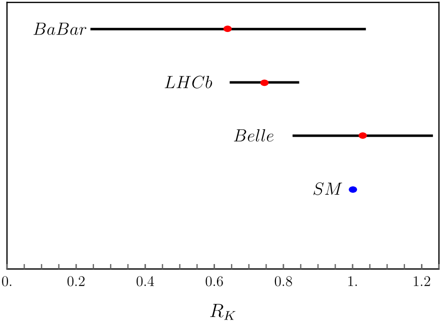
# LFU ratio - $R_K$




$$R_K = \frac{\mathcal{B}(B \to K\mu\mu)}{\mathcal{B}(B \to Kee)}$$

Average

$$R_K^{\rm avg} = 0.785 \pm 0.080$$

LHCb [1406.6482]

$$R_K^{\rm LHCb} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

BaBar[SUSY 2016]

$$R_K^{\rm BaBar} = 0.64^{+0.39}_{-0.30} \pm 0.06$$

Belle [0904.0770]

$$R_K^{\rm BaBar} = 1.03 \pm 0.19 \pm 0.06$$

SM prediction

$$R_K^{\rm th} = 1 \pm ??$$

# Theory prediction

$$R_K = 1.0000 \pm 0.0001|_{\text{[C.Bobeth,G.Hiller,G.Piranishvili]}} \pm ??|_{\mathrm{QED}}$$

Which are the sources of possible contributions to $R_K$ in $q^2 \subset [1,6]\mathrm{GeV}^2$ region?

- kinematics and form factor effects are small $\sim \frac{m_\ell^2}{q^2}$
- naive estimation of QED corrections $\sim \frac{\alpha}{\pi} \log^2\left(\frac{m_\ell^2}{q^2}\right)$

[T.Huber,T.Hurt,E.Lunghi
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Can we trust the error?

- semi-analytic calculation of radiative corrections

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$$\boxed{\ell = e \;\Rightarrow\; 10\%}$$

Can we trust the error?

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# Calculation setup

- limit $m_\ell^2 \ll q^2$
- interested to extract log-enhanced terms $\sim \frac{\alpha}{\pi} \log\left(\frac{m_\ell^2}{q^2}\right)$ and $\sim \frac{\alpha}{\pi} \log^2\left(\frac{m_\ell^2}{q^2}\right)$
  - since they depend on $m_\ell$ they can be the only terms responsible of LFU violation
  - can be extracted from term associated with collinear and soft divergences due to the photon emission
- neglect $\mathcal{O}(\alpha/\pi)$ finite corrections ($\sim 0.2\%$)
- radiation from meson leg is negligible (not log-enhanced)

# Radiator function

$\omega(x, x_\ell)$: probability density function that a dilepton system retains a fraction $\sqrt{x}$ of its original invariant mass $q_0^2$ after bremstrahlung

$$\omega(x, x_\ell) = \omega_1(x, x_\ell)\theta(1 - x - x_*) + \omega_2(x, x_\ell, x_*)\delta(1 - x)$$

- $\omega_1$: real emission

- $x = q^2/q_0^2$
- $x_\ell = m_\ell^2/q_0^2$
- $x_*$: IR regulator

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- $\omega_1$: real emission
- $\omega_2$: soft emission and virtual corrections, obtained from

$$\int_{2x_\ell}^{1} \mathrm{d}x\, \omega(x, x_\ell) = 1 + \mathcal{O}\left(\frac{\alpha}{\pi}\right)$$

- $x = q^2/q_0^2$
- $x_\ell = m_\ell^2/q_0^2$
- $x_*$: IR regulator

# Implementation of the radiator into the non radiative spectrum

Double-differential decay width

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q_0^2\mathrm{d}x}\,(B \to K\ell\ell(\gamma)) = \mathcal{F}_K^{(0)}(q_0^2)\omega(x, x_\ell)$$

$\mathcal{F}_K^{(0)}(q^2)$: non radiative spectrum of the decay $B \to K\ell\ell$
To obtain the radiative-spectrum we need to perform the following **convolution**

$$\mathcal{F}_K^{\ell}(q^2) = \int_{q^2}^{q_0^2,\mathrm{max}} \frac{\mathrm{d}q_0^2}{q_0^2}\; \mathcal{F}_K^{(0)}(q_0^2)\;\omega\left(\frac{q^2}{q_0^2}, \frac{2m_\ell^2}{q_0^2}\right)$$

where the kinematical region of integration depends on experimental cuts, namely $m_B^{\mathrm{rec}}$

# Modelling the $J/\Psi$

## Non-radiative spectrum

$$\mathcal{F}_K^{(0)}(q^2) \propto \lambda^{3/2}(q^2)|f_+(q^2)|^2 \left[|a_9(q^2)|^2 + |a_{10}|^2\right]$$

## Non-perturbative spectrum

$$a_9(q^2) = a_9^{\mathrm{pert}} + \kappa_\Psi \frac{q^2}{q^2 - m_\Psi^2 + i m_\Psi \Gamma_\Psi}$$

- $a_9^{\mathrm{pert}}$ ensures the behaviour at low $q^2$ region
- BW reproduces the presence of $J/\Psi$, $\kappa_\Psi$ normalised to $\mathcal{B}(B \to K J/\Psi)$
- relative phase between $a_9^{\mathrm{pert}}$ and BW doesn't affect the result
- we do not claim this is the "true" shape of the resonance, but still it is suitable toy to study the behaviour around the $J/\Psi$

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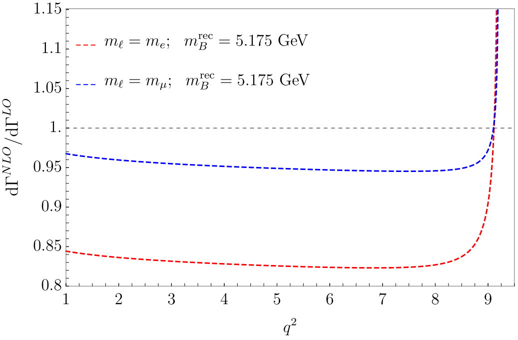
**Are we safely below the effects of the $J/\Psi$?**

# $J/\Psi$ tail

$m_B^{\rm rec}$: reconstructed mass of the $B$ meson from charged tracks

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# $J/\Psi$ tail

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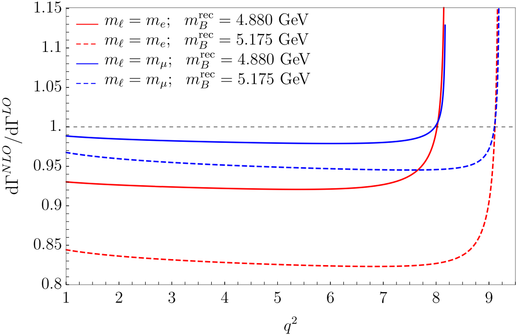
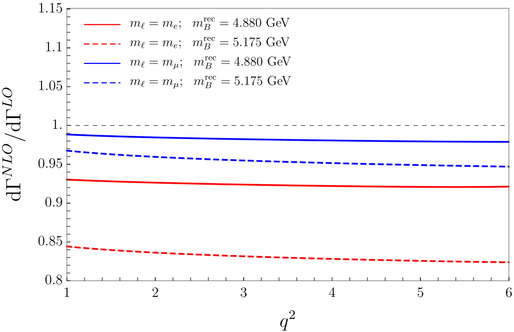



- **Key-variable**: $m_B^{\rm rec}$, that determines the size of the effect of radiation we need to take in account
- even with the looser cut $m_B^{\rm rec} = 4.880$ GeV the tail is safely above the interesting region $[1, 6]$ GeV$^2$

# $B \to K\ell\ell(\gamma)$ for $1\ \mathrm{GeV}^2 \le q^2 \le 6\ \mathrm{GeV}^2$




| $m_B^{\rm rec}$ | $\ell = e$ | $\ell = \mu$ |
|---|---|---|
| 4.880 GeV | $-7.6\%$ | $-1.8\%$ |
| 5.175 GeV | $-16.9\%$ | $-4.6\%$ |

- radiative correction can be sizable

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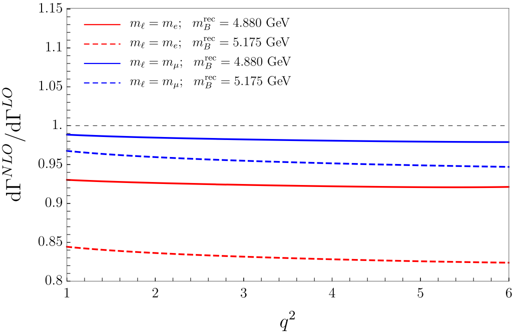


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- radiative correction can be sizable
- due to the cuts applied in the analysis the overall effect is less important

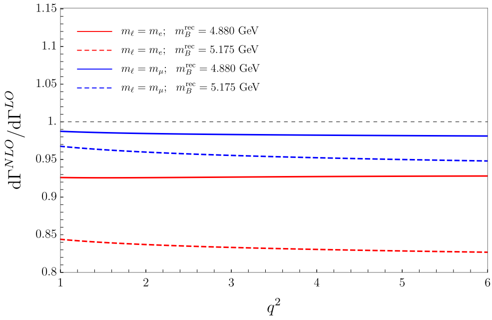
**estimate effect on $R_K$: $\Delta R_K = +3\%$**

**good agreement with PHOTOS up to a few ‰**

# Results for $B \to K^*\ell\ell(\gamma)$

Same story as for $K$...

| $m_B^{\rm rec}$ | $\ell = e$ | $\ell = \mu$ |
|---|---|---|
| 4.880 GeV | −7.3% | −1.7% |
| 5.175 GeV | −16.7% | −4.5% |




- also in the $K^*$ case the effects can be potentially sizable
- if we use the same cuts implemented in the analysis for $B \to K\ell\ell$ we still are safe from $J/\Psi$ tail and the overall effect of radiative correction is less important
- $\Delta R_{K^*} = 2.8\%$

# Summary-Part I

**1.** Taking into account

- low $q^2$ region $q^2 \subset [1,6]\mathrm{GeV}^2$
- the cuts applied by LHCb analysis

$$R_{K_{[\mathbf{1},\mathbf{6}]\mathrm{GeV}^2}} = 1.00 \pm 0.01$$

**2.** In the region below the resonances $J/\Psi$

- good agreement with PHOTOS

# Summary-Part II

If

**the current measurement is confirmed**

there is still space for new physics!